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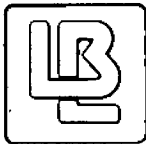
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### Author

Li, L-F.

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Ling-Fong Li and Lincoln Wolfenstein

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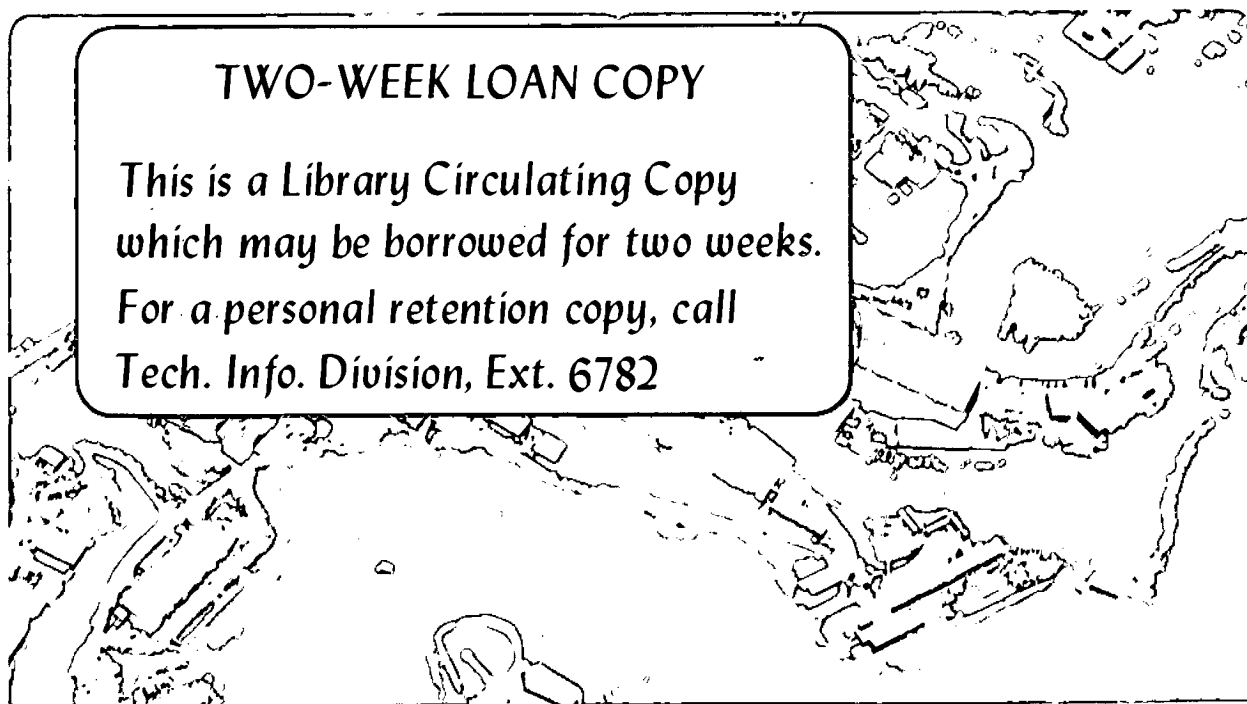
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CURRENT ALGEBRA ANALYSIS OF CP VIOLATIONS IN  $K \rightarrow 3\pi$  DECAY  
IN THE SIX-QUARK WEINBERG-SALAM MODEL\*

Ling-Fong Li  
Physics Department, Carnegie-Mellon University  
Pittsburgh, Pennsylvania 15213 and

Lawrence Berkeley Laboratory<sup>†</sup>  
University of California  
Berkeley, California 94720

and

Lincoln Wolfenstein  
Physics Department, Carnegie-Mellon University  
Pittsburgh, Pennsylvania 15213

Abstract

The standard current algebra technique is used to relate the parameter  $\eta_{+-0} \equiv \langle \pi^+ \pi^- 0 | H_W | K_S \rangle / \langle \pi^+ \pi^- 0 | H_W | K_L \rangle$  in the  $K \rightarrow 3\pi$  decay to the CP violating parameters of  $K \rightarrow 2\pi$  decays in the context of the six-quark Weinberg-Salam theory (Kobayashi-Maskawa model).

