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UNIVERSITY OF CALIFORNIA
RIVERSIDE

River Basin Economics and Management: International Trade, Allocation And
Quality

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Wen Kong

June 2015

Dissertation Committee:

    Dr. Keith Knapp, Chairperson
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Committee Chairperson

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To my parents who do not understand it at all but spare no effort to support me

To my friends who have kept me company throughout the doctorate program

ABSTRACT OF THE DISSERTATION

River Basin Economics and Management: International Trade, Allocation And Quality

by

Wen Kong

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2015
Dr. Keith Knapp, Chairperson

Water is essential to human life and activities, and rivers are an importance source of water. This dissertation addresses two problems related to rivers. First is the water quantity allocation between countries which share an international river. International resources such as water are typically subject to conflict as individual countries perceive individual gains from increased use of the resource. This inherent conflict is also reflected in analytical studies which are typically partial equilibrium and hence naturally assume that welfare functions are increasing in the resource allocation. In this setting, the question arises if there are ever circumstances such that it is in the joint self-interest of political entities to share the resource. With a two-country-two-good Ricardian trade model, the conflicts over water that naturally stem from a welfare function monotonically increasing in water could be mitigated, since the free trade welfare functions can be non-monotone when a country has absolute disadvantage in production of both goods. The welfare function is applied to a Nash bargaining game to show how it reduces conflicts over water. The results also hold when the number of production factors increases to two and number of countries sharing the river increases to three. This contributes to the literature in that it combines the general equilibrium trade model with a river sharing context, derives the welfare functions that can be utilized in a game-theoretic framework of river sharing, and demonstrates the possibility that the welfare functions are not always monotonically increasing in a country's resource. Second is the allocation and efficient usage of river water in an irrigated agricultural region, with

an application to the California lower San Joaquin River. The results show that the region is threatened by water salinity problems in times of drought and efficient use of the water could help increase aggregate irrigation benefits and improve water quality. The research incorporates a set of crop-water-salinity agricultural production functions in an integrated hydrologic-economic surface water quality model and is significant in terms of empirical originality.

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Chapter 1

Introduction

The world is facing a water crisis. Water scarcity affects every country and every human being on the planet. According to the United Nations' water statistics, "around 1.2 billion people, or almost one-fifth of the world's population, live in areas of physical scarcity, and 500 million people are approaching this situation. Another 1.6 billion people, or almost one quarter of the world's population, face economic water shortage (where countries lack the necessary infrastructure to take water from rivers and aquifers)." [1]

Scarcity leads to conflicts in water, and nowhere else are these conflicts more fierce than at rivers shared by two or more countries, in other words international rivers. There are 261 international rivers, which cover 45.3% of the land surface of the earth (excluding Antarctica) (Wolf et al., 1999). Figure 1.1 shows the international river basins by continent as delineated by the Transboundary Freshwater Dispute Database project, Oregon State University (2000).

Barrett (1994b) has also counted the number of international river basins to be over 200 and the details are shown in Table 1.1. Most of the international rivers are shared by two countries only (148 out of 200), 30 out of 200 rivers are shared by three countries. Rivers that are shared by more than five countries can be named, La Plata and Elbe (by five), Chad, Volta, Ganges-Brahmaputra and Mekong (by six), Zambezi,

Table 1.1: International Rivers Basins by Number of Sharing Countries

		Number of Countries Sharing the Basin									
Region	Area[1]	2	3	4	5	6	7	8	9	10	Total
Africa	A	3	2	6	-	2	1	-	3	-	17
	B	30	8	-	-	-	-	-	-	-	38
Americas	A	10	2	-	1	-	1	-	-	-	14
	B	43	3	-	-	-	-	-	-	-	46
Asia	A	7	5	2	-	2	-	-	-	-	16
	B	20	3	1	-	-	-	-	-	-	24
Europe	A	-	2	-	1	-	1	-	-	1	5
	B	35	5	-	-	-	-	-	-	-	40
Total	A	20	11	8	2^2	4^3	3^4	-	3^5	1^6	52
	B	128	19	1	-	-	-	-	-	-	148
		148	30	9	2	4	3	-	3	1	200

1. Area A(B) comprises more(less) than 100,000 square kilometers
2. La Plata, Elbe
3. Chad, Volta, Ganges-Brahmaputra, Mekong
4. Zambezi, Amazon, Rhine
5. Niger, Nile, Congo
6. Danube

Figure 1.1: International River Basins by Continent
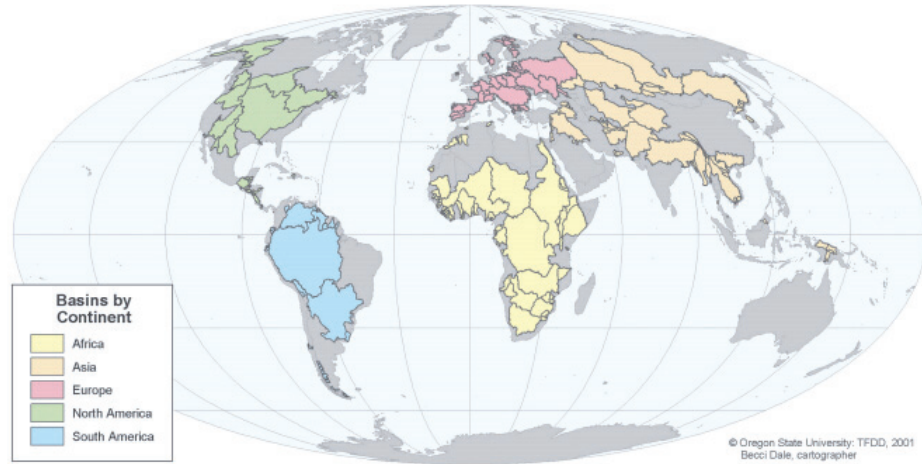
Amazon and Rhine (by seven), Niger, Nile and Congo (by nine), and Danube (by ten).

Parallel with the conflicts, countries sharing a common river are also struggling with establishing cooperation with each other. Data from the Oregon State University Transboundary Freshwater Dispute Database (TFDD) illustrates that in most of the international river basins, at least one agreement has been signed (Figure 1.2). Figures 1.3-1.7 show the international river basins with and without treaties by continent. Most of the river basins in all five continents have already established treaties.

However, there are still places where countries fail to reach an agreement to effectively share a river. The literature (Barrett (1997),Bennett et al. (1998),Pham-Do et al. (2011)) has proven that linking the water issue with non-water issues, such as international trade, hydropower production, infrastructure, national defense and etc., could help countries to establish agreements on sharing water. Table 1.2 shows issue linkage in action, of all the 646 treaties signed, 65 have non-water issues embedded, which on the one hand shows that issue linkage plays a role in reaching agreements, and

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[1]Quoted from `http://www.un.org/waterforlifedecade/scarcity.shtml`

Figure 1.2: Cooperation Over International Water

Number of Agreements per International River Basin

Number of Treaties
- 0
- 1 - 2
- 3 - 5
- 6 - 10
- 11 - 15
- 16 - 20

© Oregon State University: TFDD, 2001
Becci Dale, cartographer
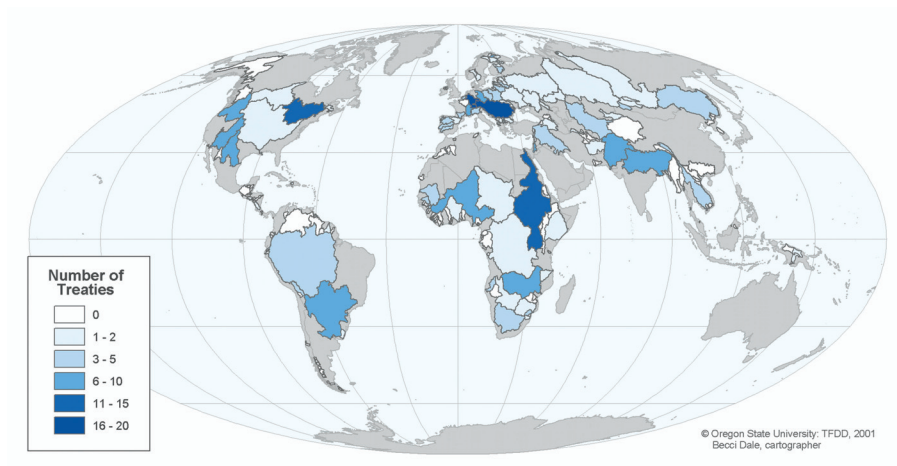
Data source: Treaties- Wolf (1999b).

Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

Figure 1.3: Agreements In Africa

Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

Figure 1.4: Agreements In Asia

Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

Figure 1.5: Agreements In Europe

Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

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Figure 1.6: Agreements In South America

Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

Figure 1.7: Agreements In North America

Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

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on the other hand, indicates that there are more room for issue linkage to take effects in establishing more treaties.

Table 1.2: The Role of Issue Linkage in International River Basin Cooperation

	Yes	65
Non Water Embedded	No	282
	NA	299
Total Number of Treaties		646

Data Source: Oregon State University, Transboundary Freshwater Dispute Database (TFDD), `http://www.transboundarywaters.orst.edu/`

Hence this dissertation focuses on the issue linkage of international trade and water allocation among riparian countries which share an international river, and develops the welfare functions for countries with different water allocation parameter values. It is shown that when countries are involved in international trade, it is possible for a country to be better off by voluntarily giving up some water and sharing water with the other country. That is, the welfare function can be decreasing in the amount of water a country obtains within a certain range of the parameter domain. This property of the welfare function would naturally reduce the conflicts over water between countries which share a river and also trade with each other.

Chapter 2 develops the basic welfare function within a two-country two-good Ricardian free trade framework with the only factor of production being water, and countries differ in production technologies. The results are applied to a simple Nash bargaining model to show that conflicts can indeed be mitigated with developed welfare functions. Chapter 3 extends the basic model in Chapter 2 in two dimensions: first the number of factors is extended to two, with water and labor; then the number of countries sharing the river is extended to three. The model is solved numerically with a general equilibrium method, and examples of the welfare functions are plotted to show that similar non-monotone property exists as with the basic one-factor two-country case.

Chapter 4 turns attention from the international water allocation problem to the water allocation efficiency within a river basin in a country, namely the lower San

Joaquin Valley in California. The chapter not only simply deals with the water quantity allocation, but also looks into the quality aspect of water, i.e. salinity, and its interactions with agricultural irrigation activities. The model aims to allocate the surface water from the lower San Joaquin River efficiently among the farming zones that divert water from this stretch of the river.

Chapter 2

Economic and Political Equilibrium for a Renewable Natural Resource with International Trade

2.1 Introduction

Water is a critical natural resource for economic activity, and is increasingly scarce due to population and economic growth, and to increasing demand for environmental amenities stemming from water in its natural state. As difficult as water allocation is within countries and other political units, allocation is even more challenging when the water source is international. Wolf et al. (1999) documents that there are "261 international rivers, which cover 45.3 % of the land-surface of the earth (excluding Antarctica)". Of these more than 200 international rivers, 148 flow through two countries, 30 through three countries and the rest through more than three countries (Barrett, 1994a). The nature of international rivers and the scarcity of water intensify the conflicts, and lead to cooperation and river basin management issues. In particular, since no supra-national government exists, international water management carries an even higher burden of cooperative self-interest than might exist in other settings which

have governmental allocation channels to oversee joint use of the resource by affected parties.

There is a substantial game-theoretic literature devoted to the design of self-enforcing agreement on water allocation (Ambec and Sprumont, 2002; Ambec and Ehlers, 2008a,b; Ansink and Ruijs, 2008; Ansink, 2009). Cooperative models use coalition formation theory to study the welfare consequences and stability of an international agreement under different circumstances. These include symmetric countries (Carraro and Siniscalo, 1993), asymmetric countries (McGinty, 2007), uncertainty (Na and Shin, 1998; Ulph, 2004; Finus and Pintassilgo, 2012), and issue linkage (Pham-Do et al., 2011). Non-cooperative analyses include Hoel (1992), Barrett (1994b), Bennett et al. (1998), and Ansink (2009). In particular, Ansink (2009) analyses self-enforcing agreements on water allocation based on the outcome of a bargaining game. Carraro and Siniscalo (1998) point out that the structure of the game involving different countries is a chicken game rather than prisoners' dilemma. At least some degree of cooperation exists.

For an international water allocation plan to be self-enforcing, the riparian countries must be better off under cooperation than non-cooperation, that is cooperation is beneficial for all participants. The construction of such a plan relies on the welfare functions of the riparian countries. The current literature normally assumes a particular welfare function without deriving it from the microeconomic foundations. In particular, Carraro and Siniscalco (1995), Ambec and Sprumont (2002) and others assume the benefit function is strictly increasing and concave in the resources allocated to a country, and Ambec and Ehlers (2008b) assume that the agents' benefit function exhibits a satiation point. However monotonic welfare functions may not always be appropriate; for instance, Carraro and Siniscalco (1997) demonstrate a humped-shaped payoff function when environmental issues are linked with R&D cooperation.

As for the stability of an international agreement, even if cooperation is beneficial, it may not be stable as countries have incentives to free ride. Other issues can be linked with the problem of interest to enhance cooperation. Carraro and Siniscalco (1997) present a model to stabilize an environmental agreement by linking it to an R&D agreement. Barrett (1997) uses trade policy in a partial equilibrium model as a threat

in achieving full cooperation to supply a global public good. Bennett et al. (1998) connect the river basin management problem to trade to improve upon the unsatisfactory "victim pay" outcome.[1]

This paper analyzes two countries with joint access to an international river and which also can or do participate in international trade. Each country has a representative household and water is the only factor of production. With this setup we consider a two-stage equilibrium model. The first stage solves a trade model to determine economic equilibrium as a function of water allocation between the two countries. The second stage then utilizes the welfare functions from the first-stage analysis to identify political equilibrium formulated as a bargaining or game-theoretic problem. In this setup, the primary question of interest is whether and under what circumstances it is ever self-interest for the two countries to cooperate over the natural resource as opposed to compete for it. The answer will turn out to be yes under some circumstances.

There are two possible spatial configurations of the countries and the river. One might be that the countries are upstream-downstream, or the second is that they might have a joint boundary along the river such as the Rio Grande between Mexico and the United States. Spatial configuration can influence initial property rights and hence influence the starting point in the second-stage political equilibrium analysis. However, the first-stage economic equilibrium analysis considers all possible water allocations, so geometry does not matter here. As a consequence, we do not consider a specific geometry of the system (upstream-downstream or riparian); the spatial configuration does not directly enter into the analysis, and the analysis is general in this regard.

Conceptually there are at least three motivations for trade between countries (Feenstra, 2004): productivity differences (Ricardian model), primary factor endowments (Hechsler-Ohlin model), or economies of scale (Krugman, 1979). While all of these are relevant to the problem at hand, here we concentrate on the Ricardian case as a reasonable starting point for understanding international natural resource allocation when countries are engaged in trade. For a given technological parameter specification,

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[1]The general literature on the design of self-enforcing International Environmental Agreements(IEAs)(Hoel, 1992; Barrett, 1994b; Batabyal, 1996; Na and Shin, 1998; Finus and Pintassilgo, 2012) is also applicable to river basin management.

the trade model is used to calculate world prices for the two goods as dependent on water allocation between the two countries. This is then used to specify country welfare as functions of the water allocation.

The results from the trade model analysis are striking: under some circumstances country welfare can be *declining* in water allocation meaning that countries could potentially gain by giving up some water. This occurs by comparative advantage when the natural resource is necessary for production and productivity differences imply that a good can be produced more effectively elsewhere. Of course, there is a limit to this process as countries have to have sufficient resources to generate exports and income to pay for imported goods/services. While it is intuitive that this phenomenon could occur, it still requires verification given the opposing forces at work, and also this phenomenon does not occur under all circumstances.

The paper next turns to a consideration of political equilibrium; this is where the system geometry may enter by defining an initial property rights allocation. We first consider a bargaining model. The analysis demonstrates that bargaining outcomes are not unique as they depend on the initial water allocation level. In some instances countries might be willing to voluntarily give up water, but not necessarily in other instances, and only up to some point. A discrete-strategy game-theory model is also considered as in previous studies (Bennett et al., 1998). There are two possible water allocation strategies under the control of one country, and autarky/free trade as the two discrete trade strategies for the both countries. Perhaps the main result here is that this may be a limited framework for analysis as the outcome depends on the discrete strategies selected, and in addition it is typically in the self-interest of countries to pursue free trade, so autarchic threats may well lack credibility.

This work contributes to the water allocation literature in several ways. This is a general equilibrium model with trade. A trade model implicitly underlies the game-theoretic models with side payments, and the analysis here allows for the fact that the terms of trade may alter for non-marginal water allocations. This in turn implies that currency and hence side-payment unit values may also alter with water allocation. The main contribution of the paper is the welfare functions. The welfare function properties

are derived from the trade model, and, most importantly, the analysis demonstrates that these functions can be non-monotone under some circumstances. This implies that joint allocation of the resource can be self-interest even with no side payments. At the very least, trade reduces the gains from additional water and thus lessens the level of conflict.[2] Finally, while we focus on international river water allocation, clearly the results are applicable to natural resources in general. Examples might be access to a joint groundwater aquifer, a common property resource such as fisheries or forests, or waste assimilative capacity of an environmental resource.

2.2   Model

We consider an international river basin where there are two countries ($i = 1, 2$) with joint access to the river and which potentially engage in Ricardian trade in produced goods. Annual water flow is W, of which country i takes $W_i = \theta_i W$. The river is assumed to be fully allocated with $\theta_1 + \theta_2 = 1$. There are also two goods, each of which is produced utilizing water with production coefficients specific to each country. Subsequent analysis considers autarky and free trade with exogenous water allocations, social welfare functions and bargaining political equilibrium with endogenous water allocations and trade policy.

For simplicity, identical household preferences

$$U_i = c_{i1}^{\alpha} c_{i2}^{1-\alpha} \tag{2.1}$$

are assumed for both countries. Here U_i is utility and c_{ij} is good j consumption in country i, with the preference parameter satisfying $0 < \alpha < 1$.

The two goods ($j = 1, 2$) are homogeneous across countries, but technologies

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[2]A further implication is that country welfare functions in a multi-country case with trade will generally depend on all the water allocations, not just the country's own allocation as in the literature. This follows because there are likely differential trading incentives for an individual country with respect to other countries, and this is affected by the water allocation. If there are n countries and water is fully allocated, then the welfare functions can be written as a function of $n-1$ allocations, leaving a single allocation in the two-country case. This is not pursued here, but could potentially be quite interesting as well as realistic.

in the two countries differ. Linear production functions are

$$y_{ij} = \beta_{ij} w_{ij} \tag{2.2}$$

where β_{ij} is country i's output coefficient to produce good j, w_{ij} is water allocated to the production of good j in country i, $i \in \{1, 2\}$ and $j \in \{1, 2\}$. Without loss of generality, we assume that

$$\frac{\beta_{12}}{\beta_{11}} < \frac{\beta_{22}}{\beta_{21}} \tag{2.3}$$

implying that country 1 has a comparative advantage in good 1 and country 2 has a comparative advantage in good 2. This comparative advantage assumption will prevail in the rest of the paper, even if one country has absolute advantage in both goods.

The resource constraint is

$$\sum_{j=1}^{2} w_{ij} \leq W_i \tag{2.4}$$

for each country. As both utility and production are increasing functions, this constraint will always be binding.

The two countries can choose to stay in autarky and produce both goods to meet the domestic demand, or they can specialize in the good that they have comparative advantage in and trade with each other in order to increase welfare. The question we want to answer is: if the gap between the two countries' productivities is substantially large, would the countries benefit by giving up water and enjoy low-cost goods imported from the other country? Hence, the following sections will derive the welfare functions for each country under both autarky and free trade as a function of the water allocation parameter. Welfare in this context is measured by consumer utility in each country.

2.3   Autarky

We first consider autarky over the entire range of exogenous water allocations. The subsequent welfare functions will be used later to demonstrate that allowing for free trade can substantially change the nature of individual countries valuation of water

allocations. Welfare functions under autarky will also be necessary for the later political equilibrium analysis.

Under autarky, each country maximizes utility subject to the technology and resource constraints with consumption just equal to production $c_{ij} = y_{ij}$. The optimization problem is then

$$\max \ U_i = c_{i1}^{\alpha} c_{i2}^{1-\alpha} \tag{2.5}$$

$$\text{s.t. } c_{ij} = y_{ij} = \beta_{ij} w_{ij} \quad j \in \{1,2\}$$

$$w_{i1} + w_{i2} = \theta_i W$$

for country $i \in \{1,2\}$ and given the allocation parameter θ_i.

The utility maximization problem is illustrated in Figure 2.1. Solving this problem gives consumptions and outputs:

Figure 2.1: Utility Maximization Under Autarky

(a) Country 1   (b) Country 2
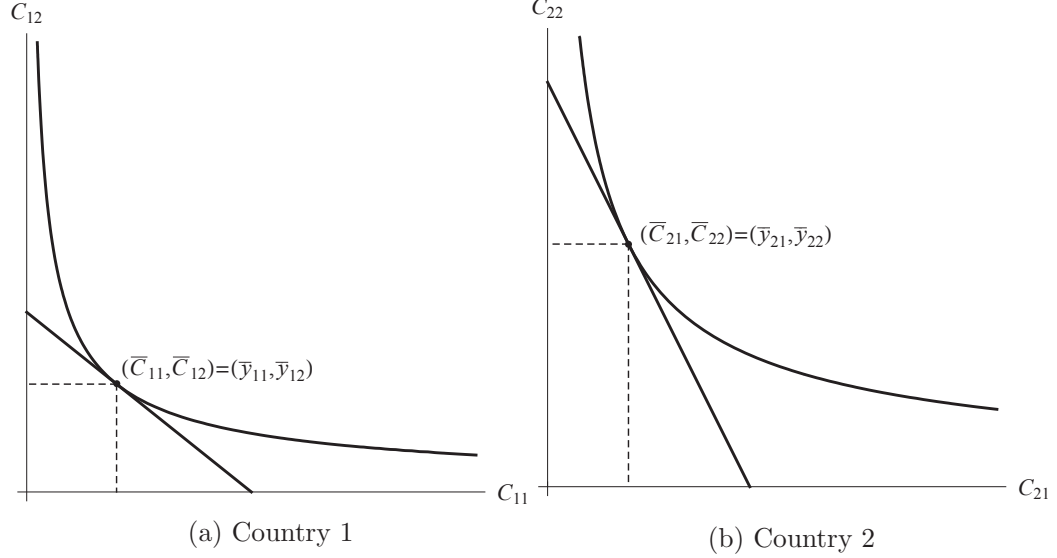
Figures generated with parameter values $\alpha = 0.4$, $\beta_{11} = 5, \beta_{12} = 4, \beta_{21} = 3, \beta_{22} = 6$.

$$\bar{c}_{i1} = \bar{y}_{i1} = \alpha \beta_{i1} \theta_i W \tag{2.6}$$

$$\bar{c}_{i2} = \bar{y}_{i2} = (1-\alpha)\beta_{i2}\theta_i W \tag{2.7}$$

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for $i \in \{1, 2\}$. Substituting the optimal consumption levels into the utility function gives the maximized utility for country i under autarky:

$$U_i^A = (\alpha\beta_{i1})^\alpha((1-\alpha)\beta_{i2})^{1-\alpha}\theta_i W. \tag{2.8}$$

The autarky price ratios are just the slope of the production possibility frontier in Figure 2.1. Hence, autarky relative prices are

$$\bar{p}_{i1}/\bar{p}_{i2} = \beta_{i2}/\beta_{i1} \tag{2.9}$$

Furthermore, as illustrated in Figure 2.2, autarky implies that both countries' welfare functions are linear and monotonically increasing in the water allocation parameter, θ_i . Both countries would be better off as they get more water to produce more goods that are only consumed domestically. Neither country would voluntarily concede to less water without any other conditions, hence, conflict arises.

Figure 2.2: Welfare Functions Under Autarky
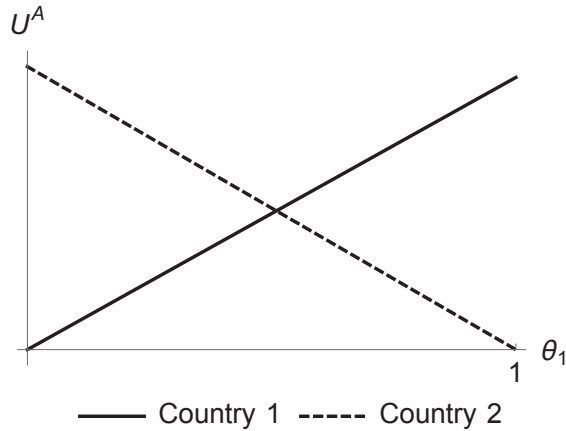
2.4 Free Trade

Now suppose the two countries engage in free trade. We want to see if free trade will give countries some leverage in negotiating the water allocation. Would the gains from free trade let the countries give up some water out of its self-interest? First,

we derive the free trade welfare as a function of θ_1. The free trade equilibrium has three cases (Feenstra, 2004). The most general case is that each of the two countries specializes in the good that they have comparative advantage in. The other two cases arise when one country is relatively large compared to the other. In those cases, if the two countries still specialize in one good, then production in the small country would not be able to meet the demand of both countries. The large country has to produce both goods while the small country still specializes in the good that it has comparative advantage in. Hence the world relative price would be the autarky relative price in the large country.

In general, the free trade utility maximization problem for country i is

$$\max \quad U_i = c_{i1}^{\alpha} c_{i2}^{1-\alpha} \tag{2.10}$$

$$\text{s.t.} \quad \tilde{p}_1 c_{i1} + \tilde{p}_2 c_{i2} = \tilde{p}_1 y_{i1} + \tilde{p}_2 y_{i2}$$

$$y_{ij} = \beta_{ij} w_{ij}$$

$$w_{i1} + w_{i2} = \theta_i W.$$

where $\tilde{p}_j$ is the free trade equilibrium world price for commodity j, which clears the world market for good j: $y_{1j} + y_{2j} = c_{1j} + c_{2j}$.

2.4.1 Intermediate water allocation

Here we consider an intermediate water allocation such that the world equilibrium price ratio $\tilde{p}_1/\tilde{p}_2$ falls between autarky prices

$$\beta_{12}/\beta_{11} = \bar{p}_{11}/\bar{p}_{12} < \tilde{p}_1/\tilde{p}_2 < \bar{p}_{21}/\bar{p}_{22} = \beta_{22}/\beta_{21} \tag{2.11}$$

with the parametric condition for this to occur to be derived later. Country 1 then specializes in good 1

$$y_{11}^* = \beta_{11}\theta_1 W, \quad y_{12}^* = 0, \tag{2.12}$$

while Country 2 specializes in good 2

$$y_{21}^* = 0, \quad y_{22}^* = \beta_{22}(1 - \theta_1)W, \tag{2.13}$$

implying that each country uses all the water assigned to it to produce the good in which it has a comparative advantage.

Each country solves the consumer optimization problem (2.10), resulting in the optimal consumption levels

$$c_{11}^* = \alpha\beta_{11}\theta_1 W, \quad c_{12}^* = \frac{\tilde{p}_1}{\tilde{p}_2}\beta_{11}(1 - \alpha)\theta_1 W, \tag{2.14}$$

$$c_{21}^* = \frac{\tilde{p}_2}{\tilde{p}_1}\beta_{22}\alpha(1 - \theta_1)W, \quad c_{22}^* = (1 - \alpha)\beta_{22}(1 - \theta_1)W. \tag{2.15}$$

Market clearing for the first good is

$$y_{11}^* = c_{11}^* + c_{21}^* \tag{2.16}$$

with market clearing for good 2 implied by Walras Law. This yields

$$\frac{\tilde{p}_1}{\tilde{p}_2} = \frac{\beta_{22}\alpha(1 - \theta_1)}{\beta_{11}(1 - \alpha)\theta_1} \tag{2.17}$$

as the world equilibrium price ratio in this particular case.

Figure 2.3 illustrates the free trade equilibrium for the two countries. Country 1 specializes in good 1, exports $(y_{11}^* - c_{11}^*)$ of good 1 to country 2 and imports (c_{12}^*) of good 2 from country 2. Country 2 specializes in good 2, exports $(y_{22}^* - c_{22}^*)$ of good 2 and imports (c_{21}^*) of good 1. This is the standard free trade case.

