Title
A HITHERTO OVERLOOKED SU3 OCTET OF REGGE POLES IMPLIED BY BOOTSTRAP DYNAMICS

Author
Pignotti, Alberto.

Publication Date
1963-12-03

UCRL-11152

University of California

Ernest O. Lawrence Radiation Laboratory

A HITHERTO OVERLOOKED SU_3 OCTET OF REGGE POLES IMPLIED BY BOOTSTRAP DYNAMICS

Berkeley, California

Rept. submitted for pub. in the Physical Review Letters.

UCRL-11152

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

A HITHERTO OVERLOOKED SU$_3$ OCTET OF REGGE POLES IMPLIED BY BOOTSTRAP DYNAMICS

Alberto Pignotti

December 3, 1963

A Hitherto Overlooked SU_3 Octet Of Regge Poles Implied by Bootstrap Dynamics*

Alberto Pignotti†

Lawrence Radiation Laboratory
University of California
Berkeley, California

December 3, 1963

ABSTRACT

The scattering of two pseudoscalar mesons belonging to the octet representation of SU_3 is studied in the framework of bootstrap dynamics. It is found that the leading Regge poles correspond to a singlet and two octet SU_3 representations. The singlet state is unambiguously identified with the Pomeranchuk trajectory, and one of the octets corresponds to the well known vector mesons. The other octet is composed of even-signature poles which are called P', Q, and R, and which, in a first approximation, coincide in position with the vector-meson trajectories. These poles are not likely to manifest themselves as low-mass particles or resonances because of their positive signature, but they are expected to influence appreciably high-energy scattering in the crossed channel. In particular, the P' pole plays the role of the second vacuum trajectory proposed by Igi, and the R is suitable to explain, through an interference with the ρ pole, the sign, value, and high-energy dependence of the difference between total pp and np cross sections.

A HITHERTO OVERLOOKED SU_3 OCTET OF REGGE POLES IMPLIED BY BOOTSTRAP DYNAMICS*

Alberto Pignotti†

Lawrence Radiation Laboratory
University of California
Berkeley, California

December 3, 1963**

In this note we want to point out the existence of an octet of even-signature Regge poles that follows from assuming two features that have been recognized to play important roles in strong interactions: SU_3 symmetry[1] and the "bootstrap" dynamical model.[2] The isospin-0 member of the octet is naturally identified with the second vacuum trajectory proposed by K. Igi,[3] whereas the Pomeranchuk Regge pole fits nicely into the dynamical scheme as belonging to the singlet representation of SU_3. The $T = 1/2$, $S = \pm 1$ and $T = 1$, $S = 0$ components of the octet are likely to produce noticeable interferences with the K^* and ρ Regge poles in high-energy scattering, thus making it possible to resolve some apparent inconsistencies that have been pointed out recently. Finally, the physical manifestations of the proposed trajectories in the resonance region may occur as spin-2^+ resonances, at energies of the order of 1.5 BeV.

We consider the scattering of two pseudoscalar mesons belonging to the octet representation of SU_3, in the limit of exact symmetry. It is well known that such a process can be written in terms of amplitudes corresponding to representations of dimensionality 1, 8, 8', 10, $\overline{10}$, and 27, and that under interchange of the final mesons the unitary spin dependence of the amplitudes A^1, A^8, and A^{27} is even, and of $A^{8'}$, A^{10}, and $A^{\overline{10}}$ odd. We denote these amplitudes A^e and A^o, respectively, and define the Froissart-Gribov continuation for any of these amplitudes as[4]

$$A^{N,(\pm)}(s,\ell) = A_t^N(s,\ell) \pm A_u^N(s,\ell) , \tag{1}$$

where

$$A^N_{t,u}(s,\ell) = \frac{1}{\pi} \int_{1+\frac{8}{s-4}}^{\infty} dz' \sum_{N'} \beta^{NN'}_{t,u} D^{N'}_{t,u}\left(s, \frac{(s-4)(z'-1)}{2}\right) Q_\ell(z') . \tag{2}$$

