Neighborhoods and Individual Preferences: A Markovian Model

Shripad Tuljapurkar
Elizabeth Bruch
Robert D. Mare

CCPR-038-08

December 2008
Latest Revised: December 2008

NEIGHBORHOODS AND INDIVIDUAL PREFERENCES: A MARKOVIAN MODEL

SHRIPAD TULJAPURKAR
Stanford University
Stanford, CA 94305-5020, USA
tulja@stanford.edu

ELIZABETH BRUCH
University of Michigan
Ann Arbor, MI 48109-2029 USA

ROBERT D. MARE
University of California Los Angeles
Los Angeles, CA 90095-1551, USA

1 Introduction

Formal demographic models often relate individual behavior to population dynamics. For example, the renewal equation connects individuals' fertility and mortality to changing population structure. Indeed, much of demographic research requires connecting some micro-level behavior with macro-level outcomes. However, our demographic models typically assume additivity, that is, transition rates do not vary with features of the population. When individuals' actions or choices are interdependent, macro-level patterns are not simply aggregates of micro-level characteristics and behavior (Coleman 1994; Granovetter 1978). There are different ways one can model interdependent behavior, including threshold, contagion, and epidemic models. All these models rest on two-way interaction: individual actions may be influenced by the actions of others who act in a given way, while changes in individual behavior alter the makeup of the population. These social interaction models account for the "emergence" of collective properties from the behavior of individuals, and can also explain why the same individuals may experience a wide range of social outcomes, depending on the structure of their interaction. In recent years, population researchers have applied interactions-based models in a number of areas of research, from studies of fertility transition to patterns of assortative mating (e.g., Kohler 2001; Goldstein and Kenney 2001).

One area where we observe interdependent behavior is in the study of neighborhood formation and change. Any person who moves is both responding to neighborhood composition and also (by leaving one neighborhood and entering another) changing neighborhood composition. Schelling (1971, 1972, 1978) laid the conceptual groundwork for modeling the relationship between individual preferences and behavior on the one hand and the evolution of neighborhoods on the other. Using rudimentary computational models applied to artificial agents, he showed how the preferences of individuals about where to live give rise to (often unanticipated) aggregate patterns of residential segregation. These patterns, moreover, may be at odds with the majority of individuals' preferences.

Schelling's work assumed that people respond to neighborhoods based on a threshold, or "tipping" function. More recently, Bruch and Mare (2006) extend Schelling's work to examine the implications of alternative assumptions about how individuals evaluate neighborhoods (based on their race-ethnic composition) for aggregate patterns of residential differentiation. They couple their model with survey data to determine what assumptions about individual preferences are most plausible.

Schelling and Bruch and Mare rely on agent-based (microsimulation) models to draw inferences about the relationship between individuals' choices about where to live and aggregate patterns of segregation. Agent-based modeling, which provides a flexible framework for capturing systemic properties of interacting individuals, has grown in popularity in recent years. But agent-based models, despite their power to model complex situations, are in themselves complex structures and the key determinants of their dynamic behavior are often hard to understand and distill into simple conclusions.

One can also capture the dynamic properties of social interactions through an analytic model. Bruch and Mare formulate a simple, stylized Markov chain model to capture the key features of their agent-based models. In this Markovian model, individual preferences in the form of probabilities of moving or staying are functions of neighborhood composition, and the equilibrium states of the model represent possible macrolevel patterns of segregation (or its absence). Here we present the results of a complete analytical treatment of the equilibrium properties of this model and some extensions. Two results are especially interesting. First, the analytics yield an exact condition for the minimum strength that individual preferences must have in order to yield stable segregation patterns. Thus we can relate observed preferences to observed macrolevel patterns of differentiation. Second, the analysis shows that under Schelling-type rules, segregation is the only stable equilibrium possible.

We show that simple formal models have the potential to usefully illuminate the relationship between individual choices and collective population-level patterns. In the conclusion, we outline next steps and future directions for this line of work.

Groups, Neighborhoods and Preferences

Groups and Neighborhoods

We are interested in preferences that express a tendency for individuals to live with other individuals with whom they identify. Individuals may identify with others along many distinct social or economic dimensions, including income, wealth, sexual preference, political views, religious beliefs, and race or ethnicity. An axis of identification that is of considerable current interest is age, illustrated by the rise of communities with age-defined residency policies and a parallel movement of older people out of their long-time residential neighborhoods. In our model, we assume that one or more such criteria are used to define two distinct groups and each individual identifies herself as a member of one group. We are interested in the distribution of individuals over spatially defined and distinct neighborhoods. Each individual's preference for living in a neighborhood depends on the fraction of same-group individuals who live there. This preference is expressed as a probability that an individual will choose a neighborhood to live in. It is important to note that these are probabilities: a strong preference does not translate into certain behavior.

Local and Global Conditions

A central idea here is that local conditions result in changes to the global distribution of individuals. Here, local means within a residential neighborhood whereas global means over all neighborhoods. The essential features of a local-global interaction can be nicely demonstrated in a model that has only two neighborhoods that we denote by numbers 1,2. There are two groups, denoted A, B and at a particular time (say an annual census) there is a global distribution of people in these groups. Globally, a fraction p_A of all group A people and a fraction p_B of all group B people live in neighborhood 1; the others are in neighborhood 2. But what matters to an individual is local information: the fraction of people in a neighborhood that is of the same type. Thus a member of group A evaluates neighborhood 1 in terms of the local proportion, f_A^1, of group A residents in that neighborhood.

The relationship between the global and the local proportions depends on the total numbers of the two groups. We suppose that there are N_A individuals in group A and N_B individuals in group B, and define the ratio $K = (N_B/N_A)$; we use the convention that $K > 1$ (simply by letting A be the group with smaller total size). Writing f_A^1, f_A^2 for the fractions of group A individuals within neighborhoods 1, 2 respectively,

$$f_A^1 = \frac{p_A N_A}{p_A N_A + p_B N_B} = \frac{p_A}{p_A + K p_B}, \tag{1}$$

and similarly

$$f_A^2 = \frac{(1 - p_A)}{(1 - p_A) + K(1 - p_B)}.$$

The fraction of group B individuals is just $f_B^1 = 1 - f_A^1$ in neighborhood 1, and is $f_B^2 = 1 - f_A^2$ in neighborhood 2. These equations tell us how local conditions change with the global distribution of people.

One feature of this local-global relationship is that when the two groups have unequal sizes, $K > 1$, there is a dilution of the effect of a global shift on local frequencies. Qualitatively, if A is less numerous than B, a small global shift of A individuals has a small effect on the local frequencies of A. More formally, equation (1) shows that $f_A^1 \approx p_A/(Kp_B)$ when K is large: as K becomes larger, a change in global frequency of group A has a smaller effect on the local frequency f_A^1. This dilution effect features in our results below.

