Lawrence Berkeley National Laboratory
Recent Work

Title

ON THE DESCRIPTION OF FERMION SYSTEMS IN BOSON REPRESENTATIONS. (IV). NUMERICAL CALCULATION OF QUADRUPOLE EXCITATIONS IN Cd, Sn, Te, Sm AND Pb

Permalink

https://escholarship.org/uc/item/6j40b5rn

Author

Sorensen, Bent.

Publication Date

1969-04-01

eScholarship.org Powered by the California Digital Library
University of California

Submitted to Nuclear Physics

UCRL-18903
Preprint

RECEIVED
LAWRENCE
RADIATION LABORATORY
1969
LIBRARY AND
DOCUMENTS SECTION

ON THE DESCRIPTION OF FERMION SYSTEMS IN BOSON REPRESENTATIONS. (IV). NUMERICAL CALCULATION OF QUADRUPOLE EXCITATIONS IN Cd, Sn, Te, Sm AND Pb

Bent Sørensen

April 1969

AEC Contract No. W-7405-eng-48

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

UCRL-18903

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

ON THE DESCRIPTION OF FERMION SYSTEMS IN BOSON REPRESENTATIONS. (IV). NUMERICAL CALCULATION OF QUADRUPOLE EXCITATIONS IN Cd, Sn, Te, Sm AND Pb[†])

Bent Sørensen[††])

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

April 1969

Abstract

Quadrupole vibrational excitations are described in a modified quadrupole plus pairing model, using the boson expansion method. The radial dependence of the quadrupole force is chosen so as to yield as average field a Wood-Saxon type potential. A qualitatively correct description is provided for the phase transition in the Sm-isotopes, but the energy spectra, transition probabilities and static quadrupole moments of nuclei near closed shells are barely consistent with experiment, the most puzzling disagreement being the sign of the static quadrupole moment of the first 2+ state in ^{122}Te.

[†]) Work performed under the auspices of the U.S. Atomic Energy Commission.

[††]) On leave from the Niels Bohr Institute, University of Copenhagen, Denmark.

1. Introduction

The use of simplified model interactions for the calculation of nuclear properties does call for the awareness of certain facts. First of all such simplified models are usually tailored to a certain set of nuclear properties and will give nonsense answers to questions which fall outside their domain. Secondly these model interactions may assume a structure of the nucleus which in certain respects is unrealistic. It should therefore be allowed to renormalize the model interaction in order to partly compensate for the difference between the actual system and that implied by the model. An example of this situation is the strength of the quadrupole interaction, which is uniquely fixed by requiring proportionality between the shape of the system and that of the average quadrupole field. Nevertheless the fact that the true interaction does create an average field which is different from that of the quadrupole force will in general imply that a choice of quadrupole strength different from the self-consistent one may give a better description of the actual system. This type of renormalization is of course not related to that required because of truncation of the configuration space.

The problems mentioned above are usually interwoven with the effects of approximations in the solution of the many-body problem to such an extent that it is impossible to obtain an independent judgement of the validity of those simplifications made in the interaction and those made in the diagonalization. We claim by the boson expansion method to have improved the treatment of the many-body problem to such an extent that we are able to make rather reliable statements concerning the model interaction itself. It is interesting to note that the application of the best available realistic two-body interactions

to the description of complex nuclear excitation has often employed inferior many-body techniques. Calculating for instance collective vibrations in RPA using effective interactions based on a Hamada-Johnson interaction[1]) one finds in several cases an imaginary energy for the lowest root, indicating merely the inappropriateness of RPA. If, on the other hand, one had used a phenomenological interaction for the RPA diagonalization, one would have chosen a renormalized interaction strength for which RPA reproduces the experimental energy, thus formally blaming the deficiency of RPA on the interaction[2]). The gain of insight achieved in this way is clearly not very big, yet such ways of proceeding did in fact for some time leave the impression that the nature of collective multipole vibrations near closed shells were fairly well understood. Only when improved experimental techniques revealed inconsistency with the theoretical predictions for higher lying parts of the collective spectrum it became obvious that something had to be modified. However, it remained nuclear, whether the problems had to be sought in the interaction or in the method of calculation, and the situation became even more uncertain, when experiments suggested large static moments in nuclei, which the theoretical models predicted should be nearly spherical in shape[3]).

Although the fourth order boson calculation presented here is far from an exact solution of the many-body problem, we think it provides a fairly clear indication of the type of results one can obtain by improving the diagonalization techniques, and it leaves a number of definite discrepancies which must be associated with details of the nuclear interaction not contained in the pairing plus quadrupole model, or at least not in the parts of these interactions which are effective in the present treatment.

In sect. 2 we discuss the model interaction employed in the calculations, and in sect. 3 the numerical results are presented after some general comments on the choice of parameters.

2. The Modified Quadrupole Plus Pairing Interaction

The general form of the interaction we are going to use is well known. It contains a single particle hamiltonian H_s containing the sum of the kinetic energies of the particles present in the system and their potential energies associated with the average field. Further a pairing interaction, H_P acting on T=1, J=0 pairs of particles with a fixed strength for each T_z, usually zero for T_z=0. The constantness of strength holds within a certain subspace in the single particle space, often assumed to be symmetric in dimension around the Fermi level. Outside this subspace no pairing force is present. Finally there is a quadrupole force H_Q acting among particle-hole pairs, so that the total hamiltonian may be written

$$H_s + H_P + H_Q = \sum_{\substack{j \\ (\nu=n,p)}} \hat{j}\, \varepsilon_j (a_j^+ \bar{a}_j)_0 + \frac{1}{4} \sum_{\substack{jj' \\ (\nu=n,p)}} \hat{j}\hat{j}'\, G_\nu$$

$$\times (a_j^+ a_{j'}^+)_0 (\bar{a}_j, \bar{a}_{j'},)_0 - \frac{\chi}{2\sqrt{5}} \sum_{\substack{j_1 j_3 \\ (\nu=n,p)}} \sum_{\substack{j_2 j_4 \\ (\nu'=n,p)}} \langle j_1 \| P_\nu(r) Y_2(\omega) \| j_3 \rangle$$

$$\times \langle j_2 \| P_{\nu'}(r') Y_2(\omega') \| j_4 \rangle \left((a_{j_1}^+ \bar{a}_{j_3})_2 (a_{j_2}^+ \bar{a}_{j_4})_2 \right)_0 , \quad (2.1)$$

where $\hat{j} = (2j+1)^{1/2}$ and $\bar{a}_{jm} = (-)^{j+m} a_{j-m}$ is the operator connected to a_{jm} by the time-reversal operation. We have assumed that χ is independent of (ν,ν'), since no definite knowledge of the opposite is available.†) The nuclear states are for brevity denoted j, but the summations shall extend

†) The summation over the isospin quantum number ν is put in parentheses below the sum signs for j-values having the same ν.

over n and ℓ as well. The pairing force H_P has proven successful in describing $J=0$ pairing type nuclear levels, but it does also imply a quadrupole particle-hole interaction because of the relation

$$(a_j^+ a_j^+)_0(\bar{a}_{j'} \bar{a}_{j'})_0 = -\sum_J \langle (jj)0(j'j')0;0|(jj')2(jj')2;0\rangle \times \left\{(a_j^+ \bar{a}_{j'})_2(a_j^+ \bar{a}_{j'})_2\right\}_0 + \text{contraction term.} \tag{2.2}$$

This addition to the quadrupole interaction will be neglected, since we try to chose $P(r)$ so as to make H_Q alone as physical as possible.

A convenient way of arriving at a representation in which we can focus on $J=2$ particle-hole operators, is to perform the BCS transformation

$$a_{jm}^+ = u_j \alpha_{jm}^+ + v_j \bar{\alpha}_{jm} \quad , \tag{2.3}$$

where $u_j^2 + v_j^2 = 1$. We thereby arrive at the hamiltonian we are going to use for the expansion in terms of quadrupole bosons

$$H = \sum_{\substack{j \\ (\nu=p,n)}} \hat{j}\, E_j\, (\alpha_j^+ \bar{\alpha}_j)_0 - \frac{\chi}{10} \sum_{\substack{Mj_1j_2(\nu=p,n) \\ j_3j_4(\nu'=p,n)}} \langle j_1 \| P_\nu Y_2 \| j_3\rangle \langle j_2 \| P_{\nu'} Y_2 \| j_4\rangle$$

$$\times \left\{ \frac{1}{2}(u_{j_1} u_{j_3} - v_{j_1} v_{j_3})(u_{j_2} u_{j_4} - v_{j_2} v_{j_4})(\alpha_{j_1}^+ \bar{\alpha}_{j_3})_{2M}^+ (\alpha_{j_2}^+ \bar{\alpha}_{j_4})_{2M}^+ \right.$$

$$- (u_{j_1} v_{j_3} + u_{j_3} v_{j_1})(u_{j_2} u_{j_4} - v_{j_2} v_{j_4}) \Big((\alpha_{j_4}^+ \alpha_{j_3}^+)_{2M}(\alpha_{j_2}^+ \bar{\alpha}_{j_4})_{2M}^+$$

$$\left. + \delta_{\nu\nu'} \frac{1}{2\hat{j}_3} (\alpha_{j_3}^+ \alpha_{j_3}^+)_{00}(\delta_{j_1j_2}\delta_{j_3j_4} - \delta_{j_1j_4}\delta_{j_2j_3}) \Big) \right.$$

$$+\frac{1}{4}(u_{j_1}v_{j_3}+u_{j_3}v_{j_1})(u_{j_2}v_{j_4}+u_{j_4}v_{j_2})\left((-)^M(\alpha^+_{j_1}\alpha^+_{j_3})_{2M}(\alpha^+_{j_2}\alpha^+_{j_4})_{2-M}\right.$$

$$\left.+(\alpha^+_{j_1}\alpha^+_{j_3})_{2M}(\alpha^+_{j_2}\alpha^+_{j_4})^+_{2M}-\frac{1}{\hat{j}_1}\delta_{\nu\nu'}(\alpha^+_{j_1}\bar{\alpha}_{j_1})_{00}(\delta_{j_1j_2}\delta_{j_3j_4}-\delta_{j_1j_4}\delta_{j_2j_3})\right)\Bigg\}$$

$$+\text{h.c.}\quad, \tag{2.4}$$

where E_j is the BCS quasi-particle energy. The hamiltonian (2.4) differs from that employed in I (ref. [4]), eq. (4.1)) in a number of details, for which we have come to the conclusion that (2.4) offers the most consistent physical picture. For the reasons given above the residual part of the pairing interaction is not allowed to add to the quadrupole interaction. This implies not only the neglect of the $J=2$ term in (2.2), but also of the non-BCS type $J=0$ pairing terms, which does contain boson terms with four $J=2$ boson operators. In writing H_Q in the quasi-particle representation we have kept the quadrupole character of each term and thus not normal ordered every term. Otherwise certain effects of the quadrupole force would by recoupling have been described by branches of $J\neq 2$, which we would neglect in our boson expansion, which for practical purposes is restricted to quadrupole bosons. However, two contraction terms appear in (2.4) which had been neglected in I. One describes the change in quasi-particle energies due to the quadrupole force, the other changes the quasi-particle number by two and hence describes the disturbance of the BCS solution induced by the presence of the quadrupole field. As the quadrupole force is of long range, the claim made in I that these effects could be assumed included in the average potential seems dubious, unless the single-particle energies were adjusted

phenomenologically. However, the calculations will show, that the influence of these contraction terms is small.

The fluctuation in particle number implied by the BCS approximation is fully inherent in our interaction (2.4), since no residual pairing interaction is present. An improved treatment in this respect will require boson expansions with both $J=0$ and $J=2$ bosons present. Without this the correct average particle number ensured for the BCS vacuum is changed by the presence of the quadrupole interaction. Our method of restoring average particle number is described in II (ref. [5])). It is there shown, that the fermion space may be mapped into the boson space in an infinite number of ways. The various expansions which are characterized by a set of parameters $\{y_j^0\}$ are given in ref. [6]). They define boson bases which are connected by unitary transformation, so the choice of parameters y_j^0 does not affect any physical results as long as no truncation is made. It is shown in II that by choosing constant $y_j^0 = y^0$ and using a suitable y^0 one can usually make the number of particles correct in one eigenstate of a given hamiltonian of the particle number violating kind, no matter whether the particle number non-conservation is due to truncations in the original fermion space or in its boson expansion. The calculations to be reported in sect. 3 show that it is not possible to make the particle number completely correct in this way, but that a certain choice of y^0, usually zero, produces a minimum in the deviation from the correct particle number, and this y^0 usually is the same for all lowlying states.

In addition to determining the boson expansions of the hamiltonian (2.4) and the number operator, we find those of the electric quadrupole operator, which as discussed in III (ref. [7])) is required for calculation of

potential energy surfaces. The anharmonic terms are kept only for the collective branch of excitation, which according to the discussion in III is chosen as that defined by a Tamm-Dankoff (TD) diagonalization. The non-collective branches are thus described by their TD wave functions and energies. Although one could improve them by adding anharmonic terms like one does for the collective branch (using the same computer code), this would not be a systematic improvement, since the higher order couplings between collective and non-collective branches is neglected. Our assumption is thus that the collective anharmonicities are more important than anharmonic couplings between collective and non-collective branches, which again are more important than non-collective anharmonicities. We feel that it is important to understand the anharmonicities of the collective branch before going into the complexities of deciding which other branches may be significantly coupled to this. It has already been mentioned in I, that we think that coupling-to-pairing excitations will influence the 0+ states in many nuclei. It is further suggested in I, that some non-collective quadrupole branches may be strongly coupled to "two-phonon" states. This seems to be true for states with TD energies close to that of collective states, according to the magnitude of coupling terms evaluated by Sano[8] using the boson expansion method.

The use of a TD collective boson implies several simplifications in the expansions of the various operators mentioned above. The explicit formulae are given in appendix 1.

2.1. AVERAGE POTENTIAL AND RADIAL SHAPE OF THE QUADRUPOLE FORCE

The single particle potential H_s may be thought of as the result of a Hartree-Fock (HF) calculation based on pure kinetic energies and the best available two-nucleon interaction. In that case the pairing plus quadrupole interactions may be visualized as approximations to the residual interaction. Such an average potential should allow a simple calculation of the properties of lowlying levels in odd nuclei adjacent to closed shell nuclei. Further the average potential should be related to the optical potentials used in the description of scattering and reaction processes involving a single nucleon. Such a relation requires knowledge of the energy dependence of the optical potential, extrapolated to the negative energy of the Fermi level. As the energy variation of the optical potential is rather slow (for positive energies) and the Fermi energy often close to zero (unbound), one may guess from the extreme lack of similarity between optical potentials calculated by the HF method (see e.g. ref. [9])), that the latter ones are not reliable enough for nuclear structure calculations at present. This conclusion is also drawn from attempts to calculate collective excitations by adding pairing plus quadrupole interaction to the HF average potential (using the boson method). We therefore use as single-particle hamiltonian the Woods-Saxon potential[10]) constructed in analogy to optical potentials. Another possibility would be to extract the single particle energies directly from one-particle stripping and pick-up experiments, using the sum-rule method of MacFarlane and French[11]), but this would not provide the single particle wavefunctions, which are essential for our application.

We then assume the spherical average potential

$$V_\nu^{sph}(r) = \frac{1}{2}(1 - \tau_{3\nu})\, V_{coul}(r) + W_\nu(1 - \frac{1}{2} v_{so} \lambda^2 (\underline{p} \times \underline{s}) \cdot \underline{\nabla}) \times (1 + \exp\{\frac{r-R_0}{a}\})^{-1} \qquad (2.5)$$

with $\lambda = (1 + A^{-1})\, \hbar/Mc$,

from which we calculate single-particle energies $\varepsilon_{n\ell j\nu}$ and wavefunctions $\psi_{n\ell j\nu}$ of the bound states. In the present application the configuration space is always truncated to an energy region around the Fermi energy, which is so small that continuum states do not have to be considered explicitly. If an enlarged configuration space were to be used, one would have to decide on a separate method for treating bound and unbound states.

