Title
The Equity Premium: A Deeper Puzzle

Author
Azeredo, Francisco

Publication Date
2007-09-01

The Equity Premium: A Deeper Puzzle*

Francisco Azeredo
Department of Economics
University of California
Santa Barbara, CA
azeredo@econ.ucsb.edu

September 18, 2007

Abstract

Traditional pre-1930 consumption measures understate the extent of serial correlation in the U.S. annual real growth rate of per capita consumption of nondurables and services due to measurement limitations in the construction of their major components. Under alternative measures proposed in this study, the serial correlation of consumption growth is found to be 0.32, contrary to the original estimate of -0.14. This new evidence implies that the class of dynamic general equilibrium models studied by Mehra and Prescott [12] generates negative equity premium for reasonable risk-aversion levels, thus further exacerbating the equity premium puzzle.

 I would like to thank Rajnish Mehra, John B. Donaldson, Edward C. Prescott, Finn Kydland, Stephen LeRoy, Marek Kapicka, Peter Rupert, Javier Birchenall, Adam Sanjurjo, Joao Rosario, James Nachbaur, Brian Lim and seminar participants at the Department of Economics at the University of California Santa Barbara for many helpful comments. The usual caveat applies. This study is part of my dissertation in the Department of Economics at the University of California, Santa Barbara.

1 Introduction

Consensus among financial economists has been that the U.S. annual real growth rate of per capita consumption of non-durables and services – hereafter, "consumption growth" – is uncorrelated over time. This consensus can be clearly found among the studies that followed Mehra and Prescott [12], where the assumption of *i.i.d.* consumption growth has been routinely used.[1]

The results documented in this paper, however, cast doubt on the consensus that consumption growth is uncorrelated over time. Under alternative measures for the food and service components for $1899-1929$, the serial correlation of consumption growth is found to be 0.32.[2]

This new evidence has interesting implications for the equity premium puzzle. If the serial correlation of consumption growth is positive, then the equity premium in the Mehra and Prescott [12] model is no longer an increasing function of the RRA coefficient. As the RRA coefficient increases, the equity premium increases initially and then declines. It is negative for reasonable values of the RRA coefficient.

More precisely, if the serial correlation of consumption growth, ρ, is greater than 0 and the RRA coefficient is greater than ρ^{-1}, then the class of dynamic general equilibrium models in Mehra and Prescott [12] cannot account for the *size* or for the *sign* of the U.S. historical equity premium. Moreover, the representative agent perceives equity to be less risky than the risk-free asset, implying that the equity premium declines with higher levels of the RRA coefficient. These findings further exacerbate the equity premium puzzle.

2 Data

This section shows that traditional pre-1930 consumption measures understate the extent of serial correlation in the U.S. annual real growth rate of per capita consumption

[1]Mehra and Prescott [12] were the first to estimate the serial correlation of consumption growth. Using the Grossman and Shiller [6] series, they found that the serial correlation of consumption growth was -0.14 for $1889-1978$. A literature review can be found in Mehra and Prescott [13, 14] and Donaldson and Mehra [5].

[2]Mehra and Prescott [12] used all available data to estimate the serial correlation of consumption growth. Their estimation procedure presupposes that the serial correlation of consumption growth is stable (constant) over time. Using modern econometric techniques for structural break testing, Azeredo [1] found that the serial correlation of consumption growth is not stable over time. A structural break point is found in 1930 which reveals a serial correlation of 0.43 for $1930-2004$ and -0.45 for $1889-1929$. A structural break is not found under the alternative measures discussed in this study.

due to measurement limitations in the construction of their food and service components. Alternative measures are suggested for these two components for 1899 – 1929 that can better account for short-term cyclical variation in consumption. Under the alternative measures for the food and service components, the serial correlation of consumption growth is found to be 0.32, contrary to original estimate of −0.14 found in Mehra and Prescott [12].

2.1 Non-durables

From 1889 to 1928, the flow of perishable and semi-durable (non-durables) goods to consumers at cost to them was constructed in a collection of publications by William H. Shaw and Simon S. Kuznets. The final version of this series can be found in an unpublished mimeograph containing the annual series underlying tables R-27 and R-28 in Kuznets [10].

