# **Lawrence Berkeley National Laboratory**

**Lawrence Berkeley National Laboratory** 

#### **Title**

Design of RF Conditioner Cavities

#### **Permalink**

https://escholarship.org/uc/item/6m92c0ck

#### **Author**

Govil, R.

#### **Publication Date**

2008-09-18



# Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

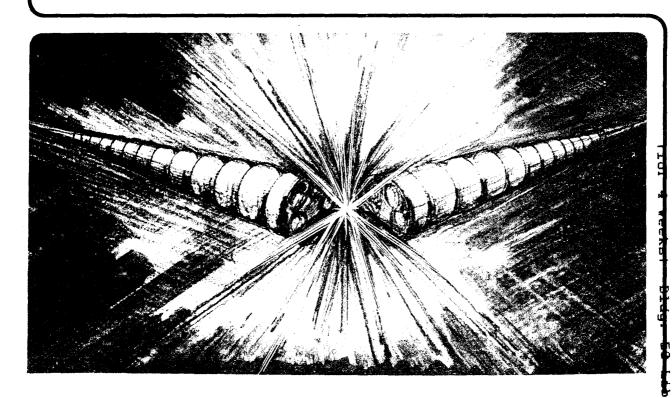
# Accelerator & Fusion Research Division

Presented at the Fourteenth International Free Electron Laser Conference, Kobe, Japan, August 23–28, 1992, and to be published in the Proceedings

## **Design of RF Conditioner Cavities**

R. Govil, R.A. Rimmer, A. Sessler, and H.G. Kirk

June 1992



| LOAN COPY

LBL-32232

# Design of RF Conditioner Cavities

#### R. Govil, R.A. Rimmer and A. Sessler\*

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

#### H.G. Kirk\*\*

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

June 1992

<sup>\*</sup>Work supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098

<sup>\*\*</sup> Work supported by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016

### Design of RF Conditioner Cavities

R. Govil, R. A. Rimmer and A. M. Sessler

Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720\*

H. G. Kirk

Brookhaven National Laboratory, Upton, NY 11973\*\*

June 1992

Theoretical studies are made of radio frequency structures which can be used to condition electron beams so as to greatly reduce the stringent emittance requirements for successful lasing in a free-electron laser. The basic strategy of conditioning calls for modulating an electron beam in the transverse dimension, by a periodic focusing channel, while it traverses a series of rf cavities, each operating in a TM<sub>210</sub> mode. In this paper, we analyze the cavities both analytically and numerically (using MAFIA simulations). We find that when cylindrical symmetry is broken the coupling impedance can be greatly enhanced. We present results showing various performance characteristics as a function of cavity parameters, as well as possible designs for conditioning cavities.

<sup>\*</sup> Work supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

<sup>\*\*</sup> Work supported by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

#### 1. Introduction

The rf conditioning of electron beams for free-electron lasers (FEL) has been discussed in recent papers [1,2]. In the proposed device, an electron beam is sent down a focusing channel, which provides an oscillation in the transverse beam radius while the beam traverses suitably positioned, and properly phased, rf cavities operating in the TM<sub>210</sub> mode. The result is the introduction of a correlation between electron amplitude of oscillation and energy such that there is very little spread in the longitudinal velocity of electrons. The FEL performance is thereby greatly enhanced.

For rf cavities operated in a  $TM_{210}$  mode, the accelerating field varies as  $r^2$  for small radius r. Such a variation is just what is desired in a conditioner, but because the electron beam can have a very small emittance, the coupling between the beam and the rf field of the conditioner cavity can be very small. Increasing this coupling, "making the shunt impedance better" for this mode, is the subject of this paper. A method for doing this is mentioned in a footnote in Ref.1, but not explored in that communication.

In Section 2 we give formulas for a right-circular cylindrical cavity. This case, which can be treated analytically, provides a "base line" for comparison of other (more complicated) structures. We compare the analytic results with numerical calculations (using MAFIA [3]). Then, in Section 3, we study the effect of indentations to the end-walls of a cavity through MAFIA simulations. Cylindrically symmetric (or ring-shaped) indentations do not provide a significant improvement in the shunt impedance. In Section 4 we break the 2D symmetry and consider 3D structures.

