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ABSTRACT

Both the tractability and the reliability of the Reggeized strip approximation are increased by normalizing the long-range part of the potential at zero energy. We make the decomposition $V(t,s)=V(t,0)+V^R(t,s)$, where s is the energy squared and t is the negative square of momentum transfer, and then calculate the first part $V(t,0)$ from a low-t partial-wave expansion in the t reaction. The other part, $V^R(t,s)$, which vanishes at $s = 0$ but which dominates at large s, continues to be calculated by the method of Chew and Jones from the leading crossed-reaction Regge trajectories. The normalization eliminates from the bootstrap calculation the need for accurate treatment of secondary trajectories that fail to reach $J = 0$; normalization, in fact, amounts to a sum over <u>all</u> poles, and is useful because poles that remain in the left half of the J plane may have an important influence on the low-energy potential while being negligible at higher energies.

I. INTRODUCTION

The version of the strip approximation discussed recently by Chew[1] and by Chew and Jones[2] gives a prescription for calculating the generalized potential from the leading Regge poles for crossed reactions. All poles must be included that reach or closely approach the right half of the J plane. The leading crossed trajectory necessarily dominates the potential at high energy, but any trajectories that come near to physical J values contribute significantly to the low-energy direct-channel potential. A good illustration occurs for the vacuum quantum numbers in a crossed reaction, where the Pomeranchuk trajectory dominates at high energies, but cannot account for the ABC virtual $J = 0$ state[3] that is important for the low-energy potential. This state must be associated with a secondary trajectory that approaches but presumably does not reach $J = 0$. It might seem necessary to have a detailed and accurate treatment of at least this secondary trajectory if the potential is to be reliable, but we show here that the low-energy combined effect of all secondary trajectories can be obtained from a conventional partial-wave expansion.

Furthermore, the method proposed here for computing the low-energy potential exhibits certain positive-definiteness features that are not apparent in the Chew-Jones prescription. In other words, the contributions from individual crossed Regge poles are correlated at low energy in an important way that may be overlooked if the poles are treated separately. At high energy, the leading crossed Regge pole must dominate, but at low energy it is safer to consider all poles at once.

The method advocated here is related to that proposed some years ago by Cini and Fubini,[4] and independently by Chew and Mandelstam.[5] We shall nowhere invoke mathematically divergent partial-wave expansions, however, and the Regge asymptotic behavior at high energy will be maintained. The old method was deficient in both these respects. Recently Balázs has proposed a strip approximation which, in many respects, is similar to ours, but which does not attempt a treatment of the high-energy potential.[6]

II. PRESCRIPTION FOR NORMALIZATION OF THE POTENTIAL

As in reference 1 and 2, we assume that the important parts of the double spectral function are confined to the six strip regions shown in Fig. 1. The meanings of the parameters s_0, s_1, t_0, t_1, u_0, and u_1 are the same as in reference 2. The s-reaction generalized potential arising from the t discontinuity is then

$$V_s^t(t,s) = \frac{1}{\pi} \int_{t_0}^{t_1} \frac{dt'}{t'-t} D_t(t',s) + \frac{1}{\pi} \int_{t_0}^{t_1} \frac{dt'}{t'-t} d_t(t',s) , \tag{II-1}$$

where

$$D_t(t,s) = \frac{1}{\pi} \int_{s_1}^{\infty} \frac{ds'}{s'-s} \rho_2(s',t) + \frac{1}{\pi} \int_{u_1}^{\infty} \frac{du'}{u'-u} \rho_4(u',t) \tag{II-2}$$

is the complete discontinuity across the t cut, while

$$d_t(t,s) = \frac{1}{\pi} \int_{u_0}^{u_1} \frac{du'}{u'-u} \rho_3(t,u') \tag{II-3}$$

is that portion of the discontinuity arising from the ρ_3 strip. The full potential $V_s^{\pm}(t,s)$, in the notation of reference 2, is obtained by combining V_s^t with $\pm V_s^u$, the latter arising from the u discontinuity. It evidently suffices to discuss V_s^t separately.

