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Authors

Chen, Mu-Chun

Ratz, Michael

Takhistov, Volodymyr

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# $R$ parity violation from discrete $R$ symmetries

Mu-Chun Chen <sup>a</sup>, Michael Ratz <sup>b</sup>, Volodymyr Takhistov <sup>a,\*</sup>

<sup>a</sup> Department of Physics and Astronomy, University of California, Irvine, CA 92697-4575, USA

<sup>b</sup> Physik-Department T30, Technische Universität München, James-Frank-Strasse 1, 85748 Garching, Germany

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## Abstract

We consider supersymmetric extensions of the standard model in which the usual  $R$  or matter parity gets replaced by another  $R$  or non- $R$  discrete symmetry that explains the observed longevity of the nucleon and solves the  $\mu$  problem of MSSM. In order to identify suitable symmetries, we develop a novel method of deriving the maximal  $\mathbb{Z}_N^{(R)}$  symmetry that satisfies a given set of constraints. We identify  $R$  parity violating (RPV) and conserving models that are consistent with precision gauge unification and also comment on their compatibility with a unified gauge symmetry such as the Pati–Salam group. Finally, we provide a counter-example to the statement found in the recent literature that the lepton number violating RPV scenarios must have  $\mu$  term and the bilinear  $\kappa LH_u$  operator of comparable magnitude.

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## 1. Introduction

Low-energy supersymmetry is still one of the most attractive schemes for physics beyond the standard model (SM). One of the striking features of supersymmetry is that it leads to precision gauge unification in the minimal supersymmetric extension of the SM, the MSSM. Supersymmetry allows for the stabilization of the electroweak scale against the grand unification scale,  $M_{\text{GUT}}$ , where the gauge couplings unify. The non-observation of the superpartners so far at the

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\* Corresponding author.

E-mail addresses: [muchunc@uci.edu](mailto:muchunc@uci.edu) (M.-C. Chen), [michael.ratz@tum.de](mailto:michael.ratz@tum.de) (M. Ratz), [vtakhist@uci.edu](mailto:vtakhist@uci.edu) (V. Takhistov).

Large Hadron Collider (LHC) [1,2] has placed significant constraints on the minimal supersymmetric models that have been generally considered.  $R$  parity violation (RPV) turns out to be an interesting alternative [3–8] to consider beyond the minimal models, since it may explain why the superpartners have not been seen at the LHC (see e.g. [9] for a recent analysis). On the other hand, in the presence of  $R$  parity violation, one should explain non-observation of nucleon decay thus far [10]. To achieve these two goals simultaneously requires, naturally, additional symmetries, with discrete symmetries being a plausible option. Alternatives include invoking minimal flavor violation [11,12].

$R$  symmetries play a special role in this context, since the order parameter for  $R$  symmetry breaking is the gravitino mass  $m_{3/2}$ .<sup>1</sup> Thus, without having to go into details of supersymmetry breaking, it is possible to estimate the amount by which discrete  $R$  symmetries are broken. In turn, this enables one to make statements about the coefficients of the effective operators that arise through  $R$  symmetry breaking. Such effective operators will be suppressed by powers of the ratio of gravitino mass and the fundamental scale,  $m_{3/2}/\Lambda$ . In general, (discrete)  $R$  symmetries are broken by the vacuum expectation value (VEV) of some “hidden sector” superpotential, which carries  $R$  charge  $2q_\theta$ , where  $q_\theta$  denotes the  $R$  charge of the superspace coordinate  $\theta$ , and possibly by further operators. This allows for the possibility of residual non- $R$   $\mathbb{Z}_M$  symmetries, in particular for  $q_\theta > 1$  [14]. In light of arguments from quantum gravity [15], we will focus on gauged discrete symmetries.

In a recent analysis, Dreiner, Hanussek and Luhn (DHL) [16] analyzed discrete  $R$  symmetries of the type described above. In their work, the  $R$  charge of the superspace coordinate  $\theta$  was restricted to 1. Further, DHL [16] allowed for the Green–Schwarz (GS) mechanism [17] to cancel the anomalies, and required that the couplings of the GS axion  $a$  to the various field strengths of the (MS)SM be universal. On the other hand, as pointed out in [18], universality of the anomaly coefficients is, strictly speaking, not a consistency condition, even though one may impose it in order not to spoil precision gauge unification [13]. Precision gauge unification may also be preserved, for example, if the scalar partner of the axion  $a$  has an expectation value that is much smaller than the axion decay constant  $f_a$ , or by an accidental cancellation of unrelated effects [19].

The purpose of this work is to complete and to extend the analysis of DHL [16] by

- allowing the superspace coordinate  $\theta$  to have an  $R$  charge that differs from 1;
- allowing for a GS cancellation of discrete anomalies with non-universal couplings of  $a$ ;
- identifying redundant symmetries in DHL [16];
- presenting a novel method allowing one to systematically identify the maximal symmetry compatible with given selection criteria.

Moreover, we will also comment on  $R$  parity conserving scenarios.

In our analysis, we consider both  $R$  and non- $R$  Abelian discrete symmetries, and impose that

1. The nucleon is sufficiently long-lived, i.e. that the dangerous operators are either forbidden by a residual  $\mathbb{Z}_M$  symmetry or sufficiently suppressed by appropriate powers of  $m_{3/2}/\Lambda$ .

<sup>1</sup> Recall that  $R$  parity is actually not an  $R$  symmetry. Rather, it is equivalent to matter parity (see e.g. [13] for a discussion).

Here,  $\Lambda$  is the cutoff scale which we take to be the Planck scale  $M_P$  unless stated otherwise. We will also discuss settings with a lower cutoff scale.

2. The discrete symmetry forbids the  $\mu$  term at the perturbative level.

Further, we demonstrate additional features that were absent from DHL [16] including

- the compatibility of charges with (partial) unification, specifically whether the matter charges commute with the Pati–Salam group  $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R^2$ ;
- a natural suppression of the neutrino masses either through the Weinberg operator or from supersymmetry breaking, thus yielding light Dirac neutrinos.

This paper is organized as follows. In Section 2, we present a novel method for classifying discrete symmetries. We comment on anomaly cancellation, provide a recipe for identifying and eliminating equivalent symmetries, and comment on the limitations of our analysis. In Section 3, we illustrate our methods by presenting models obtained for anomaly-universal as well as non-universal scenarios while considering both  $R$  parity violation and conservation. Section 4 contains our conclusions.

## 2. Classification

### 2.1. Goals of our classification

In the MSSM, the renormalizable superpotential terms consistent with the SM gauge symmetry are

$$\begin{aligned} \mathcal{W}_{\text{ren}} = & \mu H_u H_d + Y_{fg}^u Q_f \bar{U}_g H_u + Y_{fg}^d Q_f \bar{D}_g H_d + Y_{fg}^e L_f \bar{E}_g H_d \\ & + \kappa^f L_f H_u + \lambda^{fgh} L_f L_g \bar{E}_h + \lambda'^{fgh} L_f Q_g \bar{D}_h + \lambda''^{fgh} \bar{U}_f \bar{D}_g \bar{D}_h, \end{aligned} \quad (2.1)$$

where the first line denotes the usual couplings of the MSSM, while the second line contains the so-called  $R$  parity violating terms. In what follows, we will suppress the flavor indices  $f$ ,  $g$  and  $h$ . We will further assume that there is no flavor dependence of the discrete charges, i.e.  $q_Q^f = q_Q$  for all  $f$  and so on.

At the non-renormalizable level, additional  $B$  and  $L$  violating operators need to be considered (cf. e.g. [16,21–23])

$$\begin{aligned} \mathcal{O}_1 &= [QQQL]_F, & \mathcal{O}_2 &= [\bar{U}\bar{U}\bar{D}\bar{E}]_F, \\ \mathcal{O}_3 &= [QQQH_d]_F, & \mathcal{O}_4 &= [Q\bar{U}\bar{E}H_d]_F, \\ \mathcal{O}_5 &= [LH_uLH_u]_F, & \mathcal{O}_6 &= [LH_uH_dH_u]_F, \\ \mathcal{O}_7 &= [\bar{U}\bar{D}^\dagger\bar{E}]_D, & \mathcal{O}_8 &= [H_u^\dagger H_d\bar{E}]_D, \\ \mathcal{O}_9 &= [Q\bar{U}L^\dagger]_D, & \mathcal{O}_{10} &= [QQ\bar{D}^\dagger]_D, \end{aligned} \quad (2.2)$$

as well as operators of even higher dimensions.

<sup>2</sup> We do not consider compatibility of matter charges with  $SU(5)$  or  $SO(10)$  in the case of RPV. This is because, if  $\bar{U}\bar{D}\bar{D}$  is allowed, so is automatically  $LL\bar{E}$ , and vice versa. See [20] for a discussion of  $R$  parity violation in settings with GUT relations.

We will discuss settings with renormalizable baryon number violation ( $\mathcal{B}$ ), renormalizable lepton number violation ( $\mathcal{L}$ ) as well as “non-perturbative”  $B$  and  $L$  violation, which appears only after the “hidden sector” superpotential acquires its VEV. We will further comment on settings with  $R$  parity conservation. To constrain overly rapid proton decay, renormalizable  $\mathcal{B}$  operators must be forbidden in the case of the RPV setting with renormalizable  $\mathcal{L}$ , and vice versa for the RPV setting with renormalizable  $\mathcal{B}$ . Since not all of the above higher-dimensional operators shown in Eq. (2.2) are independent (see [16,21,23]), only a subset of such terms need to be considered to account for all the phenomenological constraints. In RPV setups with either renormalizable  $\mathcal{B}$  or renormalizable  $\mathcal{L}$ , one needs only to examine the existence condition for the Weinberg operator  $\mathcal{O}_5$  [24] for neutrino mass generation.

We consider different classes of models based on Abelian discrete  $R$  or non- $R$  symmetries,  $\mathbb{Z}_N^{(R)}$ , with properties specified below. We distinguish between  $\mathbb{Z}_N^{(R)}$  symmetries that are anomaly-free in the traditional sense and symmetries in which the anomalies are canceled by a non-trivial (discrete) Green–Schwarz (GS) mechanism [17]. In the second case, we discriminate between universal and non-universal couplings of the GS axion to the various field strengths of the standard model gauge group  $G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ .

To sum up, we search for, both in the anomaly-universal case and in the anomaly non-universal case, classes of models that have the following respective properties:

- (1)  $R$  parity conservation;
- (2) renormalizable RPV with  $\mathcal{L}$  and the existence of  $\mathcal{O}_5$  at the perturbative level;
- (3) renormalizable RPV with  $\mathcal{B}$  and the existence of  $\mathcal{O}_5$  at the perturbative level;
- (4) “non-perturbative”  $\mathcal{L}$  and  $\mathcal{B}$ .

## 2.2. Equivalent discrete symmetries

In order to avoid an unnecessary double-counting of symmetry patterns, we provide a recipe that allows to identify and eliminate equivalent symmetries.<sup>3</sup> This can be achieved by avoiding the following redundancies in the definition of the discrete  $\mathbb{Z}_N^{(R)}$  charges:

*Common divisors:* If the order  $N$  and all charges have a common divisor  $M$ , then the  $\mathbb{Z}_N^{(R)}$  is equivalent to a  $\mathbb{Z}_{N/M}^{(R)}$  with all charges divided by  $M$ .

*Non-trivial centers:* In the presence of an  $\text{SU}(M)$  gauge factor, acting with the center of  $\text{SU}(M)$ ,  $Z_{\text{SU}(M)} \simeq \mathbb{Z}_M$ , is always a symmetry. Thus, in the context of the standard model gauge symmetry, we can apply

- The non-trivial elements of the center  $Z_{\text{SU}(3)_C} \simeq \mathbb{Z}_3$ , which acts as  $\text{diag}(\omega, \omega, \omega)$  with  $\omega = e^{2\pi i/3}$  or  $e^{4\pi i/3}$  on the triplets. Hence, if 3 divides the order  $N$ , i.e. if  $N = 3 \cdot N'$ , with  $N' \in \mathbb{N}$ , this allows us to shift the charges according to  $(q_Q, q_{\bar{U}}, q_{\bar{D}}) \rightarrow (q_Q + N' \cdot v, q_{\bar{U}} - N' \cdot v, q_{\bar{D}} - N' \cdot v)$  with  $v \in \mathbb{Z}$ .
- The non-trivial center of  $\text{SU}(2)$ ,  $Z_{\text{SU}(2)} \simeq \mathbb{Z}_2$ . That is, equivalent charges are obtained by multiplying all doublets by  $-1$ , or adding  $N/2$  to the doublet charges, if the order  $N$  is even.

<sup>3</sup> One possible, “brute force” way of doing this consists of comparing the Hilbert bases for the Kähler potential and superpotential of the two candidate symmetries. However, this turns out to be often impractical.

We note that in the settings compatible with SU(5) unification there is a “fake”  $\mathbb{Z}_5$  which is nothing but the non-trivial center of SU(5) [25,26].

*Hypercharge shift:* One may add integral multiples of hypercharge (normalized to integer charges), arriving at an equivalent charge assignment. The freedom of performing the hypercharge shift and modding out the non-trivial centers of SU(3)<sub>C</sub> and SU(2)<sub>L</sub> are not independent.

*Coprime factors:* Multiplying all charges by a factor  $f$  that is coprime to the order  $N$ , i.e.  $\gcd(f, N) = 1$ , leads to the same symmetry. Based on this, one can show that all non- $R$  symmetries of a given order in Tables 2 and 3 in DHL [16] are equivalent. That is, rather than having 2, 3, 2, 3 or 2 non- $R$  symmetries for orders 5, 7, 8, 9 or 10, respectively, there is only one non- $R$  symmetry for each order in Table 2. Similar statements apply to the non- $R$  symmetries of Table 3. We list the truly inequivalent symmetries in Table D.1 and Table D.2.

The last statement also implies that for  $R$  symmetries of prime order, one only needs to consider the cases  $q_\theta = 0$  (which corresponds to a non- $R$  symmetry) or  $q_\theta = 1$ . This follows from the fact that the linear congruence

$$q_\theta \cdot f = 1 \pmod{N} \quad (2.3)$$

has, according to the discussion in Appendix A, a non-trivial solution with a non-trivial  $f$  that is coprime to  $N$ . More generally, for a given order  $N$ , one has to scan only the values of  $q_\theta$  that divide  $N$ , since any other  $q_\theta \neq 0$  can be rescaled to 1. Further, the case  $q_\theta = N/2$  for even  $N$  should not be considered. This is because the transformation under which  $\theta \rightarrow -\theta$  and all fermions being mapped to minus themselves is a symmetry of any supersymmetric theory. Consequently, imposing such a transformation as a symmetry does not forbid any couplings.

### 2.3. Systematic search for $\mathbb{Z}_N^{(R)}$ symmetries

Very often in model building one encounters the situation in which one wishes to forbid certain operators, such as some of the  $\mathcal{O}_i$  in (2.2), by an appropriate discrete symmetry. In most approaches, the desired symmetries and charges were found by a scan. In what follows, we will discuss a method to systematically construct  $\mathbb{Z}_N$  symmetries which allow for certain desired operators and forbid other undesired operators.

Suppose we have  $n_c$  constraints, which correspond to  $n_c$  conditions of the type

$$\sum_{j=1}^{n_q} a_{ij} q_j = 0 \pmod{N} \quad \forall 1 \leq i \leq n_c \quad (2.4)$$

on the  $n_q$  charges  $q_j$ . Here we concentrate first on the case with constraints of the equality form. Constraints in the form of inequalities will be discussed later. There are two questions to be addressed: whether the conditions can be solved in a non-trivial way, and, if continuous symmetries are not available, what is the maximal meaningful  $\mathbb{Z}_N$  symmetry that one can impose that fulfills the constraints. As we shall see, using the Smith normal form, which has been shown [26,27] to be an important tool in other applications of discrete symmetries in physics, one can find the maximal order  $N$  of the corresponding meaningful symmetry.

Let us start by clarifying what we mean by a “meaningful” symmetry. Consider a field  $\phi$  transforming under a  $\mathbb{Z}_N$  symmetry with  $\mathbb{Z}_N$  charge  $q$ , i.e.

$$\phi \xrightarrow{\mathbb{Z}_N} e^{2\pi i q/N} \phi. \tag{2.5}$$

The task is now to find a “meaningful” order  $N$  and charge  $q$  in the one-dimensional version of (2.4), i.e. in the constraint equation

$$a \cdot q = 0 \pmod N. \tag{2.6}$$

We may rephrase this as the problem of finding the maximal meaningful symmetry  $\mathbb{Z}_N$  and charge  $q$ , such that the operator  $\phi^a$  is  $\mathbb{Z}_N$  invariant. The order  $N$  is a priori unknown. However, it is evident that  $N = a$  with  $q = 1$ . If we were to choose  $N < a$ , then for any integer  $q$  which satisfies (2.6), there would be a power  $a'$  such that  $\phi^{a'}$  is  $\mathbb{Z}_N$  invariant. That is, in addition to the operator  $\phi^a$ , there will be additional operator(s)  $\phi^{a'}$  with  $a' < a$  allowed by the  $\mathbb{Z}_N$  symmetry. Hence, we should have started from (2.6) with  $a$  replaced by  $a'$ . On the other hand, choosing  $N > a$  does not add anything useful, but will require solutions to have  $q \neq 1$ . That is, we would have a redundancy and not a “meaningful” symmetry.

Let us now look at a situation where there is another field with charge  $\tilde{q}$ , fulfilling

$$\tilde{a} \cdot \tilde{q} = 0 \pmod N. \tag{2.7}$$

Using analogous arguments as above, it is straightforward to convince oneself that the maximal meaningful order is then  $N = a \cdot \tilde{a} / \text{gcd}(a, \tilde{a})$ .

These statements are almost trivial and can be straightforwardly generalized to multiple conditions of the type of (2.6). A slightly more interesting situation arises when the constraints involve more than one charge at the same time, as in (2.4). The strategy of the subsequent discussion will be to transform these equations into constraints on linear combinations of charges which are all of the form (2.6).

Let us now discuss in detail how this works. We start out by considering equalities of the type

$$q_Q + q_{\bar{U}} + q_{H_u} - 2q_\theta = 0 \pmod N, \tag{2.8}$$

a condition for the  $u$ -type Yukawa coupling to be allowed. Apart from the charges, the order  $N$  of the discrete symmetry is, as before, unknown.

We can rewrite the conditions of this type as

$$A \cdot q = 0 \pmod N, \tag{2.9}$$

where  $q$  denote the vector of the  $n_q$  charges and  $A$  is an integer matrix. If  $A$  does not have full rank, then there is at least one  $U(1)$  which one can impose in order to satisfy the conditions. The  $U(1)$  charges are given by the entries of (one of) the vector(s) in the kernel of  $A$ . In this case, one can impose an arbitrary  $\mathbb{Z}_N$  which is a subgroup of such a  $U(1)$ . We therefore specialize here on the case where  $n_c \geq n_q$  and  $A$  has full rank, such that there is no  $U(1)$  which one may impose. Note that  $A$  is not necessarily a square matrix, i.e. we also allow for more constraints than variables,  $n_c > n_q$ .  $A$  can be brought to the so-called Smith normal form,<sup>4</sup>

$$U \cdot A \cdot V = D, \tag{2.10}$$

<sup>4</sup> A mathematica package to compute the Smith normal form for integer matrices can be found at <http://library.wolfram.com/infocenter/MathSource/6621/>.

where  $U$  and  $V$  are unimodular  $n_c \times n_c$  and  $n_q \times n_q$  matrices, respectively.<sup>5</sup>  $D$  is also an integer matrix and diagonal (but not necessarily square),

$$D = \begin{pmatrix} d_1 & & & \\ & \ddots & & \\ & & d_{n_q} & \\ \hline & & & 0_{(n_c-n_q) \times n_q} \end{pmatrix}, \tag{2.11}$$

and the diagonal elements satisfy  $d_i | d_{i+1}$ , i.e. the  $i$ th element divides the  $(i + 1)$ th element. It is also possible that  $d_{i \geq n_0} = 0$  for some  $n_0 \leq n_q$ . If  $D$  has maximal rank, and if it was not for the modulo  $N$ , then the matrix equation (2.9) has only the trivial solution. However, if we only ask the conditions to be fulfilled modulo  $N$ , then the last non-trivial element  $d_{n_q}$  defines the maximal order of a meaningful  $\mathbb{Z}_N$  symmetry, i.e.  $N = d_{n_q}$ , under which the conditions encoded by (2.9) can hold for non-trivial  $q$ . This can be seen by first noting that, for  $U$  and  $V$  being invertible, (2.9) is equivalent to

$$U \cdot A \cdot q = D \cdot V^{-1} \cdot q = 0 \pmod{N}. \tag{2.12}$$

This implies that there exists a linear combination of charges with integer coefficients that sum up to  $d_{n_q} = N$ . We see immediately that  $\text{rank}(D \pmod{N}) = \text{dim}(A) - 1$ .