#### 2. RF Modes in Right-Circular Cylindrical Cavities

A right-circular cylindrical cavity is one of the very few cases for which the wave equation can be solved analytically. We consider such a cavity with radius R and length L oriented along the z-axis. For Transverse Magnetic (TM) modes, the electric and magnetic fields in the cavity can be derived from the axial electric field  $E_z$ ,

$$E_{t}(r,\varphi) = E_{0}J_{m}(k_{t}r)\cos m\varphi, \qquad (1)$$

where  $J_m$  are the first order Bessel functions,  $k_i = k_{mn} = \frac{x_{mn}}{R}$  is the transverse wave number, and  $x_{mn}$  are roots of the Bessel functions  $J_m$ . The TM modes are denoted by three subscripts, the first of which denotes the number of full periods in  $\varphi$ , the second denotes the number of radial zeros and the third denotes the number of half-period variations in the z coordinate. For the TM<sub>210</sub> mode,  $x_{2l} = 5.136$ ,  $k_{2l} = \frac{x_{2l}}{R}$  and  $k_z = 0$  and the non-zero field components are,

$$E_{z}(r,\varphi,z,t) = E_{0}J_{2}(k_{2l}r)\cos 2\varphi \ e^{-i\omega t} ,$$

$$B_{r}(r,\varphi,z,t) = i\frac{\omega}{c^{2}k_{2l}^{2}}E_{0}\frac{J_{2}(k_{2l}r)}{r}\sin 2\varphi \ e^{-i\omega t} ,$$

$$E_{\varphi}(r,\varphi,z,t) = -i\frac{\omega}{c^{2}k_{2l}}E_{0}J_{2}'(k_{2l}r)\cos 2\varphi \ e^{-i\omega t} ,$$
(2)

where  $\omega = c\sqrt{k_{2l}^2 + k_z^2}$  is the mode frequency, c the speed of light, and  $k_z$  the wave number for variations in the z-direction. The Bessel function  $J_2(x)$  varies as  $x^2/4$  for small x and is zero on axis. Since the electron beam is almost on axis, the coupling with the fields is very small.

The energy given to a particle moving parallel to the axis at a radius  $r_0 \ll R$  is,

$$\Delta W = e \int_0^L E(z) e^{i\alpha z/c} dz = e E_0 \frac{x_{2l}^2}{4} \left(\frac{r_0}{R}\right)^2 L \left[\frac{\sin(\omega L/2c)}{\omega L/2c}\right]. \tag{3}$$

The power lost at all the surfaces in the cavity is given by

$$P_{\text{loss}} = \frac{\pi \delta}{4\mu c} E_0^2 J_I^2(x_{2I}) x_{2I} R[I + L/R] , \qquad (4)$$

where  $\delta = \sqrt{\frac{2}{\mu\omega\sigma}}$  is the skin depth and  $\sigma$  the conductivity of the walls of the cavity. The shunt impedance is defined as,  $Z = (\Delta W)^2/P_{loss}$ .

#### 3. Cylindrically Symmetric Indentations

Using MAFIA simulation we examine the simple case of a cylindrically symmetric ring-shaped indentation with width  $2\rho$  and depth d, placed at radius  $r_c$  with its center at the cavity axis (Figure 1). All results for various models are obtained for an operating frequency of 2856 MHz by varying the radius to compensate for any changes in frequency resulting from modifications to the base case design (a right-circular cylindrical cavity). On studying ring-shaped indentations of varying sizes placed at different positions, we find that the shunt impedance is either only slightly higher or, in some cases, even lower than the base case. Therefore, cylindrically symmetric indentations do not provide any significant benefits in the form of increased shunt impedance.

#### 4. Three-dimensional Models

We now break the 2D symmetry of the ring-shaped indentations and study structures with four-fold symmetry which is appropriate for the TM<sub>210</sub> mode: four identical indentations are made at each end of a conditioner cavity. We vary the dimensions of the indentations, including their radial width and position, the longitudinal gap between them and the size of the beam aperture. In all cases the cavity radius and/or cavity length is adjusted to ensure that the resonant frequency remains at 2856 MHz. Two different classes of indentations are considered, both of which are adequate for the purpose.

We have verified that, up to approximately half the aperture radius, the longitudinal electric field in a conditioner cavity with indentations has a quadratic dependency. The indentations serve a function analogous to pole pieces in a static magnet, pulling the electric field in toward the center of the cavity, thus enhancing the shunt impedance. In Figure 2, we compare the structure of the longitudinal electric field for the two cases of cavities with and without wedge-shaped indentations placed about the aperture.