As previously noted, for this case to occur the world equilibrium price must be bounded by the autarky prices. Substituting (2.17) into (2.11) yields

$$\frac{\beta_{12}}{\beta_{11}} < \frac{\beta_{22}\alpha(1 - \theta_1)}{\beta_{11}(1 - \alpha)\theta_1} < \frac{\beta_{22}}{\beta_{21}} \tag{2.18}$$

Figure 2.3: Utility Maximization Under Free Trade: Case 1

(a) Country 1

(b) Country 2
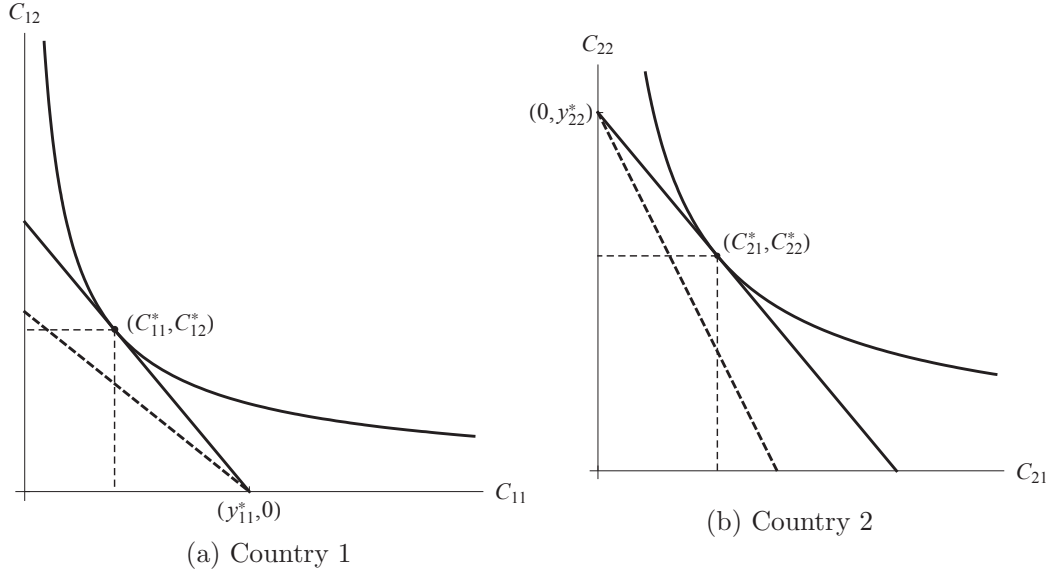
Case 1: Intermediate water allocation between two countries($\theta_1 = 0.4$). Figures generated with parameter values $\alpha = 0.4$, $\beta_{11} = 5$, $\beta_{12} = 4$, $\beta_{21} = 3$, $\beta_{22} = 6$.

and solving for θ_1 gives

$$\frac{\alpha\beta_{21}}{\alpha\beta_{21} + (1-\alpha)\beta_{11}} < \theta_1 < \frac{\alpha\beta_{22}}{\alpha\beta_{22} + (1-\alpha)\beta_{12}} \tag{2.19}$$

as the parametric condition for this free trade pattern. This requires that the water allocation θ_1 must not be too large, in which instance production of country 2 could not meet the world demand, nor can it be too small, implying that production of country 1 would be insufficient to meet world demand for good 1.

Under these conditions, we can substitute the world equilibrium price ratio (2.17) into the optimal consumption equations (2.14-2.15), and then optimal consumption into the utility functions (2.1) to find the country welfare functions. This yields

$$(U_1^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}\alpha W \tag{2.20}$$

and

$$(U_2^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}(1-\alpha)W \tag{2.21}$$

as utilities for the respective countries in free trade, case 1.

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2.4.2 Large country 1 water allocation

If θ_1 violates condition (2.19), then the world equilibrium price will not fall between the two autarky prices, and full specialization will not occur. Consider first a large θ_1

$$\theta_1 \geq \frac{\alpha\beta_{22}}{\alpha\beta_{22} + (1-\alpha)\beta_{12}} \tag{2.22}$$

implying that Country 1 gets a relatively large share of the water resource. Full specialization as in Case 1 will not occur for two reasons. First, the production of good 2 in country 2 would not meet the total demand in both countries. Second, if free trade pattern was similar to case 1, then free trade utility for country 1 would be lower than its autarky utility, $(U_1^{FT})_1 \leq U_1^A$ as implied by condition (2.22). Hence, Country 1 would not have an incentive to participate in such trade activity.

In this case, the world equilibrium price will be determined by the autarky price in country 1, since the small country 2 is not influential in world prices. Hence

$$\frac{\tilde{p}_1}{\tilde{p}_2} = \frac{\beta_{12}}{\beta_{11}} \tag{2.23}$$

defines the relative world equilibrium price. Country 2 still has a comparative advantage in good 2 in the sense that the relative price for good 2 is lower than the world price (price in country 1), so it still specializes in good 2 with production $y_{22}^* = \beta_{22}(1-\theta_1)W$.

As country 2 is small, its production cannot meet total demand: hence country 1 will produce both goods. Its consumption is equal to the autarky consumption (bundle A in Figure 2.4), while optimal consumption levels for country 2 are

$$c_{21}^* = \frac{\tilde{p}_2}{\tilde{p}_1}\beta_{22}\alpha(1-\theta_1)W, \quad c_{22}^* = (1-\alpha)\beta_{22}(1-\theta_1)W \tag{2.24}$$

which follows from (2.10) after substituting the world price (2.23). This is illustrated as bundle C in Figure 2.4.

Optimal outputs of country 1 (bundle B in Figure 2.4)

$$y_{11}^* = \beta_{11}\left(\alpha\theta_1 W + \frac{\beta_{22}}{\beta_{12}}\alpha(1-\theta_1)W\right) \tag{2.25}$$

$$y_{12}^* = \beta_{12}\left((1-\alpha)\theta_1 W - \alpha\frac{\beta_{22}}{\beta_{12}}(1-\theta_1)W\right) \tag{2.26}$$

follow from the market clearing conditions. Furthermore, these also imply that

$$w_{11}^* = \alpha\theta_1 W + \frac{\beta_{22}}{\beta_{12}}\alpha(1-\theta_1)W \tag{2.27}$$

$$w_{12}^* = (1-\alpha)\theta_1 W - \alpha\frac{\beta_{22}}{\beta_{12}}(1-\theta_1)W \tag{2.28}$$

which define water allocation within sectors in country 1. It can be verified that the total amount of water used by the two sectors equals the total amount allocated to country 1, i.e. $w_{11}^* + w_{12}^* = \theta_1 W$.

Figure 2.4: Utility Maximization Under Free Trade: Case 2

(a) Country 1   (b) Country 2
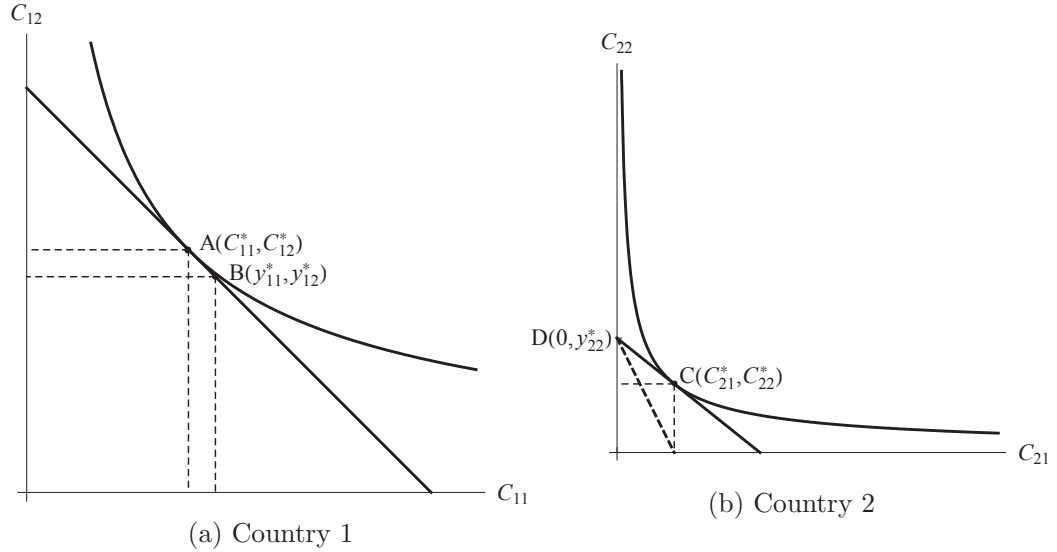
Case 2: Large country 1 water allocation($\theta_1 = 0.9$). Figures generated with parameter values $\alpha = 0.4$, $\beta_{11} = 5$, $\beta_{12} = 4$, $\beta_{21} = 3$, $\beta_{22} = 6$.

Utility of country 1 in this case equals the autarky level U_1^A. Given that the world price is β_{12}/β_{11}, we can find utility of country 2 by substituting the world price into the optimal consumption levels (2.24). Thus

$$(U_2^{FT})_2 = \left(\frac{\beta_{11}}{\beta_{12}}\alpha\right)^\alpha (1-\alpha)^{1-\alpha}\beta_{22}(1-\theta_1)W \tag{2.29}$$

gives free trade utility for country 2 under case 2. We can verify that $(U_2^{FT})_2 > U_2^A$

based on the comparative advantage assumption $\beta_{12}/\beta_{11} < \beta_{22}/\beta_{21}$. Therefore, when two countries with large enough disparities in size (in terms of water allocation) are involved in free trade, the small country (country 2 in this case) gains from free trade while the large country still gets its autarky utility.

2.4.3 Small country 1 water allocation

The case of a relatively small θ_1

$$\theta_1 \leq \frac{\alpha\beta_{21}}{\alpha\beta_{21} + (1 - \alpha)\beta_{11}} \tag{2.30}$$

is symmetric to case 2. Country 2 now becomes the large country, produces both goods and its consumption levels and utility are equal to the autarky levels. The world equilibrium price ratio

$$\tilde{p}_1/\tilde{p}_2 = \beta_{22}/\beta_{21} \tag{2.31}$$

equals country 2's autarky price ratio.

Figure 2.5 illustrates the equilibrium in this case. Country 1 specializes in producing good 1, $y_{11}^* = \beta_{11}\theta_1 W$, and its consumption levels are

$$c_{11}^* = \alpha\beta_{11}\theta_1 W \quad c_{12}^* = \frac{p_1}{p_2}\beta_{11}(1 - \alpha)\theta_1 W \tag{2.32}$$

from the utility maximization problem (2.10) with the world price ratio equal to β_{22}/β_{21}. The expression

$$(U_1^{FT})_3 = \alpha^\alpha \left(\frac{\beta_{22}}{\beta_{21}}(1 - \alpha)\right)^{1-\alpha} \beta_{11}\theta_1 W \tag{2.33}$$

gives free trade utility for country 1 in case 3.

Country 2's consumptions equal autarky consumptions and its outputs are

$$y_{21}^* = \beta_{21}\left(\alpha(1 - \theta_1)W - \frac{\beta_{11}}{\beta_{21}}(1 - \alpha)\theta_1 W\right) \tag{2.34}$$

$$y_{22}^* = \beta_{22}\left((1 - \alpha)(1 - \theta_1)W + \frac{\beta_{11}}{\beta_{21}}(1 - \alpha)\theta_1 W\right) \tag{2.35}$$

Figure 2.5: Utility Maximization Under Free Trade: Case 3

(a) Country 1
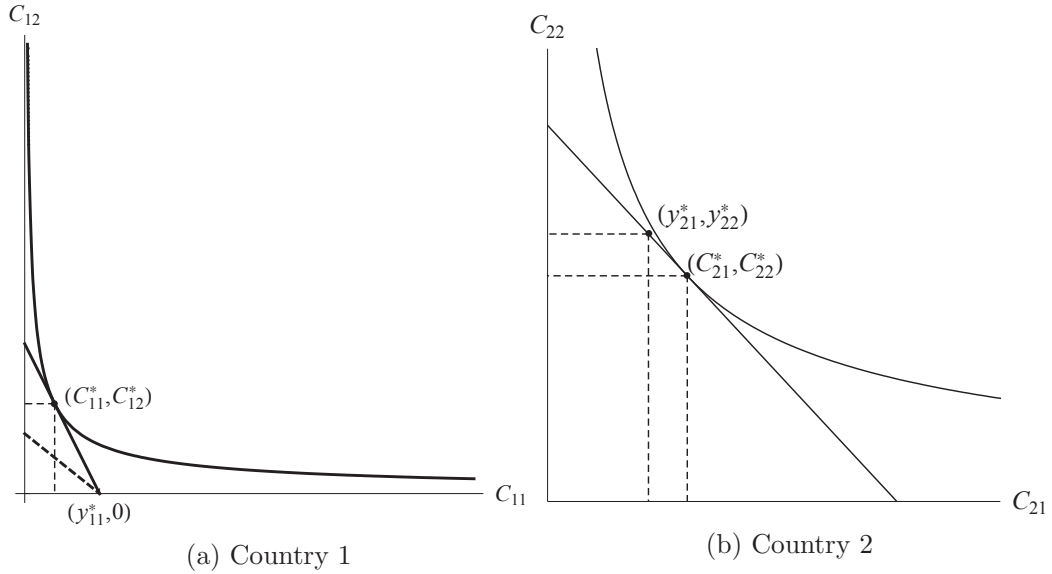
(b) Country 2

Case 3: Small country 1 water allocation($\theta_1 = 0.1$). Figures generated with parameter values $\alpha = 0.4$, $\beta_{11} = 5$, $\beta_{12} = 4$, $\beta_{21} = 3$, $\beta_{22} = 6$.

from market-clearing and the production and consumption levels of country 1. As before, a consistency check indicates that country 2's water resource constraint is satisfied by these relations.

2.5 Welfare Analysis

This section synthesizes the above three cases which are conditional on the water allocation parameter θ_1 to analyze the qualitative properties of the welfare functions. As noted above, these welfare functions give country utilities as functions of the water allocation parameter θ_1. The specific questions of interest include monotonicity of the welfare functions in water allocation, a comparison of the welfare gains from additional water allocated to a given country under autarky and free trade, and conditions under which it might be in the self-interest of countries to share water.

2.5.1 Welfare functions

We define the bounds as

$$m_1 = \frac{\alpha\beta_{21}}{\alpha\beta_{21} + (1-\alpha)\beta_{11}} \qquad m_2 = \frac{\alpha\beta_{22}}{\alpha\beta_{22} + (1-\alpha)\beta_{12}} \qquad (2.36)$$

for convenience in partitioning the water allocation space.

Country 1's welfare is

$$U_1^A = (\alpha\beta_{11})^\alpha((1-\alpha)\beta_{12})^{1-\alpha}\theta_1 W \qquad (2.37)$$

under autarky, and

$$U_1^{FT} = \begin{cases} (U_1^{FT})_3 = \alpha^\alpha \left(\frac{\beta_{22}}{\beta_{21}}(1-\alpha)\right)^{1-\alpha}\beta_{11}\theta_1 W & \text{if} \quad 0 \leq \theta_1 \leq m_1 \\[2mm] (U_1^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}\alpha W & \text{if} \quad m_1 < \theta_1 < m_2 \\[2mm] (U_1^{FT})_2 = (\alpha\beta_{11})^\alpha((1-\alpha)\beta_{12})^{1-\alpha}\theta_1 W & \text{if} \quad m_2 \leq \theta_1 \leq 1 \end{cases} \qquad (2.38)$$

in the free trade equilibrium. Likewise, country 2's welfare is

$$U_2^A = (\alpha\beta_{21})^\alpha((1-\alpha)\beta_{22})^{1-\alpha}(1-\theta_1)W \qquad (2.39)$$

under autarky, and

$$U_2^{FT} = \begin{cases} (U_2^{FT})_3 = (\alpha\beta_{21})^\alpha((1-\alpha)\beta_{22})^{1-\alpha}(1-\theta_1)W & \text{if} \quad 0 \leq \theta_1 \leq m_1 \\[2mm] (U_2^{FT})_1 = (\beta_{11}\theta_1)^\alpha(\beta_{22}(1-\theta_1))^{1-\alpha}(1-\alpha)W & \text{if} \quad m_1 < \theta_1 < m_2 \\[2mm] (U_2^{FT})_2 = \left(\frac{\beta_{11}}{\beta_{12}}\alpha\right)^\alpha(1-\alpha)^{1-\alpha}\beta_{22}(1-\theta_1)W & \text{if} \quad m_2 \leq \theta_1 \leq 1 \end{cases} \qquad (2.40)$$

under free trade.

2.5.2 Qualitative properties

We now analyze the qualitative properties of the welfare functions, in particular monotonicity. As delineated below, there are three cases to consider depending on the production parameters. Note that in all these cases, country 1 is assumed to have a comparative advantage in good 1 (Eq. 2.3), which implies that $m_1 < m_2$.

Case P1. $\beta_{11} > \beta_{21}$ and $\beta_{12} > \beta_{22}$.

In this case, country 1 not only has a comparative advantage in good 1, but also absolute advantages in both goods. It can be shown that when $\beta_{11} > \beta_{21}$, $m_1 < \alpha$, and when $\beta_{12} > \beta_{22}$, $m_2 < \alpha$. Hence, $m_1 < m_2 < \alpha$.

The welfare functions in this case are shown in Figure 2.6. The welfare function for country 1 is monotonically increasing as the water allocated to it increases. As country 2 gets increased water allocation, welfare first increases, then decreases for $\theta_1 \in (m_1, m_2)$, and then increases again.

Intuitively, since country 1 has absolute advantages in both goods, it does not have incentives to share water with the other country. The loss from sharing water cannot be offset by the gains from trade, hence the more water the better. However, for country 2, when its water allocations reaches the level of $(1 - m_2)$, it would be worse off by getting additional water. If initially, $\theta_1 \in (m_1, m_2)$, country 2 would even be better off by giving up some water. This can happen because the more water country 2 gets, the more goods it has to produce by itself. But country 2 has lower productivity coefficients, thus it could give up water to country 1 to produce at lower costs, and then gain via trade.

Case P2. $\beta_{11} < \beta_{21}$ and $\beta_{12} < \beta_{22}$.

In this case, country 2 has absolute advantages in both goods. Given that $\beta_{11} < \beta_{21}$ and $\beta_{12} < \beta_{22}$, $\alpha < m_1 < m_2$. This case is symmetric to Case P1.

Figure 2.6 shows that country 2's welfare function will be monotonically increasing with water allocated to it, θ_2, while for country 1, welfare starts to decrease when $\theta_1 > m_1$, and increases again when θ_1 exceeds m_2. There is a region where its welfare will be decreasing with the water allocation parameter θ_1. This result once again demonstrates that increased water allocations may not be welfare-enhancing in

the presence of productivity differences. This occurs because the gain from more water cannot offset the loss in trade.

Case P3. $\beta_{11} > \beta_{21}$ and $\beta_{12} < \beta_{22}$.

In this case, neither country has absolute advantages in both goods. Country 1 has a comparative advantage in good 1 and country 2 has a comparative advantage in good 2. As a result, $m_1 < \alpha < m_2$.

In Figure 2.6, when $\theta_1 \in (m_1, m_2)$, both countries' welfare functions are concave with a local maximum at $\theta_1 = \alpha$. It would be in both countries' mutual interest to set the water allocation parameter θ_1 equal to α if the welfare at the two boundary points ($\theta_1 = 0$ or $\theta_1 = 1$) does not exceed the welfare at $\theta_1 = \alpha$ (which is possible with some parameter specification). Even if the extreme welfare levels are higher, they are not necessarily attainable in reality.

Figure 2.6: Welfare functions Under Free Trade

(a) Case P1 (b) Case P2 (c) Case P3
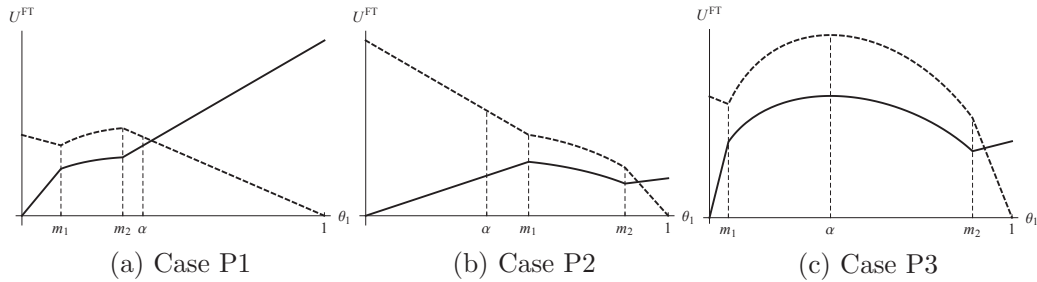
———— Country 1   - - - - - - Country 2

Case P1: Country 1 has absolute advantage in both goods, with $\alpha = 0.4, \beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$.

Case P2: Country 2 has absolute advantage in both goods, with $\alpha = 0.4, \beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$.

Case P3: Each country only has comparative advantage in one good, with $\alpha = 0.4, \beta_{11} = 10, \beta_{12} = 1, \beta_{21} = 1, \beta_{22} = 10$.

2.5.3 Water valuation under autarky and free trade

With intermediate water allocation, even if a country's welfare function under free trade is not declining, it is less steeply sloped under free trade than autarky, as illustrated by Figures 2.6, $\partial U_1^{FT}/\partial \theta_1$ is smaller than $\partial U_1^A/\partial \theta_1$ when $\theta_1 \in (m_1, m_2)$. This means that in the presence of trade, the marginal valuation of water can be lower

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with trade than without. This has two implications. First, even if it is in the country's self-interest to obtain more water, the gains are less than they would otherwise be. Thus, even if trade does not completely eliminate conflict over water, it can serve to reduce the level of conflict. Second, these results show that partial equilibrium studies could mis-estimate welfare gains if there are strong general equilibrium effects such as trade impacts.

2.5.4 Conflict and cooperation

To summarize, when one of the countries has absolute disadvantages in both goods, its welfare function will start to turn down after it gets a substantial amount of water. Because when the other country only gets a small portion of water, it will not be able to meet the demand of the large country (in terms of water) in trade. This can be seen from Case P1 and Case P2, where country 2's welfare function turns down when it has absolute disadvantages and country 1's welfare function turns down when it has absolute disadvantages.

The other country which has absolute advantages has monotonically increasing welfare function. However, the middle part of the graph has flatter slope than the other two parts, illustrating that the gains from more water is somewhat, though not completely, offset by the losses from trade when water allocation facilitates full specialization.

In the last case, when each country only has a comparative advantage in one good, the gains from trade are more obvious. Both countries' welfare functions will turn down as they get substantial amount of water. Hence, countries would agree to share the water at $\theta_1 = \alpha$. Both countries' welfare functions will reach a local maximum. Also, notice that the welfare when one country gets all the water may be greater than the sharing strategy. However, for one country to block the other country's access to the river water is not quite realistic.

2.6   The Water and Trade Game with Discrete Strategies

The welfare functions derived the from the previous trade model are now used in this and the subsequent section to analyze political strategies and equilibrium. We consider a non-cooperative approach in this section and a cooperative approach in the next section to improve upon the non-cooperative outcome.

The international cooperation literature notes the possibility of issue linkage with trade as a prominent example. For example, Kolstad (2010) discusses various forms of issue linkage with respect to transboundary pollution, while Bennett et al. (1998) and Pham-Do et al. (2011) consider issue linkage in the context of international river basins. Accordingly, we now consider political equilibrium formulated as a water and trade interconnected game. Suppose country 1 is the upstream country and country 2 is a downstream country. Country 1 has the priority to choose the water allocation by deciding how much to allocate to itself and leaves the rest to the other country, and also chooses to trade with other country or not. That is to say, country 1's strategy profile has two elements, a water allocation parameter and a trade policy. At the same time, country 2 only decides on whether to open to trade or not.

Following the literature, we formulate this as a two-player water-trade interconnected discrete game (Bennett et al., 1998). Table 2.1 shows the construction of a general payoff matrix. As illustrated in the table, country 1 can choose between two levels of water $\theta_1 = \theta_{high}$ and $\theta_1 = \theta_{low}$. The low *theta* value might be determined by rainfall and runoff occurring in each country, or it might be based on historical usage. The precise circumstances leading to this initial distribution are not relevant here, we simply take this distribution as given. The high value strategy can be conceptualized as a water diverting program. If the program is launched, then the water diverted by country 1 increases from θ_{low} to θ_{high}. Both countries choose a trade policy between free trade and autarky. Trade relations will be established only when both countries choose to trade.

In order to figure out the Nash equilibrium to this normal game, we need to analyze the welfare functions for both countries under the two scenarios of trade

Table 2.1: General Payoff Matrix

		Country 2	
		Autarky	Trade
θ_{low},	Autarky	$(U_1^A(\theta_{low}),\, U_2^A(\theta_{low}))$	$(U_1^A(\theta_{low}),\, U_2^A(\theta_{low}))$
θ_{low},	Trade	$(U_1^A(\theta_{low}),\, U_2^A(\theta_{low}))$	$(U_1^{FT}(\theta_{low}),\, U_2^{FT}(\theta_{low}))$
θ_{high},	Autarky	$(U_1^A(\theta_{high}),\, U_2^A(\theta_{high}))$	$(U_1^A(\theta_{high}),\, U_2^A(\theta_{high}))$
θ_{high},	Trade	$(U_1^A(\theta_{high}),\, U_2^A(\theta_{high}))$	$(U_1^{FT}(\theta_{high}),\, U_2^{FT}(\theta_{high}))$

Country 1 labels the rows.

relations. From Figure 2.7 that compares the welfare functions under free trade and autarky, we can see either country's free trade utility is always greater than or equal to autarky utility, hence both countries are more prone to choose a "trade" strategy to an "autarky" one when the parameters dictate that. However that does not excludes the "autarky" strategy from being chosen when the two scenarios give same welfare or when the other country already chooses autarky.

Figure 2.7: Welfare Functions Under Autarky and Free Trade

(a) Country 1   (b) Country 2

——— Free Trade   - - - - - - - Autarky
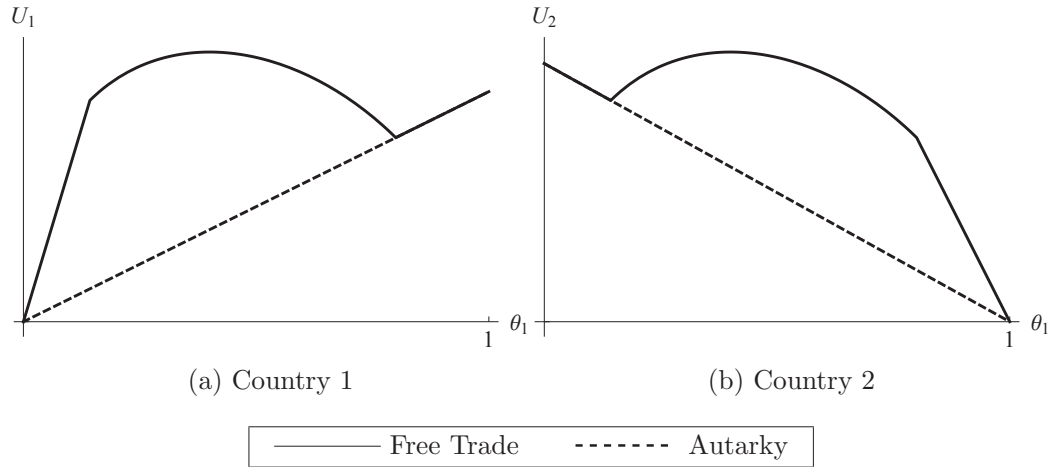
For country 1, it also needs to compare its free trade welfare for different values of water allocation parameter θ_1. Hence Figure 2.8 plots out all possible shapes of country 1's welfare functions under various parameter specifications. The free trade welfare functions for country 1 can be either increasing or decreasing, hence it is possible for country 1 to choose either the low or high value of water allocation.

From these two perspectives, the resulting equilibrium to the game will not be

unique and depends on the parameters. Therefore, we consider two numerical examples in the subsequent analysis to illustrate possible outcomes.

