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**Berkeley, California**

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ABSTRACT

The possibility of including inelastic effects within a one-channel N/D calculation is discussed, and it is seen that the one-channel calculation can agree with the Bjorken matrix calculation whenever the elastic amplitude has no zero on the physical sheet. An example indicates that this may be the case for  $\pi\pi$  scattering in the  $\rho$  channel.

## I. INTRODUCTION

The N/D procedure for calculating partial-wave scattering amplitudes has been generalized by Bjorken<sup>1</sup> to include the case of several channels. The amplitude is still written  $A = ND^{-1}$ , where  $N$  and  $D$ , as well as  $A$ , are now matrices. If we were interested only in a single elastic amplitude, say  $A_{11}$ , we might try to do a one-channel calculation which would somehow include inelastic effects. Several methods have been proposed to accomplish this;<sup>2</sup> the method I discuss is to try to express  $A_{11} = n/d$ , where  $d$  contains the entire right-hand cut, including inelastic contributions. The dispersion relation for  $d$ ,

$$d(s) = 1 + \frac{1}{\pi} \int ds' \frac{\text{Im } d(s')}{s' - s}$$

becomes

$$d(s) = 1 + \frac{1}{\pi} \int_{s_1}^{\infty} ds' \frac{n(s')}{s' - s} \text{Im} \frac{1}{A_{11}(s')} \quad (1)$$

In this paper I discuss the conditions for which  $a(s) \equiv n(s)/d(s)$  is in fact identical to the Bjorken amplitude  $A_{11}(s)$  calculated by the matrix method.

In what follows I consistently suppress the angular momentum index  $\ell$ . Also, I assume that the Bjorken method, without arbitrary constants, generates the correct amplitude.

In Section II below I discuss general criteria for determining when this one-channel calculation with inelastic effects does reproduce the Bjorken amplitude. In Section III I present a calculation designed to indicate whether the  $(\pi\pi)$  amplitude in the  $\rho$  channel does satisfy these criteria, and so could be calculated by this one-channel method.

II. DETERMINATION OF WHEN A ONE-CHANNEL  
CALCULATION IS APPROPRIATE

Let us define

$$R(s) \equiv \frac{-1}{\rho(s)} \frac{\text{Im } A_{11}(s)}{|A_{11}(s)|^2} = \frac{\sigma_{\text{tot}}}{\sigma_{\text{el}}}$$

If  $s_1$  and  $s_2 > s_1$  are the elastic and inelastic thresholds, respectively,

$$R(s) = 1, \quad s_1 < s < s_2$$

$$R(s) > 1, \quad s_2 < s$$

In terms of  $R$ , Eq. (1) can be written

$$d(s) = 1 - \frac{1}{\pi} \int_{s_1}^{\infty} ds' \frac{n(s') \rho(s') R(s')}{s' - s} \quad (2)$$

For  $R(s) \equiv 1$ , we recover the usual relation for purely elastic scattering.

Now consider the function  $\tilde{d}(s) = n(s)/A_{11}(s)$ . I assume that  $n$  is chosen so that  $\tilde{d} \rightarrow 1$  as  $s \rightarrow \infty$ , as does  $d$ ; this leaves open the possibility that  $\tilde{d}$  may have poles. By assumption,  $\tilde{d}$  has no left-hand cut, and its discontinuity across the right-hand cut is the same as that of  $d$ . So if  $\tilde{d}(s)$  has no pole on the physical sheet,  $d(s) = \tilde{d}(s)$ , and  $a(s) \equiv \left[ \frac{n(s)}{d(s)} \right] = A_{11}(s)$ . If  $\tilde{d}(s)$  does have poles, then  $d(s) \neq \tilde{d}(s)$ ,

and  $a(s) \neq A_{11}(s)$ ;  $n$  will not have superfluous poles, and so we can conclude that  $a(s) \neq A_{11}(s)$  if  $A_{11}$  has a zero on the physical sheet which is not shared by  $n$ , and  $a(s) = A_{11}(s)$  if  $A_{11}$  has no such zero. Whether or not  $A_{11}$  has a zero is of course not determined from a knowledge of  $R$  and  $n$ .