#### A. Muffin-tins

In this section, the effect of four circular "muffin-tin" shaped indentations to each end-wall of the rf cavities is examined. The centers of the muffin tins, which have radius  $\rho$ , are placed a distance  $r_c$  from the axis of the cylinder (Figures 1 and 3).

The shunt impedance varies with the offset of the muffin-tins from the axis, namely  $r_c$ . If the muffin-tins are placed adjacent to the beam port, the impedance is higher by a factor of about 20. As the muffin-tins

are moved away from the beam port, keeping all other muffin-tin parameters fixed, the impedance rises and then falls. Table 1 shows that the shunt impedance peaks around  $r_c$  /R = 20% for a muffin radius of 1.5 cm and gap of 5 mm. For most designs, the power loss at the cavity surface at radius R is negligible.

Another parameter of interest is the size of the muffin-tins, namely the radius  $\rho$ . There are two ways of varying the radius: holding  $r_c$  fixed or keeping  $r_c$  - $\rho$  constant. We start with the base case, which has  $\rho$ =0, and increase the radius. When  $r_c$  - $\rho$  is held fixed, the frequency, power loss and energy gain vary in a more systematic manner (Table 1), suggesting that  $r_c$  - $\rho$  is a more appropriate parameter for study than  $r_c$ . For both cases, the impedance is maximum for  $\rho$ =0.75 cm, that is, 12.5% of the cavity radius.

The size of the beam port has negligible effect on the shunt impedance. This implies that the beam port can be made larger without adverse effect on the shunt impedance, thus reducing the transverse coupling impedance and ensuring that the conditioner will not cause transverse beam motion [4]. The transverse wake fields in conditioner cavities with indentations are not any stronger than in right-circular cylindrical cavities.

Impedance increases monotonically as we increase the depth of indentations while keeping the cavity length fixed at 3 cm. Field-breakdown constraints place a lower limit on the size of the gap. For an indentation length of 13.5 mm and cavity length of 30 mm, Table 2 gives the optimum size and position of muffin-tins which give the highest increase in impedance: muffin-tins with a radius which is one-eighth of the outer radius should be placed one-quarter of the way from the axis.

However, if we allow the length of the rf cavity to vary, further increase in shunt impedance can be obtained by lengthening the indentations further. As the depth of indentations, d, approaches quarter wavelength (2.63 cm), we can achieve impedance as high as a factor of 500 over the right-circular cylindrical cavity. The wedge-shaped indentations, which are studied in teh next sub-section, carry this concept even further.

#### B. Wedge-shaped Indentations

We now explore conditioner cavities with wedges placed about the aperture (Figure 4). In general, we find that the smaller apertures yield the best shunt impedance. This is shown most clearly by an examination of Table 3, where the impedance increases while the cavity power loss (normalized by a factor of four to account for the four-fold symmetry) decreases for the cases with smaller cavity apertures.

Using the time-domain component of the MAFIA package, T3, we have examined the wakefield characteristics of the wedge indentations and muffin-tin cavities. In general, we find that the principal variable driving the appearance of transverse wake fields is the aperture of the cavities. The wedge configurations show slightly higher transverse fields than the corresponding base case, while the muffin-tin configurations, with shorter indentation lengths, show no perceptible difference from their base case.

#### 5. Conclusions

We have explored designs for conditioner cavities which will render electron beams suitable for subsequent lasing in a free-electron laser. We find that it is important to break the cylindrical symmetry of the rf cavities in order to obtain structures which will deliver significantly higher shunt impedance than the standard cylindrical cavity.

For similar apertures, the wedge configuration delivers better shunt impedance than the muffin-tin configuration. Significantly higher shunt impedance can be obtained with the wedge indentations by decreasing the cavity aperture. This will result, however, in higher transverse wake fields. If transverse wake fields are an issue, then one may find a better solution with the muffin-tin approach with shorter indentations, where transverse wakes can be reduced by opening the apertures with little effect on the shunt impedance.

#### 6. Acknowledgments

The authors thank Glen Lambertson for his helpful suggestions and comments.

