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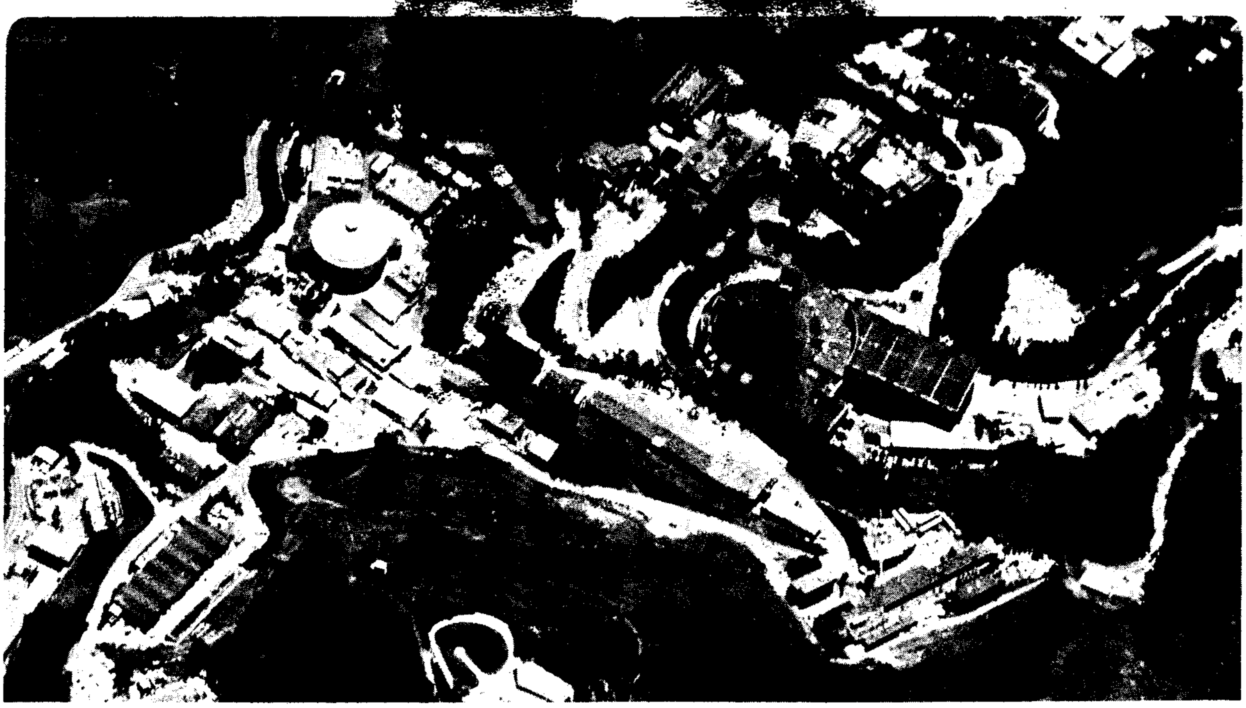

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$\pi^+\pi^0$ AND K^+K^0 MASS SPLITTINGS IN $SU(3)_c \times SU(2)_L \times U(1)$

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ABSTRACT

We compute the $\pi^+\pi^0$ and K^+K^0 mass splittings in the standard $SU(3)_c \times SU(2)_L \times U(1)$ theory of strong, electromagnetic and weak interactions of quarks. The only additional tool is PCAC, dealing with the covariant (with respect to the group $SU(2)_L \times U(1)$) divergences of the corresponding hadronic currents. The questions of cancellation of infinities and the gauge dependence are worked out in detail. All computations are reduced to those of self energy type diagrams in which the strong interactions can be easily resumed. This leads to the recovery of the results of Das et al., and Dicus and Mathur, and their justification in a modern approach. The achievements and limitations of the method are discussed.

I. Introduction

The notion of symmetry has always possessed a great power of simplification in physics. Unfortunately, Nature provides us only very seldom with exact symmetries, and, as soon as one of the most accurate was discovered, strong isospin SU(2), it was evident, from the observation of the proton-neutron and $\pi^+ - \pi^0$ mass splittings, that it was broken and that a quantitative understanding of its violation was not an easy problem.

In the continuous search of more and more exact symmetries, the physicists have been led to use them at more and more fundamental levels. Mesons and baryons are no longer thought as elementary objects, and nature was hoped to be conceptually simpler at the quark level. However, the isospin symmetry is now well known, from the up-down quark mass difference, to be also broken at this level, and the physicists are now challenged in two directions: is there exact symmetry at a deeper level of still more elementary constituents and, staying at this one, how to go up the hierarchy of particles and explain hadron physics from quark physics?

Leaving completely aside the first speculative idea of an ultimate simplification, we shall in this study address ourselves to the question of computing, in our present state of knowledge of quark interactions, an effect of isospin breaking at the hadron level. We choose the simplest objects, the pions (and the kaons), thus showing our ignorance to deal with the (still too) complicated baryons and the proton-neutron mass splitting.

As will become clear throughout this study, this is a new type of approach by the fact that the starting point is the now standard model of strong, electromagnetic and weak interaction of quarks, showing that one can deduce isospin breaking effects from this unifying gauge theory. The result itself cannot pretend to be new since we recover the old and good result of Das et al., [1] based on simple electromagnetic interactions of pions treated as fundamental objects.

As the $\pi^+-\pi^0$ mass splitting has now a rather long history, we shall first in this introduction review the main steps, contributions and problems which marked out many efforts toward its comprehension during the last 25 years. We shall after that briefly describe the main characteristics of the method used here, its achievements (finiteness, recovering of a good numerical result, detailed study of the gauge dependence), and also its limitations: a complete gauge invariant computation for physical (massive) pions is out of reach of our present computational ability; this will in particular constrain us to evaluate the electroweak contribution to the mass splittings only at the chiral limit.

At the time where one had no theory of hadrons as composite particles, it was natural to consider the electromagnetic interactions as the only responsible for the $\pi^+\pi^0$ (and p-n) mass difference. This lead to the pioneering work of refs. [2]. The dispersion technique [3] appeared here as the first of a long series of technical tools which were to be applied to this specific problem. The evaluation of the electromagnetic self-mass of pions was linked to the π electromagnetic form factor; saturation by the lowest 2π resonance at

$M = 750$ MeV (the ρ), gave the very good value $m_{\pi^+} - m_{\pi^0} = 4.95$ MeV. However, it was immediately recognized that the K^+K^0 mass splitting, with its opposite sign compared to the $\pi^+\pi^0$ was a challenge to the present status of the theory.

The next step was taken by Coleman and Glashow [4a], and Socolow [4b], in the 60's, contemporary with the great development of the SU(3) symmetry [5]. The computation of the π mass splitting is refined by taking into account higher intermediate states, and the K mass splitting problem is tried to be solved by incorporating the so-called "tadpole" diagrams, where a scalar meson octet (π') induces virtual isospin breaking transitions to the vacuum. While this addition does not modify the result for the pion mass splitting, it makes the great improvement of reversing the sign of the kaon one, though giving only half of its experimental value. Let us mention at this stage that the present translation of the "tadpole" term in the language of QCD is a product of a quark mass difference times some non perturbative vacuum expectation values of bilinear quark operators [6a]. They indeed solve the problem of the kaon mass splitting [6b] without affecting the pion one. They will be displayed explicitely in this study.

After dispersion relations and symmetry considerations, the most powerful tool to be applied to the problem was Current Algebra and PCAC [7], with the success of ref. [1], relying heavily on the newly discovered hadronic sum rules of Weinberg [8]. The underlying theory is still scalar electrodynamics for pions and the quantity computed the electromagnetic self-energy of the π's. Relying on the electromagnetic gauge invariance of the pole of the pion propagator, the choosen gauge is the Yennie gauge [9], which simplifies the computations by suppressing the $\pi\pi\gamma\gamma$ vertex. The 3 (linked) features of this approach are: (putting aside the fact that the pion was then formally treated as a fundamental field)

* it is performed at the chiral limit $m_\pi = 0$, and relies on soft pion techniques;

* it relies on both Weinberg sum rules, which are vital to ensure the convergence of the integrals involved. From the present QCD point of view, both are valid at the chiral limit; however, at that time, the use of the 2nd Weinberg sum rule was a problem because it was known to be model dependent. Consequently, there was a need to obtain convergence without it.

The next five years saw the outstanding development of gauge theories and the construction of models unifying the electromagnetic and weak interactions [10]. In this framework, Weinberg [11a] and 't Hooft [11b] draw the attention to the fact that the weak interactions are fundamental in determining mass splittings, especially in the cancellation of divergences; but most of the paper deals with the fundamental fermionic fields of the theory, and it is only in this case that the mechanism of cancellation of divergences is explicitly displayed; the question of composite particles is hardly mentioned. However, as the notion of quarks as fundamental fields was still a hypothesis, it was natural to transpose those ideas directly to the pions: five years after ref. [1], Dicus and Mathur [12a] and Yang [12b] showed that finiteness could be achieved in the so-called now Glashow-Weinberg-Salam model [10], introducing not only the photon but also the massive W and Z

bosons, without using the 2nd Weinberg sum rule. The mass of the Z was shown to act as an ultraviolet cut-off. The questions which arise nowadays about this approach are the following:

* it is still performed for soft pions;

* the choice of the unitary gauge for the W and Z propagators seems hard to justify as soon as one is not sure to compute a gauge-invariant quantity in a gauge invariant electroWEAK theory of pions; (1)

* one can check easily that the final formula doesn't give any sensible result unless one plugs in the values of masses and coupling constants extracted from <u>both</u> Weinberg sum rules, that is the same as obtained at the chiral limit, in which case one recovers the result of Das et al. up to negligible corrections. Trying to avoid the 2nd Weinberg sum rule and working with massless pions is contradictory in today's point of view;

* the mechanism proposed therein to cancel divergences is not enough in QCD (which could not be seen at that time): divergences arise from the high energy of the current-current propagators involved, while only their low energy behaviour was there plugged in (using vector meson or pole dominance).

From a modern point of view, the works that have been done up to that time suffer consequently from several drawbacks: the pions are treated as fundamental and massless, the question of the gauge is very unclear and the finiteness not established (see for example ref. [13] for a review on the subject at that time).

The years 70-75 have seen a big interest in light-cone physics, [14] and it was applied to electromagnetic and weak mass shifts [15], as an alternative to soft pion techniques. The Glashow-Weinberg-Salam model is by now a usual ingredient, together with the explicit quark realization of Current Algebra [7]. In those still prior-QCD works, one conclusion seems generally admitted: the divergences, originating from the deep-inelastic electron-hadron scattering region, may be removed by a suitable quark mass renormalization. The question of the gauge is seldom mentioned [15a].

The two last works [16] resolutely tackle the problem from the point of view of Quantum Chromodynamics. In both of them, the weak interactions are ignored and the computations performed in the Yennie gauge. However, as soon as the quarks are the fundamental fields and no longer the mesons, the mass splittings can in general receive contributions from diagrams which are not of the form of a meson self-energy and contribute to gauge dependence at order m^2. Consequently, we think that those computations are only justified at the chiral limit, in which case the 2nd Weiberg sum rule is perfectly valid [17]. While it is true (see the remark in Section 4) that in a pure QED approach and in the Yennie gauge the infinities do disappear, we shall show that it is only an artefact and that one can consider the cancellation of infinities as a consequence of the renormalizability of the $SU(3)_c \times SU(2)_L \times U(1)$ theory of strong, electromagnetic and weak interactions. In the first of those two approaches [16a] where pion and kaon mass splittings are dealt with simultaneously, Dashen's theorem [18] has to be put by hand, and the numerical final values are emphasized to depend essentially on the asymptotic behaviour of QCD, which we show to be irrelevant

because of going away in the procedure of renormalization. In the second approach, the authors advocate for some dubious cancellations of infinities, and make use of a nice variation of the Shankar method [19], but whose stability and reliability can be hardly checked. And, again, we do not agree with the claim that the pion and kaon mass differences are determined by the asymptotic behaviour of QCD.

It seems so clear that, up to now, there is no satisfactory finite computation of those mass splittings in the standard model of quarks, and that the question of the gauge has not been investigated.

In the new approach that we propose here, we shall partially fill this gap, and that between meson and quark physics. One of the main reasons why the mesons are more tractable than baryons (p,n) is that we have in this case at our disposal the powerful tool of PCAC [7], stating that the covariant, with respect to $SU(2)_L \times U(1)$, divergences of hadronic quark currents are the interpolating fields of the corresponding mesons. However, as already emphasized, we shall not be able to achieve this goal completely and to obtain a complete gauge invariant computation valid for physical (massive) mesons. Briefly, the reason for that is the following:

The mesons propagators that we study (linked by PCAC to the propagators of the divergences of the hadronic currents) have no reason to be by themselves gauge independent, but only their poles (m_π^2, m_K^2), and the residues of those poles $(f_\pi^2 m_\pi^4, f_K^2 m_K^4)$.

