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ROTATION OF MODERATELY-DEFORMED ODD-A NUCLEI†

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Abstract:

The particle-plus-rotor model has been solved for some weakly- and moderately-deformed odd-A nuclei. Some very characteristic regularities are expected to occur for the high-spin states coming from high-j orbitals, and these depend strongly on the sign of β. Application to some Au, Hg, and Tℓ nuclei shows surprising agreement, and indicates oblate shapes for these states in all these nuclei. Under certain conditions, a new coupling scheme is realized where the particle angular momentum, j, is quantized along the rotation axis rather than the symmetry axis.

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The rotational model, proposed by Bohr[1] in 1952 and developed subsequently by Bohr and Mottelson[2] and others, requires that the spectrum of an odd-A rotational nucleus corresponds to that of a particle coupled to a non-spherical rotating core. This is well borne out in the regions of "deformed" nuclei, where the particle is strongly coupled to the deformed shape. This model has not really been studied in intermediate- or weak-coupling situations, although these were discussed in the early papers. An attempt to extend the model into these regions was made by Malik and Scholz[3] in the mass region, A = 25 - 80. The results were generally encouraging, although the data were nowhere sufficient to provide a very convincing picture. Recently the striking behavior of the high-spin states in the La nuclei[4] gave the first strong indication that this model might apply outside the strong-coupling regions. In the present note, we would like to point out the general features of this model for high-j orbitals at moderate deformations, and demonstrate its applicability to the region just below lead.

The problem in applying this model to odd-A nuclei with decreasing deformations is that the Coriolis effects become very large and must be diagonalized exactly, taking into account all closely-related states. In the region just below lead we would at first hope that by choosing states of the appropriate parity, only one j-shell need be considered. This would be $h_{11/2}$ for protons and $i_{13/2}$ for neutrons. In fact, for protons, states of $h_{9/2}$ j-shell from the 82-126 shell are also observed in this region of nuclei, but are not strongly coupled to those of the $h_{11/2}$ j-shell, and thus can be diagonalized separately. We will outline briefly our method for solving this problem, discuss the results in the Au region, and then summarize the general features of the model.

For a particle coupled to an axially symmetric rotor, the Hamiltonian can be written:

$$H = H_{intr} + H_{rot} \quad , \qquad \text{with;} \qquad (1)$$

$$H_{rot} = \frac{\hbar^2}{2\mathfrak{I}} \vec{R}^2 = \frac{\hbar^2}{2\mathfrak{I}} [I(I+1)-\Omega^2] + \frac{\hbar^2}{2\mathfrak{I}} [\vec{J}^2-\Omega^2] + H_C \quad , \tag{2}$$

where the symbols have their usual meaning, H_C is the Coriolis operator, and H_{intr} is the Hamiltonian of the particle in the absence of any rotation. The solution of this Hamiltonian, with the addition of pairing effects, has been discussed previously[4] using several simple approximations in order to limit the number of variables to two; the deformation β, and the Fermi surface λ. We will follow the same procedures here, and in addition will fix β and λ, so that there remain no adjustable parameters. We fix |β| by averaging the value obtained from B(E2; 2 → 0) measurements for the two adjacent even-even nuclei. For this |β|, λ is fixed at the level corresponding to the correct number of particles according to the Nilsson diagram. Thus, for each odd-A case we calculate two spectra, corresponding to a prolate and an oblate shape for the nucleus.

There are a number of approximations in the above procedure. It is fairly easy to show that some, such as the assumption of pure j-values, are rather good. The three that seem likely to be poor at times are: a) the use of a perfect-rotor Hamiltonian for the core; b) the neglect of the effect of the odd particle on the core parameters; and c) the restriction of the core to axially-symmetric shapes. We have not made quantitive estimates of b) and c), but we have examined (a) by using:

$$H_{rot} = \frac{\hbar^2}{2\mathfrak{I}} [\vec{R}^2 + B\vec{R}^4 + C\vec{R}^6] \quad , \tag{3}$$

where B and C can be obtained from approximate fits to levels in the adjacent even-even nuclei (see the caption to Fig. 2). In the calculations discussed here, we have used Eq. (3); however, we have found that this does not cause large changes in the calculated spectra.