In Shaw [17] and Kuznets [7], the flow of non-durables goods to consumers was constructed at cost to producers using available data from the Census of Manufactures, Agriculture, and Mines. For 1889 – 1919, Kuznets modified Shaw's series in order to incorporate transportation and distribution costs, creating the flow of non-durables to consumers at cost to them.[3]

For this period, the growth rates of the Kuznets series are nearly identical to the growth rates in the Shaw series, implying that the statistical properties of the former are mainly governed by those of the latter. This result is due to the procedure used to construct the Kuznets series which implied a nearly constant mark-up for the value added by the transportation and distribution sectors.

Since food is the largest component of non-durables, more comprehensive measures for this item that account for time varying mark-ups for transportation and distribution can potentially give more plausible measures for the flow of non-durables at cost to consumers. Additionally, the small representativeness of the manufactured food of the Shaw series, suggests that the new data can better account for short-term cyclical variation in the food item, and hence, in the non-durable component – see Shaw [17].

Studies from the Department of Agriculture suggest that substantial work on estimates for transportation, processing, and distribution costs of farm products for the pre-1929 period were only available after the publications of major studies on commodity output. According to their studies, efforts by the U.S. Congress to investigate the cause of the spread between prices received by farmers and prices paid by consumers

[3] A comprehensive discussion on the construction of the Kuznets series for this period can be found in Romer [15] and in Kuznets [9].

for food items led to a better compilation of data for the study of the value added by transportation, distribution, and processing of farm products.

Though the Department of Agriculture had collected and published data on food production for many years, it did not start publishing estimates on food consumption, marketing costs, and price spreads until 1945. These estimates were published by the Department of Agriculture only after the publication of major studies on commodity output, and hence, were not incorporated into the estimates derived under the traditional methodology.[4]

Food represents by far the largest component of the non-durable series. For 1899 – 1929, the food item was a fairly constant share of the non-durables total, representing, on average, 57% of the total non-durables. Given the importance of this item, adjustments to it can have important effects on the total non-durable series. These alternative food measures are incorporated into the perishable component of the Kuznets series, replacing the old food measures – Azeredo [1] details the procedure.

2.2 Services

For 1889 – 1929, Kuznets constructed estimates for services. The method used to compute the service series varied with the availability of income data. For 1919 – 1928, the services series was measured as a residual by subtracting the flow of goods to consumers and net capital formation from national income. Estimates of national product computed using the income approach were available from Kuznets [8], permitting the computation of the flow of service to consumers at cost to them as a residual. In other words, the service component was computed by subtracting the flow of goods to consumers at cost to them and net capital formation from net national product.

The residual service measures for 1919 – 1928 present erratic year-to-year variation, despite being conceptually correct. This fact was first pointed out in Kuznets [9], where alternative services measures were also suggested for researchers interested in short-term cyclical variation. According to Kuznets [9]:

> "There is little question that this alternative series gives a more reliable picture of year-to-year variations in the flow of services to consumers than the present or the old residual series: for several reasons short-term fluctuations in the service total derived as a residual are erratic.", (page 11).

[4] See "Major Statistical Series of the U.S. Department of Agriculture: How they are constructed and used", U.S. Department of Agriculture, Agriculture Handbook No. 365. Also "Price spreads between farmers and consumers for food product, 1913–44", BAE, Sep. 1945, introduced the concept and data for the computation of the marketing bill and consumption expenditures for farm products for 1913 to 1944.

Kuznets [8] offers an explanation for the inconsistencies in the year-to-year variation of the residual series. In summary, measurement errors in the perishable, semi-durable, and durable components accumulated into the residual service measure generating a choppy series:

> "The estimated outlay on services not embodied in new commodities increases during all contractions because estimates of consumers' outlay on commodities exaggerate their levels during expansions and underestimate them during contractions. While these errors are minor compared with amounts spent on finished commodities or with total consumers' outlay, they loom large in the estimate of outlay on services, computed as the difference between consumer's outlay and the outlay on all commodities", (page 283 - 284).

The alternative series for the service sector mentioned in Kuznets [9] is the starting point of the estimates for services for $1899 - 1928$. This series was initially constructed by Lough [11] in one of the first endeavors in measuring consumption expenditures in the U.S. The series constructed by William H. Lough were later revised by Barger [2] and Dewhurst [3, 4], who further refined the original estimates. Barger [2] included measures for expenditures on natural and manufactured gas for domestic purposes.[5]

For $1889 - 1918$, national income data was not available and the services series was computed by Kuznets using a regression procedure. Deviations from trend in services were assumed to be linearly related to deviations from trend in flow of perishable, semi-durables, and durables to consumers. This relation was estimated using data for $1919 - 1941$ and applied to the $1889 - 1919$ period.[6].