The same ambiguity is present through the arbitrary CDD poles in the N/D method for the purely elastic case and in fact in the Bjorken multichannel prescription. However, in those cases arbitrary poles in the denominator function, while consistent with the dispersion relation, are usually rejected because they would not be dynamically determined. In the case under discussion, however, we must consider the possibility that although  $A_{11}$  is dynamically determined (which is what we assumed when we said that the Bjorken prescription, without arbitrary constants, would give the correct answer),  $d$  might have poles anyway. These would not be true CDD poles, since we assume that they could be calculated by the full matrix method, but they would mean that this single-channel calculation would not give the correct answer unless additional constants were inserted.

That  $a$  and  $A_{11}$  need not be identical has previously been pointed out by Squires;<sup>3</sup> however, the example which he used to show this is not conclusive. Squires studies an elastic amplitude  $A_{11}$  which, in the absence of coupling to other channels, has a zero in the physical region at  $s = \bar{s}$ . He considers turning on a small coupling to a second channel with  $s_2 < \bar{s}$ , and points out that  $A_{11}$  is unlikely to pass through zero above  $s_2$ , since unitarity would then require both the elastic and inelastic amplitudes to be zero simultaneously, which would be accidental.



He then notices that  $n$  will not be changed much (for small coupling) from what it had been for zero coupling, and concludes that  $n$ , and hence  $a = n/d$ , will pass through zero in the physical region near  $s = \bar{s}$ . This would mean that  $a \neq A_{11}$ .

This conclusion by Squires differs from the analysis presented above. It is of course possible that the zero in  $A_{11}$  which was in the physical region in the absence of coupling will move upwards onto the physical sheet, in which case it would be true that  $a \neq A_{11}$ . However, this need not happen, and Squires' arguments, based only on the absence of the zero from the physical region, do not depend on its happening. In fact, by replacing the left-hand cuts by two poles, simple examples can be constructed which satisfy Squires' assumptions and in which the zero does not move onto the physical sheet. In the case where the zero did not move onto the physical sheet, Squires would not be correct in stating that  $a \neq A_{11}$ .

For small coupling,  $A_{11}$  would be very small near  $\bar{s}$ , although it need not go through zero. Then  $R(s)$  would be large in the region near  $\bar{s}$ , and because of the  $(s' - s)$  in the denominator in Eq. (2),  $d(s)$  would be rapidly varying near  $\bar{s}$  and so could be expected to develop a zero to cancel the zero in  $n(s)$ . Then  $a(s)$  would remain finite, thus invalidating Squires' proof that  $a \neq A_{11}$ . That the zeros in  $n$  and  $d$  should appear at precisely the same place may seem accidental, until we realize that, as long as  $A_{11}$  has no zero on the physical sheet, this is exactly what Cauchy's formula tells us must happen.

To see whether this method of one-channel calculation will agree with the Bjorken method, it is sufficient to watch the zeroes of  $A_{11}$ ;

the methods agree if and only if  $A_{11}$  has no zero on the physical sheet which is not shared by  $n$ . Bander, Coulter, and Shaw<sup>4</sup> have recently found a similar difficulty with the one-channel method of Frye and Warnock,<sup>2</sup> in which  $d$  does not contain the full right-hand cut. They find examples where the Frye-Warnock amplitude ceases to agree with the Bjorken amplitude when a zero of the S matrix appears on the physical sheet.

$$1 - (\lambda_{22} - \lambda_{12}^2 \lambda_{11}^{-1}) d_2(s_2) > 0 \quad (5)$$

The inequalities (4) and (5) are true simultaneously because  $\lambda_{12}^2 \lambda_{11}^{-1}$  is large enough.

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FOOTNOTES AND REFERENCES

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