Preferences and Probabilities

We assume that individuals have a preference for a neighborhood that is determined by the proportion of people living there who belong to their own group. Preferences are not probabilities of action but are simply weights that need to be converted into probabilities. An individual who knows that the local frequency of her own group in a particular neighborhood is f will assign a preference weight $R(f)$ to that neighborhood. In this paper we consider monotonic increasing preferences,

$$R(f) = e^{b\,f}, \text{ with } b > 0. \tag{2}$$

The preference parameter b measures the strength of the tendency to associate with one's own-group.

An individual translates preferences into probabilities by comparing neighborhoods. Thus a group A individual who knows that the group A frequencies in neighborhoods 1, 2 are f_A^1, f_A^2 will choose neighborhood 1 with a probability

$$\pi_A^1 = \frac{R(f_A^1)}{R(f_A^1) + R(f_A^2)} = \frac{e^{b\,f_A^1}}{e^{b\,f_A^1} + e^{b\,f_A^2}}. \tag{3}$$

A group A individual chooses neighborhood 2 with probability $\pi_A^2 = (1-\pi_A^1)$.

The choice probability π_B^1 for group B individuals is defined similarly, but using the local frequencies $f_B^1 = (1 - f_A^1)$ and $f_B^2 = (1 - f_A^2)$; a little algebra with (3) shows that

$$\pi_B^1 = \frac{R(f_A^2)}{R(f_A^2) + R(f_A^2)} = \frac{e^{-b\,f_A^1}}{e^{-b\,f_A^1} + e^{-b\,f_A^2}}.$$

Of course, $\pi_B^2 = (1 - \pi_B^1)$. Given the local frequencies at the start of one period (say a year), these choice probabilities are the probabilities that an individual will move, during the period, to one or other neighborhood. Consequently these choice probabilities determine the global frequencies at the start of the next period.

The choice probability π_A^1 depends on the difference between the local group frequencies, $x = (f_A^2 - f_A^1)$. Because local frequencies range from 0 to 1, their difference ranges from -1 to $+1$. An important feature of equation (3) is that the choice probabilities are never either zero or one, regardless of the difference in local conditions, so long as the preference parameter b in (2) is finite. Fig. 1 illustrates the change in choice probability π_A^1 from (3) as a function of the local difference $x = (f_A^2 - f_A^1)$. The choice probability is symmetric around $x = 0$, equals 1/2 when $x = 0$, and falls as x increases. Even when $x = 1$, meaning that all group A individuals are in neighborhood 2, there is a nonzero probability of $[1/(1+\exp(-b)] < 1$ that a group A individual will choose neighborhood 2 rather than neighborhood 1. An important take-home message is that complete segregation of groups into distinct neighborhoods is not ordained by the assumptions of this model: there is always a nonzero probability that an individual will choose a neighborhood that has a lower local frequency of its own group.

Dynamics: Process and Question

Dynamic Process

Say that at time t the overall (global) proportions of group A, respectively B, individuals living in neighborhood 1 are $p_A(t)$, $p_B(t)$. Equation (1) turns these into the local frequencies of groups A and B. Equation (3) turns these into the choice probabilities $\pi_A^1(t), \pi_B^1(t)$. Between times t and $t+1$, group A individuals may move, choosing neighborhoods 1 or 2 with probabilities $\pi_A^1(t)$ and $(1 - \pi_A^1(t))$. Over the same interval, group B individuals choose neighborhoods 1 or 2 with probabilities $\pi_B^1(t)$ and $(1 - \pi_B^1(t))$. Thus at time $t+1$, the global frequencies of A and B are

$$p_A(t+1) = \pi_A^1(t), \quad p_B(t+1) = \pi_B^1(t). \tag{4}$$

These steps define a dynamic that changes global proportions $p_A(t)$, $p_B(t)$ into new global proportions $p_A(t+1)$, $q_A(t+1)$.

A nice feature of (3) is that the choice probability $\pi_A^1(t)$ depends only on the difference between local frequencies: with $x(t) = f_2^A(t) - f_1^A(t)$ we have

$$\pi_A^1(t) = \frac{1}{1+e^{-b\,x(t)}}, \quad \pi_B^1(t) = \frac{1}{1+e^{b\,x(t)}}. \tag{5}$$

One time step in the dynamics takes these values into $p_A(t+1) = \pi_A^1(t)$, $q_A(t+1)\pi_B^1(t)$, which via (3) determine the new local frequencies at $t+1$, and hence the new difference in local frequencies. Formally the dynamics can be summarized by an equation,

$$x(t+1) = H(x(t)), \tag{6}$$

where the function H can be written explicitly by putting together all the transformations (see Appendix). So the dynamics here are effectively one-dimensional and can be analyzed entirely in terms of the difference $x(t)$ in local frequencies.

Questions

The difference $x(t)$ in local frequencies measures exactly what we are after, the amount of segregation. When $x(t) = 0$ there is no segregation. When $x(t) = \pm 1$ there is complete segregation because all group A individuals are in one of the two neighborhoods. Note that segregation is symmetric in $x(t)$, in the sense that $x = -0.2$ and $x = +0.2$ represent the same level of segregation. The dynamic equations are also symmetric, as they should be, in that $H(-x) = H(x)$ in equation (6) (see Appendix for details). In our simple model we care only about the difference in neighborhood composition, so from here on, we restrict our discussion to values of $x(t) \geq 0$.

With the formal dynamics in hand, we want to answer three sets of questions.

(1) What are the equilibrium levels of segregation in the model? An equilibrium is a value x_0 that is maintained under the dynamics, so if $x(t=1) = x_0$ then $x(t) = x_0$ for all $t > 1$. In particular, are there equilibria without segregation ($x_0 = 0$), or with segregation ($x_0 > 0$), and can there be more than one equilibrium?

(2) Are the equilibria dynamically stable? If we perturb local frequencies away from an equilibrium x_0 (i.e., set $x(t=1) = x_0 + d$ with nonzero d),

then do the dynamics take the frequencies back towards the equilibrium x_0 (stability) or away from it (instability). It is useful to distinguish local stability, in which we return to x_0 if the initial change d is small, from global stability, in which we return to x_0 regardless of the initial change d.

(3) How do these properties depend on the preference strength b and the relative group size K?

Results

Equilibrium conditions

An equilibrium x_0 is maintained under the dynamics, so from (6) all equilibria are solutions of

$$x_0 = H(x_0). \tag{7}$$

Geometrically, draw a graph with values of x from 0 to 1 on the horizontal axis. An equilibrium is the intersection of the line $y_1 = x$ with the curve $y_2 = H(x)$. In the Appendix we show that $H(x)$ equals zero at $x = 0$, increases as x increases from 0 to 1, and reaches a maximum value $H(1) < 1$. Hence a no-segregation equilibrium $x_0 = 0$, always exists. An equilibrium with segregation can only exist if the slope of $H(x)$ at $x = 0$ is greater than 1; in that case there must be an equilibrium x_0 between 0 and 1.