Assuming now the quadrupole field to be related to the average potential by a self-consistency argument we can completely determine the quadrupole force. The procedure is described by Kumar and Sørensen[12]) and it consists in imposing a small quadrupole deformation on the system and requiring it to be maintained by the deformed average field produced by the quadrupole interaction. The approximations involved are i) proportionality between density and field is only secured to first order in the deformation parameter, ii) the range of the quadrupole force has to be assumed small in comparison to nuclear dimensions in order that the proportionality holds and iii) the contribution from the spin-orbit part of (2.5) is calculated only for axially symmetric deformation, considering for the deformed density distribution ρ that the third component of the projection of $(\underline{p} \times \underline{s}) \cdot \underline{\nabla}\, \rho$ on the unit vectors $\underline{n}_r$, $\underline{n}_\theta$ and $\underline{n}_\phi$ will be negligible. Under these assumptions the radial

part $P(r)$ of the quadrupole interaction (2.1) takes the form

$$P_\nu(r) = \frac{1}{2}(1 - \tau_{3\nu})\, r \frac{\partial}{\partial r} V_{coul}(r) + W_\nu \left\{ r \frac{\partial}{\partial r} - \frac{1}{2} v_{so} \lambda^2 \underline{\ell}\cdot\underline{s} \times \left(\frac{\partial}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} \right) \right\} \left(1 + \exp\left\{\frac{r-R_0}{a}\right\}\right)^{-1} . \qquad (2.6)$$

This radial function is strongly peaked at the nuclear surface R_0, in contrast to the r^2-dependence often employed for quadrupole interactions[2]). Examples of the differences in magnitudes of radial matrix elements of (2.6) and r^2 using either the corresponding Woods-Saxon wavefunctions $\psi_{n\ell j\nu}$ or harmonic oscillator wavefunctions are given in ref. [12]).

In the calculations to be presented we have used the modified quadrupole interaction implied by (2.6) and the single particle energies of the potential (2.5), but it should be mentioned that although we feel that the modification of the radial shape of the quadrupole force is essential, it is not absolutely required to use the W-S single-particle levels in our type of calculation. This is because of our neglect of any other multipole force $\lambda \neq 2$, which could be obtained in analogy to the derivation of (2.6). One may hope to be able to grossly incorporate the effects of these branches, which are not contained in the spherical W-S potential, by performing the quadrupole calculation with a renormalized set of single particle energies. Further it is probably correct to suspect that the W-S potential may not contain enough details to provide accurate predictions for quantities as sensitive as the single particle energies, whereas it is much more likely to be accurate enough for evaluating quadrupole matrix elements. One might further mention that the surface delta

interaction, which is a first order approximation to the multipole forces derived from a W-S potential, has been applied with considerable success[13]).

3. Numerical Results

3.1. CHOICE OF PARAMETERS

The parameters of the Woods-Saxon potential are given in table 1. All tables are collected in appendix 2. The single particle levels included in the calculations are given in table 2, together with effective force parameters and effective charges and masses. The pairing strengths are given the form

$$G_p = \frac{23}{A}\left(1 + 0.75\,\frac{N-Z}{A}\right)\ \text{MeV}\ ,$$
$$G_n = \frac{23}{A}\left(1 - 0.75\,\frac{N-Z}{A}\right)\ \text{MeV} \qquad (3.1)$$

in accordence with the symmetry energy in the empirical mass formula[14]). The constancy of the numerical factors appearing in (3.1) corresponds to the approximate constancy of the size of the configuration space considered explicitly. For the same reasons the quadrupole strengths $\chi_{nn} = \chi_{pp} = \chi_{np} = \chi$ have been chosen as a fixed scale factor (=3.1) times the self-consistent value[12]) implied by the proportionality between average density and field. The coefficients of the boson expanded hamiltonian are given in table 3, those of the quadrupole operator T_{2M} in table 6, both calculated on the basis of the formulae of appendix 1. A number of calculated and experimental quadrupole moments and deformation parameters are given in table 4 of appendix 2. As in figures etc. we consistently give energies in units of MeV, lengths in units of the oscillator parameter

$$b = r_0\left(\frac{4}{5}\left(\frac{2A}{3}\right)^{1/3}\right)^{1/2}\ \text{fm}\ , \qquad (3.2)$$

using the same r_0 as in evaluating matrix elements (table 1). Masses are in units of M_p and charges in units of $|e|$. Table 4 gives the electric quadrupole moment $Q_{e\ell}(2+)$ of the first 2+ state and the BE2 value for its decay to the ground state, BE2↓. We define the intrinsic electric quadrupole moment by the rotational model, $Q_0^{e\ell} = -7\, Q_{e\ell}(2+)/2$, and the RMS value of the deformation parameters by the BE2 value

$$\beta_{e\ell}^{RMS} = \frac{16\pi}{15} \left(\frac{2}{3}\right)^{1/3} \frac{(5\ BE2\downarrow)^{1/2}}{ZA^{1/3}} \quad . \tag{3.3}$$

Another value for $\beta_{e\ell}$ is extracted from the static moment

$$\beta_{e\ell} = \frac{\sqrt{5\pi}\, Q_0^{e\ell}\, b^2}{3Zr_0^2\, A^{2/3}} \quad . \tag{3.4}$$

The experimental values are based analogously on the measured BE2↓ and $Q_{e\ell}(2+)$. From the calculated $Q_m(2+)$ we define $Q_0^m = -7\, Q_m(2+)/2$ and

$$\beta_m = \frac{4\pi\, Q_0^m\, b^2}{3r_0^2\, A^{5/3}} \quad . \tag{3.5}$$

The abscissa on the potential energy plots calculated by the method described in III (ref. [7]) are intrinsic mass quadrupole moments, in some cases also a β_m scale is provided. Table 4 gives the positions of the lowest potential energy minima.

Experimental access to the value of β_m is provided by the analysis of scattering experiments in terms of a quadrupole expansion of the nuclear surface which is introduced as a deformation dependent form factor in a DWBA analysis or in the coupling potential in a coupled channel formalism.

The experimental mass quadrupole moments quoted are not obtained in this way, but are related to the electric moments by assuming $\beta_m = \beta_{e\ell}$

$$Q_m(2+) = \frac{A}{Z}\sqrt{\frac{5}{16\pi}}\, Q_{e\ell}(2+) \quad . \tag{3.6}$$

In table 5 we give expectation values of the number operator for a few lowlying calculated states.

We now present and discuss the results of the diagonalization of the boson hamiltonian and the evaluation of E2 transition probabilities, static quadrupole moments and potential energy surfaces according to the description given in III.

3.2. THE PHASE TRANSITION IN THE Sm ISOTOPES

Several pieces of evidence suggest that a major change in the magnitude of quadrupole deformation takes place in the Sm isotopes. Judging from ground state energies and lowlying quadrupole vibrational states, the transition is fairly smooth with ^{150}Sm being most far from both the rotational model and the spectra of nuclei around closed shells. Already ^{148}Sm has a fairly large 2+ quadrupole moment and the increase in both $B(E2, 2+ \rightarrow gr)$ and $Q_{e\ell}(2+)$ when going from ^{148}Sm to ^{152}Sm is again rather smooth. These remarks have been substantiated by a calculation of potential energy surfaces by Kumar and Baranger[23]) which, however, adjusted the force strengths so as to give the phase transition at the correct place rather than to reproduce the energy spectra, which would require a subsequent numerical integration[24]). We show in fig. 1 the energy spectra for $^{148-152}$Sm together with the results of our boson calculation, using the parameters listed in table 2 (case a and b in the fig.). The correction from contraction terms which is added in column b makes very little difference. In both calculations the trends of developing a ground state rotational band from the ground state, 2+ state and 4+ member of the "two-phonon" triplet, adding of course substantial components of higher phonon states, is in qualitative agreement with the observed one. In contrast to the harmonic approximation the fourth order boson results are not very sensitive to changes in the ratio of pairing to quadrupole strengths. In column c, we have increased χ by 30%, thereby improving the energies of the ground band. At the same time the higher states (β- and γ-vibrations in ^{152}Sm) gets pushed upwards, evidently (from the boson wave-function) due to the smallness of the diagonalization basis (7 bosons).

The apparent need for couplings to configurations with more than 7 bosons also suggests that the collective boson chosen becomes less and less realistic, the more deformed the system becomes. Since the non-collective states remain at approximately the same energy in all of the nuclei, we do not expect couplings to these to be important. This conclusion may not be safe, since the rotational model suggests that although the pairing gap in neighbouring spherical and deformed nuclei are of the same size, still the orbits involved may be completely different, which would also imply a change in the structure of the collective boson.

In fig. 2 we compare calculated and experimental B(E2) values and electric quadrupole moments. The agreement obtained is fair, keeping the effective charges constant (cfr. table 2). The lack of correlations in the higher calculated states expected from the discussion above imply a corresponding underestimation of the transition strengths. Interesting details can be learned from the comparison of electric and mass multipole moments and deformations given in table 4 of appendix 2. The deformation parameters $\beta_{e\ell}$ extracted from B(E2, 2+ → gr.) (β^{RMS}) and from $Q_{e\ell}(2+)$ are expected to be the same for well deformed nuclei, whereas β^{RMS} should be larger than β when approaching spherical nuclei. The isotope ^{150}Sm has $\beta > \beta^{RMS}$ in contradiction to any macroscopic model. The trends are reproduced by the anharmonic boson calculation. On the other hand the calculated mass quadrupole moments of 2+ states are about a factor of two smaller than the experimental values obtained from the assumption $\beta_m = \beta_{e\ell}$. Direct values of β_m extracted from fits by coupled channel calculations to scattering experiments using a surface expansion in the

optical potential cannot be considered very accurate, since they depend on the other optical model parameters, which have to be taken from often rather distant regions believed to be spherical. A likely reason for the shortcoming of the theoretical estimate is the explicit inclusion of only 11 j-shells, leading to large effective charges e_p, e_n and masses, which we have taken as $m_p = m_n = e_p = 3/\sqrt{2}$. With $e_n = \sqrt{2}$, we actually get approximately 65% of the contributions to the electric E2 matrix elements from neutron configurations. Thus, counting neutrons and protons on the same footing for mass E2 matrix elements does not make the difference of $\frac{A}{Z}$ implied by extracting $Q_m(2+)$ from $Q_{e\ell}(2+)$ using $\beta_m = \beta_{e\ell}$. A calculation using much larger configuration space will be necessary if one wants to test the validity of assuming $\beta_m = \beta_{e\ell}$.

Figs. 3 to 5 show the potential energy functions calculated from the boson hamiltonian and fig. 6 the $\gamma = 0°$ contours for the three Sm-isotopes. The sharp raise in Q at large $|Q|$ is presumably tied together with the too high excitation energies of β- and γ-vibrational states (fig. 1), and is therefore expected to be modified by the higher order expansion terms. Quite generally a truncated power series expansion of this kind is expected to produce too sharp minima and too step raise towards infinity when the highest order term is dominant. In order to allow a numerical integration also kinetic energy functions are needed. These are very anharmonic, as one can guess from the difference between the positions of potential energy minima and the calculated expectation values of the mass quadrupole moment (cfr. table 4). For instance the potential energy for ^{152}Sm is minimum for $\beta_m = 0.21$, whereas the value calculated with the boson wavefunctions is 0.12.

The BCS approximation used for the pairing force and the neglect of the residual pairing force causes the expectation value of the number operator, which is correct in the pure quasi-particle vacuum, to be wrong when evaluated in the quadrupole correlated boson eigenstates. The deviations of the particle numbers for a few of the lowlying eigenstates given in table 5 of appendix 2, tells whether the correlations beyond those of the underlying BCS vacuum are symmetric around the Fermi energy or not. Since each quasi-particle operator carries an incorrect particle number we expect the number-deviations to increase with excitation energy. The table shows that the particle numbers are worst for ^{148}Sm and improves towards ^{152}Sm, obviously because of the moving upwards of the Fermi energy in the direction of the center of gravity of the configuration space used. To the extent that the isotopes considered are superfluid, one will not expect even deviations in particle number of a couple of units to influence the physical predictions significantly.

3.3. THE Pb REGION

The doubly closed shell nucleus ^{208}Pb and the two adjacent even isotopes are considered in fig. 7. The energy spectra are in agreement with the experimental ones, except for the first excited 0+ state of ^{206}Pb. Since the dominant component of the first 2+ state is $(f_{5/2}^{-1} p_{1/2})$, the 0+ state becomes mainly $(f_{5/2}^{-2})$ with respect to the ^{208}Pb core. This is exactly the lowest non-collective pairing state, and the observed depression of its energy can be perfectly described by applying the residual pairing force, as shown in ref. [44]). The $j=1/2$ spin of the lowest available orbit is the special reason why in this case the pairing and quadrupole states are not two distinct levels.

The spectra are almost harmonic, in particular for ^{210}Pb, due to the fact that the $(h_{9/2})^2$ configuration has an amplitude of 0.992 in the basic boson. Transition matrix elements are given in fig. 8. The effective charges (see table 2) are rather large despite the fact that no major part of the wave function lies outside the configuration space considered. The reason is the smallness of the proton configuration space, but the relative uncorrelated wavefunctions obtained ensure that the effective charge method will work extremely well in this region. For some time it was considered puzzling that the E2 transitions in ^{208}Pb were stronger than in the neighbouring nuclei, which have much smaller 2+ energies. The reason is of course the large degeneracies of the configurations entering in the ^{208}Pb 2+ state, and as expected the static quadrupole moment is smaller here and positive in all three cases.

Equipotential surfaces for the potential energy are shown in figs. 9-11, and the $\gamma = 0^\circ$ contours in fig. 12. One should notice that the ^{210}Pb spectrum is extremely harmonic <u>despite</u> the asymmetry in the potential energy.

3.4. THE Z = 50 REGION

We first consider two N=66 nuclei, the proton closed shell nucleus ^{116}Sn and the two-proton-hole nucleus ^{114}Cd. Later we look at two N=70 nuclei, the proton closed shell nucleus ^{120}Sn and the two-proton-particle nucleus ^{122}Te. The calculated and experimental energy levels are shown in fig. 13. The neutron single particle levels of the Woods-Saxon potential do with the chosen force parameters fit the excited Sn-states reasonably well, so presumably the disagreement of energy levels for ^{114}Cd is mainly due to proton configurations. Since the proton pairing gap is extremely sensitive to both pairing strength and position of single particle levels, a slight adjustment could make the energy of the first excited 2+ state agree with experiment. However, the magnitudes of anharmonic terms would remain approximately the same and the next excited states therefore still be above twice the 2+ energy. This criticism seems to apply to all the calculated spectra and is thus directed either against the model interaction or the approximations of the boson method. We take up the subject again below after having presented the results for transition probabilities and moments. One can remark that the residual parts of the pairing interaction can explain the experimental finding of several 0+ states in the two-phonon region[44]). On the other hand the position of the third 2+ state in ^{114}Cd can not easily be understood, since both the Woods-Saxon single particle levels and another set proposed by Bès and Dussel[57]) (after making a least square fit to the energies of the odd mass Cd-isotopes) will not allow any non-collective state to come at this low energy. The BE2 systematics, shown in fig. 14,

seems even to suggest that this 2+ state is a three-phonon[†]) vibration, in which case the spectrum is not as harmonic as otherwise believed. If true this increasing depression of two- and three-phonon states with respect to both the harmonic position and our calculated values is most likely to point to a deficiency of the pairing plus quadrupole interaction since the character of the third and fourth order anharmonic terms in our hamiltonian definitely is to lower the energy of the ground state and one-phonon state and at the same time raise the energies of the two- and three-phonon states. This pattern is independent of the truncation problem arising in the diagonalization of the boson hamiltonian, being dictated by the ratios of magnitudes of third and fourth order terms (see table 3) and the overall positive signs of the fourth order terms. The observed structure would require either a third order term w^{31} much larger than the fourth order terms or alternatively negative fourth order terms (which would make it necessary to include still higher order terms).