Table 2.2 is generated with production coefficients $\beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$, which is the case shown in Figure 2.8 and Country 1 has absolute advantage in both goods. The two discrete water allocation strategies are chosen such that in this range of parameter values, country 1's welfare increases with θ_1. In this instance, higher θ_1 value will grant country 1 higher welfare no matter under autarky or free trade, hence country 1 will always choose the large allocation, and there are two pure strategy Nash equilibria: $\{(\theta_{high} = 0.3, \text{Autarky}), \text{Autarky}\}$ and $\{(\theta_{high} = 0.3, \text{Trade}), \text{Trade}\}$. The solution shows that the water allocation is unambiguously $\theta_{high} = 0.3$, but the resulting trade situation will be ambiguous. The payoffs under the two equilibria are $(1.2835, 1.3808)$ and $(1.4079, 2.1118)$ respectively. Both countries can be better off under the trade equilibrium and this equilibrium can be sustained in a repeated play, however this is beyond the scope of this paper and will not be discussed here. This kind of outcome can be realized as long as Country 1's free trade welfare function increase with θ_1, as shown in Case P1 under all water allocation strategy space, Case P2 when $\theta_{low}, \theta_{high} \in (0, m_1) \cup (m_2, 1)$ and Case P3 when $\theta_{low}, \theta_{high} \in (0, \alpha) \cup (m_2, 1)$.

Table 2.2: Payoff Matrix: Case 1

| | | | Country 2 | |
			Autarky	Trade
	$\theta_{low} = 0.2,$	Autarky	(0.8556, 1.5780)	(0.8556, 1.5780)
Country 1	$\theta_{low} = 0.2,$	Trade	(0.8556, 1.5780)	(1.2969, 1.9454)
	$\theta_{high} = 0.3,$	Autarky	(1.2835, 1.3808)	(1.2835, 1.3808)
	$\theta_{high} = 0.3,$	Trade	(1.2835, 1.3808)	(1.4079, 2.1118)

Note: Country 1's welfare increases with θ_1, with $\beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$. $\alpha = 0.4$

Table 2.3 considers the case where country 1's welfare is decreasing as θ_1 increases. Production coefficients are $\beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$ such that country 2 has absolute advantage in both goods and this correspond to the case in Figure 2.8. In this instance, higher θ_1 is better for country 1, however, it is still possible for country

Figure 2.8: Country 1's Welfare Under Free Trade: All Cases
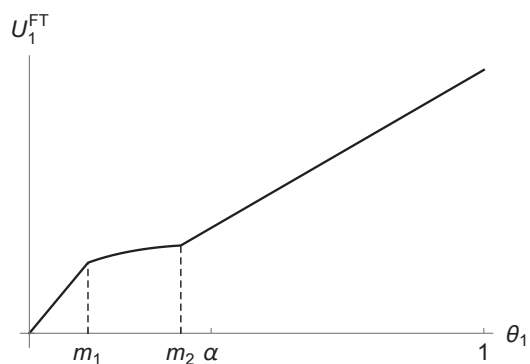
Case P1:
$\beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$
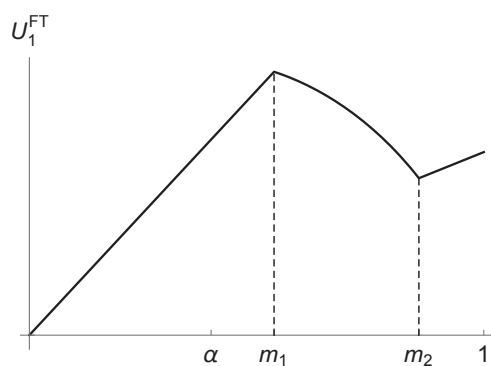
Case P2:
$\beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$
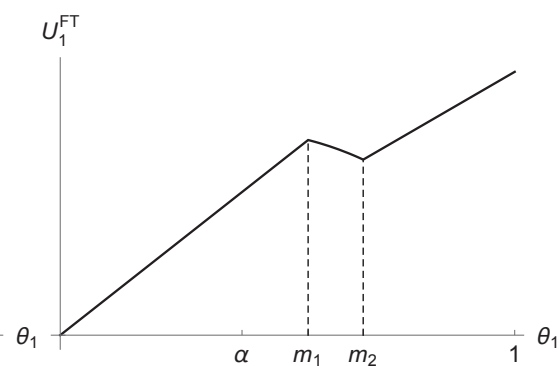
Case P2:
$\beta_{11} = 5, \beta_{12} = 3, \beta_{21} = 9, \beta_{22} = 9$
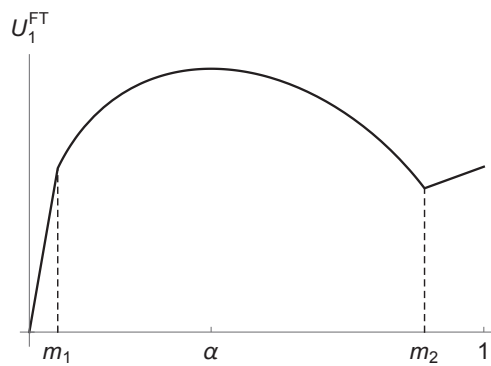
Case P3:
$\beta_{11} = 10, \beta_{12} = 1, \beta_{21} = 1, \beta_{22} = 10$
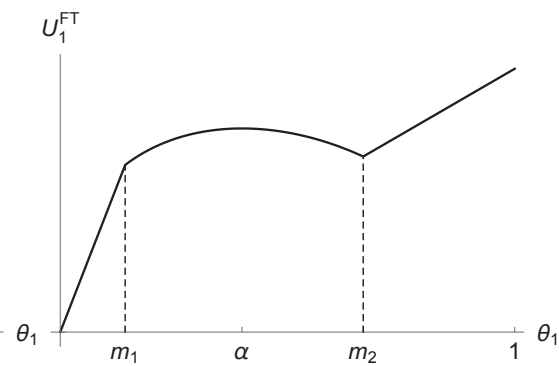
Case P3:
$\beta_{11} = 8, \beta_{12} = 3, \beta_{21} = 2, \beta_{22} = 9$

1 to select the low water allocation, i.e. to voluntarily share water with the downstream country when both countries choose to trade. As a result, the two pure strategy Nash equilibria are $\{(\theta_{high} = 0.7, \text{Autarky}), \text{Autarky}\}$ and $\{(\theta_{low} = 0.6, \text{Trade}), \text{Trade}\}$. This outcome will be more complicated than the previous case as both the water allocation parameter and the trade strategy are not unique. This can occur when country 2 has absolute advantages in both goods (Case P2 when $\theta_{low}, \theta_{high} \in (m_1, m_2)$), or when country 1 only has comparative advantage in good 1 (Case P3 when $\theta_{low}, \theta_{high} \in (\alpha, m_2)$).

Table 2.3: Payoff Matrix: Case 2

| | | | Country 2 | |
			Autarky	Trade
	$\theta_{low} = 0.6,$	Autarky	(0.5330, 1.6610)	(0.5330, 1.6610)
	$\theta_{low} = 0.6,$	Trade	(0.5330, 1.6610)	(1.2244, 1.8366)
Country 1	$\theta_{high} = 0.7,$	Autarky	(0.6218, 1.2457)	(0.6218, 1.2457)
	$\theta_{high} = 0.7,$	Trade	(0.6218, 1.2457)	(1.0958, 1.6437)

Note: Country 1's welfare decreases with θ_1, with $\beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$.

The equilibrium in Table 2.3 where countries share water and engage in free trade is facilitated by the fact that the upstream country 1 with water property rights is disadvantaged in production (either absolute disadvantage in both goods or comparative advantage in just one good). Clearly cooperation is easier to achieve when each country has leverage in some dimension.

The literature emphasizes issue linkage as a way to solve international cooperation problems (Bennett et al., 1998; Pham-Do et al., 2011), with trade policy as a specific example. The analysis in this section offers a somewhat different perspective, in that introducing trade does not give the second country any explicit leverage over the actions of the first country holding the water rights. Trade does influence the welfare function of the water-rights holding country, and in some circumstances it will be of self-interest for that country to jointly allocate water as noted previously. However, a threat by the second country to impose autarky is not credible since it is never better off doing this.

Thus, while trade can influence the political outcome, it is not necessarily through the channel of political bargaining power as in the issue linkage literature. Rather it may be through the evaluation process of individual country's welfare. The outcome is determined solely by the self-interest of the water-rights country, the other country does not have any credible bargaining power, at least within the context of the game-theoretic model here under standard rationality assumptions. Of course, richer game-theoretic models with asymmetric information or perhaps repeated play might yield a different outcome, and likewise for models with continuous, credible trade policies.

A methodological conclusion from these results is that the discrete strategy game is a limited analytical engine for this problem. The difficulty is that for a given trade model parameterization, the choice of discrete strategies is arbitrary but can influence the qualitative properties of the Nash equilibrium, i.e. whether or not joint water allocation is self-interest. Similar conclusions hold for trade policy as the game in this section only considers the extremes of autarky and free trade.

2.7 Nash Bargaining

The noncooperative approach gives non-unique outcomes. Both countries will be better off by choosing the trade equilibrium, but the autarky equilibrium is still possible to be realized. In this section, we consider the case when the autarky equilibrium is indeed realized and let this equilibrium be the initial conditions between the two countries in a cooperative Nash bargaining setting (John F. Nash, 1950). Both countries will be better off by yielding to a cooperative equilibrium. In this analysis, coercion is not possible; countries only agree to move from their initial distribution out of self-interest, that is individual rationality is satisfied. Another property that needs to be satisfied is group rationality, which means there is no other outcomes that will make both parties better off than the current equilibrium. Furthermore, we assume that the two countries have equal bargaining power.

In this bargaining problem, the two countries' preferences are given by their free trade welfare functions, as they will be cooperating with each other (trade in this

case). The payoff vector $U = (U_1^{FT}, U_2^{FT})$, a two dimensional space. When the two countries fail to reach an agreement, there will be an disagreement (conflict) payoff vector, $t = (t_1, t_2)$, which is given by the initial conditions. The Nash solution to the bargaining problem $\bar{U} = (\bar{U}_1, \bar{U}_2)$, is given by maximizing the Nash product,

$$(\bar{U}_1 - t_1)(\bar{U}_2 - t_2) = \max_{U \in P}[(U_1 - t_1)(U_2 - t_2)] \tag{2.41}$$

In a situation where production coefficients are specified like $\beta_{11} = 9, \beta_{12} = 8, \beta_{21} = 2, \beta_{22} = 6$, as in Case P1, the autarky noncooperative equilibrium in Section 2.6 gives the two countries payoffs of $(U_1^A(0.3), U_2^A(0.3)) = (1.28, 1.38)$. This pair of payoffs will be regarded as an initial condition between the two countries as they start the bargaining. It plays a role of a threat payoff vector such that when disagreement arises and negotiation fails, the initial payoffs will be the resulting payoffs. The bargaining solution is illustrated in Figure 2.9. It gives rise to a water allocation of $\theta_1 = 0.435$, with payoff vector $(\bar{U}_1, \bar{U}_2) = (1.86, 1.81)$, both countries are better off compared to the initial $(1.28, 1.38)$. Both individual and group rationality are satisfied as both individuals are better off than initial conditions and no individual can be better off without sacrificing the other. Also note that the payoff vector space is not convex due to the fact that the welfare functions are not monotone.

Consider Case P2, when country 2 has absolute advantage and production coefficients are $\beta_{11} = 4, \beta_{12} = 1, \beta_{21} = 7, \beta_{22} = 9$. The noncooperative Nash equilibrium in Section 2.6 gives an autarky payoff of $(U_1^A(0.7), U_2^A(0.7)) = (0.62, 1.25)$, and we take this as the disagreement payoff vector. The Nash bargaining solution is illustrated in Figure 2.10, which gives $\theta_1 = 0.481$ and payoff vector $(\bar{U}_1, \bar{U}_2) = (1.14, 2.16)$.

A similar analysis can be conducted for Case P3 (Figure 2.6). It is in both countries' consent to achieve an equilibrium allocation of $\theta_1^* = \alpha$ and $(\bar{U}_1, \bar{U}_2) = (2.04, 3.06)$(Figure 2.11).

There are several general conclusions from this analysis. First, there is a unique bargaining solution given model parameters and the initial allocation. We showed some

Figure 2.9: Nash Bargaining Solution: Case P1

U_2^{FT}

$\theta_1^*=0.435$

$(1.86,\ 1.81)$

$(1.28,\ 1.38)$

U_1^{FT}
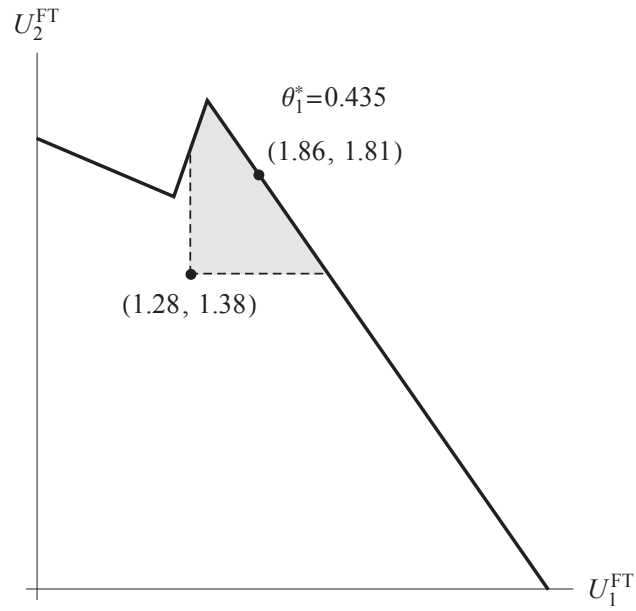
Figure 2.10: Nash Bargaining Solution: Case P2

U_2^{FT}

$\theta_1^*=0.481$

$(1.14,\ 2.16)$

$(0.62,\ 1.25)$

U_1^{FT}
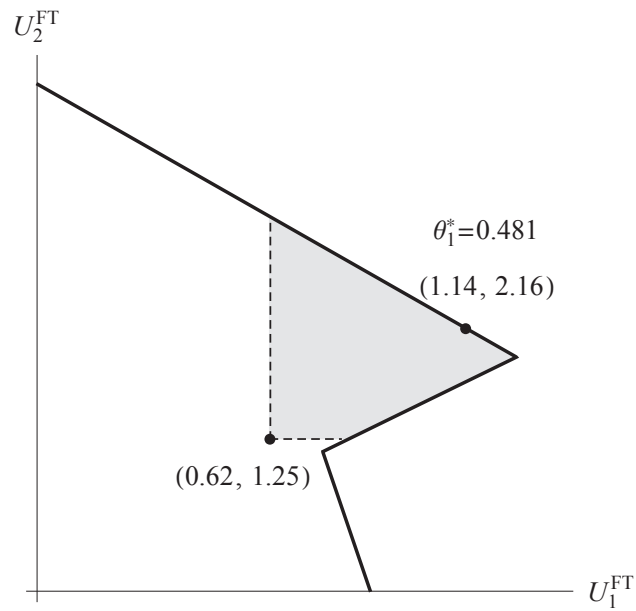
Figure 2.11: Nash Bargaining Solution: Case P3

U_2^{FT}

$\theta_1^*{=}0.4$, (2.04, 3.06)

(0.38, 1.42)

U_1^{FT}
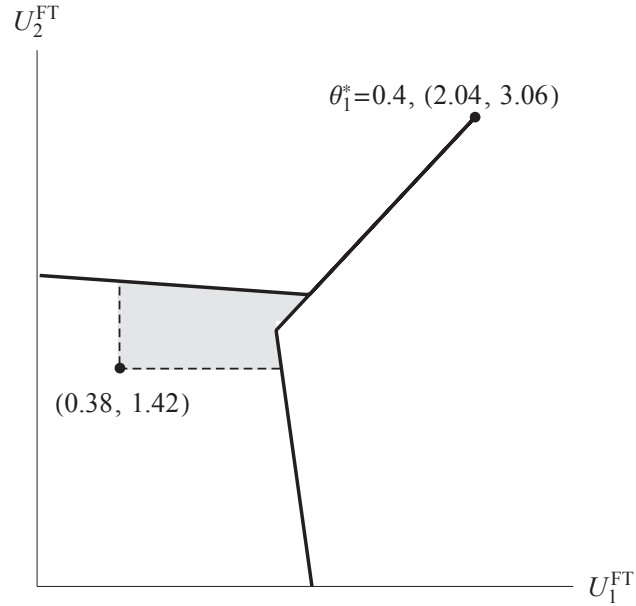
possible bargaining outcomes given certain initial allocations and production parameter specification. Second, bargaining can result in self-interested, mutually beneficial real-location due to the presence of trade. While this can occur in each of the three Cases P1-P3, it is most pronounced for the specific instance illustrated in Case P3 in which no country has absolute advantages in production. Third, a bargaining solution yields to an intermediate and equitable water allocation (θ_1 value around 0.4-0.5).

Finally, we note that some of the outcomes noted above may be specific to the particular parameterization used. While Figures 2.6-2.6 accurately convey monotonicity properties of the welfare functions for the respective Cases P1-P3, they may not be completely general with respect to the height of the endpoints relative to interior points. This can potentially affect the equilibrium outcomes in some instances.

2.8 Conclusions

The paper models water allocation for two countries which share a river and also engage in trade. Trade is a two-country/two-good Ricardian model, with water as the only factor of production and country variation in productivity as the conceptual motivation for trade. The analysis considers behavioral regimes of autarky and free

trade. In each instance, equilibrium consumptions and prices are derived for a given water allocation, and these in turn are used to derive country welfare as a function of water allocation. Game-theoretic models for political equilibrium are then formulated and analyzed utilizing the welfare functions from the economic model. Game-theoretic analysis of international water allocation has been studied in the previous literature. However, to our knowledge, the economic analysis of the welfare functions under trade and the subsequent game-theory models derived from those functions are novel.

Country welfare depends on the water allocation and the subsequent welfare functions exhibit some regularity. (1) First, consistent with standard trade theory, countries gain from free trade in the sense that the free trade welfare is larger or at least equal to autarky welfare. (2) As long as a country does not have absolute advantages in both goods, the benefit of getting more water will finally be offset by a trade loss as it gets more water, which is then reflected as a decrease in the welfare function. (3) Even if a country has absolute advantages in both goods, the benefit of getting more water will still be, though not completely, offset by the loss from trade, reflected by a flatter growth in the welfare function with intermediate water allocation.

Thus, when riparian countries are engaged in free trade, and for certain parameter specifications, there are circumstances in which country welfare can actually be decreasing in water allocation. Hence, it would be in the countries' self-interest to share water. Furthermore, even if the welfare function is increasing in water allocation, trade means that the gains from additional water can be smaller than that under autarky. This observation then serves to reduce conflict over the resource, although not necessarily eliminating it.

Political equilibrium is analyzed both as a noncooperative game and as a cooperative bargaining problem. First considered is a discrete strategy game with two water allocations for one country, and autarky/free trade options for both country. The primary conclusion here is that the trade policy of the second country may not be credible as a means of getting additional water since free trade is generally advantageous to that country over autarky. In this setting, then, the primary role of trade is not as a bargaining tool, but rather it affects country's evaluation of their welfare function

and self-interest in water allocation. Under the bargaining problem, we treat the Nash equilibrium in the noncooperative game as an initial condition, and find that both countries could be better off by moving to a cooperative bargaining solution. However, the resulting equilibrium also depends on the initial conditions and parameter specifications.

In general, moving to a general equilibrium setting can potentially be conflict-reducing, although not necessarily conflict-eliminating. This is due to the fact that in general equilibrium, there can be additional channels through which water allocation affects an entity, and some of these may be adverse. This work also implies that—in contrast to much of the literature—in the presence of trade, country welfare is a function of not only its own water allocation, but also that of the other countries. In general, if there are multiple countries involved in trade and water is fully allocated, then country welfare will be a function of water allocations among all the countries.

Chapter 3

Economic Equilibrium for Sharing an International River with International Trade: Two-Factor and Three-Country Case

3.1   Introduction

Natural resources such as rivers are gifts of nature, but the use of the resources is complicated when they lie on the boundaries of countries. Conflicts over the natural resources are inevitable. Take international rivers as an example, often cases, these rivers are shared by multiple countries, like the Mekong River in Asia flows across China, Burma, Laos, Thailand, Cambodia, and Vietnam; The Rhine River in Europe is shared by Germany, Austria, Switzerland, France, Neitherlands, and Liechtenstein; The Nile River provides water to Ethiopia, Sudan, Egypt, Uganda, Congo-Kinshasa, Kenya, Tanzania, Rwanda, Burundi, South Sudan, Eritrea. There are many more rivers like these in the world which attracts researchers' attention to mitigate the conflicts over water between countries and achieve more efficient use of the resource.

While the simple Ricardian model in Chapter 2 with water being the only factor of production, provides insight on the possible shapes of country's welfare functions of

water, in reality, countries will not just use water to produce goods, at least labor will be used. A natural question to ask, is whether adding labor into the production functions will dramatically change the results we obtained from Chapter 2. Hence, we extend the model to include labor in production, and also examine the results with three countries.

The model is solved both analytically and numerically. The analytical solution is obtained with a similar method from 2 by optimizing the utility functions. The numerical solution adopts an applied general equilibrium framework described in Cardenete et al. (2012).

The main results from the model with two factors and with three countries are consistent with the results from the one-factor Ricardian model in Chapter 2, but what slightly differs from the simple Ricardian Model in which the country's "strength" only lies in the relative productivity, is that a country's "national strength" is a combination of both its production functions and the labor endowment. The comparative advantage is not simply determined by production function either, but rather should be broadly defined by comparing the relatively price ratios which in turn depend on production coefficients, preference coefficients and also factor endowments. This consistency between the one-factor and two-factor models further confirm the conclusions from the Ricardian model, and it should still hold with multiple factors and multiple countries without surprise. Hence, an examination of a simple two-factor two-country model provides bases for international natural resource sharing in reality with more than two countries.

3.2   Two-Factor Model

Consider an international river shared by n countries. Denote countries by i, $i = 1, 2, ..., n$. In the subsequent sections, we will examine the results when $n = 2$ and $n = 3$. Each country has a representative household with identical preferences as:

$$U_i = C_{i1}^{\alpha_1} C_{i2}^{\alpha_2} \qquad (\alpha_1 + \alpha_2 = 1), \tag{3.1}$$

and with labor endowment L_i and water resource $W_i = \theta_i W$ from the international river, where W is the total annual flow of water and θ_i is the proportion of total water

that is allocated to country i. Assume that the allocation is exogenous at this point and the river is fully allocated among the n countries such that $\sum_{i=1}^{n} \theta_i = 1$.

On the production side, there are two consumption goods denoted by $j = 1, 2$, with the Cobb-Douglas production function:

$$y_{ij} = w_{ij}^{\gamma_{ij}} l_{ij}^{1-\gamma_{ij}}. \tag{3.2}$$

The production function implies that countries may differ in technology as γ_{ij} varies. The difference in technology makes trade possible between countries.

3.2.1 Autarky

Under autarky, let p_{ij} represent the price of the consumption good j in country i, and p_{iw} and p_{il} represent the factor price of water and labor in country i.

The household demand functions for goods are solved from maximizing the utility function (3.1), subject to the budget constraint

$$\sum_{j=1}^{2} p_{ij} C_{ij} = p_{iw} \theta_i W + p_{il} L_i, \tag{3.3}$$

which yields

$$C_{ij} = \alpha_1 \frac{p_{iw} \theta_i W + p_{il} L_i}{p_{ij}}. \tag{3.4}$$

The firms' profit functions are

$$\pi_{ij} = p_{ij} y_{ij} - (p_{iw} w_{ij} + p_{il} l_{ij}), \tag{3.5}$$

where the conditional factor demands can be derived from the firms' cost minimization problem, which gives

$$w_{ij} = \left(\frac{p_{il} \gamma_{ij}}{p_{iw}(1 - \gamma_{ij})} \right)^{1-\gamma_{ij}} y_{ij} \qquad l_{ij} = \left(\frac{p_{iw}(1 - \gamma_{ij})}{p_{il} \gamma_{ij}} \right)^{\gamma_{ij}} y_{ij}. \tag{3.6}$$

Hence the firms' profit maximization conditions becomes

$$p_{ij} = p_{iw} \left(\frac{p_{il}\gamma_{ij}}{p_{iw}(1-\gamma_{ij})} \right)^{1-\gamma_{ij}} + p_{il} \left(\frac{p_{iw}(1-\gamma_{ij})}{p_{il}\gamma_{ij}} \right)^{\gamma_{ij}}, \tag{3.7}$$

which is intuitive that the unit price of any commodity equals the marginal cost of production in a perfectly competitive market. This condition holds for any quantities of supply for good ij. Hence any supply will be profit maximizing and firms will supply up to the point where consumer demand is satisfied.

The autarky equilibrium is characterized by the goods market-clearing conditions

$$y_{ij} = C_{ij} \tag{3.8}$$

and the factor-marketing clearing conditions

$$\sum_{j=1}^{2} w_{ij} = \theta_i W \qquad \sum_{j=1}^{2} l_{ij} = L_i, \tag{3.9}$$

that is, both water and labor are fully employed.

Let good 2 in each country be the numeraire, i.e. $p_{i2} = 1$, then the price of good 1, factor prices, consumption and production in each country can be solved. Plug the equilibrium consumption levels into the utility function to get the indirect utility, which gives the welfare level of each country under a certain water allocation. Furthermore, the welfare function can be numerically calculated by varying the water allocation and solving for the welfare levels to yield a country welfare as a function of water allocation parameter.

3.2.2 Free Trade

Assume that goods can be traded under free trade, while factors of production are confined within the country. Different from autarky, the free trade goods prices will be equal across the two countries. Let $\tilde{p}_j$ be the world equilibrium price of good j, then

the budget constraint for country i becomes

$$\sum_{j=1}^{2} \tilde{p}_j C_{ij} = p_{iw}\theta_i W + p_{il}L_i. \tag{3.10}$$

The household's demand function also changes according to the world prices,

$$C_{ij} = \alpha_j \frac{p_{iw}\theta_i W + p_{il}L_i}{\tilde{p}_j} \tag{3.11}$$

The firms' profit maximization conditions are now

$$\tilde{p}_j - p_{iw}\left(\frac{p_{il}\gamma_{ij}}{p_{iw}(1-\gamma_{ij})}\right)^{1-\gamma_{ij}} - p_{il}\left(\frac{p_{iw}(1-\gamma_{ij})}{p_{il}\gamma_{ij}}\right)^{\gamma_{ij}} \leq 0$$

$$y_{ij} \geq 0, \quad \left(\tilde{p}_j - p_{iw}\left(\frac{p_{il}\gamma_{ij}}{p_{iw}(1-\gamma_{ij})}\right)^{1-\gamma_{ij}} - p_{il}\left(\frac{p_{iw}(1-\gamma_{ij})}{p_{il}\gamma_{ij}}\right)^{\gamma_{ij}}\right)y_{ij} = 0. \tag{3.12}$$

The complementary-slack conditions capture the possibility that under free trade, coun-
tries may specialize in one good.

The world goods market-clearing conditions

$$\sum_{i=1}^{n} C_{ij} = \sum_{i=1}^{n} y_{ij} \quad (\text{for } j=1,2), \tag{3.13}$$

and each country's factor market-clearing conditions (3.9) constitute the equilibrium
conditions. Let good 2 be the numeraire, that is $\tilde{p}_2 = 1$, then solve for the world
equilibrium price $\tilde{p}_1$ for good 1, factor prices p_{iw}, p_{il}, consumption C_{ij} and production
y_{ij} for each country. Analogous to the autarky case, the welfare level can be obtained
by plugging the consumption levels into the utility function, and the welfare function
can be computed by solving the equilibrium repeatedly with various water allocation
parameter values.

3.3   Parameter Values

The Two-factor Model uses the following four sets of parameter values shown
in Table 3.1 to generate some incomplete yet representative welfare functions for the

countries involved in sharing an international river and trading.

