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ONE PION EXCHANGE IN THE  $K^+ + p \rightarrow K^* + N^*$  REACTION

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Gerson Goldhaber

April 1963

# ONE PION EXCHANGE IN THE $K^+ + p \rightarrow K^* + N^*$ REACTION

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I would like to discuss an experiment which was carried out with the 20 inch hydrogen bubble chamber at Brookhaven in the Brookhaven-Yale beam of the AGS and the analysis of which was done at Berkeley. The people involved in this work are W. Chinowsky, S. Goldhaber, W. Lee, T. O'Halloran, and myself.

In the present talk I wish to discuss the reactions leading to four particles in the final state viz.  $K^+ + p \rightarrow K + \pi + N + \pi$ . We have found that these reactions are dominated by the double resonance production  $K^+ + p \rightarrow K^* (895) + N_{33}^* (1238)$ . Moreover, the experimental data supports a spin zero meson exchange presumably a pion, for this process.

The ratios of the cross sections for the various charge states observed:

$$K^+ + p \rightarrow K^+ \pi^- p \pi^+ \quad (1)$$

$$K^0 \pi^0 p \pi^+ \quad (2)$$

$$K^0 \pi^+ n \pi^+ \quad (3)$$

are in accord with the expectation for the production of two "quasi-particles" with isotopic spin 1/2 (the  $K^*$ ) and 3/2 (the  $N^*$ ) respectively in the total isotopic spin state  $T = 1$ . A complete list of the possible charge states together with the experimental data, is given in Table I. These are to be compared with the predictions from isotopic spin combinations as shown in Column 6. We conclude from this comparison that the reactions proceed principally via "double resonance", formation, i. e.,

$$K^+ + p \rightarrow \begin{array}{c} K^{*0} \\ \hookrightarrow K^+ + \pi^- \end{array} + \begin{array}{c} N^{*++} \\ \hookrightarrow p + \pi^+ \end{array} \quad (1)$$

We have found it convenient to represent the four particle production process discussed here in terms of the production of two "2-particle composites" with invariant masses  $m_x$  and  $m_y$  in the overall cm system considered here as variables. This

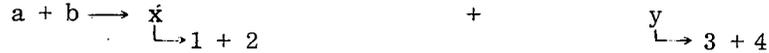
TABLE I.

CROSS-SECTIONS FOR THE VARIOUS CHARGE STATE COMBINATIONS IN THE  
REACTION  $K^+ + p \rightarrow K\pi p\pi$  AT 1.96 BEV/C

	$K^*$	$N^*$	Experimental mb.	No. Events Observed	No. Events (a) Corrected	Probability From I-Spin Composition		Experimental Normalized
							Normalized	
1.	$K^+\pi^-$	$p\pi^+$	$1.7 \pm .2$	435	435	1/2	1	1
2.	$K^0\pi^0$	$p\pi^+$	$1.3 \pm .2$	110	330	1/4	.72	.76 $\pm$ .15
2.	$K^0\pi^+$	$p\pi^0$				1/9		
3.	$K^0\pi^+$	$n\pi^+$	$0.33 \pm .1$	27	81	1/18		
4.	$K^+\pi^0$	$n\pi^+$	Unmeasurable	---	---	1/36	.06	----
5.	$K^+\pi^0$	$p\pi^0$	Unmeasurable	---	---	1/18	.11	----

( a ) Correction for the "invisible" decay mode of the  $K^0$  have been made.

description is valid irrespective of whether or not the composites form resonances at specific mass values  $m_x = M_x^*$  and  $m_y = M_y^*$ . The kinematical limits in this representation are particularly simple, namely they form a right angle isosceles triangle hereafter called Phase Space Triangle (PST). If we consider the general reaction:



then the length of each of the two sides of the PST is

$$Q = W - \sum_{i=1}^4 m_i$$

where  $W$  is the total energy in the cm of particles  $a$  and  $b$ .

Thus the values of  $m_x$  and  $m_y$  run over the intervals  $m_1 + m_2 \leq m_x \leq m_1 + m_2 + Q$  and  $m_3 + m_4 \leq m_y \leq m_3 + m_4 + Q$ , respectively. The effect of changing the incident momentum and thus  $Q$ , is then simply to move the hypotenuse of the PST leaving the two sides, as well as the location of any resonances which may occur among composites  $x$  and/or  $y$ , fixed.