We will now discuss briefly the application of this model to the Au region of the periodic table. A portion of the Nilsson diagram for protons is shown in Fig. 1, which contains particularly the $h_{9/2}$ and $h_{11/2}$ j-shells. The Tℓ and Au nuclei have λ values around 3 or 4 MeV on this chart, and this lies completely below the $h_{9/2}$ j-shell for $|\beta| \gtrsim 0.2$. In such a case the results of diagonalizing Eq. (1) do not depend very much on λ and are shown as a function of β in Fig. 2a. For the Tℓ nuclei we use $|\beta| = 0.11$, which is taken from the even-even Hg nuclei. One set of dots in Fig. 2a shows the location of the observed negative-parity levels in ^{199}Tℓ, taken as representative of the lighter Tℓ isotopes[5] and normalized to the lowest I = j calculated level (in parentheses). The rotational-bandlike character of these levels requires an oblate shape for these states according to Fig. 2a, which is consistent with the previous interpeetation.[5] We believe the explanation of: a) rotational bands in Tℓ nuclei; b) the approximate $\hbar^2/2\mathcal{I}$ value of these bands; and c) the sign and rough magnitude of the deviations from a pure-rotor spectrum, are rather convincing for a calculation with no adjustable parameters. To show that these calculations go over into the region of large deformation in reasonable fashion, we have indicated the predictions for ^{179}Re, where β is taken from ^{178}W and ^{180}Os, together with the observed states.[6] Again the order of the observed levels is correct as is the approximate spacing of the levels. This interpretation of these levels in ^{179}Re is also consistent with that previously made.[6]

The situation for the $h_{11/2}$ j-shell is shown in Fig. 2b, and is quite similar to that of Fig. 2a, except that all spins are one higher and a given level pattern occurs for the opposite sign of β. This reversal is a result of the fact that λ is now above the j-shell rather than below it. The predicted position of ^{195}Au is indicated, with $|\beta|$ taken from ^{196}Hg and ^{194}Pt, and the experimental levels[7] are again shown as dots. In this case, an oblate shape is clearly indicated by the decoupled-type Au spectrum and the order of levels and spacings are surprisingly well given. A lower λ-value should be used for ^{179}Re in both the $h_{9/2}$ and $h_{11/2}$ j-shells; however, the differences are not very large so we have not made separate figures. The previously identified[6] $\Omega = 9/2$ band in ^{179}Re is seen to be in rather good agreement with the calculations for a prolate shape. The odd-parity levels in the Ir isotopes are probably also derived from the $h_{11/2}$ j-shell, but we have not yet analyzed these states. In any case, since the Au nuclei are oblate for these levels and the Re nuclei are prolate, one might have to deal with triaxial shapes in the Ir nuclei.

Our recent results[8] on the high-spin states in the odd-A Hg nuclei indicate that decoupled bands exist, similar to those in the La isotopes.[4] A series of two or three stretched E2 transitions, whose energies approximate those of the adjacent even-even nuclei, were observed in ^{189,195,197,199}Hg. In one of these cases, the population of the known $i_{13/2}$ isomeric state was measured and shown to be large; suggesting that the E2 cascade populates this level. This is precisely what one would expect from the $i_{13/2}$ j-shell around $\beta = -0.1$ according to the present calculations. Thus in Tℓ, Hg, and Au, the levels from the high-j orbitals can be reasonably well accounted for by the particle-plus-rotor model, and consistently require oblate shapes.

The general features of the solutions in Eq. (1) can be easily recognized in Fig. 2. At large deformation (of either sign) a good rotational region occurs, corresponding to a strong-coupling between the particle and the deformation. In this coupling scheme, Ω, the projection of j on the symmetry axis, is a good quantum number of the system, requiring a mixture of R values, and the Ω-band lying lowest depends on the position of λ in the j-shell (which changes with deformation). As the deformation decreases, there is a region where the Coriolis effects, coupling the particle to the rotation and mixing Ω values, can be treated as a perturbation. At the other limit of zero deformation, there is no coupling between the particle and the (non-existent) deformation, resulting in the degeneracy of all the states corresponding to different orientations of a given j and R. The coupling scheme here has the core rotational angular momentum, R, as a good quantum number, requiring a mixture of Ω values. As the deformation increases from zero, there is a region where it can be treated as a perturbation in a particle-core weak-coupling model.[9]