Because of the adjustments to the food item mentioned above and the flaws in the residual services series for $1919 - 1929$, it became necessary to revisit the interpolation and the estimation procedures used by Kuznets to compute the service measure.[7]

[5]Other adjustments were made to automobile and life insurance, and private hospital and hotel services to better account for the value added by these components. These adjustments are discussed at length in Barger [2]. Dewhurst [3, 4] incorporated more comprehensive estimates for recreation services.

[6]For a general description of the regression procedures used by Kuznets see Kuznets [10]. Romer [15] also presents a description and critique to the Kuznets regression procedures.

[7]See Azeredo [1] for detail procedure.

2.3 Results

Table 1 presents basic statistics for the U.S. annual real growth rate of non-durables, and services. The main differences occurred in the serial correlation of the series. For $1899 - 2004$, the serial correlation of the U.S. annual real growth rate of per capita consumption growth increased from -0.15 to 0.32. The serial correlation of non-durables changed from -0.12 to 0.20, and that for services increased from -0.14 to 0.51.

As expected, their average growth rates are nearly the same under the two methodologies. For the entire sample, the standard deviation of the non-durable series is about 0.6% less under the alternative methodology, while for services it is about 0.8% less. The standard deviation of consumption growth is about 0.7% less under the alternative methodology.

3 Implications to the Equity Premium Puzzle

This section presents the implications of positive serial correlation of consumption growth for the equity premium puzzle and provides an intuitive explanation for these results.

For a serial correlation of 0.32, the equity premium is found to be negative for values of the RRA coefficient greater than approximately 3. The intuition for this result is that as risk-aversion rises, the covariance between the marginal rate of substitution and the return on equity becomes positive. The equity security is then perceived to be less risky than the risk-free asset, implying that agents will demand a rate of return on equity that is lower than the rate of return on the risk-free asset.

The fundamental asset pricing equation can be written as:

$$E_t\left[MRS_{t+1}(R_{e,t+1} - R_{f,t+1})\right] = 0 \tag{1}$$

where $R_{e,t+1}$, $R_{f,t+1}$, and MRS_{t+1} are, respectively, the return on equity, the risk-free rate, and the marginal rate of substitution at time $t+1$. Using the fact that $E[XY] = E[X]E[Y] + Cov[X,Y]$, the following expression can be derived:

$$E_t\left[R_{e,t+1}\right] - R_{f,t+1} = -Cov_t\left(MRS_{t+1}, R_{e,t+1}\right)\left\{E_t\left[MRS_{t+1}\right]\right\}^{-1} \tag{2}$$

In the Mehra and Prescott [12] model, the preferences of the consumers are represented by the CRRA class of utility functions. Under this class, $MRS_{t+1} = \beta\lambda_{t+1}^{-\gamma}$, where λ_{t+1} is consumption growth, γ is the RRA coefficient, and β is the time preference parameter.

Equation (2) can then be rewritten as:

$$E_t\left[R_{e,t+1}\right] - R_{f,t+1} = -Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, R_{e,t+1}\right)\left\{E_t\left[\beta\lambda_{t+1}^{-\gamma}\right]\right\}^{-1} \tag{3}$$

Since $E_t\left[\beta\lambda_{t+1}^{-\gamma}\right] > 0$, a positive covariance between the marginal rate of substitution and the return on equity implies a negative equity premium. In this case, equity returns are high in states when the consumer most needs a high return. Equity is then perceived to be less risky than the risk-free asset, implying a negative equity premium.

The covariance between the marginal rate of substitution and the return on equity is positive if the covariance between the marginal rate of substitution and the capital gains component of the return on equity is positive and exceeds, in absolute terms, the covariance between the marginal rate of substitution and the dividend yield component of the return on equity.

To illustrate this point, the covariance between the marginal rate of substitution and the return on equity is decomposed into two components. Recall first that the return on equity can be written as:

$$\begin{aligned} R_{e,t+1} &= \frac{P_{t+1} + D_{t+1}}{P_t} \\ &= \frac{P_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} \end{aligned} \tag{4}$$

Replacing this expression into the covariance term in equation (3):

$$Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, R_{e,t+1}\right) = Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, \frac{P_{t+1}}{P_t}\right) + Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, \frac{D_{t+1}}{P_t}\right) \tag{5}$$

The first covariance term in the right hand side of equation (5) determines the covariance between marginal rate of substitution and capital gains, $\frac{P_{t+1}}{P_t}$. This covariance can be positive or negative depending on preference and tecnology parameters. The second covariance term in equation (5) is always positive and determines the covariance between the marginal rate of substitution and the dividend yield, $\frac{D_{t+1}}{P_t}$. The covariance between marginal rate of substitution and the return on equity is positive when the former exceeds, in absolute terms, the latter.