The phase space distribution is given by  $\phi = \frac{1}{W} \int k_x k_y p_o dm_x dm_y$ , where the integral extends over the PST. Here  $k_x$  and  $k_y$  are the momenta in the cm of the composites  $x$  and  $y$  respectively, and  $p_o$  is the outgoing momentum of each of the composites in the cm of particles  $a$  and  $b$ . It is noteworthy that along each of the three sides of the PST, one of the factors in the integrand vanishes.

Let us consider that the composites  $X$  and  $Y$  form resonances of masses  $M_x^*$  and  $M_y^*$  and with full widths at half maximum  $\Gamma_x$  and  $\Gamma_y$  respectively. We can then define as the "double resonance rectangle" a rectangle of sides  $2\Gamma_x$  and  $2\Gamma_y$  centered at  $(M_x^*, M_y^*)$ . Events contained in this rectangle can then be considered as belonging to both resonances simultaneously. This selection is perhaps somewhat liberal. An alternative choice could be an ellipse with semi axes  $a = \Gamma_x$  and  $b = \Gamma_y$  respectively. The area of the rectangle occupies the fraction  $8\Gamma_x \Gamma_y / Q^2$  of the total area of the triangle.

We can consider as a "threshold" energy for the double resonance production the value  $W_A = M_x^* + M_y^*$  for which the hypotenuse crosses the point  $M_x^*, M_y^*$ . For this energy the double resonance will, of course, still be produced, however only half the area of the "double resonance rectangle" is available in this case. If we choose

Four-particle phase space triangle

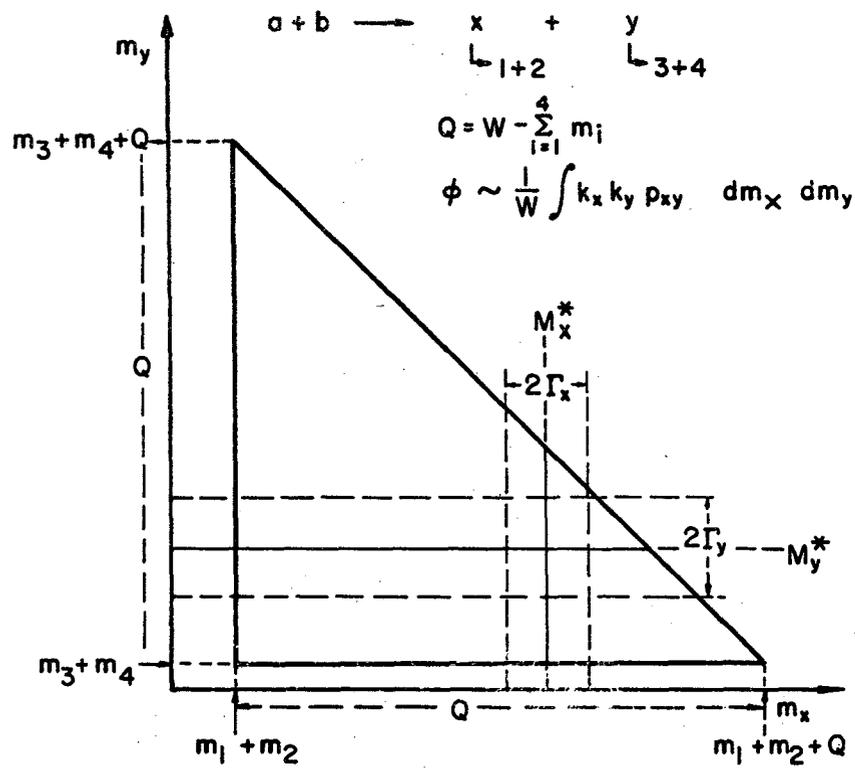


FIGURE 1.