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<sup>†</sup>Participating Guest.

The gauge theory of weak and electromagnetic interactions has not only aesthetical appeal but also quite substantial experimental support. Even though the experimental data has not ruled out all the other models, it is rather impressive that the simplest Weinberg-Salam model with 3<sup>1</sup> doublets of quarks and leptons is able to describe most, if not all, of weak interaction data accumulated over the last few years. As pointed out by Kobayashi and Maskawa,<sup>2</sup> one of the nice features of the Weinberg-Salam model with 3 left-handed doublets of quarks is the presence of the CP violation in the gauge coupling to the quarks. So far, most of the predictions on the CP violating processes from this model are hampered by the strong interaction dynamics.<sup>3</sup> Recently, there has been some experimental interest in measuring the CP violations in the  $K \rightarrow 3\pi$  decays.<sup>4</sup> In this paper, the standard current algebra technique is used in relating the CP violations in the  $K \rightarrow 3\pi$  decays to the better known CP violating  $K \rightarrow 2\pi$  decays in the context of the Weinberg-Salam model. From the successes<sup>5</sup> of the current algebra analysis of the usual CP conserving  $K \rightarrow 3\pi$  decays, one can expect that the result from this type of analysis for the CP violating part of the  $K \rightarrow 3\pi$  decays will not depend very much on the details of the strong interaction dynamics.

In the  $K \rightarrow 3\pi$  amplitudes, with the kinematics given by

$$K^i(p) \rightarrow \pi_1^a(k_1) + \pi_2^b(k_2) + \pi_3^c(k_3)$$

we can define the Lorentz invariant energy variables  $s_i$  as

$$s_i = (p - k_i)^2 \quad (1)$$

with  $s_1 + s_2 + s_3 = 3s_0 = 3(m_\pi^2 + \frac{1}{3}m_K^2)$ . The indices  $i$  label the isospin of the kaon and  $a, b, c$  label the isospin of the pions. In the usual approximation that the dependence of the Dalitz plot on the energies  $s_i$  is at most linear, the CP conserving part of the  $K \rightarrow 3\pi$  amplitudes can be parametrized as

$$\langle \pi^+ \pi^+ \pi^- | H_W^+ | K^+ \rangle = i \left[ (2\alpha_1 - \alpha_3) + (\beta_1 - \frac{1}{2}\beta_3 + \sqrt{3}\gamma_3) \left( \frac{s_3 - s_0}{m_\pi} \right) \right] \quad (2a)$$

$$\langle \pi^0 \pi^0 \pi^+ | H_W^+ | K^+ \rangle = i \left[ (-\alpha_1 + \frac{1}{2}\alpha_3) + (\beta_1 - \frac{1}{2}\beta_3 - \sqrt{3}\gamma_3) \left( \frac{s_3 - s_0}{m_\pi} \right) \right] \quad (2b)$$

$$\langle \pi^+ \pi^- \pi^0 | H_W^+ | K_1 \rangle = \frac{-2i\gamma_3}{\sqrt{3}m_\pi} (s_1 - s_2) \quad (2c)$$

$$\langle \pi^+ \pi^- \pi^0 | H_W^+ | K_2 \rangle = i \left[ (\alpha_1 + \alpha_3) - (\beta_1 + \beta_3) \left( \frac{s_3 - s_0}{m_\pi} \right) \right] \quad (2d)$$

$$\langle \pi^0 \pi^0 \pi^0 | H_W^+ | K_2 \rangle = -3i(\alpha_1 + \alpha_3) \quad (2e)$$

