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# Periodic States, Local Effects and Coexistence in the BML Traffic Jam Model

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The Biham-Middleton-Levine model (BML) is simple lattice model of traffic flow, self-organization and jamming. Recently, the conventional understanding was shown to be incomplete: rather than a sharp phase transition between free-flow and jammed, there is an additional region where convergence to intermediate states is observed, with details dependent on the aspect ratio of the underlying lattice. For aspect ratios formed by two subsequent Fibonacci numbers, intermediate states converge to ordered, periodic limit cycles (*i.e.*, periodic intermediate (PI) states). In contrast, for square aspect ratios, intermediate states typically converge to random, disordered intermediate (DI) states. We show these DI states are very robust to perturbation and occur more frequently than the conventional states for some densities. Furthermore, we report here on the discovery of PI states on square aspect ratios, showing PI states are not just an idiosyncrasy of particular aspect ratios. Finally, we investigate features that lead towards jamming and identify that local effects can dominate. A strategic perturbation of a few selected bits can change the nature of the flow, nucleating a global jam. The global parameters, density together with aspect ratio, are not sufficient to determine the full jamming outcome.

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## I. OVERVIEW: BML AND RESULTS

Transport phenomena plays an underlying role in a broad range of physical systems: traffic flow on highways [1, 2, 3, 4], congestion of packets on the Internet [5], flow of nutrients through the body [6], formation and flow of river networks [7, 8], etc. These all rely on transportation and flow of physical substances. Self-organized patterns in the flow, such as vortices and spiral waves, occur frequently. Moreover, the flow often jams (*i.e.*, comes to a complete halt) abruptly, in response to just a small change in an external control parameter. Simple models from statistical physics have been proposed which produce aspects of self-organization and the abrupt onset of jamming, in particular the Biham, Middleton, and Levine model (BML) [9]. BML is a simple cellular automata model of two-dimensional traffic flow, modeling gridlock between east-bound and north-bound “cars”. The standard understanding is that the BML model undergoes a first-order phase transition as a function of traffic density  $\rho$ , from free-flow (FF) traffic (all cars move at all times with velocity  $v = 1$ ) to a global jam (GJ) of traffic (no car ever moves, so  $v = 0$ ). Figures 1 (a) and (b) show typical FF and GJ configurations. The dynamics leading to their formation can be seen at [10]. BML has become a theoretical under-

pinning for traffic modeling, with hundreds of citations in the scientific literature referencing BML and its first-order phase transition. For comprehensive reviews see Refs. [2, 3, 4].

Recently a larger family of behaviors was discovered [11]. BML does not necessarily (or even typically, as shown below) exhibit a sharp phase transition from FF to GJ, but instead has a range of “intermediate states” with regions of FF intersecting at jammed wavefronts. The geometry and velocities of the intermediate states are intrinsically dependent upon the aspect ratio of the underlying lattice (*i.e.*, the ratio of the number of columns,  $L$ , to the number of rows,  $L'$ ). When  $L$  and  $L'$  are two subsequent Fibonacci numbers (referred to herein as “Fibonacci lattices”), the intermediate states each converge to a periodic limit cycle as was shown in [11]. The exact microscopic configuration recurs every  $\tau$  timesteps, with  $\tau$  on the order of the number of particles in the system. These periodic intermediate (PI) states have a highly regular, crisp geometric structure, as illustrated in Fig. 2 (a), with bands of free flowing traffic intersecting at jammed wave fronts that propagate smoothly through the space. (System 1 of Ref. [10] illustrates this behavior.) Furthermore, as shown in [13] and reproduced here in Fig. 2 (b), hundreds of runs were simulated for Fibonacci lattices with densities  $\rho$  between  $1/3$  and  $1/2$ . Each one con-