Table 3.1: Parameter Values for Two-Factor Model

	α_1	γ_{11}	γ_{12}	γ_{21}	γ_{22}	W	L_1	L_2
Set 1	0.4	0.3	0.4	0.2	0.8	1	1	0.8
Set 2	0.4	0.5	0.2	0.3	0.7	1	1	3
Set 3	0.4	0.5	0.2	0.3	0.7	1	1	1
Set 4	0.4	0.3	0.7	0.5	0.2	1	3	1

The parameter α_1 on the utility function is fixed at 0.4 to indicate a slight preference of good 2 over good 1 and reversing the preference would be symmetric. Total quantity of water W is fixed at 1, and the actual endowment of water allocated to each country depends on the proportion parameter θ_i. L_1, L_2 are the labor endowment of the two countries and the parameterization includes the cases of Country 1's labor endowment being larger/smaller/equal to that of Country 2's. Different values of γ represents the difference in opportunity cost of the two goods by the two countries. The opportunity cost of good 1 in Country i can be represented by the slope of the production possibility frontier in:

$$\frac{dy_{i2}}{dy_{i1}} = \frac{\gamma_{i2}w_{i2}^{\gamma_{i2}-1}l_{12}^{1-\gamma_{i2}}dw_{i2} + (1-\gamma_{i2})w_{i2}^{\gamma_{i2}}l_{i2}^{-\gamma_{i2}}dl_{i2}}{\gamma_{i1}w_{i1}^{\gamma_{i1}-1}l_{11}^{1-\gamma_{i1}}dw_{i1} + (1-\gamma_{i1})w_{i1}^{\gamma_{i1}}l_{i1}^{-\gamma_{i1}}dl_{i1}}. \tag{3.14}$$

The relative opportunity cost of good 1 between countries determines the countries' comparative advantages in goods, however at this point, the opportunity cost is ambiguous.

The two-factor model with three countries uses the parameter values in Table 3.2.

Table 3.2: Parameter Values for Two-Factor Three-Country Model

	γ_{i1}	γ_{i2}	L_i
Country 1	0.3	0.7	3
Country 2	0.5	0.2	1
Country 3	0.4	0.6	1.5
$\alpha_1 = 0.4$		$W = 1$	

3.4 Analytical Results for Two-Factor Model

3.4.1 Autarky

The autarky equilibrium can be solved analytically. In the equilibrium, the water and labor factor demand for each good in each country are characterized by:

$$w_{ij}^A = \frac{\theta_i W \alpha_j \gamma_{ij}}{\sum_{j=1}^{2} \alpha_j \gamma_{ij}} \qquad l_{ij}^A = \frac{L_i \alpha_j (1 - \gamma_{ij})}{1 - \sum_{j=1}^{2} \alpha_j \gamma_{ij}}, \tag{3.15}$$

where α_j is the coefficient on utility function, and γ_{ij} is the coefficient on production function, $i = 1, 2$ represents countries and $j = 1, 2$ represents goods. The factor demand depends on the preferences, technology and also endowment.

The equilibrium consumption and output are equal in equilibrium under autarky, and can be obtained by plugging the factor demands (3.15) into the production functions (3.2), which yields

$$y_{ij}^A = C_{ij}^A = \left(\frac{\theta_i W \alpha_j \gamma_{ij}}{\sum_{j=1}^{2} \alpha_j \gamma_{ij}} \right)^{\gamma_{ij}} \left(\frac{L_i \alpha_j (1 - \gamma_{ij})}{1 - \sum_{j=1}^{2} \alpha_j \gamma_{ij}} \right)^{1 - \gamma_{ij}}. \tag{3.16}$$

Furthermore, plugging these consumption levels into the utility function (3.1) will lead to the welfare functions under autarky for each country:

$$U_i^A = \prod_{j=1}^{2} \left(\left(\frac{\theta_i W \alpha_j \gamma_{ij}}{\sum_{j=1}^{2} \alpha_j \gamma_{ij}} \right)^{\gamma_{ij}} \left(\frac{L_i \alpha_j (1 - \gamma_{ij})}{1 - \sum_{j=1}^{2} \alpha_j \gamma_{ij}} \right)^{1 - \gamma_{ij}} \right)^{\alpha_j}. \tag{3.17}$$

The welfare function can be simplified to

$$U_i^A = \phi \theta_i^{\sum_{j=1}^{2} \gamma_{ij}\alpha_j},\tag{3.18}$$

where $\phi = \prod_{j=1}^{2}\left(\left(\frac{W\alpha_j\gamma_{ij}}{\sum_{j=1}^{2}\alpha_j\gamma_{ij}}\right)^{\gamma_{ij}}\left(\frac{L_i\alpha_j(1-\gamma_{ij})}{1-\sum_{j=1}^{2}\alpha_j\gamma_{ij}}\right)^{1-\gamma_{ij}}\right)^{\alpha_j}$ is a constant that only depends the exogenous parameter values. The welfare function implies that country i's autarky welfare is positively related with the proportion of water it gets (θ_i), and the shape of the welfare function is determined by the country's own production coefficients γ_{ij}: when $\sum_{j=1}^{2}\gamma_{ij}\alpha_j = 1$, the welfare function will be linear, and when $\sum_{j=1}^{2}\gamma_{ij}\alpha_j \neq 1$, it is nonlinear. If we were to restrict $\gamma_{ij} \leq 1$, and given $\sum_{j=1}^{2}\alpha_j = 1$, only when both $\gamma_{ij} = 1$ $(j = 1,2)$ will Country i's autarky welfare function be linear. This condition simply reduces the production to the one-factor linear production function as in Chapter 2.

The autarky relative price can also be obtained from the first order conditions:

$$\frac{p_{i1}^A}{p_{i2}^A} = \frac{\alpha_1}{\alpha_2}\frac{C_{i2}^A}{C_{i1}^A} = \frac{\alpha_1}{\alpha_2}\frac{\left(\frac{\theta_i W\alpha_2\gamma_{i2}}{\sum_{j=1}^{2}\alpha_j\gamma_{ij}}\right)^{\gamma_{i2}}\left(\frac{L_i\alpha_2(1-\gamma_{i2})}{1-\sum_{j=1}^{2}\alpha_j\gamma_{ij}}\right)^{1-\gamma_{i2}}}{\left(\frac{\theta_i W\alpha_1\gamma_{i1}}{\sum_{j=1}^{2}\alpha_j\gamma_{ij}}\right)^{\gamma_{i1}}\left(\frac{L_i\alpha_1(1-\gamma_{i1})}{1-\sum_{j=1}^{2}\alpha_j\gamma_{ij}}\right)^{1-\gamma_{i1}}}\tag{3.19}$$

It can be verified that the autarky relative price in the one-factor model $\frac{p_{i1}^A}{p_{i2}^A} = \frac{\beta_{i2}}{\beta_{i1}}$ is a special case of the price in Eq. 3.19. Hence a country's comparative advantage can be broadly defined by comparing the autarky relative prices, and we have the following definition:

Definition 1. *If the autarky relative prices for the two countries have the following relationship*

$$\frac{p_{11}^A}{p_{12}^A} < \frac{p_{21}^A}{p_{22}^A},\tag{3.20}$$

then Country 1 is said to have a comparative advantage in good 1, and Country 2 has a comparative advantage in good 2, and vice versa.

It is worth noting that the autarky relative price for each country is not constant, because it depends not only on the exogenous parameters, but also depends on θ_i. As the water allocation changes, the relationship between autarky prices may reverse,

and the comparative advantage for a country will change accordingly.

3.4.2 Free Trade

For the two-factor model, the free trade analytical solution is complicated by all the different possibilities of the relationship between the two countries' autarky relative prices. For example, Country 1's autarky price can be smaller than Country 2's for all θ_i values, and vice versa. Or Country 1's autarky price can first be smaller than Country 2's for small values of θ_i, and reverse when θ_i gets larger. Therefore, we only study one possibility in this section to illustrate some of the conclusions.

With Data Set 4, we can calculate that Country 1's autarky relative price is always smaller than Country 2's for any θ_1, as shown in Figure 3.1, Country 1's price falls below Country 2's price, which implies that Country 1 has a comparative advantage in good 1 and Country 2 has a comparative advantage in good 2. If the world equilibrium price is between the two countries' autarky prices, it can be conjectured that the two countries will specialize in the goods that they have comparative advantage in. Therefore, we first assume that the world equilibrium price satisfies

$$\frac{\alpha_1}{\alpha_2}\frac{C_{12}^A}{C_{11}^A} = \frac{p_{11}^A}{p_{12}^A} < \frac{\tilde{p}_1}{\tilde{p}_2} < \frac{p_{21}^A}{p_{22}^A} = \frac{\alpha_1}{\alpha_2}\frac{C_{22}^A}{C_{21}^A}. \tag{3.21}$$

Under this price condition, Country 1 specializes in good 1 and Country 2 specializes in good 2, i.e. the outputs of each country are

$$y_{11}^{FT} = (\theta_1 W)^{\gamma_{11}}(L_1)^{1-\gamma_{11}} \quad y_{12}^{FT} = 0 \qquad y_{21}^{FT} = 0 \quad y_{22}^{FT} = (\theta_2 W)^{\gamma_{22}}(L_2)^{1-\gamma_{22}} \tag{3.22}$$

The free trade consumption levels can be obtained by maximizing the utility function given the world prices $\tilde{p}_1$ and $\tilde{p}_2$:

$$C_{11}^{FT} = \alpha_1(\theta_1 W)^{\gamma_{11}}(L_1)^{1-\gamma_{11}} \qquad C_{12}^{FT} = \frac{\tilde{p}_1}{\tilde{p}_2}\alpha_2(\theta_1 W)^{\gamma_{11}(L_1)^{1-\gamma_{11}}}$$

$$C_{21}^{FT} = \frac{\tilde{p}_2}{\tilde{p}_1}\alpha_1(\theta_2 W)^{\gamma_{22}(L_2)^{1-\gamma_{22}}} \qquad C_{22}^{FT} = \alpha_2(\theta_2 W)^{\gamma_{22}}(L_2)^{1-\gamma_{22}}, \tag{3.23}$$

Figure 3.1: Autarky and Free Trade Prices For the Two-Factor Model
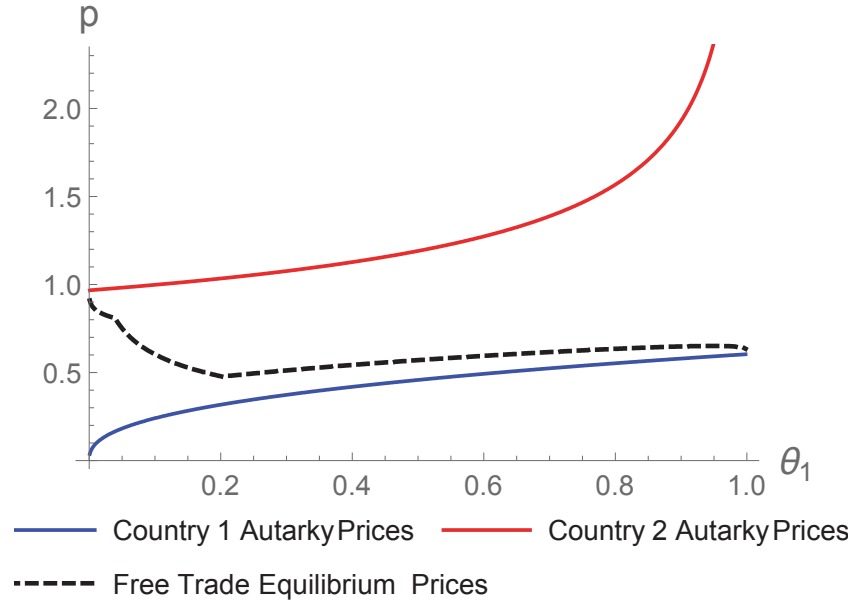
Note: Prices generated with Data Set 4.

and combined with the free trade good market clearing conditions Eq. 3.13 give the world equilibrium prices

$$\frac{\tilde{p}_1}{\tilde{p}_2} = \frac{\alpha_1(\theta_2 W)^{\gamma_{22}}(L_2)^{1-\gamma_{22}}}{\alpha_2(\theta_1 W)^{\gamma_{11}}(L_1)^{1-\gamma_{11}}}. \tag{3.24}$$

This world equilibrium price will prevail when neither country is too large, each country just specialize in one good, and the world price is between the autarky prices.

Unlike the one-factor case, in which when one country becomes a large country, the world equilibrium price will be exactly equal to that country's autarky price, as shown in Figure 3.2, in the two-factor model, when Country i becomes a large country, the world equilibrium price will be

$$\frac{\tilde{p}_1}{\tilde{p}_2} = \frac{\alpha_1}{\alpha_2}\frac{C_{i2}^{FT}}{C_{i1}^{FT}}, \tag{3.25}$$

which has a formula similar to Country i's autarky price ratio $\frac{\alpha_1}{\alpha_2}\frac{C_{i2}^A}{C_{i1}^A}$, but does not have the exact same value since the autarky and free trade consumption levels differ.

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Figure 3.2: Autarky and Free Trade Prices For the One-Factor Model
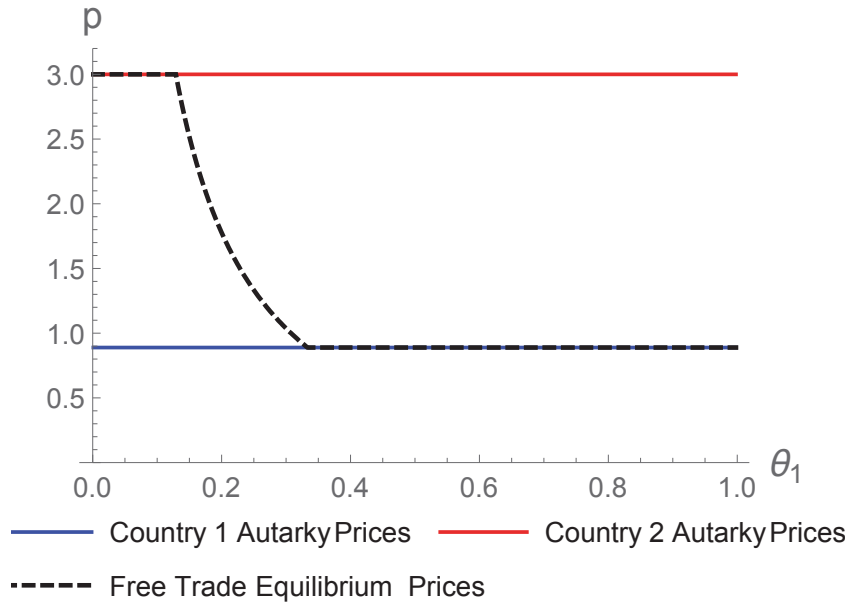
Therefore, we see in Figure 3.1, the world equilibrium price curve has two turning points: (1)Before the first turning point, θ_1 is quite small, Country 2 gets most of the water and is a large country, the world price is "quasi-same" with Country 2's autarky price. Hence it produces both goods, while Country 1 specializes in good 1; (2) After the second turning point, θ_1 becomes larger, Country 1 becomes the large country, and the world price is "quasi-same" with Country 1's autarky price. Country 2 specialize in good 2 while Country 1 produces both goods. This production patterns can be verified by plotting out each country's output along with the world equilibrium price in Figure 3.3. The production patterns change exactly at the two turning points of the world equilibrium price.

For a comparison, the production patterns and world equilibrium price for the One-factor Ricardian Model is illustrated in Figure 3.4. The production pattern also alters when the world equilibrium price changes.

Figure 3.3: Production Patterns and the World Equilibrium Price for Two-Factor Model

(a)Country 1

(a)Country 2
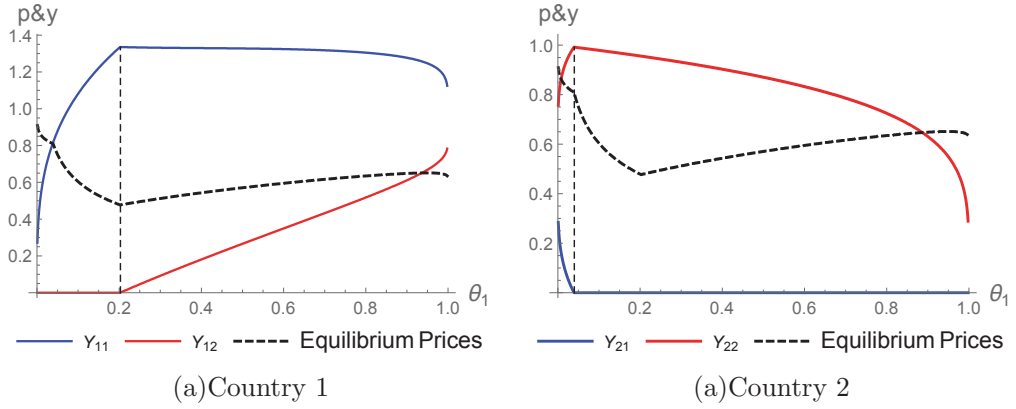
Note: Prices and outputs generated with Data Set 4.

Figure 3.4: Production Patterns and the World Equilibrium Price for One-Factor Model

(a)Country 1

(a)Country 2
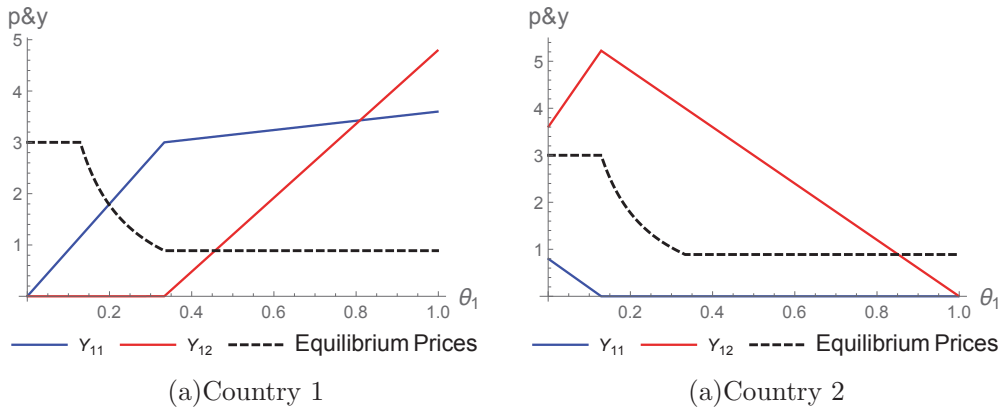
3.5 Numerical Results

3.5.1 Equilibrium under Autarky and Free Trade

The equilibrium for the model under autarky and free trade are solved numerically and repeatedly by changing the value of θ_1, i.e. changing the water allocation between the countries and examining how the equilibrium will vary with the allocation. Table 3.3 shows the equilibrium welfare levels, consumption and production levels, relative good prices, and factor prices in each country, when θ_1 ranges from 0.1 to 0.9 with a step of 0.1 and other parameters take the values in Data Set 4. In the autarky equilibrium, welfare increases with the quantity of water obtained. Each country's consumption equals production. Country 1's relative price of good 1 p_{11} is smaller than that of Country 2's p_{21} (note that good 2 in each country are set as numeraire), indicating that Country 1 has a smaller opportunity cost of good 1, i.e. a comparative advantage in good 1 in this case. Each country's water prices decreases with the quantity of water, that is, the more abundant water is, the lower price it is. The price of labor also changes with quantity of water, when water is scarce, the fixed amount of labor combined with a small amount of water lowers the price of labor.

Table 3.4 shows the equilibrium under free trade and is also generated with data set 4. The equilibrium commodity balance is characterized by Eq. 3.13. Under data set 4, Country 1 has a comparative advantage in good 1, and Country 2 has a comparative advantage in good 2, as can be seen from comparing the autarky relative prices from Table 3.3. Hence, under free trade, Country 2 specializes in good 2, Country 1 specializes in good 1 when it gets small amount of water. When Country 1 gets large amount of water, together with the large labor endowment, Country 1 becomes a big country compared to Country 2, thus produces both goods. The factor prices in each country have the same pattern as that under autarky.

In order to visualize the results, we solve the equilibrium with more θ_1 values, ranging from 0.001 to 0.999 with a step of 0.001. The results are shown with graphs in the following subsections which demonstrate the welfare function plots and their

Table 3.3: Autarky Equilibrium Results for Two-Factor Model

θ_1	U_1^A	U_2^A	$C_{11} = y_{11}$	$C_{12} = y_{12}$	$C_{21} = y_{21}$	$C_{22} = y_{22}$
0.1	0.264	0.518	0.487	0.176	0.407	0.609
0.2	0.384	0.499	0.599	0.285	0.383	0.595
0.3	0.478	0.478	0.676	0.379	0.359	0.579
0.4	0.558	0.455	0.737	0.463	0.332	0.562
0.5	0.629	0.429	0.788	0.542	0.303	0.541
0.6	0.695	0.400	0.833	0.615	0.271	0.518
0.7	0.755	0.365	0.872	0.686	0.235	0.489
0.8	0.811	0.320	0.908	0.753	0.192	0.451
0.9	0.865	0.257	0.941	0.817	0.136	0.392

θ_1	p_{11}	p_{21}	p_{1w}	p_{1l}	p_{2w}	p_{2l}
0.1	0.241	0.998	1.580	0.045	0.361	0.690
0.2	0.317	1.034	1.284	0.073	0.397	0.674
0.3	0.373	1.076	1.137	0.097	0.441	0.656
0.4	0.419	1.127	1.043	0.118	0.499	0.636
0.5	0.458	1.191	0.975	0.138	0.578	0.614
0.6	0.493	1.273	0.923	0.157	0.690	0.587
0.7	0.524	1.388	0.881	0.175	0.869	0.554
0.8	0.553	1.567	0.847	0.192	1.202	0.511
0.9	0.579	1.930	0.817	0.209	2.093	0.445

Note: Equilibrium results generated using data set 4.

Table 3.4: Free Trade Equilibrium Results for Two-Factor Model

θ_1	U_1^{FT}	U_2^{FT}	C_{11}	C_{12}	C_{21}	C_{22}	y_{11}	y_{12}	y_{21}	y_{22}
0.1	0.408	0.611	0.433	0.392	0.649	0.587	1.081	0	0	0.979
0.2	0.437	0.655	0.533	0.383	0.799	0.574	1.331	0	0	0.956
0.3	0.516	0.621	0.605	0.464	0.727	0.559	1.332	0.092	0	0.931
0.4	0.588	0.588	0.665	0.542	0.665	0.542	1.330	0.181	0	0.903
0.5	0.654	0.556	0.718	0.614	0.610	0.522	1.328	0.266	0	0.870
0.6	0.714	0.523	0.765	0.682	0.560	0.500	1.324	0.349	0	0.832
0.7	0.771	0.487	0.808	0.747	0.510	0.472	1.318	0.432	0	0.786
0.8	0.823	0.443	0.848	0.807	0.457	0.435	1.305	0.518	0	0.725
0.9	0.873	0.383	0.888	0.864	0.389	0.379	1.277	0.611	0	0.631

θ_1						$\tilde{p}_1$	p_{1w}	p_{1l}	p_{2w}	p_{2l}
0.1						0.604	1.958	0.152	0.218	0.783
0.2						0.479	0.957	0.149	0.239	0.765
0.3						0.512	0.897	0.168	0.266	0.745
0.4						0.543	0.858	0.187	0.301	0.722
0.5						0.571	0.827	0.203	0.348	0.696
0.6						0.595	0.801	0.219	0.416	0.666
0.7						0.616	0.780	0.233	0.524	0.629
0.8						0.635	0.763	0.245	0.725	0.580
0.9						0.648	0.751	0.254	1.262	0.505

Note: Equilibrium results generated using data set 4.

properties for the models with two-factor-two-country and two-factor-three-country.

3.5.2 Welfare Functions for the Two-factor Model

Figure 3.5 demonstrates the autarky welfare functions for the two countries using the four different data sets. The absolute values of the welfare functions may differ, but the properties of the functions are similar. As shown in the figure, under autarky, each country's welfare monotonically increases with the amount of water allocated to the country, which is consistent with the finding in Chapter 2, even though the autarky welfare functions no long retain linearity because of the Cobb-Douglas production functions in the two factors.

Also with Set 1 and 3, the two countries are relatively "equal" in other aspects such as production, labor endowment, so that they obtain same autarky welfare levels when the water is approximately equally allocated. While with Set 2, Country 1 has to obtain much more water than Country 2 to achieve equal autarky welfare, which implies that Country 1 is "weaker" in other aspects. This can also be revealed by the highest possible autarky welfare a country could obtain when it gets all the water. Set 4 is the reverse of Set 2.

The welfare functions under free trade will change dramatically in properties with the parameter values. However, we are able to show that it is possible for the free trade welfare function to be non-monotone with certain parameter values (Set 2 and Set 4). The resulting graph Figure 3.6 shows, with data Set 2, Country 1's welfare function is not monotonic in water and Country 2's welfare function monotonically increases with its own water. Since data Set 2 shows Country 1 is relatively "weaker" than Country 2 in combined productivity and labor endowment, hence Country 1 will be better off to give up some water when it gets substantial amount of water. Data Set 4 indicates that Country 2 is "weaker", thus Country 2 will be better off sharing the water, leading to a non-monotone welfare function for Country 2. This result is consistent with the result obtained from 2 in which a country will be better off by voluntarily giving up water when it has absolute disadvantage in the production of both goods. In the one-factor model, only relative productivity determines the qualitative properties of the welfare

Figure 3.5: Autarky Welfare Functions for the Two-Factor Model
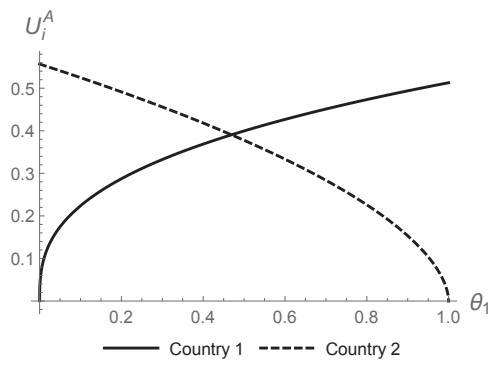
(a)Set 1
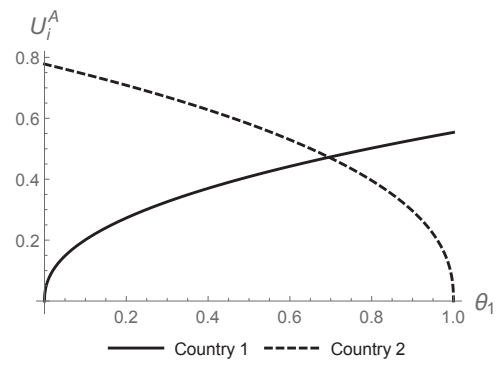
(b)Set 2
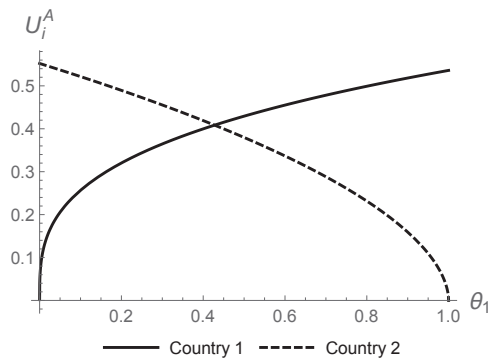
(c)Set 3
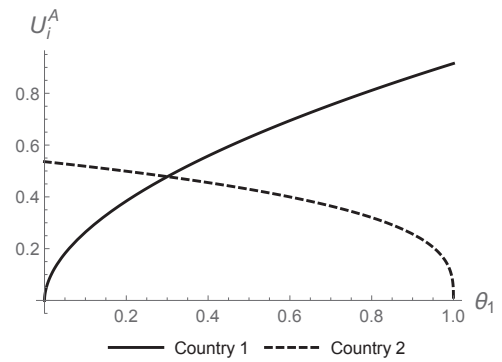
(d)Set 4

function, whereas in the two-factor model, a country's overall productivity and labor endowment combined "national strength" play together to shape the welfare function.