#### References

- 1. A. M. Sessler, D. H. Whittum, and L-H. Yu, Phys. Rev. Lett. 68, 309 (1992).
- 2. L-H. Yu, A. M. Sessler, and D. H. Whittum, "Free-Electron Laser Generation of VUV and X-Ray Radiation Using a Conditioned Beam and Ion-Channel Focusing", FEL '91, Nuclear Instr. and Methods in Phys. Research, A318, 721 (1992).
- 3. "Reports at the 1986 Stanford Linac Conference, Stanford, USA, June 2-6 1986," DESY M-86-07, June 86.
- 4. R. H. Helm and G. A. Loew, "Beam Breakup", Linear Accelerators, P. M. Lapostolle and A. L. Septier, ed., Amsterdam: North Holland Pub. Co., 1970; K. L. F. Bane, "Wakefield Effects in a Linear Collider", AIP Conf. Proc. No. 153, 971 (1987).

#### Tables

- Table 1. MAFIA results and analysis for various conditioner cavity designs with muffin-tin indentations. Cavity radius has been varied to hold the operating frequency fixed at 2856 MHz. The energy gain is calculated for an electron 2 mm away from the axis.
- Table 2. Optimum values of muffin-tin parameters obtained through MAFIA for cavity length fixed at 3 cm.
- Table 3. MAFIA results and analysis for various conditioner cavity designs with wedge-shaped indentations. Cavity radius and length have been varied to hold the operating frequency fixed at 2856 MHz. The energy gain is calculated for an electron 2 mm away from the axis.

#### Figures

- Figure 1. Side-view of a typical conditioner cavity with indentations on either side.
- Figure 2. The longitudinal component of the electric field in (a) the base case right cylindrically symmetric cavity and, (b) a cavity with wedge indentations. The beam travels parallel to the z-axis.
- Figure 3. Cross-sectional view of a typical conditioner cavity with four muffin-tin indentations and a beam aperture.
- Figure 4. Cross-sectional views of a typical conditioner cavity with wedgeshaped indentations and a beam aperture.

Table 1

Cavity Parameters					Peak E field fixed				
Cavity Parameters  R rho r_c a L			d	Power Loss	Voltage	Impedance Z			
1	rho	r_c		cm	cm	10^-10 W	10^-5 V	ohms	ratio
cm	cm_	cm	cm	CIII	Ciri	10 10 11			
8.59	_	_	0.5	3.00	_	7.13	12.3	21.2	1
0.59	_		•	0.00					
MUFFIN DISPLACEMENT r_c									
8.63	0.75	1.25	0.5	3.00	1.25	2.50	35.2	495.6	23
8.60	0.75	1.35	0.5	3.00	1.25	1.45	26.2	473.4	22
8.50	0.75	1.40	0.5	3.00	1.25	1.34	26.6	529.6	25
8.41	0.75	1.45	0.5	3.00	1.25	1.20	21.5	385.2	18
8.21	0.75	1.50	0.5	3.00	1.25	0.92	22.0	525.2	25
8.00	0.75	1.55	0.5	3.00	1.25	0.47	21.5	977.3	46
7.20	0.75	1.65	0.5	3.00	1.25	0.39	17.2	760.5	36
5.50	0.75	1.75	0.5	3.00	1.25	0.35	14.1	569.7	27
						<u>.</u>			
MUFFIN RADIUS rho ( CONSTANT r_c )							40.0	04.0	4
8.59	0	1.55	0.5	3.00	1.30	7.13	12.3	21.2	1
8.00	0.45	1.55	0.5	3.00	1.30	0.19	9.9	514.8	24
5.30	0.65	1.55	0.5	3.00	1.30	0.29	16.1	893.8	42
6.00	0.75	1.55	0.5	3.00	1.30	0.36	20.7	1177.2	55
6.50	0.85	1.55	0.5	3.00	1.30	0.50	21.1	883.4	42
8.00	0.95	1.55	0.5	3.00	1.30	0.77	24.6	781.9	37
MUFFIN RAI					4.00	7 40	12.3	21.2	1
8.59	0	0.80	0.5	3.00	1.30	7.13 0.57	17.5	541.0	25
8.50	0.45	1.25	0.5	3.00	1.30	1		1126.7	53
8.00	0.65	1.45	0.5	3.00	1.30	0.37	20.5	j.	55 55
6.00	0.75	1.55	0.5	3.00	1.30	0.36	20.7	1177.2 952.1	45
4.24	0.85	1.65	0.5	3.00	1.30	0.45	20.8	952.1	43
HALF APERT	URFa								
6.00	0.75	1.55	0.5	3.00	1.30	0.36	20.7	1177.2	55
6.00	0.75	1.55	0.6	3.00	1.30	0.36	20.7	1183.7	56
6.00	0.75	1.55	0.7	3.00	1.30	0.36	20.6	1175.5	55
6.00	0.75	1.55	0.8	3.00	1.30	0.36	20.6	1175.5	55
0.00	• • • • • • • • • • • • • • • • • • • •								
	INDENTATION LENGTH d ( CONSTANT L )						. <b>.</b>		
8.59	0.75	1.55	0.5	3.00	0.00	7.13	12.3	21.2	1
8.58	0.75	1.55	0.5	3.00	0.50	7.37	18.9	48.5	2
8.50	0.75	1.55	0.5	3.00	1.15	1.01	22.0	480.1	23
8.00	0.75	1.55	0.5	3.00	1.25	0.47	21.5	977.3	46
6.00	0.75	1.55	0.5	3.00	1.30	0.36	20.7	1177.2	55
INIDENITATIO	INDENITATION ( PAIGTIL & CONICTANT ~ )								
INDENTATION LENGTH d ( CONSTANT g ) 8.50 0.35 0.85 0.3 3.00 1.35				5.24	43.2	356.0	17		
8.50	0.35	0.85	0.3	3.20	1.45	3.92	44.6	506.8	24
8.09	0.35	0.85	0.3	3.60	1.65	0.20	43.4	9334.4	440
2.83	0.35	0.85	0.3	4.50	2.10	0.20	40.2	8024.0	378
2.00	0.00	0.00	0.0	<del></del>		<u> </u>		1	