Populations of Equal Size

For clarity, start with the case when groups A and B are equal in total numbers so $K = 1$. An *equilibrium with segregation* exists only when the preference strength

$$b > 2. \tag{8}$$

Fig. 2 illustrates, first, that when $b < 2$ only a no-segregation equilibrium exists, and is both locally and globally stable. In this case the curve of $H(x)$ lies below the 45 degree line everywhere. Fig. 2 also illustrates the case when $b > 2$, when $H(x)$ rises faster than the 45 degree line near $x = 0$ and then must fall below it as x gets large. When $b > 2$ there are two equilibria. The no-segregation equilibrium at $x = 0$ exists but is unstable: if we start at $x = 0$ any global shift in either group triggers local responses that eventually increase the level of segregation. These changes eventually drive the global and local distributions to a new equilibrium at $x_0 > 0$, as shown in Fig. 2. The equilibrium with segregation is now locally and globally stable.

Thus segregation can only be maintained if the strength of preference is high enough. The preference strength b is also the elasticity of the preference function R, and for segregation has to be greater than 2.

Populations of Unequal Size

When group B is K times as numerous overall as group A, an *equilibrium with segregation* exists only when

$$b > 2 + \frac{(K-1)^2}{2K}. \tag{9}$$

Fig. 3 illustrates how this condition operates when $K = 3$ so the critical value in (9) is 2.67. First, we try $b = 2.3$, which is sufficient for stable segregation with groups of equal size. When $K = 3$ this preference strength is not enough and only the no-segregation equilibrium exists and is stable. Fig. 3 illustrates an equilibrium with stable segregation when preference strength is above 2.67.

Writing the critical preference strength as in (9) shows that (8) is the special case for $K = 1$, and also that the critical strength increases with the ratio K of total group sizes. This increase results from the dilution effect that we discussed earlier. As one group increases in total number relative to the other, global movements have less effect on local frequencies and it takes a stronger response to maintain segregation.

Extensions

Schelling and threshold preferences

The Schelling model uses a preference function $R(f)$ as illustrated in Figure 4. The value of zero for the first segment matters but the specific value (here 0.5) for the above-threshold preference is immaterial (the value could be 1, for example). Given the own-group frequencies f_A^1 and f_A^2 for the 2 neighborhoods, this preference function can be used to find the corresponding choice probabilities in equation (1).

Figure 5 illustrates what happens by enumerating the selection probability s_1^1 for different values of f_A^1 and f_A^2. It is clear that there is a no-segregation equilibrium at $(f_A^1, f_A^2) = (1/2, 1/2)$ and two edge equilibria with complete segregation at (1,0) and (0,1). Figure 5 shows a line through (1/2, 1/2) – any perturbation away from the no-segregation equilibrium along this line is unstable. If you move up and left from (1/2, 1/2), in the first step you get $p_A = 1$ and $p_B = 0$ and thus end up at the (1,0) edge equilibrium. Conversely if you move down and right. Thus the no-segregation equilibrium is locally unstable in Schelling's model and the system will always head for an edge equilibrium.

The Schelling model with nonzero probabilities has a preference function such as the one in Figure 6. For these numerical values Figure 7 displays the choice probabilities. As before we have the no-segregation equilibrium and the 2 edge equilibria. But consider what happens when you perturb away from (1/2, 1/2). If you move up and left from (1/2, 1/2), in the first step you get $p_A = 1/3$ and $p_B = 2/3$. Therefore we have two additional locally stable equilibria indicated by the diamonds in Figure 7. Now the edge equilibria are unstable, as you can see by considering a perturbation away from (1,0) or (0,1).

This analysis can be extended to more general threshold preferences with many jumps. For example, when $R(f)$ has jumps at 0, 1/4, 1/2, 3/4, each jump of size 1/4, one can make a plot just like Figure 7, but with the square divided into 16 smaller blocks. Again, one finds equilibria at (1/2, 1/2), (1,0) and (0,1). Again the (1/2, 1/2) equilibrium is locally unstable, but this time a perturbation takes the system to a new locally stable internal equilibrium at (2/5, 3/5) or (3/5,2/5). In this case, interestingly, the edge equilibria at (1,0) and (0,1) are also locally stable.

Conclusions

We have shown that a simple analytic model may yield useful insights into segregation dynamics. Most notably, we demonstrate that, in a world where individuals' preferences follow a linear continuous function, a preference strength of $b \geq 2$ is necessary to sustain segregation. This is consistent with corrected results from Bruch and Mare (2006). We have also shown that, as the relative size of groups differs (that is, $K > 1$) more discriminatory preferences are needed to sustain segregation than when groups are of equal size. The reason for this is as follows: any movement of type A individuals in or out of neighborhoods must generate a corresponding change in neighborhood desirability, which is a function of local proportions f_A^1 and f_A^2. As K increases (and the size of group A declines relative to group B) movement by group A affects the global distribution of group A across neighborhoods, p_A, more than it affects the local representation of group A. Bruch and Mare (2006) show that segregation is maintained when changes in the size of a population at risk of moving into an area are offset by changes in the relative desirability of that area. In this case, we see that as K increases, b must also increase to produce a corresponding change in neighborhood desirability.

Bruch and Mare (2006) point out that the estimated preferences may be asymmetric, so that group 1 has preference function $R_1(f)$ (say, a quadratic $f(1-f)$) and group 2 has a different preference function $R_2(f)$ (say an exponential, e^{bf}). In this case the arguments above show that the Markov chain recursion is 2-dimensional.

A model with many neighborhoods can be formulated with the same structure as the one in this paper. We find that the instability condition (9) also guarantees instability of the no-segregation equilibrium in this case. We have not yet characterized the equilibria with segregation but further development of this case should be interesting

References

Bruch, Elizabeth, and Robert Mare. 2006. Neighborhood choice and neighborhood change. American Journal of Sociology 112: 667-709.

Coleman, James. 1994. Foundations of Social Theory. Cambridge, MA: Belknap.

Goldstein, J.R. and C.T. Kenney (2001) "Marriage Delayed or Marriage Forgone? New Cohort Forecasts of First Marriage for U.S. Women," American Sociological Review 66(4):506-519.

Granovetter, Mark. 1978. Threshold Models of Collective Behavior. American Journal of Sociology 83:1420-43.

Granovetter, Mark, and Roland Soong. 1988. Threshold Models of Collective Behavior Chinese Restaurants, Residential Segregation, and the Spiral of Silence. Sociological Methodology 18:69-104.

Kohler, Hans-Peter. 2001. Fertility and Social Interaction: An Economic Perspective. Oxford University Press.

Schelling, Thomas. 1971. Dynamic Models of Segregation. Journal of Mathematical Sociology 1:143-86.

