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ABSTRACT

The statement of the requirements for regenerative extraction of particles from a synchrocyclotron in the rapidly changing edge of the magnetic field is followed by an analysis for determining the regenerator shape and position and the rate of increase of radial oscillation amplitude. The effect of the regenerator on the axial motion is considered. The analysis is useful for extraction amplitudes of several inches. Regenerator strengths are computed as an illustrative example.

DESIGN OF REGENERATIVE EXTRACTORS FOR SYNCHROCYCLOTRONS*
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I. INTRODUCTION

In 1950 Tuck and Teng proposed the removal of synchrocyclotron beams by means of magnetic field perturbations placed near the outer radius of the cyclotron.[1] The first perturbation, the peeler, consists of a region of field that decreases radially. This is followed at some angle with a region of radially increasing field, the regenerator. An exact analysis of this system was made by LeCouteur[2] and Judd[3] for beam extraction occurring in a linear portion of the magnetic field of the cyclotron. Later, LeCouteur extended this analysis to the region where the cyclotron field was rapidly decreasing with radius.[4] Crewe and LeCouteur applied the linear analysis and extracted a portion of the circulating beam of the University of Liverpool synchrocyclotron.[5] LeCouteur and Lipton considered a numerical analysis of the deflection with only a region of increasing field as the perturbation.[6] Crewe and Kruse applied this analysis directly to the University of Chicago synchrocyclotron and removed a portion of the circulating beam with a regenerator alone.[7]

The field perturbation needed for extracting a portion of the beam of synchrocyclotron is determined by an extension of these analyses. This extension, which is intimately tied to the physical process, applies to a system using only a regenerator and is useful for extraction amplitudes of several inches.

*This work was done under the auspices of the U.S. Atomic Energy Commission.

II. METHOD OF CALCULATION

The regenerator for extracting the beam of a synchrocyclotron is designed in seven steps, as follows:

(a) The nonlinear equations of motion for the radial and axial oscillations are obtained from the Lorentz equation.

(b) The frequencies of radial and axial motion for various amplitudes of oscillations are determined from these nonlinear equations and the magnetic field shape in the region for extraction.

(c) The increase in the radial amplitude and the necessary change in radial momentum to satisfy the extraction requirements are computed by a simple matrix analysis using the radial frequencies.

(d) The field perturbation is determined from the necessary change in radial momentum.

(e) The resulting change in axial momentum is determined from the field perturbation.

(f) The growth of the axial motion from the impulses of the regenerator is computed.

(g) A suitable arrangement for the particular cyclotron is selected from the set of values of radial and axial growth as functions of the regenerator position and the initial amplitudes of oscillations.

III. THE EXTRACTION REQUIREMENTS

The extraction process begins when the precession of the particles' center of rotation about the center of the cyclotron is arrested. The maximum of the radial extent of the particles then occurs repeatedly at one azimuth. As the radial-oscillation amplitude increases, the particles move outward; a channel may be placed in this region. If the radial gain per turn is large enough and the beam is not greatly attenuated by axial blowup, the particles may "step over" the channel wall and give efficient beam removal.

The extraction requirements, simply stated, are:

(a) the precession must be arrested,

(b) sufficient gain per turn must be obtained,

(c) losses owing to axial blowup must be minimized.

To understand the objective and method of calculation one needs to note that under (a) the particles repeatedly cross the synchronous radius at the same point in the cyclotron. This point may be designated by its angle, and the regenerator and channel positions referred to it. The synchronous radius R is the radius at which the particles circulate when they have no radial-oscillation amplitude, and it is also the radius at which the regenerator field starts. Radial displacement is measured from the synchronous radius. The synchronous radius suitable for deflection in the cyclotron is just inside the radius at which normal beam destruction occurs.

IV. THE NONLINEAR EQUATIONS OF MOTION

In the static magnetic field of the cyclotron, the force on a moving charged particle is given by the Lorentz equation,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{d}{dt} \vec{p} \;,$$

where q is the charge, $\vec{v}$ is the velocity, $\vec{B}$ is the magnetic field, $\vec{p}$ is the momentum, and t is the time. Component equations of this equation are written in cylindrical coordinates for particular convenience in cyclotron problems. In the extraction process, the particle energy may be considered as unchanging, because the energy gain over the few turns is small compared with the total energy. Thus the particle speed is a constant during the extraction process. The speed of particles is expressed as the square root of the sum of the squares of the azimuthal, radial, and axial velocities. This is used to replace time, as the independent variable, by azimuthal angle θ. We denote the synchronous radius by R and define the radius of the particle measured from the center of the cyclotron as $r = R + \rho$ and the axial displacement by z. We combine the component equations and obtain to squared terms the following equations:

$$\frac{d\rho^2}{d\theta^2} + (1-n)\,\rho = \left(2n - 1 + \frac{m}{2}\right)\frac{\rho^2}{R} - \left(\frac{m}{2} + \frac{n}{2}\right)\frac{Z^2}{R} + \frac{1}{2R}\left(\frac{d\rho}{d\theta}\right)^2 - \frac{1}{2R}\left(\frac{dZ}{d\theta}\right)^2 , \quad (1)$$

$$\frac{d^2Z}{d\theta^2} + nZ = -\,(2n + m)\,\frac{\rho Z}{R} + \frac{1}{R}\,\frac{dZ}{d\theta}\,\frac{d\rho}{d\theta} , \quad (2)$$

where

$$n = -\frac{R}{B_Z}\,\frac{\partial B_Z}{dr} , \qquad m = -\frac{R^2}{B_Z}\,\frac{\partial^2 B_Z}{dr^2} . \quad (3)$$

V. FREQUENCIES OF OSCILLATION

Using the nonlinear Eqs. (1) and (2) and the magnetic field data in Fig. 1, we determine the frequencies of oscillation of the radial and axial motions. We use this information in following the particles through their paths within the cyclotron field and through the regenerator field.

The path may be divided into two parts, that part in which the particles are beyond the synchronous orbit, and that part in which the particles are between the synchronous orbit and the machine center. These parts are bounded by the two points at which the synchronous orbit is crossed by the particle orbit.

The method of determining the required perturbation may be illustrated by the following example. In Fig. 1 we select a synchronous orbit just inside the n = 0.2 resonance-loss radius. The synchronous orbit R is 79.8 inches where the n value is 0.155; n is greater beyond and less towards the center of the cyclotron. The dotted lines show an approximation to the change in n as the radius of the orbit increases and decreases. The change in n is related to m, which is defined in Eq. (3) above. The approximations that we use are

$n = 0.155$ at $R = 79.8$ inches;

$m/R = 0.055$ per in. for $78 \text{ in.} \leq r \leq 79.8$ in.

$= 0.138$ per in. for $r > 79.8$ in.

$= 0$ for $r < 78$ in.

The angular frequency of oscillation as a function of the amplitude of the particle motion and the nonlinearities of the magnetic field is found by the approximation technique of Kryloff and Bogoliuboff.[8] We compute these angular frequencies separately for each portion of the cycle and obtain the following equations for the radial motion in the median plane, Z = 0:

$$\frac{d\phi}{d\theta} = 0.919 - 0.019\,\rho \text{ for } r \geq R,$$

$$\frac{d\phi}{d\theta} = 0.919 + 0.009\,\rho \text{ for } r < R \text{ and for } \rho \leq 1.8 \text{ inches},$$

where ρ is the amplitude of oscillation. For our example we obtain the following dependence on amplitude as shown in Table I.

Setting $d\phi/d\theta = \omega_r$, we find that the radial motion whose approximation is a sinusoid of the angle ϕ is related to the angle in the cyclotron by $\phi = \omega_r\,\theta$.

Table I

Amplitude dependence of the oscillation frequencies

Amplitude (in.)	$\sqrt{1-n_{eff}}$ $= d\phi/d\theta$ $(r > R)$	$\sqrt{1-n_{eff}}$ $= d\phi/d\theta$ $(r < R)$	$\sqrt{n_{eff}}(r > R)$	$\sqrt{n_{eff}}(r < R)$
0	0.919	0.919	0.393	0.393
0.25	0.914	0.922	0.406	0.387
0.5	0.909	0.924	0.417	0.382
1.0	0.900	0.928	0.435	0.372
1.5	0.891	0.933	0.454	0.359
2.0	0.881	0.936	0.473	0.352
2.5	0.871	0.936	0.492	0.352

VI. RADIAL MOTION

We now consider the oscillation of the particles in the radial direction and with no axial motion. In accordance with the extraction requirements, the particles must cross the synchronous radius R at a specific point. The analysis is repeated for several amplitudes of radial oscillation and for several positions of the regenerator.

Figure 2 shows the motion of the particles without and with the perturbation. When the particles move freely, their periods are greater than that of the cyclotron and they precess. The half periods are different owing to the different radial positions of the particles. We use the frequency appropriate to the particle position.