The constraint equation is now brought to the diagonal form in the ‘‘charge basis’’  $q' = V^{-1} \cdot q$  that are linear combinations of  $q_i$ 's with integer coefficients,

$$V^{-1} \cdot q = \begin{pmatrix} k_1 \frac{d_{n_q}}{d_1} \\ \vdots \\ k_{n_q} \frac{d_{n_q}}{d_{n_q}} \end{pmatrix}. \tag{2.13}$$

The possibly inequivalent charges are thus given by

$$q = V \cdot \begin{pmatrix} k_1 \frac{d_{n_q}}{d_1} \\ \vdots \\ k_{n_q} \frac{d_{n_q}}{d_{n_q}} \end{pmatrix}. \tag{2.14}$$

If we shift the charges  $q_i$  by integral multiples of  $N$ , the r.h.s. of (2.13) will shift by integral multiples of  $\text{gcd}((V^{-1})_{i1}, \dots, (V^{-1})_{in_q}) \cdot N$  with  $N = d_{n_q}$ . Such shifts of the charges will obviously lead to the same  $\mathbb{Z}_N$  symmetry. However, since  $V^{-1}$  is unimodular,  $\text{gcd}((V^{-1})_{i1}, \dots, (V^{-1})_{in_q}) = 1$  for all  $i$ , and we can take  $k_i$  to lie only between 1 and  $d_i$ . We thus obtain the charges for the maximal meaningful symmetry  $\mathbb{Z}_{N=d_{n_q}}^{(R)}$  with the desired properties.

We note that there exist more symmetries that fulfill the conditions. Specifically, these additional symmetries can be obtained by dividing the order  $d_{n_q}$  by one of its divisors  $\delta_i$ . Then, (2.9) will still be fulfilled modulo  $N' = d_{n_q} / \delta_i$ . However, not all of these symmetries possess all of the properties that  $\mathbb{Z}_N$  might have.

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<sup>5</sup> Recall that unimodular matrices are integer matrices with determinant  $\pm 1$ . The inverses of such matrices are also integer.



If there are inequalities, such as

$$3q_Q + q_L - 2q_\theta = p \neq 0 \pmod N, \tag{2.15}$$

all one has to do is to add

$$3q_Q + q_L - 2q_\theta - p = 0 \pmod N \tag{2.16}$$

to the equation, regard  $p$  as an extra charge, and project on solutions which give  $p \neq 0 \pmod N$ . This will not lead to any new constraints. Therefore, one can just scan the symmetries obtained from the imposed equalities and explore the possible  $p$  values.

One can also determine the order  $M$  in the inhomogeneous equation

$$A \cdot q = b \pmod M \tag{2.17}$$

with some  $n_c$ -dimensional vector  $b$ . After bringing  $A$  to Smith normal form and multiplying Eq. (2.17) with  $U$  from the left, we obtain

$$D \cdot q' = b' \pmod M \tag{2.18}$$

with

$$b' = U \cdot b \quad \text{and} \quad q' = V^{-1} \cdot q. \tag{2.19}$$

If  $n_c > n_q$ ,  $b'$  can have non-trivial entries at the positions  $n_q + 1, \dots, n_c$ . Then, a solution is only possible if  $M$  divides  $b'_{n_q+1}, \dots, b'_{n_c}$ . Hence, we see that the maximal meaningful order may even be even more constrained for inhomogeneous equations. An application of our methods will be discussed in Section 3.4.3.

In conclusion, we have looked at symmetries that fulfill certain constraint equations. We have focused on systems in which the constraint equations do not allow for continuous or  $U(1)$  solutions. We have then shown that the maximal meaningful order of  $\mathbb{Z}_N$  symmetries compatible with the constraints can be read off from the Smith normal form (2.10) of the matrix encoding the constraint equations, and is given by the last diagonal element  $d_{n_q}$  (cf. Eq. (2.11)).

### 2.4. Anomaly (non-)universality

As mentioned, anomalies for discrete symmetries can be canceled by a discrete version of the Green–Schwarz (GS) mechanism [17]. This, however, may destroy the beautiful picture of the MSSM gauge coupling unification if the anomalies are not universal, i.e. if the GS axion couples with different coefficients to the various field strength terms of the SM gauge group factors.

We start out by discussing anomaly (non-)universality. For a  $U(1)_{(R)}$  symmetry, the relevant anomaly coefficients are

$$\begin{aligned} A_3 &= \frac{1}{2} \sum_f [2q_Q^f + q_{\bar{U}}^f + q_D^f - 4q_\theta] + 3q_\theta \\ &= \frac{3}{2} [2q_Q + q_{\bar{U}} + q_D] - 3q_\theta, \end{aligned} \tag{2.20a}$$

$$\begin{aligned} A_2 &= \frac{1}{2} \left[ q_{H_u} + q_{H_d} - 2q_\theta + \sum_f (3q_Q^f + q_L^f - 4q_\theta) \right] + 2q_\theta \\ &= \frac{1}{2} [q_{H_u} + q_{H_d} + 3(3q_Q + q_L)] - 5q_\theta, \end{aligned} \tag{2.20b}$$

$$\begin{aligned}
 A_1 &= \frac{1}{2} \left[ q_{H_u} + q_{H_d} - 2q_\theta + \frac{1}{3} \sum_i (q_Q^f + 8q_{\bar{U}}^f + 2q_{\bar{D}}^f + 3q_L^f + 6q_{\bar{E}}^f - 20q_\theta) \right] Y_L^2 \\
 &= \frac{3}{10} [q_{H_u} + q_{H_d} + q_Q + 8q_{\bar{U}} + 2q_{\bar{D}} + 3q_L + 6q_{\bar{E}} - 22q_\theta].
 \end{aligned}
 \tag{2.20c}$$

In the second line of each equation we switched to family-independent charges.  $q_\theta$  denotes the charge of the superspace coordinate  $\theta$ , i.e.  $q_\theta = 0$  for a non- $R$  symmetry.  $Y_L$  controls the normalization of hypercharge, i.e.  $Y_L^2 = 3/5$  if  $U(1)_Y$  is part of a unified  $SU(5)$  symmetry.

By imposing the existence of the Yukawa couplings we can eliminate the charges of the  $\bar{U}$ ,  $\bar{D}$  and  $\bar{E}$  fields,

$$q_{\bar{U}} \equiv -q_Q - q_{H_u} + 2q_\theta, \tag{2.21a}$$

$$q_{\bar{D}} \equiv -q_Q - q_{H_d} + 2q_\theta, \tag{2.21b}$$

$$q_{\bar{E}} \equiv -q_L - q_{H_d} + 2q_\theta. \tag{2.21c}$$

where ‘ $\equiv$ ’ means ‘equal modulo  $N$ ’. After eliminating  $q_{\bar{U}}$ ,  $q_{\bar{D}}$  and  $q_{\bar{E}}$  via (2.21), the anomaly coefficients (2.20) become

$$A_3 = -\frac{3}{2}(q_{H_u} + q_{H_d} - 2q_\theta), \tag{2.22a}$$

$$A_2 = \frac{1}{2}[q_{H_u} + q_{H_d} + 3(q_L + 3q_Q) - 10q_\theta], \tag{2.22b}$$

$$A_1 = -\frac{3}{10}[7(q_{H_u} + q_{H_d}) + 3(q_L + 3q_Q) - 10q_\theta]. \tag{2.22c}$$

By allowing for different couplings of the axion  $a$  to the field strengths of  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$ , it is always possible to cancel the anomalies with the Green–Schwarz mechanism [18, 19]. However, if one is to preserve gauge coupling unification in a natural way, the anomalies need to be universal, i.e.

$$A_3 = A_2 = A_1. \tag{2.23}$$

For a discrete  $\mathbb{Z}_N^R$  symmetry, the coefficients in (2.20) are only defined up to modulo

$$\eta = \begin{cases} N/2 & \text{if } N \text{ is even,} \\ N & \text{if } N \text{ is odd.} \end{cases}
 \tag{2.24}$$

The anomaly universality condition (2.23) then boils down to

$$A_3 \equiv A_2 \equiv A_1, \tag{2.25}$$

where now ‘ $\equiv$ ’ means modulo  $\eta$ .

Let us note that in DHL [16] the anomaly universality condition has been taken to be  $A_3 \equiv A_2$ . However, it is crucial to include the anomaly coefficient due to  $U(1)_Y$ , the  $A_{U(1)_Y-U(1)_Y-\mathbb{Z}_N^R}$ , particularly when addressing compatibility with gauge coupling unification. Therefore, we will employ in the first part of our analysis the universality condition (2.25).

The discrete anomaly universality conditions can be rewritten as

$$A_3 - A_2 = -2q_{H_d} - 2q_{H_u} - \frac{3}{2}q_L - \frac{9}{2}q_Q + 8q_\theta \equiv 0, \tag{2.26a}$$

$$A_3 - A_1 = \frac{3}{10}(2q_{H_d} + 2q_{H_u} + 3q_L + 9q_Q) \equiv 0. \tag{2.26b}$$

It is interesting to note that the second universality condition does not distinguish between  $R$  and non- $R$  symmetries, since it is independent of  $q_\theta$ . By using the freedom of shifting  $q_{H_u}$  and  $q_L$  by integral multiples of the order  $N$ , we can shift the l.h.s. of (2.26a) by integral multiples of  $N/2$  and the l.h.s. of (2.26b) by integral multiples of  $3N/10$ . These equations then become equivalent to the so-called linear congruences (see Appendix A)

$$Nx = 2(A_3 - A_2) \pmod N, \tag{2.27a}$$

$$3Nx = 10(A_3 - A_1) \pmod{10N}. \tag{2.27b}$$

Since  $\text{gcd}(N, N) = \text{gcd}(3N, 10N) = N$ , one obtains the constraints

$$2(A_3 - A_2) = -4q_{H_d} - 4q_{H_u} - 3q_L - 9q_Q \stackrel{!}{=} 0 \pmod N, \tag{2.28a}$$

$$10(A_3 - A_1) = 3(2q_{H_d} + 2q_{H_u} + 3q_L + 9q_Q) \stackrel{!}{=} 0 \pmod N. \tag{2.28b}$$

These constraints are now of the same type as the conditions for operators in the superpotential or Kähler potential to be allowed.