Here we have taken the pseudoscalar mass to be one, s is the square of the energy in the center-of-mass system, $D_t^{N'}(s,t)$ and $D_u^{N'}(s,u)$ are the absorptive parts in the crossed t and u channels corresponding to the representation N', and $\beta^{NN'}_{t,u}$ are the crossing matrices in unitary spin space. Bose statistics, and the symmetry properties of $A^{e,o}$ mentioned above imply

$$A^{e,(-)}(s,\ell) \equiv A^{o,(+)}(s,\ell) \equiv 0$$

and therefore

$$A^{e,(+)}(s,\ell) = 2\, A_t^e(s,\ell)$$

$$A^{o,(-)}(s,\ell) = 2\, A_t^o(s,\ell) ,$$

which shows that the effect of the u channel can be represented by a factor of two in the contribution from the t channel. Furthermore, we can drop the signature index $(\pm)$, because for nonzero amplitudes it is determined by the evenness or oddness of the representation.

The so-called "bootstrap" philosophy now requires that all Regge poles appearing in the above amplitudes be bound states caused by the exchange of the same poles in the crossed t channel.[2] In the following we do not attempt to solve the self-consistency equations that arise from this principle, but rather to examine what can be said about the solution without actually solving the problem.

For this purpose we adopt the criterion that the sign of the force due to a Regge-pole exchange is given by the sign of the Born approximation for the exchange of the physical particles lying on the Regge trajectory. We obtain the Born approximation by keeping only a resonant partial wave in the expansion for $D_t^{N'}$ in Eq. (2). This yields

$$A^N_{B.A.}(s,\ell) = \gamma \; \beta_t^{NN'}(2\ell_t + 1)\, P_{\ell_t}\left(1 + \frac{2s}{t_R - 4}\right) Q_\ell\left(1 + \frac{2t_R}{s - 4}\right) \frac{4t_R}{(s-4)(t_R-4)^{\frac{1}{2}}} \tag{3}$$

for the exchange of a resonance of energy squared t_R, width γ, and spin ℓ_t, belonging to the N' representation in the t channel.[5] It is well known that a positive Born approximation above threshold corresponds to an attractive force. Inspection of Eq. (3) therefore shows that the sign of the crossing matrix element $\beta_t^{NN'}$ alone determines the sign of the force and that a positive element corresponds to attraction.[6]

We now go back to general arguments, which do not depend further on the Born approximation. In our problem the crossing matrix, worked out by Neville, is[7]

$$\beta_t^{NN'} = \begin{pmatrix} 1/8 & 1 & 1 & 5/4 & 5/4 & 27/8 \\ 1/8 & -3/10 & 1/2 & -1/2 & -1/2 & 27/40 \\ 1/8 & 1/2 & 1/2 & 0 & 0 & -9/8 \\ 1/8 & -2/5 & 0 & 1/4 & 1/4 & -9/40 \\ 1/8 & -2/5 & 0 & 1/4 & 1/4 & -9/40 \\ 1/8 & 1/5 & -1/3 & -1/12 & -1/12 & 7/40 \end{pmatrix}, \qquad (4)$$

for $N,N' = 1,\ 8,\ 8',\ 10,\ \overline{10}$, and 27. By examining this matrix we find the following features:

(a) The leading Regge trajectory corresponds to the singlet representation of SU_3. In fact, if we consider the forces arising in the various states in the s channel from any given t channel exchange, we see that the resulting force in the singlet state is as strong or stronger than the force in any other state. The quantum numbers $S = 0$, $T = 0$, $G = +$, and signature $\tau = +$ associated with the singlet representation coincide with those of the Pomeranchuk trajectory, thus making the correspondence unambiguous.[8]

(b) The exchange of the singlet state does not play an important dynamical role, because it contributes the same force to all states.

(c) The 8' representation, which is naturally associated with the ρ, K^*, $\overline{K}^*$, and ϕ vector mesons,[9] is apt to be self-consistent. This was noted by Neville, who also pointed out that the same is true for the 10 and $\overline{10}$ representations.[7] However, Chew has given arguments that show that a resonant solution for the 10 and $\overline{10}$ states almost certainly implies a violation of the Froissart limit, and that we should therefore expect the completely self-consistent solution to contain no 10 and $\overline{10}$ resonances.[10] If we adopt this point of view and look again at the crossing matrix (4) emphasizing the forces due to the exchanges of the established 1 and 8' states, (columns 1 and 3), we find the following feature.