Identifying these poles or residues implies computations performed in QCD at $Q^2 = m_\pi^2(m_K^2)$, untractable because they involve diagrams which cannot be reduced to that of simple hadron self-energy and where strong interactions cannot be resumed (see Fig. 1). The compromise that we shall take is to compute those propagators at $Q = 0$, that is with mesons off mass shell. So doing, it turns out that, and this is the big advantage of this method, all electroweak contributions to the mass splittings can be expressed as self-energy type diagrams (see Fig. 2), involving 2 quarks and a gauge (or Higgs) boson, with external energy momentum 0, identical to vacuum fluctuations, and where all strong interactions can be very easily resumed at all orders. As a consequence, they can be handled by introducing the appropriate hadronic resonnances. We obtain trivially in addition the "tadpole" terms. The price to pay for this simplification is that, not computing with pions (kaons) on mass shell, one expects a gauge dependence of the electroweak terms of order $m_\pi^2(m_K^2)$. The explicit computations show that this is indeed what happens, and we recover in the pion case the old result of Das et al. [1] and Dicus and Mathur [12a] up to gauge dependent terms of order m_π^2, corrections of order $(m_u + m_d)^2$, and the "tadpole" terms. As we are computing a physical quantity, this is the signal that our calculation is only valid for massless pions, which also makes the $(m_u + m_d)^2$ corrections irrelevant except for giving an order of magnitude of the uncertainty of our result. The tadpole term, gauge independent, is shown to play no role in the pion case and at the opposite to be crucial for the kaon case. It displays explictly the role of the up and down quark

mass difference in the hadronic mass shifts. The electroweak contribution in the kaon case is also to be computed at the chiral limit, which puts it equal to the pion one, as stated by Dashen's theorem [18].

We obtain $m_{\pi^+} - m_{\pi^0} = 4.95$ MeV, which, taking into account the expected incertitude of order $\frac{m_\pi^2}{M_\rho^2}$, is in good agreement with the experimental result of 4.6 MeV.

The paper is organized as follows:

* in the second section, we present a general overview of the method together with considerations about the problem of gauge invariance;

* the third section is devoted to technical computations: detailed computation of the divergences of hadronic currents in the standard model, of the 2-point function $\tilde{\psi}(0)$ relative to those divergences, detailed discussion of the Higgs sector and its non cancellation of the gauge dependence of the electroweak sector. The results will be seen to be still plagued with infinities, proportional to $(m_d + m_u)^2$, originating from the asymptotic behaviour of current-current correlation functions in QCD;

* the fourth section studies the cancellation of infinities and recovers the result of Das et al. [1] and Dicus and Mathur [12a];

* the fifth section deals briefly with the kaon case, where the same treatment has been applied;

* finally, we conclude with a general discussion, and possible extensions of the work.

II. General Overview of the Method

The purpose of this section is to present in a non technical way the main features of the method and to provide a qualitative understanding of it.

We shall deal explicitly with the pion case; the kaon case can be treated exactly in the same way.

Starting from the $SU(3)_c \times SU(2)_L \times U(1)$ Lagrangian of the strong, electromagnetic and weak interactions of quarks, we identify by the PCAC hypothesis the propagators of the charged and neutral pions with those of the covariant (with respect to the gauge group $SU(2)_L \times U(1)$) divergences of the appropriate hadronic axial currents:

$$\begin{aligned} A^{\mu(1+i2)}(x) &= \bar{u}(x)\gamma^\mu\gamma_5\, d(x) \\ A^{\mu(3)}(x) &= \frac{1}{2}\left(\bar{u}(x)\gamma^\mu\gamma_5 u(x) - \bar{d}(x)\gamma^\mu\gamma_5\, d(x)\right) . \end{aligned} \tag{2.1}$$

While in a pure Quantum Chromodynamics approach, the (ordinary) divergences receive contributions from the quark mass terms only, they may be written in our general approach in the form:

$$\partial_\mu A^\mu = M + G + H, \tag{2.2}$$

where M represents the contribution from the mass terms (generated through Yukawa couplings to a doublet of Higgs fields), G from the gauge bosons of the $SU(2)_L \times U(1)$ group, and H from the Higgs fields. Accordingly, the covariant derivative of any axial current A^μ with respect to $SU(2)_L \times U(1)$ may be

written:

$$D_\mu A^\mu = \partial_\mu A^\mu - G - H = M , \tag{2.3}$$

which is the same expression as that of the ordinary derivative in a theory of only strongly interactive massive quarks. Equation (2.3) ensures the conservation of the axial current at the chiral limit. (The only discussion can be about the Higgs contribution: it turns out (as will be shown in Section 4) that the removing of divergences dictates the presence of M only in $D_\mu A^\mu$.)

Defining the $\tilde{\psi}(q^2)$'s as the propagators of the covariant divergences of the hadronic currents:

$$\tilde{\psi}^{(i)}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T D_\mu A^{\mu(i)}(x)\, D_\nu A^{\nu(i)+}(0)|0\rangle , \tag{2.4}$$

and $\psi(q^2)$'s as those of the ordinary $\partial_\mu A^\mu$'s, they are related through Eq. (2.3) by:

$$\begin{aligned} \tilde{\psi}(q^2) = \psi(q^2) - i\int d^4x e^{iq\cdot x}\{&\langle 0|T(G+H)(x)(G+H)^+(0)|0\rangle \\ &+ \langle 0|T(G+H)(x)M^+(0)|0\rangle + \langle 0|TM(x)(G+H)^+(0)|0\rangle\}. \end{aligned} \tag{2.5}$$

The advantage of making the ψ's appear explicitly is that they will easily generate the "tadpole" terms through the use of the Current Algebra Ward Identities [7] that they satisfy. On the other hand, as they can also be thought of, by PCAC, as the propagators of pions, the $\tilde{\psi}$'s may be written as:

$$\tilde{\psi}^{(1+i2)}(q^2) = \frac{2f^2_{\pi^+} m^4_{\pi^+}}{-q^2 + m^2_{\pi^+}} + G.T^{(1+i2)} + \text{continuum},$$

$$\tilde{\psi}^{(3)}(q^2) = \frac{f^2_{\pi^0} m^4_{\pi^0}}{-q^2 + m^2_{\pi^0}} + G.T^{(3)} + \text{continuum}. \qquad (2.6)$$

We shall always in the following neglect the contributions of the continuum.

The presence in Eq. (2.6) of $SU(2)_L \times U(1)$ gauge dependent terms (GT) is a priori unavoidable in any propagator, and only the poles and the corresponding residues are expected to be gauge independent. So, a gauge independent computation of the pion mass splitting would necessitate an identification of the poles or the residues of the $\tilde{\psi}$'s, which unavoidably requires computations, performed at $q^2 = m^2_\pi$, of diagrams involving two quarks and one gauge or Higgs boson line (at the order g^2 of electroweak interactions we shall be working at). Those cannot in general be reduced to self-energy type diagrams (Fig. 2), and we do not know how to tackle those depicted in Fig. 1, whose resummation at all orders of strong interactions is untractable. We consequently need a method in which strong and electroweak interactions factorize at order g^2 of $SU(2)_L \times U(1)$, and where we can safely resume the first, leading to diagrams of the type depicted in Fig. 3.

This tremendous simplification can be achieved by working at $Q = 0$, that is slightly off mass shell. However, the price to be paid for that is a gauge dependence of order m^2_π in the electroweak contribution to the mass splitting:

$$\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0) = 2(f^2_{\pi^+} m^2_{\pi^+} - f^2_{\pi^0} m^2_{\pi^0}) + GT(\mathcal{O}(m^2_\pi)). \qquad (2.7)$$

As we are evaluating a physical quantity, this means that the electroweak part of our result can only be computed and trusted at the chiral limit.

The tadpole term is not subject to this restriction; we shall see that it is proportional to the gauge invariant and renormalization-group invariant combination of $(m_d - m_u)$ times some quark vacuum condensates. Actually, defining the chiral limit as the limit where the Yukawa couplings go to zero does not mean that the tadpole terms are absent. Once we admit the presence of a quark condensation, the quarks are expected to acquire non vanishing masses from this non perturbative mechanism, and, subsequently, a difference between u and d quark masses may be understood as an effect of the electroweak interactions. A deeper discussion [20] of this point is outside the scope of this work, and for practical computations, we whall maintain the tadpole term, essential in the understanding of the kaon mass splitting.

However, the limitation mentioned above for the electroweak part makes our computation only trustable up to a relative incertitude of order m^2_π/M^2, where M is a typical hadronic scale involved in the calculation (the mass of the ρ or A_1 meson).

It is also clear from Eq. (2.7), that in order to display the quantity $m^2_{\pi^+} - m^2_{\pi^0}$, we shall have to make the assumption of the equality of the hadronic parameters f_{π^+} and f_{π^0}, that we are unable to compute in our present state of knowledge. The program that we shall fulfill is accordingly the computation of

$$2f_\pi^2\,(m^2_{\pi^+} - m^2_{\pi^0}) = \psi^{(1+i2)}(0) - 2\psi^{(3)}(0) - i\int d^4x\, e^{iqx}$$
$$\{\langle 0|T(G + H)(x)(G + H)^+(0)|0\rangle + \langle 0|T(G + H)(x)M^+(0)|0\rangle$$
$$+ \langle 0|TM(x)(G + H)^+(0)|0\rangle\}^{(1+i2)-2(3)}_{q=0}, \qquad (2.8)$$

and the extraction of a finite result from the above formula. Indeed the resummation of diagrams displayed in Fig. 2 into those of Fig. 3 is not enough to ensure finiteness before working at the chiral limit; while the contributions from low energy hadronic poles and resonances give back the result of ref. [12a], the high energy contributions of the corresponding current-current hadronic propagators extracted from the asymptotic behaviour of Quantum Chromodynamics [17] give rise to divergences proportional to $(m_u + m_d)^2$. We shall show, and this will be the purpose of the fourth section, that one can indeed obtain a finite result <u>before</u> working at the chiral limit, condition that we shall have finally to fulfill due to the finite gauge dependence that we shall be left with afterwards (in this respect, Section IV has no influence on the final result).

As we are working in a renormalizable theory, all infinities which may appear in the computation of a physical quantity from the bare Lagrangian can be eliminated by only a suitable redefinition of the parameters of the original Lagrangian. This can be achieved by introducing suitable counterterms respecting its symmetries, the contribution of which may be simply subtracted from the bare (infinite) result to obtain the physical (finite) result. Now, the counterterms associated to any parameter may

be computed at a given (for example 2nd) order of the coupling constant by evaluating the contribution of

$$\frac{i}{2}\int d^4y\,\mathcal{L}_{int}(x)\,\mathcal{L}_{int}(y) \qquad (2.9)$$

to this parameter. In the case we are concerned in, we shall directly evaluate the contribution of the T-product Eq. (2.9) to the mass splitting, thus short cutting the lengthy step of working out explicitly all the conterterms needed in the $SU(3)_c \times SU(2)_L \times U(1)$ theory, and consider that as strictly equivalent to the usual procedure. It turns out that this contribution to the mass splitting is exactly of the same form as the bare result; we shall consequently obtain the physical (finite) result by simply subtracting from the last one its infinite part.

We shall next have to operate technically the removal of the infinities; in the absence, in this special case of vacuum fluctuations, of a well defined algebraic procedure, we shall only be able to achieve this up to corrections of order $(m_u + m_d)^2$, which anyhow are irrelevant according to the expected uncertainty of our computations, as mentioned above.

After those many comments, we now turn to detailed computations displayed in the next sections.

III. Computation of the 2-Point Functions of the Current Divergences

We shall develop in this section all the formal apparatus leading to the formula:

$$\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0) = (m_u - m_d)\langle \bar{u}u - \bar{d}d\rangle$$

$$+ e^2\int \frac{d^4q}{(2\pi)^4}\left[(D^{\gamma}_{\mu\nu}(q) - D^{Z}_{\mu\nu}(q))\left(2\,\underset{VV}{\Pi}^{(3)\mu\nu}(q) - \underset{AA}{\Pi}^{(1+i2)\mu\nu}(q)\right)\right]$$

$$- 2i\sqrt{2}\, G_F(m_u + m_d)^2 \int \frac{d^4q}{(2\pi)^4}\left[(D^{P^{(3)}}(q) - D^S(q))D^{\Phi^3}(q) + c_1^2\, D^S(q)D^{\Phi\pm}(q)\right], \quad (3.1)$$

where the first term represents the "tadpole" contribution, the second the $SU(2)_L \times U(1)$ gauge bosons contribution and the third that of the Higgs bosons.

The notations in (3.1) are the following:

G_F is the Fermi constant:

$$G_F = \frac{1.02\ 10^{-5}}{Mp^2} \quad (M_p \text{ is the mass of the proton}). \quad (3.2)$$

c_1 is the cosine of the Cabibbo angle.

γ and Z denote respectively the photon and Z gauge boson, D^{γ} and D^Z their propagators, and the doublet of Higgs fields is chosen as follows:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi^+ = \phi^1 + i\phi^2 \\ \phi^0 = V + H + i\phi^3\end{pmatrix}, \quad (3.3)$$

such that the vacuum expectation value of Φ is:

$$\Phi = \begin{pmatrix} 0 \\ V/\sqrt{2}\end{pmatrix}. \quad (3.4)$$

D^{ϕ^3} and $D^{\phi^\pm}$ are the corresponding propagators.