In reference 2 the discontinuity D_t , as well as d_t , was approximated for all s in terms of the leading Regge poles in the t reaction. Here we propose instead to write

$$D_t(t',s) = D_t(t',0) + D_t^R(t',s) \ , \tag{II-4}$$

and to calculate $D_t(t,0)$ through the partial-wave expansion

$$D_t(t',0) = \sum_{\ell_t = 0}^{\infty} (2\ell + 1) \text{ Im } A_{\ell_t}(t') \ . \tag{II-5}$$

This expansion converges for the real positive range of t' needed in the first term of the right side of Eq. (II-1). Furthermore, since only $t' \leq t_1$ is required, a reasonably small number of partial waves in Eq. (II-5) will yield an adequate approximation.

In the notation of reference 2, the Chew-Jones prescription for $V_s^{\ t}$ was as follows:

$$V_s^{\ t}(t,s) \approx \sum_j \left\{ \frac{1}{\pi} \int_{s_1}^{\infty} \frac{ds'}{s'-s} R_j(s',t) + \frac{\xi_j}{\pi} \int_{u_1}^{\infty} \frac{du'}{u'-u} [R_j(u',t) - R_j(u',t')] \right\}$$

$$+ \sum_k \frac{\xi_k}{\pi} \int_{t_1}^{\infty} \frac{dt'}{t'-t} R_k(t',u') \ . \tag{II-6}$$

The sum over j arises from the Regge poles of the t reaction (the ρ_2 and ρ_4 strips), and corresponds to the first term on the right side of Eq. (II-1), while the sum over k , arising from the u-reaction poles (the ρ_3 strip), corresponds to the second term of Eq. (II-1). Our proposal here simply amounts to subtracting from the j sum in Eq. (II-6) its value at $s = 0$ and then adding back to Eq. (II-6) the expression

$$\frac{1}{\pi} \int_{t_0}^{t_1} \frac{dt'}{t'-t} D_t(t',0) \qquad \text{(II-7)}$$

as evaluated from Eq. (II-5). From the practical standpoint of numerical calculations, such a prescription presents no difficulties not already present in the scheme of reference 2.

III. DISCUSSION

What has been gained by the modified prescription and how is it related to earlier schemes? We have now normalized the long range potential [i.e., that arising in Eq. (II-1) from $t' < t_1$] to a reliable value at zero energy. Presumably, an adequate low-energy result can be achieved by keeping a sufficient number of the t-reaction Regge poles in Eq. (II-6), but the following circumstance is noteworthy: If we keep only the leading crossed pole with the vacuum quantum numbers in the t channel (the Pomeranchuk trajectory), we obtain a negative (repulsive) potential at $s = 0$. For $\pi\pi$ scattering, however, where the t reaction is also elastic, the discontinuity $D_t^{I_t=0}(t',0)$ is positive definite, being proportional to the total cross section in the $I_t = 0$ state. The new prescription, which guarantees the satisfaction of this positive definite condition, evidently has an advantage over the Chew-Jones prescription, where such a feature must arise from a not yet understood correlation between the parameters of different Regge poles. Taken alone, the leading pole, in at least the example above, violates the condition of positiveness.

On the other hand, of course, the leading crossed Regge poles generally must be dominant at high energy, and we have not disturbed this aspect of the

ERRATUM:

In the second integral on the right-hand side of Eq. (II.1) the limits should be $\int_{t_1}^{\infty}$ rather than $\int_{t_0}^{t_1}$.

Chew-Jones prescription. Throughout the interval of t for which the strip approximation is designed to be reliable, the leading trajectories lie to the right of $J = 0$ in the angular-momentum plane. Their contribution therefore increases with s and overwhelms the energy-independent normalizing term introduced here. It appears from rough estimates, in fact, that this constant term represents only a small fraction of the total potential in the region s of the order of magnitude s_1 which is crucial in dynamical calculations of resonance and bound-state energies. Nevertheless, the low-energy potential remains important for resonance widths (coupling constants) and for non resonance scattering.

It counts as a definite practical advantage in bootstrap calculations to deal only with Regge poles that clearly manifest themselves at high energy. The choice of trial parameters for such trajectories can be motivated by experimental information and the results of calculations can be checked more thoroughly. In the $\pi\pi$ amplitude, for example, the only Regge poles that seem likely to be significant for our present scheme are the P(Pomeranchuk), the P' (Igi-Pignotti),[7] and the ρ, for each of which a substantial amount of information already exists concerning trajectories and residues. In contrast, we know little about the ABC pole, because it has a negligible influence on forward and backward peaks in high-energy elastic scattering.