(d) The forces on the 8 channel are equal to those on the 8' state. We are thus led to predict an 8 Regge pole, coinciding in position with the known 8'. This statement is exact in the absence of inelastic channels and if we consider only 1 and 8' exchanges. However, it is clear that if an 8 Regge pole is present, we must include it as an acting force too. We expect this force to be small, because even if this pole manifests itself as a spin-two resonance, the latter will occur at a considerably higher energy.[11,12] In fact, if we displaced horizontally the Pomeranchuk trajectory requiring it to cross $\mathrm{Re}\,\alpha = 1$ at the average energy-squared of the 8' vector mesons, it would cross $\mathrm{Re}\,\alpha = 2$ at approximately $(1.5\ \mathrm{BeV})^2$.[13] A quantitative estimate of the force due to the exchange of a spin-2 resonance at that energy should properly take into account the effect of the Regge behavior in order to avoid embarrassing divergences, and is beyond the scope of this note. We only remark that neglecting distant signularities has been a basic approximation in the use of dispersion relations, and that in our case not only the real part of the pole position is large, but also the imaginary part is probably substantial, suggesting that the physical manifestations of the pole are weak. Therefore, we are inclined to consider the 8 exchange force as a perturbation that will not profoundly change the established pattern. The crossing matrix (4) shows that the effect of the 8 exchange tends to split the 8' and 8 Regge poles by raising the 8' and depressing the 8 itself. Of course, because SU_3 symmetry is not exact, we also expect some splitting among the different isomultiplets in our octet.

We have succeeded in showing that nothing spectacular should be expected from the proposed octet of Regge poles in the region of low-energy resonances, thus explaining why they have not been found there. On the other hand, the effect of these poles is likely to be important on scattering processes in the crossed channels. In fact, our octet accommodates naturally as its isospin-0 member a pole that was proposed more than one year ago by Igi, in order to avoid a discrepancy between a theoretical prediction and the experimental values of the total πp cross section.[3] The poles proposed by Igi and by us have similar positions at $t = 0$, and both have quantum numbers $S = 0$, $T = 0$, $G = +$, and $\tau = +$. However, we should remark that in our scheme the second vacuum trajectory belongs to a different SU_3 representation than the Pomeranchuk pole, and thus we avoid the feature of having two significant poles with exactly the same quantum numbers.

In addition, Dr. Chew has pointed out that another apparent anomaly in high-energy scattering may be explained by the inclusion of the $T = 1$ member of our octet, which we denote by R.[14] This is the fact, discussed by Sharp and Wagner[15] and by Phillips,[16] that the contribution of the ρ Regge pole to high-energy total pp, np, and $p\bar{p}$ cross sections has been inferred from experimental data to be abnormally small, and of a sign opposite to that expected from the assumption of real analyticity of the factorized residues. Introducing our R pole, with the quantum numbers $S = 0$, $T = 1$, $G = -$ and $\tau = +$, we can write

$$\sigma^T(pp) = \mathcal{R}^P_{(s)} + \mathcal{R}^{P'}_{(s)} - \mathcal{R}^{\omega}_{(s)} - \mathcal{R}^{\rho}_{(s)} + \mathcal{R}^{R}_{(s)}$$

$$\sigma^T(np) = \mathcal{R}^P_{(s)} + \mathcal{R}^{P'}_{(s)} - \mathcal{R}^{\omega}_{(s)} + \mathcal{R}^{\rho}_{(s)} - \mathcal{R}^{R}_{(s)}$$

$$\sigma^T(p\bar{p}) = \mathcal{R}^P_{(s)} + \mathcal{R}^{P'}_{(s)} + \mathcal{R}^{\omega}_{(s)} + \mathcal{R}^{\rho}_{(s)} + \mathcal{R}^{R}_{(s)}$$

where the real analiticity of the factorized residues implies that the functions $\mathcal{R}(s)$ are positive. It is clear that a small positive value of $\sigma^T(pp) - \sigma^T(np)$ for momenta in the region of 2 to 3 BeV can be explained by an approximate cancellation occurring between the contributions of the ρ and R poles, both having "normal-sized" residues with the expected reality-analyticity properties.[17] We also note that the introduction of the R trajectory preserves the feature of having a large value for $\sigma^T(p\bar{p}) - \sigma^T(np)$. The R pole should also contribute appreciably to the differential cross sections for KN and NN charge-exchange scattering,[18] thus requiring revision of some results obtained for the ρ trajectory from the assumption that it alone contributed.[16,19]