$D^{P^{(3)}}$ is the propagator of the pseudoscalar bilinear quark operator.

$$P^{(3)}(x) = \frac{1}{2}\,(\bar{u}(x)\gamma^5 u(x) - \bar{d}(x)\gamma^5 d(x)) \quad (3.5)$$

and D^S the sum $\frac{1}{2}D^{S^{(0)}} + \frac{1}{3}D^{S^{(8)}} + \frac{1}{6}D^{S^{(15)}}$ of propagators of the scalar bilinear quark operators $S^{(0)}$, $S^{(8)}$ and $S^{(15)}$ carrying the flavor SU(4) indices 0, 8 and 15, whose complete expressions may be found in Eqs. (3.19). $\underset{VV}{\Pi}^{(i)\mu\nu}$ and $\underset{AA}{\Pi}^{(j)\mu\nu}$ are the two point functions of the hadronic vector and axial currents respectively, carrying the flavor SU(4) quantum numbers i or j, for example:

$$\underset{AA}{\Pi}^{(1+i2)\mu\nu}(q) = i\int d^4x\, e^{iq\,x}\ \langle 0|T\bar{u}(x)\gamma^{\mu}\gamma_5 d(x)\ \bar{d}(0)\gamma^{\nu}\gamma_5 u(0)|0\rangle\ . \quad (3.6)$$

In order to simplify the formulas, all quark mass differences will be dropped in the electroweak contributions. We shall furthermore restrict ourselves to 2 generations of quarks.

The Higgs sector will be investigated in detail, in connection with the question of gauge dependence.

For the sake of completeness and self-sufficiency of the paper, we shall write in detail the notations and conventions used throughout the paper, especially for the Lagrangian. We shall also write in full generality the expressions for the divergences of the hadronic currents as obtained from the complete Lagrangian [21].

We shall show how, from Eq. (3.1) the result of Ref. [12a] may be recovered, but no longer justified due to infinities arising in particular from the asymptotic behaviour of the $\Pi^{\mu\nu}$'s.

1) The Langrangian

a) The kinetic terms and the gauge field sector

We have the following contributions:

$$\circ L_{Y.M} = -\frac{1}{4} G^{(a)}_{\mu\nu} G^{\mu\nu(a)}; \tag{3.7}$$

$G^{(a)}_{\mu\nu}$ is the gluon field tensor and a color index (a = 1 ... 8).

$$\circ L_{\gamma} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}; \tag{3.8}$$

$F_{\mu\nu}$ is the electromagnetic field tensor.

$$\circ L_{kin.fermions} = i\bar{\psi}\gamma_\mu \partial^\mu \psi \tag{3.9}$$

$$\psi = \begin{pmatrix} u \\ c \\ d \\ s \end{pmatrix} .$$

$$\circ L_{W,Z} = -\frac{1}{2} W^+_{[\mu,\nu]} W^{-[\mu,\nu]} - \frac{v^2 e^2}{4 \sin^2\theta_W} W^+_\mu W^{\mu -}$$
$$- \frac{1}{4} Z_{[\mu,\nu]} Z^{[\mu,\nu]} - \frac{v^2 e^2}{8 \sin^2\theta_W \cos^2\theta_W} Z_\mu Z^\mu; \tag{3.10}$$

V is defined in (3.3) and (3.4), θ_W is the Weinberg angle.

$$\circ L_{int\ SU(3)_c} = i\bar{\psi}\gamma_\mu(-ig_s \frac{1}{2} \lambda^{(a)} G^{\mu(a)}\psi; \tag{3.11}$$

g_s is the strong coupling constant

$G^{\mu(a)}$ is the gluon field,

the $\lambda^{(a)}$'s are the 8 Gell-Mann matrices.

$$\circ L_{int\ SU(2)_L\times U(1)} = e\Big\{A_\mu J^{\mu em.} + \frac{1}{2 \sin\theta_W \cos\theta_W} \bar{\psi}\gamma_\mu Z^\mu \frac{1-\gamma_5}{2} \mathbb{N}\psi - \frac{\sin^2\theta_W}{\sin\theta_W \cos\theta_W} \bar{\psi}\gamma_\mu Z^\mu \mathbb{Q}\psi$$
$$+ \frac{1}{\sqrt{2}\sin\theta_W}\left(\bar{\psi}\gamma_\mu W^{\mu+} \frac{1-\gamma_5}{2} \mathbb{C}\psi + \bar{\psi}\gamma_\mu W^{\mu-} \frac{1-\gamma_5}{2} \mathbb{C}^+\psi\right)\Big\} . \tag{3.12}$$

A^μ is the photon field, J^{em}_μ the electromagnetic hadronic current.

$$\mathbb{N} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} . \tag{3.13a}$$

$$\mathbb{Q} = \begin{pmatrix} 2/3 & & & \\ & 2/3 & & \\ & & -1/3 & \\ & & & -1/3 \end{pmatrix} \text{ is the charge matrix.} \tag{3.13b}$$

$$\mathbb{C} = \left(\begin{array}{c|c} 0 & C \\ \hline 0 & 0 \end{array}\right), \qquad (3.13c)$$

where C is the standard Cabibbo matrix, $C = \begin{pmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{pmatrix}$.

b) <u>The gauge fixing terms</u>

We write them in the form

$$L_{GF} = -\frac{1}{2a_\gamma} C_A^2 - \frac{1}{2a_W}(C_Z^2 + 2C_+C_-), \qquad (3.14)$$

with

$$\begin{aligned} C_A &= \partial_\mu A^\mu, \\ C_Z &= \partial_\mu Z^\mu - a_W M_Z \phi^3, \\ C_+ &= \partial_\mu W^{\mu+} - a_W M_W \phi^+, \\ C_- &= \partial_\mu W^{\mu-} - a_W M_W \phi^-. \end{aligned} \qquad (3.15)$$

M_Z, M_W are the masses of the Z and W bosons respectively. ϕ^1, ϕ^2, ϕ^3 defined in (3.3) and (3.4) are the 3 Higgs-Goldstone bosons.

c) <u>The Ghost sector</u>

We shall not need any explicit computation involving ghost fields at the order of the electroweak interactions that we shall be working at.

d) <u>The Higgs sector</u>

The quarks will receive masses through Yukawa couplings to the doublet of Higgs fields. The process of diagonalization of the mass matrix, giving rise to the mixing matrix, is well known. We refer, for example, the reader to Ref. [22]. We shall only write here the final result using the conventions used by most physicists [23].

One obtains:

$$\begin{aligned} L_{\psi\text{-Higgs}} = &-\bar\psi \mathbb{M}\psi - \sqrt{\sqrt{2}\,G_F}\,\bar\psi M\psi H + i\sqrt{\sqrt{2}\,G_F}\,\bar\psi \mathbb{N}\,\mathbb{M}\gamma_5\psi\phi^3 \\ &+ \sqrt{2}\sqrt{\sqrt{2}\,G_F}\left[\bar\psi \left(\begin{array}{c|c} 0 & C^{-1} \\ \hline 0 & 0 \end{array}\right) \mathbb{M}\,\frac{1+\gamma_5}{2}\,\psi\phi^+ \right. \\ &- \bar\psi\mathbb{M}\left(\begin{array}{c|c} 0 & C^{-1} \\ \hline 0 & 0 \end{array}\right) \frac{1-\gamma_5}{2}\,\psi\phi^+ \\ &\left. - \bar\psi\left(\begin{array}{c|c} 0 & 0 \\ \hline C & 0 \end{array}\right) \mathbb{M}\,\frac{1+\gamma_5}{2}\,\psi\phi^- + \bar\psi\mathbb{M}\left(\begin{array}{c|c} 0 & 0 \\ \hline C & 0 \end{array}\right) \frac{1-\gamma_5}{2}\,\psi\phi^- \right] \end{aligned} \qquad (3.16)$$

where we introduced the diagonalized mass matrix $\mathbb{M}$:

$$\mathbb{M} = \begin{pmatrix} m_u & & & \\ & m_c & & \\ & & m_d & \\ & & & m_s \end{pmatrix} \qquad (3.17)$$

For practical computations, it is also very useful to put this part of the Lagrangian in the following form:

$$
\begin{aligned}
L_{\psi\text{-Higgs}} = & -\Bigg[(m_u - m_d)S^{(3)} + (m_u + m_d)\left(\frac{S^{(0)}}{\sqrt{2}} + \frac{S^{(8)}}{\sqrt{3}} + \frac{S^{(15)}}{\sqrt{6}}\right) \\
& + m_s\left(\frac{S^{(0)}}{\sqrt{2}} - \frac{2S^{(8)}}{\sqrt{3}} + \frac{S^{(15)}}{\sqrt{6}}\right) \\
& + m_c\left(\frac{S^{(0)}}{\sqrt{2}} + \sqrt{\frac{3}{2}}\, S^{(15)}\right)\Bigg](1 + \sqrt{\sqrt{2}\, G_F}\, H) \\
& + i\sqrt{\sqrt{2}\, G_F}\Bigg[(m_d - m_u)\left(\frac{P^{(0)}}{\sqrt{2}} + \frac{P^{(8)}}{\sqrt{3}} + \frac{P^{(15)}}{\sqrt{6}}\right) - (m_d + m_u)P^{(3)} \\
& + m_s\left(\frac{P^{(0)}}{\sqrt{2}} - \frac{2P^{(8)}}{\sqrt{3}} + \frac{P^{(15)}}{\sqrt{6}}\right) - m_c\left(\frac{P^{(0)}}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}}\, P^{(15)}\right)\Bigg]\phi^3 \\
& + \sqrt{\sqrt{2}\, G_F}\Big\{[c_1(m_d - m_u)S^{(1+i2)} + c_1(m_d + m_u)P^{(1+i2)} \\
& + s_1(m_d - m_c)S^{(11+i12)} + s_1(m_d + m_c)P^{(11+i12)} \\
& - s_1(m_s - m_u)S^{(4+i5)} - s_1(m_s + m_u)P^{(4+i5)} \\
& + c_1(m_s - m_c)S^{(13+i14)} + c_1(m_s + m_c)P^{(13+i14)}]\phi^+ \\
& + [c_1(m_d - m_u)S^{(1-i2)} - c_1(m_d - m_u)P^{(1-i2)} \\
& + s_1(m_d - m_c)S^{(11-i12)} - s_1(m_d + m_c)P^{(11-i12)} \\
& - s_1(m_s - m_u)S^{(4-i5)} + s_1(m_s + m_u)P^{(4-i5)}
\end{aligned}
$$

$$
+ c_1(m_s - m_c)S^{(13-i14)} - c_1(m_s + m_c)P^{(13-i14)}]\phi^-\Big\} \tag{3.18}
$$

where the S's and the P's are scalar and pseudoscalar diquark operators. We list them explicitly below for the scalar case:

$$
\begin{aligned}
S^{(1+i2)} &= \bar{u}d & S^{(3)} &= \frac{1}{2}(\bar{u}u - \bar{d}d) \\
S^{(4+i5)} &= \bar{u}s & S^{(8)} &= \frac{1}{2\sqrt{3}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \\
S^{(6+i7)} &= \bar{d}s & S^{(15)} &= \frac{1}{2\sqrt{6}}(\bar{u}u + \bar{d}d + \bar{s}s - 3\bar{c}c) \\
S^{(9+i10)} &= \bar{u}c & S^{(0)}\,(\text{singlet}) &= \frac{1}{2\sqrt{2}}(\bar{u}u+\bar{d}d+\bar{s}s+\bar{c}c) \\
S^{(11+i12)} &= \bar{c}d & & \\
S^{(13+i14)} &= \bar{c}s & &
\end{aligned} \tag{3.19a}
$$

so that one has in particular

$$
\begin{aligned}
\frac{S^{(0)}}{\sqrt{2}} + \frac{S^{(8)}}{\sqrt{3}} + \frac{S^{(15)}}{\sqrt{6}} &= \frac{1}{2}(\bar{u}u + \bar{d}d) \\
\frac{S^{(0)}}{\sqrt{2}} - \frac{2S^{(8)}}{\sqrt{3}} + \frac{S^{(15)}}{\sqrt{6}} &= \bar{s}s \\
\frac{S^{(0)}}{\sqrt{2}} - \sqrt{\frac{3}{2}}\, S^{(15)} &= \bar{c}c
\end{aligned} \tag{3.19b}
$$

2) <u>Computation of the divergences of the hadronic axial currents</u>[2]