Schelling, Thomas. 1972. A Process of Residential Segregation: Neighborhood Tipping. Pp. 157-184 in A. Pascal (ed.) Racial Discrimination in Economic Life. Lexington, MA: D.C. Heath.

Schelling, Thomas. 1978. Micromotives and Macrobehavior. New York: Norton and Company.

Appendix

We use the notation in the text and present mathematical details behind our results. The difference $x(t)$ between neighborhoods in own-group frequency follows text equation (6). We write the function H first as a function of the global proportions

$$H(p_A, p_B) = K\ \frac{M(p_A, p_B)}{D(p_A, p_B)}, \tag{A-10}$$

where the functions M, D are given by

$$M(p_A, p_B) = p_B(1-p_A) - p_A(1-p_B),\ D(p_A, p_B) = (p_A + K p_B)[(1-p_A) + K(1-p_B)]. \tag{A-11}$$

The dynamics use the fact that the right side depends only on $x(t)$,

$$\begin{aligned} x(t+1) &= K\ \frac{M(p_A(t+1), p_B(t+1))}{D(p_A(t+1), p_B(t+1))}, \\ &= H\left(p_A(x(t)), p_B(x(t))\right). \end{aligned} \tag{A-12}$$

At an equilibrium

$$x = H\left(p_A(x), p_B(x)\right).$$

Local stability: suppose there is an equilibrium at x_0. Local stability requires that

$$\left| \left(\frac{dH}{dx} \right)_{x=x_0} \right| < 1. \tag{A-13}$$

We now prove some facts that will be useful in examining the equilibria of the Markov chain. Text equation (4) and (5) relate $p_A(x(t))$ and $p_B(x(t))$ to $x(t)$. Leaving out the t,

$$p_A(-x) = 1 - p_A(x) = p_B(x) = 1 - p_B(-x). \tag{A-14}$$

It follows that $p_A(0) = p_B(0) = 1/2$.

Next observe that

$$\begin{aligned} D(x) &= (p(x) + K\, p_B(x))\ (p_A(-x) + K\, p_B(-x)), \\ &= (p_A(x) + K\, p_B(x))\ (p_B(x) + K\, p_A(x)), \\ &= K + (K-1)^2\, p_A(x) p_B(x). \end{aligned} \tag{A-15}$$

The first of these equations shows that $D(x) = D(-x)$ and $D(0) = 0$. The last equation tells us that when $K = 1$ we have $D(x) = K = 1$ for all x, and when $K \geq 1$ that $D(x) > K \geq 1$ for all x.

Turning to $M(p_A, p_B)$ we have

$$M(x) = p_B(x)p_A(-x) - p_A(x)p_B(-x) = p_B^2(x) - p_A^2(x) = 1 - 2p_A(x). \quad \text{(A-16)}$$

Therefore $M(x) = -M(-x)$, $M(0) = 0$, and $M(1) = [1 - 2p_A(1)] < 1$ (the last follows because $p_A(1) < (1/2)$ from text equation (5)).

These properties imply that $H(x) = K\,M(x)/D(x) = -H(-x)$. The slope of H is

$$\frac{dH}{dx} = \frac{1}{D}\frac{dM}{dx} - \frac{H}{D}\frac{dD}{dx}. \quad \text{(A-17)}$$

Differentiation yields

$$D\,\frac{dH}{dx} = Kbp_B\left[2 + \frac{K(K-1)^2(p_A - p_B)^2}{D}\right] > 0, \quad \text{(A-18)}$$

which shows that for $x \geq 0$, H is increasing in x. When $x = 1$ we have

$$H(1) = K\,M(1)/D(1) \leq M(1) < 1, \quad \text{(A-19)}$$

where we have used the fact that $D(1) \geq K$ and $M(1) < 1$, as shown earlier.

For any value of K the slope of $H(x)$ at $x = 0$ is

$$\left(\frac{dH}{dx}\right)_{x=0} = K\left(\frac{dM}{dx}\right)_{x=0} = \frac{2\,b\,K}{(K+1)^2}. \quad \text{(A-20)}$$

This and the local stability condition (A-13) at $x = 0$ yield text equation (8) and (9). Global stability follows from local stability by looking at the geometry of the curve $H(x)$ and the resulting signs of $x(t+1) - x(t)$.

Figure Legends

Fig. 1. The probability π_A^1 that a group A individual chooses neighborhood 1, computed from text equation (5) as a function of the difference $x = f_A^2 - f_A^1$ in local frequencies of group A. Note that $\pi_A^1 > 0$ even when $x = 1$.

Fig. 2. Equilibria with equal group sizes ($K = 1$). The blue line is the 45 degree line, say $y = x$. The magenta line is the function $H(x)$ computed

for preference strength $b = 1.5$ which is below the threshold, so there is only one intersection between the lines at the no-segregation equilibrium $x = 0$. The red line is the function $H(x)$ computed for preference strength $b = 2.3$ which is above the threshold, so there are two intersections between the lines, the unstable no-segregation equilibrium at $x = 0$ and stable segregation at x 0.58.

Fig. 3. Equilibria with unequal groups, when group B is $K = 3$ times as numerous as group A. The blue line is the 45 degree line, say $y = x$. The magenta line is the function $H(x)$ computed for preference strength $b = 2.3$ which is above the threshold for $K = 1$ but below the threshold for $K = 3$, so there is only one intersection between the lines at the no-segregation equilibrium $x = 0$. The red line is the function $H(x)$ computed for preference strength $b = 2.97$ which is above the threshold for $K = 3$, so there are two intersections between the lines, the unstable no-segregation equilibrium at $x = 0$ and stable segregation at x 0.64.