The E2 data of the two N=66 systems shown in fig. 14 shows that the calculated ^{116}Sn 2+ quadrupole moment has the correct sign, which implies that the neutron configurations in ^{114}Cd, which presumably are not very much changed by the p-n quadrupole interaction, do also contribute by a positive amount to the 2+ quadrupole moment. The proton contribution coming from the

---

[†])We use for simplicity the expression n-phonon state to denote the collective state whose largest component is that with n bosons.

$(g_{9/2})^{-2}$ configuration is negative but because of the large degeneracy of this level (and the absence of close levels other than j=1/2) the 2+ wavefunction does not contain very much proton correlation, which again implies that the proton and neutron contributions to $Q_{e\ell}(2+)$ almost cancel, in contradiction to most of the experimental indications.†) A plausible way to obtain a large negative $Q_{e\ell}(2+)$ is to lower the proton pairing gap as mentioned above, in order to make the 2+ state mainly a proton state and mainly a $(g_{9/2})^{-2}$ configuration (this may also involve weakening the parts of the interaction acting on neutrons). Thereby we can explain both the quadrupole moment (especially if the configuration space is enlarged so as to lower the neutron effective mass; the net contribution to Q(2+) from higher proton configurations is hopefully small (hopefully because the lowest configurations not included in the present calculation contribute by a positive amount) and formally also the almost harmonic two-phonon states. For the higher part of the excitation spectrum large anharmonicities appear as mentioned still to be necessary. The calculated B(E2) values are in reasonable agreement with experiments, leaving open the question of which excited 0+ state belongs to the collective branch. The boson wavefunctions are fairly rapidly converging (for instance the amplitudes in the second 2+ state

†) One of the experimental determinations of the static quadrupole moment (ref. [31]) disagrees beyond experimental errors with the remaining ones. The authors quote two values consistent with their experiment, and in preliminary communications they quoted the opposite sign (given in parentheses on fig. 14).

of the two possible 6-phonon components are -0.0161 and -0.0106, those of the three possible 7-phonon components -0.0279, 0.0007 and 0.0003), the expansion of the hamiltonian (table 3) less so. However, since the small third order term and positive fourth order terms are true features of the model interaction, we will have to conclude that it is to blame for the failure to describe this nucleus. The quadrupole operators converge somewhat better than the hamiltonian (table 6), but the ideal hamiltonian for describing ^{114}Cd would be one in which the hamiltonian has extremely small anharmonic terms but where M^{21} of the proton quadrupole operator is large. Alternatively the contributions to the two-phonon states from higher order terms in the hamiltonian have to cancel accidentially. This statement is in agreement with the results of the third order calculation performed earlier[45]).

Our claim made earlier about the mass deformation β_m being badly determined by the coupled channels calculations of inelastic scattering angular distributions using an expansion of the nuclear surface in terms of deformation parameters can be substantiated by comparing wavefunctions obtained for ^{114}Cd by such fits (ref. [35]) using the formalism of ref. [56])) with ours. The amplitude of the pure two-boson component in the wavefunctions of the "two-phonon" J=0, 2 and 4 states implied by the $\beta_{\lambda J}$ parameters are 0.50, 0.56 and 0.78 as compared to 0.60, 0.86 and 0.86 in the boson wavefunctions. In view of the earlier discussion the extremely large admixtures implied by the analysis of the (pp') scattering experiment appears to be unreasonable and ours even too large.

The ^{114}Cd and ^{116}Sn potential energies, which are shown in figs. 15 - 17, at least qualitatively show how the excitation spectrum can be

nearly harmonic, although $Q=0$ is a maximum. At least for Sn the structure in V_{pot} is hidden by the zero point motion, and only the asymmetry of the otherwise regular well is responsible for the static quadrupole moments.

We now turn to the two $N=70$ nuclei, also shown in fig. 13. Here the agreement of excitation energies is much better, with the exception of the second 2+ in ^{122}Te, and so are the B(E2) values (fig. 18). A reason for the boson method to work better for ^{122}Te than for ^{114}Cd is the much larger amount of admixtures in the proton part of the wave function. In contrast to all other cases included in this series of examples the deviation in ground state particle number is for ^{122}Te smallest not for $y^0 = 0$ but for $y^0 = 0.05$. However this one-parameter adjustment of particle number is as anticipated in III achieved by a decrease in correlation, which is insignificant for the energy spectrum but decreases the transition probabilities by as much as 25%. We think that the influence of y^0 on the transition operator is unphysical and therefore use the $y^0 = 0$ operators (table 6 and fig. 18).

The ^{120}Sn $Q_{e\ell}(2+)$ is too large to be in agreement with experiments which claim that the static 2+ quadrupole moment is nearly the same for all the Sn-isotopes and therefore close to the revised value of the Rutgers reorientation experiments[37]). A much more striking disagreement occurs in ^{122}Te, where the magnitudes are in agreement but the signs different. Here no cancellations take place as in ^{114}Cd, so the quoted experimental sign is really controversial. One may argue by again assuming the neutron $Q(2+)$ contribution to be nearly identical to the (experimental) ^{120}Sn value, i.e. between -1.8 and +5.4. The proton contributions from the main components

are all positive (both from shell-model estimates and the boson calculation) and fairly large. It is thus completely impossible from the theoretical picture employed to explain a large negative 2+ moment.

Theoretical potential energy surfaces are shown in fig. 19 to 21. If the neutron contributions are reduced so as to yield the experimental $Q_{e\ell}(2+)$ in Sn, we expect the Te deformation to be only half of the calculated one. Other calculations with certain anharmonic effects have been made in this region, none of which included excitation of more than $2p2h$ [46-48]. These methods neglect what corresponds to our third order terms and thus do not give correct static quadrupole moments. This can, however, be healed and the diagonalization methods are attractive in allowing second order admixtures between collective and non-collective states. The projection used by ref. [48]) further removes spurious states, which is found to be important. There are in the 2 plus 4 quasi-particle diagonalization method two sources of redundancies, one connected with the over-completeness of the $4qp$ basis and another with the BCS type of spurious. The former is taken care of by the boson expansion method, whereas the particle number projection may improve the boson wave functions. This, however, can not be a systematic improvement, since the structure information of projected wavefunctions cannot be larger than that contained in the generator states. We therefore think that the only real improvement is explicitly to include the residual parts of the pairing interaction in the boson hamiltonian (or, of course, not to use a number non-conserving basis). We have tried in part to do this, including contributions from the residual pairing interaction to the J=2

bosons, but it became evident, that only by explicitly considering both $J=0$ and $J=2$ boson quanta can a real improvement be obtained.

4. Conclusions

By performing calculations ranging from a doubly closed shell nucleus which presumably has less correlations than any other (since p-n correlations are weaker in ^{208}Pb than in lighter closed-shell nuclei) to a quadrupole deformed nucleus (^{152}Sm) we believe to have learned a number of regularities of the boson method which enables us further to draw definite conclusions concerning the model interaction.

In Pb it seems as if the truncation of the configuration space made is allowable, and both the 4th order truncation of the boson expansions and the 7th order truncation of the collective diagonalization basis are high enough or even higher than necessary. The couplings between collective and non-collective branches are small but so are the collective correlation effects. Otherwise any deviations between calculated and experimental quantities (e.g. the energy difference between the ^{206}Pb and ^{210}Pb 2^+ states must be ascribed to deficiencies in the interaction.

In the Sn region we expect the onset of inaccuracy of the 4th order truncation of the hamiltonian, whereas the 7th order boson basis still do quite well for lowlying states, judging from the magnitudes of 6- and 7-boson components in the wave functions. The discrepancies for the second and higher excited states could have some origin in couplings to non-collective states, but in conjunction with the spectacular disagreement with respect to static quadrupole moments (and some BE2 values) for the Cd and Te cases, it seems inevitable not be conclude that the interaction lacks fundamental parts (or that the experiments are incorrect, which might be a possibility in the case of the ^{122}Te reorientation experiment (only one, unpublished

result), but less likely in ^{114}Cd, where a large number of experiments disagree with our Q(2+) value and only one agrees within uncertainties). This inappropriateness of the pairing plus quadrupole interaction could never have been revealed in a harmonic-type approximation, since the experimental spectrum up to two-phonon levels appears harmonic whereas in fact we have shown that this is not the prediction when using the P+Q interaction. In cases like this one could take advantage of working with a phenomenologically chosen collective boson and use the quadrupole particle-phonon coupling scheme to calculate the anharmonicities and transition matrix elements[55]).

Finally, in the Sm-region neither an enlarged configuration space nor couplings to non-collective degrees of freedom can be considered unimportant as one goes towards more deformed isotopes, but one may hope that the large number of important couplings to rather uniformly spaced levels of different properties make the renormalization methods work decently. However, we here experience the natural limitation of renormalization based on simple scale factors, e.g. in the difference between calculated mass and electric quadrupole moments, which probably is unphysical.

The convergence of the collective hamiltonians are actually better in Sm than in the Sn region, but due to the large ratio w^{20}/w^{21} the 7th order truncation of the boson basis becomes increasingly insufficient from ^{148}Sm to ^{152}Sm. Thus only the calculated ground state band is even qualitatively correct, and it is in this situation no longer possible to make any statements concerning the model interaction.

The author wants to thank Dr. J. Gunn for patience and help in handling the lengthy computer codes made for these calculations, which were all

run at the NEUCC and NORDITA-Niels Bohr Institute facilities in Copenhagen. Stimulating discussions with Dr. F. Dönau and Professor B. Mottelson are highly appreciated.

Appendix 1

TD Normal Mode Boson Expansion

After the hamiltonian (2.4) has been subjected to the basic boson expansion defined in I, we define the TD representation by the unitary transformation

$$c_n^+(JM) = \sum_{ij} r_{ij}(n)\, b^+(ij\ JM) = 2 \sum_{i\leqslant j} \frac{r_{ij}(n)\, b^+(ij\ JM)}{1+\delta_{ij}} \tag{A.1}$$

where

$$r_{ji}(n) = \theta_{ij}\, r_{ij}(n) \quad , \quad \theta_{ij} = -(-)^{i+j} \quad \text{and}$$

$$1 = \sum_{ij} r_{ij}(n)^2 = 2 \sum_{i\leqslant j} \frac{r_{ij}(n)^2}{1+\delta_{ij}} \quad . \tag{A.2}$$

For the TD diagonalization

$$H_{TD} = 2 \sum_{\substack{ij\ (\nu=p,n) \\ i'j'(\nu=p,n)}} z^{2l}(ij\ i'j') \sum_M b^+(ij\ 2M)\, b(ij\ 2M)$$

$$= \sum_n E_n \sum_M c_n^+(2M)\, c_n(2M) \tag{A.3}$$

we need the coefficients

$$z^{21}(ij\ i'j') = \delta_{ii'}\,\delta_{jj'}\,\delta_{\nu\nu'}\left(E_i + \frac{\chi}{10\hat{j}^2}\sum_{j''}\delta_{\nu''\nu}(u_j v_{j''} + u_{j''}v_j)^2\, P^2_{jj''}\right)$$

$$- \frac{\chi}{20}(1-2y^0)(u_i v_j + u_j v_i)(u_{i'}v_{j'} + u_{j'}v_{i'})\, P_{ij}\, P_{i'j'}$$

$$- \frac{5\chi}{2}\,\delta_{jj'}\,\delta_{\nu\nu'}\sum_{j''}\delta_{\nu''\nu}(u_i u_{j''} - v_i v_{j''})(u_{i'}u_{j''} - v_{i'}v_{j''})\, P_{ij''}\, P_{i'j''}$$

$$\times\, W(ij\ 22;2j'')W(i'j'\ 22;2j'') - \frac{5\chi}{2}\,\delta_{\nu\nu'}(u_i u_{j'} - v_i v_{j'})$$

$$\times\,(u_j u_{i'} - v_j v_{i'})\, P_{ij'}\, P_{i'j}\, W(ij\ 22;2j')W(i'j\ 22;2j') \quad , \tag{A.4}$$

where the Bohr-Mottelsen reduced matrix elements[14]) are

$$P_{ij} = \langle i\|P_\nu\, Y_2\|j\rangle = \theta_{ij}\langle j\|P_\nu\, Y_2\|i\rangle = \hat{i}\,\frac{\langle im'|P_\nu\, Y_{2M}|jm\rangle}{\langle jm2M|im'\rangle} \tag{A.5}$$

It is seen that the non-diagonal part of z^{21} does not consist of a single separable term, for which reason the TD equation can not be reduced to a dispersion relation.

The lowest root of the TD diagonalization defines the collective boson c^+_{2M}, in terms of which the anharmonic part of the collective hamiltonian (eq. (2.14) of III) is expressed with coefficients

$$w^{20} = -\chi\,(1-2y^0)\,X^2/\sqrt{5} + \frac{1}{4}\chi\, X_4\,\sqrt{5} \tag{A.6}$$

$$w^{31} = -\chi\, X_3\sqrt{\frac{5}{2-4y^0}} + 2\chi\ X\, X_2\ \sqrt{5(2-4y^0)} \tag{A.7}$$

$$w_J^{41} = 5 \chi \hat{J} X X_{5J} \tag{A.8}$$

$$w_J^{42} = -25 \chi \hat{J} W(2222;2J) X_1^2 + 5 \chi \hat{J} \chi$$

$$\times \sum_{J'} \hat{J}'^2 W(222\,2;J'J) X_{5J'} \tag{A.9}$$

where

$$X = \sum_{i \leqslant j(\nu=n,p)} \frac{r_{ij}}{1+\delta_{ij}} (u_i v_j + u_j v_i) P_{ij} \quad , \tag{A.10}$$

$$X_1 = \sum_{i \leqslant j(\nu=n,p)} \frac{r_{ij}}{1+\delta_{ij}} \sum_{j(\nu')} \delta_{\nu'\nu} W(ij\ 22,2j') \{r_{j'j}(u_i u_{j'} - v_i v_{j'}) P_{ij'}$$

$$+ \theta_{ij} r_{j'i} (u_j u_{j'} - v_j v_{j'}) P_{jj'}\} \quad , \tag{A.11}$$

$$X_2 = \sum_{ij(\nu=n,p)} (u_i u_j - v_i v_j) P_{ij} \sum_{j(\nu')} \delta_{\nu'\nu} r_{ij'} r_{jj'} W(ij'\ 22;2j) \quad , \tag{A.12}$$

$$X_3 = \sum_{i\leqslant j(\nu=n,p)} \frac{r_{ij}}{1+\delta_{ij}} \left(\sum_{j'(\nu')} \delta_{\nu'\nu}\, r_{jj'}\, r_{ij'}\, W(ij'\ 22;2j) \right) \left\{ \frac{1}{\hat{i}^2} \sum_{j''(\nu'')} \delta_{\nu''\nu} \right.$$

$$\times (u_{j''}v_i + u_i v_{j''})(u_{j''}v_i - v_{j''}v_i)\, P^2_{j''i} + \frac{1}{\hat{j}^2} \sum_{j''(\nu'')} \delta_{\nu''\nu}\, (u_{j''}v_j + u_j v_{j''})$$

$$\left. \times (u_{j''}u_j - v_{j''}v_j)\, P^2_{j''j} \right\} \quad , \tag{A.13}$$

$$X_4 = \sum_{i\leqslant j(\nu=n,p)} \frac{r_{ij}}{1+\delta_{ij}} \sum_{i'\leqslant j'(\nu')} \delta_{\nu'\nu} \frac{r_{i'j'}}{1+\delta_{i'j'}} \left\{ (u_{i'}v_j + u_j v_{i'})(u_i v_{j'} + u_{j'}v_i) \right.$$

$$\times P_{i'j}\, P_{ij'} \sum_J \hat{J}^2 \left(W(i'jJ2;2i)W(i'j'J2;2i) + W(j'iJ2;2j) \right.$$

$$\left. \left. \times W(j'i'J2;2j) \right) + \theta_{i'j'}(u_{j'}v_j + u_j v_{j'})(u_i v_{i'} + u_{i'}v_i)\, P_{j'j}\, P_{ii'} \right.$$