Table 2

Cavity Parameters	Value			
Frequency	f	2856 MHz		
Cavity Length	L	3 cm		
Indent Depth	d	1.3 cm		
Cavity Radius	R	6.00 cm		
Muffin Diameter	ρ	0.75 cm		
Muffin Position	r_c	1.55 cm		
Aperture Radius	а	0.5 - 1.0 cm		
Impedance	Z	50 times base case		

Table 3

Cavity Parameters in cm						Peak E		
Aperture Radius a	Cavity Radius R	Cavity Length L	Pole Length d	Gap Length g	Pole Width w	Power Loss E 10^-10 W	Energy Gain 10^-5 eV	Improvement in Impedance over base case
1.00	8.505	3.25	-	-	-	7.480	9.9	1
1.00	5.320	4.50	1.25	1.00	1.0	0.548	23.7	78
1.00	4.985	3.25	1.13	0.50	1.0	0.376	17.1	59
0.50	3.600	5.30	1.65	1.00	1.0	0.408	65.2	795
0.50	3.300	5.60	1.80	1.00	0.5	0.352	74.4	1200
0.50	5.515	3.25	1.13	0.50	2.0	0.916	47.2	186
0.50	3.900	4.90	1.95	0.50	0.5	0.208	49.9	914
0.25	2.950	5.10	2.05	0.50	0.5	0.328	138.0	4431