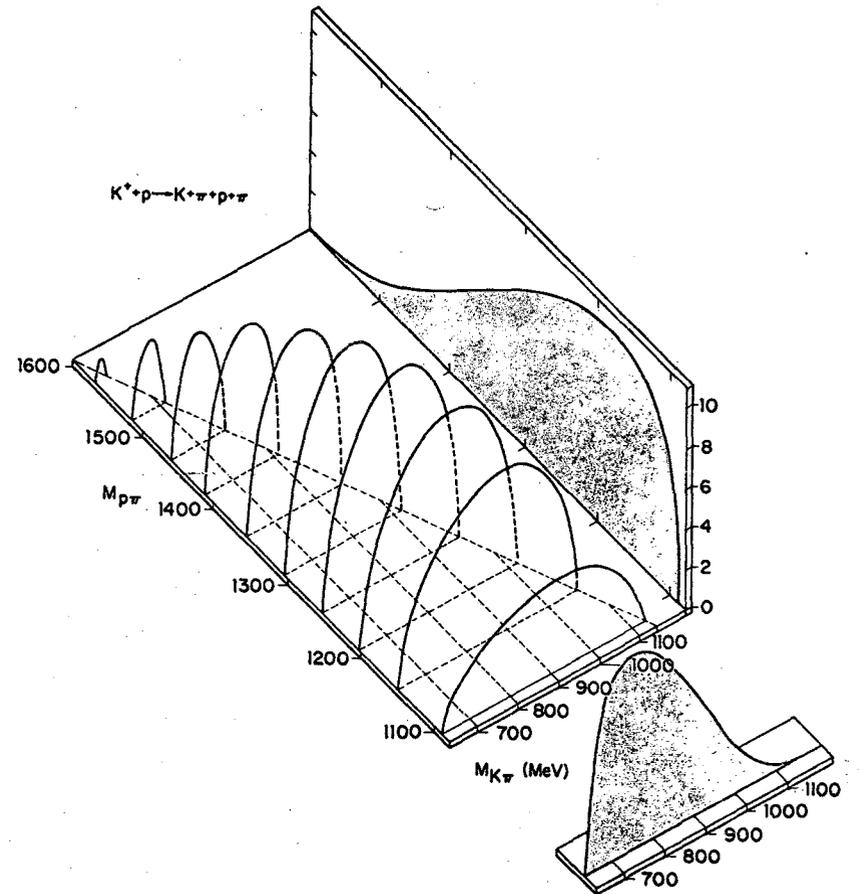


FIGURE 2

$W_B = M_x^* + M_y^* + \Gamma_x + \Gamma_y$  the hypotenuse crosses the upper edge of the double resonance rectangle. This corresponds roughly to the situation I will describe here.

In Figure 1 the PST corresponding to the general case is illustrated. In Figure 2 the phase space distribution over the PST is computed for reaction 1. The shaded areas correspond to the projections on the two mass axes.

It must be noted here that a similar description will also hold for 5 or more particles. In the case of 5 particles we can form one 3 particle and one 2 particle composite e.g.,



The kinematical limits will still be determined by an isosceles triangle where the generalization is straight-forward.

Returning to our own reaction, there are three ways (channels) in which the final state particles can be "paired" off into two particle composites for each of the charge states (1), (2), and (3). Direct evidence for double resonance production follows from the details of the events in charge state (1) which represents the largest sample of events and permits unambiguous assignment of the resonance states.

In Figures 3a, b, and c, we show the distribution of events for the various possible "two particle" composites in charge state (1), viz:

$$K^{*0} (K^+ + \pi^-) + N^{*++} (p + \pi^+) \quad (1a)$$

$$(K^+ + \pi^+) + N_{33}^{*0} (p + \pi^-) \quad (1b)$$

$$\rho^0 (\pi^+ + \pi^-) + (p + K^+) \quad (1c)$$

The corresponding three PST's as well as the projections on the respective mass axes are also shown together with the calculated phase space distributions. Channel 1a corresponds to double resonance production. Defining events with  $840 \leq M_{K^+ \pi^-} \leq 940$  to lie within the  $K^*$  resonance and events with  $1130 \leq M_{p \pi^+} \leq 1300$  to lie within the  $N_{33}^*$  resonance, we find 64% of all the events to lie within the double resonance yielding a cross section  $\sigma(K^* N^*) = 1.1 \pm 0.2$  mb. Channel 1b corresponds to single resonance formation in the  $p\pi^-$  channel. This is a small effect and occurs only in about 10% of the events. (See Figure 3b). No evidence for a resonance in the  $K\pi$   $T = 3/2$  system is observed. In channel 1c  $\rho^0$  production is energetically possible but is strongly suppressed