If λ and the sign of β locate a nucleus near the high-Ω levels of a j-shell (in Fig. 1 for the Au region this would be prolate for $h_{11/2}$ and oblate, for $h_{9/2}$) the above two perturbation regions merge into each other, and one shifts rather suddenly from the strong-coupling to the weak-coupling region around $|\beta| = 0.1$. On this side rotational bands exist for $|\beta| > \sim 0.1$, with a normal spin sequence and $\Omega = j$ at low β-values, changing to $\Omega = j - 1$ as λ moves closer to that state. For a nucleus situated near the low-Ω levels (opposite to the above shapes) there is a broad region between $\beta \sim 0.1$ and $\beta \sim 0.3$ where neither of these coupling schemes applies. This is a region of intermediate coupling, and an outstanding feature here is the occurrence of the decoupled band of high-spin states ($I = j, j+2, j+4, \ldots,$ with energy spacings like those of the even-even core).[4] In this region a new coupling-scheme is approximated[10], in which α, the projection of j on the rotation axis, is a good quantum number, requiring mixtures of both R and Ω. The rotation axis can lie anywhere in the plane perpendicular to the symmetry axis. The most favorable states are the ones where j is aligned with this axis as well as possible ($\alpha = j$), and the high-spin decoupled states all have this configuration coupled with different average values of R to make up the different I-values. For $|\beta| > 0.3$, the coupling scheme on this side again tends toward pure-Ω values, approaching $\Omega = 1/2$ or $3/2$ on the right side of Fig. 2a, and $\Omega = 5/2$ on the left side of Fig. 2b, as determined by the position of λ.

We have seen that a model which treats consistently both deformation and rotation for nuclei with small deformations can lead to level schemes quite different from those obtained by simply constructing rotational bands on Nilsson levels. Under some conditions a new coupling scheme is realized where the projection of j on the rotation axis, rather than on the symmetry axis, is a good quantum number of the system. There is growing evidence that at least high-spin states from high-j orbitals can be represented by such a model for $|\beta| \gtrsim 0.1$. Many experiments are required in order to test the nature of these high-spin states in greater detail, to study the lower-spin states from the high-j orbitals, and to explore the applicability of this model to some lower-j orbitals. Such a simple picture is not likely to be sufficient in great detail, but the implied basic unity of the so-called "vibrational" region with the rotational one is very appealing.