where  $\alpha_1, \beta_1, \alpha_3, \beta_3$  come from the transition into the  $I=1$  state of 3 pions caused by  $\Delta I=1/2$  ( $\Delta I=3/2$ ) part of the CP conserving weak Hamiltonian  $H_W^+$  and  $\gamma_3$  comes from the  $I=2$  state of 3 pions. The CP eigenstates of the neutral kaons are defined by

$$|K_1\rangle = \frac{1}{\sqrt{2}} (|K_0\rangle + |\bar{K}_0\rangle) \quad |K_2\rangle = \frac{1}{\sqrt{2}} (|K_0\rangle - |\bar{K}_0\rangle) \quad (3)$$

with

$$|\bar{K}_0\rangle = \text{CP} |K_0\rangle$$

Note that the CP even state  $K_1$  can decay into  $\pi^+ \pi^- \pi^0$  state through the  $\Delta I=3/2$  part of the Hamiltonian without CP violation. Similarly, the CP violating part of the  $K \rightarrow 3\pi$  amplitudes can be parametrized as

$$\langle \pi^+ \pi^+ \pi^- | H_W^- | K^+ \rangle = i^2 \left[ (2\alpha_1' - \alpha_3') + (\beta_1' - \frac{1}{2}\beta_3' + \sqrt{3}\gamma_3') \left( \frac{s_3 - s_0}{m_\pi} \right) \right] \quad (4a)$$

$$\langle \pi^0 \pi^0 \pi^+ | H_W^- | K^+ \rangle = i^2 \left[ (-\alpha_1' + \frac{1}{2}\alpha_3') + (\beta_1' - \frac{1}{2}\beta_3' - \sqrt{3}\gamma_3') \left( \frac{s_3 - s_0}{m_\pi} \right) \right] \quad (4b)$$

$$\langle \pi^+ \pi^- \pi^0 | H_W^- | K_1 \rangle = i^2 \left[ (\alpha_1' + \alpha_3') - (\beta_1' + \beta_3') \left( \frac{s_3 - s_0}{m_\pi} \right) \right] \quad (4c)$$

$$\langle \pi^+ \pi^- \pi^0 | H_W^- | K_2 \rangle = i^2 \left[ \frac{-2\gamma_3'}{\sqrt{3}m_\pi} \right] (s_1 - s_2) \quad (4d)$$

$$\langle \pi^0 \pi^0 \pi^0 | H_W^- | K_1 \rangle = 3(\alpha_1' + \alpha_3') \quad (4e)$$

where  $H_W^-$  is the CP violating part of the weak Hamiltonian  $H_W$  and  $\alpha_i', \beta_i', \gamma_i'$  are defined in a similar way as  $\alpha_i, \beta_i, \gamma_i$ . The parameters  $\alpha_i', \alpha_3', \text{etc.}$  are real if final state interactions are neglected but in general have complex phases completely determined by CPT and unitarity from the final state strong interaction S matrix. The final  $3\pi$  states can be classified by the isospin and the permutation symmetry of 3 pions. To a good approximation<sup>7</sup> the phase is determined by the isospin and the permutation symmetry of the  $3\pi$  states; thus there is one phase for all the  $\alpha_i$ 's,

the amplitudes of the completely symmetrical I=1  $3\pi$  state, one for all the  $\beta$ 's, the amplitudes of the I=1  $3\pi$  state with mixed symmetry, and one for all the  $\gamma$ 's, the amplitudes of the I=2 state.