Performing an infinitesimal axial flavor SU(n) transformation on the quark field ψ, the Lagrangian gets modified by an amount δL which is the divergence of the corresponding current:

$$\delta L^{(i)} = \partial_\mu A^{\mu(i)} \; . \qquad (3.20)$$

We find the general formula:[3]

$$\begin{aligned}
\partial_\mu A^{\mu(i)} = {} & i\bar\psi\gamma_5[T^{(i)},\mathbb{M}]_+\psi + ie\Big\{\bar\psi\not{A}\gamma_5[T^{(i)},\mathbb{Q}]\psi \\
& + \frac{1}{2\sin\theta_W\cos\theta_W}\;\bar\psi\not{Z}\gamma_5\frac{1-\gamma_5}{2}\;[T^{(i)},\mathbb{N}]\psi \\
& - \frac{\sin^2\theta_W}{\sin\theta_W\cos\theta_W}\bar\psi\not{Z}\gamma_5[T^{(i)},\mathbb{Q}]\psi \\
& + \frac{1}{\sqrt{2}\sin\theta_W}\Big(\bar\psi\not{W}^+\gamma_5\frac{1-\gamma_5}{2}[T^{(i)},\mathbb{C}]\psi \\
& + \bar\psi\not{W}^-\gamma_5\frac{1-\gamma_5}{2}[T^{(i)},\mathbb{C}^+]\psi\Big)\Big\} \\
& + i\sqrt{\sqrt{2}\,G_F}\Big\{\bar\psi\gamma_5[T^{(i)},\mathbb{M}]_+\psi H - i\bar\psi[T^{(i)},\mathbb{N}\,\mathbb{M}]_+\psi\phi^3 \\
& - \sqrt{2}\,\bar\psi\frac{1+\gamma_5}{2}\left[T^{(i)},\left(\begin{array}{c|c}0&C^{-1}\\\hline 0&0\end{array}\right)\mathbb{M}\right]_+\psi\phi^+ \\
& - \sqrt{2}\,\bar\psi\frac{1-\gamma_5}{2}\left[T^{(i)},\mathbb{M}\left(\begin{array}{c|c}0&C^{-1}\\\hline 0&0\end{array}\right)\right]_+\psi\phi^+ \\
& + \sqrt{2}\,\bar\psi\frac{1+\gamma_5}{2}\left[T^{(i)},\left(\begin{array}{c|c}0&0\\\hline C&0\end{array}\right)\mathbb{M}\right]_+\psi\phi^- \\
& + \sqrt{2}\,\bar\psi\frac{1-\gamma_5}{2}\left[T^{(i)},\mathbb{M}\left(\begin{array}{c|c}0&0\\\hline C&0\end{array}\right)\right]_+\psi\phi^-\Big\} \; . \qquad (3.21)
\end{aligned}$$

$T^{(i)}$ is $1/2\lambda^{(i)}$, where the $\lambda^{(i)}$'s are the generalized Gell-Mann matrices. In the particular cases $i = 1 + i2$ and $i = 3$ we are interested in, we get:

$$\begin{aligned}
\partial_\mu A^{\mu(1+i2)} = {} & i(m_u + m_d)\bar u\gamma_5 d \\
& - ie\Big\{\bar u\not{A}\gamma_5 d + \frac{1}{2\sin^2\theta_W\cos^2\theta_W}\left((1-2\sin^2\theta_W)\bar u\not{Z}\gamma_5 d - \bar u\not{Z}d\right) \\
& - \frac{c_1}{\sqrt{2}\sin\theta_W}\left(\bar u\not{W}^-\gamma_5\frac{1-\gamma_5}{2}u - \bar d\not{W}^-\gamma_5\frac{1-\gamma_5}{2}d\right) \\
& + \frac{s_1}{\sqrt{2}\sin\theta_W}\left(\bar u\not{W}^-\gamma_5\frac{1-\gamma_5}{2}c + s\not{W}^-\gamma_5\frac{1-\gamma_5}{2}d\right)\Big\} \\
& - i\sqrt{\sqrt{2}\,G_F}\Big\{-(m_u + m_d)P^{(1+i2)}H - i(m_d - m_u)S^{1+i2}\phi^3 \\
& + \sqrt{2}\Big[c_1(m_u + m_d)\left(\frac{S^{(0)}}{\sqrt{2}} + \frac{S^{(8)}}{\sqrt{3}} + \frac{S^{(15)}}{\sqrt{6}}\right) \\
& + c_1(m_u - m_d)\left(\frac{P^{(0)}}{\sqrt{2}} + \frac{P^{(8)}}{\sqrt{3}} + \frac{P^{(15)}}{\sqrt{6}}\right) \\
& - s_1(m_c + m_d)\frac{S^{(9+i10)}}{2} - s_1(m_c - m_d)\frac{P^{(9+i10)}}{2} \\
& + s_1(m_u + m_s)\frac{S^{(6-i7)}}{2} + s_1(m_u - m_s)\frac{P^{(6-i7)}}{2}\Big]\phi^-\Big\} \; ,
\end{aligned}$$

(3.22)

$$\partial_\mu A^{\mu(3)} = i(m_u \bar{u}\gamma_5 u - m_d \bar{d}\gamma_5 d)$$

$$+ \frac{ie}{\sqrt{2}\sin\theta_W}\left[c_1\left(\bar{u}\not{W}^+\gamma_5 \frac{1-\gamma_5}{2} d - \bar{d}\not{W}^-\gamma_5 \frac{1-\gamma_5}{2} u\right)\right.$$

$$+ \frac{s_1}{2}\left(\bar{u}\not{W}^+\gamma_5 \frac{1-\gamma_5}{2} s - \bar{s}\not{W}\gamma_5 \frac{1-\gamma_5}{2} u\right.$$

$$\left.\left. - \bar{c}\not{W}^+\gamma_5 \frac{1-\gamma_5}{2} d + \bar{d}\not{W}^-\gamma_5 \frac{1+\gamma_5}{2} c\right)\right]$$

$$-i\sqrt{\sqrt{2}\,GF}\left\{(m_d - m_u)\left(\frac{P^{(0)}}{\sqrt{2}} + \frac{P^{(8)}}{\sqrt{3}} + \frac{P^{(15)}}{\sqrt{6}}\right) H - (m_d + m_u)P^{(3)}H\right.$$

$$+ i(m_u + m_d)\left(\frac{S^{(0)}}{\sqrt{2}} + \frac{S^{(8)}}{\sqrt{3}} + \frac{S^{(15)}}{\sqrt{6}}\right)\Phi^3 + i(m_u - m_d)S^{(3)}\Phi^3$$

$$+ \frac{\sqrt{2}}{2}\left\{+ \frac{s_1}{2}(m_u + m_s)(S^{(4+i5)}\Phi^+ - S^{(4-i5)}\Phi^-)\right.$$

$$+ \frac{s_1}{2}(m_s - m_u)(P^{(4+i5)}\Phi^+ + P^{(4-i5)}\Phi^-)$$

$$+ \frac{s_1}{2}(m_d + m_c)(S^{(11+i12)}\Phi^+ - S^{(11-i12)}\Phi^-)$$

$$\left.\left. + \frac{s_1}{2}(m_d - m_c)(P^{(11+i12)}\Phi^+ + P^{(11-i12)}\Phi^-)\right\}\right\} . \qquad (3.23)$$

3) <u>Computation of the propagators $\psi(0)$ of the divergences of the currents</u>

They will be computed by using the Current Algebra Ward Identities: [7]

$$q_\mu q_\nu \underset{AA}{\Pi}^{\mu\nu}(q) \equiv q^2 \tilde{\Pi}^0(q^2)$$

$$= \psi(q^2) - q_\nu \int d^4x e^{iq\cdot x}\delta(x_0)\,\langle 0|[A^0(x), A^{\nu+}(0)]|0\rangle$$

$$- i\int d^4x e^{-iq\cdot x}\delta(x_0)\,\langle 0|[\partial_\mu A^\mu(0), A^{0+}(x)]|0\rangle , \qquad (3.24)$$

where $\underset{AA}{\Pi}^{\mu\nu}$ has been defined in (3.6); we have used its decomposition:

$$\underset{AA}{\tilde{\Pi}}^{\mu\nu}(q^2) = -\left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\tilde{\Pi}^1(q^2) + \frac{q^\mu q^\nu}{q^2}\tilde{\Pi}^0(q^2) . \qquad (3.25)$$

$\tilde{\Pi}^1$ and $\tilde{\Pi}^0$ are orthogonal in the $J = 1$ and $J = 0$ channels. From (3.23), one deduces at $q = 0$:

$$\psi(0) = i\int d^4x\,\delta(x_0)\,\langle 0|[\partial_\mu A^\mu(0), A^{0+}(x)]|0\rangle . \qquad (3.26)$$

The commutator $[\partial_\mu A^\mu(0), A^{0+}(x)]$ appearing in (3.26) is entirely determined by the anticommutation relations of the fermionic fields. As already mentioned, we shall omit hereafter the electroweak terms proportional to any quark mass difference. We obtain:

$$\psi^{(1+i2)}(0) = -(m_u + m_d)\langle \bar{u}u + \bar{d}d\rangle$$

$$+ e\left[\langle \bar{u}\not{A}u - \bar{d}\not{A}d\rangle + \frac{1}{2}\frac{1 - 2\sin^2\theta_W}{\sin\theta_W\cos\theta_W}\langle u\not{Z}u - d\not{Z}d\rangle\right.$$

$$- \frac{1}{2\sin\theta_W\cos\theta_W}\langle u\not{Z}\gamma_5 u - d\not{Z}\gamma_5 d\rangle$$

$$+\frac{2c_1}{\sqrt{2}\,\sin\theta_W}\langle \bar{d}\not{W}^{-}\frac{1-\gamma_5}{2}u\rangle$$

$$-\frac{s_1}{\sqrt{2}\,\sin\theta_W}\langle \bar{d}\not{W}^{-}\frac{1-\gamma_5}{2}c-\bar{s}\not{W}^{-}\frac{1-\gamma_5}{2}u\rangle\Big]$$

$$-\sqrt{\sqrt{2}\,G_F}\Big[(m_u+m_d)\langle(\bar{u}u+\bar{d}d)H\rangle$$

$$+\sqrt{2}\langle(c_1(m_u+m_d)\bar{d}u+\frac{s_1}{2}(m_c+m_d)\bar{d}c$$

$$-\frac{s_1}{2}(m_u+m_s)\bar{s}u)\phi^{-}\rangle\Big], \qquad (3.27)$$

$$\psi^{(3)}(0) = -(m_u\langle\bar{u}u\rangle+m_d\langle\bar{d}d\rangle)$$

$$-\frac{e}{\sqrt{2}\,\sin\theta_W}\Big[-c_1\langle\bar{u}\not{W}^{+}\frac{1-\gamma_5}{2}d+\bar{d}\not{W}^{-}\frac{1-\gamma_5}{2}u\rangle$$

$$-\frac{s_1}{4}\langle\bar{u}\not{W}^{+}\frac{1-\gamma_5}{2}s+\bar{s}\not{W}^{-}\frac{1-\gamma_5}{2}u\rangle$$

$$+\frac{s_1}{4}\langle\bar{c}\not{W}^{+}\frac{1-\gamma_5}{2}d+\bar{d}\not{W}^{-}\frac{1-\gamma_5}{2}c\rangle\Big]$$

$$-\sqrt{\sqrt{2}\,G_F}\Big[\langle(m_u\,\bar{u}u+m_d\,\bar{d}d)H\rangle-i\langle(m_u\,\bar{u}\gamma_5u-m_d\bar{d}\gamma_5d)\phi^3\rangle$$

$$+\frac{\sqrt{2}}{2}\frac{s_1}{4}((m_u+m_s)\langle\bar{u}s\phi^{+}-\bar{s}u\phi^{-}\rangle$$

$$+(m_d+m_c)\langle\bar{c}d\phi^{+}-\bar{d}c\phi^{-}\rangle)\Big], \qquad (3.28)$$

where the notation ⟨ ⟩ means "vacuum expectation value". In the last equation, the terms involving charged Higgs field cancel by the relations:

$$\langle\bar{u}s\phi^{+}\rangle=\langle\bar{s}u\phi^{-}\rangle \text{ etc. } \ldots, \qquad (3.29)$$

easily deducible, for example, from the expansion procedure that we shall use in the following.

The relevant combination for the pion mass splitting writes:

$$\psi^{(1+i2)}(0)-2\psi^{(3)}(0)=(m_u-m_d)\langle\bar{u}u-\bar{d}d\rangle$$

$$+e\Big[\langle\bar{u}\not{A}u-\bar{d}\not{A}d\rangle+\frac{1-2\sin^2\theta_W}{2\sin\theta_W\cos\theta_W}\langle\bar{u}\not{Z}u-\bar{d}\not{Z}d\rangle$$

$$-\frac{1}{2\sin\theta_W\cos\theta_W}\langle\bar{u}\not{Z}\gamma_5u-\bar{d}\not{Z}\gamma_5d\rangle$$

$$+\frac{2c_1}{\sqrt{2}\,\sin\theta_W}\langle\bar{u}\not{W}^{+}\frac{1-\gamma_5}{2}d\rangle$$

$$-\frac{s_1}{\sqrt{2}\,\sin\theta_W}\langle\bar{d}\not{W}^{-}\frac{1-\gamma_5}{2}c-\bar{c}\not{W}^{+}\frac{1-\gamma_5}{2}d$$

$$+\bar{u}\not{W}^{+}\frac{1-\gamma_5}{2}s-\bar{s}\not{W}^{-}\frac{1-\gamma_5}{2}u\rangle\Big]$$

$$-\sqrt{\sqrt{2}\,G_F}\Big[i(m_u+m_d)\langle(\bar{u}\gamma_5u-\bar{d}\gamma_5d)\phi^3\rangle$$

$$+\sqrt{2}(c_1(m_u+m_d)\langle\bar{d}u\phi^{-}\rangle+\frac{s_1}{2}(m_c+m_d)\langle\bar{d}c\phi^{-}\rangle$$

$$-\frac{s_1}{2}(m_u+m_s)\langle\bar{s}u\phi^{-}\rangle)\Big]. \qquad (3.30)$$

Now, the meaning of the vacuum expectation values of the type $\langle \bar{q} \not{A} q \rangle$ etc... is easily made explicit by a perturbative expansion at next order of the electroweak interactions. This leads immediately to the vacuum fluctuation-type diagrams at zero external momentum displayed in Fig. 2 (as we shall treat the pure quark vacuum condensates at a phenomenological level, we shall not address ourselves in this work to their possible interpretation).

