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Dynamical electroweak symmetry breaking by a neutrino condensate

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Abstract

We show that the electroweak symmetry can be broken in a natural and phenomenologically acceptable way by a neutrino condensate. Therefore, we assume as particle content only the chiral fermions and gauge bosons of the Standard Model and in addition right-handed neutrinos. A fundamental Higgs field is absent. We assume instead that new interactions exist that can effectively be described as four-fermion interactions and that can become critical in the neutrino sector. We discuss in detail the coupled Dirac–Majorana gap equations which lead to a neutrino condensate, electroweak symmetry breaking and via the dynamical see-saw mechanism to small neutrino masses. We show that the effective Lagrangian is that of the Standard Model with massive neutrinos and with a composite Higgs particle. The mass predictions are consistent with data.

1. Introduction

The generalization of renormalizable relativistic gauge theories to the Standard Model (SM) was very successful and has been confirmed experimentally in an impressive way, including detailed tests of radiative corrections.[1] However, it is important to keep in mind that the mechanism of electroweak (EW) symmetry breaking is still mostly untested. The postulated Higgs particle has so far not been observed and there is only indirect evidence from quantum corrections that a SM Higgs boson should be lighter than about 200 GeV [2].

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[1] Recent measurements of small deviations of $g-2$, A_{FB} and $\sin^2\Theta_W$ in the neutrino sector may be the first signs of physics beyond the SM [1].

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The Higgs sector has furthermore well-known theoretical problems, especially the gauge hierarchy problem, which strongly suggest that new physics exists which is connected to the mechanism of EW symmetry breaking. Whatever the correct symmetry breaking mechanism is, it must satisfy by now a number of stringent direct and indirect constraints. Given the success of the SM it is, however, immediately clear how an alternative symmetry breaking scenario can be consistent with data. In the limit where new physics decouples, it just has to reproduce effectively the SM Higgs sector with a light Higgs particle [3]. If the model has such a decoupling limit, as in the case discussed in this paper, then deviations from the SM can be understood as a departure from the decoupling limit.

Motivated by the evidence for neutrino masses, we discuss the possibility that the EW symmetry is broken dynamically by a neutrino condensate. This would normally lead to neutrino masses of the order of the symmetry breaking scale, i.e., $\mathcal{O}(200\,\mathrm{GeV})$. Neutrinos may, however, possess both Dirac and Majorana mass terms and the dynamical generation of large Dirac mass terms leads via the see-saw mechanism [4–6] to small, phenomenologically acceptable neutrino masses. A composite Higgs particle will emerge that is not affected by the see-saw mechanism, i.e., it will have a mass of the order of the EW symmetry breaking scale. The low-energy effective Lagrangian in the decoupling limit is therefore the SM, with a composite Higgs instead of a fundamental scalar.

A heavy Dirac neutrino mass is similar to the heavy top mass of the order of the EW scale, which gave rise to speculations that top condensation might be responsible for EW symmetry breaking [7,8]. However, top condensation is not viable in its simplest version, since it predicts too large top and Higgs masses. Different non-minimal models are in principle viable [9–12], and the possibility that third-generation neutrinos contribute to top condensation was studied by Martin [13]. The condensation of a full fourth generation (including neutrinos of the fourth generation) [14] was also discussed in this context. We study the case where only neutrinos are responsible for the dynamical breakdown of the EW symmetry. We briefly discuss a mixed case as well where combined neutrino and top condensation leads to an effective two-Higgs scenario with a leptonic and a hadronic Higgs particle.

The paper is organized as follows: in Section 2 we discuss the condensation of neutrinos, i.e., we study the relevant system of coupled gap equations in combination with the see-saw mechanism in the proper mass eigenstate basis. Afterwards, we solve the gap numerically. The following section contains the phenomenology and the predictions arising from the renormalization group improved compositeness conditions. Section 4 contains a short discussion of the option that all three generations of neutrinos condense simultaneously and in Section 5 we outline briefly the possibility of a combined neutrino-top condensation scenario.

2. Neutrino condensation

We assume as mentioned that some physics exists at high energies which yields an effective four-fermion picture similar to weak interactions at low energies. However, contrary to weak interactions we assume that certain four-fermion couplings become strong enough to trigger the formation of condensates, thus giving masses to some of the fermions via gap equations. The remaining fermions could, e.g., obtain masses from further four-fermion couplings, which only subdominantly contribute to the gap. For top condensation it was shown how this can be justified in the context of broken renormalizable gauge theories at high energies, for example, in the framework of so-called top-color models [15] or $U(1)$ models [16]. In this spirit, we consider the particle content of the SM extended by three right-handed neutrinos but without a fundamental Higgs field. Instead of the SM Higgs field we assume four-fermion couplings involving the lepton doublets and the right-handed neutrinos. In addition, since the right-handed neutrinos are singlets under the SM gauge group and since there is no protective symmetry, we assume them to have huge Majorana masses.

