Title

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Comment on "Application of Linear Elastic Fracture Mechanics to the Quantitative Evaluation of Fluid-Inclusion Decrepitation" by A. Lacazette

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In a recent paper, Lacazette (1990) invoked principles of linear elastic fracture mechanics [LEFM] to help constrain conditions at which fluid inclusions decrepitate (i.e., fracture). The approach presented is both interesting and novel. However, some fundamental concepts of fracture mechanics were improperly applied, and as a result, many deductions in the paper are unfounded. Two points in particular will be addressed in this letter: (1) the use of incorrect expressions for the stress concentrations near voids with small cracks emanating from their surfaces, and (2) the attempt to relate void volumes to fracture mechanics parameters. To illustrate our points, we examine the elastic stresses around cracks and voids. For simplicity and clarity we focus on the circumferential stress about uniformly pressurized cracks and voids in infinite bodies under no remote stress.

As noted by Lacazette, traditional LEFM treats *cracks*, features that have infinitely sharp tips. This assumption of sharp tips causes the near-tip elastic stresses to be singular. The stress intensity factor is a measure of the strength of this singularity. As an example of the use of the stress intensity factor concept, consider the circumferential stress $\sigma_{\theta\theta}$ a small distance r from the tip of an infinitely deep fracture of half-length a (Fig. 1). The internal crack pressure is P. For $r \ll a$, the circumferential elastic stress is (Lawn and Wilshaw, 1975)

$$\sigma_{\theta\theta} = \frac{K_I}{(2\pi r)^{1/2}} \cos(3\theta/2) \ , \tag{1}$$

where the mode I stress intensity factor K_I is given by

$$K_I = \sqrt{\pi}(P)a^{1/2} \ . \tag{2}$$

The form of this expression for the stress intensity factor is identical to that given by equation 1 of Lacazette (1990):

$$K_I = -Y\,\sigma\, b^{1/2}\,. \tag{3}$$

The difference in sign between equations 2 and 3 arises because Lacazette considered compressive stresses to be positive, whereas Lawn and Wilshaw (1975) consider tensile stresses as positive. The term $\sqrt{\pi}$ in equation (2) equals 1.77, which is the shape factor Y given by Lacazette in his Fig. 2 for a "tunnel crack" (the value 1.17 given in the text is therefore a typographical error). Note that the "crack-size" parameter b must be identified as the half-length of the crack in order for equation 3 to be correct.

Now consider the circumferential stress around a pressurized circular hole of radius R and infinite length (Fig. 1b) and a spherical void of radius R. According to Timoshenko and Goodier (1970), the circumferential stress at a radial distance r from the edge of the hole is

$$\sigma_{\theta\theta} = P\left[\frac{R}{r+R}\right]^2 \tag{4}$$

and the circumferential stress about the spherical void is

$$\sigma_{\theta\theta} = 0.5P\left[\frac{R}{r+R}\right]^3 . \tag{5}$$

Comparison of equations 1, 4, and 5 shows that the stress distribution around a sharp crack differs markedly from that around a circular or a spherical void. The stress concentration near the crack tip is singular, can be described by a stress intensity factor, and depends on crack length. The stress concentration around the circular and spherical voids, however, are finite, cannot be described by a stress intensity factor, and are independent of void size. Linear elastic fracture mechanics in general, and stress intensity factors and the associated "shape factors" in particular, can not be applied to circular and spherical voids.

The concepts of LEFM *can* be applied to cylindrical and spherical voids that have small cracks emanating from their boundaries. These are presumably geometries contemplated by Lacazette, although they are mentioned only in the caption to his Fig. 1, and not in the text. For example, consider an infinitely long circular hole of radius R, with two small cracks each extending a short distance a from the hole (Fig. 3). The stress intensity factor at the tip of these cracks is a complicated function of both R and a (Tada et al., 1973). For infinitesimally short cracks (i.e., $a \ll R$), the mode I stress intensity factor is given by

$$K_I = 2.243\sqrt{\pi}\, P a^{1/2} = 3.98\, P a^{1/2} , \tag{6}$$

where P is the pressure in the void and the tiny cracks. The numerical factor 3.98 was cited as the shape factor Y by Lacazette, who used the radius R of the hole as the length term b. However, the length term that appears in equation 6 for K_I is the length a of the small cracks that extend from the hole, *not* the radius of the hole. In fact, since the factor 3.98 is correct only in the limit of $a \ll R$, the two dimensions R and a are not even of the same order of magnitude. An analogous situation holds for a vanishingly small annular crack emanating from a pressurized spherical void (Murakami, 1987). If one attempts to relate K_I to the void radius, one finds that for vanishingly small cracks that K_I goes to zero (Murakami, 1987, p. 872). The varied conclusions drawn regarding shape factors and fracture mechanics failure criteria for circular and spherical voids are thus unfounded.