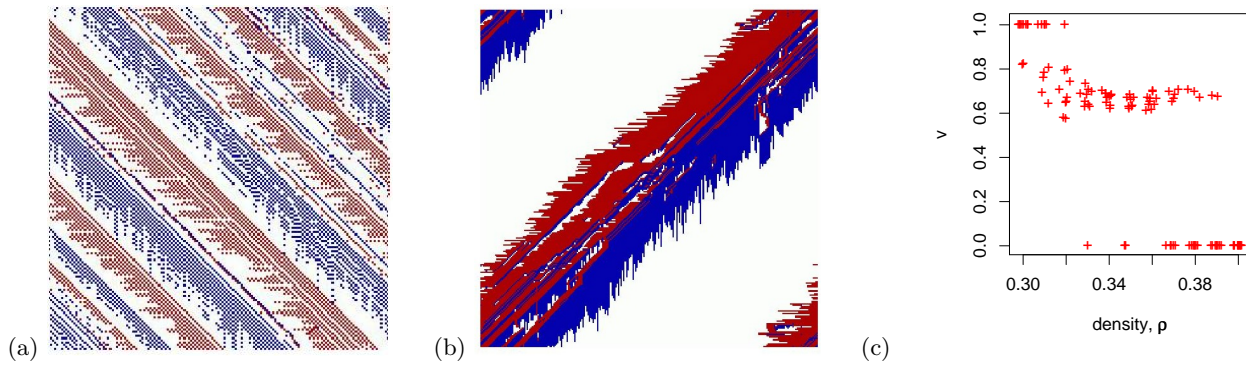


FIG. 1: Typical configurations for the BML model on an  $L \times L$  lattice, with  $L = 256$ . (a) The free-flow (FF) phase reached by any random initial configuration with small density,  $\rho$ . Self-organized bands of separated east- and north-bound cars allow all particles to advance each update, thus  $v = 1$ . (b) The global jam (GJ) phase reached by any random initial configuration for larger  $\rho$ . Here all cars are immobilized, thus  $v = 0$ . The conventional belief is that there is a first-order phase transition between these two behaviors as a function of  $\rho$ , with critical density  $\rho_c \approx 0.35$ . (c) Numerical results show otherwise. Each point represents one individual random realization run until convergence. FF ( $v = 1$ ) and GJ ( $v = 0$ ) states do occur, but less frequently than intermediate states.

verged to a periodic limit cycle with either  $v \approx 0.70$  or  $v \approx 0.36$  (see [11] for a derivation of these velocities). The conventional behaviors (FF or GJ) were never once observed and do not seem to be in the accessible phase space for these densities. Thus, by changing the underlying lattice aspect (from square to Fibonacci), one can considerably delay the onset of when complete jamming is first observed, and instead observe high-throughput flow in a regime where previously it was believed the system would fully jam.

On square aspect ratios (*i.e.*,  $L = L'$ , the configuration typically studied in the literature), intermediate states are also observed. Their geometric structure, illustrated by Fig. 3, is qualitatively similar to Fig. 2 (a), but in addition most states have disordered chains of particles, interspersed at random, moving throughout the space which preclude such states from being on periodic limit cycles. (System 3 of [10] illustrates the dynamics.) Thus we distinguish Disordered Intermediate (DI) from Periodic Intermediate (PI) states. Unlike the PI states which have converged to a fixed limiting behavior, we cannot rule out the possibility that the DI states are metastable long-lived transients. Previous evidence [11, 12] strongly suggested that to reach a periodic limit cycle (*i.e.*, a PI state) required that  $L$  and  $L'$  be relatively prime to each other. We report here on the discovery of PI states on square lattices: PI states exist in the conventional BML model, and are not just an idiosyncrasy of rela-

tively prime aspect ratios. We show that PI states on square aspect ratios are rare (we observed only two in a sample of 361 random realizations), yet DI states are extremely common (over 60% of random realizations with  $\rho = 0.36$  converge to DI states). In addition, we quantify how stable the DI states are to perturbation. Here too, the results are consistent with DI states comprising over 60% of the ensemble of realizations.