In order to show the essential aspects of such a scenario, we consider first the case where only one of the four-fermion couplings drives the condensation, while the others vanish. Thus, the four-fermion Lagrangian is

$$\mathcal{L}_{4\mathrm{f}} = G^{(\nu)}(\overline{\ell_{\mathrm{L}}}\nu_{\mathrm{R}})(\overline{\nu_{\mathrm{R}}}\ell_{\mathrm{L}}), \tag{1}$$

where we have omitted the $SU(2)$ indices and where ℓ_{L}, ν_{R} stand for the relevant neutrino degrees of freedom. Moreover, we assume the Majorana mass matrix to be diagonal so that the condensing pair of neutrinos can be studied independently. Therefore, we need to consider only one Majorana mass term,

$$-\mathcal{L}_{\mathrm{M}} = \frac{1}{2}M\,\overline{\nu_{\mathrm{R}}}\nu_{\mathrm{R}}^{\mathrm{C}} + \mathrm{h.c.} \tag{2}$$

We will see that this describes the most interesting features of neutrino condensation. More general scenarios will be discussed briefly in Sections 4 and 5.

The question whether a non-perturbative solution for the ground state exists in the presence of the huge Majorana mass, i.e., if the gap equation has a non-trivial solution, will be studied in Section 2.2. If the gap equation produces a fermion condensate which is a doublet under $SU(2)_{\mathrm{L}}$ and which carries a suitable $U(1)_{\mathrm{Y}}$ charge, then it is immediately clear that the $SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}$ gauge symmetry is broken. For the chosen NJL-like interaction a composite Higgs particle emerges. This is visualized in Fig. 1, where a massive scalar pole and three massless Goldstone bosons are produced by the summation of a certain class of diagrams with dynamical fermion propagators. The Higgs mechanism and the "eating" of the Goldstone bosons is illustrated in Fig. 2. For more details see, e.g., [8].

We analyze now the gap equations of our model with an explicit Majorana mass term for ν_{R}. If a non-trivial solution produces dynamically a large Dirac mass term, it breaks the EW symmetry. Then the presence of the huge singlet Majorana mass will lead to a dynamical see-saw mechanism with small neutrino masses. A computation of the gap equation in the basis of mass eigenstates must therefore include in a self-consistent way the possibility of

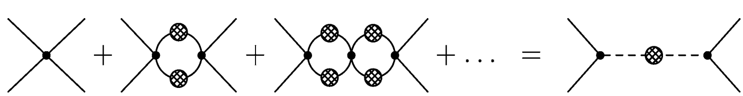

Fig. 1. The exchange of a virtual composite Higgs scalar can be seen as a sum over all loop contributions involving the four-fermion vertex in the so-called bubble sum approximation. Hatched blobs denote full propagators.

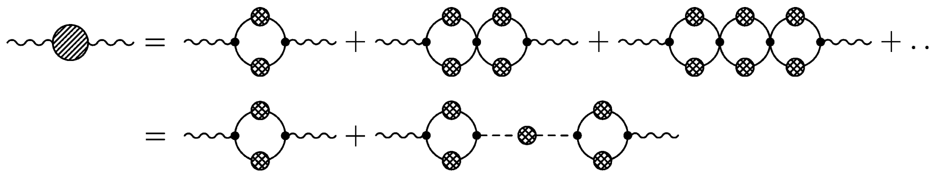

Fig. 2. Dynamical generation of gauge boson masses. The bubble sum in the second line is expressed in terms of the composite Higgs propagator from Fig. 1 in the third line. The shaded blob on the left side is the OPI 2-point vertex function.

a dynamically generated Dirac mass term,

$$-\mathcal{L}_{\mathrm{D}} = D\overline{\nu_{\mathrm{L}}}\nu_{\mathrm{R}} + \text{h.c.} \tag{3}$$

2.1. Mass eigenstates and eigenvalues

For any value of D, the mass eigenstates are two Majorana fermions, given by

$$\begin{pmatrix} \lambda \\ \rho \end{pmatrix} = U \cdot \begin{pmatrix} \lambda' \\ \rho' \end{pmatrix} \tag{4}$$

with $\lambda' := \nu_{\mathrm{L}} + \nu_{\mathrm{L}}^{\mathrm{C}}$ and $\rho' := \nu_{\mathrm{R}} + \nu_{\mathrm{R}}^{\mathrm{C}}$ and the orthogonal matrix

$$U = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} =: \begin{pmatrix} c & s \\ -s & c \end{pmatrix}. \tag{5}$$

The corresponding mass eigenvalues

$$m_\lambda = \frac{1}{2}\bigl(M - \sqrt{4D^2 + M^2}\bigr), \tag{6a}$$

$$m_\rho = \frac{1}{2}\bigl(M + \sqrt{4D^2 + M^2}\bigr) \tag{6b}$$

are related to φ by

$$\varphi = \arctan\sqrt{-\frac{m_\lambda}{m_\rho}}. \tag{7}$$

For convenience, we rewrite the singlet Majorana mass term (2) as well as the neutrino part of the four-fermion interaction (1) in terms of Majorana fermions,

$$-\mathcal{L}_{\mathrm{M}} = \frac{1}{2}M\overline{\nu_{\mathrm{R}}}\nu_{\mathrm{R}}^{\mathrm{C}} + \text{h.c.} = \frac{1}{2}M\overline{\rho'}P_{\mathrm{R}}\rho'^{\mathrm{C}} + \text{h.c.}, \tag{8}$$

$$\mathcal{L}_{4\nu} = G^{(\nu)}(\overline{\nu_{\mathrm{L}}}\nu_{\mathrm{R}})(\overline{\nu_{\mathrm{R}}}\nu_{\mathrm{L}}) = G^{(\nu)}\bigl(\overline{\lambda'}P_{\mathrm{R}}\rho'\bigr)\bigl(\overline{\rho'}P_{\mathrm{L}}\lambda'\bigr). \tag{9}$$