The experimental quantity of interest is

$$\eta_{+0}(s_1, s_2, s_3) = \frac{\langle \pi^+ \pi^- | H_W | K_S \rangle}{\langle \pi^+ \pi^- | H_W | K_L \rangle} \quad (5)$$

where

$$|K_S\rangle = (|K_1\rangle + \rho |K_2\rangle) / \sqrt{1 + |\rho|^2} \quad (6a)$$

$$|K_L\rangle = (|K_2\rangle + \rho |K_1\rangle) / \sqrt{1 + |\rho|^2} \quad (6b)$$

Because of the CP-conserving amplitude (2c) the quantity  $\eta_{+0}$  is not in general a CP-violating quantity. We therefore limit ourselves to the case  $s_1 = s_2$  to eliminate the amplitude (2c) so that  $\eta_{+0}$  depends only on  $s_3$ . From Eqs.(5)-(6) we then find

$$\eta_{+0}(\gamma) = \rho + i \frac{(\alpha_1' + \alpha_3') - (\beta_1' + \beta_3')\gamma}{(\alpha_1 + \alpha_3) - (\beta_1 + \beta_3)\gamma} \quad (7)$$

$$\gamma \equiv (s_3 - s_0) / m_\pi^2$$

The isospin decomposition of the  $K \rightarrow 2\pi$  amplitudes are

$$\langle \pi^+ \pi^0 | H_W | K^+ \rangle = \frac{1}{\sqrt{2}} f_3 \quad (8a)$$

$$\langle \pi^+ \pi^- | H_W | K_0 \rangle = \sqrt{\frac{1}{6}} f_1 + \frac{1}{\sqrt{12}} f_3 \quad (8b)$$

$$\langle \pi^0 \pi^0 | H_W | K_0 \rangle = \sqrt{\frac{1}{6}} f_1 + \frac{1}{\sqrt{3}} f_3 \quad (8c)$$

for the CP conserving part and

$$\langle \pi^+ \pi^0 | H_W | K^+ \rangle = \frac{i}{2} \sqrt{\frac{3}{2}} g_3 \quad (9a)$$

$$\langle \pi^+ \pi^- | H_W | K_0 \rangle = i \left( -\frac{1}{\sqrt{6}} g_1 + \frac{1}{\sqrt{12}} g_3 \right) \quad (9b)$$

$$\langle \pi^+ \pi^- | H_W | K_0 \rangle = i \left( \frac{1}{\sqrt{6}} g_1 + \frac{1}{\sqrt{3}} g_3 \right) \quad (9c)$$

for the CP violating part. The parameters  $f_1, g_1$  denote the  $\Delta I=1/2$  transitions and  $f_3, g_3$  denote the  $\Delta I=3/2$  transitions. They are related to the standard notation for the  $K \rightarrow 2\pi$  amplitudes by

$$\begin{aligned} f_1 &= -2 \operatorname{Re} a_0 e^{i\delta_0} & f_3 &= 2 \operatorname{Re} a_2 e^{i\delta_2} \\ g_1 &= -2 \operatorname{Im} a_0 e^{i\delta_0} & g_3 &= 2 \operatorname{Im} a_2 e^{i\delta_2} \end{aligned} \quad (10)$$

where

$$\langle I = n | H_W | K_0 \rangle = a_n e^{i\delta_n} \quad (11)$$

The experimentally measured quantities are

$$\eta_{+-} = \epsilon + \epsilon' / (1 + \omega / \sqrt{2}) \quad (12a)$$

$$\eta_{00} = \epsilon - 2\epsilon' / (1 - \sqrt{2} \omega) \quad (12b)$$

where<sup>9</sup>

$$\epsilon = \rho + i g_1 / f_1 \quad (13a)$$

$$\sqrt{2} \epsilon' = -i \frac{f_3}{f_1} (g_3 / f_3 - g_1 / f_1) \quad (13b)$$

$$\omega = -f_3 / f_1$$

The weak Hamiltonian responsible for  $K \rightarrow 3\pi$  decays in the six-quark Kobayashi-Maskawa model can be written  $H_W = H_W^+ + H_W^-$  with

$$H_W^+ = \frac{G_F}{\sqrt{2}} A \bar{d} \gamma^\lambda (1-\gamma_5) \{ (\bar{u}u - c\bar{c}) + K (c\bar{c} - t\bar{t}) \} \gamma_\lambda (1-\gamma_5) s \quad (14a)$$