For square aspect ratios, DI, PI and GJ phases can all be observed at the same value of density. Thus we now know that density together with aspect ratio influence the jamming outcome, but are not “control parameters”, sufficient to fully predict it. *We show that local effects play a fundamental role in determining the outcome.* A strategic perturbation of just a few bits (which cannot, by definition, alter global properties such as density) can make a realization flip from the DI to the GJ state.

## II. IMPLEMENTATION PROCEDURE

The BML model consists of two species of “cars” moving on a two-dimensional square lattice with periodic boundary conditions. “Red” cars want to move eastward. “Blue” cars want to move northward. And they alternate attempts to do so. First all the red cars in synchrony attempt to advance one lattice site to the east. Any car succeeds so long as the site it wants to occupy is currently empty (no red or blue car is already occu-

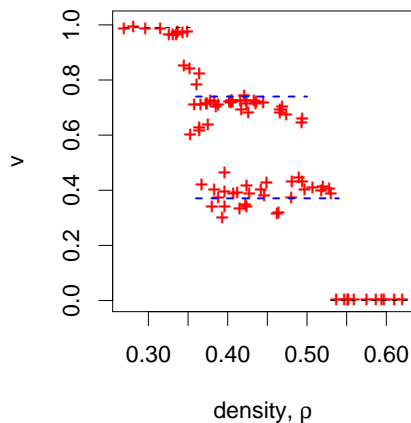
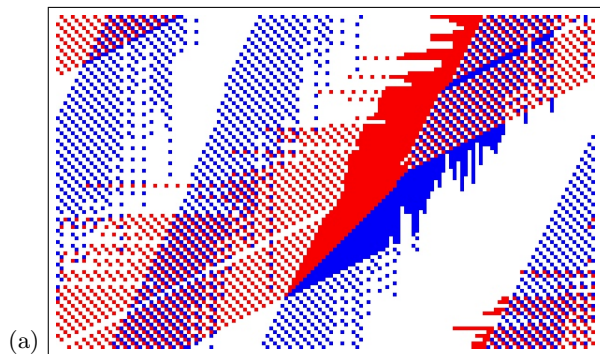


FIG. 2: (a) Typical PI state on a Fibonacci lattice for a realization with  $L = 144$  and  $L' = 89$ . (b) Experimental results for  $L = 89$  and  $L' = 55$ . Each point represents one individual realization. For densities between  $\rho \approx 0.35$  and  $\rho \approx 0.5$ , each realization converges to a PI state.

pying it). Then all the blue cars in synchrony attempt the corresponding advance north-ward. Cars that advance are said to have velocity  $v = 1$ . Stationary (*i.e.*, “blocked”) cars have  $v = 0$ . We can think of the BML lattice as having a traffic light at each site, with all lights synchronously timed to alternate between east-ward and north-ward flow. The dynamics is fully deterministic. The only randomness is in the initial condition, when an empty lattice is populated uniformly at random with density  $\rho/2$  of cars of each species. Thus, total car density is  $\rho$ .

We implement the BML model using a square aspect ratio, with  $L = L' = 128$ , initialized at random with density  $\rho \approx 0.36$ . We first generate an ensemble of 45 such random realizations of the system and run each one until at least the “relaxation time” of the system  $\tau_c =$

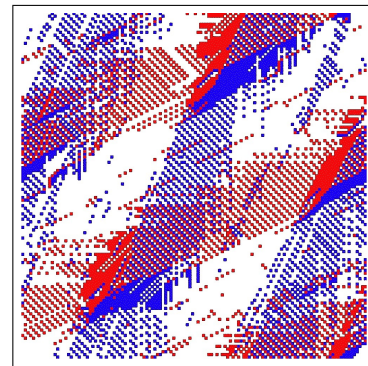


FIG. 3: A typical DI state with  $L = L' = 128$  and  $\rho = 0.36$ . Chains of particles are interspersed randomly throughout, in contrast to the PI state shown in Fig. 2 (a).