In addition to the GS anomaly cancellation, we shall comment on conditions for anomaly cancellation in the traditional sense, i.e.  $A_3 \equiv A_2 \equiv A_1 \equiv 0$ . This condition is equivalent to demanding anomaly universality and  $A_3 \equiv 0$ . Consequently,

$$2A_3 = -3(q_{H_u} + q_{H_d} - 2q_\theta) \stackrel{!}{=} 0 \pmod N. \tag{2.29}$$

Thus, an anomaly-free symmetry can only forbid the  $\mu$  term under certain conditions. For  $N = 3$ , the condition is trivially fulfilled. If 3 divides  $N$ , then the solutions are given by  $(q_{H_u} + q_{H_d} - 2q_\theta) \in \frac{N}{3} \cdot \{0, 1, 2\}$ . As an example of the latter case, consider an  $SU(5)$  compatible  $\mathbb{Z}_6^R$  symmetry of [28]. With the field charges of  $(q_{H_u}, q_{H_d}, q_\theta) = (4, 0, 1)$ , condition (2.29) is satisfied while the  $\mu$  term is forbidden.

### 2.5. Limitations of analysis

While our aim is to provide a general analysis of discrete symmetries of the MSSM with the properties discussed above, we note that our approach is not without limitations. In particular, if there exist additional states at lower energies, one can have effective operators which appear to have a total  $R$  charge different from the one of the superpotential and which are endowed with large coefficients.

As an example, consider the MSSM with a  $\mathbb{Z}_4^R$  symmetry in which the dangerous operator  $\overline{U}\overline{U}\overline{D}\overline{E}$  arise in the Kähler potential with a highly suppressed coefficient of  $m_{3/2}/M_{\text{P}}^3$ . This is based on the model which will be specified in Table 3.1 and assumes that the operator results from integrating out heavy state(s) in the UV theory.

On the other hand, adding a color triplet  $X$  and an anti-triplet  $\overline{X}$ , both with  $R$  charge 0, we obtain additional allowed terms

$$\Delta\mathcal{W} = m_{3/2}X\overline{X} + \overline{D}\overline{E}X + \overline{U}\overline{U}\overline{X}, \tag{2.30}$$

where we omitted the coefficients. After integrating out  $X$  and  $\overline{X}$  we get

$$\Delta\mathcal{W}_{\text{eff}} = \frac{1}{m_{3/2}}\overline{U}\overline{U}\overline{D}\overline{E}, \tag{2.31}$$

which is a dangerous proton decay operator with a large coefficient. On the other hand, the  $\overline{U}\overline{U}\overline{D}\overline{E}$  operator has  $R$  charge 0, i.e. according to our previous arguments we expect it to be suppressed. To clarify this point, we note that this operator is still  $\mathbb{Z}_4^R$  covariant since we can write (2.31) as [28,29]

$$\Delta\mathcal{W}_{\text{eff}} = e^{\beta S} \overline{U}\overline{U}\overline{D}\overline{E}, \quad (2.32)$$

where  $S$  is the superfield that contains the axion and  $\beta$  is a coefficient. Note that this effective term has the opposite sign in the exponential than the usual instanton contributions.

Our analysis thus only applies if there are no extra states below the fundamental scale  $\Lambda$ . Similar conclusions arise in the recently proposed models of “dynamical  $R$  parity violation” [30].

### 3. Models

#### 3.1. Examples of maximal meaningful symmetries

First, as a cross-check of the algorithm, we have confirmed the maximal meaningful order of a  $\mathbb{Z}_N^R$  symmetry in the MSSM that allows Yukawa couplings and the Weinberg operator  $\mathcal{O}_5$ , which is in agreement with previous analyses. Specifically, the maximal meaningful order

- (i) for matter field charges satisfying SU(5) relations is 24 [14,28];
- (ii) for matter field charges satisfying SO(10) relations is 4 [14,29].

An explicit example for how the algorithm works can be found in Appendix B.

As another illustration, we consider symmetries compatible with Pati–Salam partial unification. We demand the existence of the Yukawa couplings as well as the Weinberg operator. Starting with the Pati–Salam charge relations

$$q_Q = q_L \quad \text{and} \quad q_{\overline{U}} = q_{\overline{D}} = q_{\overline{E}}, \quad (3.1)$$

the maximal symmetry order is found to be 60. One of the inequivalent charge assignments for a  $\mathbb{Z}_{60}^R$  symmetry is given by

$$q_\theta = 1, \quad q_{H_u} = q_{H_d} = 59 \equiv -1, \quad (3.2a)$$

$$q_Q = q_L = 2, \quad q_{\overline{U}} = q_{\overline{D}} = q_{\overline{E}} = 1. \quad (3.2b)$$

The  $\mu$  term is forbidden; however, unlike in the case of the SO(10) and SU(5) compatible symmetries discussed above, it does not appear at linear order in  $m_{3/2}$ .

#### 3.2. Pati–Salam compatible settings

In contrast to SU(5) and SO(10), the Pati–Salam (PS) partial unification [31] can be reconciled more easily with RPV. We note that the Pati–Salam group evades the no-go theorems for  $R$  symmetries in four-dimensional GUT models [32]. RPV models with an underlying PS symmetry have not been extensively studied, a gap which we aim to fill.

Specifically, we consider 4D Pati–Salam models with  $G_{\text{PS}} = \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R$  spontaneously broken to  $G_{\text{SM}}$  by the VEV of a  $D$ -flat combination of  $(\mathbf{4}, \mathbf{1}, \mathbf{2}) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$  Higgses with  $R$  charge 0. This VEV may then explain the effective coupling  $\overline{U}\overline{D}\overline{D}$  or  $LL\overline{E}$ . In addition, we

would need Higgses in the  $(\mathbf{6}, \mathbf{1}, \mathbf{1})$  and  $(\mathbf{1}, \mathbf{1}, \mathbf{1})$  representations with  $R$  charge 2. Pati–Salam models of this type have been derived from the heterotic string [33].

We note that since the PS group does not fully unify into a single gauge group, one can allow for different couplings of the GS axion to the different SM gauge factors. In other words, PS does not lead to anomaly universality, which is consistent with the fact that the PS symmetry does not imply gauge coupling unification.

Let us now have a look at RPV models which are compatible with PS. As a first example, we show that

$$\left. \begin{array}{l} \text{PS compatibility} \\ \text{allow } \bar{U}\bar{D}\bar{D} \\ \text{forbid } LH_u \end{array} \right\} \curvearrowright \text{Weinberg operator is forbidden.} \tag{3.3}$$

Starting with the PS compatibility, which implies

$$q_Q = q_L, \quad q_{\bar{U}} = q_{\bar{D}} = q_{\bar{E}}, \quad \text{and} \quad q_{H_u} = q_{H_d}, \tag{3.4}$$

one can now write down the conditions for the  $\bar{U}\bar{D}\bar{D}$  operator being allowed and the  $LH_u$  term being forbidden,

$$-3q_{H_u} - 3q_L + 4q_\theta = 0 \pmod N \quad (\bar{U}\bar{D}\bar{D}), \tag{3.5}$$

$$q_{H_u} + q_L - 2q_\theta \neq 0 \pmod N \quad (LH_u). \tag{3.6}$$

Here we have taken into account the conditions for the existence of the Yukawa couplings by the means of (2.21). This leads to

$$2q_{H_u} + 2q_L - 2q_\theta \neq 0 \pmod N, \tag{3.7}$$

which forbids the Weinberg operator. This result may be interpreted as the statement that PS compatible  $\mathcal{B}$  RPV models tend to favor Dirac neutrino masses.

### 3.3. Scenarios with anomaly universality

#### 3.3.1. Effective $R$ parity conservation ( $\text{RPC}_{\text{eff}}$ )

We start by looking at scenarios which effectively preserve  $R$  parity, in which the usual  $R$  parity violating operators are forbidden. However, we do not explicitly impose  $R$  parity. For our search, we forbid dimension 4 and 5 RPV operators in the superpotential, as well as the perturbative level  $\mu$  term. This leads to the following criteria

$$\text{RPC}_{\text{eff}} \curvearrowright \begin{cases} 2q_{H_d} + q_{H_u} + 3q_Q - 4q_\theta \neq 0 \pmod N & (\bar{U}\bar{D}\bar{D}), \\ q_L - q_{H_d} \neq 0 \pmod N & (LL\bar{E}), \\ q_{H_u} + q_{H_d} - 2q_\theta \neq 0 \pmod N & (H_u H_d), \\ q_L + q_{H_u} - 2q_\theta \neq 0 \pmod N & (LH_u), \\ 2q_{H_d} + 2q_{H_u} + q_L + 3q_Q - 6q_\theta \neq 0 \pmod N & (\bar{U}\bar{U}\bar{D}\bar{E}), \\ 3q_Q + q_L - 2q_\theta \neq 0 \pmod N & (QQQL), \\ 3q_Q + q_{H_d} - 2q_\theta \neq 0 \pmod N & (QQQH_d). \end{cases} \tag{3.8}$$

With the various dimension 5 operators being related (see Section 2.1), prohibiting the  $QQQH_d$  term also automatically forbids  $\mathcal{O}_{10}$ . Similarly, forbidding  $LH_u$  implies the absence of the operators  $\mathcal{O}_4$ ,  $\mathcal{O}_7$ ,  $\mathcal{O}_8$  and  $\mathcal{O}_9$ . Additionally, we will discuss whether a given solution is compatible with the type I seesaw mechanism, i.e. if it satisfies

Table 3.1

Anomaly-universal  $R$  parity conserving symmetry  $\mathbb{Z}_4^R$ .