$$\left. \times \sum_J \hat{J}^2 \left(W(j'jJ2;2i)W(j'i'J2;2i) + W(i'iJ2;2j)W(i'j'J2;2j) \right) \right\} \quad , \tag{A.14}$$

$$X_{5J} = \sum_{ij(\nu=n,p)} (u_i v_j + u_j v_i)\, P_{ij} \sum_{i'j'(\nu')} \delta_{\nu'\nu}\, r_{i'j'}\, r_{ij'}\, r_{i'j}\, W(ijJ2;2i')$$

$$\times W(ij'J2;2i') \quad , \quad J = 0,\ 2,\ 4 \quad . \tag{A.14}$$

It should be mentioned that there is a great simplicity in the equations above, which is not present if the RPA representation is used (as it was for some of the examples shown in III). In connection with eq. (2.4) the effects of two contraction terms were pointed out. These effects, which are referred to as self-energy corrections in sect. 3, are connected with the second term in line one of eq. (A.4) and with the first term in eq. (A.7). Our computer code for calculating the matrix elements (A.4) and the anharmonic coefficients (A.6) to (A.9) has the option of leaving out the self-energy terms. The first term in w^{20} (eq. (A.6)) describe the "backward-going graphs" of RPA, whereas the second term contains fourth order contribution to the stability criterion. No similar fourth order contributions are present in the leading order term, which is simply

$$w^{21} = \frac{\sqrt{5}}{2} E_n \quad . \tag{A.16}$$

Together with the boson expansion of the hamiltonian we get that of the number operator

$$N = n + N^0 + N^{21} (c^+ \bar{c})_0 + N^{31}\{(c^+ c^+ \bar{c})_0 + (c^+ \bar{c} \bar{c})_0\} \quad , \tag{A.17}$$

$$N^0 = y^0 \sum_{j(\nu=n,p)} \hat{j}^2 (u_j^2 - v_j^2) \quad , \tag{A.18}$$

$$N^{21} = 2\sqrt{5} \sum_{j(\nu=n,p)} (u_j^2 - v_j^2) \sum_{j'(\nu')} \delta_{\nu\nu'} r_{jj'}^2 \quad , \tag{A.19}$$

$$N^{31} = \frac{10\sqrt{5}}{(2-4y^0)^{1/2}} \sum_{j(\nu=n,p)} u_j v_j \sum_{i'j'(\nu')} \delta_{\nu'\nu} W(j'i'\ 22;2j)\ r_{i'j} r_{jj'}\ r_{i'j'} \tag{A.20}$$

and the quadrupole moment operator

$$T_{2M} = T^{10}\ (c^+ + \bar{c})_{2M} + T^{21}\ (c^+ \bar{c})_{2M} + \sum_J T_J^{31} \Big[((c^+ c^+)_J \bar{c}) + (c^+ (\bar{c}\ \bar{c})_J) \Big]_{2M} \tag{A.21}$$

$$T^{10} = -\frac{(2-4y^0)^{1/2}}{\sqrt{5}} \sum_{i\leq j(\nu=n,p)} (1+\delta_{ij})^{-1} r_{ij}\ t_{ij}\ (u_i v_j + u_j v_i) \quad , \tag{A.22}$$

$$T^{21} = \frac{10}{\sqrt{5}} \sum_{ij(\nu=n,p)} t_{ij}\ (u_i u_j - v_i v_j) \sum_{j'(\nu')} \delta_{\nu'\nu}\ r_{ij'}\ r_{jj'}\ W(ij'\ 22;2j) \quad , \tag{A.23}$$

$$T_J^{31} = \frac{5\hat{J}}{(2-4y^0)^{1/2}} \sum_{ij(\nu=n,p)} t_{ij}\ (u_i v_j + u_j v_i) \sum_{J'} \hat{J}'^2\ W(2222;J'J) \times \sum_{i'(\nu')} r_{i'j} W(ijJ'2;2i') \sum_{j'(\nu')} \delta_{\nu'\nu}\ r_{ij'}\ r_{i'j'}\ W(ij'J'2;2i') \quad , \tag{A.24}$$

where

$$t_{ij} = t \langle i \| r^2 Y_2(\omega) \| j \rangle \tag{A.25}$$

is defined in terms of effective charges

$$t = (e_p \delta_p + e_n \delta_n)/b^2 \tag{A.26}$$

if T_{2M} is the electric quadrupole operator $\mathcal{M}(E2M)$ and in terms of an effective mass

$$t = m/b^2 \tag{A.27}$$

if T_{2M} is the mass quadrupole operator Q_{2M}.

Appendix 2

Tables

Table 1. Parameters of average potential (2.5). In order to describe the symmetry energy we choose a potential depth $W_\nu = W_0 + W_1 \frac{\underline{t}\cdot\underline{T}}{A}$ ($\nu = 1$ for proton, -1 for neutron), where $\underline{T}$ is the isospin of the A-particle system and $\underline{t}$ that of the particle. The radius characterizing the density is assumed to be of the form $R_0 = r_0 A^{1/3}$ as also the Coulomb radius $R_c = r_c A^{1/3}$.

W_0	W_1	v_{s0}	r_0	a	r_c
-51 MeV	132.4 MeV	32	1.27 fm	0.67 fm	1.27 fm

Table 2. Single-particle levels explicitly included in the boson calculations. Their energies were determined by the Woods-Saxon potential specified in table 1. Further the effective quadrupole strength χ, the effective charges e_p; e_n and the effective mass $\underline{m}$ defined through eqs. (A.26) and (A.27). The pairing strengths are given by eq. (3.1).

	proton levels	neutron levels	χ MeV^{-1}	e_p	e_n	m
^{114}Cd	$0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$	$0g_{9/2}, 1d_{5/2}, 0g_{7/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}$	0.00151	1.5	1.0	1.5
^{116}Sn	$0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}, 1d_{5/2}, 0g_{7/2}$	$1d_{5/2}, 0g_{7/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}, 1f_{7/2}$	0.00144	1.5	1.0	1.5
^{120}Sn	$0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}, 1d_{5/2}, 0g_{7/2}$	$1d_{5/2}, 0g_{7/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}, 1f_{7/2}$	0.00142	1.5	1.0	1.5
^{122}Te	$0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}, 1d_{5/2}, 0g_{7/2}$	$0g_{9/2}, 1d_{5/2}, 0g_{7/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}$	0.00135	1.5	1.0	1.5
^{148}Sm	$0g_{7/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}$	$1f_{7/2}, 0h_{9/2}, 2p_{3/2}, 0i_{13/2}, 1f_{5/2}, 2p_{1/2}$	0.00102	$3/\sqrt{2}$	$\sqrt{2}$	$3/\sqrt{2}$
^{150}Sm	$0g_{7/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}$	$1f_{7/2}, 0h_{9/2}, 2p_{3/2}, 0i_{13/2}, 1f_{5/2}, 2p_{1/2}$	0.00103	$3/\sqrt{2}$	$\sqrt{2}$	$3/\sqrt{2}$
^{152}Sm	$0g_{7/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 0h_{11/2}$	$1f_{7/2}, 0h_{9/2}, 2p_{3/2}, 0i_{13/2}, 1f_{5/2}, 2p_{1/2}$	0.00104	$3/\sqrt{2}$	$\sqrt{2}$	$3/\sqrt{2}$
^{206}Pb	$0h_{11/2}, 0h_{9/2}, 1f_{7/2}$	$0h_{9/2}, 1f_{7/2}, 0i_{13/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2}, 0i_{11/2}, 2d_{5/2}, 3s_{1/2}$	0.0005	1.725	1.15	1.725
^{208}Pb	$0h_{11/2}, 0h_{9/2}, 1f_{7/2}$	$0i_{13/2}, 1g_{9/2}, 0i_{11/2}$	0.0005	1.725	1.15	1.725
^{210}Pb	$0h_{11/2}, 0h_{9/2}, 1f_{7/2}$	$0i_{13/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2}, 0i_{11/2}, 2d_{5/2}, 3s_{1/2}$	0.0005	1.725	1.15	1.725

UCRL-18903

Table 3. Boson expansion coefficients for the collective hamiltonian (III, eq. (2.14)), corresponding to the parameter choices of the preceeding two tables[†]).

	w^{20}	w^{21}	w^{31}	w_0^{41}	w_2^{41}	w_4^{41}	w_0^{42}	w_2^{42}	w_4^{42}
^{114}Cd	-1.643	0.685	0.098	0.250	0.137	0.167	0.211	0.146	0.205
^{116}Sn	-2.162	1.215	-0.289	0.370	0.212	0.236	0.303	0.214	0.300
^{120}Sn	-1.249	1.384	-0.870	0.290	0.179	0.125	0.197	0.147	0.230
^{122}Te	-1.535	0.850	-0.774	0.259	0.110	0.158	0.176	0.162	0.183
^{148}Sm	-0.930	0.755	-0.448	0.215	0.102	0.119	0.151	0.124	0.162
^{150}Sm	-1.267	0.742	-0.697	0.219	0.114	0.113	0.148	0.124	0.164
^{152}Sm	-1.582	0.728	-0.684	0.193	0.108	0.092	0.132	0.103	0.150
^{206}Pb	-0.145	1.086	-0.326	0.112	0.047	0.048	0.045	0.071	0.062
^{208}Pb	-0.620	4.791	-0.171	0.094	0.058	0.004	0.044	0.032	0.076
^{210}Pb	-0.118	0.844	-0.148	0.056	0.025	0.038	0.038	0.038	0.037

[†]) The sign of w^{31} depends on a phase choice in the BCS transformation. It is the relative sign of w^{31} and the E2 operator (M^{10}) which mainly determines the sign of the quadrupole moment.

Table 4. Electric and mass quadrupole matrix elements and deformations. The quantities are defined in sect. 3-1. The experimental BE2's are from the compilation of ref. [17]), the $Q_{e\ell}(2+)$'s from ref. [25]), and the incomplete selection of β_m quasi-experimental values (sign undetermined) from refs. [26,35,40,42,43,49-54,59]).

	$Q_{e\ell}(2+)$		BE2↓		$Q_0^{e\ell}$		$Q_m(2+)$		Q_0^m		$\beta_{e\ell}^{RMS}$		$\beta_{e\ell}$		β_m Ex				β_m Th	β for V_{min} Th
	Ex	Th	Ex	Th	Ex	Th	Ex	Th	Ex	Th	Ex	Th	Ex	Th	n	p	d	α		
^{114}Cd	-10.9 ±3	-2.9	37.2 ±3	26.8	38.2	10.3	-8.1	-1.1	28.4	3.9	0.17	0.15	0.15	0.04	0.17 0.22				0.02	0.10
^{116}Sn	1.8 ±4	4.4	16.0 ±2	26.0	-6.3	-15.4	1.3	1.9	-4.6	-6.7	0.11	0.14	-0.02	-0.06	0.13 0.14			0.11	-0.03	-0.10
^{120}Sn	1.8 ±4	8.1	15.0 ±2	16.3	-6.3	-28.3	1.4	3.6	-3.5	-9.0	0.10	0.11	-0.02	-0.11	0.12	0.12	0.12	0.10 0.12	-0.06	-0.09
^{122}Te	-9.0 ±4	13.7	41.7 ±4	46.2	31.5	-47.9	-6.9	5.6	24.1	-19.6	0.16	0.17	0.11	-0.17					-0.10	-0.13
^{148}Sm	-12.2 ±7	-12.1	49.7 ±5	47.2	42.4	42.3	-9.2	-4.9	32.2	17.2	0.14	0.14	0.12	0.12					0.06	0.10
^{150}Sm	-20.3 ±3	-19.6	73.2 ±4	93.0	71.0	68.5	-15.5	-8.0	54.2	28.0	0.17	0.19	0.20	0.19					0.10	0.16
^{152}Sm	-29.9 ±10	-23.5	187.0 ±6	141.4	104.5	82.3	-23.2	-9.4	81.1	32.9	0.27	0.24	0.29	0.23				0.21	0.12	0.21
^{206}Pb		4.9	6.0 +1,-2	5.9		-17.1		2.3		-8.1	0.03	0.03		-0.03		0.05			-0.02	0
^{208}Pb		1.4		12.3		-4.9		0.6		-2.1		0.05		-0.01		0.04			-0.005	0
^{210}Pb		2.9		5.2		-10.2		1.4		-4.9		0.03		-0.03					-0.01	0

Table 5. Deviations from the correct ones of particle numbers for the boson wave function, i.e. $\langle N-n \rangle$ where N is given by eq. (A.17) of appendix 1.

	Eigenstate J_i			
	0_1	0_2	2_1	4_1
^{114}Cd	-0.559	-1.561	-1.994	-3.788
^{116}Sn	0.145	0.695	0.656	1.325
^{120}Sn	-0.002	0.152	0.046	0.129
^{122}Te	-0.738	0.650	0.929	2.762
^{148}Sm	0.412	1.304	1.966	4.022
^{150}Sm	0.177	0.233	0.563	0.983
^{152}Sm	0.148	0.143	0.392	0.633
^{206}Pb	-0.012	-1.214	-1.407	-3.723
^{208}Pb	0	0	0	0
^{210}Pb	0.025	2.351	2.827	7.424

Table 6. Boson expansion coefficients for the collective part of the $\lambda = 2$ transition operator, eq. (A.21). The parameters are again given in table 1 and 2. The unit is b^{-2} but no effective mass or charge has been multiplied in.

	protons					neutrons				
	T^{10}	T^{21}	T^{31}_0	T^{31}_2	T^{31}_4	T^{10}	T^{21}	T^{31}_0	T^{31}_2	T^{31}_4
^{114}Cd	0.759	-0.344	-0.060	-0.045	-0.058	2.440	0.234	-0.132	-0.089	-0.128
^{116}Sn	0.468	0.050	-0.004	-0.003	-0.007	2.985	0.169	-0.233	-0.159	-0.226
^{120}Sn	0.341	0.026	-0.001	-0.001	-0.002	2.468	0.923	-0.226	-0.146	-0.263
^{122}Te	0.980	0.327	-0.055	-0.052	-0.060	2.257	0.372	-0.129	-0.089	-0.128
^{148}Sm	-1.033	-0.095	0.016	0.011	0.018	-2.071	-0.684	0.246	0.185	0.260
^{150}Sm	-1.192	-0.129	0.028	0.019	0.031	-2.385	-0.875	0.205	0.147	0.222
^{152}Sm	-1.318	-0.159	0.038	0.026	0.042	-2.596	-0.674	0.141	0.098	0.158
^{206}Pb	0.097	0.002	-0.001	-0.001	-0.002	1.906	1.650	-0.368	-0.298	-0.433
^{208}Pb	0.501	0.028	-0.096	-0.062	-0.161	2.120	0.464	-0.093	-0.061	-0.155
^{210}Pb	0.076	0.001	-0.000	-0.000	-0.000	1.783	0.903	-0.295	-0.221	-0.285

References

1) T. Kuo and G. Brown, Nucl. Phys. A114 (1968) 241

2) L. Kisslinger and R. Sorensen, Rev. Mod. Phys. 35 (1963) 853; R. Uher and R. Sorensen, Nucl. Phys. 86 (1966) 1

3) J. deBoer et al., Phys. Rev. Letters 14 (1965) 564

4) B. Sørensen, Nucl. Phys. A97 (1967) 1

5) B. Sørensen, Nucl. Phys. A119 (1968) 65

6) B. Sørensen, Progr. Theoret. Phys. 39 (1968) 1468

7) B. Sørensen, Nucl. Phys.