To satisfy the extraction requirement the period must be reduced to that of the cyclotron. Since the inward motion is not affected, the half period associated with it is only a function of the amplitude and is $\pi/\omega_r(r < R)$. Requiring that the particle cross the synchronous orbit at 0, 2π, etc. as it moves outward enables us to determine the value of θ_2:

$$\theta_2 = \pi \left(2 - \frac{1}{\omega_r\ (r < R)} \right) .$$

This value varies with the amplitude because $\omega_r(r \lessgtr R)$ is a function of the amplitude. The values for $\theta = 0,\ 2\pi$, etc. correspond to the unique point of the motion. We now ask what change in radial momentum must occur impulsewise at θ_1 to bring the particles to the synchronous radius at θ_2, this being the function of the regenerator.

We consider the motion of the particle in the three intervals suggested by Fig. 2(b). For convenience we use a simple matrix calculation over these intervals. The motion of a particle over the interval $0 \leq \theta \leq \theta_1$ may be expressed in the matrix notation

$$\begin{pmatrix} \rho_1 \\ \rho'_1 \end{pmatrix} = \begin{pmatrix} \cos\ \omega_r\ \theta_1, & \frac{1}{\omega_r}\ \sin\ \omega_r\ \theta_1 \\ -\ \omega_r\ \sin \omega_r\ \theta_1, & \cos\ \omega_r\ \theta_1 \end{pmatrix} \begin{pmatrix} \rho_0 \\ \rho'_0 \end{pmatrix}$$

where $\omega_r = \omega_r\ (r > R)$ and ρ_0 and $\rho'_0 = d\rho_0/d\theta$ are the values at the beginning of the interval and ρ_1 and ρ'_1 are the values at the end of the interval.

The action of the regenerator causes the initial conditions for the second interval to be different from those at the end of the first interval by changing ρ' without changing ρ. For the second interval, $\theta_1 < \theta < \theta_2$, we have

$$\begin{pmatrix} \rho_2 \\ \rho'_2 \end{pmatrix} = \begin{pmatrix} \cos\ \omega_r\ (\theta_2-\theta_1), & \frac{1}{\omega_r}\ \sin \omega_r\ (\theta_2-\theta_1) \\ -\ \omega_r\ \sin \omega_r\ (\theta_2-\theta_1), & \cos\ \omega_r\ (\theta_2-\theta_1) \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho'_1 + \delta(\rho') \end{pmatrix},$$

where $\omega_r = \omega_r\ (r > R)$ and $\delta(\rho')$ is the change in the slope by the regenerator.

Over the third interval, $\theta_2 \leq \theta \leq \theta_3 < 2\pi$, we have a similar expression,

$$\begin{pmatrix} \rho_3 \\ \rho'_3 \end{pmatrix} = \begin{pmatrix} \cos \omega_r\ (\theta_3-\theta_2), & \frac{1}{\omega_r}\ \sin \omega_r\ (\theta_3-\theta_2) \\ -\ \omega_r\ \sin \omega_r\ (\theta_3-\theta_2), & \cos\ \omega_r\ (\theta_3-\theta_2) \end{pmatrix} \begin{pmatrix} \rho_2 \\ \rho'_2 \end{pmatrix}$$

where $\omega_r = \omega_r\ (r < R)$ and ρ_3 and ρ'_3 are the initial values for the next cycle. We note $\omega_r\ (\theta_3-\theta_2) = \pi$, thus the third matrix has the values of

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\begin{pmatrix} \rho_3 \\ \rho'_3 \end{pmatrix} = \begin{pmatrix} -\rho_2 \\ -\rho'_2 \end{pmatrix}$$

This merely changes the sign of the position and slope, therefore we may evaluate the effect of the regenerator by considering the conditions at 0 and θ_2 by noting the change of sign.

At the beginning of the initial interval, we have $\rho_0 = 0$, $\rho'_0 = \rho'_0$, and at the end of the second interval we have $\rho_2 = 0$, $\rho'_2 = \rho'_2$. The ratio $-\rho'_2/\rho'_0$ is proportional to the ratio of the amplitudes before and after passing through the regenerator and is the radial increase per turn; we call it α. The radial momentum change, $\delta(\rho')$, is required to give $\rho_2 = 0$. Putting these conditions in the matrix equations and simplifying the expressions, we obtain

$$\alpha = + \frac{\sin \omega_r \theta_1}{\sin \omega_r (\theta_2 - \theta_1)}, \qquad \omega_r = \omega_r \ (r > R), \tag{4}$$

$$\delta(\rho') = - \frac{\rho'_0 \sin \omega_r \theta_2}{\sin \omega_r (\theta_2 - \theta_1)}, \qquad \omega_r = \omega_r \ (r > R). \tag{5}$$

One computes the radial increase per turn, α, and the required change in radial momentum, $\delta(\rho')$, for several regenerator positions and the different amplitudes of radial oscillations. These are shown in Fig. 3. The smaller the interval $\theta_2 - \theta_1$, the greater the increase α and regenerator strength.