Field	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

Table 3.2

Anomaly-universal effective  $R$  parity conserving symmetry  $\mathbb{Z}_{12}^R$ .

Field	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$
$\mathbb{Z}_{12}^R$	4	4	0	0	4	6	10	1

$$2q_L + 2q_{H_u} - 2q_\theta = 0 \pmod N \quad (LH_u LH_u). \tag{3.9}$$

The minimal (GS) anomaly-universal solution which satisfies conditions (3.8) is a  $\mathbb{Z}_4^R$  symmetry whose charge assignment is specified in Table 3.1 and Hilbert basis [34] provided in Appendix E.1. We see that this symmetry does indeed contain  $R$  parity, as there is a residual  $\mathbb{Z}_2$  after  $R$  symmetry breaking.

This  $\mathbb{Z}_4^R$  is nothing but the well-known  $\mathbb{Z}_4^R$  symmetry [35], which was found to be the unique anomaly-free  $\mathbb{Z}_N^R$  solution that commutes with  $SO(10)$  [28,29]. We note, however, that [29] obtained this symmetry by imposing criteria different from ours.<sup>6</sup> The Weinberg operator as well as the Giudice–Masiero mechanism [36] for generating an effective  $\mu$  term are both compatible with this symmetry.

Interestingly, there exist solutions which ensure  $R$  parity conservation before SUSY breaking, but do not contain an actual  $R$  parity. A  $\mathbb{Z}_{12}^R$  symmetry with the charges given in Table 3.2 is of this type. The  $R$  parity violating operators get induced after the “hidden sector” superpotential acquires its VEV, and appear thus with coefficients that are given by (high) powers of  $m_{3/2}/M_P$ . One thus obtains a Froggatt–Nielsen-like [37] suppression of these operators.

### 3.3.2. $B$ violation at the renormalizable level

For the case of baryon number violating RPV setting, we impose the existence of the  $\bar{U}\bar{D}\bar{D}$  operator, and, at the same time, the absence of the  $LL\bar{E}$  term. Following DHL [16], the full set of phenomenological constraints can be specified as

$$\cancel{\mathcal{B}} \text{ RPV} \curvearrowright \begin{cases} 2q_{H_d} + q_{H_u} + 3q_Q - 4q_\theta = 0 \pmod N & (\bar{U}\bar{D}\bar{D}), \\ q_L - q_{H_d} \neq 0 \pmod N & (LL\bar{E}), \\ q_{H_u} + q_{H_d} - 2q_\theta \neq 0 \pmod N & (H_u H_d), \\ q_L + q_{H_u} - 2q_\theta \neq 0 \pmod N & (LH_u), \\ 3q_Q + q_L - 2q_\theta \neq 0 \pmod N & (QQQL). \end{cases} \tag{3.10}$$

Additionally, we will require that the  $LH_u H_d H_u$  term be absent.<sup>7</sup> This results in

<sup>6</sup> The analysis of [29] imposed compatibility with  $SO(10)$ , GS anomaly cancellation, absence of the  $\mu$  term before  $R$  symmetry breaking, existence of the Yukawa couplings and the presence of the Weinberg operator. On the other hand, we have obtained this result by extending the analyses of [21,23] to allow for the Green–Schwarz mechanism, Yukawa couplings and by forbidding the relevant dimension 4 and 5 RPV operators as well as the  $\mu$  term in the superpotential.

<sup>7</sup> If the  $\cancel{\mathcal{B}}$  operator  $LH_u H_d H_u$  is allowed, its combination with the  $\mathcal{B}$  term  $\bar{U}\bar{D}\bar{D}$  could result in a fast proton decay. DHL [16] argue that, since the relevant  $\bar{U}\bar{D}\bar{D}$  coupling contributing to such process is  $\lambda''_{112}$ , which is already strongly bounded by the experiment [8,38,39], the  $LH_u H_d H_u$  operator needs not be explicitly forbidden. However, we will take on a more conservative position, and impose its absence.

Table 3.3  
Anomaly-universal  $\mathcal{B}$  RPV symmetry  $\tilde{\mathbb{Z}}_8^R$ .

Field	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$
$\tilde{\mathbb{Z}}_8^R$	4	6	6	4	6	0	0	1

$$q_L + q_{H_d} + 2q_{H_u} - 2q_\theta \neq 0 \pmod N \quad (LH_uH_dH_u). \tag{3.11}$$

A complete list of unique (GS) anomaly-universal solutions up to order 12, satisfying the constraints of (3.10) and (3.11), can be found in Table C.1 in Appendix C. This set contains a  $\tilde{\mathbb{Z}}_8^R$  symmetry with the charge assignment of Table 3.3. This  $\tilde{\mathbb{Z}}_8^R$  symmetry is not only compatible with the Pati–Salam group, but also allows for the Giudice–Masiero mechanism to be implemented.

### 3.3.3. $L$ violation at the renormalizable level

Similarly, we can identify (GS) anomaly-universal symmetries which violate lepton number at the renormalizable level and satisfy the appropriate phenomenological constraints. However, a straightforward argument [20] appears to demonstrate that all such symmetries are disfavored. Let us review this in more detail.

The argument of [20] follows from the observation that if the Yukawa couplings, the  $\mu$  term as well as any of the trilinear leptonic RPV couplings are allowed in the (perturbative) superpotential, so will be the  $\kappa LH_u$  term. This leads to the expectation that  $\mu \sim \kappa$ . In more detail, equation 9 of [20] states that

$$q_L + q_{H_u} = q_Q + q_L + q_{\bar{D}} + q_\mu = 2q_L + q_{\bar{E}} + q_\mu, \tag{3.12}$$

where  $q_\mu = q_{H_u} + q_{H_d} - 2q_\theta$  and the Yukawa conditions (2.21) have been used. This implies that, if the  $\mu$  term is allowed, which implies that  $q_\mu = 0$ , the charges of  $LH_u$ ,  $QL\bar{D}$  and  $LL\bar{E}$  coincide. Therefore, all these symmetries are simultaneously allowed or forbidden by the symmetry. On the other hand, if we demand that  $QL\bar{D}$  and/or  $LL\bar{E}$  appear at the renormalizable level, i.e.  $q_Q + q_L + q_{\bar{D}} = 2q_\theta$  and/or  $2q_L + q_{\bar{E}} = 2q_\theta$ , then  $q_L + q_{H_u} = q_{H_u} + q_{H_d}$  such that one expects  $\mu$  and  $\kappa$  to be of comparable orders.

Furthermore, suppose that the  $\mu$  term originates from the Kähler potential, while the trilinear leptonic RPV couplings are allowed in the (perturbative) superpotential, i.e. before  $R$  symmetry breaking, as in the previous case. The same line of reasoning as above shows that  $\kappa LH_u$  will also be effectively generated with the size  $\mu \sim \kappa$ , leading again to the same conclusion. Since the above argument applies to all  $\mathcal{L}$  RPV models which have lepton number violating couplings present before  $R$  symmetry breaking, these scenarios are disfavored and we shall not consider them further. Because neutrino mass generation from the bilinear  $\mathcal{L}$  term is a popular mechanism in RPV settings (e.g. [4,40–42]), the above conclusion argument may be interpreted as a problematic feature on a large class of models.

Let us comment, however, that the argument of [20] is limited in the following sense. The central assumption of the argument is that lepton number violating couplings are present in the superpotential before  $R$  symmetry breaking. In contrast, if we require that both the  $\mu$  as well as the  $\mathcal{L}$  RPV terms arise only after  $R$  symmetry breaking, the conclusion that any model with lepton number violation must have  $\mu \sim \kappa$  can be evaded. We will demonstrate this with an explicit example in Section 3.4.3, where the operators arise with different powers of  $m_{3/2}/M_P$ , thus leading to very different sizes of  $\mu$  and  $\kappa$ .

Table 3.4

Anomaly non-universal  $R$  parity conserving symmetry  $\mathbb{Z}_8^R$ .

Field	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$
$\mathbb{Z}_8^R$	1	1	1	1	1	0	0	1

Table 3.5

Anomaly non-universal  $\mathcal{B}$  RPV symmetry  $\tilde{\mathbb{Z}}_4^R$ .

Field	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$
$\tilde{\mathbb{Z}}_4^R$	0	2	2	1	1	0	0	1

### 3.4. Settings with anomaly non-universality

As already mentioned, the Green–Schwarz anomaly cancellation may be satisfied without requiring universality, if the GS axion couples differently to each MSSM field strengths [13,18]. Although this may spoil precision gauge unification, dropping universality constraint leads to new solutions. Such scenarios are not compatible with either SU(5) or SO(10). However, as will be shown below, there exist solutions based on the Pati–Salam group with non-universal anomalies.

#### 3.4.1. Effective $R$ parity conservation

Abandoning anomaly universality, the lowest order solution which satisfies the constraints of (3.8) and is compatible with the Pati–Salam group is a  $\mathbb{Z}_8^R$  symmetry with the charge given in Table 3.4. Clearly, after  $R$  symmetry breaking, there is a residual  $\mathbb{Z}_4$  symmetry which contains matter parity as a subgroup.

#### 3.4.2. $B$ violation at renormalizable level

Imposing the phenomenological constraints of (3.10) and (3.11) but allowing for anomaly coefficients to be non-universal, the minimal solution with baryon number violation is a  $\tilde{\mathbb{Z}}_4^R$  symmetry with the charge assignment of Table 3.5. This solution allows for the Giudice–Masiero mechanism as well as the Weinberg operator. However, in contrast to the  $\mathbb{Z}_8^R$  symmetry in the anomaly-universal case discussed in Section 3.3.2, it does not commute with Pati–Salam.

#### 3.4.3. “Non-perturbative” $B$ and $L$ violation

Another interesting scenario is a setup where  $R$  parity violation appears after  $R$  symmetry breaking. However, as we shall prove, there are no phenomenologically viable anomaly-universal “non-perturbative” RPV symmetries.