The above expansion is performed using the Lagrangian exhibited in the subsection III.1. We obtain:

$$\psi^{(1+i2)}(0) - 2\psi^{(3)}(0) = (m_u - m_d)\langle \bar{u}u - \bar{d}d \rangle$$

$$+ e^2 \int \frac{d^4q}{(2\pi)^4} \Bigg\{ 2(D^{\gamma}_{\mu\nu}(q) - D^{Z}_{\mu\nu}(q)) \, \Pi^{(3)\,\mu\nu}_{VV}(q)$$

$$+ \frac{1}{2\sin^2\theta_W \cos^2\theta_W} D^{Z}_{\mu\nu}(q) \left[\Pi^{(3)\,\mu\nu}_{AA}(q) + \Pi^{(3)\,\mu\nu}_{VV}(q) \right]$$

$$- \frac{1}{2} \frac{c_1^2}{2\sin^2\theta_W} D^{W}_{\mu\nu}(q) \left[\Pi^{(1+i2)\,\mu\nu}_{AA}(q) + \Pi^{(1+i2)\,\mu\nu}_{VV}(q) \right] \Bigg\}$$

$$-2i\sqrt{2}\, G_F (m_u + m_d)^2 \int \frac{d^4q}{(2\pi)^4} D^{\Phi^3}(q) D^{P^{(3)}}(q), \qquad (3.31)$$

where the propagator of any pseudoscalar diquark operator is defined as:

$$D^P(q) = i\int d^4x \, e^{iqx} \langle (\bar{q}_1(x)\gamma_5 q_2(x))(\bar{q}_1(0)\gamma_5 q_2(0))^+ \rangle$$

$$= -i\int d^4x \, e^{iqx} \langle \bar{q}_1(x)\gamma_5 q_2(x) \bar{q}_2(0)\gamma_5 q_1(0) \rangle \; . \qquad (3.32)$$

To deduce eq. (3.31), we have used the relation:

$$(1 - 2\sin^2\theta_W)^2 - 1 = -4\sin^2\theta_W \cos^2\theta_W. \qquad (3.33)$$

From (3.31), we recover the fact that the contributions to the pion mass splitting are to be expected at second order of the electroweak interactions; the linear dependence in e and $\sqrt{G_F}$ of (3.30) was simply an artefact of our method.

4) <u>Computation of the other contributions to $\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0)$</u>

According to (2.8) one may write

$$\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0) = \psi^{(1+i2)}(0) - 2\psi^{(3)}(0) - (\phi^{(1+i2)}(0) - 2\phi^{(3)}(0)), \qquad (3.34)$$

with:

$$\phi(0) = \phi_G(0) + \phi_H(0) + \phi_{GH}(0) + \phi_{GM}(0) + \phi_{HM}(0); \qquad (3.35)$$

$$\phi(0)_G = i\int d^4x \, \langle 0 | T G(x) G^+(0) | 0 \rangle ,$$

$$\phi(0)_H = i\int d^4x \, \langle 0 | T H(x) H^+(0) | 0 \rangle ,$$

$$\phi(0)_{GH} = i\int d^4x [\langle 0 | T G(x) H^+(0) | 0 \rangle + \langle 0 | T H(x) G^+(0) | 0 \rangle] ,$$

$$\phi(0)_{GM} = i\int d^4x [\langle 0 | T G(x) M^+(0) | 0 \rangle + \langle 0 | T M(x) G^+(0) | 0 \rangle],$$

$$\phi(0)_{HM} = i\int d^4x [\langle 0 | T H(x) M^+(0) | 0 \rangle + \langle 0 | T M(x) H^+(0) | 0 \rangle]. \qquad (3.36)$$

We shall show that among those 5 additional contributions to $\tilde{\psi}^{(1+i2)}-2\tilde{\psi}^{(3)}$ only ϕ_G and ϕ_H are non vanishing at the order of the electroweak interactions (e^2) and chiral symmetry breaking (m^2) we are working.

a) <u>Computation of</u> ϕ_G

Using the definitions (2.2) and (3.36) and the expressions (3.21), (3.22) for the divergences of currents, we get:

$$\phi_G^{(1+i2)}(0) - 2\phi_G^{(3)}(0) = e^2\int \frac{d^4q}{(2\pi)^4}$$

$$\Big[(D^{\gamma}_{\mu\nu}(q) - D^{Z}_{\mu\nu}(q))\ \Pi^{(1+i2)\mu\nu}_{VV}(q)$$

$$+ \frac{1}{4\sin^2\theta_W\cos^2\theta_W}\ D^Z_{\mu\nu}(q)\left(\Pi^{(1+i2)\mu\nu}_{AA}(q) + \Pi^{(1+i2)\mu\nu}_{VV}(q)\right)$$

$$+ \frac{c_1^2}{2\sin^2\theta_W}\ D^W_{\mu\nu}(q)\left(\Pi^{(3)\mu\nu}_{AA}(q) - \Pi^{(1+i2)\mu\nu}_{AA}(q) + \Pi^{(3)\mu\nu}_{VV}(q) - \Pi^{(1+i2)\mu\nu}_{VV}(q)\right)$$

$$+ \frac{s_1^2}{2\sin^2\theta_W}\frac{1}{4}D^W_{\mu\nu}(q)\Big(\Pi^{(9+i10)\mu\nu}_{AA}(q) - \Pi^{(11+i12)\mu\nu}_{AA}(q)$$

$$+ \Pi^{(9+i10)\mu\nu}_{VV}(q) - \Pi^{(11+i12)\mu\nu}_{VV}(q) + \Pi^{(6-i7)\mu\nu}_{AA}(q) - \Pi^{(4+i5)\mu\nu}_{AA}(q)$$

$$+ \Pi^{(6+i7)\mu\nu}_{VV}(q) - \Pi^{(4+i5)\mu\nu}_{VV}(q)\Big)\Big]. \tag{3.37}$$

b) <u>Computation of</u> ϕ_H

From (2.2), (3.36), (3.21) and (3.22), we obtain

$$\phi_H^{(1+i2)}(0) - 2\phi_H^{(3)}(0) = \sqrt{2}\,G_F\, i\int \frac{d^4q}{(2\pi)^4}$$

$$(m_u + m_d)^2\Big[D^H(q)\left(D^{P^{(1+i2)}}(q) - 2D^{P^{(3)}}(q)\right)$$

$$- 2D^{\phi^3}(q)\left(\frac{D^{S^{(0)}}(q)}{2} + \frac{D^{S^{(8)}}(q)}{3} + \frac{D^{S^{(15)}}(q)}{6}\right)\Big]$$

$$+ D^{\phi^{\pm}}(q)\Big[2c_1^2(m_u + m_d)^2\left(\frac{D^{S^{(0)}}(q)}{2} + \frac{D^{S^{(8)}}(q)}{3} + \frac{D^{S^{(15)}}(q)}{6}\right)$$

$$+ \frac{s_1^2}{2}(m_c + m_d)^2\left(D^{S^{(9+i10)}}(q) - D^{S^{(11+i12)}}(q)\right),$$

$$+ \frac{s_1^2}{2}(m_u + m_s)^2\left(D^{S^{(6+i7)}}(q) - D^{S^{(4+i5)}}(q)\right)\Big]. \tag{3.38}$$

As the leading mass dependence has already been extracted in (3.38), we may compute all propagators at the chiral limit, in which case one has the relations:

$$D^{P^{(1+i2)}} = 2D^{P^{(3)}},$$

$$D^{S^{(9+i10)}} = D^{S^{(11+i12)}},$$

$$D^{S^{(6+i7)}}(q) = D^{S^{(4+i5)}}(q)\ ,$$

$$D^{S^{(0)}}(q) = D^{S^{(8)}}(q) = D^{S^{(15)}}(q)\,. \qquad (3.39)$$

Equation (3.38) consequently simplifies to

$$\phi_H^{(1+i2)}(0) - 2\phi_H^{(3)}(0) = \sqrt{2}\,G_F\, i\int \frac{d^4q}{(2\pi)^4}\,(m_u + m_d)^2$$

$$\left[-2D^{\phi^3}(q)\left[\frac{D^{S^{(0)}}(q)}{2} + \frac{D^{S^{(8)}}(q)}{3} + \frac{D^{S^{(15)}}(q)}{6}\right]\right.$$

$$\left. + 2c_1^2\, D^{\phi^{\pm}}(q)\left[\frac{D^{S^{(0)}}(q)}{2} + \frac{D^{S^{(8)}}(q)}{3} + \frac{D^{S^{(15)}}(q)}{6}\right]\right]. \qquad (3.40)$$

c) <u>Computation of</u> $\phi_{GH}(0)$ and $\phi_{GM}(0)$

* The nonvanishing diagrams which can arise in ϕ_{GH} only appear at higher order of electroweak interactions. So we forget them here.

* In ϕ_{GM}, due to the different behaviour of G and M under the charge conjugation, the contributions of $i\int d^4x\langle 0|TG(x)M^+(0)|0\rangle$ turns out to be exactly the opposite of that of $i\int d^4x\langle 0|TM(x)G^+(0)|0\rangle$. So ϕ_{GM} plays no role too.

d) <u>Computation of</u> ϕ_{HM}

We get:

$$\phi_{HM}^{(1+i2)}(0) - 2\phi_{HM}^{(3)}(0) = i\sqrt{\sqrt{2}\,G_F}\,(m_u + m_d)^2 \int d^4x$$

$$\Big\{\langle P^{(1+i2)}(x)H(x)P^{(1-i2)}(0)\rangle + \langle P^{(1+i2)}(x)H(0)P^{(1-i2)}(0)\rangle$$

$$-2\langle P^{(3)}(x)H(x)P^{(3)}(0)\rangle - 2\langle P^{(3)}(x)H(0)P^{(3)}(0)\rangle$$

$$+ 2i\left(\left\langle\left[\frac{S^{(0)}(x)}{\sqrt{2}} + \frac{S^{(8)}(x)}{\sqrt{3}} + \frac{S^{(15)}(x)}{\sqrt{6}}\right]\phi^3(x)P^{(3)}(0)\right\rangle\right.$$

$$\left. + \left\langle\left[\frac{S^{(0)}(x)}{\sqrt{2}} + \frac{S^{(8)}(x)}{\sqrt{3}} + \frac{S^{(15)}(x)}{\sqrt{6}}\right]\phi^3(0)P^{(3)}(0)\right\rangle\right)$$

$$-\sqrt{2}c_1\left(\left\langle P^{(1+i2)}(x)\phi^+(0)\left[\frac{S^{(0)}}{\sqrt{2}}(0) + \frac{S^{(8)}}{\sqrt{3}}(0) + \frac{S^{(15)}}{\sqrt{6}}(0)\right]\right\rangle\right.$$

$$\left. - \left\langle\left[\frac{S^{(0)}}{\sqrt{2}}(x) + \frac{S^{(8)}}{\sqrt{3}}(x) + \frac{S^{(15)}}{\sqrt{6}}(x)\right]\phi^-(x)P^{(1-i2)}(0)\right\rangle\right)\Big\}\ ; \qquad (3.41)$$

which expression is as usual to be interpreted at next order of electroweak interactions. This leads to a sum of diagrams of the type displayed in Fig. 4. A detailed inspection shows that they all cancel or vanish at the chiral limit, at which we may compute them since the leading mass dependence has already been extracted in (3.41).

e) <u>Electroweak corrections to the $\langle\bar{q}q\rangle$ vacuum condensates</u>

Working consistently at the second order of electroweak interactions needs looking at the electroweak corrections to the quark condensates appearing in the "tadpole" term. One generates diagrams of the type displayed in Fig. 5. However, the $(m_d - m_u)$ factor in this contribution entails its neglect as proportional to a quark mass difference.