8) M. Sano, JAERI-1158 (1968) p. 71

9) J. Parikh and J. Svenne, Phys. Rev. 174 (1968) 1343

10) R. Woods and D. Saxon, Phys. Rev. 95 (1964) 577

11) J. French and M. MacFarlane, Nucl. Phys. 26 (1961) 168

12) K. Kumar and B. Sørensen

13) R. Arvieu and S. Moszkowski, Phys. Rev. 145 (1966) 830; A. Plastino et al., Phys. Rev. 145 (1966) 837

14) A. Bohr and B. Mottelson, Nuclear Structure, Benjamin, New York, to be published

15) R. Keddy et al., Nucl. Phys. A113 (1968) 676

16) D. Eccleshall et al., Nucl. Phys. 78 (1966) 481

17) P. Stelson and L. Grodzins, Nucl. Data 1 (1965) 21

18) J. Simpson et al., Nucl. Phys. A94 (1967) 177

19) G. Goldring et al., Proc. of 3rd Conf. on Reaction between Complex Nuclei, U. C. Press, Berkeley, 1963, p. 278

20) J. deBoer et al., Phys. Rev. 134 (1964) B1032

21) G. Goldring and U. Smilansky, Phys. Rev. Letters 16 (1965) 151

22) L. Varnell et al., Nucl. Phys. A127 (1969) 270

23) K. Kumar and M. Baranger, Nucl. Phys. A110 (1968) 529

24) M. Baranger and K. Kumar, Nucl. Phys. A122 (1968) 241 and 273

25) J. Eichler and J. de Boer, in Advances in Nucl. Phys., Vol. 1, (ed. M. Baranger and E. Vogt) Plenum, New York 1968

26) D. Hendrie et al., Phys. Letters 26B (1968) 127

27) P. Stelson, Proc. Summer Study Group on Phys. of Emp. Tand. V. d. Graff Region, Vol. 3, p. 1005, BNL 1965

28) R. Stockstad et al., Nucl. Phys. A92 (1967) 319

29) J. Glenn and J. Saladin, Phys. Rev. Letters 19 (1967) 33

30) G. Schilling et al., Phys. Rev. Letters 19 (1967) 318

31) J. Simpson et al., Phys. Letters 27B (1968) 633

32) A. Bäcklin et al., Nucl. Phys. 80 (1966) 154; R. Smither, Phys. Rev. 124 (1961) 183

33) P. Stelson and F. McGowan, Phys. Rev. 110 (1958) 489

34) P. Stelson and F. McGowan, Phys. Rev. 121 (1965) 209

35) P. Stelson et al., Nucl. Phys. A119 (1968) 14

36) A. Kleinfeld et al., Bull. Am. Phys. Soc. 12 (1967) 564

37) A. deBoer, private communication

38) F. McGowan and P. Stelson, Phys. Rev. 115 (1958) 489

39) P. Stelson et al., Phys. Rev. 170 (1968) 1172

40) K. Yagi et al., Nucl. Phys. A111 (1968) 129

41) Gangskii et al., Sovj. Phys. JETP 15 (1962) 711

42) P. Stelson et al., Nucl. Phys. 68 (1965) 97

43) N. Baron et al., Phys. Rev. 146 (1966) 861

44) B. Sørensen, to be published

45) B. Sørensen, Phys. Letters 24B (1967) 328

46) T. Tamura and T. Udagawa, Nucl. Phys. 53 (1964) 33

47) L. Hsu and J. French, Phys. Letters 19 (1965) 135

48) P. Ottaviani et al., Phys. Rev. 153 (1967) 1138

49) V. Lewis et al., Nucl. Phys. A117 (1968) 673

50) V. Edwards, Nucl. Phys. A101 (1967) 17

51) M. Koike et al., Phys. Letters 26B (1968) 288

52) W. Makofske et al., Phys. Rev. 174 (1968) 1429

53) I. Kumabe et al., J. Phys. Soc. Japan 25 (1968) 14

54) O. Jarvis et al., Nucl. Phys. A102 (1967) 625

55) B. Mottelson, "private" communication

56) T. Tamura, Suppl. Prog. Theor. Phys. 37 and 38 (1966) 383

57) D. Bès and G. Dussel, Nucl. Phys.

58) G. Igo, private communication

59) G. Vallois et al., Phys. Letters 24B (1967) 512

Figure Captions

Fig. 1. Comparison between J = 0, 2, 4 energy levels in ^{148,150,152}Sm from experiments and from boson calculations without (columns a and c) and with (column b) the contraction terms discussed in sect. 2. Columns a and b correspond to the parameter choice given in appendix 2, whereas the quadrupole strength χ has been increased by 30% for column c. Dashed lines represent states which do not belong to the collective quadrupole branch. In the theoretical spectra only J = 2 non-collective (TD) states are shown.

Fig. 2. B(E2) values and static quadrupole moments in ^{148,150,152}Sm. The energies are not to scale. The experimental information is labelled corresponding to refs. [15]) (a), [16]) (b), [17]) (c), [18]) (d), average of [19-21]) (e) and finally the number in parentheses represent relative BE2 values for each upper level taken from ref. [22]). The calculated values correspond to the calculation labelled a in fig. 1, for which parameters are listed in appendix 2.

Fig. 3. Equipotential surfaces for ^{148}Sm.

Fig. 4. Equipotential surfaces for ^{150}Sm.

Fig. 5. Equipotential surfaces for ^{152}Sm.

Fig. 6. Potential energy at $\gamma = 0°$ for Sm-isotopes.

Fig. 7. Comparison of J = 0, 2, 4 experimental levels in ^{206,208,210}Pb with those of the boson calculation (including the contraction terms, using parameters listed in tables 1 and 2). Dashed lines in the theoretical spectra indicate the lowest non-collective 2+ states. The experimental levels of ^{210}Pb, all the highlying ones in ^{208}Pb and a few levels in

^{206}Pb have only been observed in two-neutron transfer reactions[58]) and may hence represent states not belonging to the quadrupole branch, at the same time as this explains the non-observation of the multiphonon levels in ^{210}Pb, where the pureness of the harmonic picture will imply very weak transfer cross sections.

Fig. 8. B(E2) values and static quadrupole moments in Pb-isotopes. Energies are not to scale. The experimental B(E2) is from ref. [17]).

Fig. 9. Equipotential surfaces for ^{206}Pb.

Fig. 10. Equipotential surfaces for ^{208}Pb.

Fig. 11. Equipotential surfaces for ^{210}Pb.

Fig. 12. Potential energy at $\gamma = 0°$ for 3 Pb-isotopes.

Fig. 13. Comparison of J = 0, 2, 4 experimental levels in ^{114}Cd, ^{116,120}Sn and ^{122}Te with those of the boson calculation (without contraction terms, parameters of table 1 and 2). Dashed lines represent non-collective states. A number of such 0+ states in the experimental spectrum are known to be pairing states.

Fig. 14. B(E2) values and static quadrupole moments in ^{114}Cd and ^{116}Sn. The energies are not to scale. Labels correspond to refs. [3]) (a and d), [25]) (b, this represents an average of refs. [18,27-30]), of which at least one completely disagrees with the adopted BE2 values), [31]) (c and d), [17]) (e), [32]) (f), [33,34]) (g), [36]) (h), [37])(i), [38]) (j) and [39]) (k). Numbers in parentheses are relative values. According to ref. [37]), the Q(2+) value of ref. [36]) seems to be in error. The ambiguity of the sign of $Q_{e\ell}(2+)$ from ref. [31]) is discussed in the text.

Fig. 15. Equipotential surfaces for ^{114}Cd.

Fig. 16. Equipotential surfaces for ^{116}Sn.

Fig. 17. Potential energy at $\gamma = 0°$ for two N=66 nuclei.

Fig. 18. B(E2) values and static quadrupole moments in ^{120}Sn and ^{122}Te. The energies are not to scale. Labels correspond to refs. [39] (a), [17] (b), [37] (c), [27] (d), [34] and [41] (e). The calculated transitions in ^{122}Te are taken for $y^0 = 0$ rather than 0.05, which would reduce the important ones by about 25%.

Fig. 19. Equipotential surfaces for ^{120}Sn.

Fig. 20. Equipotential surfaces for ^{122}Te.

Fig. 21. Potential energy at $\gamma = 0°$ for two N=70 nuclei.

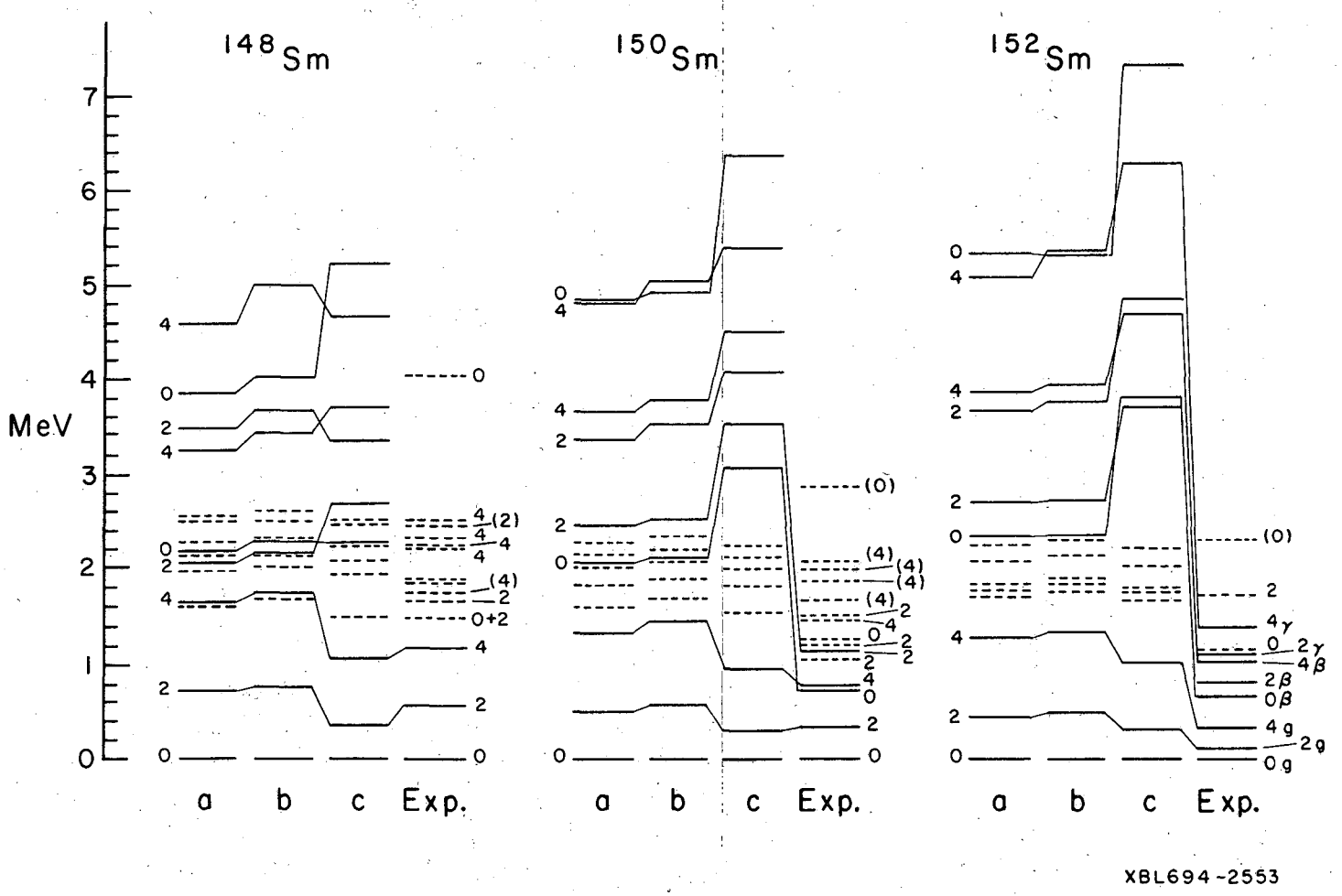
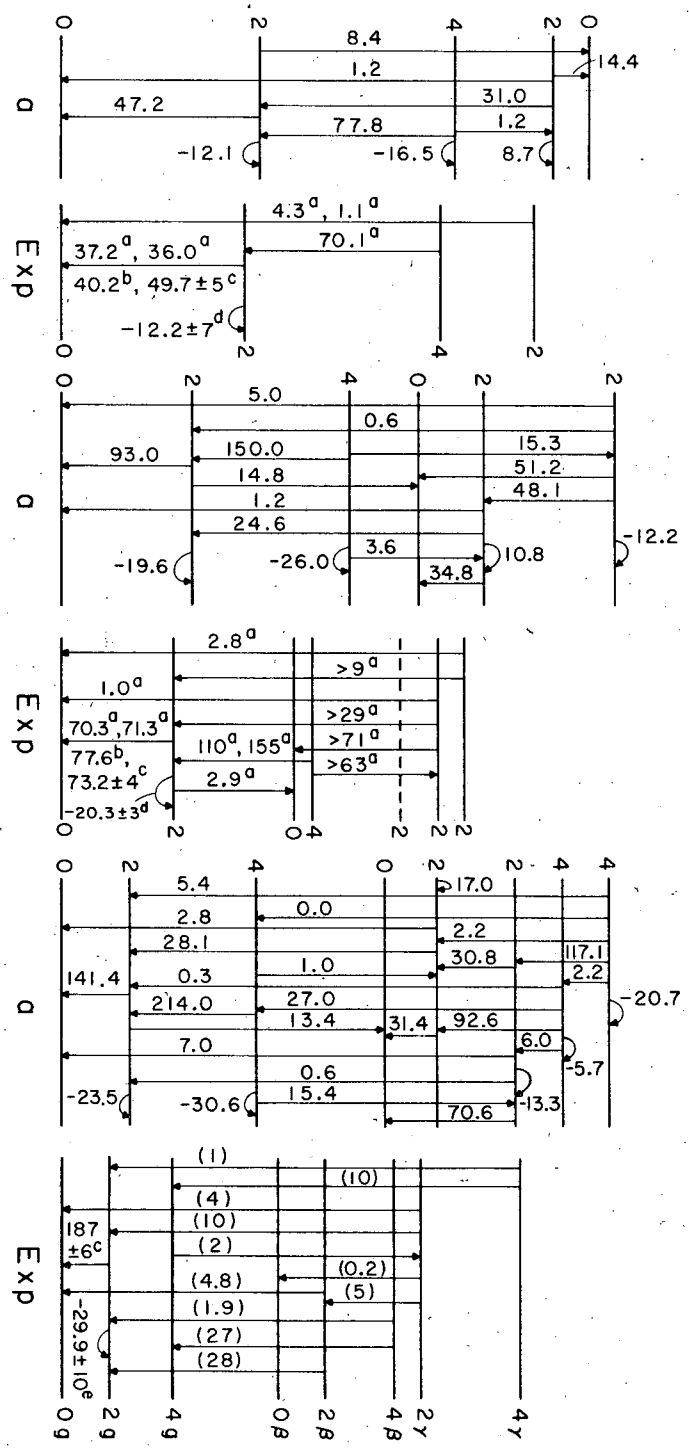

148Sm
150Sm
152Sm
MeV
0
1
2
3
4
5
6
7
a
b
c
Exp.
(0)
(2)
(4)
0+2
0γ
2γ
4γ
0β
2β
4β
0g
2g
4g
XBL694-2553

Fig. 1.



XBL694-2554

Fig. 2.