VII. REGENERATOR FIELD

The required change in particle motion is used to determine the magnetic-field perturbation of the regenerator. An impulse computation is made as follows:

$$m \frac{dr}{dt} = m \frac{d\rho}{dt} = m \omega_0 \frac{d\rho}{d\theta} .$$

The change in the radial momentum is

$$-m\,\omega_0\,\delta(\rho') = m\,\omega_0 \left[\left(\frac{d\rho}{d\theta}\right)_{out} - \left(\frac{d\rho}{d\theta}\right)_{in}\right]$$

The force exerted by the regenerator multiplied by the time of action equals the change in radial momentum. The force is $e\widetilde{B}_Z v$ and the time is $r\widetilde{\theta}/v$, where v is the velocity of the particle, $\widetilde{B}_Z$ is the regenerator field, e is the electronic charge, and $\widetilde{\theta}$ is the angle over which the regenerator extends.

The force times the time is then

$$e\widetilde{B}_Z r\widetilde{\theta} = -m\omega_0\,\delta(\rho') ,$$

so we have

$$\delta(\rho') = -\frac{e\widetilde{B}_Z r\,\widetilde{\theta}}{m\omega_0} .$$

Multiplying and dividing by B_0, the magnetic field at R, we obtain

$$\delta(\rho') = -\frac{eB_0}{m}\,\frac{1}{\omega_0}\,\frac{\widetilde{B}_Z r\,\widetilde{\theta}}{B_0} .$$

However, we have

$$\frac{eB_0}{m} = \omega_0$$

so that we get

$$\delta(\rho') = -\frac{\widetilde{B}_Z r\,\widetilde{\theta}}{B_0} \tag{6}$$

Thus the product of the regenerator field and angle of its extent is

$$\widetilde{B}_Z\widetilde{\theta} = -\frac{\delta(\rho')\,B_0}{r} . \tag{7}$$

The simple impulse computation gives the value $\widetilde{B}_Z\widetilde{\theta}$ which is the integrated regenerator field over the angle of the regenerator. The total magnetic field in the regenerator region is increased by the regenerator field so as to decrease the radius of curvature of the particles and to turn them back to the synchronous radius. This approximation must be examined more carefully for a regenerator of large azimuthal extent. When B_0 is in gauss, $\widetilde{B}_Z\widetilde{\theta}$ is in gauss-radians.

If we consider a regenerator placed at $120^{\circ} = \theta_1^{\beta}$, from Fig. 3 we obtain

$$-\delta(\rho') = a\rho + b\rho^2$$

$$= 0.695\,\rho + 0.02\,\rho^2,$$

and from Fig. 1 we find, at R = 79.8 in. and B_0 = 21,730 gauss,

$$\widetilde{B_Z}\widetilde{\theta} = \frac{0.695\,\rho + 0.02\,\rho^2}{r}\quad B_0 .$$

This is given for our example in Table II.

Table II

Regenerator strength

ρ (in)	$\delta(\rho')$	$\widetilde{B_Z}\widetilde{\theta}$ (kilogauss-degrees)
0.5	0.36	5.6
1.0	0.72	11.0
1.5	1.08	16.5
2.0	1.47	22.3
2.5	1.70	25.6

VIII. EFFECT OF REGENERATOR FIELD ON AXIAL MOTION

The axial momentum is given by

$$m\frac{dZ}{dt} = m\ \omega_0\ \frac{dZ}{d\theta} .$$

The change in axial momentum is

$$m\omega_0\ \delta(Z') = m\,\omega_0\left[\left(\frac{dZ}{d\theta}\right)_{out} - \left(\frac{dZ}{d\theta}\right)_{in}\right].$$

The impulse time is the same as for the radial computation and the force is given by $F_Z = ev\widetilde{B_r}$, where $\widetilde{B_r}$ is the radial component of the magnetic-field perturbation. Using the relation $\nabla \times B = 0$, we obtain

$$B_r = \frac{\partial \widetilde{B_Z}}{\partial r}\ Z .$$

The change in momentum is then

$$m\,\omega_0\,\delta(Z') = \frac{e\,\partial\widetilde{B}_Z}{\partial r}\;\widetilde{\theta} Z r\ ,$$

where $\frac{\partial \widetilde{B}_Z}{\partial r}\ \widetilde{\theta}$ is integrated over the regenerator region, and thus we have

$$\delta(Z') = \frac{erZ}{m\omega_0}\ \frac{\partial\widetilde{B}_Z}{\partial r}\ \widetilde{\theta}\ .$$