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References

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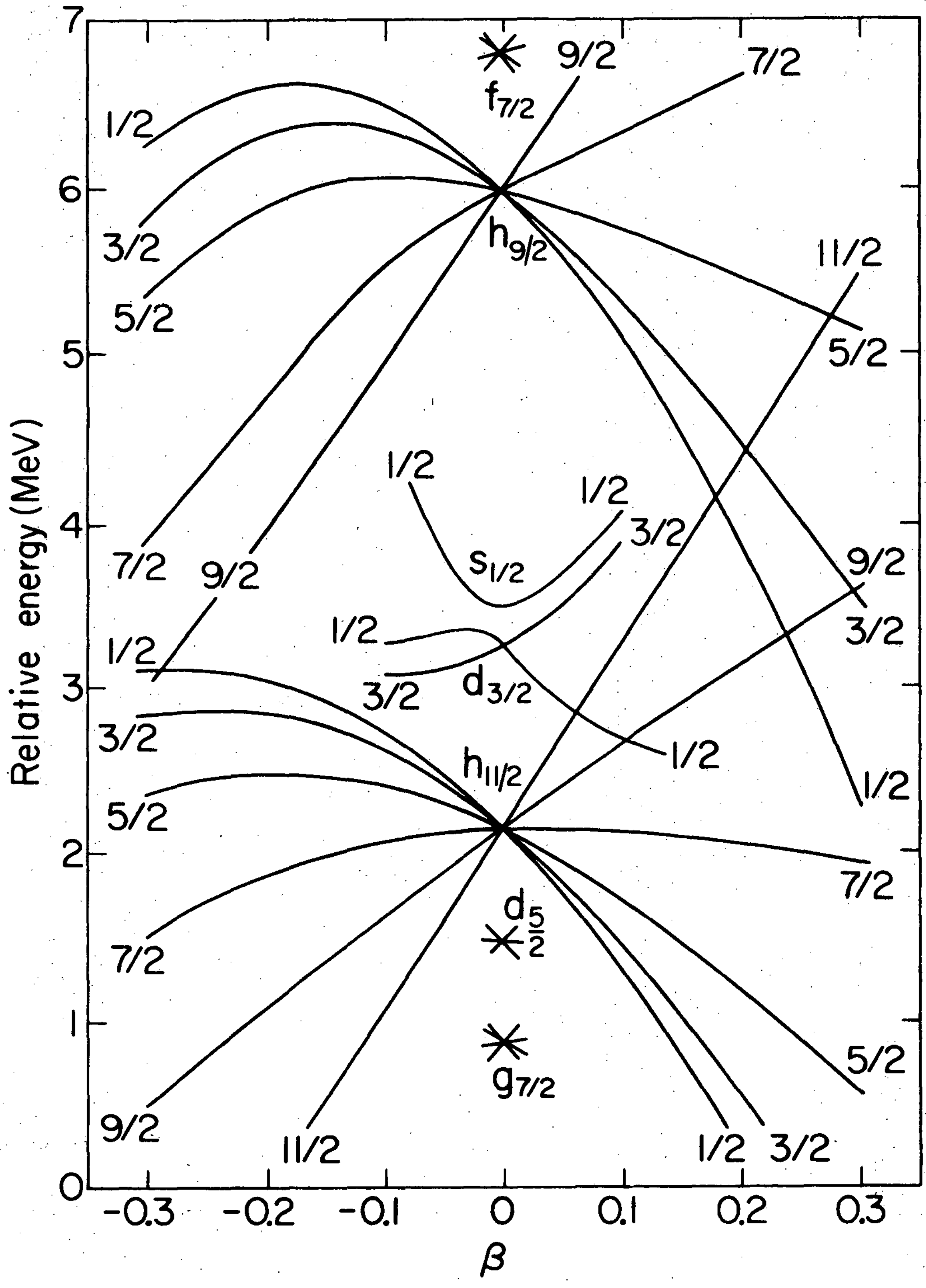
1. A. Bohr, Dan. Mat. Fys. Medd. 26, no. 14 (1952).
2. A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. 27, no. 16 (1953).
3. F. B. Malik and W. Scholz, Phys. Rev. 150, 919 (1966); 176, 1355 (1968) and others.
4. F. S. Stephens, R. M. Diamond, J. R. Leigh, T. Kammuri, and K. Nakai, Phys. Rev. Lett. 29, 438 (1972).
5. J. O. Newton, S. D. Cirilov, F. S. Stephens, and R. M. Diamond, Nucl. Phys. A148, 593 (1970).
6. J. R. Leigh, J. O. Newton, L. A. Ellis, M. C. Evans, and M. J. Emmott, Nucl. Phys. A183, 177 (1972).
7. J. Frana, A. Spalek, M. Fiser, and A. Kokes, Nucl. Phys. A165, 625 (1972).
8. D. Benson, Jr., M. R. Maier, R. M. Diamond, and F. S. Stephens, to be published.
9. A. de-Shalit, Phys. Rev. 122, 1530 (1961).
10. The concept that the whole spectrum in this region of β might be represented by a new and simple coupling scheme came out of discussions with A. Bohr and B. R. Mottelson. Mottelson has developed a slightly different scheme from the one described here, in which j is approximately perpendicular to the symmetry axis and oscillates about that position, while the core rotates in the plane perpendicular to the general direction of j.

Figure Captions

Fig. 1. A portion of the Nilsson diagram for protons, where only the $h_{9/2}$ and $h_{11/2}$ j-shells have been fully drawn. The ground states of the Tℓ and Au nuclei are in the $s_{1/2}$ and $d_{3/2}$ j-shells, which have been partially drawn.

Fig. 2. Solutions to Eq. (1) for the yrast states from the $h_{9/2}$ and $h_{11/2}$ j-shells, with $\lambda \approx 3.5$ MeV relative to Fig. 1. For H_{rot}, Eq. (3) was used, with $B/A = (1.6 \times 10^{-2})(1 - 14\beta^2)$ and $C/A = (8.8 \times 10^{-5})(1 - 14\beta^2)$ and $B=C=0$ if $|\beta| > 0.267$. The measured negative-parity levels in ^{199}Tℓ, ^{195}Au, and ^{179}Re are shown as dots, with dashed lines connecting each dot to the calculated level having that spin value.