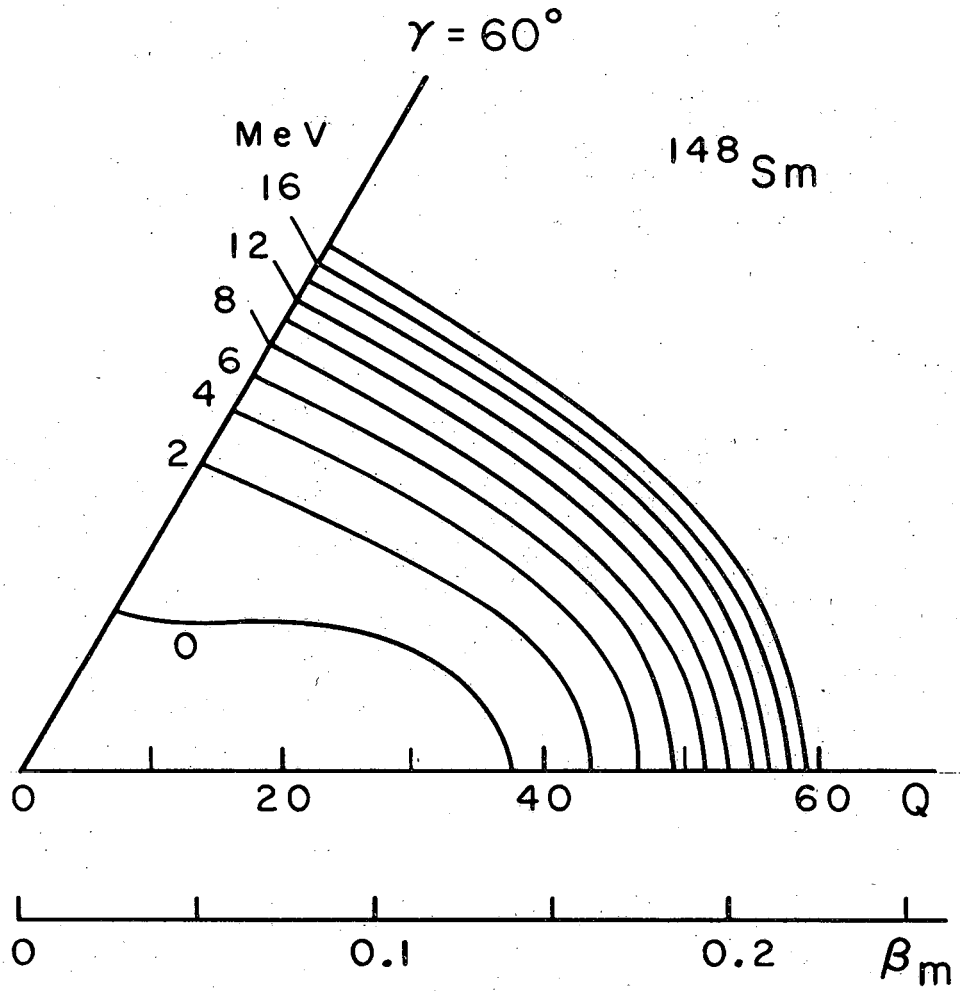
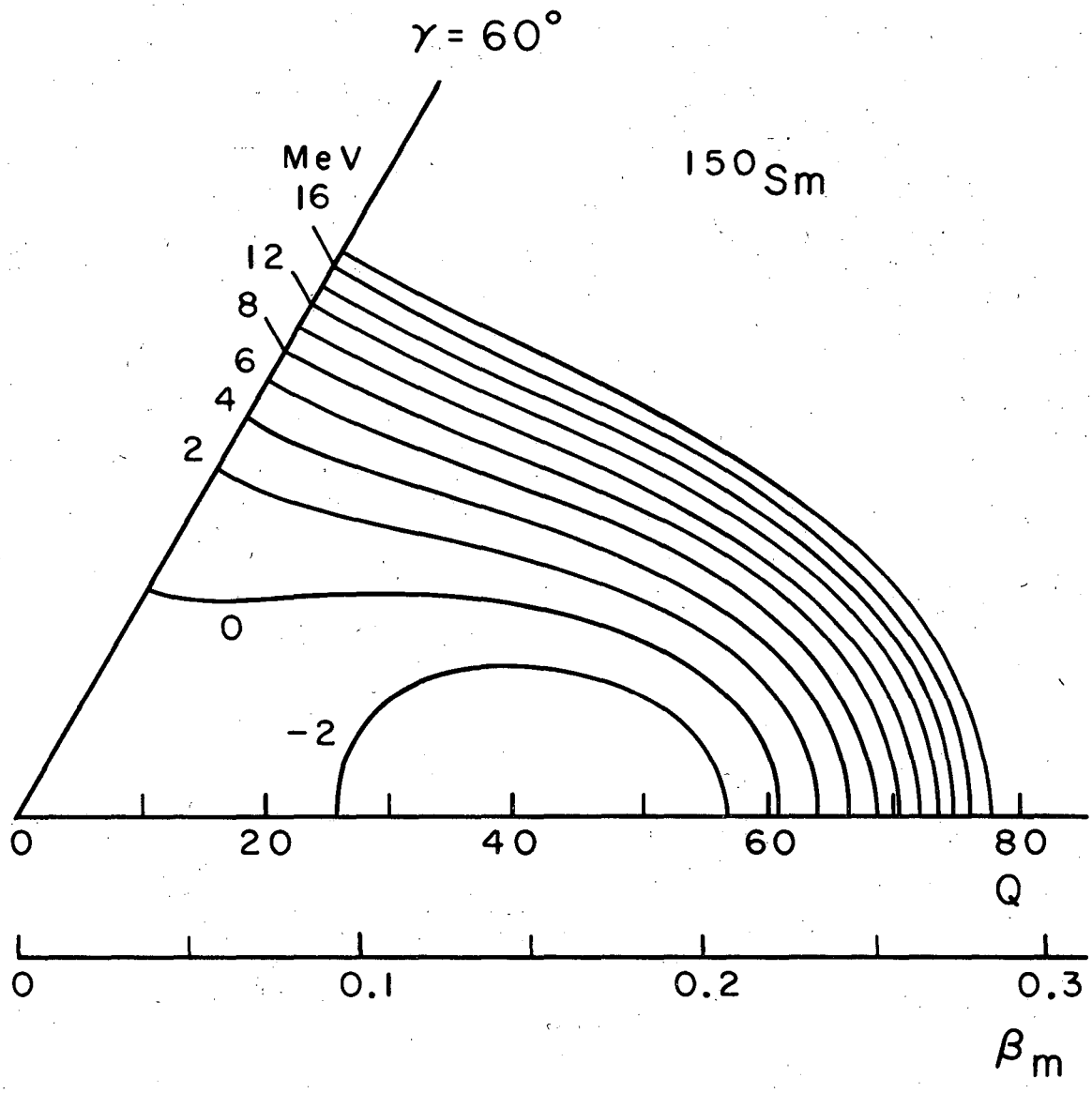


Fig. 3.



XBL694-2557

Fig. 4.

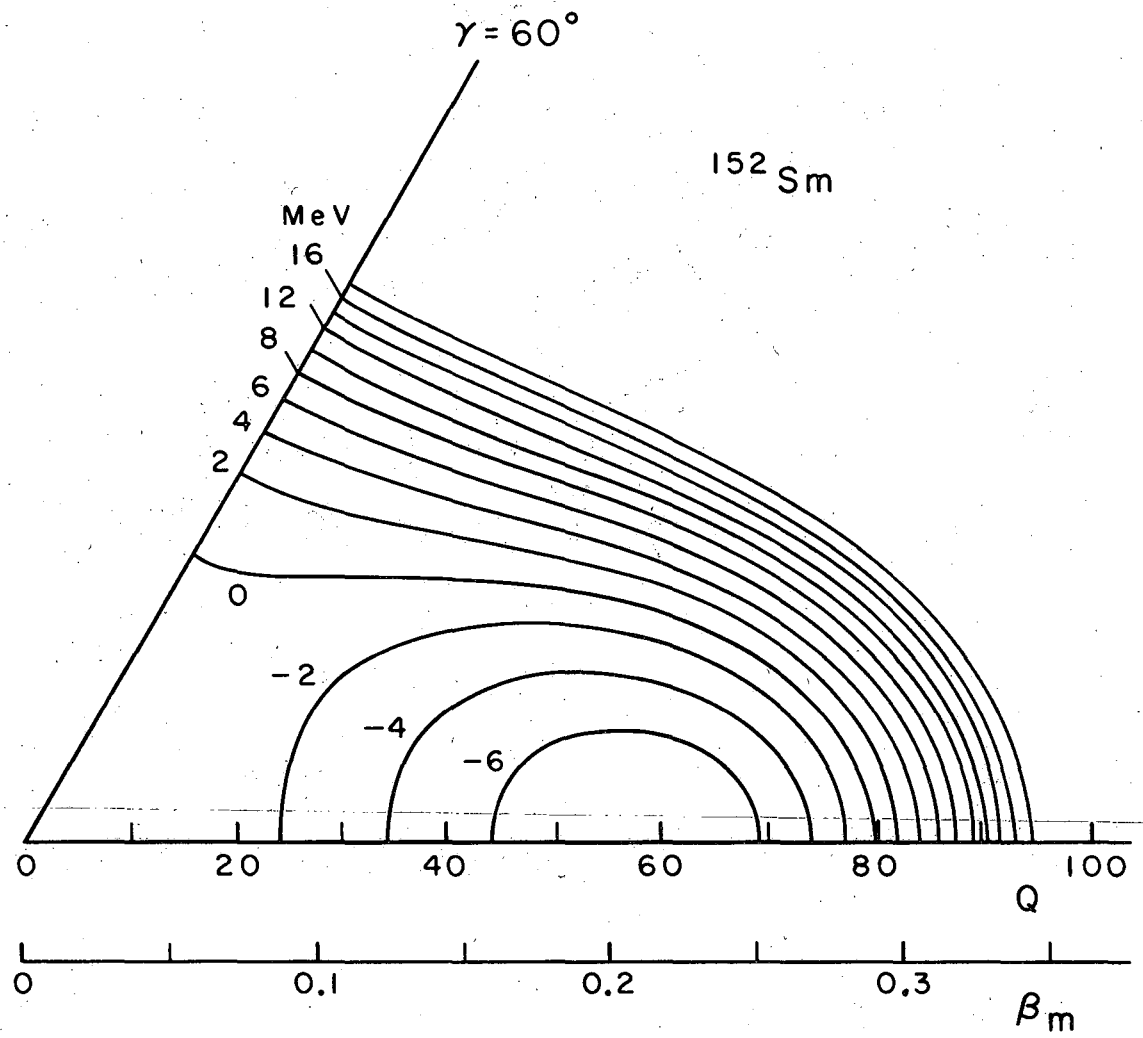

γ = 60°
152 Sm
MeV
16
12
8
6
4
2
0
-2
-4
-6
0
20
40
60
80
100
Q
0
0.1
0.2
0.3
βm
XBL694-2558

Fig. 5.

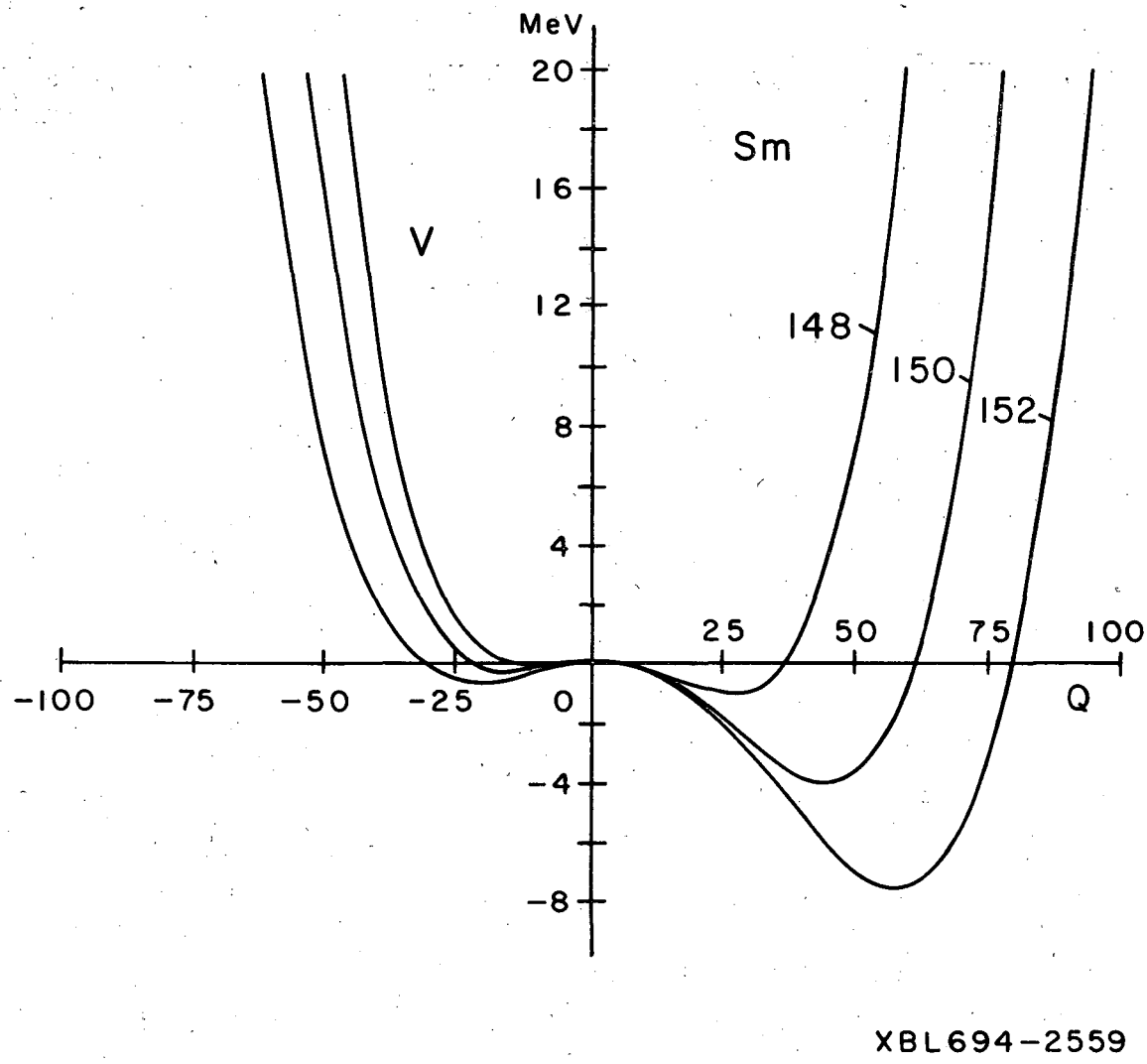

MeV
20
16
12
8
4
-4
-8
Sm
V
148
150
152
25
50
75
100
-100
-75
-50
-25
0
Q
XBL694-2559

Fig. 6.

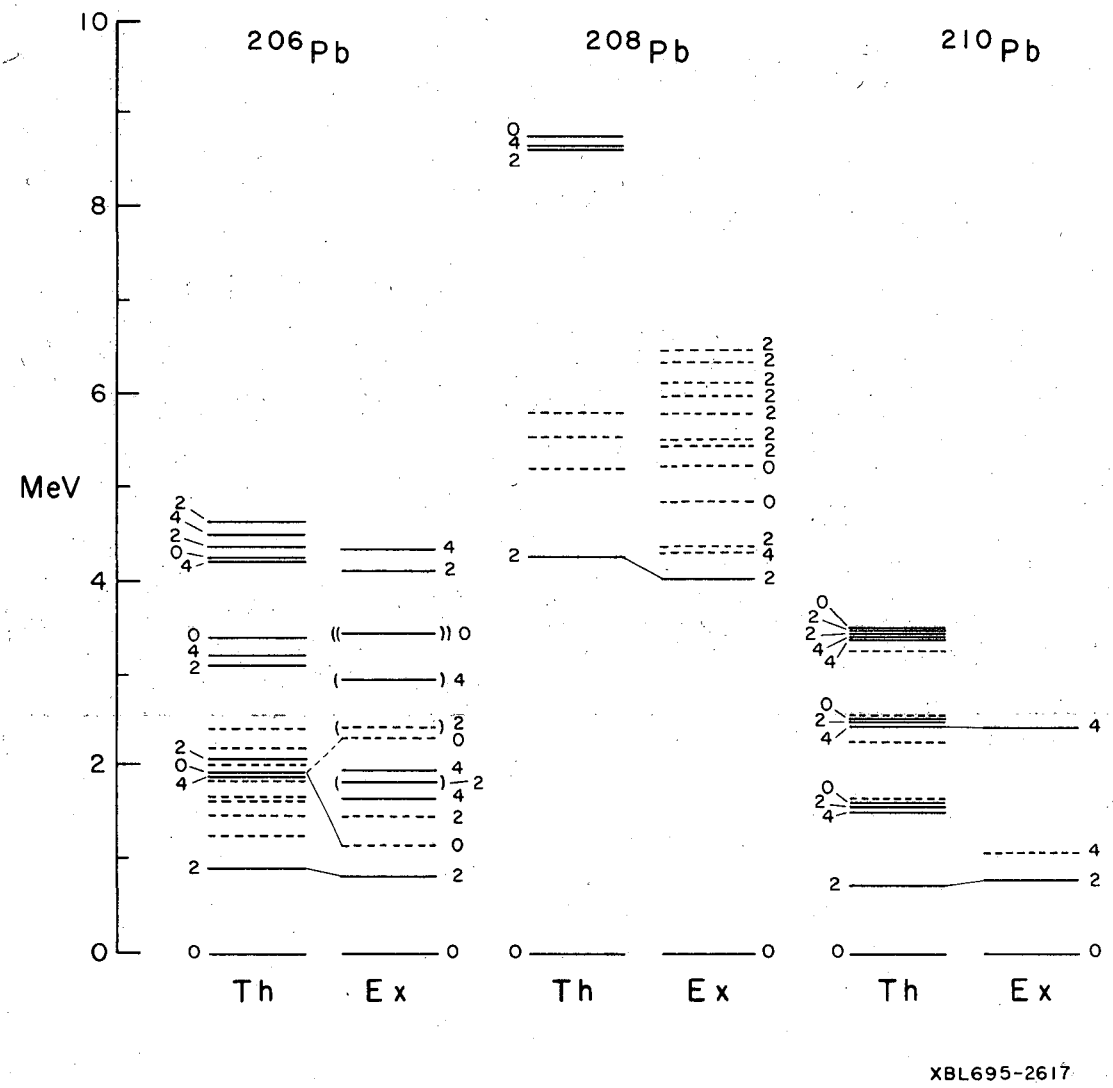


Fig. 7.