200,000 time-steps, where this value of  $\tau_c$  was determined in [11] to exceed the convergence time for such system sizes. All simulations were implemented in Python. The numerical results presented in Fig. 4 and Fig. 5 alone required over a month of non-stop running time, using an 8-processor, dual-core, 2.4GHz computer.

### III. RESULTS

#### A. Accessibility of intermediate states

Of the 45 random initial conditions, 14 of them converge to the jammed state ( $v = 0$ ). The remaining 31 converge to the DI state (with  $v \approx 2/3$ ). Figure 4 is a plot of the exact converged velocity for each sample. Thus for  $L = L' = 128$  and densities  $\rho \approx 0.36$ , more than 68% of the random initial conditions converge to the DI state. Less than 32% of them converge to the expected GJ behavior of  $v = 0$ .

#### B. Robustness of intermediate states

We wish to understand how robust the intermediate states are to perturbation. If we flip the identity of some of the cars (*i.e.*, from red-to-blue and blue-to-red), how likely is the perturbed system to then jam? We consider the subset composed of the 31 DI realizations discussed above. For each realization, we choose some fraction of the particles uniformly at random (ensuring equal numbers of red and blue), and exchange their color. Then we run the perturbed system for a second  $\tau_c$  timesteps and measure the new velocity.

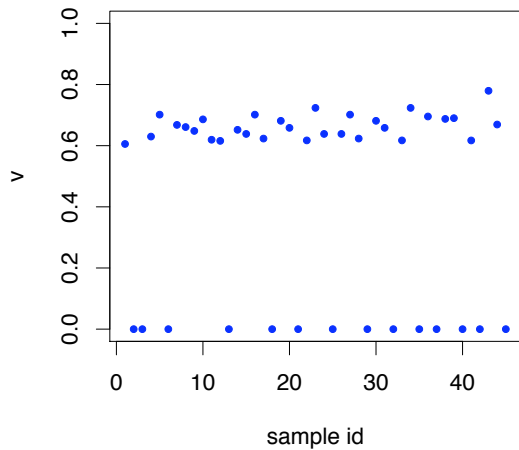


FIG. 4: Experimental results for the 45 realizations with  $\rho = 0.36$  on an  $L = L' = 128$  lattice. Each point represents one individual realization started from a random initial condition and run until time  $\tau_c$ . Only 14 realizations converge to the expected GJ state. The remaining 31 converge to the DI state.

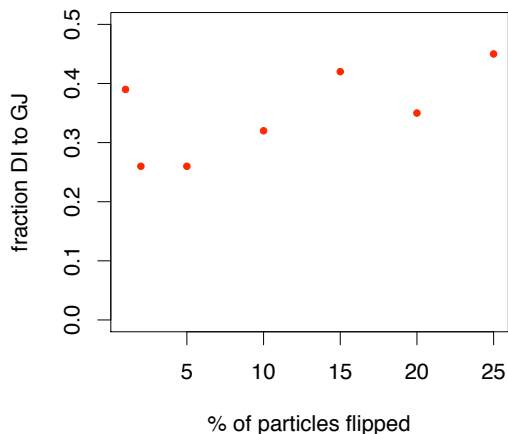


FIG. 5: The fraction of realizations which flip from DI to GJ as a function of the extent of the perturbation.

Figure 5 is a plot of the fraction of intermediate states which changed from DI to GJ as a function of the percentage of the particles whose identities were flipped. Between 25% and 45% of the realizations flip from DI to GJ when perturbed, but the majority remain in the DI phase. We are finding the intermediate states are extremely robust! Note, in Fig. 5, random perturbations as large as 25% of the particles are considered and there is a weak trend indicating a system is more likely to reach a GJ with increased perturbation. But, as will be presented in Sec. IV, a tiny *strategic* perturbation can take a realization from the DI to the GJ phase.