In the Mehra and Prescott [12] model, $P_t = w_{\rho,\gamma}(\lambda_t)D_t$ and $D_t = C_t$, where $w_{\rho,\gamma}(\lambda_t)$ is the price-dividend ratio as a function of consumption growth, λ_t, the RRA coefficient, γ, and the serial correlation, ρ. Substituting these expressions into equation (5):

$$Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, R_{e,t+1}\right) = \left\{Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, w_{\rho,\gamma}(\lambda_{t+1})\lambda_{t+1}\right) + Cov_t\left(\beta\lambda_{t+1}^{-\gamma}, \lambda_{t+1}\right)\right\} w_{\rho,\gamma}^{-1}(\lambda_t) \tag{6}$$

If $\rho > 0$ and $\gamma > 1$, then the price-dividend ratio, $w_{\rho,\gamma}(\lambda_{t+1})$, is counter-cyclical to consumption growth. That is, when consumption growth is high, the price-dividend ratio is low, and vice-versa. For a large enough risk-aversion, capital gains are also counter-cyclical to consumption growth and induce the covariance between the marginal rate of substitution and the return on equity to be positive, and consequently, the equity premium to be negative.

Figure 1 shows that for a serial correlation of 0.32, the equity premium is negative for a RRA coefficient greater than 3, approximately. Thus for a reasonable level of risk-aversion the equity premium is negative. For the post-1930 period, the serial correlation of consumption growth is 0.41. In this case, the equity premium is negative for a RRA coefficient of 2.5, approximately.

More generally, under the parameters of the traditional methodology, the equity premium increases for higher values of the RRA coefficient, γ. Under the parameters of the alternative methodology, as the RRA coefficient increases, the equity premium attains a maximum at approximately half of the inverse of the serial correlation coefficient. Then the equity premium starts to decline as risk aversion increases. The equity premium is negative for a risk aversion value greater than approximately the inverse of the serial correlation coefficient.

Figure 2 shows the equity premium for a reasonable range of values of the RRA coefficient and the serial correlation. The time-preference parameter was kept at 0.98 and the unconditional mean and standard deviation of consumption growth maintained at 1.8% and 3.0%. In this figure, the serial correlation and the RRA coefficient range from -0.30 to 0.45 and 1 to 5, respectively. For a large enough RRA coefficient, but still within reasonable values commonly used in the literature, the equity premium declines as the serial correlation increases from -0.30 to 0.45.[8]

[8]The existence of negative equity premium under positive serial correlation of consumption growth was not captured in the extensive robustness checks provided in Mehra and Prescott [12]. Salyer [16] provided numerical examples showing that the equity premium can be negative under positive serial correlation of consumption growth. A detail characterization is presented in Azeredo [1].

4 Conclusion

In this paper, it was argued that traditional pre-1930 consumption measures understate the extent of the serial correlation of consumption growth. Alternative measures for the food and service components were proposed for $1899 - 1928$. These measures can better account for short-term cyclical variation in consumption. Under the alternative measures, the serial correlation of consumption growth is found to be 0.32, contrary to the original estimate of -0.14 in the traditional series.

The implications of positive serial correlation of consumption growth to the equity premium puzzle were discussed. It was shown that under positive serial correlation of consumption growth, the class of model studied by Mehra and Prescott [12] generates negative equity premium for reasonable values of RRA coefficient, thus further exacerbating the equity premium puzzle.

References

[1] F. Azeredo. Essays on aggregate economics and finance. *Ph.D. Dissertation, Department of Economics, University of California, Santa Barbara*, 2007.

[2] H. Barger. *Outlay and Income in the United States*, volume 4 of *Conference on Research in National Income and Wealth, Studies in Income and Wealth.* National Bureau of Economic Research, New York, 1942.

[3] J. F. Dewhurst. *America's Needs and Resources.* The Twentieth Century Fund, New York, 1947.

[4] J. F. Dewhurst. *America's Needs and Resources: A New Survey.* The Twentieth Century Fund, New York, 1955.

[5] J. B. Donaldson and R. Mehra. *Risk Based Explanations of the Equity Premium*, volume in Rajnish Mehra ed., Handbook of Investments: The Equity Risk Premium. North - Holland Elsevier, Amsterdam, 2007.