$$H_W^- = \frac{G_F}{\sqrt{2}} i A K' \bar{s} \gamma^\lambda (1-\gamma_5) (c\bar{c} - t\bar{t}) \gamma_\lambda (1-\gamma_5) s \quad (14b)$$

where  $A$ ,  $K$ , and  $K'$  involve the three mixing angles and the CP-violating phase  $\delta$ .<sup>10</sup> With the phase convention chosen by Kobayashi and Maskawa,<sup>2</sup> the CP-violating piece  $H_W^-$  satisfies the  $\Delta I=1/2$  rule so that

$$\alpha_3' = \beta_3' = \gamma_3' = g_3 = 0 \quad (15)$$

and only the CP-conserving piece  $H_W^+$  violates the  $\Delta I=1/2$  rule.

Since both  $H_W^+$  and  $H_W^-$  consist purely of left-handed currents, we have for both the usual relation

$$[Q_5^Y, H_{pc}] = - [Q^Y, H_{pv}] \quad (16)$$

where  $H_{pv}$  is the parity-violating piece and  $H_{pc}$  the parity-conserving piece of the Hamiltonian and

$$Q_5^Y = \int d^3x A_0^Y(x)$$

$$Q^Y = \int d^3x V_0^Y(x)$$

The soft-pion limits of the  $K \rightarrow 3\pi$  amplitude can then be written using the usual current algebra technique as

$$\lim_{k_3 \rightarrow 0} \langle \pi_1^\alpha \pi_2^\beta \pi_3^\gamma | H_{pc} | K_i \rangle =$$

$$- \frac{i}{f_\pi} \langle \pi_1^\alpha \pi_2^\beta | [Q_5^Y, H_{pc}] | K_i \rangle =$$

$$\frac{i}{f_\pi} \langle \pi_1^\alpha \pi_2^\beta | [Q^Y, H_{pv}] | K_i \rangle \quad (17)$$

When the isospin operator  $Q^Y$  acts on the states  $|K_i\rangle$  and  $|2\pi\rangle$ , the  $K \rightarrow 3\pi$  amplitudes are related to the  $K \rightarrow 2\pi$  amplitudes in the soft-pion limits. For example,

$$\langle \pi_1^+ \pi_2^- \pi_3^0 | H_{pc} | K_0 \rangle \xrightarrow{k_1 \rightarrow 0} \frac{-i}{f_\pi} \left[ \langle \pi^0 \pi^0 | H_{pv} | K_0 \rangle + \langle \pi^+ \pi^- | H_{pv} | K_0 \rangle \right]$$

$$\xrightarrow{k_2 \rightarrow 0} \frac{i}{f_\pi} \left[ \langle \pi^0 \pi^0 | H_{pv} | K_0 \rangle + \langle \pi^+ \pi^- | H_{pv} | K_0 \rangle \right]$$

$$\xrightarrow{k_3 \rightarrow 0} \frac{i}{f_\pi} \frac{1}{2} \langle \pi^+ \pi^- | H_{pv} | K_0 \rangle - \frac{1}{\sqrt{2}} \langle \pi^+ \pi^0 | H_{pv} | K^+ \rangle \quad (18)$$

The linear dependence on the energies of the  $K \rightarrow 3\pi$  amplitudes given in Eqs. (2) and (4) can then be used to extrapolate from the unphysical soft-pion limits to the physical region. In terms of the parameters previously

defined, we then obtain

$$\frac{\alpha_1}{f_1} = -\frac{1}{f_\pi} \frac{1}{6\sqrt{3}} \quad (19a)$$

$$\frac{\beta_1}{f_1} = \frac{\beta_1'}{g_1} = \frac{1}{f_\pi} \frac{1}{2\sqrt{3}} \left[ \frac{m_\pi^2}{m_k^2 - m_\pi^2} \right] \quad (19c)$$