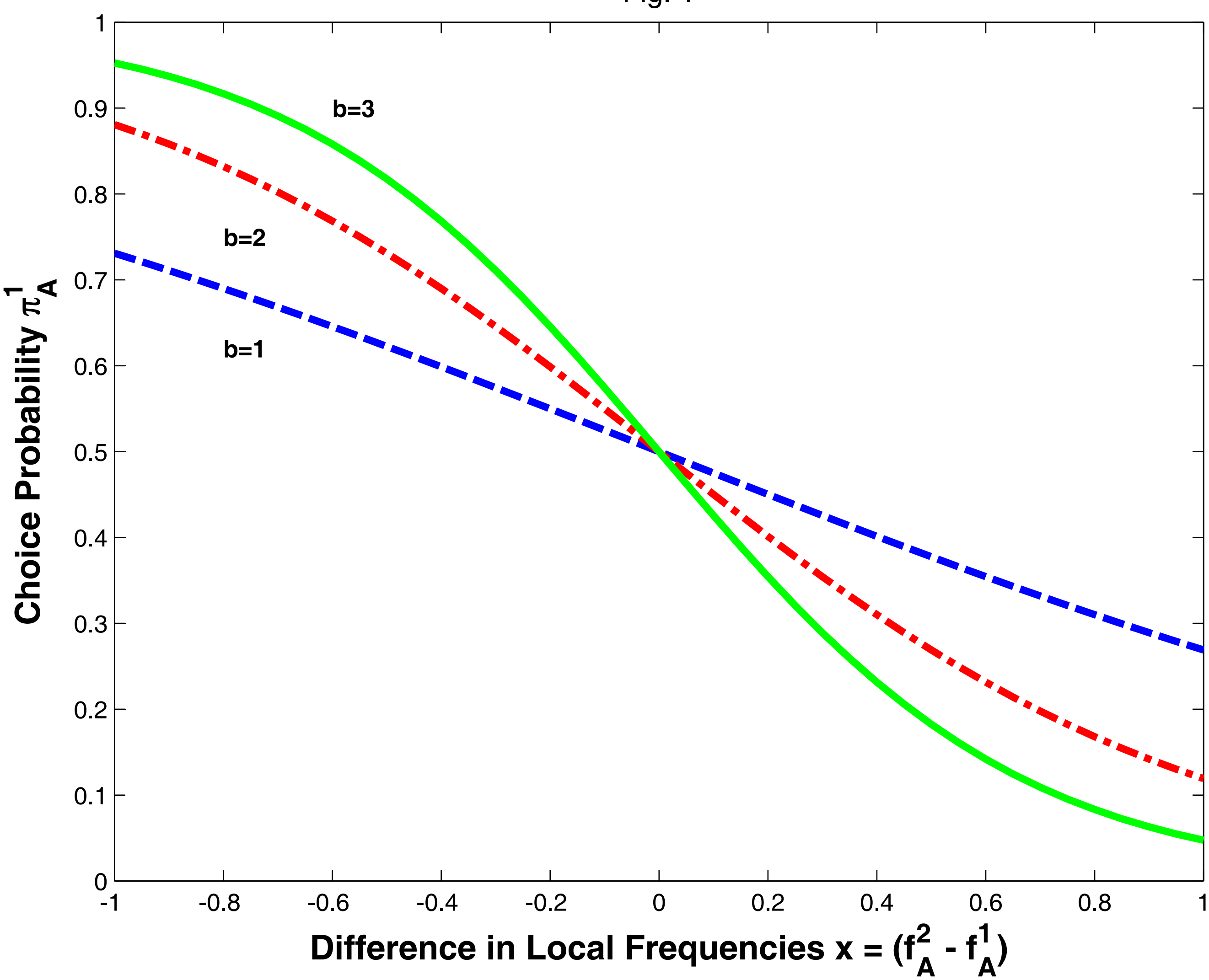
Fig. 4. A threshold preference function, based on Schelling's work, that jumps from weight zero to weight 1 as the local frequency rises above 0.5.

Fig. 5. Choice probabilities (large numbers on graph) as a function of local frequencies, using the preference function in Fig. 4.

Fig. 6. A different threshold preference function, based on Schelling's work, that jumps from weight 0.5 to weight 1 as the local frequency rises above 0.5.

Fig. 7. Choice probabilities (large numbers on graph) as a function of local frequencies, using the preference function in Fig. 5.

Fig. 1



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
b=3
b=2
b=1
Choice Probability π_A^1
Difference in Local Frequencies $x = (f_A^2 - f_A^1)$

Fig. 2

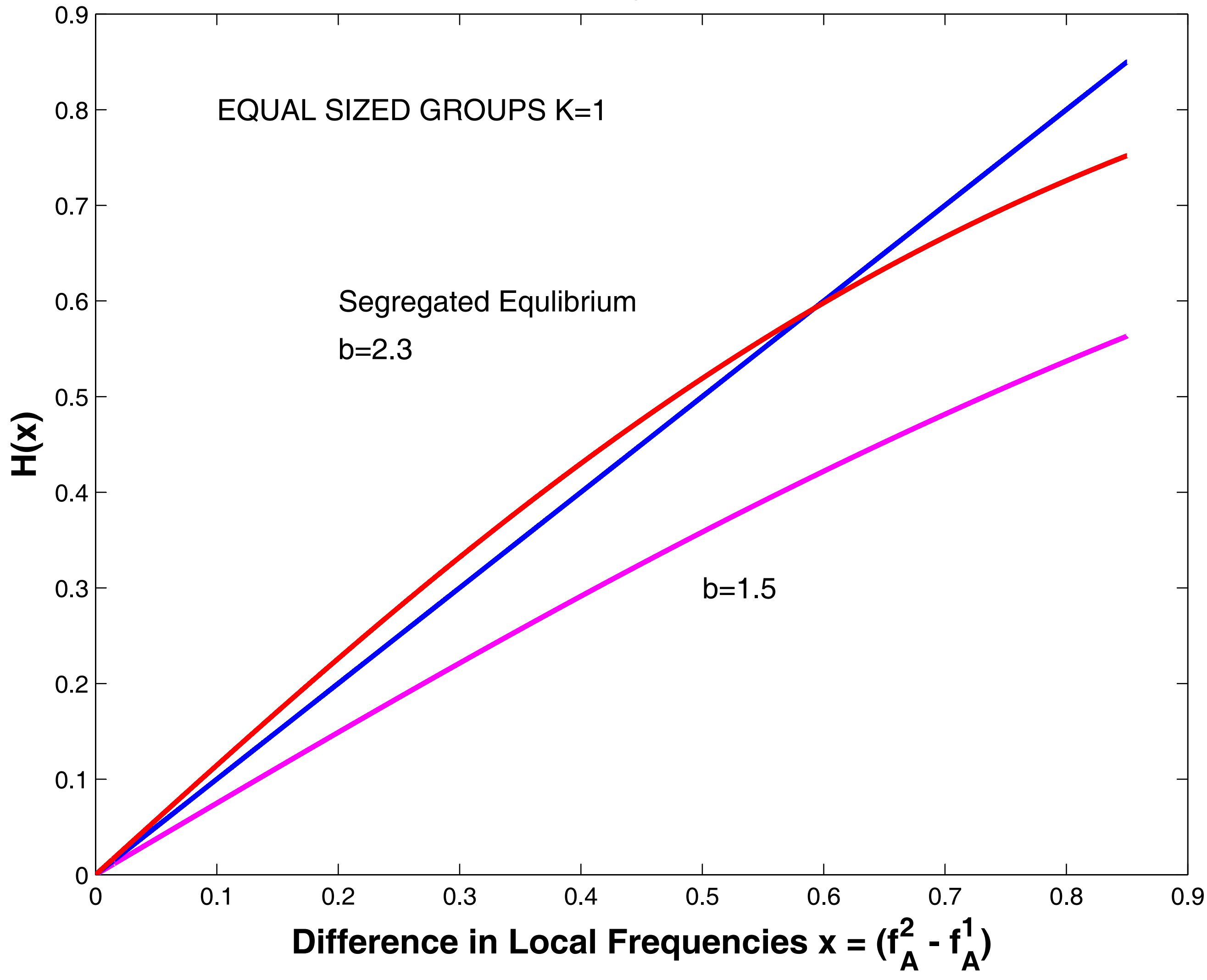

EQUAL SIZED GROUPS K=1
Segregated Equlibrium
b=2.3
b=1.5
H(x)
Difference in Local Frequencies x = (f^2_A - f^1_A)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Fig. 3