[6] S. J. Grossman and R. Shiller. The determinants of the variability of stock prices. *American Economic Review*, 71:222–227, 1981.

[7] S. S. Kuznets. *Commodity Flow and Capital Formation.* Number 34 in General Series. NBER, New York, N.Y., 1938.

[8] S. S. Kuznets. *National Income and Its Composition: 1919 – 1938.* Number 40 in 1. National Bureau of Economic Research, New York, N.Y., 1941.

[9] S. S. Kuznets. *National Product since 1869*. Number 46 in General Series. NBER, New York, N.Y., 1946.

[10] S. S. Kuznets. *Capital in the American Economy: Its Formation and Financing*. Studies in Capital Formation and Financing. Princeton University Press (NBER), Princeton, N.J., 1961.

[11] W. H. Lough and M. R. Gainsbrugh. *High-level Consumption*. McGraw-Hill Book Company, Inc. New York and London, 1935.

[12] R. Mehra and E. C. Prescott. The equity premium: A puzzle. *Journal of Monetary Economics*, 15:145–161, March 1985.

[13] R. Mehra and E. C. Prescott. *The Equity Premium Puzzle in Retrospect*. in George M. Constantinides, Milton Harris, and Rene Stulz, eds., Handbook of the Economics of Finance, Volume 1B, North - Holland Elsevier, Amsterdam, 2003.

[14] R. Mehra and E. C. Prescott. *The Equity Premium: ABCs*. in Rajnish Mehra ed., Handbook of Investments: The Equity Risk Premium, North - Holland Elsevier, Amsterdam, 2007.

[15] C. D. Romer. The prewar business cycle reconsidered: New estimates of the gross national product, 1869 – 1908. *Journal of Political Economy*, 97(1):1–37, February 1989.

[16] K. D. Salyer. Overlapping generations and representative agent models of the equity premium: Implications from a growing economy. *Canadian Journal of Economics*, 21:565–578, 1988.

[17] W. H. Shaw. *The Value of Commodity Output since 1869*. Number 48 in General Series. New York: NBER, 1947.

A Tables and Figures

U.S. Annual Real Growth Rate of Consumption of:

		Non-durables	Services	Non-durables and Services (per capita)
			1899 – 1929	
Avg.	Old	2.68	4.14	1.67
	New	2.75	4.05	1.68
Std.	Old	4.38	5.76	4.30
	New	3.22	4.14	3.12
Corr.	Old	-0.33	-0.39	-0.39
	New	-0.04	0.40	0.14
			1899 – 2004	
Avg.	Old	2.76	3.70	1.90
	New	2.78	3.67	1.90
Std.	Old	3.36	3.64	3.10
	New	2.73	2.85	2.44
Corr.	Old	-0.12	-0.14	-0.15
	New	0.20	0.51	0.32
			1930 – 2004	
Avg.		2.79	3.51	1.99
Std.		2.53	2.10	2.11
Corr.		0.31	0.60	0.41

Table 1: Average (Avg.), Standard Deviation (Std.), and Serial Correlation (Corr.)