$$\frac{\alpha_3}{f_3} = \frac{\alpha_3'}{g_3} = -\frac{1}{f_\pi} \frac{1}{3\sqrt{6}} \quad (19b)$$

$$\frac{\beta_3}{f_3} = \frac{\beta_3'}{g_3} = -\frac{1}{f_\pi} \frac{1}{4\sqrt{6}} \left[ \frac{m_\pi^2}{m_k^2 - m_\pi^2} \right] \quad (19d)$$

$$\frac{\gamma_3}{f_3} = \frac{\gamma_3'}{g_3} = \frac{1}{f_\pi} \left[ \frac{9}{8} \sqrt{2} \right] \left[ \frac{m_\pi^2}{m_k^2 - m_\pi^2} \right] \quad (19e)$$

We first apply these results to the center of the Dalitz plot where

$y=0$ . From Eqs.(7) and (15)

$$\eta_{+-0}(0) = \rho + i \alpha_1' / (\alpha_1 + \alpha_3) \quad (20)$$

We now use one of the soft-pion results, the first part of Eq.(19a)

$$\alpha_1' / \alpha_1 = g_1 / f_1 \quad (21)$$

In general, soft-pion results have problems with phases; thus we do not expect from unitarity that  $\alpha_1$  has the same phase as  $f_1$  as implied by Eq.(19a). However, Eq.(21) does not have this problem since both sides are real by unitarity. Using this result together with Eq.(13a) we obtain

$$\eta_{+-0}(0) - \epsilon = i \frac{g_1}{f_1} \frac{\alpha_3}{\alpha_1 + \alpha_3} \quad (22)$$

The result that this difference is purely imaginary has been noted before.<sup>9</sup>

Using Eqs.(12) and (13) to eliminate  $g_1/f_1$  and remembering  $g_3=0$  we find

$$\eta_{+-0} - \eta_{+-} = -(\eta_{+-} - \eta_{00}) \left\{ \frac{1}{3} + \frac{\sqrt{2}}{3} \frac{\alpha_3/\alpha_1}{f_3/f_1} \right\} \quad (23)$$

where we have neglected terms of the order  $\alpha_3/\alpha_1$  and  $f_3/f_1$  compared to unity. Since  $\alpha_3/\alpha_1$  and  $f_3/f_1$  are known approximately from experiment, Eq.(23) may be used directly to estimate  $\eta_{+-0} - \eta_{+-}$ . By combining Eqs.(19a) and (19b) we can obtain a second soft-pion result

$$\alpha_3/\alpha_1 = \sqrt{2} f_3/f_1 \quad (24)$$

Substituting Eq.(24) into Eq.(23) we obtain

$$\eta_{+-0} - \eta_{+-} = \eta_{00} - \eta_{+-} \quad (25)$$

or  $\eta_{+-0} = \eta_{00}$ . Equation (24) unlike Eq.(21) is inconsistent with the unitarity constraints since the left-hand side is real whereas the right-hand side has the phase factor  $e^{i(\delta_3 - \delta_0)}$ . Thus, Eq.(25) also is incorrect as to phase. If we use only the magnitude relation in Eq.(24) we obtain

$$|\eta_{+-0} - \epsilon| = \frac{2}{3} |\eta_{+-} - \eta_{00}| \quad (26)$$

Final-state interaction effects would also be expected to affect the magnitude relation of Eq.(24); both theory and experiment indicate it should be good within a factor of two so that we expect Eq.(26) to be valid within this factor.