Finally, we want to mention that the last members of our octet, with quantum numbers $S = \pm 1$, $T = 1/2$, and $\tau = +$, which we denote by Q and $\bar{Q}$, are expected to contribute to the high-energy differential cross sections for processes involving strangeness exchange, such as $NK \rightarrow \Sigma\pi$ and $p\bar{p} \rightarrow \Sigma\bar{\Sigma}$. Inclusion of the Q trajectory may explain, for instance, why the angular distribution for the latter process could not be fitted with the K^* Regge pole exchange alone.[20]

I wish to express my sincere gratitude to Professor Chew for his advice and encouragement, and to acknowledge a fruitful suggestion by Dr. Charles Zemach.

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U. S. Atomic Energy Commission.

† Fellow of the Consejo Nacional de Investigaciones Científicas y Técnicas of Argentina. Present address: Facultad de Ciencias Exactas y Naturales, Departamento de Física, Perú 222, Buenos Aires, Argentina.

1. M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished), and Y. Ne'eman, Nucl. Phys. 26, 222 (1961).
2. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961) and 8, 41 (1962).
3. K. Igi, Phys. Rev. Letters 9, 76 (1962) and Phys. Rev. 130, 820 (1963).
4. M. Froissart, Report to the La Jolla Conference on Theoretical Physics, June 1961 (unpublished), and V. N. Gribov, Zh. Eksperim. i Teor. Fiz. 41, 667 and 1962 (1961).
5. We have used here the approximation of neglecting the imaginary part of the pole position.
6. Similar arguments have been used repeatedly to make qualitative dynamical considerations. See for instance G. F. Chew, S. C. Frautschi, and S. Mandelstam, Phys. Rev. 126, 1202 (1962), and G. F. Chew, Phys. Rev. Letters 9, 233 (1962).
7. D. Neville, Phys. Rev. 132, 844 (1963).
8. G. F. Chew (Lawrence Radiation Laboratory, Berkeley, private communication), has conjectured that the presence of the strongest forces in the state with the quantum numbers of the vacuum may be of more general validity than expressed here.

9. We consider that the Φ belongs to the vector meson octet, and that the ω is not coupled to the pseudoscalar-pseudoscalar channel. For a discussion of the possible Φ-ω mixing see for instance, J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).

10. These arguments are based on the realization that if the 10 and $\overline{10}$ states admit a resonant solution similar to the 8', the ratio of the forces on the singlet and 8' states due to vector-meson exchanges would be 7, whereas if the 10 and $\overline{10}$ resonances are excluded, it would be just 2 (see the crossing matrix in Eq. 4). Present calculations indicate that the latter factor is adequate to produce the experimental splitting between the Pomeranchuk and vector-meson poles; conversely, given the existence of the vector mesons, a factor of 7 would imply a violation of the Froissart limit (G. Chew, Lawrence Radiation Laboratory, private communication).

11. We do not consider the exchange of a physical scalar particle on this trajectory, because we expect Re α to vanish for negative values of t.

12. The possible two-pseudoscalar meson decays of this octet of resonances would be $\pi\pi$, $\eta\eta$, and $K\overline{K}$ for the $T=0$ member; $K\pi$ and $K\eta$ for the $T=1/2$; and $\pi\eta$ and $K\overline{K}$ for the $T=1$ (G parity forbids two-pion decay in the latter case).

13. We are assuming here that the f^0 resonance lies on the Pomeranchuk trajectory, as indicated by the present experimental evidence.

14. G. F. Chew, Lawrence Radiation Laboratory, private communication.

15. D. H. Sharp and W. C. Wagner, Phys. Rev. 131, 2224 (1963).

16. R. Phillips, Phys. Rev. Letters 11, 442 (1963).

17. At higher momenta the experimental situation is still unclear. However, the corrected np data seem to tend to cross above the pp values. [See A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters 9, 32 (1962).] Our model is consistent with such an effect, because we predict a somewhat larger value for $\alpha_\rho(0)$ than $\alpha_R(0)$, and we therefore expect the ρ pole to dominate at sufficiently high energies.

18. Note that G parity forbids the R pole to contribute to πN scattering.

19. I. Muzinich, Phys. Rev. Letters 11, 88 (1963).

20. J. L. Gervais and R. Omnès, Ecole Polytechnique, Paris, France, private communication.