Relative energy (MeV)
β
f7/2
h9/2
s1/2
d3/2
h11/2
d5/2
g7/2
1/2
3/2
5/2
7/2
9/2
11/2
0
1
2
3
4
5
6
7
-0.3
-0.2
-0.1
0.1
0.2
0.3
XBL725-2887

Fig. 1

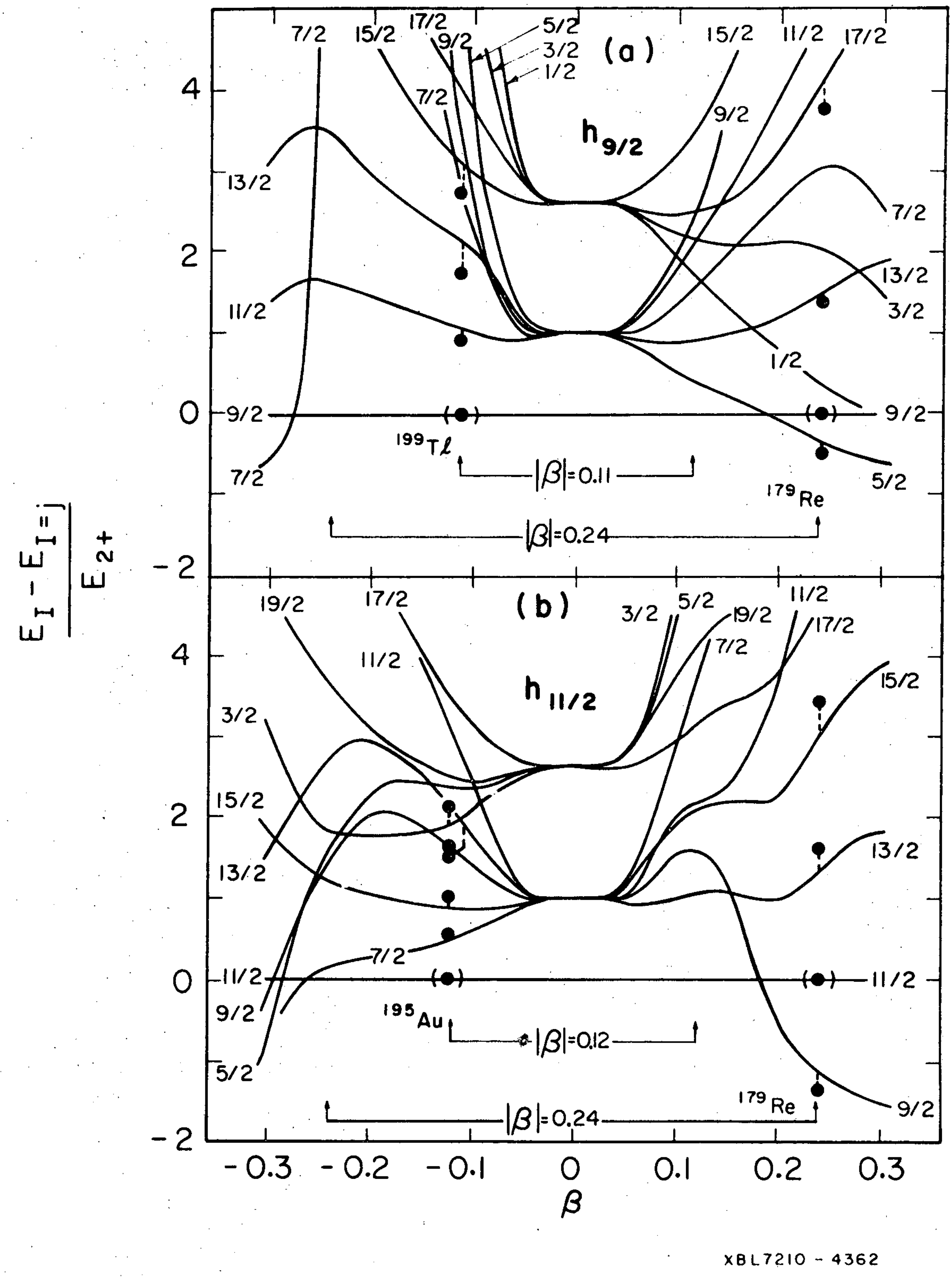

(a)
$h_{9/2}$
^{199}Tl
$|\beta|$=0.11
$|\beta|$=0.24
^{179}Re
(b)
$h_{11/2}$
^{195}Au
$|\beta|$=0.12
$|\beta|$=0.24
^{179}Re
$\frac{E_I - E_{I=j}}{E_{2+}}$
β
XBL7210 - 4362

Fig. 2

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