In the same way we can estimate  $\eta_{+-0}$  at other values of  $y$ . Using

Eqs.(19) in the form  $\beta'/\beta_1 = \alpha_1'/\alpha_1$  we find in place of Eq.(23)



$$\eta_{+-0} - \eta_{+--} = -(\eta_{+-} - \eta_{00}) \left\{ \frac{1}{3} + \frac{\sqrt{2}}{3} \frac{\alpha_3/\alpha_1}{f_3/f_1} - \frac{\sqrt{2}}{3} \frac{\beta_3/\alpha_1}{f_3/g_1} \gamma \right\} \quad (27)$$

where we have also neglected  $\beta_1/\alpha_1$  and  $\beta_3/\alpha_1$  compared to unity. Again this result is in terms of CP-conserving observables. From Eqs.(19c) and (19d)

$$\frac{\beta_3/\alpha_1}{f_3/f_1} = \frac{15}{4} \sqrt{2} \frac{m_\pi}{m_K} \quad (28)$$

so that Eq.(27) becomes

$$\eta_{+-0} - \eta_{+--} = -(\eta_{+-} - \eta_{00}) \left\{ 1 - \frac{5}{2} \frac{s_3 - s_0}{m_K - m_\pi} \right\} \quad (29)$$

Equation (29) like Eq.(25) is inconsistent with the unitarity constraints. It should, however, give the approximate magnitude of the variation of  $\eta_{+-0}$  over the Dalitz plot.

Recently, it has been suggested<sup>12</sup> that the penguin diagrams coming from the gluon exchange might help to explain the  $\Delta I=1/2$  rule in the non-leptonic decay. If an effective Hamiltonian is defined as involving purely left-handed pieces plus the penguin operator

$$H' = C[\bar{s}\gamma_\mu(1 - \gamma_5)\lambda^\alpha d](\bar{u}\gamma^\mu\alpha u + \bar{d}\gamma^\mu\lambda^\alpha d + \dots) + h.c.$$

where C is some constant and  $\lambda^\alpha$  is the color SU(3) matrix, our results still hold. This is because H' also satisfies the commutation relation of

Eq.(16) because the factor  $(\bar{u}\gamma^\mu\lambda^\alpha u + \bar{d}\gamma^\mu\lambda^\alpha d)$  is an isospin singlet.

Our major result is given as either Eq.(23) or (26). This shows that  $\eta_{+-0} - \eta_{+--}$  (or  $\eta_{+-0} - \epsilon$ ), which vanishes in the superweak model, is expected to be approximately equal in magnitude to  $(\eta_{+-} - \eta_{00})$  in the KM model. Experimentally we know that  $|\eta_{+-} - \eta_{00}|$  is less than .06 times  $|\eta_{+-}|$  and theoretically we expect a small non-zero value of the order of .01 to .05, the exact value depending on the KM parameters and the treatment of penguin terms.<sup>13</sup> Thus, an experimental determination of  $\eta_{+-0} - \eta_{+--}$  to the same accuracy as  $\eta_{+-} - \eta_{00}$  would be of interest. Proposed experiments<sup>14</sup> aim at measuring  $\eta_{00}/\eta_{+-}$  to 1% but the most optimistic proposal to measure  $\eta_{+-0}$  aims at a 25% accuracy. From the point of view of the KM model, this  $\eta_{+-0}$  proposal is therefore not competitive.

The reason for the small value of  $\eta_{+-} - \eta_{00}$  is well-known: as long as the  $\Delta I=1/2$  rule holds at least as well for CP-odd amplitudes as it does for CP-even,  $\epsilon'$  is bound to be small. From a general phenomenological point of view, this approximate  $\Delta I=1/2$  rule does not require that  $|\eta_{+-0} - \epsilon|$  be small compared to  $\epsilon$ . However, for the relatively simple KM Hamiltonian this follows from the soft-pion arguments; for more complicated Hamiltonians this need not be true. In particular, for the  $SU(2)_L \times SU(2)_R$  model of CP violation,<sup>15</sup>  $\eta_{+-} - \eta_{00}$  vanishes but  $|\eta_{+-0} - \epsilon|$  may be of the order of magnitude of  $\epsilon$  itself.

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