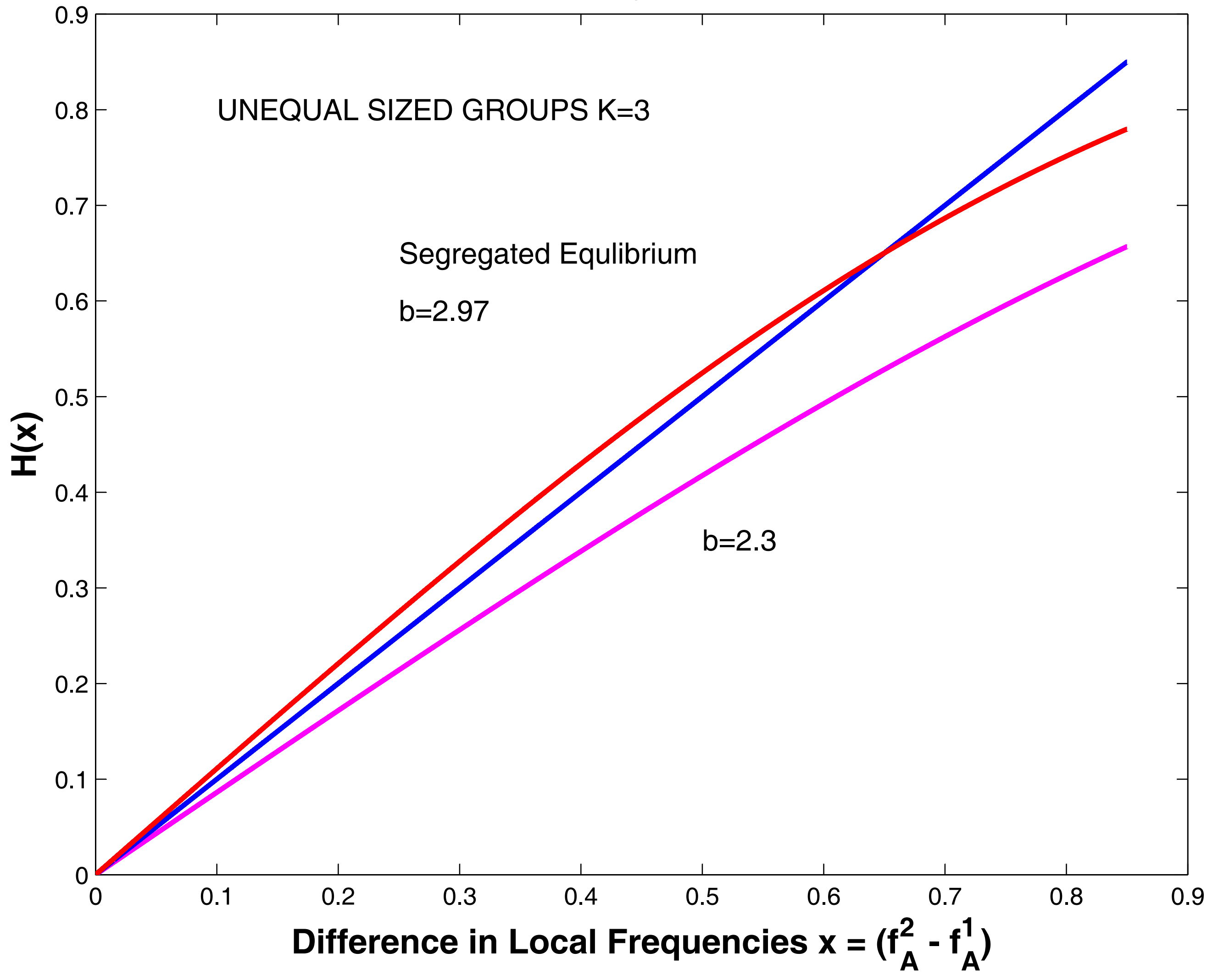

UNEQUAL SIZED GROUPS K=3
Segregated Equlibrium
b=2.97
b=2.3
H(x)
Difference in Local Frequencies x = (f_A^2 - f_A^1)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Fig. 4. SCHELLING PREFS I