Let us impose anomaly universality (2.28) as well as “non-perturbative”  $B$  and  $L$  violation. The latter condition amounts to the requirement that the total charges of  $LL\bar{E}$  and  $\bar{U}\bar{D}\bar{D}$  are 0. Since the number of variables is larger than the number of independent equations, we can impose a U(1) symmetry to satisfy all the constraints. Using the homogeneous version of the method developed in Section 2.3, we obtain a single U(1) solution which has  $q_\theta = 0$ , a continuous non- $R$  symmetry. However, as pointed out in [14], to forbid the  $\mu$  term, one needs an  $R$  symmetry with  $q_\theta \neq 0$ . Thus, we will look for discrete  $R$  symmetries, subject to the constraints



Table 3.6  
Anomaly non-universal “non-perturbative”  $\mathcal{B}'$  and  $\mathcal{L}'$  symmetry  $\mathbb{Z}_3^R$ .

Field	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$
$\mathbb{Z}_3^R$	1	1	1	1	1	0	0	1

$$-4q_{H_d} - 4q_{H_u} - 3q_L - 9q_Q + 16q_\theta = 0 \pmod N, \tag{3.13a}$$

$$6q_{H_d} + 6q_{H_u} + 9(q_L + 3q_Q) = 0 \pmod N, \tag{3.13b}$$

$$-q_{H_d} + q_L + 2q_\theta = 0 \pmod N \quad (LL\bar{E}), \tag{3.13c}$$

$$-2q_{H_d} - q_{H_u} - 3q_Q + 6q_\theta = 0 \pmod N \quad (\bar{U}\bar{D}\bar{D}). \tag{3.13d}$$

Instead of treating  $q_\theta$  as an extra variable, we can treat it as a constant, and thus make use of the inhomogeneous variant of the method of Section 2.3. Thus, one can rewrite (3.13) as

$$A \cdot q = b \pmod N \tag{3.14}$$

with

$$A = \begin{pmatrix} -4 & -4 & -3 & -9 \\ 6 & 6 & 9 & 27 \\ -1 & 0 & 1 & 0 \\ -2 & -1 & 0 & -3 \end{pmatrix}, \quad q = \begin{pmatrix} q_{H_d} \\ q_{H_u} \\ q_L \\ q_Q \end{pmatrix} \quad \text{and} \quad b = q_\theta \cdot \begin{pmatrix} -16 \\ 0 \\ -2 \\ -6 \end{pmatrix}. \tag{3.15}$$

Bringing  $A$  to the Smith normal form,  $D = U \cdot A \cdot V$ , we can rewrite (3.14) as

$$Dq' = b' \pmod N \quad \text{with} \quad q' = V^{-1}q \quad \text{and} \quad b' = Ub, \tag{3.16}$$

where

$$D = \text{diag}(1, 1, 1, 0) \quad \text{and} \quad b' = (2q_\theta, 2q_\theta, 0, -24q_\theta). \tag{3.17}$$

Clearly, the last equation  $0 \cdot q'_4 = -24q_\theta \pmod N$  has only a solution if  $N$  is a divisor of 24. We therefore conclude that the order of a discrete  $R$  symmetry that is consistent with our constraints has to divide 24. The charges are then subject to the constraints

$$q_{H_d} - q_L = 2q_\theta \pmod N, \tag{3.18a}$$

$$q_{H_u} + 2q_L + 3q_Q = 2q_\theta \pmod N, \tag{3.18b}$$

$$q_L + 3q_Q = 0 \pmod N. \tag{3.18c}$$

Subtracting the last equation from the next-to-last one, we see that

$$q_{H_u} + q_L = 2q_\theta \pmod N. \tag{3.19}$$

From this we see that all such symmetries allow for the  $\kappa H_u L$  term in the superpotential, and are, therefore, phenomenologically not viable. We have hence proved that phenomenologically viable, anomaly-universal discrete  $R$  symmetries that give rise to “non-perturbative”  $B$  and  $L$  violation do not exist.

On the other hand, abandoning the anomaly universality condition allows us to construct such models. For instance, a simple  $\mathbb{Z}_3^R$  symmetry can give rise to scenarios with “non-perturbative”  $B$  and  $L$  violation. The charge assignment for  $\mathbb{Z}_3^R$  can be found in Table 3.6 and the Hilbert basis

in Appendix E.2. If we assume that the  $\mathbb{Z}_3^R$  breaking is controlled by the gravitino mass  $m_{3/2}$ , we obtain effective RPV operators of the form

$$\mathcal{W}_{\text{eff}}^{\text{RPV}} \supset \frac{m_{3/2}}{M_{\text{P}}} LL\bar{E} + \frac{m_{3/2}}{M_{\text{P}}} QL\bar{D} + \frac{m_{3/2}}{M_{\text{P}}} \bar{U}\bar{D}\bar{D}. \quad (3.20)$$

For  $m_{3/2} \sim \text{TeV}$ , this implies that the couplings  $\lambda$ ,  $\lambda'$  and  $\lambda''$  are of the order  $10^{-15}$ . In addition, there might exist further flavor suppression for the couplings of the lighter generations. The  $LH_u$  term is suppressed by  $m_{3/2}^2/M_{\text{P}}$ , while the  $\mu$  term is of the order  $m_{3/2}$ . We have thus obtained a scenario with (sufficiently suppressed) lepton number violation that provides, in some sense, a counter-example to the statement in [20] that in such scenarios  $\kappa \sim \mu$ . Further,  $QQQL$  as well as  $\bar{U}\bar{U}\bar{D}\bar{E}$  are suppressed even further by  $m_{3/2}^2/M_{\text{P}}^3$ . Even though the charges in this case commute with  $\text{SO}(10)$ , precision gauge coupling unification might be regarded as an accident in this setting, as discussed in Section 2.4.

Even with  $R$  parity preserved at the perturbative level, because of the presence of the “non-perturbative” RPV, a sizable proton decay rate may still exist. Namely, the combination of  $LQ\bar{D}$  and  $\bar{U}\bar{D}\bar{D}$  operators will lead to the usual  $p \rightarrow e^+\pi^0$  proton decay channel. In our model, the relevant RPV couplings are predicted to be  $\lambda' \sim \lambda'' \sim 10^{-15}$ , which leads to an estimate on their combined strength  $\lambda' \cdot \lambda''$  to be of order of  $10^{-30}$ . This value is to be compared to the experimental limit of  $\lambda' \cdot \lambda'' \lesssim 10^{-27}$  [43].

We thus have provide a simple symmetry that gives rise to hierarchically small  $R$  parity violation. The LSP will be unstable. However, the gravitino will still be a good dark matter candidate, as its decay rate will go like  $m_{3/2}^5/M_{\text{P}}^4$ , where the Planck suppression originates both from the fact that the gravitino interacts only gravitationally and that the various  $\lambda$  couplings go like  $m_{3/2}/M_{\text{P}}$ . A complete survey of the phenomenology of this scenario is beyond the scope of the present analysis.

#### 4. Summary

The huge ratio between the GUT and electroweak scales allows us to give compelling arguments for the observed longevity of the nucleon which is somewhat hard to understand in extensions of the SM with low cut-off, where higher-dimensional baryon and lepton number violating operators are not very much suppressed. The traditional approach in supersymmetric model building is to invoke matter or  $R$  parity [44], amended by baryon triality [21,23]. More recently, a  $\mathbb{Z}_4^R$  symmetry [29,35], which also solves the  $\mu$  problem [28,29], has been proposed. These symmetries remain the simplest options to explain the longevity of the proton in supersymmetric extensions of the SM.

On the other hand, nature might have taken a different route, and the  $B$  or  $L$  symmetries may be violated. In this study, we have explored discrete  $R$  symmetries which explain a sufficient suppression of nucleon decay operators. In settings with such symmetries,  $R$  parity violation is related to supersymmetry breaking, i.e. RPV couplings are suppressed by appropriate powers of  $m_{3/2}/\Lambda$ . In the course of our work, we completed and extended the analysis of DHL [16] surveying viable RPV symmetries of the MSSM. We found that in some cases the symmetries are incompatible with the Weinberg operator such that Dirac neutrino masses appear to be favored. We allowed for  $q_\theta \in \mathbb{N}$  as well as symmetries with non-universal anomalies. We identified redundant symmetries in DHL [16] and found some new solutions. The most “appealing” solution that emerges for a given set of assumptions is depicted in Figs. 1 and 2. Further, we have developed a novel algorithm based on the Smith normal form, which allows to identify the maximal

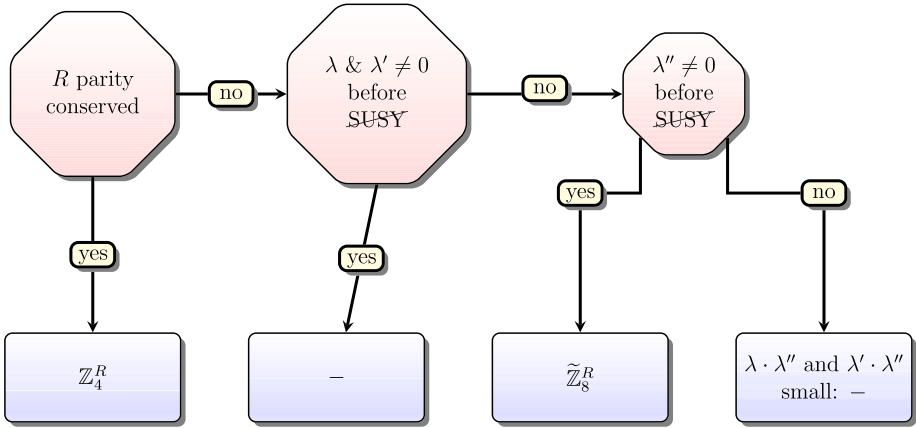


Fig. 1. Summary of our results. We present the simplest discrete  $R$  symmetries with universal anomalies and the specified properties. The symbol “–” indicates the absence of a solution.