The Feynman rules for the interactions of the mass eigenstates are derived from these Lagrangians by inserting the relations of Eq. (4).

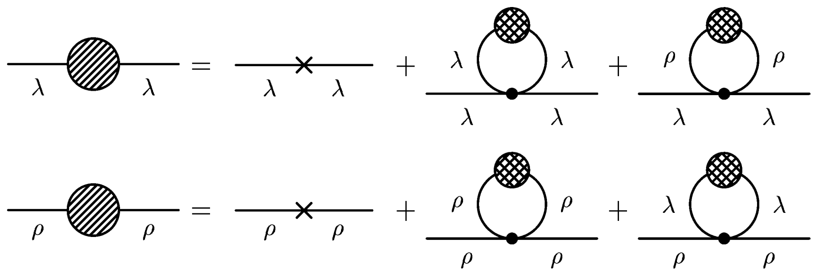

Fig. 3. Gap equations for m_λ, the mass eigenvalue of the light neutrino, and m_ρ, the mass eigenvalue of the heavy neutrino. The explicit Majorana mass from Eq. (8) is indicated by a cross. The shaded blobs on the left side are the OPI 2-point vertex functions, whereas the hatched blobs on the right side are full propagators.

2.2. *The coupled gap equations*

The gap equations for the masses m_λ and m_ρ, corresponding to Fig. 3, are

$$m_\lambda = 2G^{(\nu)}c^2s^2\big[m_\lambda I_{\text{gap}}(m_\lambda) - m_\rho I_{\text{gap}}(m_\rho)\big] + s^2M, \tag{10a}$$

$$m_\rho = 2G^{(\nu)}c^2s^2\big[m_\rho I_{\text{gap}}(m_\rho) - m_\lambda I_{\text{gap}}(m_\lambda)\big] + c^2M, \tag{10b}$$

where we have introduced

$$\frac{1}{2}I_{\text{gap}}(m) := -\frac{\Lambda^2}{16\pi^2}\left[1 - \frac{m^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m^2}+1\right)\right]. \tag{11}$$

Λ is the condensation scale, which acts as a cutoff. As $m_\lambda + m_\rho = M$ due to Eqs. (6), the gap equations are linearly dependent, so that it is sufficient to solve one of them. Note that since the mass eigenvalues are given by m_λ and m_ρ, non-trivial solutions also imply a dynamically generated Dirac mass D. Such solutions are found to exist indeed, as will be shown in Section 2.3. Remarkably, in this kind of gap equations, the heavy Majorana degree of freedom plays an essential role and does not decouple.

To check the self-consistency of our calculation, we consider the gap equation for a bilinear term that contains the fields λ and ρ as shown in Fig. 4. As we are working in the mass eigenstate basis, this term, which corresponds to an off-diagonal entry $m_{\lambda\rho}$ in the mass matrix, has to vanish identically. We obtain

$$\begin{aligned} m_{\lambda\rho} &= G^{(\nu)}[\cot\varphi - \tan\varphi]c^2s^2\big[m_\lambda I_{\text{gap}}(m_\lambda) - m_\rho I_{\text{gap}}(m_\rho)\big] + csM \\ &= \frac{1}{2}\cot\varphi\big[m_\lambda - s^2M\big] + \frac{1}{2}\tan\varphi\big[m_\rho - c^2M\big] + csM. \end{aligned} \tag{12}$$

Using the gap equations (10) as well as the relations (6) and (7) for the mass eigenvalues, it follows that $m_{\lambda\rho}$ vanishes as required. This confirms that our solution is self-consistent.

2.3. *Numerical solution of the gap equation*

The gap equation (10a) for m_λ can be considered as an equation for $G^{(\nu)}$ and D, if fixed values are assigned to M and Λ. For $M = 10^{14}$ GeV and $\Lambda = 10^{16}$ GeV, the solution is shown in Fig. 5. Instead of $G^{(\nu)}$, we plot the dimensionless coupling constant

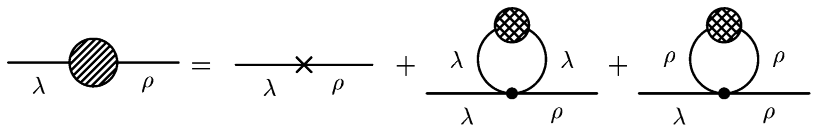

Fig. 4. Self-consistency check for the off-diagonal elements of the gap equations in the mass eigenstate basis. We find that the right-hand side vanishes as required.