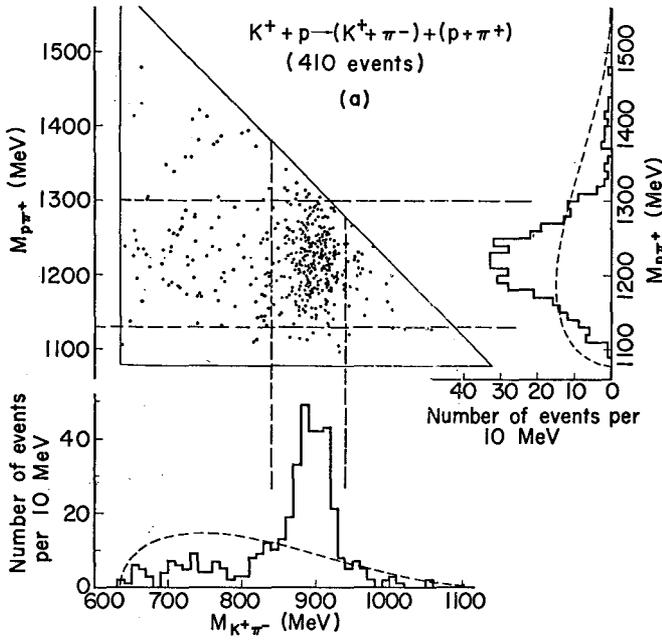


FIGURE 3(a)

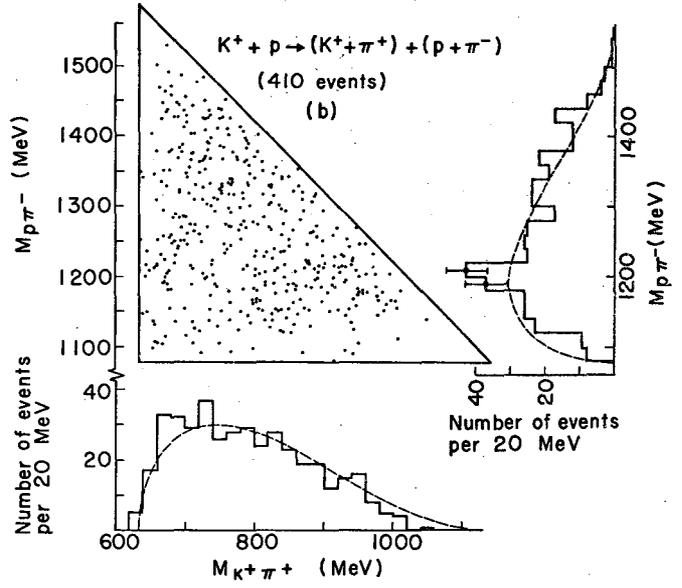


FIGURE 3(b)

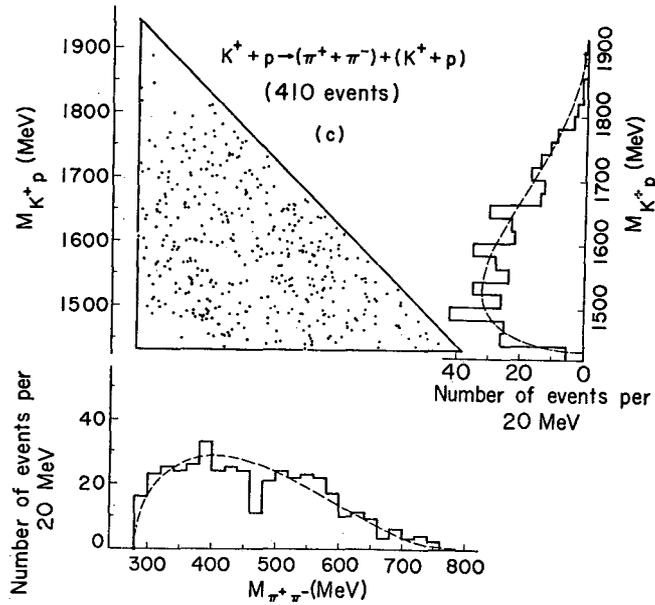


FIGURE 3(c)

by phase space. No evidence for  $\rho^0$  production was observed, neither is there any evidence for a positive strangeness hyperon ( $K^+p$ ) in the  $T = 1$  state. It is noteworthy that the reflection of the dominant resonances for channel 1a do not give rise to appreciable deviations from phase space in the other two channels.