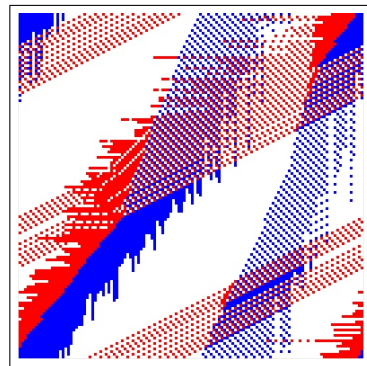


FIG. 6: A typical PI state on a square lattice for a realization with  $L = L' = 128$  and  $\rho = 0.36$ . System 2 of [10] illustrates the dynamics.

### C. PI states on square lattices

Two of the DI states, when perturbed slightly (less than 5%), actually went from DI to PI. For these states  $v \approx 0.5$  and the period,  $\tau \approx 25,000$  timesteps (about four times the number of particles in the system). A snapshot of one of the PI realizations is shown in Fig. 6. (System 2 of Ref [10] is a movie of the dynamics.) Our total sample space includes 361 realizations (*i.e.*, the original ensemble of 45 plus all the perturbed realizations). Of these 361, only two went to a PI state.

The PI states observed on a square aspect ratio are very fragile. Flipping just 1% of the particles takes the system to the DI phase. On Fibonacci lattices the PI behavior is more complex. If just one particle of a particular PI realization is perturbed, that realization jumps to a different PI state (*i.e.*, a different periodic limit cycle), but remains in the PI phase. This same result holds even when the perturbation is large. Thus, an individual PI state is fragile, yet the PI phase is extremely robust. (The phase space breaking into distinct clusters is reminiscent of ergodicity breaking observed in spin-glasses [14] and random constraint satisfaction problems, see *e.g.*, [15].)

## IV. LOCAL PROPERTIES TRIGGER JAMMING

Jamming and transport play an underlying role in a wide array of fundamental processes, hence identifying properties that enhance, delay or trigger jamming is an important problem. As shown in Sec. III, systems with  $L/L' = 1$  and the same exact  $\rho$  may converge to GJ, to DI or to PI states. Thus,  $\rho$ , together with  $L/L'$ , predict

some, but not all, properties of the jamming outcome. Our aim is to identify the additional factors that influence jamming – to find properties that differentiate between realizations which, for the same  $\rho$  and  $L/L' = 1$ , converge to GJ from those which converge to DI states. We investigate a range of potential factors, amongst them:

- The maximum density observed in any one row or column. The assumption is that a highly populated row or column could nucleate a jam.
- The standard deviation of the distribution of individual row (and column) densities. The assumption is that large variance (many high density lines mixed with low density ones) can nucleate a global jam.
- The difference between the total number of red- versus blue-cars. The assumption is that a disproportionate representation of one color could nucleate a jam. Note, each site is occupied initially at random with probability  $\rho$ , hence the total number of occupied sites is a random variable.

First we focus on the original ensemble of 45 realizations (defined in Sec. II). We found no correlation between the values of any of these properties listed above and the likelihood to converge to a GJ, rather than a DI, state. Furthermore, we found no correlation between any of these properties and the value of the converged velocity (Fig. 4 is a plot of the latter). These macroscopic properties are independent from the likelihood to jam. Similar results were found in a previous study on Fibonacci lattices, where there are two types of PI states (high- and low-velocity), and we found that none of the properties above differentiated between them [16].

We then focus on comparing individual realizations to their perturbed variants (as defined in Sec. IIIB). In particular we consider realizations that, with just a 1% perturbation, change from DI to GJ. We find the properties discussed above do not differentiate between the original and perturbed realizations. To clarify whether these particular realizations that went from DI to GJ are more susceptible to jamming we tried different random 1% tweaks – some nucleated a jam, but most did not. Sometimes we happen to perturb the key particles and nucleate a jam, but the majority of the time we do not.