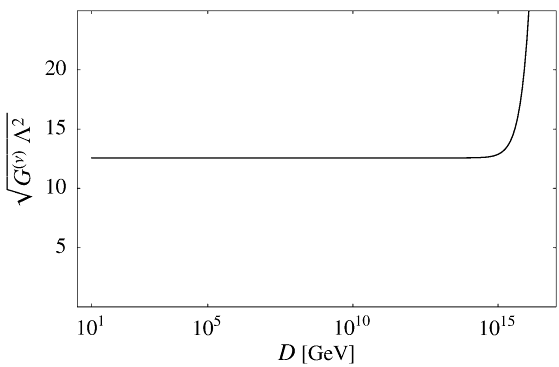

Fig. 5. Characteristic numerical solution of the gap equation for the four-fermion coupling $G^{(\nu)}$ and the dynamically generated Dirac mass D with a Majorana mass $M = 10^{14}$ GeV and a condensation scale $\Lambda = 10^{16}$ GeV.

$g = \sqrt{G^{(\nu)}\Lambda^2}$. We find non-trivial solutions for D, if the coupling g is larger than a critical value. This result is quite similar to top condensation [7], even though the right-handed neutrino has a large Majorana mass. Note, however, that in order to obtain correct solutions, the exact relations (6) for the mass eigenvalues have to be used rather than an expansion in powers of D/M.

In order to obtain a Dirac mass of the order of the EW scale, fine-tuning is required in the bubble sum approximation as can be seen from the extremely small slope of the graph in Fig. 5. Even if the same fine-tuning is present in the exact gap equation, loop corrections, which destabilize the hierarchy in the usual perturbative framework of the SM, do not pose an additional problem here. In other words, the dynamical scenario under consideration cannot explain why the hierarchy is large, but it does explain why it remains large. In that sense the hierarchy is stable and hence the model might even be considered to be on equal footing with the solution of the hierarchy problem by supersymmetry.

3. Phenomenology

3.1. *Effective low-energy theory*

The above results for m_ρ and m_λ were obtained in the bubble approximation. In order to obtain more reliable low-energy results, the renormalization group (RG) running of the effective theory has to be taken into account. Following the procedure used in [7], we show first that the low-energy effective theory of the model with four-fermion interactions is just the SM with right-handed neutrinos.[2] Therefore, we look at the original Lagrangian at the condensation scale Λ,

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{M}} + G^{(\nu)}(\overline{\ell_{\text{L}}}\nu_{\text{R}})(\overline{\nu_{\text{R}}}\ell_{\text{L}}), \tag{13}$$

where $\mathcal{L}_{\text{kin}}$ contains kinetic terms for the gauge bosons, the usual SM fermions and the right-handed neutrinos. There is no Higgs field present. We rewrite $\mathcal{L}$ in terms of a static (non-propagating) scalar auxiliary field ϕ,

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{M}} - (\overline{\ell_{\text{L}}}\phi\nu_{\text{R}} + \text{h.c.}) - \frac{1}{G^{(\nu)}}\phi^\dagger\phi. \tag{14}$$

This Lagrangian is equivalent to the Lagrangian in Eq. (13), which can be seen by exploiting the equations of motion for the auxiliary field,

$$\phi = -G^{(\nu)}\overline{\nu_{\text{R}}}\ell_{\text{L}}. \tag{15}$$

The same result can be found by integrating out the auxiliary field in the path integral formalism.

At scales below Λ, the dynamics of the theory will induce all renormalizable and gauge invariant terms that are allowed by symmetries, including a kinetic term and a quartic self-interaction for ϕ. Thus, the Lagrangian becomes

$$\begin{aligned}\mathcal{L} = {} & \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{M}} + Z(D_\mu\phi)^\dagger\big(D^\mu\phi\big) - (\overline{\ell_{\text{L}}}\phi\nu_{\text{R}} + \text{h.c.}) \\ & + \widetilde{m}^2\phi^\dagger\phi - \frac{\tilde{\lambda}}{4}\big(\phi^\dagger\phi\big)^2.\end{aligned} \tag{16}$$

Note that Z, $\widetilde{m}$ and $\tilde{\lambda}$ are running quantities, even though their dependence on the energy scale μ is not written explicitly. For $\mu \to \Lambda$ the Lagrangian (16) has to become identical to the one of Eq. (14), which leads to the following boundary (compositeness) conditions for the RG evolution:

$$Z \xrightarrow{\mu\to\Lambda} 0, \tag{17a}$$

$$\widetilde{m}^2 \xrightarrow{\mu\to\Lambda} -\frac{1}{G^{(\nu)}}, \tag{17b}$$

$$\tilde{\lambda} \xrightarrow{\mu\to\Lambda} 0. \tag{17c}$$

Eq. (16) is already very similar to the SM Lagrangian. The auxiliary field has acquired a kinetic term, i.e., it has become a propagating composite Higgs field, but its Lagrangian is not yet written in the usual normalization where $Z \equiv 1$. To fix this we perform the rescaling

$$\phi \longrightarrow \frac{1}{\sqrt{Z}}\phi \equiv y_\nu\phi, \tag{18}$$

[2] We demonstrate this for the simplified case of Section 2, where all non-critical four-fermion couplings vanish. By including such couplings, the missing Yukawa interactions and masses can easily be incorporated.