Again multiplying and dividing by B_0, we obtain

$$\delta(Z') = \frac{eB_0}{m}\ \frac{1}{\omega_0}\ \frac{r}{B_0}\ Z\ \frac{\partial\widetilde{B}_Z}{\partial r}\ \widetilde{\theta} = \frac{rZ}{B_0}\ \frac{\partial\widetilde{B}_Z}{\partial r}\ \widetilde{\theta}\ , \tag{8}$$

where we use $\widetilde{B}_Z$, the regenerator field calculated above, to obtain

$$\frac{\partial\widetilde{B}_Z}{\partial r} = \frac{B_0}{\widetilde{\theta}}\ \frac{d}{d\rho}\left(\frac{a\rho + b\rho^2}{r}\right)\ ,$$

so we have

$$\delta(Z') = \frac{Z}{R}\ (aR + 2bR\rho + b\rho^2)\ .$$

This disturbance is a function of axial amplitude, and for a given axial amplitude is rather constant as the particle radius changes, because the slope of the regenerator field changes slowly. It will, however, vary with regenerator position. The axial amplitude at which the particle crosses the regenerator is very important, and particles with axial oscillation of large amplitude are lost.

IX. AXIAL MOTION

The axial motion under the influence of the rapidly falling field and the perturbation of the regenerator may be examined by a matrix analysis of the axial motion in a manner similar to that for the radial motion. The bounds of stability are determined from the trace of the matrix[9] formed by the product of the matrices for each interval of the orbit and the regenerator impulse. The motion is bounded when the trace is less than 2 in magnitude, and for this condition the increase in axial amplitude can be estimated.

The amplitude of the axial motion, $\mathcal{Z}$, is given by

$$\mathcal{Z}_i^2 = z_i^2 + \frac{z_i'^2}{n} ,$$

where z_i and z_i' are the amplitude and slope at any point i in the orbit, and n is the magnetic field parameter given by Eq. (3). The ratio $\mathcal{Z}_{i+1}/\mathcal{Z}_i$ is the change in amplitude in one turn around the cyclotron. This may be averaged over many turns and all possible phases of regenerator encounter to give the maximum growth in axial amplitude for a given regenerator in the particular falling magnetic field. This has been done by LeCouteur.[6] Using his analysis, we obtain the ratios for each regenerator position as listed in Table III.

Table III

Amplitude ratios for various regenerator positions.

θ_1 = Regenerator position	$\frac{\mathcal{Z}\ \text{max.}}{\mathcal{Z}\ \text{initial}} = \frac{\text{maximum Amplitude}}{\text{Initial Amplitude}}$ for ρ = 0 in.	1 in.	2 in.
90°	1.86	1.65	1.55
100°	1.93	1.71	1.59
110°	2.04	1.84	1.72
120°	2.27	2.05	1.94
130°	2.75	2.48	2.31

These ratios are computed for several radial amplitudes with the assumption that $\sqrt{n_{eff}}$ used in determining the axial period is found from $\sqrt{1-n_{eff}}$ as computed for the radial motion, both are listed in Table I.

The ratios as computed for Table III are too conservative for axial motion in excess of a fraction of an inch. The magnetic field shape off the median plane becomes unfavorable for axial focusing, as the magnetic field on the median plane rapidly changes slope. This is shown by a Taylor-series expansion about the mid-plane and the synchronous orbit. The computation of $\sqrt{n_{eff}}$ for the axial motion, using Eq. (2) by applying the Kryloff-Bogoliuboff technique, shows a much less rapid change than in Table I. Neither set of values of $\sqrt{n_{eff}}$ approximates the focusing that particles experience off the median plane. The axial loss must be evaluated by orbit computations in the magnetic field of interest.

The ratios in Table III, which are only a suggestion of the blow-up by each different regenerator position, certainly must be small for adequate regenerator performance.

X. SELECTION OF FINAL ARRANGEMENT

The selection of the regenerator position may now be made according to the following criteria:

(a) The minimum gain per turn permitted for a given channel, starting angle, wall thickness, and entrance radius.

(b) The amount of beam loss one is willing to accept for the distribution, known or assumed, of axial oscillation amplitudes.

(c) The field perturbations that may be produced in the cyclotron gap by the means at one's disposal.

Efficient extractions may be more easily achieved by changing the value of the synchronous radius at which the regenerator starts, but if one is required to reduce the synchronous radius, then the energy of the extracted beam is reduced.