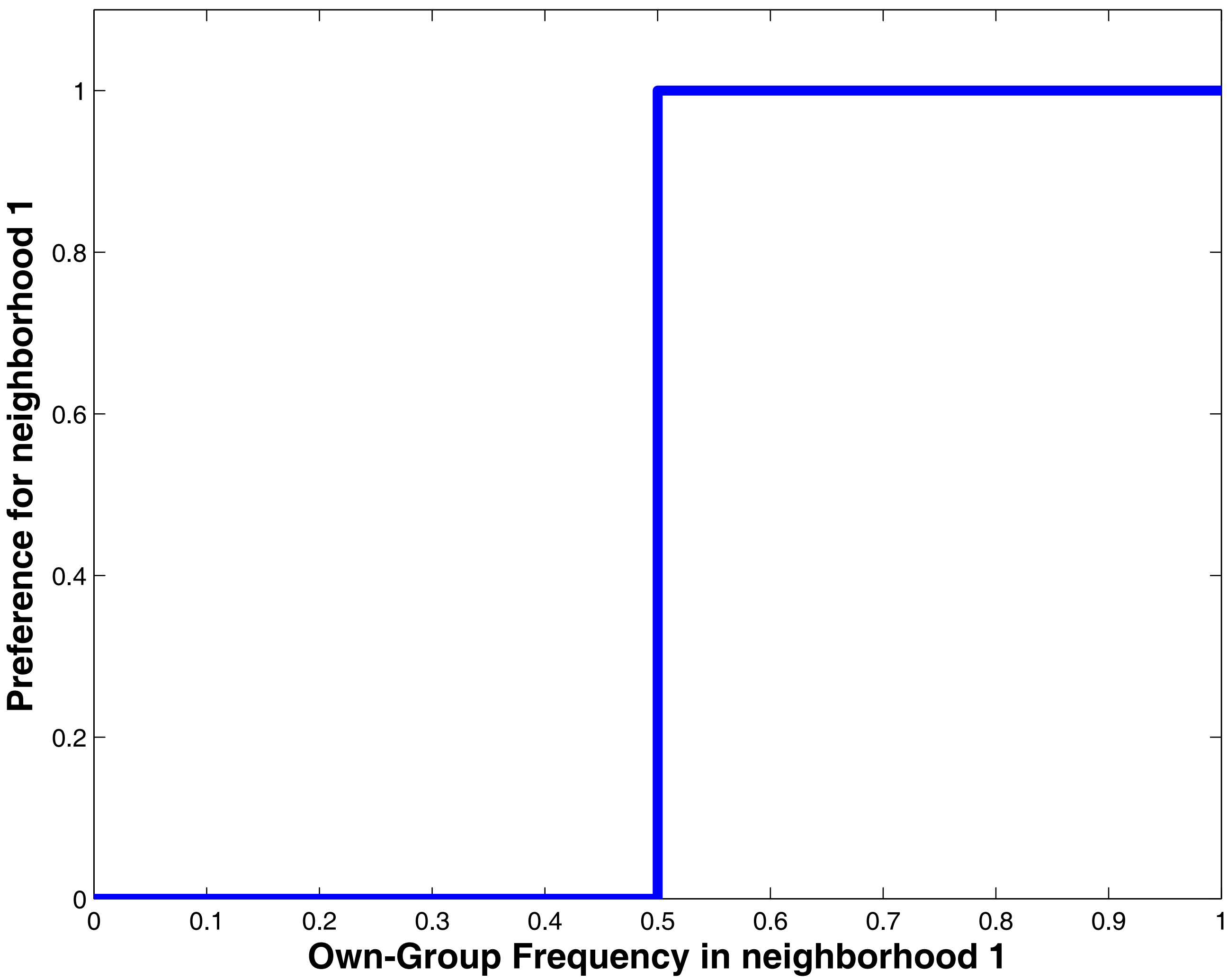


Fig. 5. EQUILIBRIA SCHELLING PREFS I

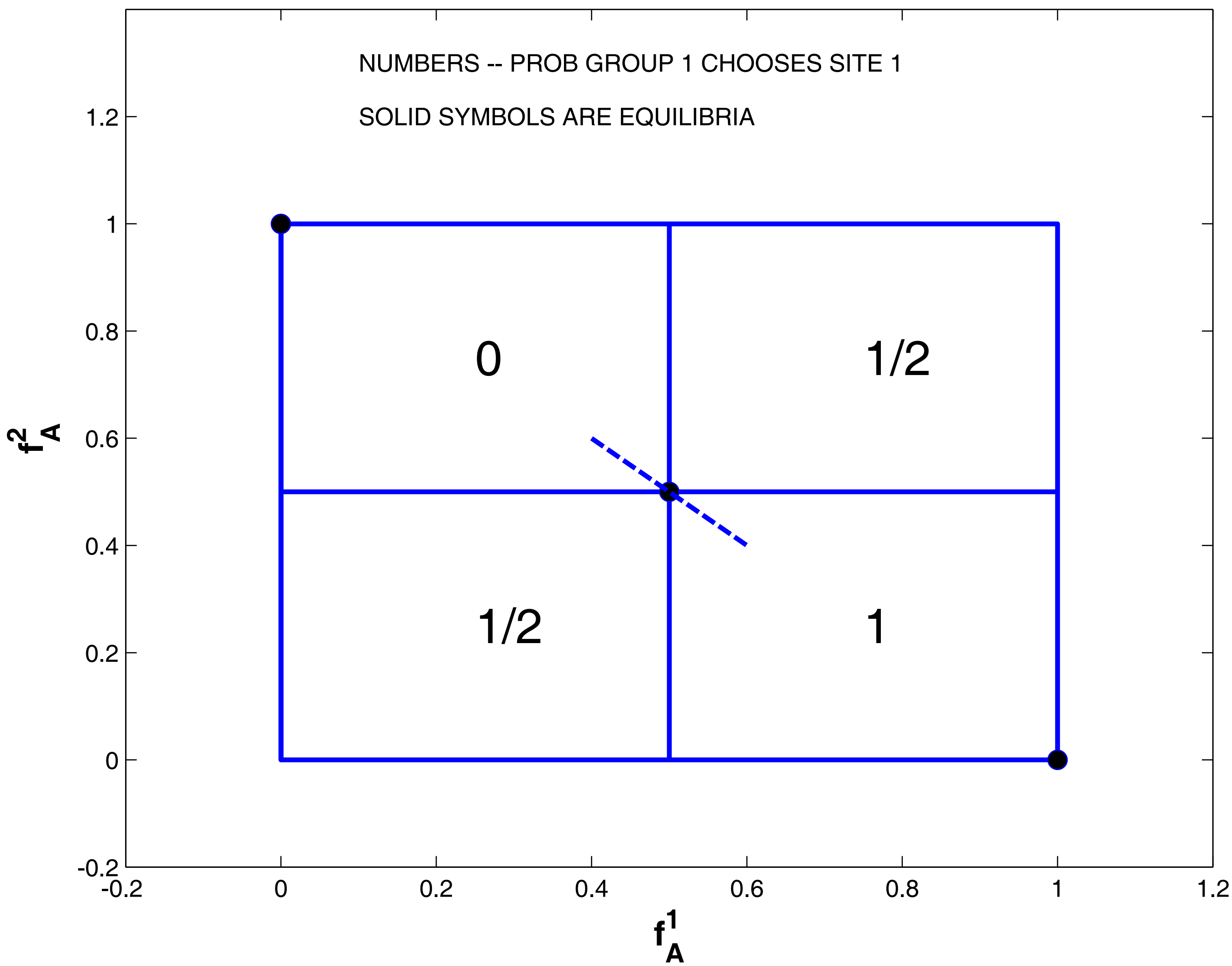
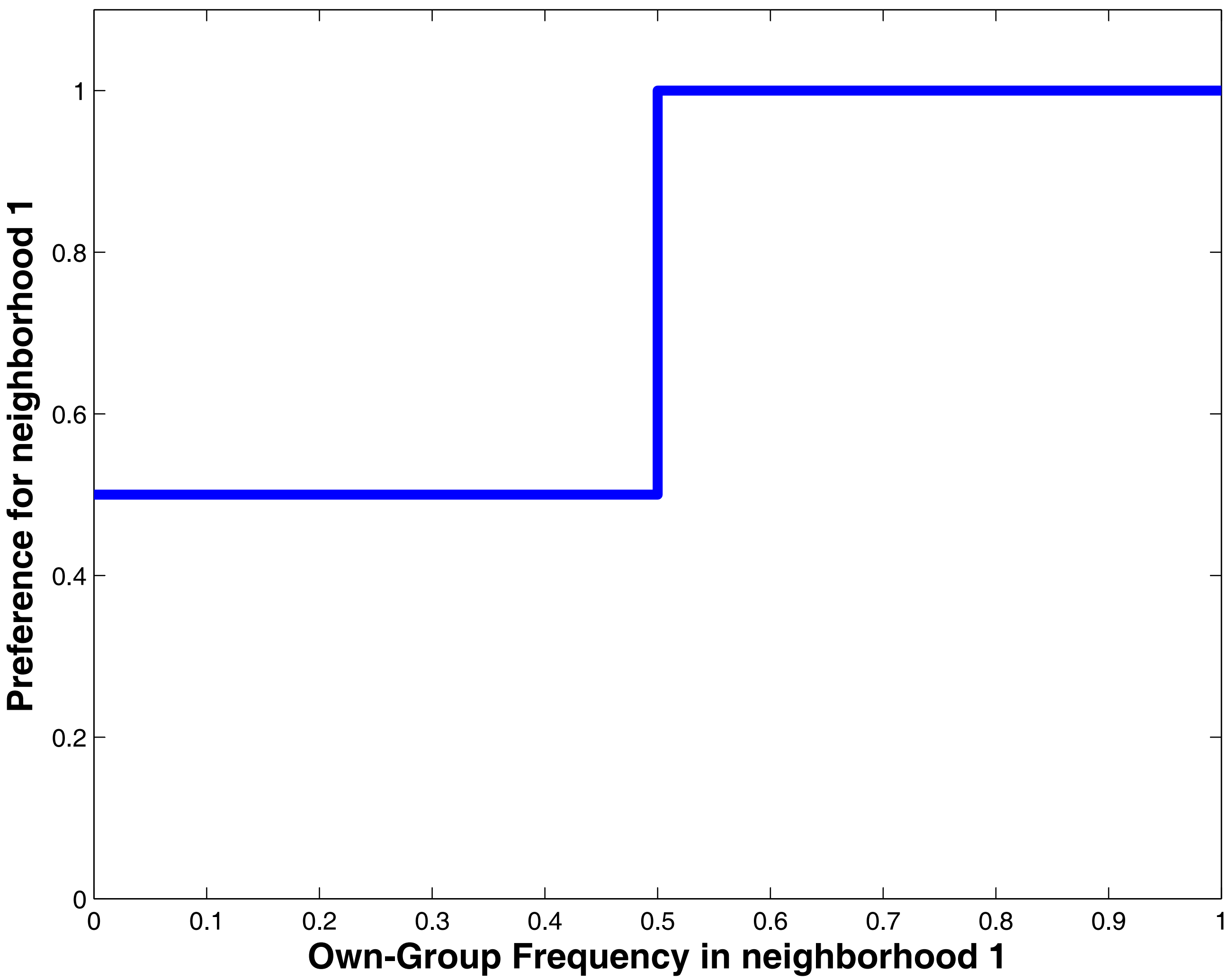