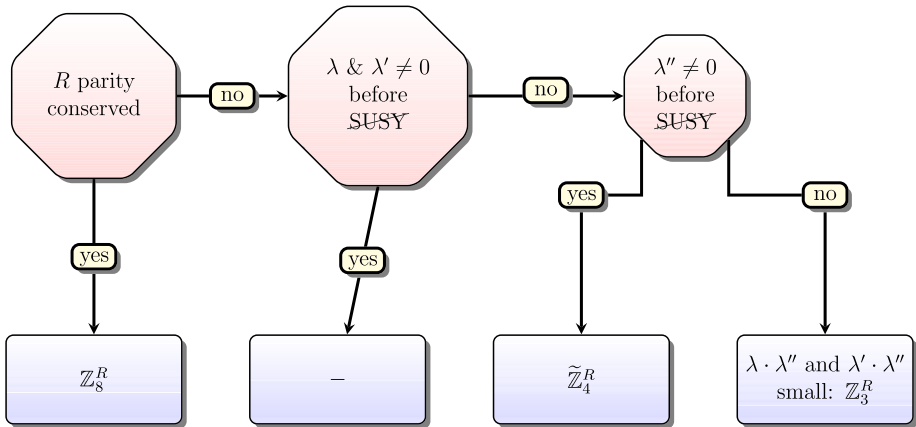


Fig. 2. Summary of our results. We present the simplest discrete  $R$  symmetries with non-universal anomalies and the specified properties. The symbol “–” indicates the absence of a solution.

meaningful symmetry for a given set of constraints. We also specified the conditions for a given set of symmetries to be equivalent.

We have also identified a simple  $\mathbb{Z}_3^R$  symmetry that ensures  $R$  parity conservation before supersymmetry breaking. The coefficients of the  $R$  parity violating operators are consequently suppressed by the small ratio  $m_{3/2}/M_P$ . This symmetry provides us, in some sense, with a counter-example to the statement in the recent literature that in lepton number violating scenarios, the  $\mu$  term and the bilinear  $\kappa LH_u$  must be of comparable magnitude.

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## Appendix A. Basic facts on congruences

In general, the linear congruence

$$ax = b \pmod{M} \quad (\text{A.1})$$

has solutions if and only if  $b$  is divisible by  $\gcd(M, a)$ , in which case there are  $\gcd(M, a)$  solutions modulo  $M$ . Further, it is true that if

$$a = b \pmod{N} \quad \text{and} \quad c = d \pmod{N} \quad (\text{A.2})$$

then

$$a + c = b + d \pmod{N}, \quad (\text{A.3a})$$

$$a \cdot c = b \cdot d \pmod{N}. \quad (\text{A.3b})$$

## Appendix B. Example of systematic search for $\mathbb{Z}_N^{(R)}$

In this example, we will impose anomaly universality, existence of the  $\overline{U}\overline{D}\overline{D}$  term, as well as Pati–Salam compatibility  $q_L = q_Q$  and the GM condition  $q_{H_u} + q_{H_d} = 0 \pmod{N}$ . After imposing these conditions, we are left with the two charges  $\{q_i\} = \{q_Q, q_\theta\}$ .

The conditions are then encoded in the matrix equation

$$A \cdot q = 0 \pmod{N} \quad \text{with} \quad A = \begin{pmatrix} 12 & -16 \\ 0 & 48 \\ -6 & 8 \end{pmatrix}. \quad (\text{B.1})$$

The Smith normal form of  $A$  is then given by the matrices

$$D = \begin{pmatrix} 2 & 0 \\ 0 & 144 \\ 0 & 0 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & -24 \\ 1 & 0 & 2 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}. \quad (\text{B.2})$$

We see that the maximal meaningful  $\mathbb{Z}_N$  symmetry has  $N = 144$ . The corresponding charges are given by

$$q_Q = -72 \cdot k_1 + 4 \cdot k_2 \pmod{144}, \quad (\text{B.3a})$$

$$q_\theta = -72 \cdot k_1 + 3 \cdot k_2 \pmod{144}, \quad (\text{B.3b})$$

where  $k_1 \in \{1, 2\}$  and  $k_2 \in \{1, \dots, 144\}$ . However, as discussed in Section 2.2, many different  $k_i$  lead to physically equivalent symmetries. The resulting inequivalent symmetries, with the  $\mu$  term forbidden, are shown in Table B.1.

Table B.1

$\mathbb{Z}_N^R$  symmetries with renormalizable  $\bar{U}\bar{D}\bar{D}$ , matter charges that commute with PS and the Higgs charges which fulfill the GM condition  $q_{H_u} + q_{H_d} = 0 \pmod N$ .

$N$	$q_Q$	$q_{\bar{U}}$	$q_{\bar{D}}$	$q_L$	$q_{\bar{E}}$	$q_{H_u}$	$q_{H_d}$	$q_\theta$
4	0	2	2	0	2	0	0	1
8	4	6	6	4	6	0	0	1
9	1	5	5	1	5	0	0	3
12	4	2	2	4	2	0	0	3
16	4	6	6	4	6	8	8	1
18	1	14	14	1	14	9	9	3
24	4	2	2	4	2	0	0	3
36	4	2	2	4	2	0	0	3
48	4	2	2	4	2	0	0	3
72	4	2	2	4	2	0	0	3
144	4	2	2	4	2	0	0	3

### Appendix C. $\mathbb{Z}_N^{(R)}$ symmetries of $B$ violating settings

Here we list the  $\mathbb{Z}_{N \leq 12}^{(R)}$  inequivalent symmetries of settings with renormalizable  $\mathcal{B}$ .

Table C.1

Anomaly-universal  $\mathcal{B}$  symmetries up to order 12. We specify the residual symmetry after the breaking of the  $R$  symmetry, and show in the  $W$  column if the Weinberg operator  $LH_u LH_u (\mathcal{O}_5)$  is allowed. The last column indicates whether or not a non-trivial GS mechanism is at work.

Symmetry										Residual symmetry								$W$	GS
$N$	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\theta$	$N'$	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$			
5	2	2	0	2	0	3	0	1	–								–	✓	
6	1	2	5	1	5	3	0	0	6	1	2	5	1	5	3	0	–	✓	
6	1	0	1	3	5	1	0	1	2	1	0	1	1	1	1	0	✓	✓	
6	1	4	3	3	1	5	0	2	2	1	0	1	1	1	1	0	✓	✓	
8	4	6	6	4	6	0	0	1	–								–	✓	
9	1	2	8	1	8	6	0	0	9	1	2	8	1	8	6	0	–	–	
9	1	5	5	1	5	0	0	3	3	1	2	2	1	2	0	0	–	–	
10	2	2	0	2	0	8	0	1	–								–	✓	
10	7	2	5	7	5	3	0	1	2	1	0	1	1	1	1	0	–	✓	
12	2	2	0	2	0	10	0	1	–								–	–	
12	0	10	2	4	10	4	0	1	–								–	✓	
12	0	10	2	8	6	4	0	1	–								–	✓	
12	2	2	0	10	4	10	0	1	–								–	–	
12	0	6	6	4	2	0	0	3	3	0	0	0	2	1	0	0	–	✓	

### Appendix D. RPV symmetries of DHL

In what follows, we list and comment on the non-redundant symmetries found by DHL [16]. We explicitly state whether these allow for a Giudice–Masiero mechanism, i.e. if  $q_{H_u} + q_{H_d} = 0$

mod  $N$ . We further specify the order  $N'_{\mathcal{W}}$  of the residual symmetry that is left after the “hidden sector” superpotential acquires a VEV.<sup>8</sup>

D.1. B violating settings

Table D.1

$\mathbb{Z}_{N \leq 12}^{(R)}$  symmetries for B violating settings. Compatibility with the Weinberg operator (W) and  $LH_u H_d H_u$  ( $\mathcal{O}_6$ ) are indicated. We specify if anomalies (A) vanish (0), are GS universal (U), or fulfill neither of these conditions (-).

$N$	$\theta$	$p$	$n$	$m$	$N'_{\mathcal{W}}$	W	$\mathcal{O}_6$	GM	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	A
5	0	1	3	1	5	-	✓	-	0	4	3	1	2	1	2	-
5	1	3	4	0	-	-	-	-	0	0	1	3	3	2	1	U
6	0	1	3	0	6	-	-	-	0	0	3	2	1	0	3	U
6	1	3	3	2	2	✓	-	-	0	4	5	0	5	4	3	U
7	0	1	3	6	7	-	-	-	0	1	3	3	0	6	4	-
7	1	1	3	1	-	-	-	✓	0	6	5	3	2	3	4	-
7	1	2	6	0	-	✓	-	-	0	0	1	6	2	2	1	-
7	1	5	1	4	-	-	-	-	0	3	3	1	2	6	6	-
8	0	1	3	6	8	-	-	-	0	2	3	4	7	6	5	-
8	1	1	3	0	2	-	-	-	0	0	5	4	1	2	5	-
8	1	4	4	2	-	-	-	✓	0	6	6	0	6	4	4	U
8	1	6	2	6	-	-	-	-	0	2	4	0	4	0	6	-
9	0	1	3	6	9	-	-	-	0	3	3	5	7	6	6	0
9	1	1	3	8	-	-	-	-	0	1	5	5	0	1	6	-
9	1	2	6	5	-	-	-	-	0	4	8	1	7	7	3	-
9	1	3	0	2	-	✓	-	-	0	7	2	6	5	4	0	-
9	1	5	6	5	-	-	-	-	0	4	8	7	1	7	3	-
9	1	6	0	2	-	-	✓	-	0	7	2	3	8	4	0	-
9	1	7	3	8	-	-	-	-	0	1	5	8	6	1	6	-
10	0	1	3	6	10	-	-	-	0	4	3	6	7	6	7	-
10	0	2	6	2	5	-	✓	-	0	8	6	2	4	2	4	-
10	1	1	3	8	2	✓	-	-	0	2	5	6	9	0	7	-
10	1	3	9	0	2	-	-	-	0	0	1	8	3	2	1	U
10	1	5	5	2	2	-	-	-	0	8	7	0	7	4	5	-
10	1	8	4	0	-	-	-	-	0	0	6	8	8	2	6	U

D.2. L violating settings

Table D.2

$\mathbb{Z}_{N \leq 12}^{(R)}$  symmetries for L violating settings. The columns have the same meaning as in Table D.1.