5) Final expression (non renormalized) for $\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0)$

According to Eqs. (3.33), (3.34), we have to subtract (3.37) and (3.40) from (3.31); we get:

$$\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0) = (m_u - m_d)\langle \bar{u}u - \bar{d}d\rangle$$

$$+ e^2\int \frac{d^4q}{(2\pi)^4}\left\{\left(D^{\gamma}_{\mu\nu}(q) - D^{Z}_{\mu\nu}(q)\right)\left[2\,\overset{(3)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) - \overset{(1+i2)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q)\right]\right.$$

$$+\left(\frac{-c_1^2}{4\sin^2\theta_W}D^{W}_{\mu\nu}(q) + \frac{D^{Z}_{\mu\nu}(q)}{4\sin^2\theta_W\cos^2\theta_W}\right)\left[2\,\overset{(3)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q)\right.$$

$$\left. - \overset{(1+i2)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) + 2\,\overset{(3)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q) - \overset{(1+i2)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q)\right]$$

$$- \frac{s_1^2}{8\sin^2\theta_W}D^{W}_{\mu\nu}(q)\left[\overset{(9+i10)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q) - \overset{(11+i12)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q)\right.$$

$$+ \overset{(9+i10)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) - \overset{(11+i12)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) + \overset{(6-i7)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q)$$

$$\left.\left. - \overset{(4+i5)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q) + \overset{(6-i7)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) - \overset{(4+i5)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q)\right]\right\}$$

$$+ 2i\sqrt{2}\,G_F(m_u + m_d)^2\int \frac{d^4q}{(2\pi)^4}\left\{D^{\phi^3}(q)\left[\frac{D^{S^{(0)}}(q)}{2} + \frac{D^{S^{(8)}}(q)}{3}\right.\right.$$

$$\left. + \frac{D^{S^{(15)}}(q)}{6} - D^{P^{(3)}}(q)\right]$$

$$\left. - c_1^2 D^{\phi^{\pm}}(q)\left[\frac{D^{S^{(0)}}(q)}{2} + \frac{D^{S^{(8)}}(q)}{3} + \frac{D^{S^{(15)}}(q)}{6}\right]\right\}. \qquad (3.42)$$

In the approximation of neglecting quark mass differences, we consistently take:

$$2\,\overset{(3)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) = \overset{(1+i2)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q)\,, \qquad \overset{(4+i5)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) = \overset{(6-i7)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q)\,,$$

$$\overset{(9+i10)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) = \overset{(11+i12)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q)\,,$$

(and the same in the axial cases), (3.43)

so that the W contribution and part of the Z drop out, and we recover the formula announced in (3.1).

6) Discussion and comments

a) From (3.1), one sees that the "tadpole" term, and consequently the influence of the up-down quark mass difference, is strongly suppressed in the pion case by the factor $(\langle uu\rangle - \langle dd\rangle)$. Indeed, the SU(2) symmetry breaking at the level of the vacuum quark condensates is not expected to exceed 1 or 2%. [24]

b) The γ-Z contribution is proportional to e^2, while the Higgs is proportional to $G_F\,(m_u + m_d)^2$. One so expects a strong suppression of the last contribution with respect to the first, and the non-compensation of the gauge dependence arising from the γ and Z propagators.

c) At this point, one must remark the difference between the computation we are doing and that of the radiative corrections to the pole of a fermion propagator as performed in ref. [25]. In this last work, the Higgs and ghost fields play a fundamental role in the cancellation of the gauge dependence of the gauge fields; the tadpole diagrams (do not confuse with the abusive expression of "tadpole" terms that we have been using before), were also shown to be essential. Actually, expanding (3.30) at next order of the electroweak interactions does indeed generate tadpole diagrams involving fermions, Higgs, gauge and ghosts fields, as shown in Fig. 6. However, due to the always appearing combination $(\underbrace{\bar{u}u} - \underbrace{\bar{d}d})$ or $(\underbrace{\bar{u}\gamma_5 u} - \underbrace{\bar{d}\gamma_5 d})$ on one side of the diagrams, we expect them to be suppressed by at least one power of the quark mass difference (many of them vanish also by Furry's theorem).

Now, apart in the above tadpole diagrams, ghosts fields can only appear at higher orders of electroweak interactions by a loop insertion in the gauge field propagators of Fig. 2.

d) Forgetting for a while the Higgs contribution, expected to be strongly suppressed (it is obviously divergent, but this problem will be tackled in the next section), let us concentrate on the γ-Z contribution.

With the choice of gauge fixing terms (3.14), (3.15), the photon and Z boson propagators read:

$$D^{\gamma}_{\mu\nu}(q) = -\frac{i}{q^2}\left[\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + a_\gamma \frac{q_\mu q_\nu}{q^2}\right], \qquad (3.44)$$

$$D^{Z}_{\mu\nu}(q) = -i\left[\frac{g_{\mu\nu} - \dfrac{q_\mu q_\nu}{q^2}}{q^2 - M_Z^2} + a_Z \frac{\dfrac{q_\mu q_\nu}{q^2}}{q^2 - a_Z M_Z^2}\right]. \qquad (3.45)$$

Using furthermore the decomposition (3.25) of the $\Pi_{\mu\nu}$'s, we obtain:

$$\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0)\left(\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0)\right)_{\psi,Z}$$

$$= -ie^2\int \frac{d^4q}{(2\pi)^4}\left\{(-3)\left(\frac{1}{q^2} - \frac{1}{q^2 - M_Z^2}\right)\left(2\,\overset{(3)}{\underset{VV}{\tilde{\Pi}}}{}^{1}(q^2) - \overset{(1+i2)}{\underset{AA}{\tilde{\Pi}}}{}^{1}(q^2)\right)\right.$$

$$\left. + \left(\frac{a_\gamma}{q^2} - \frac{a_Z}{q^2 - a_Z M_Z^2}\right)\left(2\,\overset{(3)}{\underset{VV}{\tilde{\Pi}}}{}^{0}(q^2) - \overset{(1+i2)}{\underset{AA}{\tilde{\Pi}}}{}^{0}(q^2)\right)\right\}, \qquad (3.46)$$

where we can also use Eqs. (3.43) to only make appear the combination (3) of the $\tilde{\Pi}$'s.

The procedure used in Ref. [12a] is the saturation of the $\tilde{\Pi}$'s by the low lying resonnances and pole. In the narrow width approximation:

$$\underset{VV}{\overset{(3)}{\tilde{\Pi}}}{}^{1}(q^2) = \frac{M_\rho^4}{g_\rho^2}\ \frac{1}{-q^2 + M_\rho^2}\ ,$$

$$\underset{AA}{\overset{(3)}{\tilde{\Pi}}}{}^{1}(q^2) = \frac{M_{A_1}^4}{g_{A_1}^2}\ \frac{1}{-q^2 + M_{A_1}^2}\ , \tag{3.47}$$

$$\underset{AA}{\overset{(3)}{\tilde{\Pi}}}{}^{0}(q^2) = \frac{f_\pi^2 m_\pi^2}{-q^2 + m_\pi^2}\ .$$

Using CVC to set $\underset{VV}{\overset{(3)}{\tilde{\Pi}}}{}^{0} = 0$, and taking $a_\gamma = a_Z = a$ for simplification, we obtain immediately:

$$\left[\tilde{\psi}^{(1+i2)}(0) - 2\tilde{\psi}^{(3)}(0)\right]_{\gamma,Z} = 2ie^2\int \frac{d^4q}{(2\pi)^4}\left\{\frac{3M_Z^2}{q^2(q^2 - M_Z^2)}\left(\frac{M_\rho^4}{g_\rho^2}\ \frac{1}{q^2 - M_\rho^2} - \frac{M_{A_1}^4}{g_{A_1}^2}\ \frac{1}{q^2 - M_{A_1}^2}\right) + \frac{a^2}{q^2(q^2 - aM_Z^2)}\ \frac{f_\pi^2 m_\pi^2}{q^2 - m_\pi^2}\right\}, \tag{3.48}$$

which gives the finite result of ref. [12a] up to a gauge dependent correction proportional to m_π^2.

However, in QCD, one has the relations [17]:

$$2\underset{VV}{\overset{(3)}{\tilde{\Pi}}}{}^{1}(q^2) - \underset{AA}{\overset{(1+i2)}{\tilde{\Pi}}}{}^{1}(q^2) \sim q^2\ \frac{1}{\pi}\int_{(m_u+m_d)^2}^{\infty} dt\ \frac{1}{t - q^2}\ \frac{3}{12\pi}\left(\frac{3(m_u + m_d)^2}{2t} + O\left(\frac{m^4}{t^2}\right)\right), \tag{3.49a}$$

$$2\underset{VV}{\overset{(3)}{\tilde{\Pi}}}{}^{0}(q^2) - \underset{AA}{\overset{(1+i2)}{\tilde{\Pi}}}(q^2) \sim q^2\ \frac{1}{\pi}\int_{(m_u+m_d)^2}^{\infty} dt\ \frac{1}{t - q^2}\left(-\frac{3}{8\pi}\ \frac{(m_u + m_d)^2}{t} + O\left(\frac{m^4}{t^2}\right)\right). \tag{3.49b}$$

When plugged into Eq. (3.46), one sees immediately that they give rise to logarithmically divergent integrals proportional to $(m_d + m_u)^2$. The result (3.48) needs consequently some more work to be justified.

Before entering the process of the removal of the divergences, let us note that, looking only at the photon sector and taking the Yennie gauge $a = 3$ does result in the suppression of divergences. This makes the link with previous works [1,12a]. However as lengthy discussed before, this is no longer justified when the weak interactions are turned on.

It must also be stressed that, as well known, all the divergences disappear at the chiral limit.

IV. About Infinities. Final Result

We shall follow the line of approach described in the second section, and show that it is possible to obtain a finite result for the pion mass splitting before working at the chiral limit. This chapter will however provide only a conceptual improvement since, at the end, the chiral limit will be needed in order to achieve gauge independence, condition expected from the beginning due to our "off mass shell" computation, and the result (3.48) recovered.

As emphasized previously, we shall show that the contribution to the mass splitting originating from

$$S_2(x) = \frac{i}{2} \int d^4y \; T \; L_{int}(x) L_{int}(y) \tag{4.1}$$

is identical to that obtained from the bare Lagrangian; the corresponding infinities, equivalent to those computed from the counterterms of the $SU(3)_c \times SU(2)_L \times U(1)$ theory, may be subsequently simply subtracted from the bare result.

The divergences of the hadronic currents receive contributions from (4.1), that we shall denote C, such that Eq. (2.2) must be changed into:

$$\partial_\mu A^\mu = M + G + H + C , \tag{4.2}$$

and the expression of the covariant divergence Eq. (2.3) into:

$$D_\mu A^\mu = M + C . \tag{4.3}$$

Accordingly, in Eq. (2.5), each M term has to be replaced by $M + C$. All the C terms are second order in the electroweak interactions; as this is the order we are working up to, all the crossed terms in $C \times (G + H)$ in the computation of $\tilde{\psi}$ can be dropped, and only the additional contributions to $\psi(0)$ from (4.1) have to be computed. Let us denote them $\psi_c^{(1+i2)}(0)$ and $\psi_c^{(3)}(0)$.

Here again, all quark mass differences will be dropped.

1) Computation of $\psi_c^{(1+i2)}(0) - 2\psi_c^{(3)}(0)$.

a) The γ - Z sector

For the sake of briefness, we shall omit the W sector. From Eq. (3.20) applied to (4.1), the contributions of (4.1) to the divergences of the hadronic currents can be written:

$$\begin{aligned} C_{\gamma Z}^{(1+i2)}(x) = -e^2 \frac{i}{2} \int d^4y \; & A_\mu(x) \, A^{\mu(1+i2)}(x) A_\nu(y) J^{\nu em}(y) \\ & + A_\mu(x) J^{\mu em}(x) A_\nu(y) A^{\nu(1+i2)}(y) \\ & + \frac{ie^2}{4 \sin^2\theta_W \cos^2\theta_W} \int d^4y \Big\{ \Big[(V_\mu^{(1+i2)}(x) Z^\mu(x) \\ & - (1 - 2\sin^2\theta_W) \, A_\mu^{(1+i2)}(x) Z^\mu(x) \Big] J_\nu^n(y) Z^\nu(y) \\ & + J_\mu^n(x) Z^\mu(x) \Big[V_\nu^{(1+i2)}(y) Z^\nu(y) \\ & - (1 - 2\sin^2\theta_W) A_\nu^{(1+i2)}(y) Z^\nu(y) \Big] \Big\} , \end{aligned} \tag{4.4}$$

$$C^{(3)}_{\gamma Z}(x) = 0,$$

where the A's without isospin index stand for the photon field, and J^n_μ is the neutral weak current. The computation of the commutators involved in the Ward Identity (3.26) is somewhat lengthy but straightforward: it involves only standard Current Algebra commutations relations. We get:

$$\psi^{c(1+i2)}_{\gamma Z}(0) - 2\psi^{c(3)}_{\gamma Z}(0) = e^2 \int \frac{d^4q}{(2\pi)^4}$$

$$D^{\gamma}_{\mu\nu}(q)\left[2 \underset{VV}{\overset{(3)}{\Pi}}{}^{\mu\nu}(q) - \underset{AA}{\overset{(1+i2)}{\Pi}}{}^{\mu\nu}(q)\right]$$

$$+ \frac{1}{4\sin^2\theta_W \cos^2\theta} D^Z_{\mu\nu}(q)\left\{2 \underset{AA}{\overset{(3)}{\Pi}}{}^{\mu\nu}(q)\right.$$

$$\left.\left. - \underset{VV}{\overset{(1+i2)}{\Pi}}{}^{\mu\nu}(q) - (1 - 2\sin^2\theta_W)^2\left(2 \underset{VV}{\overset{(3)}{\Pi}}{}^{\mu\nu}(q) - \underset{AA}{\overset{(1+i2)}{\Pi}}{}^{\mu\nu}(q)\right)\right]\right\}. \tag{4.5}$$

Using the relation (3.33), (4.5) reduces to

$$\psi^{c(1+i2)}_{\gamma Z}(0) - 2\psi^{c(3)}_{\gamma Z}(0) = e^2 \int \frac{d^4q}{(2\pi)^4}\left(D^{\gamma}_{\mu\nu}(q) - D^Z_{\mu\nu}(q)\right)$$

$$\left[2 \underset{VV}{\overset{(3)}{\Pi}}{}^{\mu\nu}(q) - \underset{AA}{\overset{(1+i2)}{\Pi}}{}^{\mu\nu}(q)\right], \tag{4.6}$$

which is exactly the same as the $\gamma - Z$ contribution in (3.1).

b) The Higgs sector

By the same procedure, one finds:

$$C^{(1+i2)}_H(x) = 0$$

$$C^{(3)}_H(x) = \frac{i}{2}\sqrt{2}\, G_F\, i\int d^4y[(m_u \bar{u}u(x) + m_d \bar{d}d(x))\phi^3(x)(m_u\, u\gamma_5 u(y)$$

$$- m_d\, \bar{d}\gamma_5 d(y))\phi^3(y) + (m_u\, \bar{u}\gamma_5 u(x)$$

$$- m_d\, \bar{d}\gamma_5 d(x))\phi^3(x)(m_u\, \bar{u}u(y) + m_d\, \bar{d}d(y))\phi^3(y)], \tag{4.7}$$

and

$$\psi^{c(1+i2)}_H(0) - 2\psi^{c(3)}_H(0) = 2i\sqrt{2}\, G_F(m_u + m_d)^2 \int \frac{d^4q}{(2\pi)^4}$$

$$\left\{D^{\phi^3}(q)(D^S(q) - D^{P^{(3)}}(q)) - c_1^2 D^{\phi^\pm}(q) D^S(q)\right\}, \tag{4.8}$$

which is again the same as the Higgs contribution in (3.1). (Recall that D^S means $\frac{D^{S(0)}}{2} + \frac{D^{S(8)}}{3} + \frac{D^{S(15)}}{6}$.)