which leads to the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{M}} + (D_\mu\phi)^\dagger\big(D^\mu\phi\big) - y_\nu(\overline{\ell_{\text{L}}}\phi\nu_{\text{R}} + \text{h.c.}) \\ + m^2\phi^\dagger\phi - \frac{\lambda}{4}\big(\phi^\dagger\phi\big)^2, \tag{19}$$

where we have defined

$$m^2 := y_\nu^2\widetilde{m}^2 \quad \text{and} \quad \lambda := y_\nu^4\tilde{\lambda}. \tag{20}$$

Thus, we have recovered the SM extended by right-handed neutrinos, which proves that the Lagrangian (13) yields exactly the same low-energy physics, but with additional constraints on the parameters, namely the compositeness conditions. From Eqs. (17) and (18) we find that the compositeness conditions for the couplings and the mass parameter become

$$\frac{1}{y_\nu^2} \xrightarrow{\mu\to\Lambda} 0, \tag{21a}$$

$$\frac{m^2}{y_\nu^2} \xrightarrow{\mu\to\Lambda} -\frac{1}{G^{(\nu)}}, \tag{21b}$$

$$\frac{\lambda}{y_\nu^4} \xrightarrow{\mu\to\Lambda} 0. \tag{21c}$$

The low-energy neutrino and Higgs masses are now obtained from the RG equations for the SM extended by right-handed neutrinos. For the RG analysis in see-saw models, it is crucial to integrate out the right-handed neutrinos and the corresponding part of the neutrino Yukawa coupling matrix at the mass thresholds. The computation of the RG evolution requires the β-functions of all gauge and Yukawa couplings (including those of the neutrinos), of the quartic Higgs coupling and of the dimension 5 neutrino mass operator (see, e.g., [17,18]). For completeness, we have listed the relevant β-functions in Appendix A. When integrating out the right-handed neutrinos, the dimension 5 neutrino mass operator has to be matched at each threshold [19]. In this framework, we calculate the low-energy parameters, starting with the compositeness conditions at the condensation scale and solving the relevant systems of coupled differential equations. The results for the Higgs mass and for the mass of the neutrino participating in condensation turn out to be not very sensitive to the exact boundary conditions (21b) and (21c) due to the quasi-fixed-point behavior that arises once Eq. (21a) is satisfied.

3.2. Neutrino masses at low energy

The RG evolution of the Majorana mass of the light neutrino participating in condensation is shown in Fig. 6. It can be seen that a large, non-perturbative neutrino Yukawa coupling is in agreement with the current limits on neutrino masses, if the see-saw scale M is large enough. Since the neutrino Yukawa coupling at the condensation scale has to be non-perturbatively large and is thus not a free parameter of the model, an allowed range for the neutrino mass translates into an allowed range for M. For instance, with $y_\nu \in [2, 5]$, $M \in [10^{14}, 10^{15.5}\ \text{GeV}]$ at the condensation scale, which we have chosen to be $\Lambda = 10^{16}$ GeV, we obtain a range $m_\nu \in [0.02, 1.36\ \text{eV}]$ for the neutrino mass at low

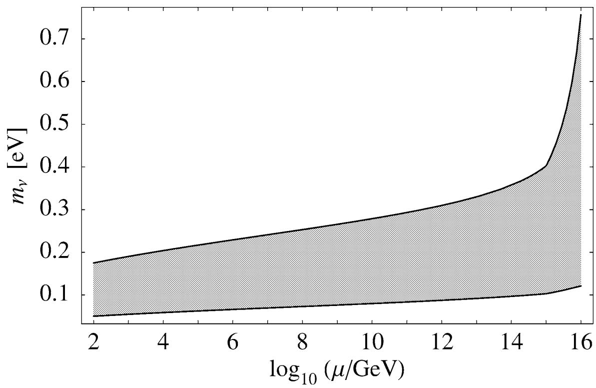

Fig. 6. Running of the mass of the neutrino participating in condensation. The starting values for the neutrino Yukawa coupling are in the range $y_\nu \in [2, 5]$ at the scale of new physics, Λ. The Majorana mass of the right-handed neutrino is $M = 10^{15}$ GeV at Λ, which we have chosen to be 10^{16} GeV in this example. The gray region shows the possible values of the neutrino mass with the above range of initial values. The resulting neutrino mass is in agreement with the current limits. It can of course be raised or lowered by varying M and further depends on Λ, which is a free parameter of the model. The kink in the evolution of the neutrino mass corresponds to the mass threshold at $\mu = M$. Below M, the heavy singlet is integrated out, producing an effective dimension 5 operator which yields a see-saw suppressed Majorana mass.

energy. Except for the neutrino and the top, the Yukawa couplings of the fermions have been omitted.