We would now like to consider to what extent the reaction discussed here proceeds through a one pion exchange. In an earlier communication we have shown that the spin of the  $K^*$  is one, by utilizing the observed anisotropy in the  $K^*$  decay distribution. We would now like to make this same argument, and utilize the alignment of the  $K^*$  which is experimentally observed to probe up to what values of  $\Delta^2$  zero spin meson exchange is responsible for the reactions we observed. In Figure 4 we show a scatter diagram of the distribution of the  $K$  scattering angle,  $\alpha$ , in the  $K^*$  center of mass, plotted against the square of the 4-momentum transfer  $\Delta^2$ . As can be noted from this diagram, as well as from the projections of sections I and II below, for the lowest values of  $\Delta^2$ , up to values of  $\Delta^2 = 25 \mu^2$ , the angular distribution follows essentially a pure  $\cos^2 \alpha$  distribution. (Figure 2, part II)

The actual expansion in terms of  $f(\alpha) = a + b \cos \alpha + c \cos^2 \alpha$  gives the coefficients  $a = 0.1 \pm 0.04$ ,  $b = 0.07 \pm 0.08$ , and  $c = 1.0 \pm 0.14$ . While for higher values of  $\Delta^2$  (Figure 2 part I) the distribution is consistent with an isotropic distribution, an analysis for this case gives  $a = 1.11 \pm 0.18$ ,  $b = 0.06 \pm 0.23$ , and  $c = 1.0 \pm 0.48$ . We can thus consider this as an indication that one pion exchange completely dominates the reaction up to  $\Delta^2$  values of  $25 \mu^2$  and that other diagrams must begin to play a part around these values of  $\Delta^2$ .

Thus, on comparing the experimental cross section with a one pion exchange calculation, we will only consider the corresponding low  $\Delta^2$  values. Further corroboration for the one pion exchange process is shown in Figure 5 where we have plotted the Treiman-Yang angle distribution. This distribution is found to be consistent with isotropy as is required for the exchange of a spin zero particle.

Finally, we compare the experimental differential cross section in the center of mass system for the double resonance region with the results of a calculation based on the one-pion exchange model. The calculation due to S. Berman takes explicit account of