We come back at this step to the remark made in Section 2 about the presence or not of the Higgs term H together with the mass term M in the covariant derivative of the current (2.3): the choice (2.3) entails the existence of ϕ_H computed in Section III.4b, so that it gives a finite result when the infinite part of (4.7) is subtracted. In the other choice, as ϕ_H would not have been present, we would have been left with the extra contribution (4.8) alone.

2) <u>Final finite expression for</u> $2f_\pi^2(m_{\pi^+}^2 - m_{\pi^0}^2)$

From now onwards, as we have shown its finiteness, we shall forget the Higgs contribution, much smaller than the gauge bosons one. We are now left with the practical question of computing the finite part of:

$$2f_\pi^2(m_{\pi^+}^2 - m_{\pi^0}^2) = (m_u - m_d)\langle\bar{u}u - \bar{d}d\rangle$$
$$+ e^2\int\frac{d^4q}{(2\pi)^4}\left[D_{\mu\nu}^{\gamma}(q) - D_{\mu\nu}^{Z}(q)\right]\left[2\,\overset{(3)}{\underset{VV}{\Pi}}{}^{\mu\nu}(q) - \overset{(1+i2)}{\underset{AA}{\Pi}}{}^{\mu\nu}(q)\right] . \quad (4.9)$$

The usual way of renormalizing Feynman diagrams is not implementable here: indeed, in addition to the fact that we are dealing with vacuum fluctuations, which is not a standard case, we have, in order to obtain a realistic picture, to resume the strong interactions at all orders, introducing Vector Meson and Pole dominance, and thus to depart from a description in terms of the fundamental fields of the theory alone.

Nevertheless, the $\tilde{\Pi}$'s may be physically described as sums of two types of functions:

- at low q^2, they are well described by Vector Meson and pole dominance, giving functions $\tilde{\Pi}^1(q^2)$ and $\tilde{\Pi}^0(q^2)$ rapidly damped in q^2 (like $1/q^2$ in the narrow width approximation see (3.47)).

- we must add to them, at high q^2, functions driven by

the asymptotic behaviour of QCD. Only those last ones generate infinities, and, as we have seen in (3.49), they are proportional to $(m_u + m_d)^2$.

We propose as a method to extract the finite part of (4.9) simply forgetting those second contributions. From what just preceeds, we expect our procedure to differ with respect to a more orthodox renormalization procedure by the order of $(m_u + m_d)^2/M^2$, where M is a typical hadronic mass involved (that of the ρ or the A_1).

This leaves (3.46) as the final result, where the $\tilde{\Pi}$'s are saturated by the pion, ρ and A_1 only. Then, as already discussed at length, the residual gauge dependence proportional to m_π^2 forces us to evaluate this electroweak contribution only at the chiral limit, and trust our result only up to an accuracy of $(m_u + m_d)^2/M^2$.

Performing explicitly the integrations, we obtain finally:

$$f_\pi^2(m_{\pi^+}^2 - m_{\pi^0}^2) = \frac{1}{2}(m_u - m_d)\langle\bar{u}u - \bar{d}d\rangle$$
$$+ 3\alpha\left[\frac{m_{A_1}^4}{g_{A_1}^2}\frac{m_Z^2}{m_Z^2 - m_{A_1}^2}\,\ell n\frac{m_{A_1}^2}{m_Z^2} - \frac{m_\rho^4}{g_\rho^2}\frac{m_Z^2}{m_Z^2 - m_\rho^2}\,\ell n\frac{m_\rho^2}{m_Z^2}\right]_{\text{chiral limit}} \quad (4.10)$$

which is (but for the tadpole term), the result of Ref. [12a]. Using the relations [8]:

$$\frac{m_\rho^2}{g_\rho^2} - \frac{m_{A_1}^2}{g_{A_1}^2} = f_\pi^2 ,$$

$$\frac{m_\rho^4}{g_\rho^2} - \frac{m_{A_1}^4}{g_{A_1}^2} = 0, \tag{4.11}$$

given by the 1st and 2nd Weinberg sum rules,(4.10) reduces to

$$f_\pi^2(m_{\pi^+}^2 - m_{\pi^0}^2) = \frac{1}{2}(m_u - m_d)\langle \bar{u}u - \bar{d}d\rangle$$

$$+ 3\alpha\left[\frac{m_\rho^4}{g_\rho^2}\,\ell n\,\frac{m_\rho^2/g_\rho^2}{m_\rho^2/g_\rho^2 - f_\pi^2} + O\left(\frac{m_\rho^2}{m_Z^2}\,\ell n\,\frac{m_Z^2}{m^2}\right)\right] . \tag{4.12}$$

Using the present experimental value $\frac{g_\rho^2}{4\pi} \approx 2.36$, we get:

$$m_{\pi^+} - m_{\pi^0} = \frac{1}{4f_\pi^2 m_\pi}(m_u - m_d)\langle \bar{u}u - \bar{d}d\rangle + 4.95\text{ MeV}, \tag{4.13}$$

where, as already mentioned, the tadpole term gives a negligible contribution. (The results 4.12 and 4.13 are slightly different from those of ref. [1] because we didn't use the relation [26] $m_\rho^2/g_\pi^2 = 2f_\pi^2$ employed therein.)

V. The K^+K^0 Mass Difference

The treatment of the K^+K^0 case can be done in a way exactly similar to $\pi^+\pi^0$. In order not to uselessly lengthen this paper, we shall not go here explicitly through all the steps exposed in the previous sections: we shall in particular forget the Higgs sector and directly compute the finite electroweak contribution arising from the low energy behaviour of the hadronic 2-point functions. And, again, but for the expressions of the divergences of the hadronic currents that we give in full generality, we shall drop all dependence in the quark mass differences.

1) The divergence of the hadronic currents $A_\mu^{(4+i5)}$ and $A_\mu^{(6+i7)}$

The hadronic currents having the quantum numbers os the K^- and K^0 are respectively:

$$A_\mu^{(4+i5)}(x) = \bar{u}(x)\gamma_\mu\gamma^5 s(x),$$

$$A_\mu^{(6+i7)}(x) = \bar{d}(x)\gamma_\mu\gamma^5 s(x). \tag{5.1}$$

Using the Lagrangian displayed in Section III.1, we obtain through (3.21);

$$\partial_\mu A^{\mu(4+i5)} = i(m_u + m_s)\bar{u}\gamma_5 s$$

$$-ie\left[\bar{u}\not{A}\gamma_5 s + \frac{1}{\sin\theta_W \cos\theta_W}\bar{u}\not{Z}\gamma_5\frac{1-\gamma_5}{2}s\right.$$

$$- \frac{\sin^2\theta_W}{\cos\theta_W \sin\theta_W} \bar{u} \not{Z} \gamma_5 s$$

$$- \frac{1}{\sqrt{2}\sin\theta_W} \left\{ s_1 \left(\bar{u} \not{W}^- \gamma_5 \frac{1-\gamma_5}{2} u - \bar{s} \not{W}^- \gamma_5 \frac{1-\gamma_5}{2} s \right) \right.$$

$$\left. - c_1 \left(\bar{d} \not{W}^- \gamma_5 \frac{1-\gamma_5}{2} s - \bar{u} \not{W}^- \gamma_5 \frac{1-\gamma_5}{2} c \right) \right\} \tag{5.2}$$

$$\partial_\mu A^{\mu(6+i7)} = i(m_d + m_s) \bar{d} \gamma_5 s$$

$$- \frac{ie}{\sqrt{2}\sin\theta_W} \left[c_1 \left(\bar{u} \not{W}^+ \gamma_5 \frac{1-\gamma_5}{2} s - \bar{d} \not{W}^- \gamma_5 \frac{1-\gamma_5}{2} c \right) \right.$$

$$\left. - s_1 \left(\bar{c} \not{W}^+ \gamma_5 \frac{1-\gamma_5}{2} s + \bar{d} \not{W}^- \gamma_5 \frac{1-\gamma_5}{2} u \right) \right] . \tag{5.3}$$

2) <u>Computation of</u> $M^2_{K^+} - M^2_{K^0}$

With the same notations as in the previous sections:

$$\tilde{\psi}^{(i)}(0) = \psi^{(i)} - \phi^{(i)}(0). \tag{5.4}$$

We find:

$$2f_K^2(m^2_{K^+} - m^2_{K^0}) = \tilde{\psi}^{(4+i5)}(0) - \tilde{\psi}^{(6+i7)}(0)$$

$$= (m_d - m_u)\langle \bar{u}u + \bar{s}s \rangle$$

$$+ e^2 \int \frac{d^4q}{(2\pi)^4} \left[D^\gamma_{\mu\nu}(q) - D^Z_{\mu\nu}(q) \right] \left[\overset{(3)}{\Pi}{}^{\mu\nu}_{VV}(q) \right.$$

$$\left. + \overset{(8)}{\Pi}{}^{\mu\nu}_{VV}(q) - \overset{(4+i5)}{\Pi}{}^{\mu\nu}_{AA}(q) \right] , \tag{5.5}$$

where, again, the contribution from the W gauge boson has disappeared. In the same way as before, divergences arise because of the asymptotic behaviour of the $\Pi^{\mu\nu}$'s; indeed, in QCD, one has [17]:

$$\overset{(3)}{\tilde{\Pi}}{}^1_{VV}(q^2) + \overset{(8)}{\tilde{\Pi}}{}^1_{VV}(q^2) - \overset{(4+i5)}{\tilde{\Pi}}{}^1_{AA}(q^2)$$

$$\sim q^2 \frac{1}{\pi} \int_{t_0}^\infty dt \, \frac{1}{t - q^2} \, \frac{3}{12\pi} \left(\frac{3(m_u + m_s)^2}{2t} + O\left(\frac{m^4}{t^2}\right) \right)$$

$$\overset{(3)}{\tilde{\Pi}}{}^0_{VV}(q^2) \; \overset{(8)}{\tilde{\Pi}}{}^0_{VV}(q^2) - \overset{(4+i5)}{\tilde{\Pi}}{}^0_{AA}(q^2)$$

$$\sim q^2 \frac{1}{\pi} \int_{t_0}^\infty dt \, \frac{1}{t - q^2} \left(- \frac{3}{8\pi} \frac{(m_u + m_s)^2}{t} + O\left(\frac{m^4}{t^2}\right) \right) . \tag{5.6}$$

Consequently, the finite electroweak contribution will be computed at the chiral limit, which makes it equal to the $\pi^+\pi^0$ case. Here, however, the expected uncertainty is of order $\frac{M_K^2}{M^2}$, which means that we have little control on the accuracy of our result. Dashen's theorem [18], stating that

$$(m^2_{\pi^+} - m^2_{\pi^0})_{EW} = (m^2_{K^+} - m^2_{K^0})_{EW} \tag{5.7}$$

emerges naturally in the framework exposed here, but with an uncertainty that can be of order 25%. From (5.5), we see that the essential difference with respect to the pion case is the "tadpole" term:

$$(m^2_{K^+} - m^2_{K^0})_{tad} = \frac{1}{2f^2_K}(m_d - m_u)\langle \bar{u}u + \bar{s}s\rangle. \qquad (5.8)$$

There is no longer suppression of the influence of the up-down quark mass difference, and, as the quark condensates are known to be negative, this term can reverse the sign of the K^+K^0 mass splitting with respect to the pion's. The expression (5.8), together with the acceptance of Dashen's theorem, is at the root of the absolute lower bounds on the up-down quark mass difference that the authors of ref. [6] obtained by QCD sum rules techniques.