Diagonal ordering may be of fundamental importance to the BML model. The self-organization leading to FF involves formation of left-sloping (NW-to-SE) diagonals of the same species (Figure 1 (a) illustrates the end result of this process, with separated left-sloping bands of the different species). Jams, in contrast, form along right-

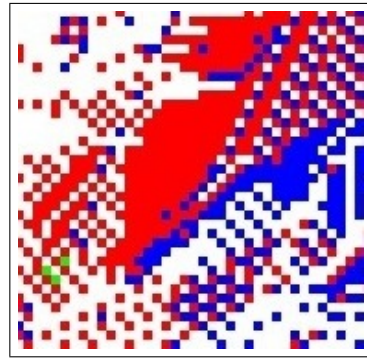


FIG. 7: Closeup of the DI state shown in Fig. 3, with  $L = L' = 128$  and  $\rho = 0.36$ . Perturbing just three particles (colored green, lower left hand corner), will make the new system converge to GJ. (System 4 of Ref. [10] is a movie of the dynamics, started from the perturbation.)

sloping (NE-to-SW) diagonals. (Figure 1 (b) illustrates the end result of a global jam). During the evolution of any realization, from the random initial condition to the final state, we see a competition between these two types of diagonals emerging and interfering with each other. The left-sloping diagonals lead towards  $v = 1$ , the right-sloping towards  $v = 0$ . Note, Austin and Benjamini recently completed an analysis of BML based on diagonals [17]. They showed that, for an  $L \times L$  system with total number of particles  $N = L/2$  the system always converges to  $v = 1$ . Of course,  $\rho = 1/(2L)$  is quite far from the regime of interest ( $0.3 < \rho < 0.6$ ), yet [17] is one of the only rigorous proofs for behavior of the BML model.

For both DI and PI states, there are regions of left-sloping bands of FF intersecting at right-sloping jammed interfaces. By perturbing just a few particles in nearby left-sloping bands, we can nucleate a jam. Figure 7 shows an example realization. We start from the same system shown in Fig. 3 and then exchange the velocity of just *three* particles along ordered left-sloping diagonals, which makes the realization flip from the DI to GJ phase. (System 4 of Ref. [10] is a movie of the process.) Here, perturbing all three particles is necessary in order to induce the GJ phase, however we did observe another realization where changing just *one* particle made the realization flip from DI to GJ. (Note that a 1% perturbation of a realization with  $L = L' = 128$  and  $\rho = 0.36$  amounts to perturbing approximately 60 particles. Perturbing three particles represents just a 0.01% change.) We believe that pinpointing exactly how to quantify the change in-

duced by the local perturbation is a difficult challenge, and leave this as an open question.

## V. DISCUSSION

The phase space of the BML model is much richer than previously expected. A range of intermediate behaviors are observed, with the exact details dependent, in part, on the aspect ratio of the underlying lattice. For Fibonacci lattices, three distinct regimes are observed as a function of increasing  $\rho$ : 1) FF, 2) PI and 3) GJ. For square aspect ratios the regimes observed are: 1) FF, 2) coexistence of FF, PI and DI, and GJ and 3) GJ.

The intermediate states on the square aspect ratio comprise a significant fraction of the accessible phase space. For  $\rho = 0.36$ , over 60% of the states observed are in the DI phase. Furthermore, results from perturbing DI states also are consistent with such phases comprising over 60% of the phase space. For Fibonacci lattices, studied at a range of densities ( $1/3 < \rho < 1/2$ ), the entire phase space consists only of PI states.

Car density,  $\rho$ , together with aspect ratio influence the

jamming outcome, but are not sufficient to fully determine it. Local properties also play an important role, and we present evidence that diagonal ordering is an additional, fundamental consideration. Our current study deals with perturbing a realization after it has converged to the DI phase. Far more powerful (and difficult) would be to understand what to alter in the random initial condition to control the phase that realization ultimately converges to.

Perhaps a practical consideration, is that one can observe high-throughput flow in a regime where previously it was believed a system would jam and no flow at all could be achieved. By changing the underlying lattice aspect ratio (from a square to a Fibonacci lattice), one can considerably delay the onset of when complete jamming is first observed.

## VI. ACKNOWLEDGMENTS

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