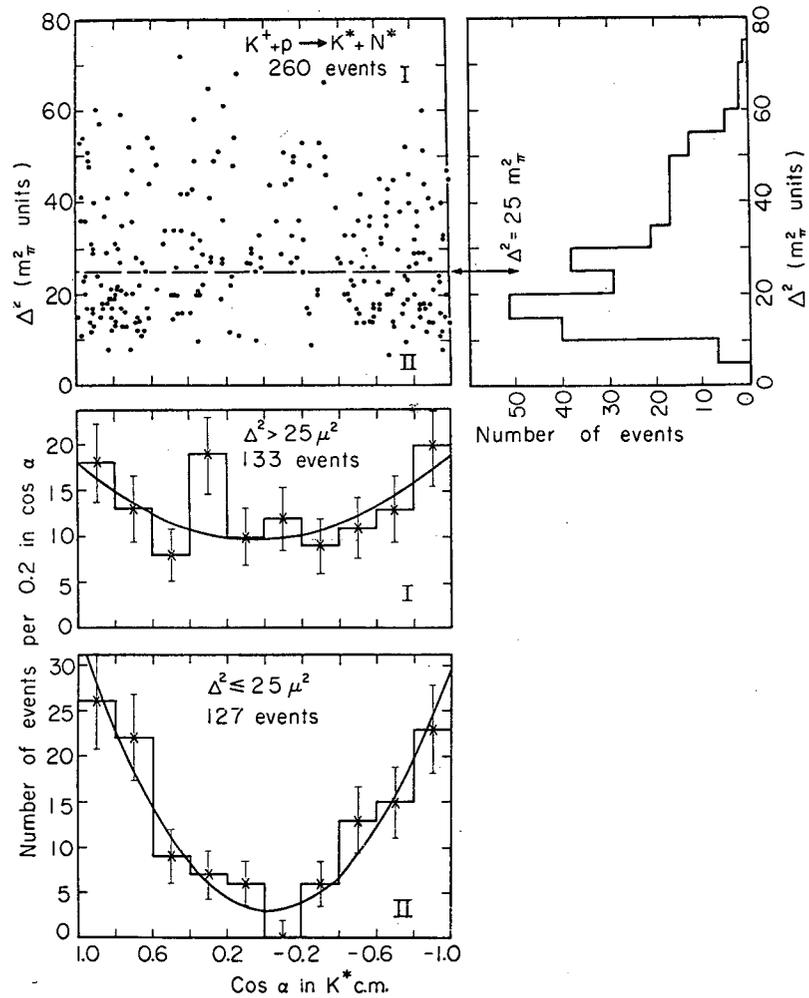


FIGURE 4

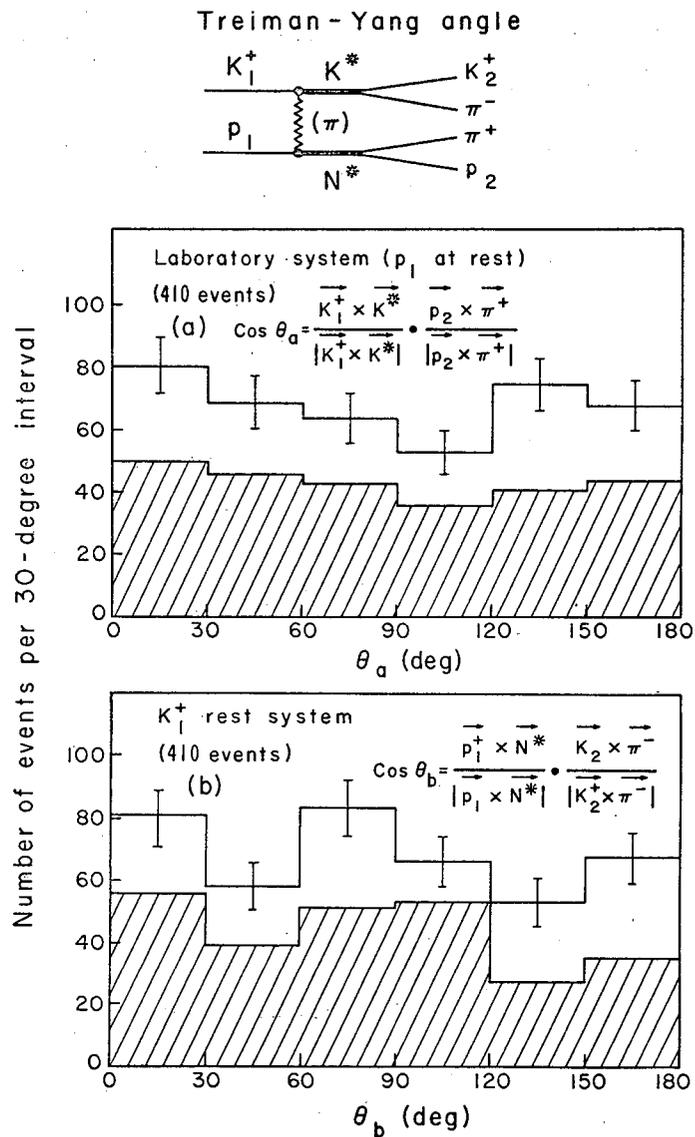


FIGURE 5.

the spins 1 and 3/2 of the two respective resonances produced.

The calculated differential cross section is:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{cm}} = \frac{\delta}{6m^{*2}M^{*2}W^2} \cdot \frac{g_{K^*}^2}{4\pi} \cdot \frac{g_{N^*}^2}{4\pi} \left(\frac{1}{\Delta^2 + m_\pi^2}\right)^2$$

$$\left\{ [\Delta^2 + (m^* + m)^2] \cdot [\Delta^2 + (m^* - m)^2] \right\} \cdot \left\{ [\Delta^2 + (M^* + M)^2]^2 \cdot [\Delta^2 + (M^* - M)^2] \right\} \quad (1)$$

where the coupling constants

$$\frac{g_{K^*}^2}{4\pi} = \Gamma_{K^*} m^{*2} / p_K^3 \sim 1.8 \quad (\text{For } K^+ \pi^- \text{ mode only})$$

and

$$\frac{g_{N^*}^2}{4\pi} = \frac{3}{2} \Gamma_{N^*} M^{*2} / p_N^3 \sim 24.0$$

are determined from the decay of the  $K^*$  and  $N^*$  respectively.