3.3. Higgs mass prediction

The Higgs mass can be predicted due to the quasi-fixed point in the RG evolution of the Higgs self-coupling. The running of the Higgs mass from the condensation scale Λ to the EW scale is shown in Fig. 7 for a wide range of parameters M, y_ν and λ at the scale Λ. For $\Lambda = 10^{16}$ GeV, we obtain Higgs masses in the range $170\ \mathrm{GeV} \lesssim m_H \lesssim 195\ \mathrm{GeV}$. One should keep in mind that this prediction depends on the condensation scale Λ, which is a free parameter in our model. As in Section 3.2, except for the neutrino and the top, the Yukawa couplings of the fermions have been omitted.

4. Three-neutrino condensation

The model discussed so far can be extended to the case where all three generations of neutrinos participate in the condensation. In general, the low-energy theory contains several Higgses. However, if the four-fermion couplings satisfy a "factorization relation" [13],

$$\sum_{f,g,h,i=1}^{3} G^{(\nu)}_{fghi} \left(\overline{\ell^f_{\mathrm{L}}} \nu^g_{\mathrm{R}}\right)\left(\overline{\nu^h_{\mathrm{R}}} \ell^i_{\mathrm{L}}\right) = \left(\sum_{f,g=1}^{3} h^{(\nu)}_{fg} \overline{\ell^f_{\mathrm{L}}} \nu^g_{\mathrm{R}}\right)\left(\sum_{h,i=1}^{3} h^{(\nu)*}_{ih} \overline{\nu^h_{\mathrm{R}}} \ell^i_{\mathrm{L}}\right), \tag{22}$$

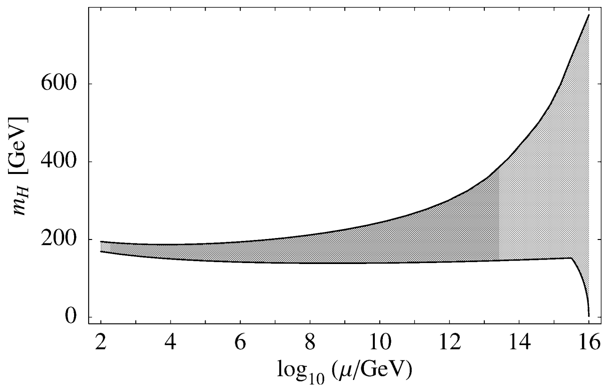

Fig. 7. RG evolution of the "Higgs mass" $m_H(\mu) = \sqrt{\lambda(\mu)/2}\, v_{\mathrm{EW}}$ (with $v_{\mathrm{EW}} = 246$ GeV), which equals at the EW scale the physical Higgs mass. The quasi-fixed point leads to a rather narrow range for the mass at the EW scale, in this example 170 GeV $\lesssim m_H \lesssim$ 195 GeV. Here we chose $\Lambda = 10^{16}$ GeV for the scale of new physics. The input parameters at this energy were varied within the intervals $y_\nu \in [2, 5]$, $M \in [10^{14}, 10^{15.5}$ GeV] and $\lambda \in [0, 20]$. We further allowed $m_t \in [170, 180$ GeV]. The gray region shows the possible values of the Higgs mass with these ranges of initial values.

then by following the steps of Eqs. (13)–(15), the Lagrangian can be rewritten in terms of *one* auxiliary field $\Phi \sim \sum_{fg} h^{(\nu)}_{fg} \overline{\ell^f_{\mathrm{L}}} \nu^g_{\mathrm{R}}$. Hence we obtain again a one-Higgs model after condensation analogous to Section 3.1. The gap equations can be treated as in Section 2. If three neutrinos condense, the infrared quasi-fixed point of the RG evolution leads to three heavy Dirac mass eigenvalues. The Majorana mass term of the right-handed neutrino in Eq. (2) must now be generalized to a mass matrix M_{ij} for three right-handed neutrinos, with entries which are unprotected by symmetries, leading to three heavy eigenvalues. The degeneracy or hierarchy of the see-saw scales is then conveyed rather directly into the full light neutrino mass pattern. The Higgs mass is almost unchanged compared to the results of Section 3.3, in spite of the contribution from the Yukawa couplings of the additional neutrinos. Using the same input parameters as in Fig. 7, now with three equally large Dirac masses for the neutrinos, we find 175 GeV $\lesssim m_H \lesssim$ 195 GeV.

5. Combined neutrino and top condensation

Besides the neutrino condensate discussed in Section 2, there can be a further condensate connected to the top quark. This means that in addition to the four-fermion coupling of Eq. (1) for the neutrino, there is a corresponding term for the top quark, $G^{(t)}(\overline{q_{\mathrm{L}}} t_{\mathrm{R}})(\overline{t_{\mathrm{R}}} q_{\mathrm{L}})$, and a mixed term $G^{(\nu t)}(\overline{\ell_{\mathrm{L}}} \nu_{\mathrm{R}})(\overline{t_{\mathrm{R}}} q_{\mathrm{L}})$. If $G^{(\nu)}$, $G^{(t)}$ and $G^{(\nu t)}$ become critical, then in general two independent condensates form, similar to the case of combined top and bottom condensation [9]. Note that for $(G^{(\nu t)})^2 = G^{(\nu)} \cdot G^{(t)}$ this is a one-Higgs scenario, which coincides with the model discussed by Martin [13]. However, in general we are dealing with an effective Two Higgs Doublet Model (2HDM), where the effective Lagrangian does not coincide with the 2HDMs usually discussed, which are phenomenologically severely constrained. The scalars which are generated dynamically