Figure 3.6: Free Trade Welfare Functions for the Two-Factor Model

(a)Set 2   (b)Set 4
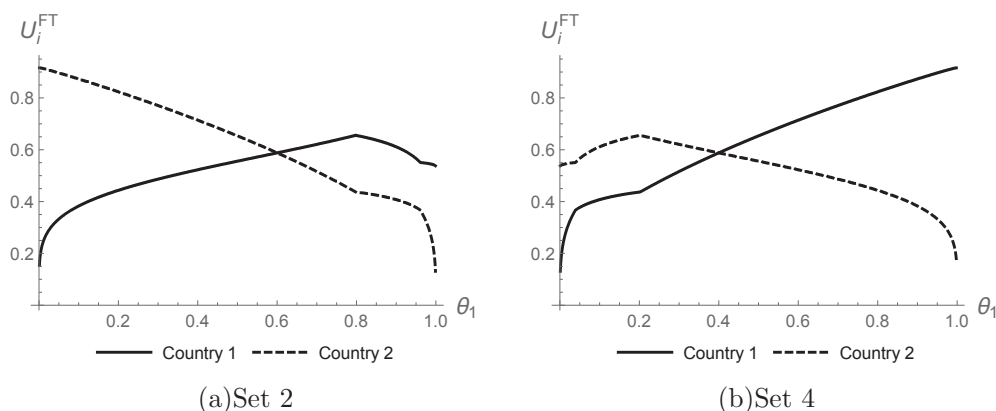
Figure 3.7 puts the welfare function under free trade and autarky for each country in the same graph to show the gains from trade. For both countries, the free trade welfare dominates the autarky welfare. This plot is generated with data Set 4 for which Country 1 is the "stronger" country and when approximately $\theta_1 > 0.2$, it will become a large country and start to produce both goods (See also Table 3.4). But different from the one-factor case in which the large country has same welfare under free trade and autarky, here, even if a country no longer specialize in both goods, the free trade welfare is still *strictly* higher than the autarky welfare. This is because even a country is a large country, the world equilibrium price is only "quasi-same" with its autarky price in formula, and the actual value of the world price is still smaller than its autarky price (see Figure 3.1), which makes gains from trade plausible. Therefore, the two-factor model provides more incentive for the countries to participate in trade, which is also more realistic.

3.5.3   Comparative Statics

In this subsection, we do a comparative statics analysis by varying the labor endowment of $L1$ and $L2$, and the Cobb-Douglas production coefficient γ_{11} and γ_{12}.

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Figure 3.7: Welfare Functions Under Free Trade and Autarky

(a)Country 1   (b)Country 2
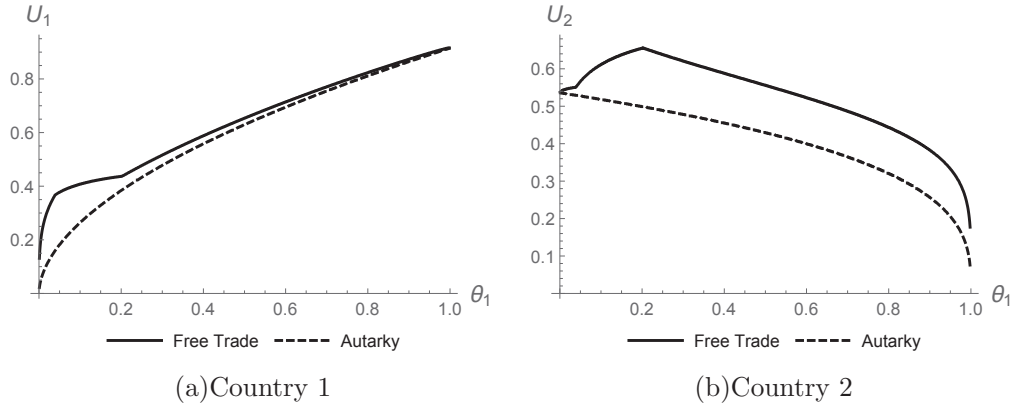
Note: Figure generated with data set 4.

Figure 3.8 shows the effects of Country 1's labor endowment (L_1) change on both countries' welfare functions. Under autarky, each country's welfare only depends on its own labor and water endowments, as a results, L_1 change doesn't affect Country 2's welfare function whereas Country 1's autarky welfare function shifts up as L_1 increases, which is straightforward that a country will benefit from a large labor force holding everything else constant. However, when the two countries involve in free trade, increasing L_1 would not only boost Country 1's own welfare, but Country 2's as well. The results imply that a country could benefit from a large (in terms of labor endowment) neighbor.

If Country 2's labor endowment L_2 changes, the impacts on each country's welfare would be similar to those when L_1 changes (Figure 3.9). Yet we can draw one more conclusion from the L_2 change, since more dramatic increases in L_2 are considered. The labor endowment of Country 2 in Set 4 is originally 1, and together with its productivity, Country 2 is "weaker" compared to Country 1, and is at a disadvantageous position in sharing the river (it is willing to give up water for higher welfare at some point.) However, when Country 2's labor endowment increases sufficiently, the comparison of national strength between countries reverse, thus reversing the the qualitative properties of the welfare functions. For instance, at $L_2 = 1$, Country 1's free trade welfare function monotonically increases with water, while Country 2's is non-monotone; at $L_2 = 10$, however, Country 2's welfare becomes increasing in its own water and Country 1's wel-

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fare becomes non-monotone. To conclude, the monotonicity of the free trade welfare function is not only affected by the relative productivity (or comparative advantages) for countries as in the one-factor model, but also can be changed by the labor endowment. A country with poor productivity in both goods, but with a sufficiently large population can still stand at an advantageous position in sharing the international river.

Figure 3.8: Effect of L_1 Change on Welfare

(a) Country 1: Autarky

(b) Country 2: Autarky

(c) Country 1: Free Trade

(d) Country 2: Free Trade
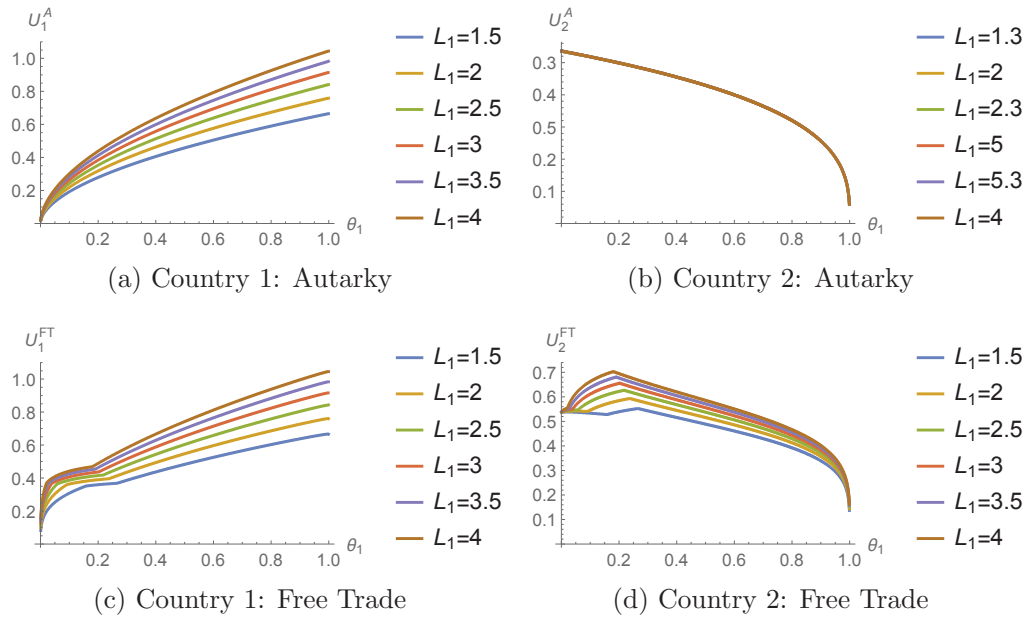
Figure 3.10 shows the impacts of the Cobb-Douglas production coefficient γ_{11} change on the autarky and free trade welfare functions of the two countries. γ_{11} is the production coefficient on water for Country 1 to produce good 1. Three effects can be summarized. First, a smaller coefficient on water is beneficial to a country, at a given level of water allocation, Country 1's welfare is larger as γ_{11} becomes smaller, whether under autarky or free trade. Second, one country's productivity change won't affect the other country's autarky welfare, but will have same impact on the other country's free trade welfare, that is, a smaller γ_{11} also benefits Country 2. Furthermore, the productivity coefficient change alters the relative productivity between countries, and thus changes the monotonicity of the free trade welfare functions.

Figure 3.9: Effect of L_2 Change on Welfare

(a) Country 1: Autarky

(b) Country 2: Autarky

(c) Country 1: Free Trade

(d) Country 2: Free Trade

Figure 3.10: Effect of γ_{11} Change on Welfare

(a) Country 1: Autarky

(b) Country 2: Autarky

(c) Country 1: Free Trade
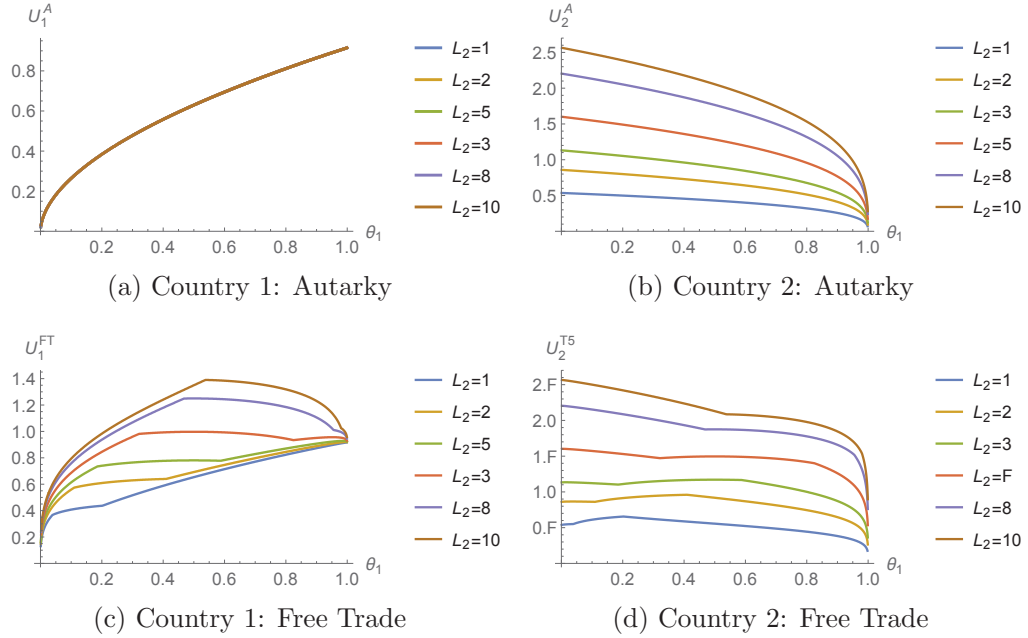
(d) Country 2: Free Trade
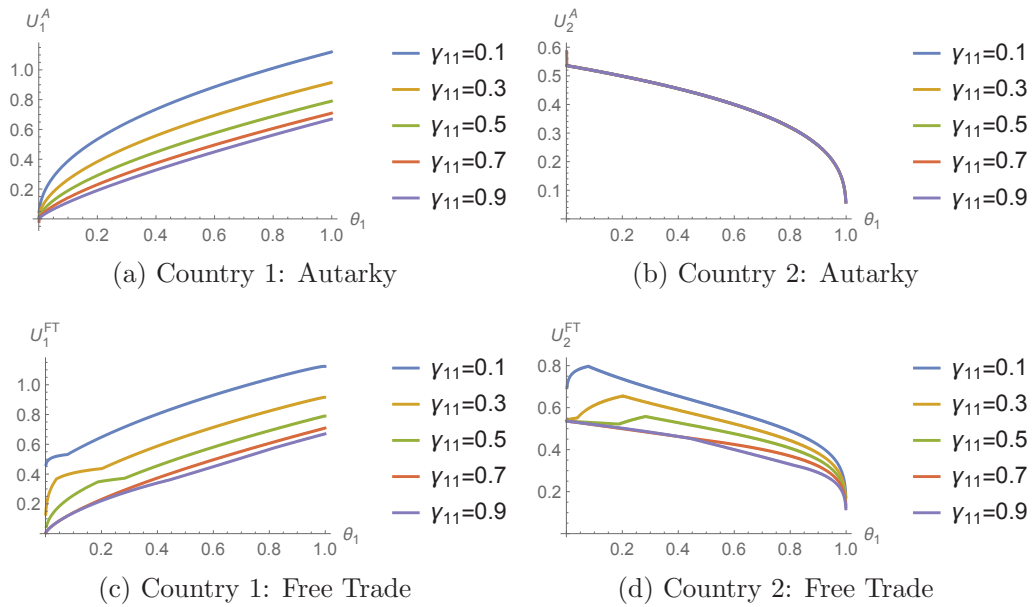
Figure 3.11 shows the effects of γ_{12} change on welfare functions. The effects on free trade welfare functions are more ambiguous, but the main results remain the same with that of γ_{11}.

Figure 3.11: Effect of γ_{12} Change on Welfare

(a) Country 1: Autarky

(b) Country 2: Autarky

(c) Country 1: Free Trade

(d) Country 2: Free Trade
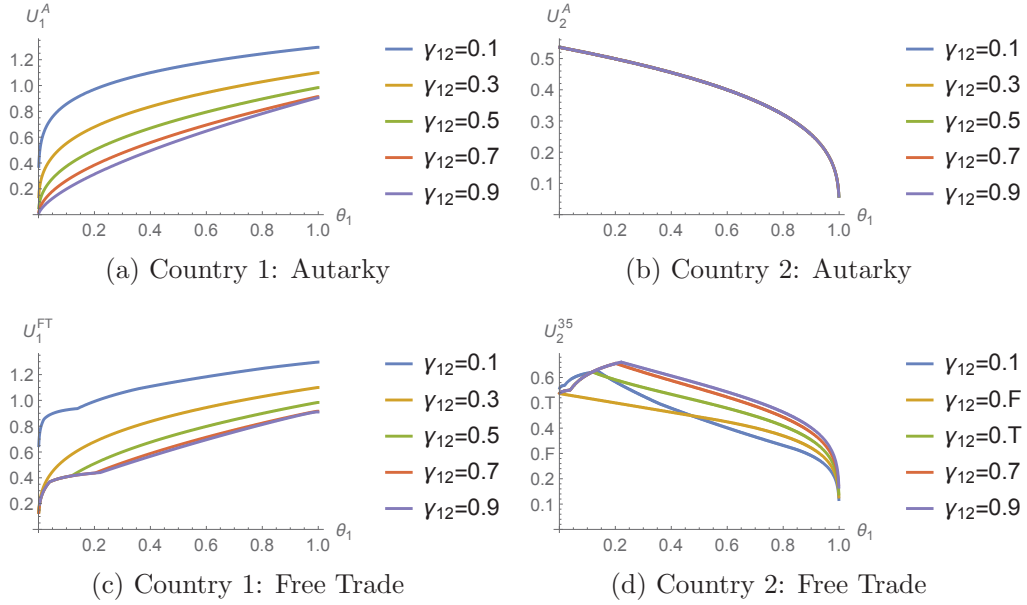
3.5.4 Welfare Functions for the Three-Country Model

In the two-factor model with three countries, the total quantity of water is also fully allocated such that $\sum_{i=1}^{3} \theta_i = 1$. The autarky welfare functions in Figure 3.12 and free trade welfare functions in Figure 3.13 are plotted for a given θ_2 value of 0.3, 0.4, 0.5, 0.6, and θ_1 can take a value up to $1 - \theta_2$.

The welfare functions show that a country's autarky welfare only depends positively on the amount of water it gets, i.e. Country i's autarky welfare function is only a function of θ_i.

The free trade welfare functions illustrate two points. First, in the three country case, it is still possible for a country's free trade welfare to be declining in the amount of water it gets, as can be seen from Country 1's welfare function when $\theta_2 = 0.3$, it is very obvious that the welfare function starts to turn down with θ_1 at some point. Sec-

Figure 3.12: Autarky Welfare Function for the Three-Country Model

(a)Country 1   (b)Country 2   (c)Country 3
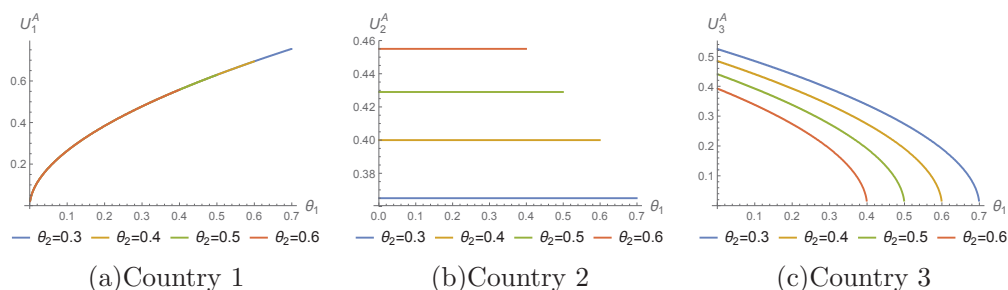
ond, a country's welfare not only depends on its own water, but also depends the water allocated to the other two countries. For example, for Country 1, its welfare function not only changes with θ_1, but also is shifted up when θ_2 decreases (which further proves the first point that Country 1 can be better off by giving up water to Country 2); and for Country 2, for a given θ_2 values, its welfare still changes with θ_1. In other words, a country's free trade depends on two parameters θ_1 and θ_2 when there are three countries. We can infer with more confidence that when there are n countries sharing the river, the free trade welfare function will be dependent on θ_1 through θ_{n-1}. This implies that with more than two countries, the main properties of the welfare functions still hold.

Figure 3.13: Free Trade Welfare Function for the Three-Country Model

(a)Country 1   (b)Country 2   (c)Country 3
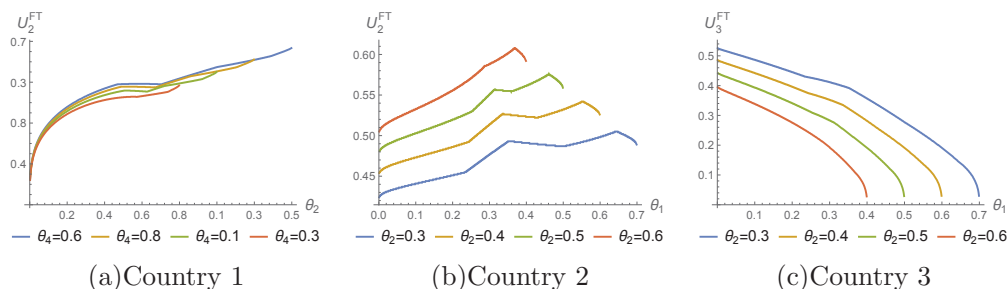
3.6   Conclusion

The two factor model in this Chapter extends the one-factor Ricardian Model in Chapter 2 by incorporating both labor and water into a Cobb-Douglas production function.

The autarky situation is solved analytically to show that the welfare is positively correlated with the amount of water a country gets and is also non-linear with the exception that the Cobb-Douglas production coefficient for both goods equals 1, i.e. the model degenerates to the one-factor model.

The free trade situation is analyzed with an example data set given the immense possibilities. With this data set (Set 4), Country 1's autarky relative price of good 1 is below that of Country 2's, indicating a comparative advantage in good 1 for Country 1. The production patterns in each country will depend on the size of the country in terms of water endowment and how its autarky price compares to the free trade world equilibrium relative price. If Country 2 is large and world price is "quasi-same" with its autarky price, Country 2 will produce both goods and Country 1 specializes in good 1 (the good that it has comparative advantage in); If Country 1 is large and world price is "quasi-same" with its autarky price, the production pattern reverses; If the world price falls between the two countries' autarky prices, each country specializes in the comparative advantaged good. This result is parallel with the result from the one-factor model, but the comparative advantage would be broadly defined by the relative prices in the two-factor model.

The two-factor model is also solved completely numerically for plotting out the welfare functions. The autarky welfare function plot is consistent with the analytical results. The properties of the free trade welfare functions can be summarized into two points: (1)The free trade welfare strictly dominates the autarky welfare; (2) It is possible to obtain a non-monotone free trade welfare function in the amount of water as long as one country is "stronger" than the other country in the productivity of all goods and labor endowment, i.e. the relative productivity between the two countries affects the monotonicity of the welfare function, and different from the one-factor Ricardian Model, the labor endowment may also play a role in the two-factor model.

A comparative statics analysis of changing labor endowments and the Cobb-Douglas production coefficients leads to several conclusions. First, one country's welfare functions under both autarky and free trade boost with the increase of its own labor endowment and the decrease of the Cobb-Douglas production coefficient on water; Sec-

ond, under free trade, a country could benefit from the other country's increase in labor endowment and decrease in the Cobb-Douglas production coefficient on water; Third, the change in both labor endowment and the Cobb-Douglas coefficient alters the relative autarky price ratios between countries, hence will change the monotonicity of the free trade welfare functions.

Finally, an illustration of the welfare functions with three countries verifies that the main properties of the welfare functions also hold with more than two countries, i.e. welfare depends on allocation of all countries instead of just own water allocation, which differs from the literature.

Chapter 4

An Integrated Hydrologic-Economic Water Quality Model: Application to the San Joaquin River Basin

4.1   Introduction

Lack of fresh water, lack of drainage, the presence of high water tables, and salinization of soil and groundwater resources are some of the major factors that are endangering the irrigated agriculture in arid and semi-arid areas (Schoup et al., 2005), like the San Joaquin Valley in the southern part of the California's Central Valley. The lower San Joaquin Valley is one of the most productive farming areas in the United States, and it is mainly irrigated by the lower San Joaquin River and its tributaries, but now the San Joaquin River ranks No.1 in America's Most Endangered Rivers of 2014, due to outdated water management and excessive diversions.[1]

Excessive diversion causes the river flow to fall and threatens the water quality. Salinity, as one important aspect of water quality (which is measured in electrical con-

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[1]America's Most Endangered Rivers For 2014 ranked by American Rivers, `http://www.americanrivers.org/endangered-rivers/2014-report/san-joaquin/`

ductivity (EC)), will be higher when water flow falls and salts becomes more condensed. Human activities, such as irrigation, on the one hand are affected by the quality of the water (e.g. crop yield will be affected by salinity), on the other hand, can also exacerbate the water quality by more saline return flows. This is the reason that river water quality needs to be improved for higher irrigation benefits and irrigation activities also needs to be efficiently managed to reduce the negative impacts on water quality.

In this chapter, we choose the lower San Joaquin River and the lower San Joaquin Valley, which is irrigated by this stretch of the river as the targeted area to apply the economic-hydrologic integrated water quality model. The model combines two strands of the literature. Lee and Howitt (1996) optimizes the river quality, resource allocation, crop production levels and total expenditures for control using a nonlinear programming model applied to the Colorado River Basin to empirically determine quality standards and address the salinity externalities in this area. Rosegrant et al. (2000) maximize the aggregate benefits of water use in irrigation, municipal use, and hydropower generation, while maintaining the water balances in river reaches, aquifers, agricultural, municipal and industrial demand sites, and applies the model to the Maipo river basin in Chile. Both of the papers use an empirically estimated production function described in Letey and Dinar (1986). While our model adopts the basic framework of an integrated economic-hydrologic water quality model, it uses the crop production functions developed in Kan et al. (2002). Schwabe et al. (2006) applies this crop production in a model with a closed drainage basin with groundwater aquifer to mitigate salinity problems.

The model in this chapter combines the economic-hydrologic model in surface water management with a crop-water-salinity production function developed from a steady-state seasonal model (Kan et al., 2002). It is empirically significant in that it is first applied to the lower San Joaquin River Region.

4.2   The Area of Interest: Lower San Joaquin Valley

The San Joaquin River originates from the Sierra Nevada and flows northward towards the Sacramento-San Joaquin River Delta, where it meets the Sacramento river.

The entire river winds 366 miles and irrigates a rich agricultural region, the San Joaquin Valley. The area addressed in this paper is the Lower San Joaquin Valley which starts from the point where the tributary Merced joins the main stream and up to the point near the Delta, after Stanislaus River joins the San Joaquin River, which is also the location of the Vernalis Monitoring Station. This stretch of the San Joaquin River has three major tributaries, Merced, Tuolumne and Stanislaus, and flows past three counties, Merced, Stanislaus and San Joaquin, as shown in the right panel Figure 4.1. Figure 4.2 shows the key monitoring sites for the stretch from the mouth of the Merced River to the monitoring site at Vernalis. [2]

Figure 4.1: The Area of Interest
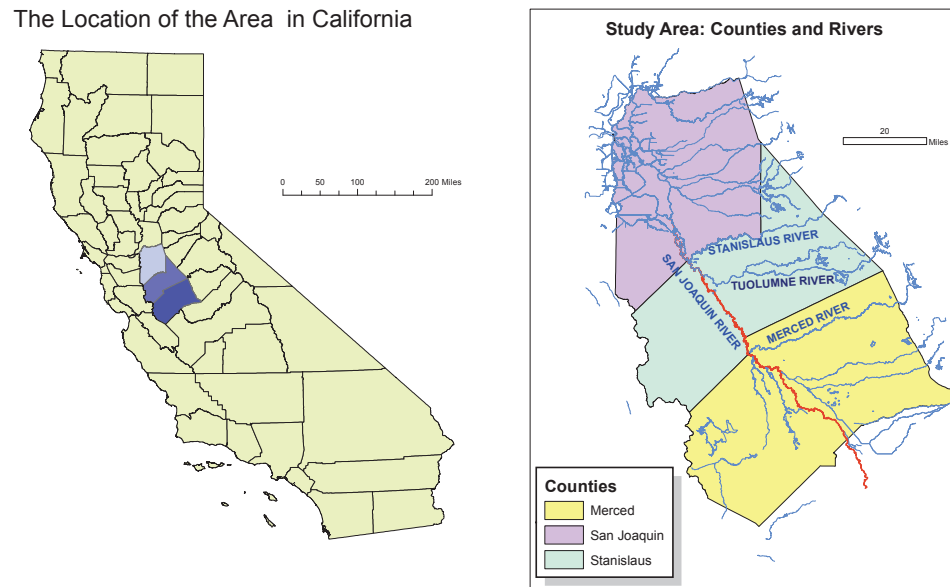
Figure 4.2: Key Monitoring Sites On the Reach of Interest
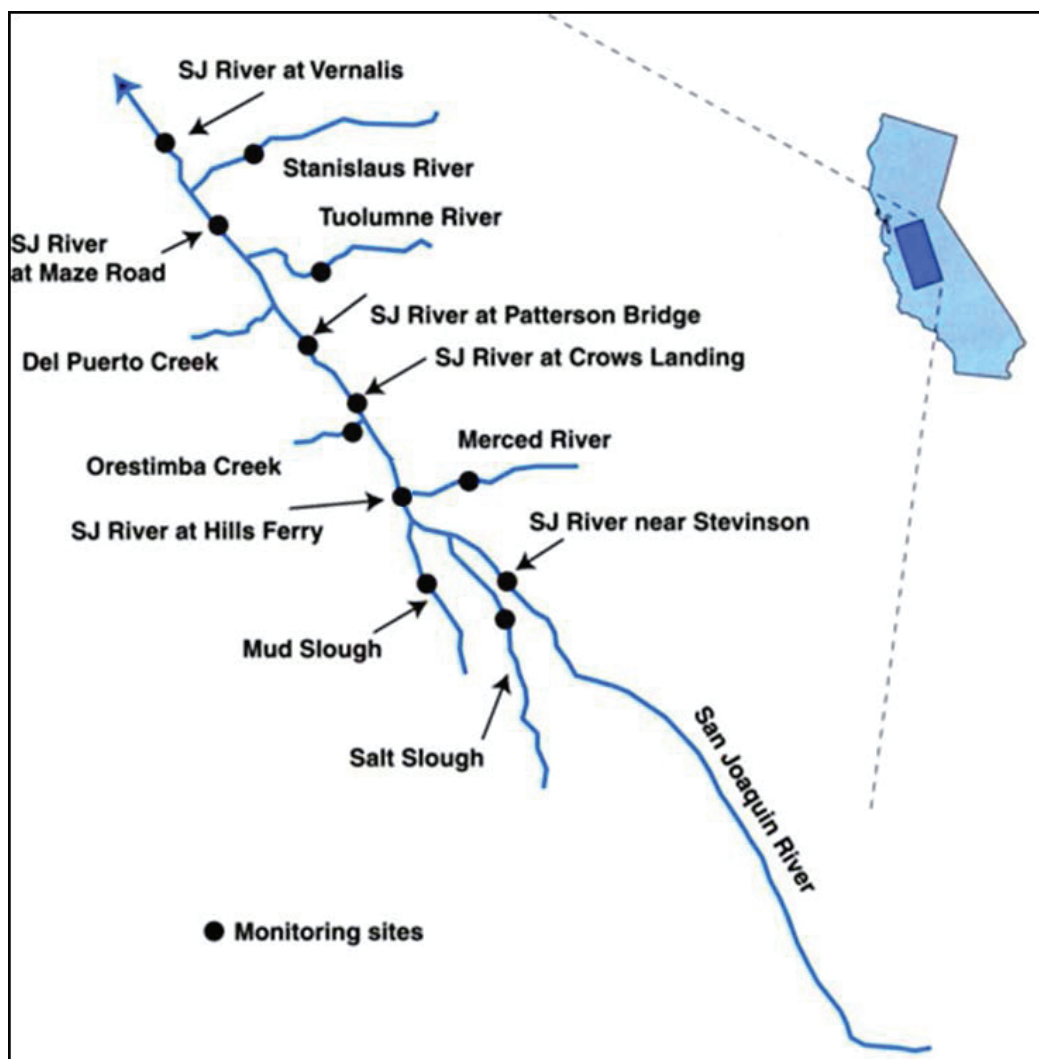
The river reach under investigation is from the mouth of Merced to the monitoring site at Vernalis.
Source: Central Valley Salinity Alternatives for Long-term Sustainability (CV Salts: http://cvsalinity.org/)

Figure 4.3: Schematics of the Conceptual Model
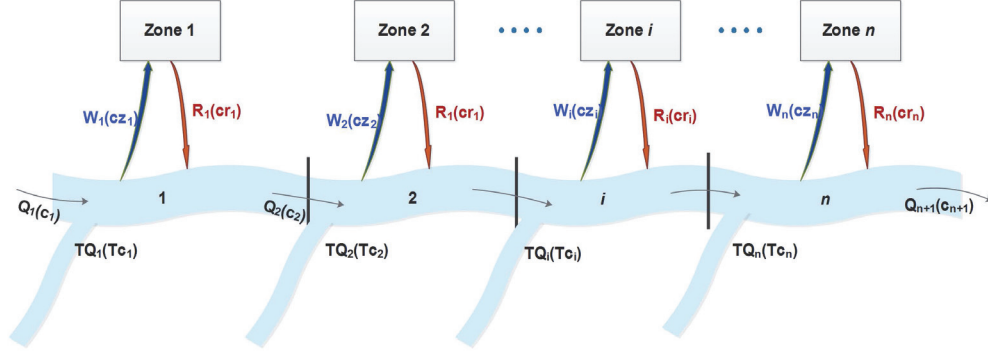
4.3 Model Framework and Assumptions

The model is of a static nature and considers a period of one year. Figure 4.3 demonstrates the schematics of the model. We divide the entire stretch of river under study into n equal reaches by length, and the corresponding farming area irrigated by this stretch of the river also into n equal zones by area. The purpose of doing this is to manually make all the other factors equal for each reach and zone so as the only difference between zones are the relative spatial locations, hence to which any potential differences in results may contribute.