XI. INFLUENCE OF THE CHANNEL ON THE BEAM

As the particle amplitude is increased by the regenerator action the particle approaches the magnetic channel, and prior to entering the channel, the particle experiences the field perturbation from the channel. This becomes serious as the radial oscillation amplitudes become large, because the uncorrected field is appreciable only near the channel wall itself. The impulse of the uncorrected channel perturbation may be computed as above. The extraction requirements may be achieved in a manner that compensates for the disturbance of the channel. The computation enables one to modify the regenerator field to account for the channel effect and, thus, the maximum effort of corrective shimming for the channel is not required. Reduction of the effort is desirable because the corrective shimming is difficult and increases the magnetic field in the channel lessening its effectiveness.

This work was done under the auspices of the U.S. Atomic Energy Commission.

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Figure Captions

Fig. 1. The magnetic field and the n value vs radius in the extraction region.

Fig. 2. The radial motion vs cyclotron angle θ. (a) Motion without regenerator, (b) motion with regenerator causing period of radial motion to be one turn in the cyclotron.

Fig. 3. Gain per turn and regenerator strength vs radial displacement into regenerator for several regenerator positions.

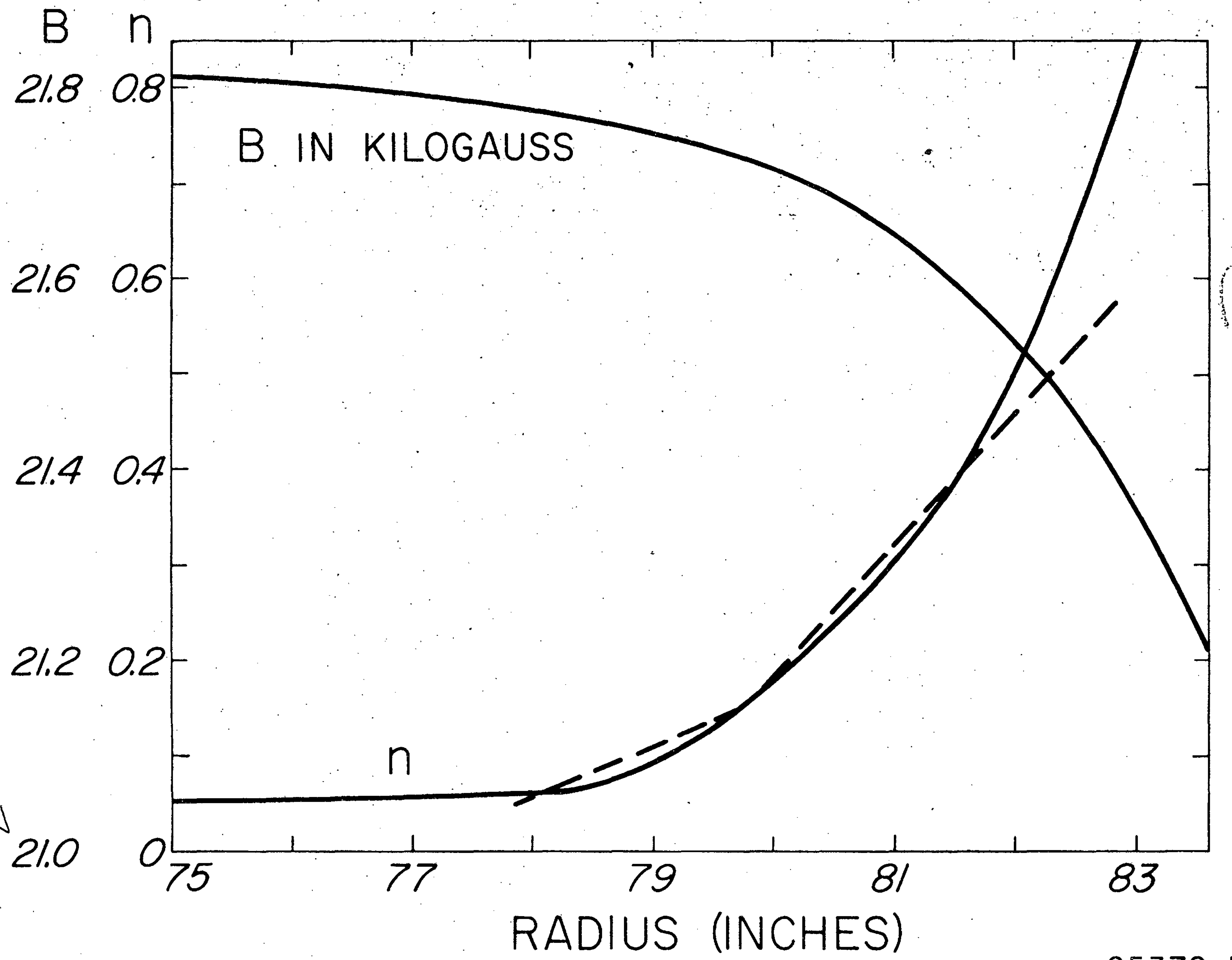
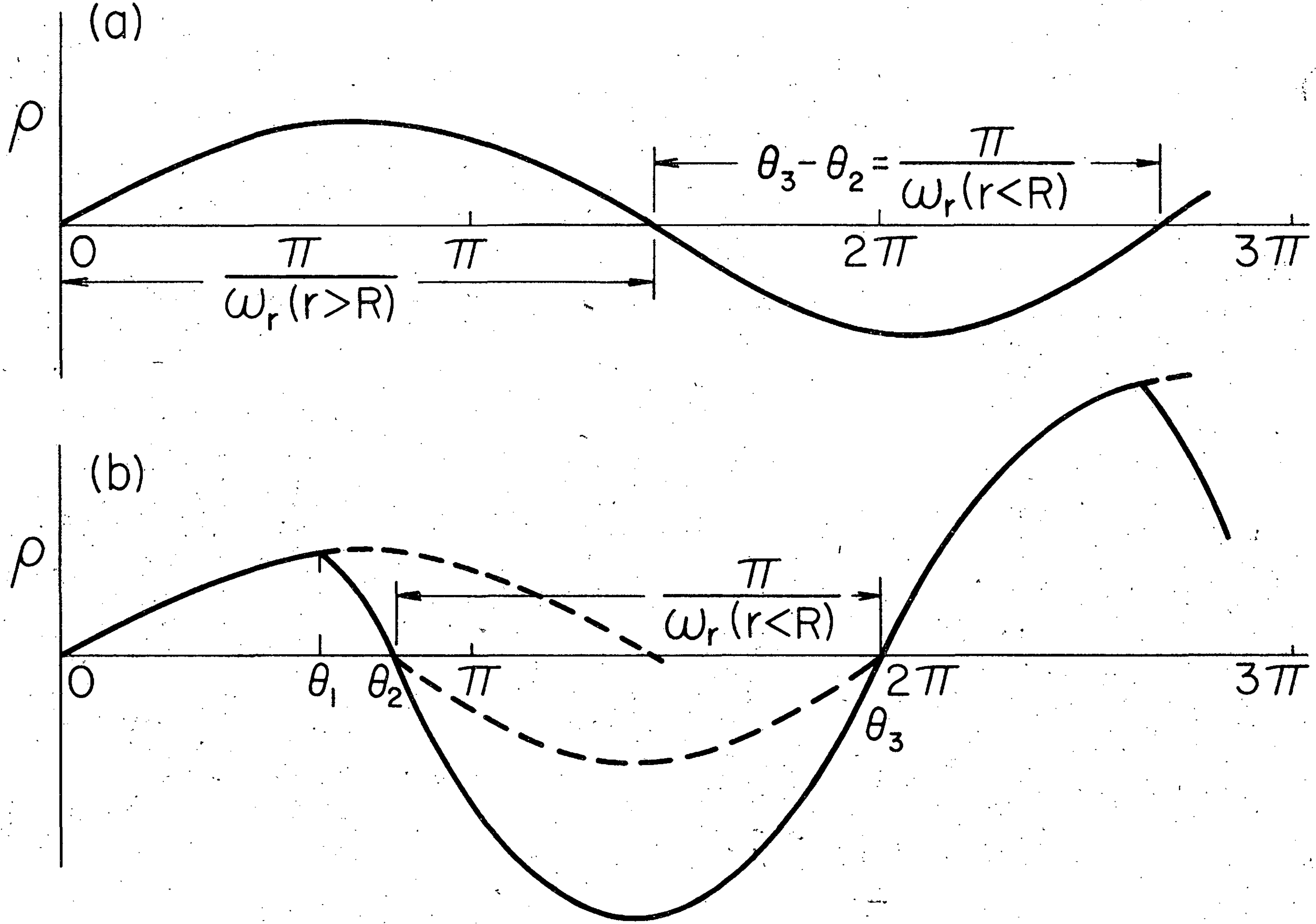