V. Conclusion and Outlook

We have presented in this work an explicit computation of the $\pi^+\pi^0$ and K^+K^0 mass splittings in the standard $SU(3)_c \times SU(2)_L \times U(1)$ theory of quarks. The problems of infinities and gauge invariance have been investigated in detail; while one has been able to get rid of infinities, the goal of performing a completely gauge invariant computation for massive pions and kaons has been only partially achieved.

Some improvements can also certainly be made by working outside the narrow width approximation and introducing higher resonnances in the process of resummation of the strong interactions; this is however expected to lead only to small numerical modifications and not to alter the basic properties of this approach. The above computations are one of the few examples where hadronic properties can be explicitly calculated from a quantum field theory of subconstituents. The most ambitious attempt in this direction is certainly, up to now, the QCD sum rules approach of Shifman, Vainshtein and Zakharov [26]. However, in this approach, an additional scale always subsist, whose choice still has some degree of arbitrariness. A recent attempt to deal with the $\pi^+\pi^0$ mass splitting by QCD sum rules [27] underwent, among others, the problem of that choice. We have here got free of that restriction.

Our calculations are also a precise example where the three types of interactions known at the level of elementary particles cannot be disassociated: we deal with hadronic currents extracted from the QCD Lagrangian, but the electroweak corrections to their divergences are the essential ingredients in the computations of the mass splittings; weak bosons are intimately linked with the photon, couple with the same strength and play a crucial role to ensure convergence; strong interactions resummation makes the link with hadronic physics.

The vacuum fluctuation diagrams that we have shown to be relevant here are now known to play an important role in chiral symmetry breaking and mass generation [28]; within a less formal approach, we have recovered the possibility that an electroweak mass can be generated by vacuum fluctuations involving massless constituents.

Several extensions of this work come to mind: we think that the computations of the mass splittings of heavier mesons and that of proton-neutron must present many analogies with those presented above; the difficulty to circumvent is then the uncertainty in using, or the necessity of replacing, the PCAC relations: the D, F..... mesons can hardly be thought of as Goldstone particles, and the association of the proton or the neutron with some interpolating hadronic current is questionable. But it is not impossible that other tools can be used [29]. It is also attractive to consider the possibility that all electroweak mass splittings are controlled by the above type of diagrams, and this may be the point of unification.

°The next extension which one may think of is a hypothetical strong interaction contribution to the masses of the pions

(kaons...)(that one knows from PCAC to be proportional to the square root of the quark masses). However, computing the above diagram replacing the electroweak bosons by a gluon is restricted by the strong assumption of the "most attractive channel", stating that the one gluon exchange is a trustable approximation.

°The last and most speculative extension is certainly asking the question of the origin of the quark masses and their splittings: do vacuum fluctuations of subconstituents play a similar role, eventually at the price of introducing some new type of interactions [30], in particular can the up-down mass splitting be understood by analogy with the present work? We hope that the next future will bring a deeper understanding of those most fascinating and challenging questions of today's physics.

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Footnotes

1. An attempt in that direction for the proton-neutron case has been made in Ref. [31a], and for pions in Refs. [31b].
2. Those equations were first written by R. Coquereaux. We are very indebted to him for having communicated them to us.
3. The notation $[\ \ ]_+$ means "anticommutator" throughout the paper.

References

1. T. Das, G. S. Guralnik, V. S. Mathur, F.E. Low and J.E. Young, Phys. Rev. Lett. 18 (1967) 759.
2. a. Riazuddin, Phys. Rev. 114 (1959) 1184.
   b. S. K. Bose and R.E. Marshak, Nuov. Cim. 25 (1962) 529.
   c. V. Barger and E. Kazes, Nuov. Cim. 28 (1963) 385.
3. "Relations de dispersions et particules elementaires", Les Houches summer school (1960), C. de Witt & R. Omnes eds. (Paris: Hermann, New York: Wiley (1960)).
4. a. S. Coleman and S.L. Glashow, Phys. Rev. 134 (1964) B671.
   b. R. H. Socolow, Phys. Rev. 137 (1965) B1221.
5. M. Gell Mann and Y. Neeman, "The Eightfold Way" (N.A. Benjamin Inc., New York-Amsterdam (1964)).
6. a. J. Gasser and H. Leutwyler, Physics Reports 87C (1982) 79 (Many informations about electromagnetic mass splittings can be found therein.)
   b. S. Narison, N. Paver, E. de Rafael and D. Treleani, Nucl. Phys. B112 (1983) 365.
7. a. M. Gell Mann, Physics 1 (1964) 63.
   b. S. L. Adler and R. F. Dashen, "Current algebra and its applications to particle physics" (Benjamin, New York 1968).
   c. V. de Alfaro, S. Fubini, G. Furlan and C. Rossetti "Currents in Hadron Physics" (North Holland, Amsterdam, 1973).
8. S. Weinberg, Phys. Rev. Lett. 18 (1967) 507.
9. H. M. Fried and D. R. Yennie, Phys. Rev. 112 (1958) 1391.

10. a. S. Weinberg, Phys. Rev. Lett., 19 (1967) 1264.
    b. S. Glashow, Nucl. Phys. 22 (1961) 579.
    c. A. Salam, in "Elementary Particle Physics" (N. Svartholm. Ed. 1968).
11. a. S. Weinberg, Phys. Rev. Lett., 29 (1972) 388.
    b. G. 't Hooft, Nucl. Phys., B35 (1971) 167
12. a. D. A. Dicus and V. S. Mathur, Phys. Rev. D7 (1973) 525.
    b. T. C. Yang, Phys. Rev. D7 (1973) 3130.
13. A. Zee, Physics Reports 3C (1972) 129.
14. C. Preparata, "The physics of the Light Cone" in "Properties of the Fundamental Interactions", 1971 International School of Subnuclear physics, Erice. Zichichi ed. (Editrice Compositori, Bologna) and references therein.
15. a. Fayyazuddin and Riazuddin, Phys. Rev. D8 (1973) 510.
    b. Fayyazuddin and Riazuddin, Phys. Rev. D8 (1973) 1179.
    c. Fayyazuddin and Riazuddin, Phys. Rev. D8 (1973) 4635.
    d. Fayyazuddin and Riazuddin, Nucl. Phys. B74 (1974) 397.
    e. L. Palla and G. Pocsik, Nucl. Phys. B83 (1974) 168.
    f. L. Palla, J. Phys. G: Nucl. Phys., 1 (1974) 606.
16. a. K. Schilcher and M.D. Tran, Phys. Rev. D23 (1981) 258.
    b. N. F. Nasrallah, N. A. Papadopoulos and K. Schilcher, Z. Phys. C 16 (1983) 323.
17. E. G. Floratos, S. Narison and E. de Rafael, Nucl. Phys. B155 (1979) 115.
18. R. Dashen, Phys. Rev. 183 (1969) 1245.

19. R. Shankar, Phys. Rev. D15 (1977) 755.

20. S. Weinberg, "The problem of mass", in Festschrift for I. I. Rabi (N.Y. Academy of Sciences, New York, 1978).

21. R. Coquereaux, (unpublished).

22. M. J. Veltman, "Gauge theories of weak interactions", in the proceedings of the summer institute on particle physics, SLAC Report 239 (1981).

23. See for example: "Table of particle properties", Phys. Lett. 111B (1982).

24. M. A. Shifman, A. I.Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.

25. S. Weinberg, Phys. Rev. D7 (1973) 2887.

26. a. K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16 (1966) 259.
    b. Riazuddin and Fayyazuddin, Phys. Rev. 147 (1966) 1071.

27. R. Coquereaux, B. Machet and E. de Rafael, (unpublished).

28. a. S. Weinberg, Phys. Rev. D13 (1976) 974.
    b. M. E. Peskin, Nucl. Phys. B175 (1980) 197.
    c. J. Preskill, Nucl. Phys. B177 (1981) 21.
    d. S. Dimopoulos, S. Raby and G. L. Kane, Nucl. Phys. B182 (1981) 77.
    e. M. E. Peskin, "Chiral symmetry and chiral symmetry breaking", Lectures at Les Houches summer school 1982, preprint SLAC-Pub 3021 (1982).

29. I. Kimel, Phys. Rev. D27 (1983) 2129.

30. U. Baur and H. Fritzsch Max Plank Institute preprint MPI-PAE/PTh 37/83.

31. a. T. Hagiwara and B. W. Lee, Phys. Rev. D7 (1973) 459.
    b. I. Bars and K. Lane, Phys. Rev. D8 (1973) 1169.
    I. Bars and K. Lane, Phys. Rev. D8 (1973) 1252.

Figure Captions

Fig. 1 A typical diagram which can contribute at $q \neq 0$.

Fig. 2 Type of diagrams we are left with at $q = 0$.

Fig. 3 Resummation at all orders of strong interactions of the diagrams of Fig. 2.

Fig. 4 Type of diagram contributing to ϕ_{HM}.

Fig. 5. Diagrams contributing to the electroweak corrections to the quark vacuum condensates.

Fig. 6 Tadpole type diagrams.

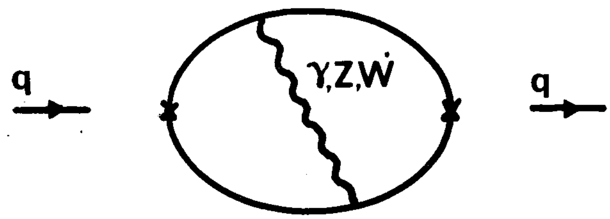

FIGURE 1

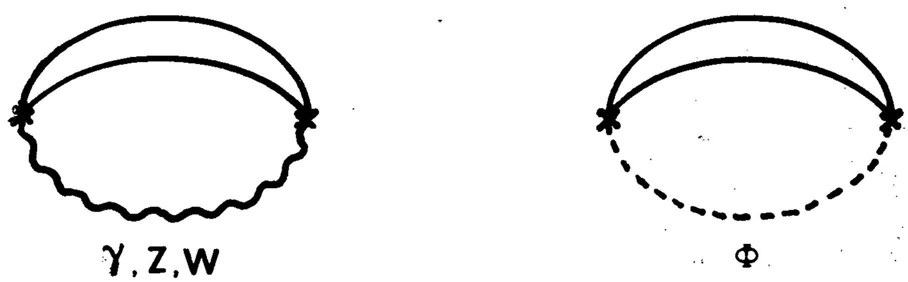

FIGURE 2

FIGURE 3

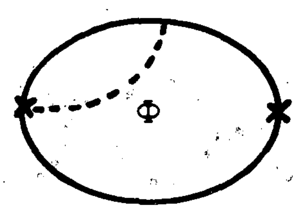

FIGURE 4

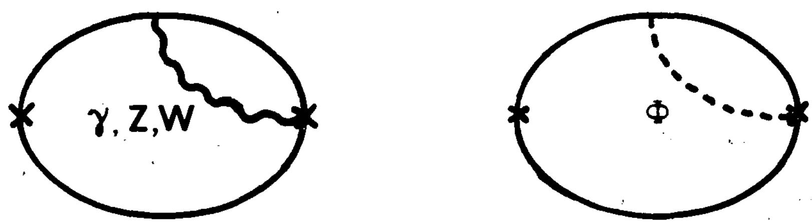

FIGURE 5

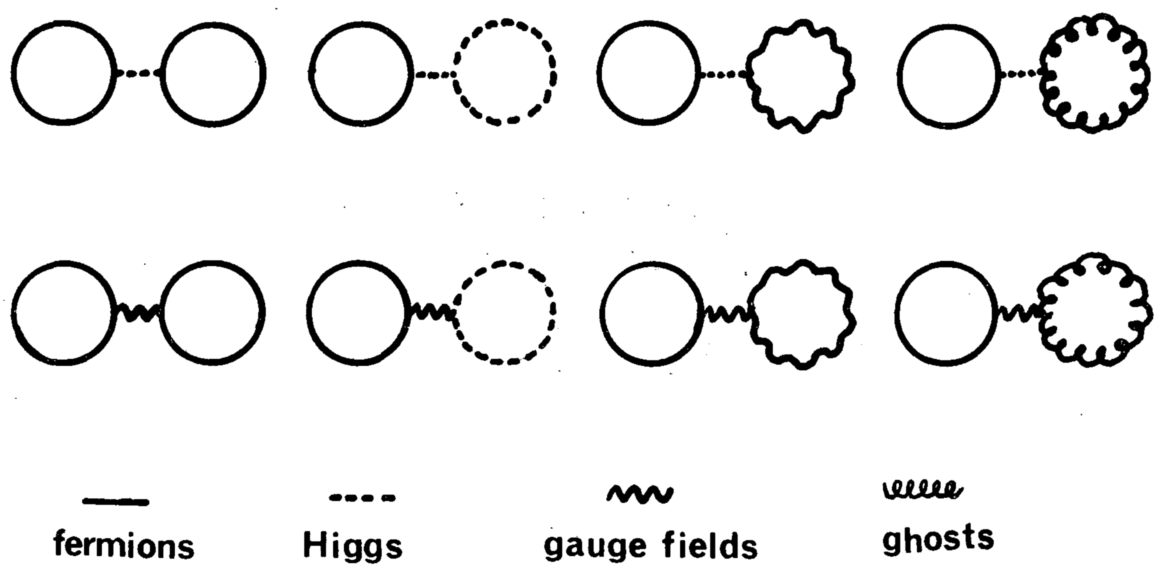

FIGURE 6

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