At the starting point of the stretch, Q_1 quantity of water with salinity c_1 comes into the system, and are exogenous data. Quantity Q_i with salinity c_i coming to reach i $(i = 2, ..., n)$ will be endogenously determined by the model. There may be exogenous tributary water TQ_i with salinity Tc_i joining the main stream ($TQ_i = 0$ and Tc_i refers to no tributary in this reach). Each Zone i diverts total quantity of water W_i with salinity cz_i for irrigation purposes, where cz_i comes from blending the main stream water and tributary water evenly as in:

$$cz_i = \frac{Q_i c_i + TQ_i Tc_i}{Q_i + TQ_i}. \tag{4.1}$$

After irrigation activities performed at Zone i, a total quantity of return flows R_i with

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[2]The area is also referred to as the North San Joaquin Valley, see the University of California, Davis Cost and return studies for fruit, vegetable, field, tree and vine crops, and animal commodities, http://coststudies.ucdavis.edu/current.php.

salinity cr_i drains to river reach i.[3] The return flows are the total amount of water applied to the crops less the amount of water transpired by the plants or evaporated, i.e. the amount of water through evapotranspiration. Figure 4.4 illustrates the evapotranspiration process and the hydrology between the farms and the river.[4]

Figure 4.4: Evapotranspiration and Hydrology
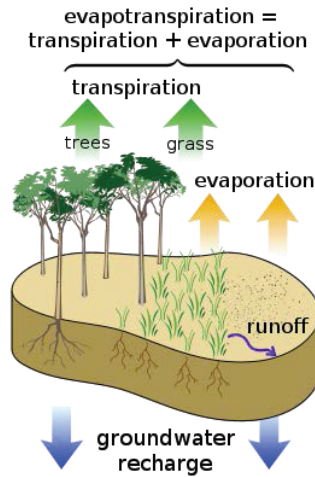
Figure Source: "Water Budgeting and Evapotranspiration"(http://support.rachio.com/article/282-water-budgeting-and-evapotranspiration-watersmart)

To summarize, the model employs several simplifying assumptions:

1. Surface water is the only source of irrigation. Groundwater and precipitation are not considered.

2. The salinity of the return flows are computed from a steady state model, that is the soil salinity in the root zone remain constant over time.

3. There are no distributional losses of transferring water from the river to the farming zones.[5]

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[3]See Appendix C for the measurement unit of all the variables throughout the paper.

[4]"Evapotranspiration (ET) is a term used to describe the water consumed by plants over a period of time. Evapotranspiration is the water loss occurring from the processes of evaporation and transpiration. Evaporation occurs when water changes to vapor on either soil or plant surfaces. Transpiration refers to the water lost through the leaves of plants. "(Definition from http://ccc.atmos.colostate.edu/~coagmet/extended_etr_about.php)

[5]Chakravorty and Gong (2015) discusses water allocation under distribution losses. The distribution losses usually occur where the infrastructure to transfer water is not well established in some developing countries, which is not the case in California where the distributional losses are negligible.

4. Return flows from the farming zones completely returns to the river.

The first assumption restrains the model to focus on the allocation of surface water among a number of farming zones lined up along the stretch of the river. All the zones have same distance to the bank and hence same water extraction cost. Zones may also use groundwater for irrigation, but different zones are at equivalent conditions for using groundwater, i.e. same pumping cost and opportunity cost. Therefore, omitting groundwater usage will not change our results dramatically. This scenario is different from the case examined by Pongkijvorasin and Roumasset (2015) where farms spread away from the surface water source, which triggers the use of groundwater for irrigation when a farm is sufficiently far away from the surface water source. Precipitation is not considered either for the semi-arid climate in California.

The steady state in the second assumption implies a balance condition for salt mass: the salt brought into the farm soil by applying saline surface water completely returns to the river through return flows, i.e. no salt is held up in the soil. Return flows are normally smaller in quantity than total applied water, which implies that return flows are more saline than applied surface water ($cr_i > cz_i$). On the other hand, since all the salt in the diverted water returns, the salt mass in the river also remain constant unless tributaries bring in new salts.

For the fourth assumption, water can return to the river directly through runoff or indirectly through deep percolation to the ground water aquifer first and finally drains to the river, as shown in Figure 4.4, deep percolation recharge the groundwater aquifer temporarily, but water finally goes back to the river.

4.4   Economic-Hydrologic Relations of the Model

The following subsections describes the agronomic production relations, hydrologic balance equations and other constraints that will be used in the various branches of the model with some minor modifications in some branch.

4.4.1 Agronomic Production Relations

Evapotranspiration(et) is directly linked to the growth and productivity of a crop and the applied water quantity and quality affects evapotranspiration. It follows that the yield of a crop depends on the amount of water applied and the quality of water, i.e. salinity here. Suppose farms apply water depth w (feet/acre) with salinity cz (dS/m) into the root zone, obtains yield y (ton/acre/year) and also generates return flows r (feet/acre). The relations are given by

$$y = f(w, cz), \tag{4.2}$$

$$r = g(w, cz). \tag{4.3}$$

The crop yield and return flows are both functions of water application depth and irrigation water salinity. The explicit forms of equations are shown in (4.4-4.5) following Kan et al. (2002). Marketable yield (y) is linearly related to the vegatative growth or evapotranspiration (et):[6]

$$y = \alpha_1 + \alpha_2 et(w, cz), \tag{4.4}$$

and

$$r = w - et(w, cz), \tag{4.5}$$

where α_1 and α_2 are parameters.

Evapotranspiration is positively related to water application and negatively related to water salinity in the following equation (Kan et al., 2002):

$$et = \frac{\bar{e}}{1 + \beta_1(cz + \beta_2 w^{\beta_3})^{\beta_4}}, \tag{4.6}$$

where $\bar{e}$ (feet/year) is the maximum evapotranspiration without water and salinity stresses, $\beta_1, \beta_2, \beta_3, \beta_4$ are parameters. Plugging (4.6) into (4.4-4.5) gives the yield and return flow functions to be estimated.

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[6]For some crops, like cotton, excessive irrigation will bring down yield, making the crop production function hump-shaped, which can be captured by a quadratic term in et. However, a linear relationship is sufficient for the crops studied in this paper, which will be specified later.

The parameters are estimated using a nonlinear least squares method with data generated from a steady-state seasonal model (Kan et al., 2002). From the relationship between evapotranspiration, water depth and water quality (salinity), it can be expected that β_3 is negative and $\beta_1, \beta_2, \beta_4$ are positive. Evapotranspiration differs among crops with irrigation systems (irrigation system uniformity), hence, the equations are estimated for each crop-irrigation system combination. All parameters vary across crops with different irrigation systems.[7]

4.4.2   Profit Relations

On each farming zone i, farmers face a collection of crops $j = 1, ..., J$ and irrigation systems $k = 1, ..., K$ and are free to mix the crops with irrigation systems. Figure 4.5 illustrates the crop-irrigation system pattern with $J = 2$ and $K = 2$. A farming zone is segmented into fields with acreage x_{ijk}, each field is irrigated with water with salinity cz_i to the depth w_{ijk}, of which e_{ijk} is evaporated and transpired and r_{ijk} is returned to the surface water system through deep percolation and runoff.

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[7]Irrigation systems that applies water more uniformly on the crops will improve plant evapotranspiration and reduce drainage, i.e. enhance water use efficiency. See Dinar and Letey (1996, Chapter 3) for a definition of irrigation uniformity.

Figure 4.5: Crop-Irrigation Pattern on Zone i
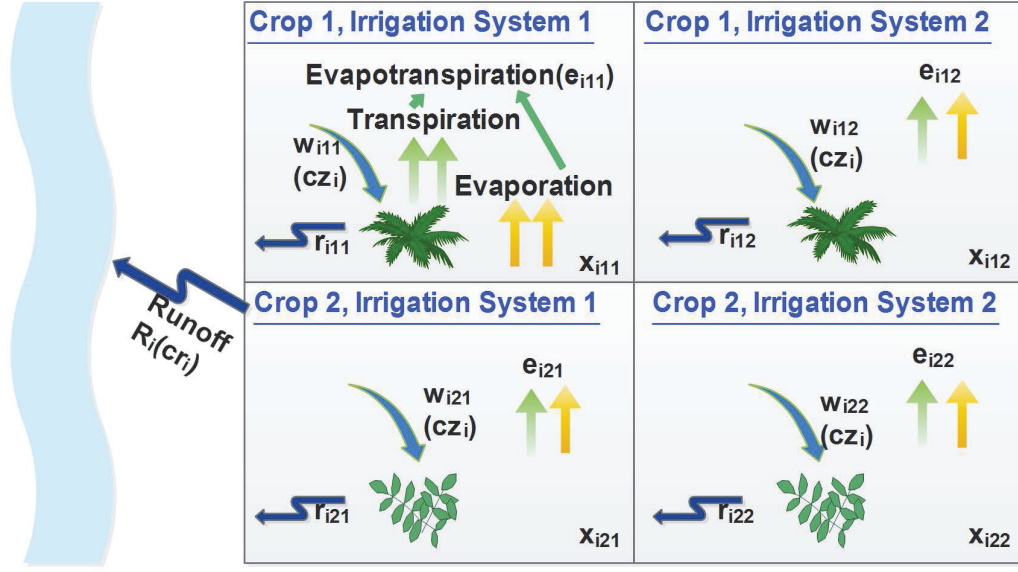
Note: Number of crops=2 number of irrigation system=2, for illustration.

Per-acre profit for a certain crop field is

$$\pi_{ijk} = p_j y_{ijk} - \gamma_{jk} - cost_i^w w_{ijk}, \tag{4.7}$$

where p_j is the market price for crop j, γ_{jk} is per-acre nonwater production cost, $cost_i^w$ is cost of obtaining surface water for zone i. Based on the assumption that all the farming zones line up along the river with equal distance to the bank, zones have same water extraction cost.

Total irrigation profit for zone i is the product of per-acre profit and acreage, summed over all crops and irrigation systems:

$$\pi_i = \sum_{j=1}^{J} \sum_{k=1}^{K} \pi_{ijk} x_{ijk}. \tag{4.8}$$

Zones will be choosing types of crops (j), irrigation systems (k) and corresponding acreage (x) for each crop-irrigation-system combination, water applied (w feet/acre/year) to each field in order to maximize this irrigation profit, while maintaining the constraints elaborated in the following subsections.

4.4.3 Constraints on Land and Water

Farm zones faces land and water constraints. Crop field land is limited to the total arable land $(\bar{x}_i)$ on zone i:

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \leq \bar{x}_i. \qquad (4.9)$$

Except for the constraints on the total arable land, the acreage for a certain crop may also be constrained by some rotation constraints, i.e. different crops have different growing seasons, while planning the acreage for a crop, other crops may have occupied the land and still in growing season. The rotation constraints can simply be expressed by:

$$\underline{x}_{ij} X_i^{cropped} \leq x_{ijk} \leq \bar{x}_{ij} X_i^{cropped}, \qquad (4.10)$$

where $X_i^{cropped} = \sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk}$ is the actual total cropped acreage on Zone i, $\underline{x}_{ij}$ and $\bar{x}_{ij}$ are the lower and upper bound for total acreage of crop j on Zone i as a proportion of the actual total cropped land. The data for the bounds will be explained in the data section.

The total amount of water that a zone can divert from the river cannot exceed the volume of water that is flowing into the current reach (from both mainstream and tributary), i.e.

$$W_i \leq Q_i + TQ_i. \qquad (4.11)$$

(4.12) illustrates that total quantity of water applied to all the crop fields amounts to the total quantity diverted by the zone, which means all the water diverted is used for irrigation and there is no loss of water in application:

$$\sum_{j=1}^{J}\sum_{k=1}^{K} w_{ijk} x_{ijk} = W_i. \qquad (4.12)$$

Total return flow out of zone i is defined as the return depth r multiplied by acreage,

summed over all crop-irrigation-system combinations:

$$\sum_{j=1}^{J} \sum_{k=1}^{K} r_{ijk} x_{ijk} = R_i. \tag{4.13}$$

4.4.4 Hydrologic Relations in the River

Figure 4.3 also depicts the hydrologic relations in the model. The water balance for reach i requires that the quantity of water that flows into the reach (mainstream inflow, possible tributary inflow, and farm return flows) equals the water that leaves the reach (the water diverted by the zone and water that flows to the next reach). That gives the water balance equation as:

$$Q_i + TQ_i + R_i = W_i + Q_{i+1}, \qquad \text{for} \quad i = 1, .., n. \tag{4.14}$$

Analogy applies to the salt mass balance equation for reach i:

$$Q_i c_i + TQ_i Tc_i + R_i cr_i = W_i cz_i + Q_{i+1} c_{i+1}, \tag{4.15}$$

where cz_i defined in (4.1) is the salinity of mixing the mainstream and tributary, and is the salinity of the water that is actually applied to crops on the farm zones.

Since soil salinity in the root zone is in steady state, the salt mass that enters into Zone i would completely come out of the zone through return flows. This implies that $W_i cz_i = R_i cr_i$. It gives the equation for return flow salinity

$$cr_i = \frac{cz_i W_i}{R_i}, \tag{4.16}$$

which indicates that the return flow would normally be more saline than the water applied to crops, with the same salt mass dissolved in lesser amount of water. With more saline return flows from each zone, the water downstream would be expected to have a higher salinity than upstream water, unless there are high quality tributaries joining the mainstream.

The steady state condition also reduces (4.15) to

$$Q_i c_i + T Q_i T c_i = Q_{i+1} c_{i+1}. \tag{4.17}$$

4.5 Data and Sources

4.5.1 Agricultural Data

Table 4.1 lists out the data related to agricultural production. According to a district/grower survey (Anderson, 2014), the total farm land irrigated by the reach from the Merced River to Vernalis Station is 55037 acres and cropping patterns show that the top four crops in terms of acreage in this area are almonds, alfalfa, tomato and wheat, which account for 68.2% of the total acreage. For simplicity, we will be using these four crops to represent all the crops in this area. Furthermore, since almonds are perennial plants, we fix the acreage of almonds at 13498 acres as the data, with applied water depth be 4.25 (feet/acre/year) and return flow from irrigating almonds be 1.06 (feet/acre/year), and leave the rest three crops, alfalfa, tomato, and wheat, at the choice of the zone planners, i.e. the decision of planting or not planting the crop and the acreage of the crop will be endogenously determined. [8]

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[8] Applied water depth and return flow data for almonds are calculated from Cost and return studies for fruit, vegetable, field, tree and vine crops, and animal commodities by University of California, Davis, http://coststudies.ucdavis.edu/current.php.. Applied water depth is the average of two irrigation systems, micro sprinkler irrigation and flood irrigation, while return flow is calculated based on the average water use efficiency, 75%.

Table 4.1: Agricultural Data

Cropping Patterns and Acreage[a]		Acreage
Crop		
Almonds		13498
Alfalfa		9666
Tomatoes		7744
Wheat		6636
Other		17493
Total		55037

		Alfalfa	Tomato	Wheat
Market Price[b]($/ton)		109.3	55.2	133.3
Non-water production cost[c]($/acre/year)	Furrow2	396.80	636.80	194.80
	Furrow4	409.90	661.20	204.50
	Sprinkler	434.60	718.80	235.50
	Lepa	492.30	704.90	288.60
	Lin	483.20	701.50	286.90
	Drip	538.60	751.20	390.90
Cost of Obtaining Water [d]($/acre-ft)				30

[a] Chester Anderson, District/Grower Survey: Cropping Patterns and Salinity in Reach 83 of the San Joaquin River, Merced River to Vernalis

[b] Schwabe et al. (2006)

[c] Schwabe et al. (2006).

[d] University of California, Davis Cost and return studies for fruit, vegetable, field, tree and vine crops, and animal commodities, `http://coststudies.ucdavis.edu/current.php`.

The current cropping pattern also gives the data for bounds of the rotation constraints. Since in the model we are using the four main crops to represent all the crops in the region, we will be using the proportion of the current acreage of alfalfa, tomato and wheat in the total land of the three, as the upper bound of the cropped

Figure 4.6: Annual Net Benefit Function and Water Demand Function for Zone
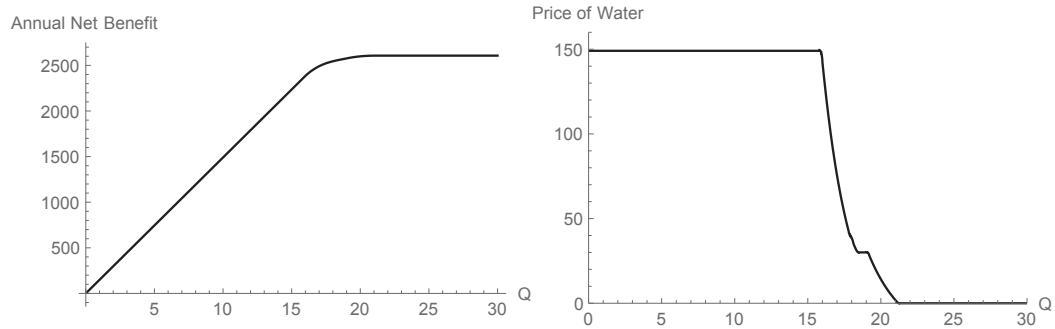
land. For example, alfalfa's acreage, 9666, takes up 40.2% of the total land for alfalfa, tomato and wheat, then 40.2% will be the maximum ratio of alfalfa in actual cropped land in the rotation constraint. The lower bound in the rotation constraint will be zero.

The market price for the alfalfa, tomato, wheat, and the non-water production cost for these three crops with six irrigation system combinations, furrow 1/2 mile, furrow 1/4 mile, sprinkler, lepa, lin and drip (irrigation efficiency ranging from low to high) in the Table 4.1 come from Schwabe et al. (2006) and are in 2000 dollars. The related data for almond is not necessary, since acreage for almond is fixed, its yield and production cost will not enter into the objective function.

The cost of obtaining water is assumed to be constant across the zones, and is at $30/acre-feet.

Using the agricultural data, we can plot out each zone's annual net benefit function, which is total crop revenue less water and non-water production cost, and also the water demand function. To illustrate, Figure 4.6 shows the annual net benefit and water demand for a typical zone when the whole region is divided into ten zones. The annual net benefit function is the objective function and is first increasing and then constant, that is, it has a satiable point. Without other constraints, it can be expected that zones will extract water up to the satiable point as long as the water supply is sufficient.

4.5.2 Hydrologic Data

The hydrologic data needed for the model are the flow and salinity (measured in electrical conductivity (EC)) data at the beginning of the river stretch (the mouth of the Merced River), and at the two tributaries of Tuolumne and Stanislaus. Data at Vernalis is needed for purpose of checking the performance of the model. Data comes from the 14-year average (2000-2013) of USGS National Water Information System. Figure 4.7 shows the daily flow (measured in cubic-feet/second) and daily maximum and minimum EC at each spot. Table 4.2 summarizes the average.

Table 4.2: Summary of Flow and EC Data at Various Spots

	Merced	Tuolumne	Stanislaus	Vernalis
Flow (Acre-feet/yr)	1,055,545	797,180	545,532	2,590,568
EC (dS/m)	1.26	0.129	0.098	0.632

4.6 Benchmark Model: Pure Allocation

We start at a baseline model where the river water is not saline and irrigation will not affect the salinity of the river. The purpose of the model is to investigate how surface water will be allocated among spatially distributed farming zones from upstream to downstream without the interference of salt, and then use the result as a benchmark to compare with that when surface water is indeed saline. The salinity of surface water is fixed at 0.07 (dS/m) across the region, which is considered as high quality fresh water prevailing in the San Joaquin River. Furthermore, tributaries are not considered in this pure water model in order to have a straightforward water allocation result.

4.6.1 Common Property: Fresh Water

In a common property setting, each zone will treat the river as a common pool of resource, maximizing own annual irrigation profits (4.8), subject to land constraint (4.9), water use constraint (4.12) and (4.11), rotation constraint (4.10), and

Figure 4.7: Flow and Salinity Patterns at "Flow-In" Spots

(a) Flow at Mouth of Merced

(b) EC at Mouth of Merced

(c) Flow at Tuolumne

(d) EC at Tuolumne

(e) Flow at Stanislaus

(f) EC at Stanislaus
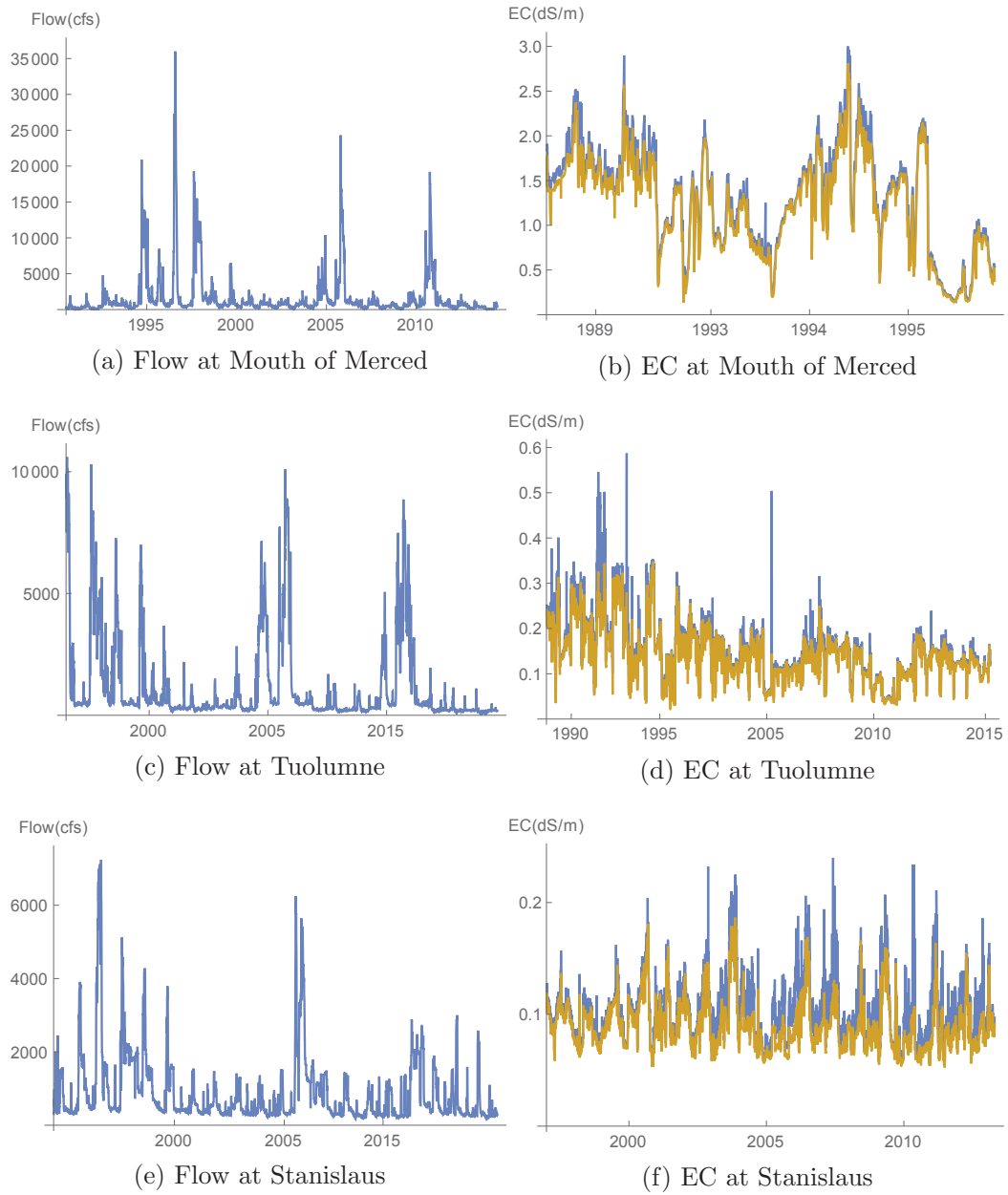
production relations (4.4) and (4.6), without considering the spatial externality to the downstream zones. The total water available to the next zone is calculated through the water balance equation 4.14. The results for each zone are then solved recursively.

As elaborated in the model Section 4.3, the whole region is divided into n equal size zones, each zone chooses from the same types of crops and irrigation systems, and faces same water extraction costs, non-water production costs, and crop prices. Table 4.3 shows model simulated zone profits, total diverted water, total return flows and crop pattern, with different numbers of zone segmentation n and initial incoming water flows Q_1. For different numbers of zone segmentation, similar patterns occur. When $Q_1 = 200$, which is a sufficient amount of water for all zones to reach the satiation annual net benefit, all zones will have same profits, total diverted water, total return flows, and crop patterns. It is also worth noting that for each crop, all zones choose the least efficient irrigation system, furrow 1/2 mile, which is intuitive when water is sufficient and zones do not have incentive to conserve water. However, when $Q_1 = 130$, the last zone does not get as much water as it desires, it switches to more water efficient irrigation systems comparing to previous zones, and obtaining a smaller profit, resulting in an equity issue simply due to spatial differences.