With regard to the ρ, it should be noted that trajectories with negative signatures that lie to the right of $J = 0$ do not necessarily violate the condition of positiveness above; here, the Chew-Jones prescription may be adequate. Finally, for the $I_t = 2$ state, the modified prescription leads to use of the (presumably) small contribution [Eq. (II-5)] while the second term in Eq. (II-4) drops completely because no trajectories come to the right of $J = 0$.

We close by emphasizing the connection of our proposal here with the original prescription of Cini-Fubini[4] and Chew-Mandelstam,[5] recently revived by Balázs.[6] The prescription was to evaluate the entire long-range potential [i.e., the first term on the right-hand side of Eq. (II-1)] through the series expansion,

$$D_t(t',s) = \sum_{\ell_t=0}^{\ell_{max}} (2\ell + 1) \operatorname{Im} A_{\ell_t}(t') P_{\ell_t}(1 + s/2q_{t'}^{2}) . \qquad \text{(III-1)}$$

This series had to be terminated because it diverges for $s > s_1$, but it was impossible to decide how to choose ℓ_{max}. Even though for $s << s_1$ the series converges rapidly and the choice of ℓ_{max} is unimportant, in the crucial region s of the order of magnitude s_1 the prescription fails to have a meaning. The normalization given here by Eq. (II-5) looks similar to (III-1) but, with $s = 0$, it does not share the difficulties of the latter. As emphasized earlier, the partial-wave expansion in Eq. (II-5) is strongly convergent.

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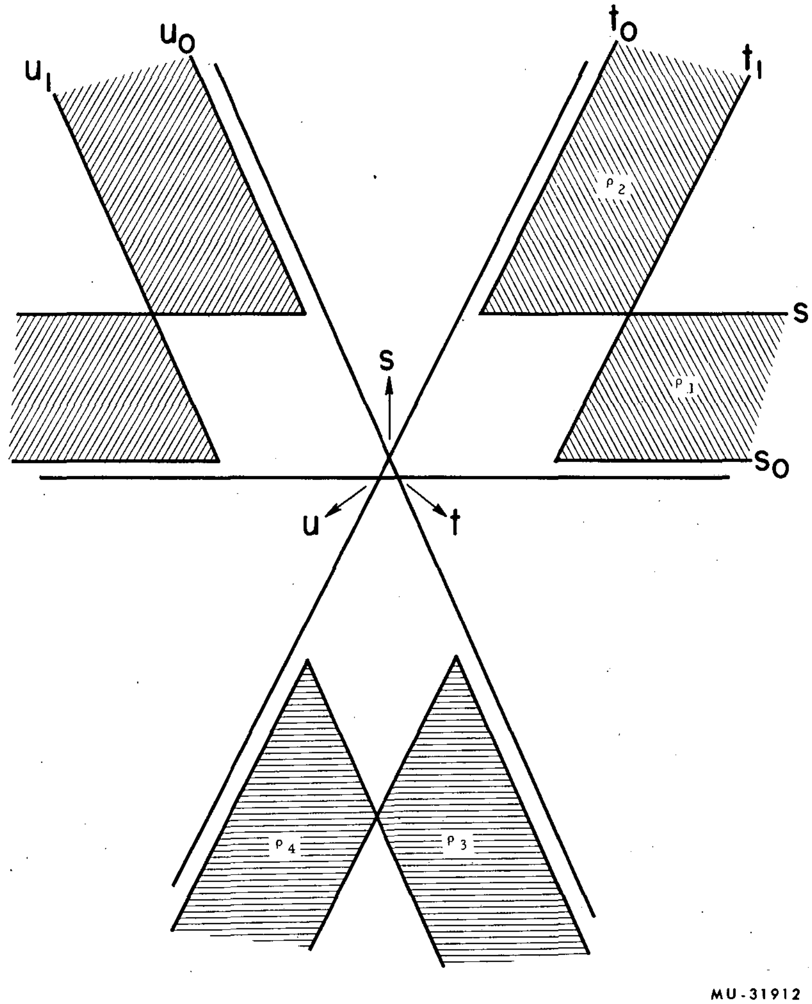

FIG 1