While it may be possible to estimate the volumes of individual fluid inclusions, fracture mechanics criteria depend on the lengths of associated cracks, lengths which may be difficult to estimate. A rigorous application of the principles of linear elastic fracture mechanics to circular and spherical voids with vanishingly small cracks emanating from their boundaries does not lead to any correlation between void size and rupture stress. This obviates the use of equation 3 to assign equivalent radii to fluid inclusions of various shapes (including some with infinite lengths and hence infinite volumes).

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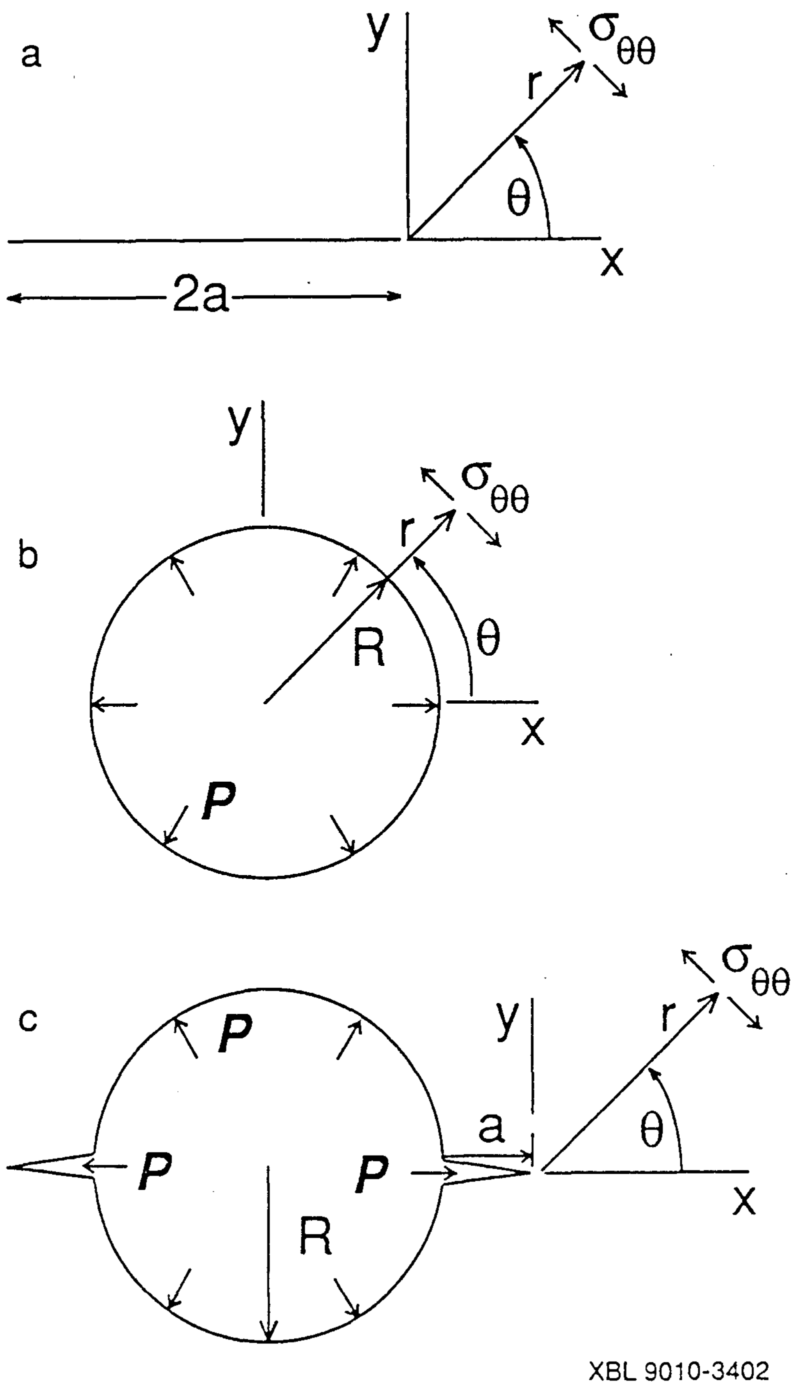

Fig. 1 Geometric parameters and reference frames for comparing the circumferential stress $\sigma_{\theta\theta}$ near a) an infinitely deep crack, b) an infinitely deep circular hole, c) an infinitely deep circular hole from which two infinitely deep cracks emanate. The pressure in all of these voids is uniform and of magnitude P.

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