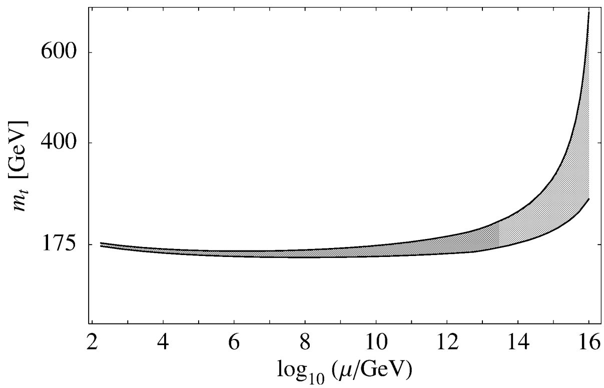

Fig. 8. Running of the "top mass" $m_t = y_t(\mu) v_{\mathrm{EW}} \sin(\beta)/\sqrt{2}$ in a 2HDM with a leptonic and a hadronic Higgs, formed by a neutrino and a top condensate, and with $\tan\beta = 1.3$ at the electroweak scale. We have chosen the value of $\tan\beta$ such that the quasi-fixed point of the RG evolution reproduces $m_t \approx 175$ GeV. The initial range for the top Yukawa coupling at the scale of new physics, chosen to be $\Lambda = 10^{16}$ GeV in this example, is $y_t \in [2, 5]$. The gray region contains the resulting values for the top mass.

here are a "leptonic Higgs", ϕ_ν, and a "hadronic Higgs", ϕ_t. The mixed coupling $G^{(\nu t)}$ leads to a term of the form $m_{\nu t}^2 \phi_\nu^\dagger \phi_t$ in the effective Higgs potential. Besides, four-Higgs interactions with odd numbers of ϕ_ν and ϕ_t become allowed in the low-energy effective theory.

An interesting feature of 2HDMs with a leptonic and a hadronic Higgs is that the correct top mass can now be obtained dynamically as a quasi-fixed point of the RG evolution. This has been found to be impossible in minimal models with pure top condensation, where the predicted top mass is too large. In the two-Higgs case, the necessary additional degree of freedom is the ratio of the VEVs, $\tan\beta$. A top mass of approximately 175 GeV is obtained for $\tan\beta \approx 1$, as illustrated in Fig. 8. This also yields large Dirac masses for the neutrinos, but due to the see-saw suppression their physical masses remain tiny.

Another issue that has to be addressed is the Higgs mass spectrum, as massless or very light Higgses would be in conflict with experiments. As suggested by the analysis of the case with a top and a bottom condensate [9], it should be possible to obtain phenomenologically viable Higgs masses by making the parameter $m_{\nu t}^2$ sufficiently large. Hence, it seems worthwhile to study 2HDMs with neutrino and top condensates in more detail.

6. Discussion and conclusions

In this article we have studied the possibility that the EW symmetry is broken dynamically by the formation of a neutrino condensate. We have started from the SM with right-handed neutrinos, but without a fundamental Higgs field. Instead of the Higgs field we have postulated strong attractive four-fermion interactions. In addition, we have included huge Majorana masses for the right-handed neutrinos, since there is no protective symmetry. The analysis of the coupled Dirac–Majorana gap equations has shown that non-trivial solutions exist, which dynamically produce a large Dirac neutrino mass and which break the EW symmetry in the desired way. However, due to the presence of the huge Majorana mass term we obtain physical neutrino masses in the phenomenologically allowed region via the see-saw mechanism. We have illustrated in the auxiliary field formalism that the NJL-type interactions lead to a composite scalar sector which resembles a Higgs Lagrangian with certain boundary conditions, i.e., predictions. In particular, the compositeness conditions require that the Yukawa coupling y_ν of the condensing neutrino becomes non-perturbative at high energies. To evaluate the predictions, we have translated these boundary conditions in the framework of the low-energy effective Lagrangian into high-energy boundary conditions for the renormalization group running.

For our minimal model discussed in Sections 2 and 3 the effective Lagrangian is the SM with massive neutrinos. The boundary conditions and the infrared quasi-fixed-point behavior of the Higgs self-coupling and of the relevant Yukawa coupling y_ν lead to two predictions for a given condensation scale Λ. For $\Lambda = 10^{16}$ GeV, typical values of $y_\nu(\Lambda)$ and $\lambda(\Lambda)$ consistent with the boundary conditions, and for $M \in [10^{14}, 10^{15.5}\ \text{GeV}]$, we have found a Higgs mass in the range 170 GeV $\lesssim m_H \lesssim$ 195 GeV, which is in agreement with current experimental bounds. Moreover, we have found that an upper bound for the neutrino mass of the order of 1 eV (0.1 eV) translates into a lower bound for the see-saw scale M of the order of 10^{14} GeV (10^{15} GeV).