$$\text{and} \quad \delta = \left( \frac{(W^2 + M^{*2} - m^{*2}) - 4M^{*2}W^2}{(W^2 + M^2 - m^2)^2 - 4M^2W^2} \right)^{1/2} \cdot \frac{1}{(M^* + M)^2 - m_\pi^2}$$

Here  $W$  is the total cm energy,  $m^*$ ,  $M^*$ ,  $m$  and  $M$  are the masses of the  $K^*$ ,  $N^*$  and  $K$  and  $N$  respectively,  $p_K$  and  $p_N$  are the momenta of the  $K^*$  and  $N^*$  decay products in their respective cm system. In this equation the last two factors in brackets result from summing over final state spin directions of  $K^*$  and  $N^*$  respectively.

The experimental distribution including all double resonance events is shown in Figure 6. The three solid curves A, B, and C represent attempts to fit the data with pion exchange models. Curve B is obtained by evaluating equation (1) with  $\Delta^2 = -m_\pi^2$  in the spinfactors so that the momentum transfer dependence is contained inly in the propagator, viz:  $1/(\Delta^2 + m_\pi^2)^2$ . The resulting equation corresponds, then, to the form originally proposed by Chew and Low. Curve A gives the results of evaluating equation (1) including the proper spin factors. Comparison with the experimental data shows that this calculation gives too high a value for the cross section and does not reproduce the experimental angular distribution. To obtain a quantitative fit to the data we multiply equations (1) by a form factor,  $F^2(\Delta^2)$ . The exact choice of this form factor is somewhat arbitrary. We have chosen a one parameter expression similar to the nucleon form factor with the