B
n
21.8 0.8
21.6 0.6
21.4 0.4
21.2 0.2
21.0 0
B IN KILOGAUSS
n
75
77
79
81
83
RADIUS (INCHES)
25379-1

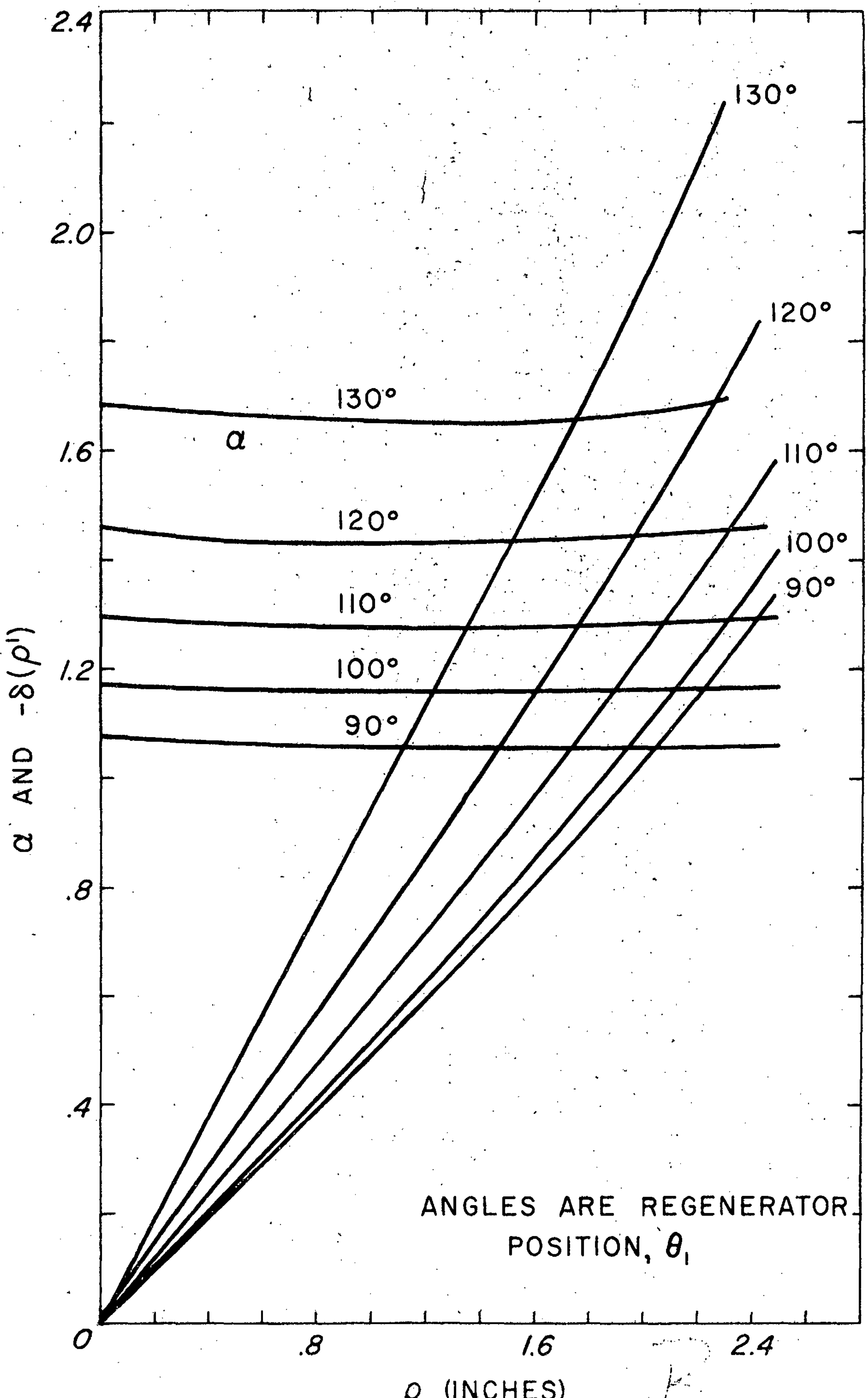
Fig. 1



(a)
ρ
$\theta_3 - \theta_2 = \frac{\pi}{\omega_r(r<R)}$
0
$\frac{\pi}{\omega_r(r>R)}$
π
2π
3π
(b)
ρ
$\frac{\pi}{\omega_r(r<R)}$
0
θ_1
θ_2
π
2π
θ_3
3π

Fig. 2

25380-1


2.4
2.0
1.6
1.2
.8
.4
0
.8
1.6
2.4
130°
120°
110°
100°
90°
α
α AND -δ(ρ')
ANGLES ARE REGENERATOR POSITION, θ1
ρ (INCHES)
Fig. 3

25381-1