4.6.2 Efficiency: Fresh Water

Efficient usage of the surface water requires that the aggregate irrigation profits of all zones

$$\sum_i \pi_i = \sum_i^n \sum_j^J \sum_k^K (p_j y_{ijk}(w_{ijk}, cz_i) - \gamma_{jk} - cost_i^w w_{ijk}) x_{ijk} \qquad (4.18)$$

are maximized, while still satisfying each zone's land (4.9), water (4.12, 4.11), rotation (4.10) and production constraints (4.4, 4.6), and at the same time maintaining the water quantity balance condition for each reach of the river (4.14).

The results are shown in the Table 4.4. The efficiency results are the same with common property when $Q_1 = 200$, since at this amount of water, there is no scarcity problem. However, when initial inflow Q_1 decreases to 130, scarcity problem arises.

Table 4.3: Common Property Results with Fresh Water

Zones	π_i	W_i	R_i	Crop Patterns						
				Crop	Irr-system	w	e	r	x	y
$n = 10$ $Q_1 = 200$	1-10	3076.07	20.80	8.05	Alfalfa	Fur 1/2	3.98	2.49	1.49	1669	7.62
$n = 10$ $Q_1 = 130$	1-9	3076.07	20.80	8.05	Tomato	Fur 1/2	4.45	1.95	2.50	1337	48.35
					Wheat	Fur 1/2	2.15	1.46	0.69	1146	2.00
	10	2886.66	15.27	3.15	Alfalfa	Fur 1/4	2.49	2.23	0.26	1669	6.83
					Tomato	Lin	2.71	1.93	0.78	1337	47.58
					Wheat	Fur 1/4	1.53	1.31	0.22	1146	1.77
$n = 6$ $Q_1 = 200$	1-6	5126.79	34.66	13.42	Alfalfa	Fur 1/2	3.98	2.49	1.49	2783	7.62
$n = 6$ $Q_1 = 130$	1-5	5126.79	34.66	13.42	Tomato	Fur 1/2	4.45	1.95	2.50	2229	48.35
					Wheat	Fur 1/2	2.15	1.46	0.69	1910	2.00
	6	4579.95	23.77	4.31	Alfalfa	Fur 1/4	2.15	2.06	0.09	2783	6.15
					Tomato	Lin	2.50	1.91	0.59	2229	46.95
					Wheat	Fur 1/4	1.39	1.19	0.19	1910	1.58

Note: The initial inflows to the whole region $Q_1 = 200$ and $Q_1 = 130$ represent the quantity of water that are sufficient/insufficient for all the zones to achieve their satiation annual net benefit. The choice of the numbers are arbitrary, but they are sufficient to illustrate the two different cases of water abundance and water scarcity. $n = 6$ and $n = 10$ are also two examples of zone segmentation which demonstrate that similar profit distributions and crop patterns prevail among zones whereas the actual number of zones doesn't matter.

Different from the common property case in which the upstream zones act exactly the same as if there were no scarcity, under efficiency use, the upstream zones will divert less water comparing to the common property scenario (20.12 compare to 20.80 when $n = 10$, and 32.64 compare to 34.66 when $n = 6$), leaving more water for the last zone. The upstream zones achieve less water diversion by applying less water per acre, but maintaining the same crop patterns with least efficient irrigation systems. Nevertheless, the aggregate irrigation profits for all zones are increased by \$34.9/year ($n = 10$) or \$107.38/year ($n = 6$) from the common property results. The spatial disadvantage for the last zone also diminishes, because the profit disparity between the upstream zones and the last zone narrows, even though it does not completely vanish. The reason might be that the return flows from the upstream zones can always be reused by downstream zones, whereas the return flow from the last zone is not utilized for irrigation any more, and hence has shadow values. It would be reasonable for the efficiency allocation to lean toward the upstream zones.

4.7 The Main Model: Irrigated Farming with Saline Surface Water

In the main model, we will incorporate the salinity in the crop productions, and also the salinity balances in the river. In order to simulate the river conditions as close to reality as possible, we will use real data for the quantity and quality (salinity) of initial inflow, tributaries data is also included in the model. In the fresh water case, we show that the number of zones doesn't affect the pattern of the results, therefore, in the this section, we only present the results for $n = 10$ for succinctness and without loss of generality.

4.7.1 Common Property: Saline Water

The common property case would be similar to that with fresh water, but now the surface water salinity is not fixed at 0.07 (dS/m), it starts at an average of 1.26 (dS/m) at the mouth of Merced, and varies downstream with the irrigation activities by

Table 4.4: Efficiency Results with Fresh Water

Zones	π_i	W_i	R_i	Crop Patterns						
				Crop	Irr-system	w	e	r	x	y
$n = 10$ $Q_1 = 200$ 1-10	3076.07	20.80	8.05	Alfalfa	Fur 1/2	3.98	2.49	1.49	1669	7.62
				Tomato	Fur 1/2	4.45	1.95	2.50	1337	48.35
				Wheat	Fur 1/2	2.15	1.46	0.69	1146	2.00
1-9	3072.99	20.12	7.45	Alfalfa	Fur 1/2	3.72	2.46	1.26	1669	7.53
				Tomato	Fur 1/2	4.41	1.95	2.47	1337	48.33
$n = 10$				Wheat	Fur 1/2	1.98	1.43	0.55	1146	1.95
$Q_1 = 130$ 10	2949.28	16.00	3.67	Alfalfa	Fur 1/4	2.73	2.32	0.42	1669	7.08
				Tomato	Lin	2.86	1.94	0.92	1337	47.91
				Wheat	Fur 1/4	1.63	1.37	0.27	1146	1.85
$n = 6$ $Q_1 = 200$ 1-6	5126.79	34.66	13.42	Alfalfa	Fur 1/2	3.98	2.49	1.49	2783	7.62
				Tomato	Fur 1/2	4.45	1.95	2.50	2229	48.35
				Wheat	Fur 1/2	2.15	1.46	0.69	1910	2.00
1-5	5107.61	32.64	11.68	Alfalfa	Fur 1/2	3.53	2.44	1.09	2783	7.46
				Tomato	Fur 1/2	4.39	1.95	2.44	2229	48.31
$n = 6$				Wheat	Fur 1/2	1.82	1.39	0.43	1910	1.88
$Q_1 = 130$				Alfalfa	Fur 1/4	2.44	2.21	0.23	2783	6.76
6	4783.23	25.20	5.09	Tomato	Lin	2.67	1.93	0.75	2229	47.50
				Wheat	Fur 1/4	1.51	1.30	0.21	1910	1.74

the farming zones along the river. Each zone maximizes its own annual net irrigation profit independently, subjecting to land, water, rotation and production constraints. The initial flow and salinity are exogenous to the first zone. The quantity and salinity of the outflow from a zone can be calculated from the water balance (4.14) and salinity balance (4.15) condition in the river. The problem is then solved sequentially by zones, with the quantity and salinity of the outflow for the previous zone being the exogenous quantity and salinity of the inflow for the next zone.

The results are shown in three tables.

Table 4.5 summarizes the zone-level profit, salinity of irrigation water, total diverted water, total return flows, and quantity and salinity of water flowing out of the corresponding river reach for each of the ten zones. The results can be interpreted from two perspectives: (1) First is the impact of water salinity on the irrigation profits. The total diverted water and total return flows is positively correlated with the applied water salinity (cz_i), while profit is negatively correlated with applied water salinity, which indicates that as applied water gets more saline, a zone needs more water to irrigate same acreage of crop fields, thus reduces profits. Above that, the total profit and total diverted water for each zone are very close besides some minor variations due to the salinity difference of applied water, and combined with the fresh water common property results, we can conjecture that should the salinity be the same for every zone, they would divert same amount of water and obtain same profit, which indicates that the quantity of water is sufficient for the irrigation activities along the river, but rather it is the quality/salinity of the water that is hindering irrigation. It is also foreseeable that once the region is in lack of water, i.e. in times of drought, common property will result in potential unequal distribution of water resources between upstream and downstream zones. (2) Second is the impact of irrigation activities on the quantity and quality of surface water. The quantity of the water flowing out of a zone decreases and the salinity increases as the river goes downstream, the off-trend change to the quantity and quality of water is due to the two tributaries Tuolumne and Stanislaus. Overall, irrigation has a negative impact on the quality of water.

Table 4.6 Shows the crop pattern of Zone 1 under common property, the crop

Table 4.5: Common Property Results with Saline Water: Zone-level Summary

Zone i	π_i	cz_i	W_i	R_i	Q_{i+1}	c_{i+1}
1	2958.749	1.2600	22.224	9.593	1042913	1.2753
2	2957.039	1.2753	22.243	9.612	1030283	1.2909
3	2955.281	1.2909	22.261	9.633	1017654	1.3069
4	2953.474	1.3069	22.281	9.654	1005028	1.3233
5	2951.616	1.3233	22.300	9.675	992403	1.3402
6	2949.704	1.3402	22.320	9.697	979780	1.3574
7	2947.735	1.3574	22.341	9.720	967159	1.3751
8	3006.541	0.8121	21.687	9.007	1751660	0.8180
9	3005.943	0.8180	21.694	9.015	1738981	0.8239
10	3022.666	0.6506	21.493	8.797	2271817	0.6542

Table 4.6: Common Property Results with Saline Water: Crop Pattern

Crop	Irrigation System	yield(y) (ton/acre/yr)	water applied(w) (ft/yr)	ET(e) (ft/yr)	Acreage(x) (acre)
Alfalfa	Fur 1/2	7.455	4.291	2.437	1669.9
Tomato	Fur 1/2	47.756	5.097	1.934	1337.6
Wheat	Fur 1/2	1.992	2.186	1.456	1146.5

pattern for other zones would be similar with same crops, irrigation systems, and acreage, but minor variations in applied water depth, evapotranspiration and per-acre yield. The table shows that zones choose to irrigate the three crops with Furrow 1/2 mile irrigation system, which is the least efficient.

Table 4.7 evaluates the performance of the model by showing the quantity and quality of water at three monitoring sites predicted by the model and comparing with the data, which is shown in the parenthesis. The three monitoring sites are Crowns Landing, Patterson and Vernalis, which locates approximately at the end of the 2nd, 4th, and 10th zone. The table shows that the model predictions are close to the monitored data, with the quantity biased upward and salinity biased downward. The closeness of the common property model prediction and the data also implies that the irrigation

activities along the river might be inefficient and needs regulation.

Table 4.7: Common Property Results with Saline Water: Model vs. Data

Water Flow and Salinity (Model vs. Data[a])

	Flow	Electrical Conductivity
	(10^6 Acre-feet/yr)	(dS/m)
SJR near Crowns Landing	1.0429	1.2909
(end point of 2nd zone)	(1.1193)	(1.038)
SJR near Patterson	1.0050	1.3233
(end point of 4th zone)	(N.A.)	(1.29)
SJR near Vernalis	2.2718	0.6542
(end point of 10th zone)	(2.5906)	(0.632)

[a] Data Source: USGS National Water Information System (water-data.usgs.gov/nwis). Data are shown in parenthesis.

4.7.2 Efficiency: Saline Water

The efficiency model maximizes the total irrigation profits from the n zones 4.18 and satisfying all the hydrologic (including water and salinity balance equations for all reaches), agrinomic production, water and land use constraints, and rotation constraints as stated. The results are the same with common property because of the abundance of water quantity on average. However, efficient usage of water would be helpful to increase aggregate profit and reduce water salinity in times of drought. We consider a drought with only 30% of the average flow at the three water-incoming spots Merced, Tuolumne and Stainislaus, the corresponding salinity levels are calibrated by the salt mass balance (Table 4.8).

Table 4.8: Flow and EC at Various Spots In Drought

	Merced	Tuolumne	Stanislaus
Flow(Acre-feet/yr)	316,663.5	239,154	163,659.6
EC(dS/m)	4.2	0.43	0.3267

The results for common property and efficiency under drought conditions are shown in the same Table 4.9 for comparison. Aggregate profit increases from 25056.71 ($/year) under common property to 25058.04 ($/year) under efficiency, and zone profit variance also drops, implying a more equal distribution. For water quality, the water leaving the river system at the end has a lower salinity level of 2.4851 (dS/m) under efficiency, and salinity of applied water is also lower for each zone along the river.

However, even if there is improvement from efficiency against common property, we can see that the differences are very small. This may be due to the fact that the quantity of water is quite large for the area considered so that the model just reallocate the water between zones to reach efficiency, while the choices of crops and irrigation systems are not altered, and the major problem–water salinity is not tackled. Reallocation of water would be of limited use to solve the problem, especially in periods of drought when salt is condensed and the salinity problem is exacerbated. Other methods aiming at reducing salinity or cope with salinity might be considered, like water treatment before drainage and switching to more salt-tolerant crops.

Table 4.9: Comparison of Common Property and Efficiency under Droughts

Zone	Common Property				Efficiency			
	π_i	W_i	R_i	cz_i	π_i	W_i	R_i	cz_i
1	2519.475	25.281	13.110	4.2000	2519.03	24.959	12.822	4.2000
2	2487.332	25.335	13.199	4.3679	2487.07	25.045	12.940	4.3674
3	2451.700	25.390	13.294	4.5492	2451.64	25.137	13.068	4.5482
4	2412.061	25.448	13.396	4.7455	2412.20	25.237	13.207	4.7440
5	2367.804	25.507	13.504	4.9588	2368.15	25.346	13.359	4.9567
6	2318.202	25.568	13.620	5.1911	2318.73	25.464	13.526	5.1885
7	2262.390	25.630	13.744	5.4450	2263.09	25.593	13.709	5.4419
8	2720.199	24.263	11.877	3.0387	2720.34	24.241	11.857	3.0378
9	2707.218	24.350	11.978	3.1207	2707.38	24.342	11.970	3.1197
10	2810.324	23.597	11.117	2.4348	2810.41	23.597	11.116	2.4343
Aggregate Profit	25056.71				25058.04			
Salinity at the End				2.4857				2.4851

4.8   Conclusions

This chapter looks into the relationship between the water quantity and quality of the lower San Joaquin River and the irrigation activities by the surrounding farming zones, using an economic-hydrologic integrated model in which we maximize irrigation profits, subject to agronomic production, land, water, rotation and hydrologic constraints. We start with a pure water allocation problem to show that common property would result in unequal distribution of the water resource, and create a downstream disadvantage for the last zone in face of water shortage, while efficiency usage could help to reduce the discrepancy between upstream and downstream zones, but not eliminating it. Then the main model with saline water which uses data at the three exogenous water-incoming spots at the mouth of Merced (the starting point), Tuolumne tributary, Stanislaus tributary demonstrates that on average, the quantity of water is sufficient for farming, but the increase in salinity of the surface water reduces irrigation profits. On the other hand, irrigation further increases salinity downstream. Even though the water at the exit of Vernalis meets the quality standard (0.7 dS/m), the salinity problem in most of the stretch of the river cannot be overlooked, because the two high quality tributaries joins the main stream near the end of the river help substantially to meet the standard.

The common property results simulate the current water quantity and quality conditions in the lower San Joaquin River quite well, which means the current river usage by irrigation might need regulation. An experiment with drought data confirms that it is more of a water quality problem than a quantity problem that needs to be tackled right now at the lower San Joaquin River.

And a final note, while this model uses the data in lower San Joaquin River, it is applicable to other farming areas which irrigates with surface water.

Chapter 5

Conclusions

This dissertation addresses two issues related to the quantity and quality aspects of rivers.

Chapter 2 deals with the quantity allocation of international rivers in a context where two countries trade goods and services but also have joint access to a scarce resource (e.g. an international river basin). The analysis is based on a two-stage equilibrium model. Economic equilibrium with two-good, two-country Ricardian trade is solved given a specific resource allocation. The trade model is then used to generate country welfare functions as a function of the allocation. These welfare functions then enter into a game-theoretic model to determine political equilibrium.

The results are striking. In the autarchic case, country welfare is increasing in water allocation as expected. However, when trade is allowed, then in some instances (under which a country has absolute disadvantage in both goods or comparative advantage in just one good), we find that the welfare functions can be non-monotone; that is, starting from some initial allocation, it can actually be in the self-interest of one country to give up water to another country. Furthermore, there can be instances in which the highest level of welfare for one country is achieved with joint use of the resource as opposed to having a full allocation of the resource. At a minimum, where productivity coefficients imply a comparative advantage such that trade occurs, then the level of conflict as measured by the gains from an additional allocation of the resource, will be reduced.

In general, the analysis indicates that some of the perceived conflict may be due to a narrow (partial equilibrium) focus on the natural resource. When the analytical and policy framework is broadened out to a more comprehensive general equilibrium framework, then the level of the conflict (gains from an increased allocation of the resource) may very well be reduced or even - in some cases - alleviated.

The analysis uses an international trade model to analyze the problem of sharing a natural resource and demonstrate that linking international trade with water resource allocation problem will mitigate the conflict over water from a general equilibrium perspective. The welfare functions derived from the analysis can serves as payoff functions in a subsequent game-theoretic model of international river sharing, instead of assuming a certain welfare function. The differences between welfare functions derived in this Chapter and those normally assumed in the literature are: they are not always increasing in the quantity of resource obtained by a country, and can be non-monotone under some circumstances; they do not only depend on the water a country obtains, but also depends on the water the other country obtains, that is, they depend on how the water is allocated between countries (the water allocation parameters).

Model in Chapter 3 deals with problem under same context, but with two factors of production. The model is solved analytically and numerically and results are qualitatively parallel with those from the one-factor model. The autarky welfare function monotonically increases with a country's water resource obtained, and the free trade welfare function can be non-monotone when a country has absolute disadvantage in both goods, though the definition of absolute disadvantage in this model differs from the one-factor model in that it not only depends on the production coefficients, but also depends on preferences, and labor endowments. These results still hold when the number of countries sharing the river is increased to three, which further confirm the conclusions from Chapter 2 and show the results are not confined to one factor of production and two countries.

For future research on this topic, a more complete theoretical analysis on the properties of free trade welfare functions with two factors can be explored. And the welfare functions can also be applied to an actual setting where several countries share

an international river to solve for the allocation equilibrium among countries.

Chapter 4 is independent from the previous chapters, but still in the realm of rivers, where river water is used for irrigation in arid or semi-arid areas. Irrigation activities and water quality are two closely interrelated subjects. Salinity, one measure of water quality, plays an important to affect agricultural production, and in turn affected by irrigated agriculture. We develop an integrated economic-hydrologic surface water quality model to maximize irrigation profits while maintaining the water quantity balance and salinity balance in the river system, and apply the model to the lower San Joaquin Valley, where the salinity of irrigation water, the impacts of irrigation on water quality, and the efficient use of surface water are of great concerns. The Model divides the river basin area into several equal farming zones and each zone faces a selection of crops and irrigation systems, then decides the acreage for each crop and the amount of water to apply in order to maximize irrigation profits. First, a benchmark model with pure water allocation is provided to illustrate unequal distribution of water to the downstream farming zones in face of water scarcity. Then, the saline water quality and quantity problem is solved both as a common property problem and an efficiency problem. The results suggest that the lower San Joaquin River suffers from salinity problem when there is drought and efficiency usage of water will improve upon common property, but with limited capacity. The salinity of the river at Vernalis (the end of the stretch) normally can meet the government water quality standard due to the two tributaries joining the mainstream towards the end of the stretch, but the upstream middle reach still has high salinity, suggesting that water quality standards are not only needed at the end of river, but also along the river in the middle.

The model is a combination of the crop-water-salinity production functions used in groundwater management literature and the hydrologic-economic model in surface water. The main contribution of the model is to provide novel empirical research for the lower San Joaquin River to address the salinity issue and water allocation for irrigation purposes in this area. While the model is simplified in the hydrological balance in the surface water system, it captures the essence of the major irrigation activity and motion of the river. The model can be extended in several ways by adding more details

and assumptions. For example, consider water conveyance loss from the river to irrigated areas; a joint-use of surface water and groundwater for irrigation; consider water reuse, evaporation ponds and other methods for drainage rather than directly draining to the river; a tax for drainage may be used to achieve efficiency for policy purposes.

All in all, as a research object, a river provides many topics to investigate on and this dissertation only touches on two of them. These two topics can be overlapping in the sense that the quality aspect of the river can also be taken into account when dealing with the allocation of international river water between sharing countries and efficient use of surface water for irrigation purposes can also occur in international river basins. The influences of rivers to human beings, the economy and the environment are immense, and the study on rivers are worthwhile to enhance the well-being of people and countries.

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Appendix A

GAMS Program for Solving the Simple Ricardian Model

The following code solves the Ricardian Model under autarky and free trade, in Chapter 2 with two goods, two countries and one factor, with water allocation parameter values θ_1 ranging from 0.001 to 0.999.

A.1 Autarky

```
Sets i countries  /c1*c2/
     j goods       /g1*g2/;

Table beta(i,j) output coefficients on production functions
      g1    g2
c1    5     4
c2    3     6;

Parameter theta(i) water allocation parameter
          alpha(j) Cobb-Douglas Utility function parameter
          /g1 0.4
           g2 0.6/;
Scalar    Wtot   total water available /1/
          theta1_val  "changing theta1 values";

Variables
OBJ         maximizing dummy
U(i)        Country i's utility function
P(i,j)      prices for good j in country i
PW(i)       prices for water in country i
```

```
Y(i,j)        total output for good j in country i
C(i,j)        consumption of good j in country i
W(i,j)        firms water demand to produce good j in country i;

Equations
PRICES(i,j)   price formation (zero profit condition)
HOUSEDEM(i,j) households demand for goods
WDFAC(i,j)    demand for water for each good
EQGOODS(i,j)  equilibrium for goods
EQFACTORS(i)  equilibrium for water in country i(full employment)
MAXIMAND      auxiliar objective function
Utility(i)    country i's utility function  ;

PRICES(i,j)..    P(i,j)=E=PW(i)/beta(i,j);
HOUSEDEM(i,j)..  C(i,j)=E=alpha(j)*PW(i)*theta(i)*Wtot/P(i,j);
WDFAC(i,j)..     W(i,j)=E=Y(i,j)/beta(i,j);
EQGOODS(i,j)..   Y(i,j)=E=C(i,j);
EQFACTORS(i)..   Sum(j,W(i,j))=E=theta(i)*Wtot;
MAXIMAND..       OBJ=E=1;
Utility(i)..     U(i)=E=Prod(j,C(i,j)**alpha(j));

Model RicardianAutarky  /ALL/;

SCALAR LB lowerbound  /1E-4/;
P.lo(i,j)=LB;
Y.lo(i,j)=LB;
W.lo(i,j)=LB;
C.lo(i,j)=LB;
P.fx('c1','g2')=1;
P.fx('c2','g2')=1;

For(theta1_val=0.001 to 0.999 by 0.001,
    theta('c1')=theta1_val;
    theta('c2')=1-theta1_val;

    Solve RicardianAutarky Maximizing OBJ using NLP;
     );
```

A.2   Free Trade

```
Sets i countries  /c1*c2/
     j goods       /g1*g2/;

Table beta(i,j) output coefficients on production functions
      g1    g2
c1    9     8
```

```
c2    2    6;

Parameter theta(i) water allocation parameter
          alpha(j) Cobb-Douglas Utility function parameter
          /g1 0.4
           g2 0.6/;
Scalar    Wtot   total water available /1/
          theta1_val   varying theta1 values;


Variables
OBJ         maximizing dummy
U(i)        Country i's utility function
P(j)        world equilibrium prices for good j
PW(i)       prices for water in country i
C(i,j)      consumption of good j in country i
W(i,j)      firms water demand to produce good j in country i;

Positive Variables
Y(i,j)      total output for good j in country i ;

Equations
PRICES(i,j)    price formation (zero profit condition)
SLACK(i,j)     Complementary Slack conditions for good j in country i
HOUSEDEM(i,j) households demand for goods
WDFAC(i,j)     demand for water for each good
EQGOODS(j)     equilibrium for good j
EQFACTORS(i)  equilibrium for water in country i(full employment)
MAXIMAND       auxiliar objective function
Utility(i)     country i's utility function  ;

PRICES(i,j)..    P(j)=L=PW(i)/beta(i,j);
SLACK(i,j)..      (P(j)-PW(i)/beta(i,j))*Y(i,j)=E=0;
HOUSEDEM(i,j).. C(i,j)=E=alpha(j)*PW(i)*theta(i)*Wtot/P(j);
WDFAC(i,j)..     W(i,j)=E=Y(i,j)/beta(i,j);
EQGOODS(j)..     Sum(i,Y(i,j))=E=Sum(i,C(i,j));
EQFACTORS(i)..   Sum(j,W(i,j))=E=theta(i)*Wtot;
MAXIMAND..       OBJ=E=1;
Utility(i)..     U(i)=E=Prod(j,C(i,j)**alpha(j));

Model RicardianFreeTrade  /ALL/;
option nlp=knitro;

SCALAR LB lowerbound  /1E-4/;
P.lo(j)=LB;
W.lo(i,j)=LB;
C.lo(i,j)=LB;
P.fx('g2')=1;
Y.l(i,j)=LB;
```

```
For(theta1_val=0.001 to 0.999 by 0.001,
   theta('c1')=theta1_val;
   theta('c2')=1-theta1_val;

   Solve RicardianFreeTrade Maximizing OBJ using NLP;
   );
```

Appendix B

GAMS Program for Solving the Two-Factor/Three Country Model

The following code solves the two-factor model under autarky and free trade in Chapter 3. The code for three-country model can be easily obtained by adapting the country index to three and adding the corresponding parameter values, and thus is not presented here.