We have also briefly studied the possibility that more than one neutrino condenses. This leads in general to several Higgses and under a factorization condition to a one-Higgs scenario. Furthermore, we have outlined the possibility of a combined neutrino-top condensation, which corresponds without further assumptions to a two-Higgs model. In addition, it might be possible to extend neutrino condensation to a supersymmetric model.

We have not attempted to embed these scenarios into a larger framework where the four-fermion terms are generated in strongly coupled broken gauge theories as it was done in “top color” theories [15] in the case of top condensation.[3] This should be interesting, since such a framework would, for example, allow to address the question if the gauge couplings of the extended gauge sector unify above the condensation scale. In addition, threshold effects near the condensation scale (in combination with extended gauge sectors or by themselves) might affect unification. Such threshold corrections are generally expected to be large in this non-perturbative scenario.

To conclude, we have introduced a dynamical realization of electroweak symmetry breaking with massive neutrinos, where the SM Higgs particle emerges from neutrino condensation, leading to predictions for the Higgs mass and for the see-saw scale.

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[3] Note that such scenarios can possess the attractive property of generating Yukawa couplings via gauge couplings.

Appendix A. Renormalization group equations

In the SM extended by right-handed neutrinos one has to consider several effective theories, corresponding to the ranges between the non-degenerate eigenvalues of the Majorana mass matrix M. At the thresholds, the heavy degrees of freedom are successively integrated out. As introduced in [19], a superscript (n) denotes a quantity between the nth and the $(n+1)$th mass threshold. With this definition, the β-functions for the Yukawa coupling matrices are given by

$$16\pi^2 \overset{(n)}{\beta}_{Y_\nu} = \overset{(n)}{Y_\nu}\left\{\frac{3}{2}\left(\overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu}\right) - \frac{3}{2}\left(Y_e^\dagger Y_e\right) - \frac{3}{4}g_1^2 - \frac{9}{4}g_2^2 + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right]\right\}, \tag{A.1a}$$

$$16\pi^2 \overset{(n)}{\beta}_{Y_e} = Y_e\left\{\frac{3}{2}Y_e^\dagger Y_e - \frac{3}{2}\overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} - \frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right]\right\}, \tag{A.1b}$$

$$16\pi^2 \overset{(n)}{\beta}_{Y_d} = Y_d\left\{\frac{3}{2}Y_d^\dagger Y_d - \frac{3}{2}Y_u^\dagger Y_u - \frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right]\right\}, \tag{A.1c}$$

$$16\pi^2 \overset{(n)}{\beta}_{Y_u} = Y_u\left\{\frac{3}{2}Y_u^\dagger Y_u - \frac{3}{2}Y_d^\dagger Y_d - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right]\right\}. \tag{A.1d}$$

The β-function for the Majorana mass matrix M reads

$$16\pi^2 \overset{(n)}{\beta}_M = \left(\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger}\right)\overset{(n)}{M} + \overset{(n)}{M}\left(\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger}\right)^T, \tag{A.2}$$

and the RG evolution of the quartic Higgs self-coupling is determined by[4]

$$\begin{aligned}16\pi^2 \overset{(n)}{\beta}_\lambda = {} & 6\lambda^2 - 3\lambda\left(3g_2^2 + g_1^2\right) + 3g_2^4 + \frac{3}{2}\left(g_1^2 + g_2^2\right)^2 \\ & + 4\lambda\,\mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right] \\ & - 8\,\mathrm{Tr}\left[Y_e^\dagger Y_e Y_e^\dagger Y_e + \overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu} + 3Y_d^\dagger Y_d Y_d^\dagger Y_d + 3Y_u^\dagger Y_u Y_u^\dagger Y_u\right].\end{aligned} \tag{A.3}$$

[4] To our knowledge, this β-function has not yet been written explicitly in the literature.

Finally, the β-function for the effective neutrino mass operator reads

$$\begin{aligned}16\pi^2 \overset{(n)}{\beta}_\kappa = {} & -\frac{3}{2}\left(Y_e^\dagger Y_e\right)^T \overset{(n)}{\kappa} - \frac{3}{2} \overset{(n)}{\kappa}\left(Y_e^\dagger Y_e\right) + \frac{1}{2}\left(\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu\right)^T \overset{(n)}{\kappa} + \frac{1}{2} \overset{(n)}{\kappa}\left(\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu\right) \\ & + 2\,\mathrm{Tr}\left(Y_e^\dagger Y_e\right) \overset{(n)}{\kappa} + 2\,\mathrm{Tr}\left(\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu\right) \overset{(n)}{\kappa} + 6\,\mathrm{Tr}\left(Y_u^\dagger Y_u\right) \overset{(n)}{\kappa} \\ & + 6\,\mathrm{Tr}\left(Y_d^\dagger Y_d\right) \overset{(n)}{\kappa} - 3g_2^2 \overset{(n)}{\kappa} + \lambda \overset{(n)}{\kappa}. \end{aligned} \tag{A.4}$$

The one-loop β-functions for the gauge couplings are of course unchanged compared to the SM.

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