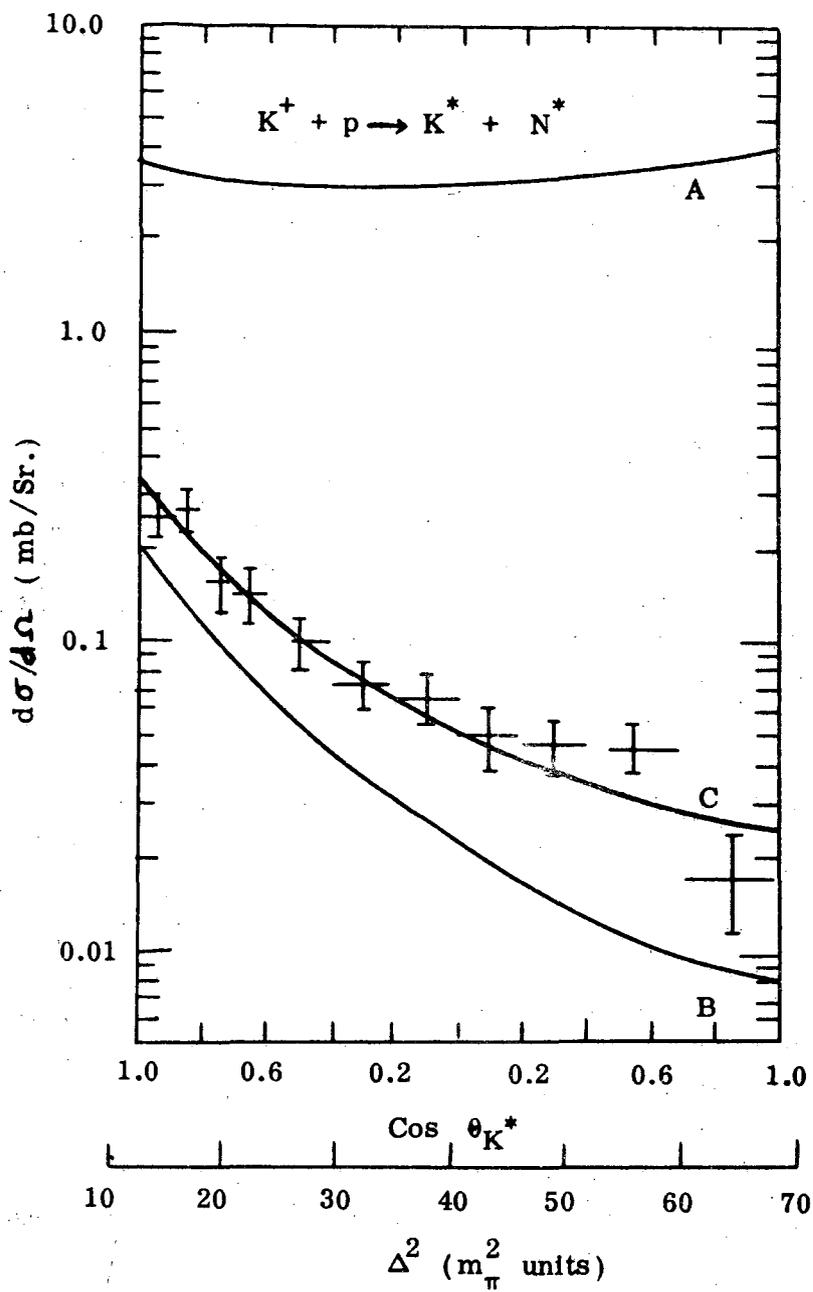


FIGURE 6.

condition that  $F(\Delta^2) \rightarrow 1$  as  $\Delta^2 \rightarrow -m_\pi^2$ , viz:

$$F(\Delta^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + \Delta^2}$$

By adjusting the parameter,  $\Lambda$ , we have obtained the fit shown in Figure 6, Curve C.

The resulting value of  $\Lambda$  is  $\Lambda = 2.6 m_\pi$ . It should be re-emphasized here that agreement with the OPE model is expected only to hold up to  $\Delta^2$  of  $25 m_\pi^2$ , we consider thus the apparent fit to the data for  $\Delta^2 \geq 25 m_\pi^2$  somewhat fortuitous.

#### DISCUSSION:

RILEY: We have heard it mentioned several times today that no resonance have been found in the  $K^+p$  scattering, although groups have looked for them. I just want to say that at Brookhaven we have been measuring  $K^+p$  and  $K^+d$  total cross sections in the region between 2 and 4 BeV, and have come to the conclusion they vary by less than a half a millibarn over the entire range, between 1 and 2 BeV which is not quite so well investigated but certainly at the high momenta no structure appears to within half a millibarn.

GOLDHABER: We measured  $K^+ + d$  interactions as well and found that the same is true at lower momenta also. We have not observed any resonance in the  $K^+ + p$  ( $T = 1$ ) nor in the  $K^+ + n$  ( $T = 0$  or  $1$ ) system for masses up to 1.8 BeV. Thus neither of the strange mass plus one systems seems to have any resonances in this region.

WALKER: I want to see whether I understood your data correctly. If you evaluate for example the  $K\pi$  cross section assuming a resonance in the  $K\pi$  system and don't put in any form factor for off the mass shell correction, do you get a cross section which is higher than the expected resonant value?

GOLDHABER: We have made various calculations of that kind. You're referring to something like the Salzman equation?

WALKER: Right.

## DISCUSSION (cont)

GOLDHABER: Are you referring to an equation without taking into account the spin factor at the vertices?

WALKER: Yes.

GOLDHABER: We have carried out calculations based on the Dalzmanns' equation as well. This involves the cross section for  $K\pi$  scattering and the cross section for  $\pi^+p$  scattering and then a propagator. Using the physical cross sections for  $\pi^+p$  scattering and a  $K\pi$  cross section given by a Breit-Wigner formula with the appropriate resonance parameters, we get a result which follows Curve B Figure 6 and thus agrees with the shape of the experimental angular distribution but is too low in absolute value. If you put in terms involving  $(p_{\text{off}}/p_{\text{on}})$  at each vertex this gives results very similar to the spin factors. Here too, the cross section comes out too high, i. e., as in Curve A, so this also, as far as we can tell, requires a form factor.

WALKER: In the nucleon-nucleon work, they put in a form factor also, or essentially off the mass shell corrections, and just looking at your formulas, your cut off agrees reasonably well with what is found in the nucleon-nucleon case.

GOLDHABER: At CERN, Ferrari and Selleri have used form factors and have gotten agreement with experimental distributions. We have tried to see if we can find one universal form factor which will fit all experiments, but so far this has not worked.

WALKER: Yes, in the  $\pi-\pi$  case you are about a factor of 30% low, in the case of the  $\rho$ . Nearly the same form factor will fix that to make that cross section agree with what we expect it should be.

GOLDHABER: I see, if we can settle on one form factor that would be very interesting.

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