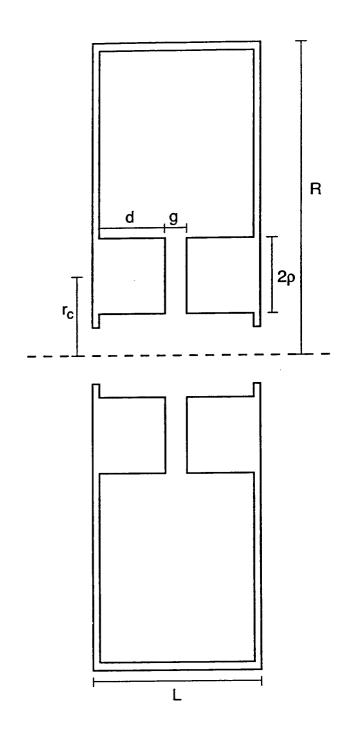


Fig. 1

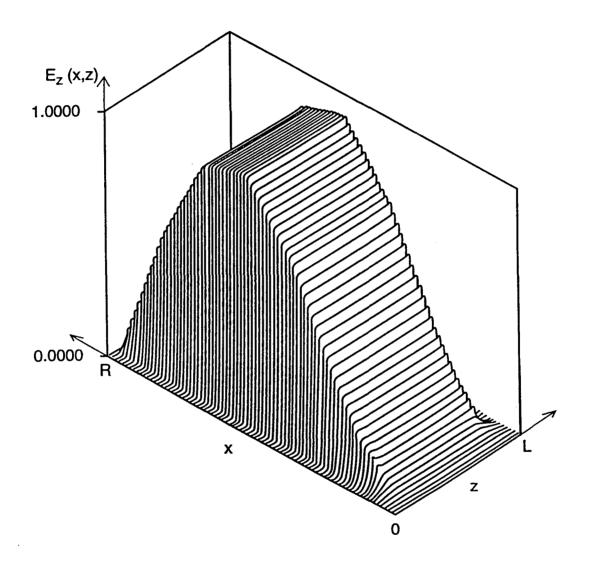


Fig. 2a

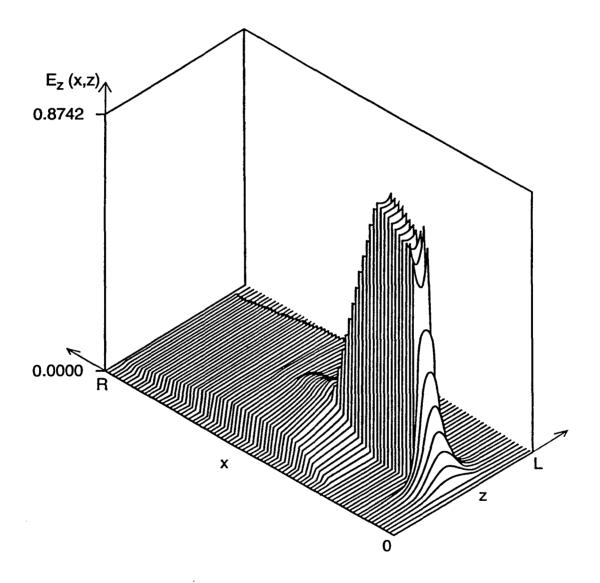
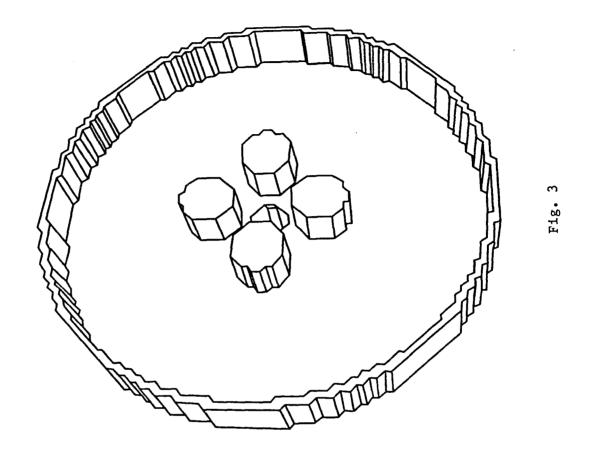


Fig. 2b



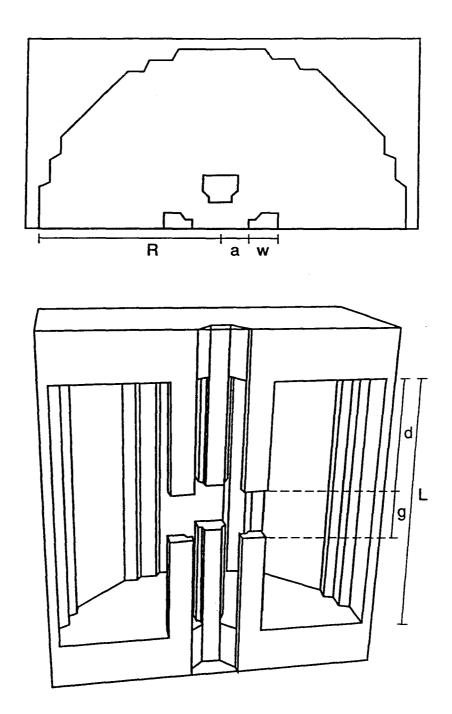


Fig. 4