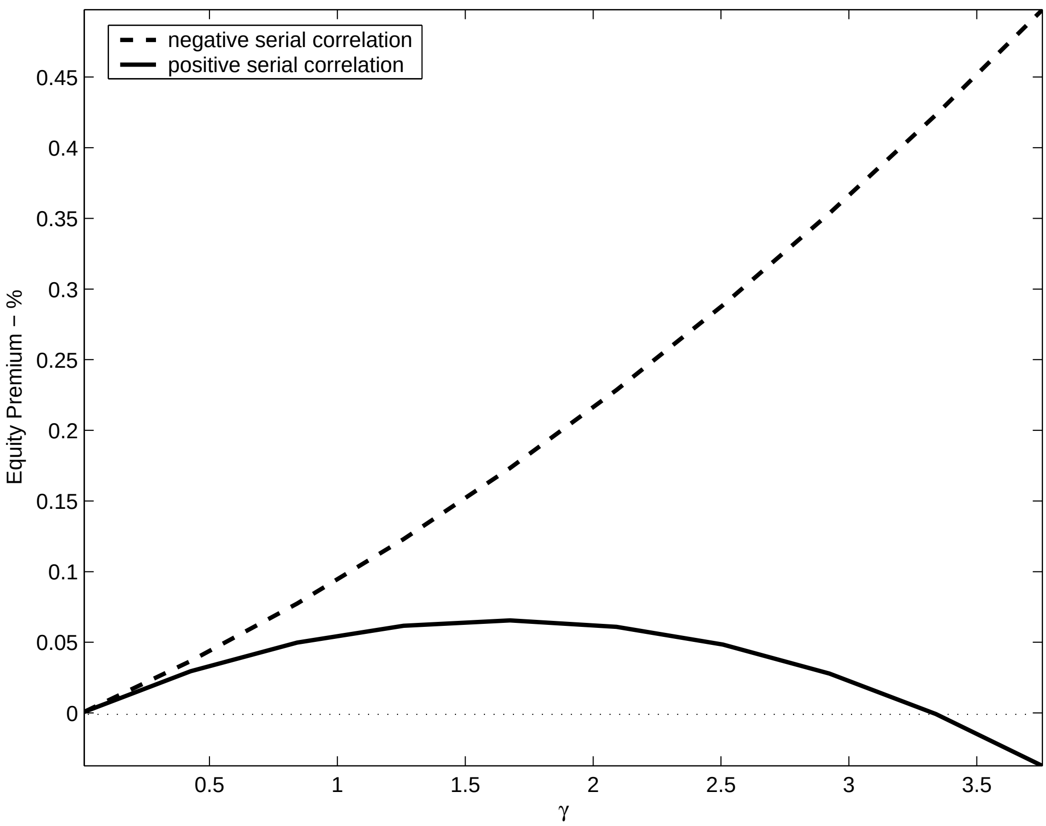

Figure 1: The Equity premium as a function of the RRA coefficient, γ, under positive and negative serial correlation

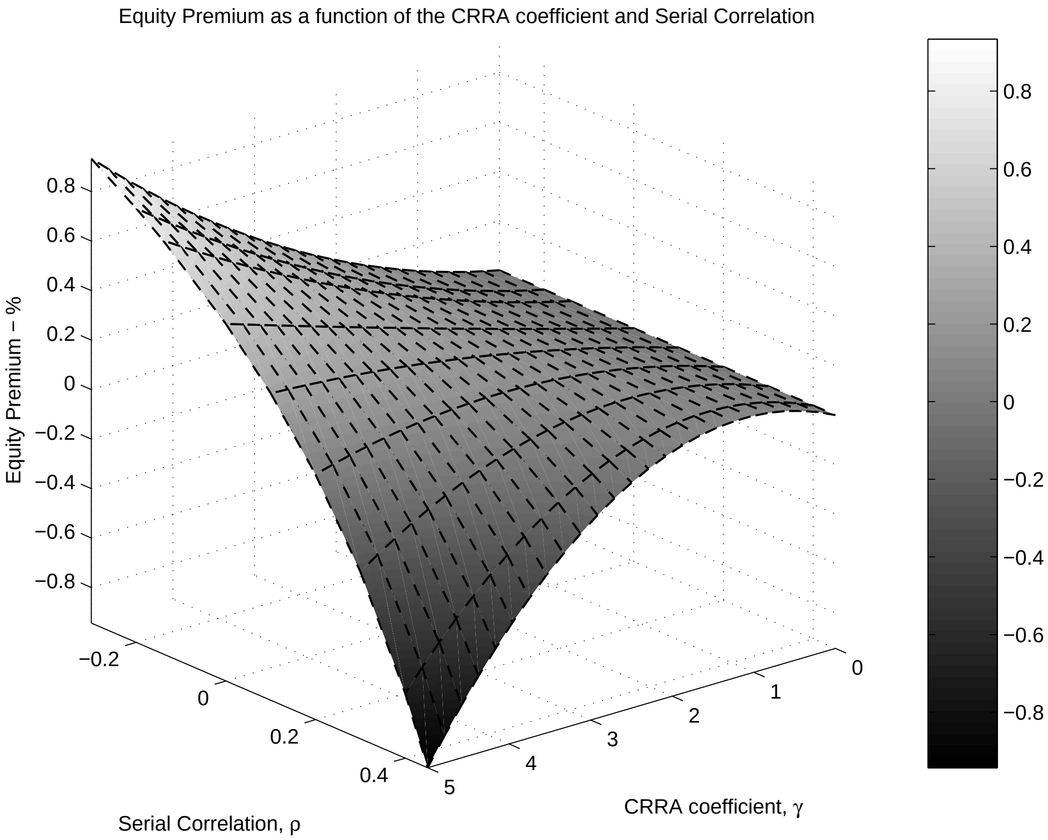

Figure 2: The Equity Premium as a function of the serial correlation and the RRA coefficient