B.1 Autarky

```
Sets i countries  /c1*c2/
     j goods       /g1*g2/
     k factors     /water,labor/;

Table gamma(i,j) Cobb-Douglas coefficients on prod functions
      g1    g2
c1    0.3   0.7
c2    0.5   0.2;

Parameter theta(i) water allocation parameter
          alpha(j) Cobb-Douglas Utility function parameter
          /g1 0.4
           g2 0.6/;
Scalar    Wtot   total water available /1/
          theta1_val  changing theta1 values;
```

```
Table Endow(i,k) endowment of factors in the two countries
      labor
c1      4
c2      1;

Variables
OBJ         maximizing dummy
U(i)        Country i's utility function
P(i,j)      prices for good j in country i
PFAC(i,k)   prices for factor in country i
Y(i,j)      total output for good j in country i
C(i,j)      consumption of good j in country i
w1(i,j)     water demanded to produce one unit of output
l1(i,j)     labor demanded to produce one unit of output
w(i,j)      firms water demand to produce good j in country i
l(i,j)      firms labor demand to produce good j in country i;

Equations
HOUSEDEM(i,j) households demand for goods
WDFAC1(i,j)   water demanded to produce one unit of output
LDFAC1(i,j)   labor demanded to produce one unit of output
WDFAC(i,j)    factor demand for water
LDFAC(i,j)    factor demand for labor
EQGOODS(i,j)  equilibrium for goods
EQWATER(i)    equilibrium for water in country i(full employment)
EQLABOR(i)    equilibrium for labor in country i(full employment)
PRICES(i,j)   price formation (zero profit condition)
MAXIMAND      auxiliar objective function
Utility(i)    country i's utility function ;

HOUSEDEM(i,j)..  C(i,j)=E=alpha(j)*Sum(k,PFAC(i,k)*Endow(i,k))/P(i,j);
WDFAC1(i,j)..    w1(i,j)=E=((PFAC(i,'labor')*gamma(i,j))/
                 (PFAC(i,'water')*(1-gamma(i,j))))**(1-gamma(i,j));
LDFAC1(i,j)..    l1(i,j)=E=((PFAC(i,'water')*(1-gamma(i,j)))/
                 (PFAC(i,'labor')*gamma(i,j)))**gamma(i,j);
WDFAC(i,j)..     w(i,j)=E=w1(i,j)*Y(i,j);
LDFAC(i,j)..     l(i,j)=E=l1(i,j)*Y(i,j);
EQGOODS(i,j)..   Y(i,j)=E=C(i,j);
EQWATER(i)..     Sum(j,w(i,j))=E=Endow(i,'water');
EQLABOR(i)..     Sum(j,l(i,j))=E=Endow(i,'labor');
PRICES(i,j)..    P(i,j)=E=PFAC(i,'water')*w1(i,j)+PFAC(i,'labor')*l1(i,j);
MAXIMAND..       OBJ=E=1;
Utility(i)..     U(i)=E=Prod(j,C(i,j)**alpha(j));


Model TwoFactorAutarky  /ALL/;
option nlp=knitro;
```

```
SCALAR LB lowerbound  /1E-4/;
P.lo(i,j)=LB;
PFAC.lo(i,k)=LB;
Y.lo(i,j)=LB;
C.lo(i,j)=LB;
w.lo(i,j)=LB;
l.lo(i,j)=LB;
w1.lo(i,j)=LB;
l1.lo(i,j)=LB;
P.fx(i,'g2')=1;

for(theta1_val=0.001 to 1.000 by 0.001,
    theta('c1')=theta1_val;
    theta('c2')=1-theta1_val;
    Endow(i,'water')=theta(i)*Wtot;

    Solve TwoFactorAutarky Maximizing OBJ using NLP;
  );
```

B.2 Free Trade

```
Sets i countries  /c1*c2/
     j goods       /g1*g2/
     k factors     /water,labor/;

Table gamma(i,j) output coefficients on production functions
      g1    g2
c1    0.3   0.7
c2    0.5   0.2;

Parameter theta(i) water allocation parameter
          alpha(j) Cobb-Douglas Utility function parameter
          /g1 0.4
           g2 0.6/;
Scalar    Wtot   total water available /1/
          theta1_val  varying theta1 values;

Table Endow(i,k) endowment of factors in the two countries
      labor
c1      4
c2      1;

Variables
OBJ         maximizing dummy
U(i)        Country i's utility function
P(j)        world equilibrium prices for good j
PFAC(i,k)   prices for factor in country i
```

```
C(i,j)        consumption of good j in country i
w1(i,j)       water demanded to produce one unit of output
l1(i,j)       labor demanded to produce one unit of output
w(i,j)        firms water demand to produce good j in country i
l(i,j)        firms labor demand to produce good j in country i;

Positive Variables
Y(i,j)        total output for good j in country i ;

Equations
Utility(i)    country i's utility function
HOUSEDEM(i,j) households demand for goods
WDFAC1(i,j)   water demanded to produce one unit of output
LDFAC1(i,j)   labor demanded to produce one unit of output
WDFAC(i,j)    factor demand for water
LDFAC(i,j)    factor demand for labor
EQGOODS(j)    equilibrium for good j
EQWATER(i)    equilibrium for water in country i(full employment)
EQLABOR(i)    equilibrium for labor in country i(full employment)
PRICES(i,j)   price formation (zero profit condition)
SLACK(i,j)    Complementary Slack conditions for good j in country i
MAXIMAND      auxiliar objective function;

Utility(i)..    U(i)=E=Prod(j,C(i,j)**alpha(j));
HOUSEDEM(i,j).. C(i,j)=E=alpha(j)*Sum(k,PFAC(i,k)*Endow(i,k))/P(j);
WDFAC1(i,j)..   w1(i,j)=E=((PFAC(i,'labor')*gamma(i,j))/
                (PFAC(i,'water')*(1-gamma(i,j))))**(1-gamma(i,j));
LDFAC1(i,j)..   l1(i,j)=E=((PFAC(i,'water')*(1-gamma(i,j)))/
                (PFAC(i,'labor')*gamma(i,j)))**gamma(i,j);
WDFAC(i,j)..    w(i,j)=E=w1(i,j)*Y(i,j);
LDFAC(i,j)..    l(i,j)=E=l1(i,j)*Y(i,j);
EQGOODS(j)..    Sum(i,Y(i,j))=E=Sum(i,C(i,j));
EQWATER(i)..    Sum(j,w(i,j))=E=Endow(i,'water');
EQLABOR(i)..    Sum(j,l(i,j))=E=Endow(i,'labor');
PRICES(i,j)..   P(j)=L=PFAC(i,'water')*w1(i,j)+PFAC(i,'labor')*l1(i,j);
SLACK(i,j)..    (P(j)-PFAC(i,'water')*w1(i,j)-
                 PFAC(i,'labor')*l1(i,j))*Y(i,j)=E=0;
MAXIMAND..      OBJ=E=1;

Model TwoFactorFreeTrade  /ALL/;
option nlp=knitro;

SCALAR LB lowerbound  /1E-4/;
P.lo(j)=LB;
PFAC.lo(i,k)=LB;
C.lo(i,j)=LB;
w.lo(i,j)=LB;
l.lo(i,j)=LB;
w1.lo(i,j)=LB;
```

```
l1.lo(i,j)=LB;
P.fx('g2')=1;
Y.l(i,j)=LB;

For(theta1_val=0.001 to 1.000 by 0.001,
    theta('c1')=theta1_val;
    theta('c2')=1-theta1_val;
    Endow(i,'water')=theta(i)*Wtot;

    Solve TwoFactorFreeTrade Maximizing OBJ using NLP;
   );
```

Appendix C

List of Variables and Measurement Units for the San Joaquin River Model

The following list provides all the variables and their measurement units used in the San Joaquin River Model.

Hydrology-related variables for Zone i ($i = 1, 2, ..., n$)

- Q_i: Quantity of water coming into reach i(acre-feet/year)

- TQ_i: Quantity of tributary water coming into reach i(acre-feet/year)

- c_i, Tc_i,cz_i,cr_i: Salinity of water at the start of reach i, for tributary i, applied to Zone i, returned from Zone i (dS/m)

- W_i: Quantity of water diverted by Zone i (acre-feet/year)

- R_i: Quantity of water returned from Zone i

Agricultural Production-related variables and parameters:

- w_{jk}: applied water depth for crop j with irrigation system k (feet/acre/year)

- r_{jk}: return flow depth for crop j with irrigation system k (feet/acre/year)

- y_{jk}: yield of crop j with irrigation system k (ton/acre/year)

- et: evapotranspiration of a crop (feet/year)

- $\bar{e}$: maximum evapotranspiration of a crop without water or salinity stresses (feet/acre/year)

    Irrigation Profits-related variables and parameters:

- p_j: market price of crop j ($/ton)

- $cost_i^w$: cost of water for zone i ($/acre-feet)

- γ_{jk}: nonwater production cost for crop j and irrigation system k ($/acre)

- x_{ijk}: acreage for crop j with irrigation system k on zone i (acre)

Appendix D

Agronomic Crop Production Functions in the San Joaquin River Model

The general production function and evapotranspiration function for crops are

$$y = \alpha_1 + \alpha_2 et$$

and

$$et = \frac{\bar{e}}{1 + \beta_1 (cz + \beta_2 w^{\beta_3})^{\beta_4}}$$

The parameter values for the crops, alfalfa, tomato and wheat with six different irrigation systems, furrow 1/2 mile, furrow 1/4 mile, sprinkler, lepa, lin and drip are shown in the Table D.1:[1]

---

[1]Source: Schwabe et al. (2006)

Table D.1: Agricultural Production Function Parameters

Crops	Irrigation Systems	α_1	α_2	$\bar{e}$	β_1	β_2	β_3	β_4
Alfalfa	Furrow2	0	3.0588	2.625	0.00101517	24.47456	-1.14915	2.45054
	Furrow4	0	3.0588	2.625	0.00115378	24.50579	-1.24722	2.42762
	Sprinkler	0	3.0588	2.625	0.00130071	24.70739	-1.34854	2.40472
	Lepa	0	3.0588	2.625	0.00143618	25.03550	-1.44350	2.38510
	Lin	0	3.0588	2.625	0.00152906	25.33633	-1.51080	2.37256
	Drip	0	3.0588	2.625	0.00152906	25.33633	-1.51080	2.37256
Tomatoes	Furrow2	-24.5280	37.3820	1.969	0.00107786	21.84846	-1.45658	2.37239
	Furrow4	-24.5310	37.3840	1.969	0.00105183	21.58911	-1.56788	2.40312
	Sprinkler	-24.5290	37.3830	1.969	0.00103073	21.41255	-1.67634	2.42978
	Lepa	-24.5310	37.3850	1.969	0.00101808	21.39608	-1.78428	2.45099
	Lin	-24.5310	37.3840	1.969	0.00099509	21.25147	-1.85735	2.47145
	Drip	-24.5310	37.3840	1.969	0.00099509	21.25147	-1.85735	2.47145
Wheat	Furrow2	-0.2390	1.5322	1.539	0.00003763	76.77791	-1.79560	2.43794
	Furrow4	-0.2829	1.5625	1.539	0.00012591	73.06529	-2.33597	2.19203
	Sprinkler	-0.3420	1.6021	1.539	0.00002789	52.53395	-2.04445	2.68065
	Lepa	-0.4192	1.6542	1.539	0.00001958	44.37918	-2.09997	2.85659
	Lin	-0.4940	1.7042	1.539	0.00002417	44.26459	-2.45720	2.85266
	Drip	-0.4940	1.7042	1.539	0.00002417	44.26459	-2.45720	2.85266

Appendix E

GAMS Program for the San Joaquin River Model

E.1   Common Property: Fresh Water

The code for solving the fresh water common property problem is as follows:

```
Sets        i "irrigation zones" /1*10/
            j "all crops" /alfalfa,tomato,wheat/
            k "irrigation systems" /fur2,fur4,spr,lepa,lin,drip/;

Scalars     wcost_val   "cost of water from the river($/acre-ft)"
            Xtot_val    "total arable land at zone i(acre)"
            c           "salinity of water"/0.07/
            Q0          "initial quantity of water into the river" /130/
            walmond     "applied water depth for almond(ft/yr)" /4.25/
            ralmond     "return flow for almond(ft/yr)" /1.06/
            xalmond     "acerage for almond (acre)"/1.3498/;

Parameters p(j)         "market price of crop j($)" /alfalfa 109.3,
                                                     tomato   55.2,
                                                     wheat   133.3/
            Xmax(j)     "maximum acreage proportion of crop j"
                                                     /alfalfa 0.402,
                                                      tomato  0.322,
                                                      wheat   0.276/
            Xmin(j)     "minimum acreage proportion of crop j"
            Xtot(i)     "total arable land for each zone"
            wcost(i)    "cost of water for each zone";
Xmin(j)=0;
Xtot(i)=5.5037;
```

```
wcost(i)=30;

Table      gamma(j,k)   "nonwater production costs"
           fur2       fur4        spr        lepa        lin        drip
alfalfa   396.80     409.90     434.60     492.30     483.20     538.60
tomato    636.80     661.20     718.80     704.90     701.50     751.20
wheat     194.80     204.50     235.50     288.60     286.90     390.90 ;

Table      alpha1(j,k) "parameter 1 in crop production function"
           fur2        fur4         spr        lepa         lin
alfalfa    0           0           0           0           0
tomato    -24.5280    -24.5310    -24.5290    -24.5310    -24.5310    -
wheat     -0.2390     -0.2829     -0.3420     -0.4192     -0.4940     -

Table      alpha2(j,k) "parameter 2 in crop production function"
           fur2        fur4         spr        lepa         lin
alfalfa    3.0588      3.0588      3.0588      3.0588      3.0588
tomato    37.3820     37.3840     37.3830     37.3850     37.3840
wheat      1.5322      1.5625      1.6021      1.6542      1.7042

Table      emax(j,k)  "maximum evapotranpiration without stress"
           fur2        fur4         spr        lepa         lin
alfalfa    2.625       2.625       2.625       2.625       2.625
tomato     1.969       1.969       1.969       1.969       1.969
wheat      1.539       1.539       1.539       1.539       1.539

Table      beta1(j,k) "parameter 1 for evapotranspiration function"
           fur2        fur4         spr        lepa         lin
alfalfa    0.00101517  0.00115378  0.00130071  0.00143618  0.00152906
tomato     0.00107786  0.00105183  0.00103073  0.00101808  0.00099509
wheat      0.00003763  0.00012591  0.00002789  0.00001958  0.00002417

Table      beta2(j,k) "parameter 2 for evapotranspiration function"
           fur2        fur4         spr        lepa         lin
alfalfa   24.47456    24.50579    24.70739    25.03550    25.33633
tomato    21.84846    21.58911    21.41255    21.39608    21.25147
wheat     76.77791    73.06529    52.53395    44.37918    44.26459

Table      beta3(j,k) "parameter 3 for evapotranspiration function"
           fur2        fur4         spr        lepa         lin
alfalfa   -1.14915    -1.24722    -1.34854    -1.44350    -1.51080
tomato    -1.45658    -1.56788    -1.67634    -1.78428    -1.85735
wheat     -1.79560    -2.33597    -2.04445    -2.09997    -2.45720

Table      beta4(j,k) "parameter 4 for evapotranspiration function"
           fur2        fur4         spr        lepa         lin
alfalfa    2.45054     2.42762     2.40472     2.38510     2.37256
tomato     2.37239     2.40312     2.42978     2.45099     2.47145
wheat      2.43794     2.19203     2.68065     2.85659     2.85266
```

```
Variables
zonepi        "total profit for a zone($/yr)"
Q             "quantity of water that flow out of river reach i"
fieldpi(j,k)  "per-acre profit($/acre/yr)"
WQtot         "total quantity of water diverted by zone i(acre-ft/yr)"
RQtot         "total return flows from zone i(acre-ft/yr)"
y(j,k)        "per-acre yield(ton/acre/yr)"
e(j,k)        "per-acre evapotranpiration(ft/yr)"
r(j,k)        "per-acre return flow(ft/year)"
Qnext         "quantity of water flowing to next reach(acre-ft/yr)"
cnext         "salinity of water flowing to next reach(dS/m)"
xfld          "actual cropped acreage";

Positive Variables
w(j,k)        "per-acre applied water depth(ft/yr)"
x(j,k)        "land for crop j with irrigation system k, at zone i(acre)";

w.lo('alfalfa',k)=1.3;
w.up('alfalfa',k)=7.0;
w.lo('tomato',k)=1.6;
w.up('tomato',k)=6.5;
w.lo('wheat',k)=1.35;
w.up('wheat',k)=3.65;
x.lo(j,k)=0.0001;

w.l(j,k)=3;
x.l(j,k)=0.1;
Q.fx=Q0;

Equations
et(j,k)           "evapotranspiration function"
yield(j,k)        "yield function"
return(j,k)       "return flow function"
fldprofit(j,k)    "field profit function"
zoneprofit        "zone profit function"
xfldeqn           "cropped acreage"
landconst         "arable land constraint at zone i"
Xmaxconst(j)      "maximum acreage proportion of crop j"
Xminconst(j)      "minimum acreage proportion of crop j"
watertot          "total diverted water by zone i"
returntot         "total return flow from zone i"
waterconst        "water constraint total water diversion"
waterbalance      "water balance for the current reach";

et(j,k)..         e(j,k)=e=emax(j,k)/(1+beta1(j,k)*(c+beta2(j,k)
                        *w(j,k)**beta3(j,k))**beta4(j,k))  ;
yield(j,k)..      y(j,k)=e=alpha1(j,k)+alpha2(j,k)*e(j,k);
```

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```
return(j,k)..     r(j,k)=e=w(j,k)-e(j,k);
fldprofit(j,k).. fieldpi(j,k)=e=p(j)*y(j,k)-gamma(j,k)-wcost_val*w(j,k);
zoneprofit..      zonepi=e=sum((j,k),fieldpi(j,k)*x(j,k));
xfldeqn..         xfld=e=sum((j,k),x(j,k));
landconst..       xfld+xalmond=l=Xtot_val;
Xmaxconst(j)..    sum(k,x(j,k))=l=Xmax(j)*xfld;
Xminconst(j)..    sum(k,x(j,k))=g=Xmin(j)*xfld;
watertot..        sum((j,k),w(j,k)*x(j,k))+walmond*xalmond=e=WQtot;
returntot..       sum((j,k),r(j,k)*x(j,k))+ralmond*xalmond=e=RQtot;
waterconst..      WQtot=l=Q;
waterbalance..    Q-WQtot+RQtot=e=Qnext;

Model commonpropertyfresh /all/;

loop(i,
     wcost_val=wcost(i);
     Xtot_val=Xtot(i);

     Solve commonpropertyfresh using nlp maximizing zonepi;

     Q.fx=Qnext.l;
     );
```

E.2   Common Property:Saline Water

The code for saline water common property problem adapts the fresh water common property problem in the following three ways.

(1)Delete the following code from Scalar part:

```
Scalars   c       "salinity of water"/0.07/
```

(2)Add the following code in corresponding parts:

```
Scalars   c0      "salinity of initial water flow" /1.26/
          TQ_val "volume of tributary"
          Tc_val "salinity of tributary"
          cz      "salinity of water applied by zones";

Parameters TQ(i)  "quantity of tributary water flow into each zone"
                             /8 797.180,
                              10 545.532/
           Tc(i)  "salinity of tributary water flow into each zone"
                             /8 0.129,
                              10 0.098/;
Variables  c       "salinity of water into of river reach i"
```

```
c.fx=c0;

Equations
salinitybalance    "salinity balance of the reach";

salinitybalance.. TQ_val*Tc_val+Q*c=e=Qnext*cnext;
```

(3)Make adaptions for the following code:

```
Scalars      Q0 "initial quantity of water into the river" /1055.545/;

et(j,k)..     e(j,k)=e=emax(j,k)/(1+beta1(j,k)*(cz+beta2(j,k)
                        *w(j,k)**beta3(j,k))**beta4(j,k)) ;
waterconst.. WQtot=l=Q+TQ_val;
waterbalance..TQ_val+Q-WQtot+RQtot=e=Qnext;

Model commonpropertysaline /all/;

loop(i,
     wcost_val=wcost(i);
     Xtot_val=Xtot(i);
     TQ_val=TQ(i);
     Tc_val=Tc(i);

     cz=(TQ_val*Tc_val+Q.l*c.l)/(TQ_val+Q.l);

     Solve commonpropertysaline using nlp maximizing zonepi;

     Q.fx=Qnext.l;
     c.fx=cnext.l
    );
```

E.3   Efficiency: Fresh Water

The code for solving the fresh water efficiency problem is as follows:

```
Sets i        farming zones & river reaches /1*10/
    im1(i)    farming zones & river reaches except the last one  /1*9/
    j         all crops /alfalfa,tomato,wheat/
    k         irrigation systems /fur2,fur4,spr,lepa,lin,drip/;

Scalars    c            "salinity of water"/0.07/
           Q0           "initial quantity of water into the river" /130/
           walmond      "applied water depth for almond(ft/yr)" /4.25/
           ralmond      "return flow for almond(ft/yr)" /1.06/
           xalmond      "acerage for almond (acre)"/1.3498/;
```

```
Parameters p(j)          "market price of crop j($)" /alfalfa 109.3,
                                                       tomato  55.2,
                                                       wheat   133.3/
            Xmax(j)      "maximum acreage proportion of crop j"
                                                       /alfalfa 0.402,
                                                         tomato  0.322,
                                                         wheat   0.276/
            Xmin(j)      "minimum acreage proportion of crop j"
            Xtot(i)      "total arable land for each zone"
            wcost(i)     "cost of water for each zone";
Xmin(j)=0;
Xtot(i)=5.5037;
wcost(i)=30;


Table     gamma(j,k)  "nonwater production costs"
          fur2       fur4        spr        lepa       lin        drip
alfalfa   396.80     409.90     434.60     492.30     483.20     538.60
tomato    636.80     661.20     718.80     704.90     701.50     751.20
wheat     194.80     204.50     235.50     288.60     286.90     390.90 ;


Table     alpha1(j,k) "parameter 1 in crop production function"
          fur2       fur4         spr        lepa        lin
alfalfa    0           0           0          0           0
tomato   -24.5280    -24.5310    -24.5290   -24.5310    -24.5310    -
wheat    -0.2390     -0.2829     -0.3420    -0.4192     -0.4940     -


Table     alpha2(j,k) "parameter 2 in crop production function"
          fur2       fur4         spr        lepa        lin
alfalfa   3.0588      3.0588      3.0588     3.0588      3.0588
tomato    37.3820     37.3840     37.3830    37.3850     37.3840
wheat     1.5322      1.5625      1.6021     1.6542      1.7042


Table     emax(j,k) "maximum evapotranpiration without stress"
          fur2       fur4         spr        lepa        lin
alfalfa   2.625       2.625       2.625      2.625       2.625
tomato    1.969       1.969       1.969      1.969       1.969
wheat     1.539       1.539       1.539      1.539       1.539


Table     beta1(j,k) "parameter 1 for evapotranspiration function"
          fur2       fur4         spr        lepa        lin
alfalfa   0.00101517 0.00115378  0.00130071 0.00143618  0.00152906
tomato    0.00107786 0.00105183  0.00103073 0.00101808  0.00099509
wheat     0.00003763 0.00012591  0.00002789 0.00001958  0.00002417


Table     beta2(j,k) "parameter 2 for evapotranspiration function"
          fur2       fur4         spr        lepa        lin
alfalfa   24.47456   24.50579    24.70739    25.03550    25.33633
```

```
tomato   21.84846   21.58911   21.41255   21.39608   21.25147
wheat    76.77791   73.06529   52.53395   44.37918   44.26459


Table      beta3(j,k) "parameter 3 for evapotranspiration function"
           fur2         fur4         spr        lepa          lin
alfalfa  -1.14915   -1.24722   -1.34854   -1.44350   -1.51080
tomato   -1.45658   -1.56788   -1.67634   -1.78428   -1.85735
wheat    -1.79560   -2.33597   -2.04445   -2.09997   -2.45720


Table      beta4(j,k) "parameter 4 for evapotranspiration function"
           fur2         fur4         spr        lepa          lin
alfalfa  2.45054    2.42762    2.40472    2.38510    2.37256
tomato   2.37239    2.40312    2.42978    2.45099    2.47145
wheat    2.43794    2.19203    2.68065    2.85659    2.85266


Positive Variables
w(i,j,k)        "per-acre applied water depth(ft/yr)"
x(i,j,k)        "acreage for crop j with irr-system k, at zone i" ;


Variables
xfld(i)         "cropped acreage at zone i"
WQtot(i)        "total water diverted by zone i(acre-ft/yr)"
RQtot(i)        "total return flows from zone i(acre-ft/yr)"
Q(i)            "quantity of water that flow into river reach i"
Qout            "quantity of water flow out of the whole region"
y(i,j,k)        "per-acre yield(ton/acre/yr)"
e(i,j,k)        "per-acre evapotranpiration(ft/acre/yr)"
r(i,j,k)        "per-acre return flow(ft/acre/yr)"
zonepi(i)       "total profit for zone i($/yr)"
fieldpi(i,j,k)  "per-acre profit($/acre/yr)"
totalprofit     "aggregate region profit($/yr)";


Equations
et(i,j,k)         "evapotranspiration function"
yield(i,j,k)      "yield function"
return(i,j,k)     "return flow function"
fldprofit(i,j,k)  "field profit function"
zoneprofit(i)     "zone profit function"
xfldeqn(i)        "cropped acreage equation"
landconst(i)      "arable land constraint at zone i"
xmaxconst(i,j)    "maximum acreage proportion for crop j at zone i"
xminconst(i,j)    "minimum acreage proportion for crop j at zone i"
WQtotdef(i)       "total diverted water for zone i"
waterconst(i)     "water constraint at zone i"
returndef(i)      "return flow constraint from zone i"
waterbalance(i)   "water balance equation in the river"
Qoutdef           "quantity of water out of the whole region"
objective         "objective function";
```

```
et(i,j,k)..        e(i,j,k)=e=emax(j,k)/(1+beta1(j,k)*(c+beta2(j,k)
                             *w(i,j,k)**beta3(j,k))**beta4(j,k));
yield(i,j,k)..     y(i,j,k)=e=alpha1(j,k)+alpha2(j,k)*e(i,j,k);
return(i,j,k)..    r(i,j,k)=e=w(i,j,k)-e(i,j,k);
fldprofit(i,j,k)..fieldpi(i,j,k)=e=p(j)*y(i,j,k)-gamma(j,k)
                             -wcost(i)*w(i,j,k);
zoneprofit(i)..    zonepi(i)=e=sum((j,k),fieldpi(i,j,k)*x(i,j,k));
xfldeqn(i)..       xfld(i) =e= sum((j,k),x(i,j,k));
landconst(i)..     xfld(i) + xalmond =l= Xtot(i);
xmaxconst(i,j)..   sum(k,x(i,j,k))=l=Xmax(j)*xfld(i);
xminconst(i,j)..   sum(k,x(i,j,k))=g=Xmin(j)*xfld(i);
WQtotdef(i)..      sum((j,k),w(i,j,k)*x(i,j,k))+walmond*xalmond
                   =e=WQtot(i);
returndef(i)..     sum((j,k),r(i,j,k)*x(i,j,k))+ralmond*xalmond
                   =e=RQtot(i);
waterconst(i)..    WQtot(i) =l= Q(i);
waterbalance(i)$im1(i)..  Q(i)-WQtot(i)+RQtot(i)=e=Q(i+1);
Qoutdef..          Q('10')-WQtot('10')+RQtot('10')=e=Qout;
objective..        totalprofit=e=sum(i,zonepi(i));


w.lo(i,'alfalfa',k)=1.3;
w.up(i,'alfalfa',k)=7.0;
w.lo(i,'tomato',k)=1.6;
w.up(i,'tomato',k)=6.5;
w.lo(i,'wheat',k)=1.35;
w.up(i,'wheat',k)=3.65;
x.lo(i,j,k)=0.0001;

Q.fx('1')=Q0;

w.l(i,j,k)=3.0;
e.l(i,j,k)=emax(j,k)/(1+beta1(j,k)*(c+beta2(j,k)
           *w.l(i,j,k)**beta3(j,k))**beta4(j,k)) ;
y.l(i,j,k)=alpha1(j,k)+alpha2(j,k)*e.l(i,j,k);
r.l(i,j,k)=w.l(i,j,k)-e.l(i,j,k);
x.l(i,j,k)=0.1;
WQtot.l(i)=100.0;
RQtot.l(i)=50;
loop(i$im1(i),Q.l(i+1)=Q.l(i)-WQtot.l(i)+RQtot.l(i));

Model   freshwaterefficiency /all/;
Solve   freshwaterefficiency using nlp maximizing totalprofit;
option nlp=knitro;
```

E.4 Efficiency: Saline Water

The code for saline water efficiency problem adapts the fresh water efficiency problem in the following three ways.

(1)Delete the following code from Scalar part:

```
Scalars   c       "salinity of water"/0.07/
```

(2)Add the following code in corresponding parts:

```
Scalars    c0      "salinity of initial water flow" /1.26/

Parameters TQ(i)  "quantity of tributary water flow into each zone"
                              /8 797.180,
                              10 545.532/
          Tc(i)  "salinity of tributary water flow into each zone"
                              /8 0.129,
                              10 0.098/;
Variables
c(i)        "salinity of water into river reach i"
cz(i)       "salinity of water applied to zone i"
cout        "salinity of water flow out of the whole region"


Equations
salinitybalance(i)   "salt mass balance in the river"
coutdef              "salinity of water out of the whole region"
mixture(i)           "salinity of mixture of mainstream and tributary"

salinitybalance(i)$im1(i).. Q(i+1)*c(i+1) =e= Q(i)*c(i) + TQ(i)*Tc(i) ;
coutdef..          cout*Qout =e= Q('10')*c('10') + TQ('10')*Tc('10');
mixture(i)..       (TQ(i)+Q(i))*cz(i) =e= Q(i)*c(i)+TQ(i)*Tc(i);

c.fx=c0;
cz.l(i)=1.0;
```

(3)Make adaptions for the following code:

```
Scalars    Q0  "initial quantity of water into the river" /1055.545/;


et(i,j,k)..                e(i,j,k)=e=emax(j,k)/(1+beta1(j,k)*
                                   (cz(i)+beta2(j,k)*w(i,j,k)
                                   **beta3(j,k))**beta4(j,k));
waterconst(i)..         WQtot(i)=l=Q(i)+TQ(i);
waterbalance(i)$im1(i)..TQ(i)+Q(i)-WQtot(i)+RQtot(i)=e=Q(i+1);
```

```
loop(i$im1(i),Q.l(i+1)=TQ(i)+Q.l(i)-WQtot.l(i)+RQtot.l(i));

Model efficiencysaline /all/;
Solve efficiencysaline using nlp maximizing totalprofit;
```