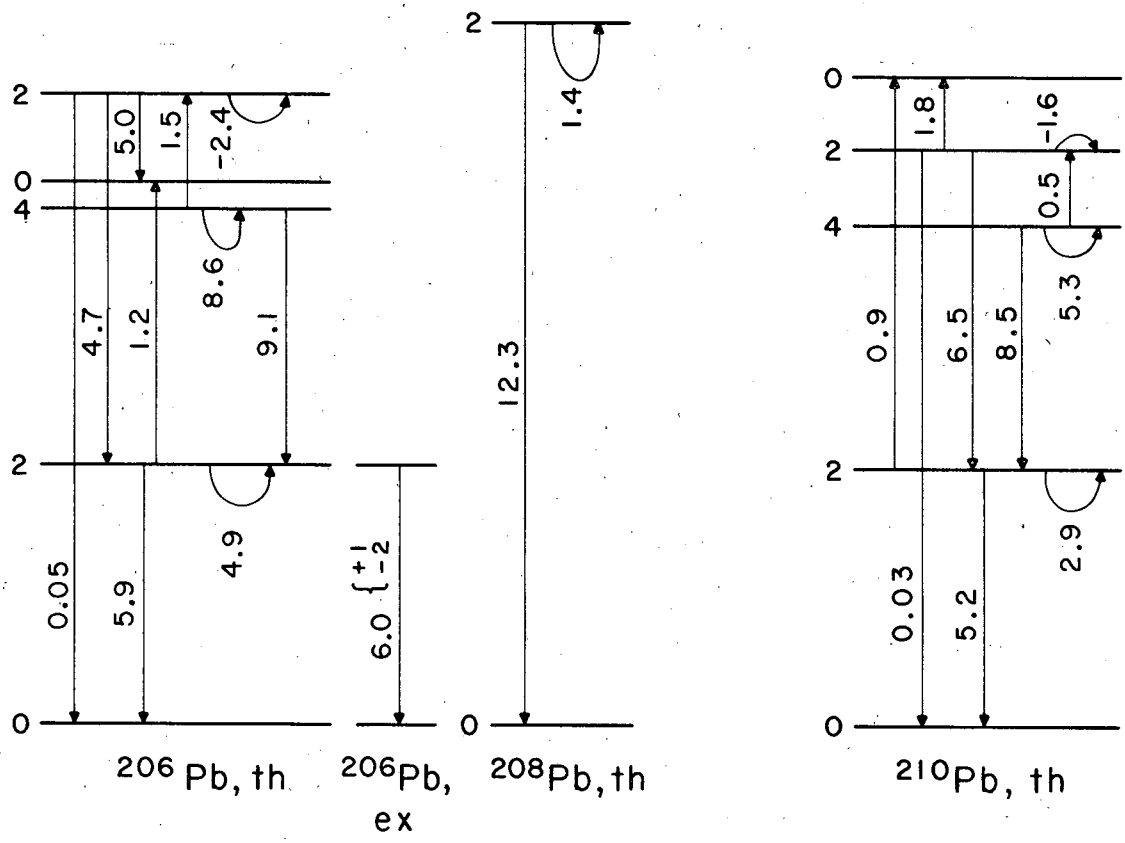

2
0
4
2
0
5.0
1.5
-2.4
8.6
4.7
1.2
9.1
4.9
0.05
5.9
6.0 {+1 -2
2
0
1.4
12.3
0
2
0
4
2
0
1.8
-1.6
0.5
5.3
0.9
6.5
8.5
2.9
0.03
5.2
206 Pb, th
206Pb, ex
208Pb, th
210Pb, th
XBL695-2618

Fig. 8.

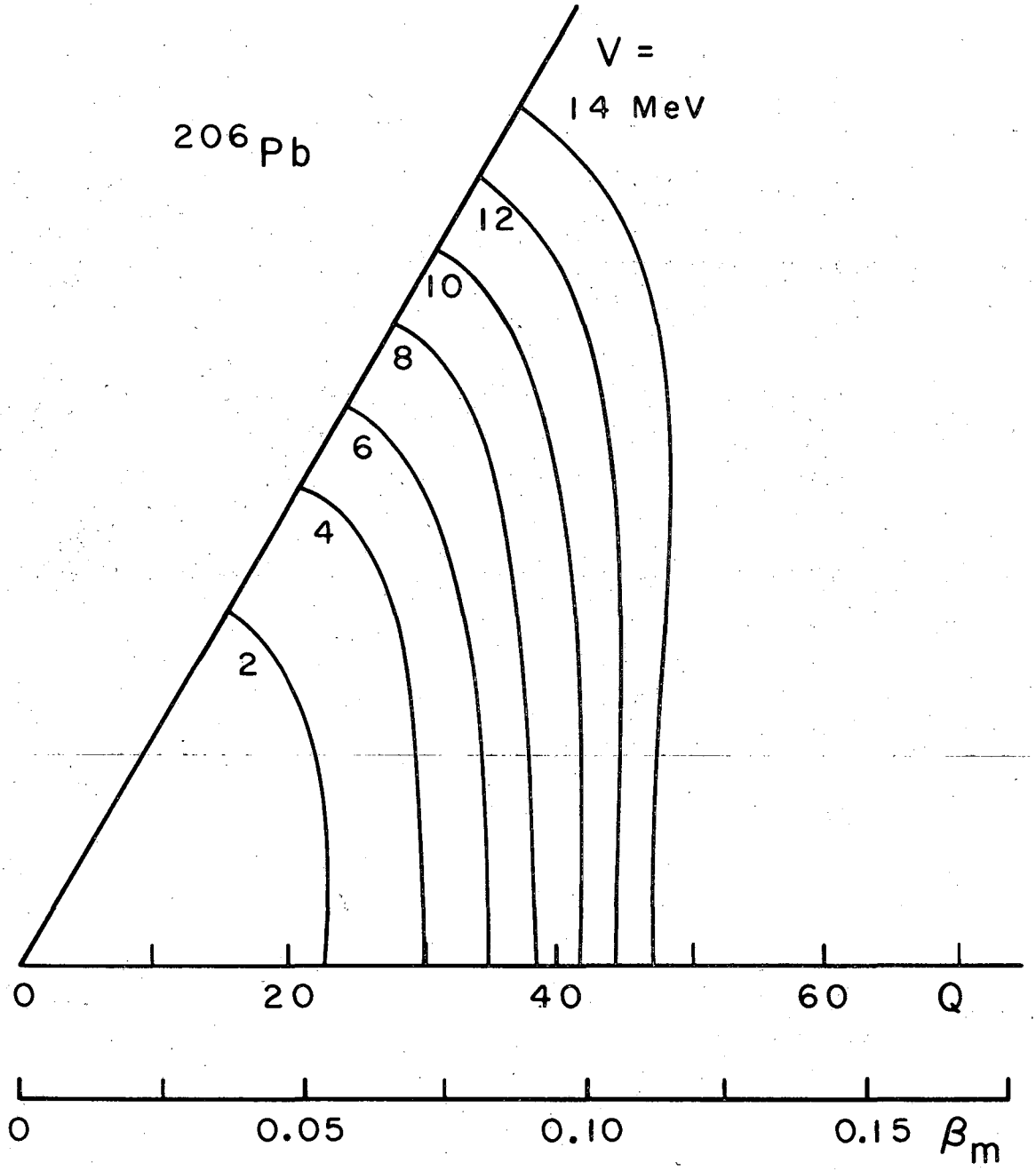

V =
14 MeV
^{206}Pb
12
10
8
6
4
2
0
20
40
60
Q
0
0.05
0.10
0.15
β_m
XBL695-2620

Fig. 9.

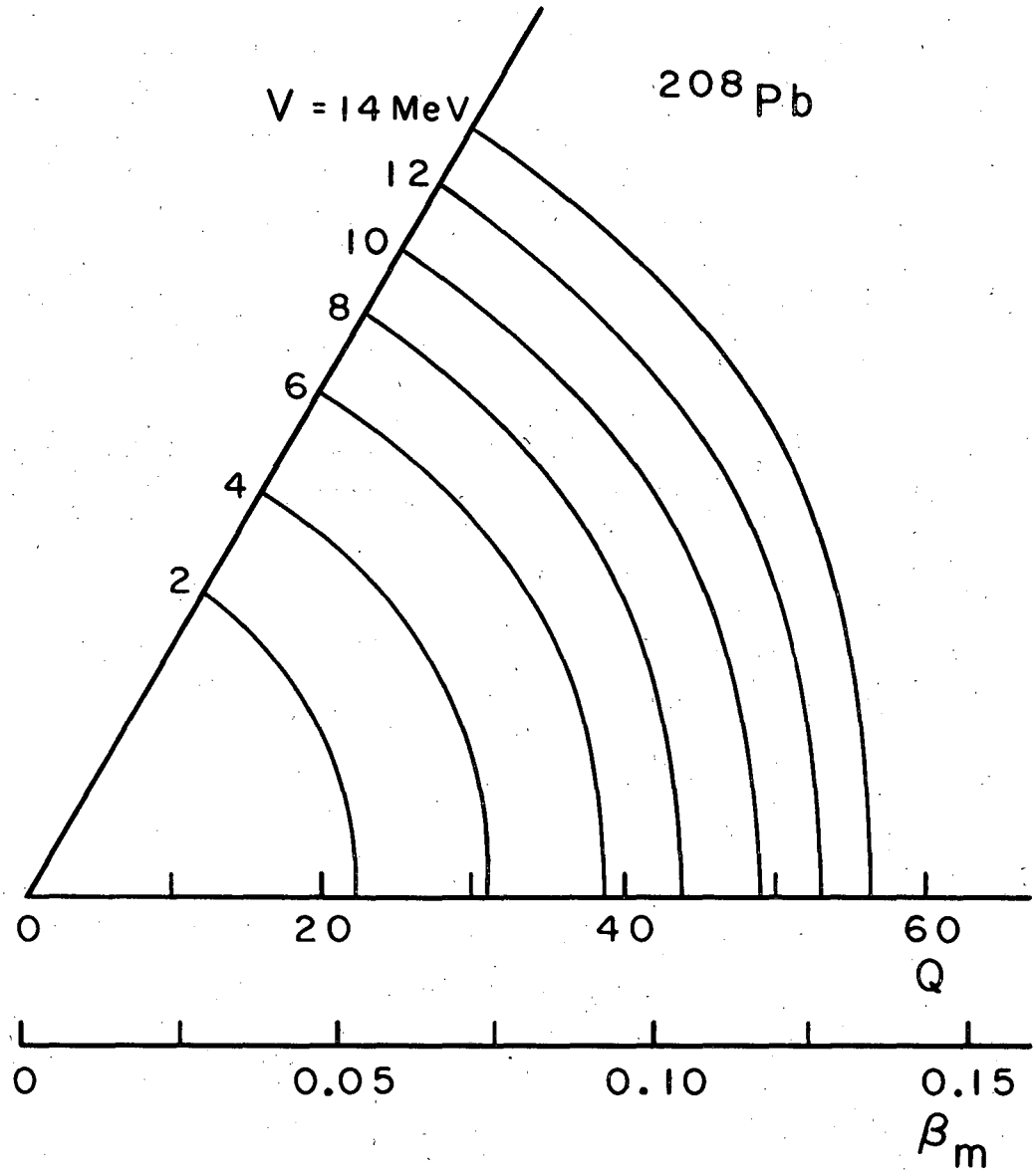


Fig. 10.

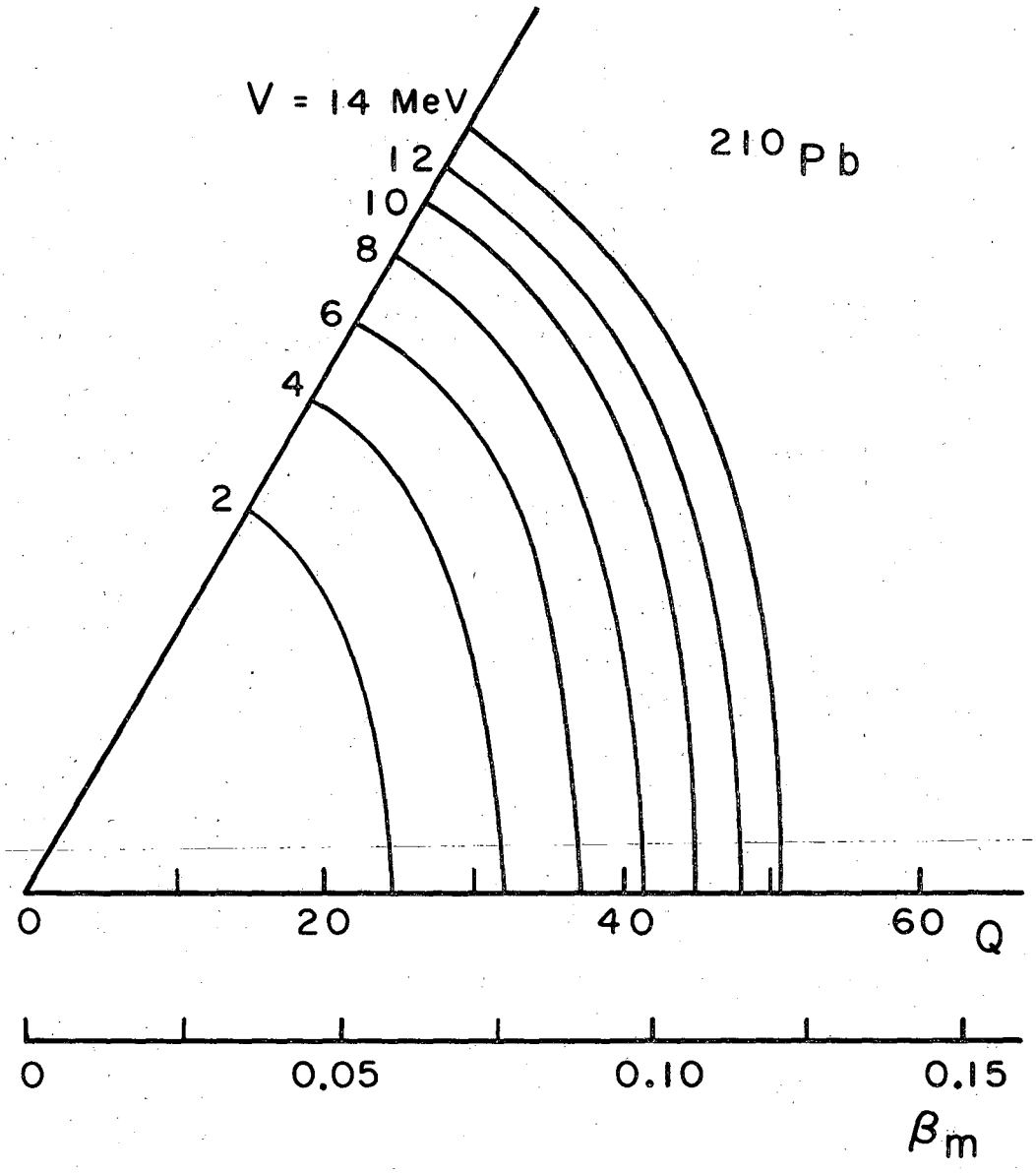


Fig. 11.