Fig. 6. SCHELLING PREFS II



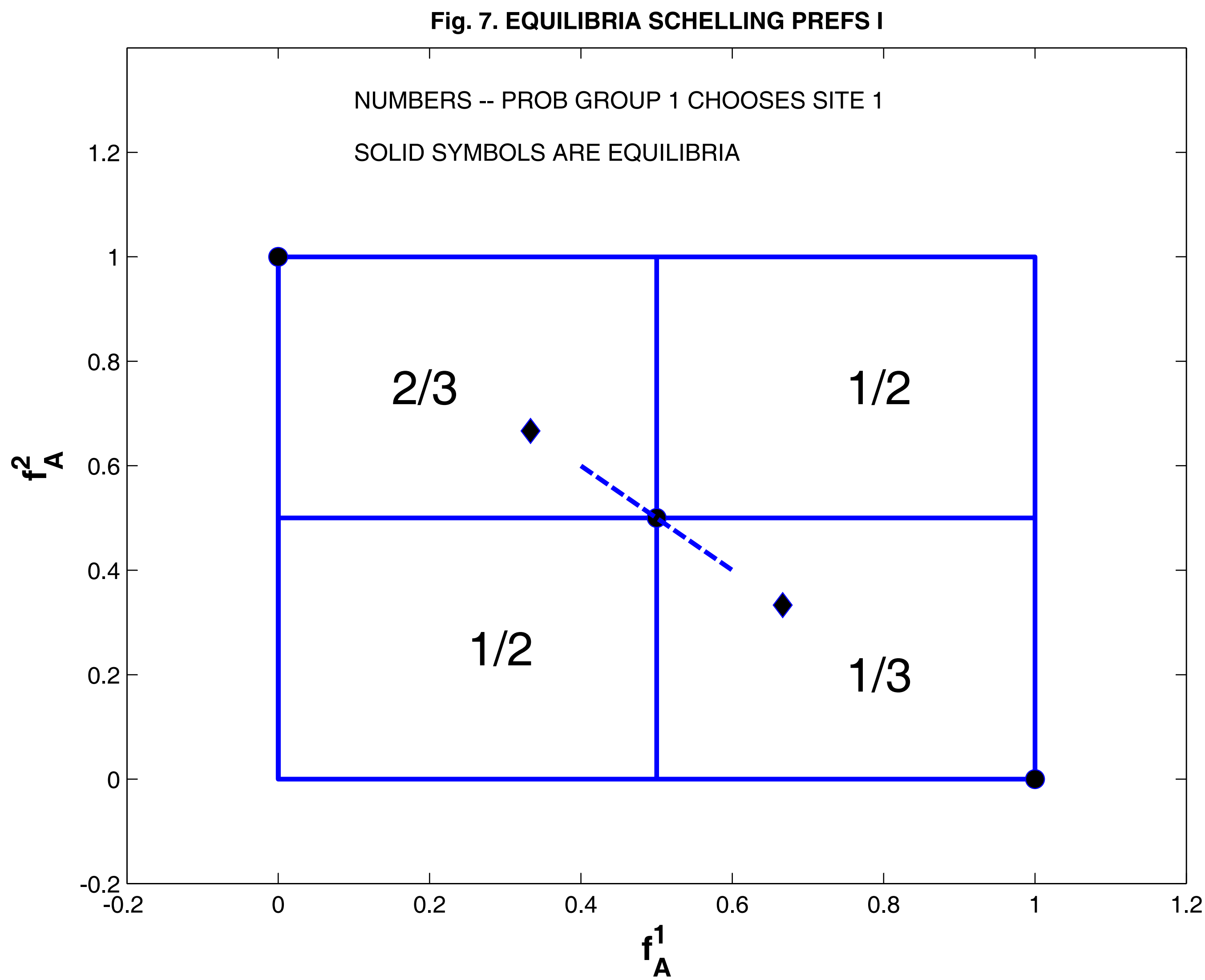


Fig. 7. EQUILIBRIA SCHELLING PREFS I
NUMBERS -- PROB GROUP 1 CHOOSES SITE 1
SOLID SYMBOLS ARE EQUILIBRIA
2/3
1/2
1/2
1/3
f_A^2
f_A^1
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.2
0
0.2
0.4
0.6
0.8
1
1.2