$N$	$\theta$	$p$	$n$	$m$	$N'_{\mathcal{W}}$	W	$\mathcal{O}_2$	GM	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	A
3	1	2	0	1	-	✓	✓	-	0	2	1	1	0	0	1	0
4	1	1	3	1	2	✓	✓	-	0	3	2	0	2	3	0	-
5	0	1	3	2	5	-	-	-	0	3	4	1	3	2	1	-
5	1	3	4	3	-	-	-	-	0	2	4	3	1	0	3	U
5	1	4	2	0	-	✓	✓	-	0	0	3	4	4	2	4	-

<sup>8</sup> Note that in DHL [16] different residual symmetries are discussed. The charge of the supersymmetry breaking spurion is adjusted in such a way that the  $\mu$  term can be generated through the Giudice–Masiero mechanism. However, in addition to the breaking of the symmetry by the  $F$  term VEV, there will always be the breaking due to expectation value of the “hidden sector” superpotential.

Table D.2 (continued)

$N$	$\theta$	$p$	$n$	$m$	$N'_{\mathcal{W}}$	W	$\mathcal{O}_2$	GM	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	A
6	0	1	3	1	6	✓	–	–	0	5	4	2	2	1	2	U
6	1	2	0	4	–	✓	✓	–	0	2	4	4	0	0	4	0
6	1	3	3	5	2	✓	–	–	0	1	2	0	2	1	0	U
6	1	5	3	1	2	✓	✓	–	0	5	4	4	0	3	4	U
7	0	1	3	0	7	–	–	–	0	0	4	3	1	0	3	–
7	1	1	3	2	–	–	–	✓	0	5	6	3	3	4	3	–
7	1	2	6	2	–	–	–	–	0	5	3	6	4	4	6	–
7	1	5	1	2	–	–	–	–	0	5	1	1	0	4	1	–
7	1	6	4	2	–	✓	✓	–	0	5	5	4	1	4	4	U
8	0	1	3	7	8	–	–	–	0	1	4	4	0	7	4	–
8	1	1	3	1	2	–	–	–	0	7	6	4	2	3	4	–
8	1	3	1	7	2	✓	✓	–	0	1	6	4	2	1	4	–
8	1	4	4	6	–	–	–	✓	0	2	2	0	2	0	0	U
8	1	6	2	4	–	–	–	–	0	4	2	0	2	6	0	–
9	0	1	3	7	9	–	–	–	0	2	4	5	8	7	5	0
9	1	1	3	0	–	–	–	–	0	0	6	5	1	2	5	–
9	1	2	6	7	–	✓	–	–	0	2	1	1	0	0	1	–
9	1	3	0	5	–	–	–	–	0	4	5	6	8	7	6	–
9	1	5	6	1	–	✓	–	–	0	8	4	7	6	3	7	–
9	1	6	0	8	–	–	–	–	0	1	8	3	5	1	3	–
9	1	7	3	6	–	–	–	–	0	3	3	8	4	8	8	–
9	1	8	6	4	–	✓	✓	–	0	5	7	4	3	6	4	–
10	0	1	3	7	10	–	–	–	0	3	4	6	8	7	6	–
10	1	1	3	9	2	–	–	–	0	1	6	6	0	1	6	–
10	1	3	9	3	2	–	–	–	0	7	4	8	6	5	8	U
10	1	4	2	0	–	✓	✓	–	0	0	8	4	4	2	4	–
10	1	5	5	7	2	–	–	–	0	3	2	0	2	9	0	–
10	1	8	4	8	–	–	–	–	0	2	4	8	6	0	8	U
10	1	9	7	5	2	✓	✓	–	0	5	8	4	4	7	4	–

### Appendix E. Hilbert bases

The Hilbert basis method [34] allows us to construct a complete basis for the gauge invariant monomials  $\mathcal{M}_i$  of fields appearing in the superpotential. In the presence of  $R$  symmetries, the  $\mathcal{M}_i$  decompose into homogeneous and inhomogeneous monomials, where allowed superpotential terms are of the form

$$\mathcal{W} \supset \mathcal{M}_{\text{inhom}} \cdot \prod_i \mathcal{M}_{\text{hom},i}^{n_i}. \tag{E.1}$$

Below, we provide the lowest order Hilbert bases for two discrete  $R$  symmetries.

#### E.1. $R$ parity conserving $\mathbb{Z}_4^R$

Inhomogeneous terms:

$$\begin{aligned} & (L\bar{E}H_d), \quad (Q\bar{D}H_d), \quad (Q\bar{U}H_u), \quad (LLH_uH_u), \quad (\bar{E}\bar{E}H_dH_dH_dH_d), \\ & (\bar{U}\bar{U}\bar{D}\bar{D}\bar{D}\bar{D}), \quad (\bar{U}\bar{D}\bar{D}LL\bar{E}), \quad (Q\bar{U}\bar{D}\bar{D}\bar{D}L), \quad (LLLL\bar{E}\bar{E}), \\ & (Q\bar{D}LLL\bar{E}), \quad (QQ\bar{D}\bar{D}LL), \quad (Q\bar{U}\bar{U}\bar{D}\bar{D}\bar{E}H_d), \quad (QQQQ\bar{D}LH_d), \end{aligned}$$

$$\begin{aligned}
& (QQQL\bar{L}\bar{E}H_d), \quad (\bar{U}\bar{U}\bar{U}L\bar{E}\bar{E}H_u), \quad (\bar{D}\bar{D}\bar{D}LLH_u), \quad (QQQQ\bar{U}LH_u), \\
& (\bar{U}\bar{U}\bar{U}\bar{E}\bar{E}\bar{E}H_dH_d), \quad (QQ\bar{U}\bar{U}\bar{E}\bar{E}H_dH_d), \quad (\bar{U}\bar{D}\bar{D}\bar{D}\bar{D}H_dH_d), \\
& (QQQQ\bar{U}\bar{E}H_dH_d), \quad (QQQQQ\bar{Q}H_dH_d), \quad (Q\bar{U}\bar{U}\bar{D}\bar{D}LH_uH_u), \\
& (QQQLLLH_uH_u), \quad (\bar{U}\bar{U}\bar{U}\bar{D}\bar{D}\bar{D}H_uH_u).
\end{aligned}$$

Homogeneous terms:

$$\begin{aligned}
& (H_uH_d), \quad (\bar{U}\bar{U}\bar{D}\bar{E}), \quad (Q\bar{U}L\bar{E}), \quad (QQ\bar{U}\bar{D}), \quad (QQQL), \\
& (\bar{D}\bar{D}\bar{D}LH_d), \quad (\bar{U}\bar{D}\bar{D}LH_u), \quad (LL\bar{L}\bar{E}H_u), \quad (Q\bar{D}LLH_u), \\
& (LL\bar{E}\bar{E}H_dH_d), \quad (Q\bar{D}L\bar{E}H_dH_d), \quad (QQ\bar{D}\bar{D}H_dH_d), \quad (\bar{U}\bar{D}\bar{D}\bar{E}H_dH_d), \\
& (Q\bar{U}L\bar{E}H_uH_d), \quad (QQ\bar{U}\bar{D}H_uH_d), \quad (QQQLH_uH_d), \quad (QQ\bar{U}\bar{U}H_uH_u), \\
& (\bar{U}\bar{U}\bar{U}\bar{E}H_uH_u), \quad (Q\bar{U}\bar{E}\bar{E}H_dH_dH_d), \quad (QQQ\bar{E}H_dH_dH_d), \\
& (LL\bar{L}\bar{E}H_uH_uH_d), \quad (Q\bar{D}LLH_uH_uH_d), \quad (Q\bar{U}LLH_uH_uH_u), \\
& (QQQLH_uH_uH_dH_d), \quad (LLLH_uH_uH_uH_u).
\end{aligned}$$

## E.2. Non-perturbative RPV $\mathbb{Z}_3^R$

Inhomogeneous terms:

$$\begin{aligned}
& (L\bar{E}H_d), \quad (Q\bar{U}H_u), \quad (Q\bar{D}H_d), \quad (LLH_uH_u), \quad (QQQQ\bar{U}), \\
& (QQQLLH_u), \quad (QQ\bar{U}\bar{U}\bar{E}), \quad (QQQL\bar{E}H_dH_d), \quad (Q\bar{U}LL\bar{E}H_u), \\
& (QQ\bar{U}\bar{D}LH_u), \quad (\bar{U}\bar{U}\bar{U}\bar{E}\bar{E}), \quad (\bar{D}\bar{D}\bar{D}LL), \quad (QQQQQQLL), \\
& (QQQLLH_uH_uH_d), \quad (\bar{U}\bar{U}\bar{D}L\bar{E}H_u), \quad (Q\bar{U}\bar{U}\bar{D}\bar{D}H_u), \quad (QQQQ\bar{U}LL\bar{E}), \\
& (QQQQQ\bar{U}\bar{D}L), \quad (\bar{D}\bar{D}\bar{D}LLH_uH_d), \quad (QQ\bar{U}\bar{U}LL\bar{E}\bar{E}), \\
& (QQQQ\bar{U}\bar{U}\bar{D}\bar{D}), \quad (QQQ\bar{U}\bar{U}\bar{D}L\bar{E}), \quad (QQQLLLL\bar{E}H_u).
\end{aligned}$$

Homogeneous terms:

$$\begin{aligned}
& (H_uH_d), \quad (QQQH_d), \quad (LL\bar{E}), \quad (Q\bar{D}L), \quad (Q\bar{U}\bar{E}H_d), \quad (\bar{U}\bar{D}\bar{D}), \\
& (LL\bar{E}H_uH_d), \quad (QQQH_uH_dH_d), \quad (Q\bar{U}LH_uH_u), \quad (Q\bar{D}LH_uH_d), \\
& (LLLH_uH_uH_u), \quad (\bar{D}\bar{D}\bar{D}H_dH_d), \quad (\bar{U}\bar{U}\bar{D}H_uH_u), \quad (QQQLL\bar{E}H_d), \\
& (QQQQ\bar{U}LH_u), \quad (QQQQ\bar{D}LH_d), \quad (QQQLLLH_uH_u), \quad (Q\bar{U}LL\bar{E}\bar{E}H_d), \\
& (QQ\bar{U}\bar{U}L\bar{E}H_u), \quad (QQQ\bar{U}\bar{U}\bar{D}H_u), \quad (QQQ\bar{U}\bar{D}\bar{D}H_d), \quad (QQ\bar{U}\bar{D}L\bar{E}H_d).
\end{aligned}$$

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