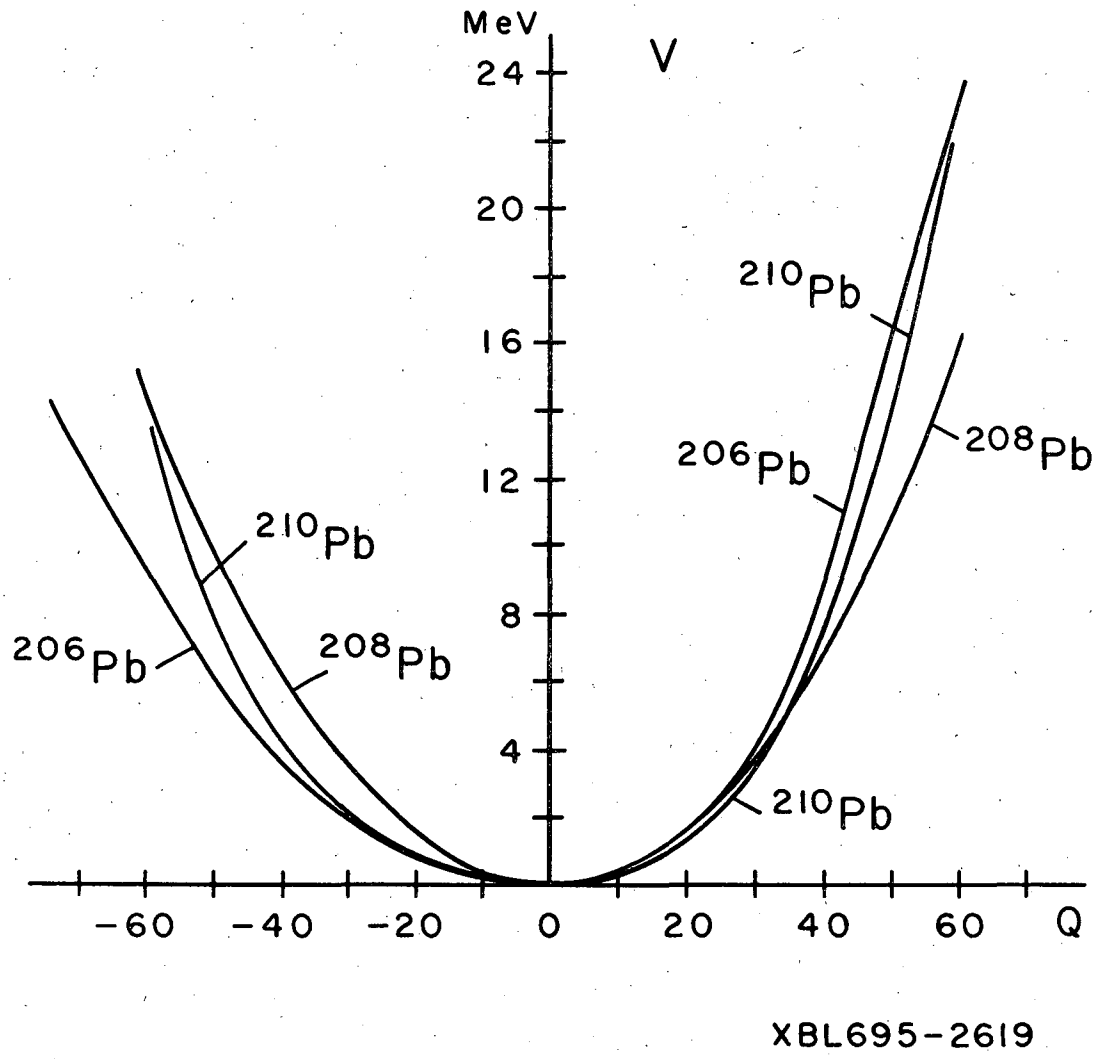

MeV
V
24
20
16
12
8
4
210Pb
206Pb
208Pb
208Pb
206Pb
210Pb
-60
-40
-20
0
20
40
60
Q
XBL695-2619

Fig. 12.

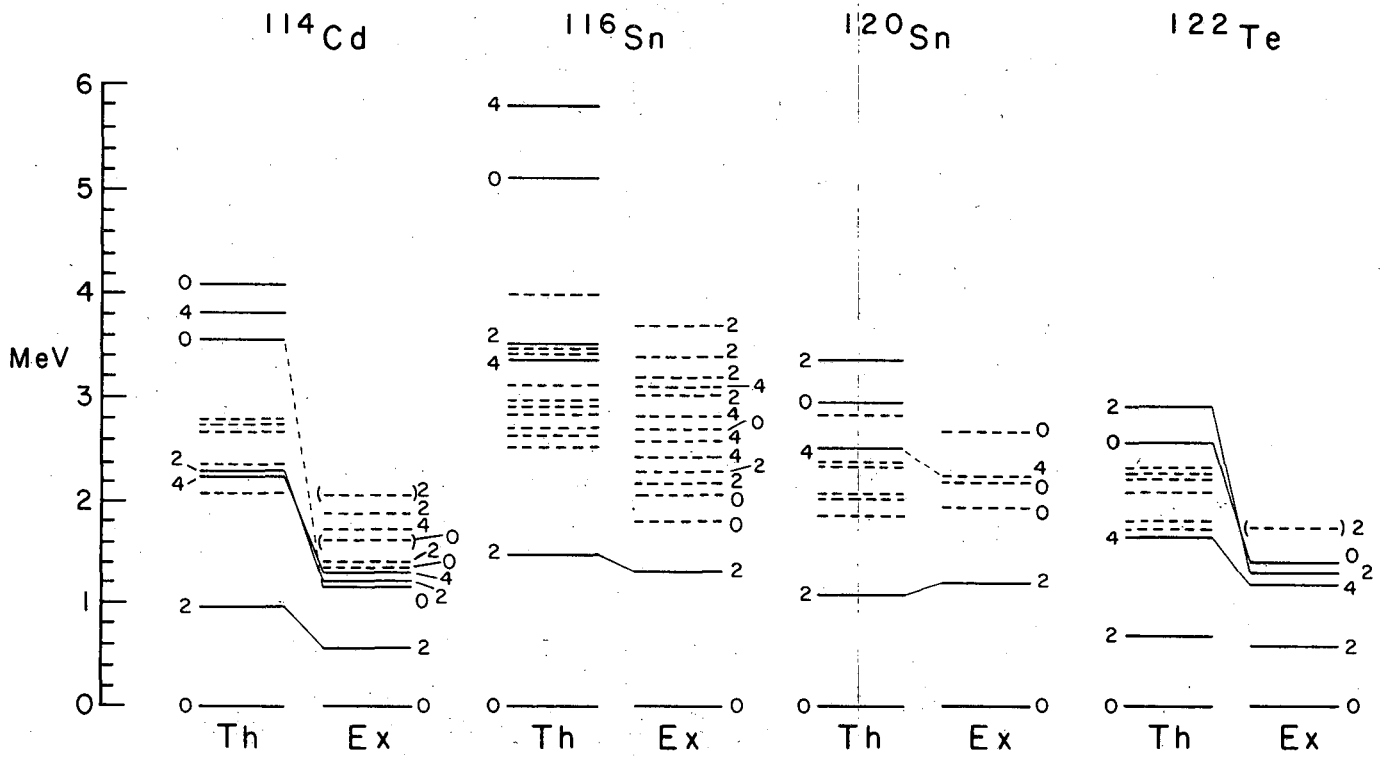


Fig. 13.

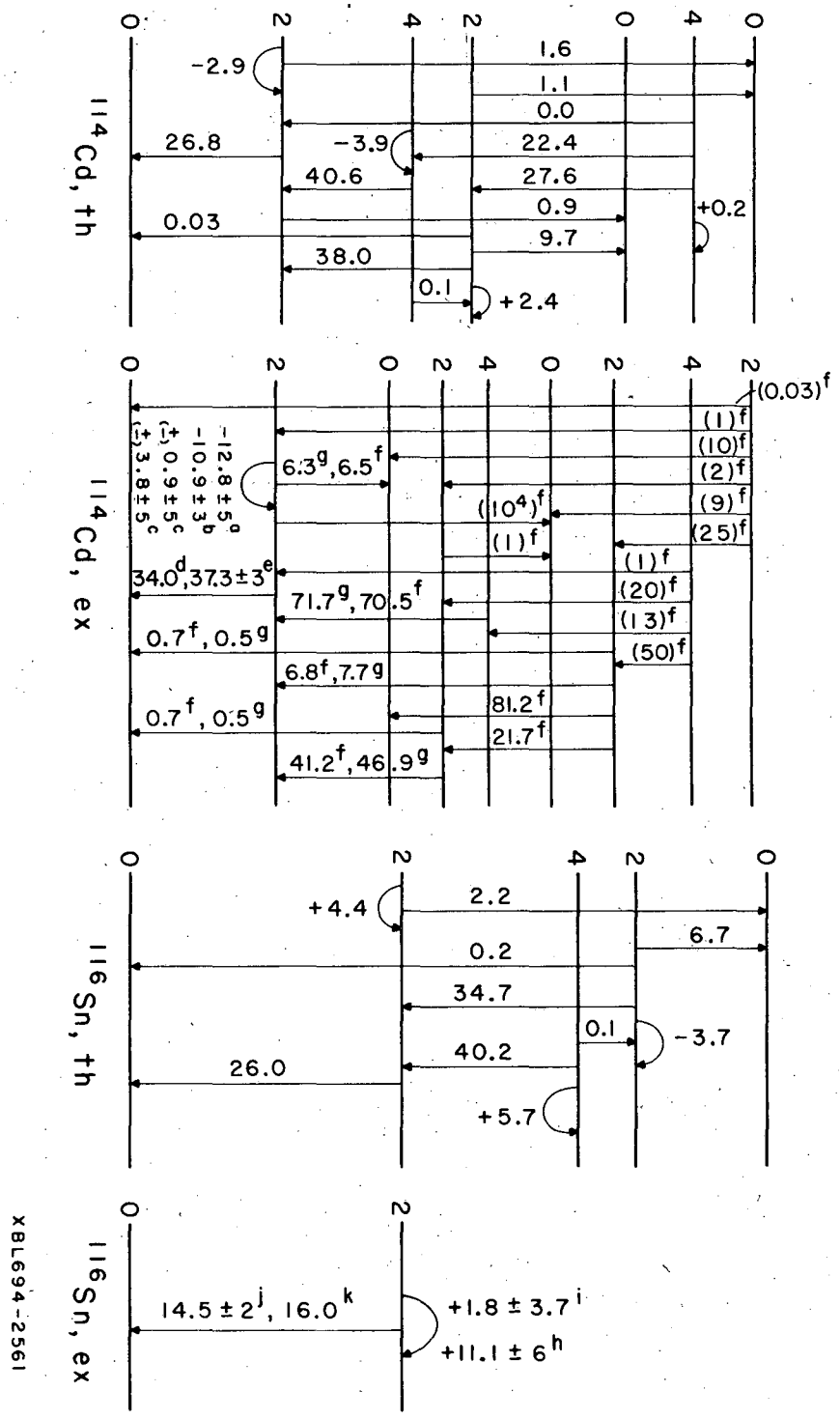

114Cd, th
0
2
4
2
0
4
0
-2.9
1.6
1.1
0.0
26.8
-3.9
22.4
40.6
27.6
0.9
+0.2
0.03
9.7
38.0
0.1
+2.4
114Cd, ex
0
2
0
2
4
0
2
4
2
(0.03)f
(1)f
(10)f
(2)f
(9)f
(25)f
(1)f
(20)f
(13)f
(50)f
(10^4)f
(1)f
81.2f
21.7f
(±)3.8±5c
(±)0.9±5c
-10.9±3b
-12.8±5a
6.3g,6.5f
34.0d,37.3±3e
71.7g,70.5f
0.7f,0.5g
6.8f,7.7g
0.7f,0.5g
41.2f,46.9g
116Sn, th
0
2
4
2
0
+4.4
2.2
6.7
0.2
34.7
0.1
-3.7
40.2
26.0
+5.7
116Sn, ex
0
2
14.5±2j, 16.0k
+1.8±3.7i
+11.1±6h
XBL694-2561

Fig. 14.

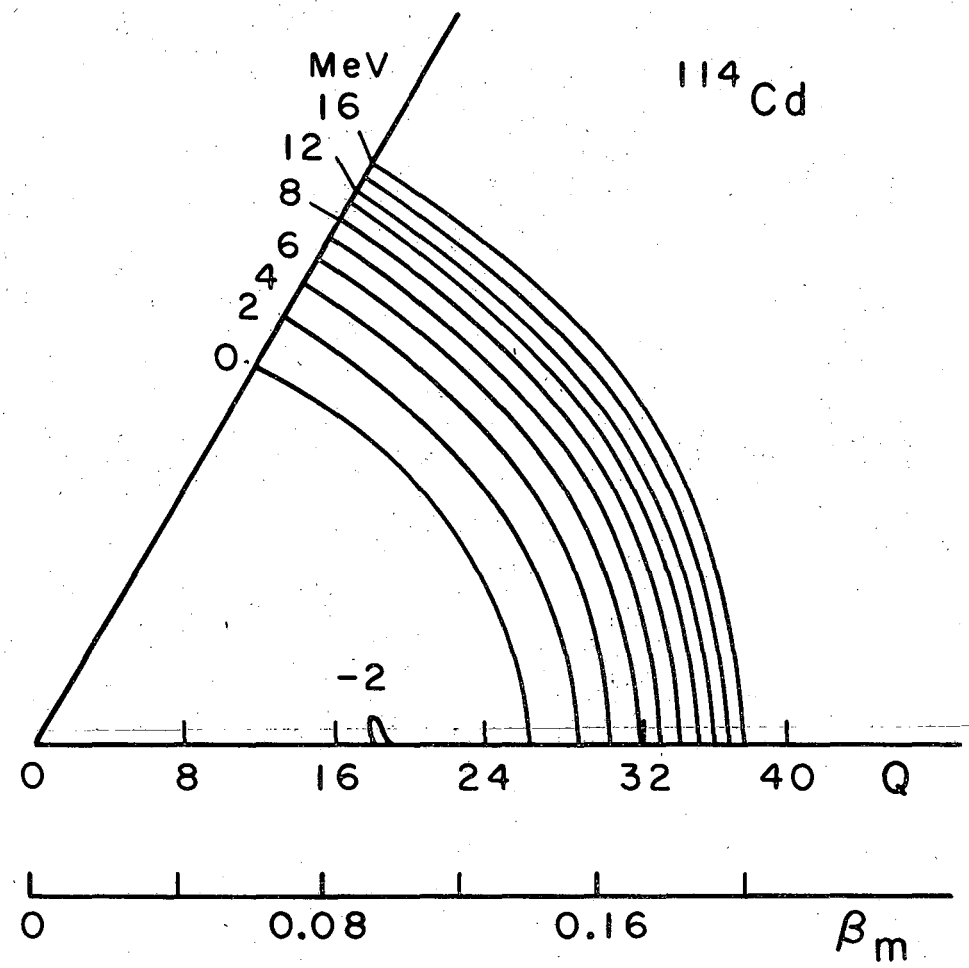

MeV
16
12
8
6
4
2
0
-2
^{114}Cd
0
8
16
24
32
40
Q
0
0.08
0.16
β_m
XBL694-2562

Fig. 15.

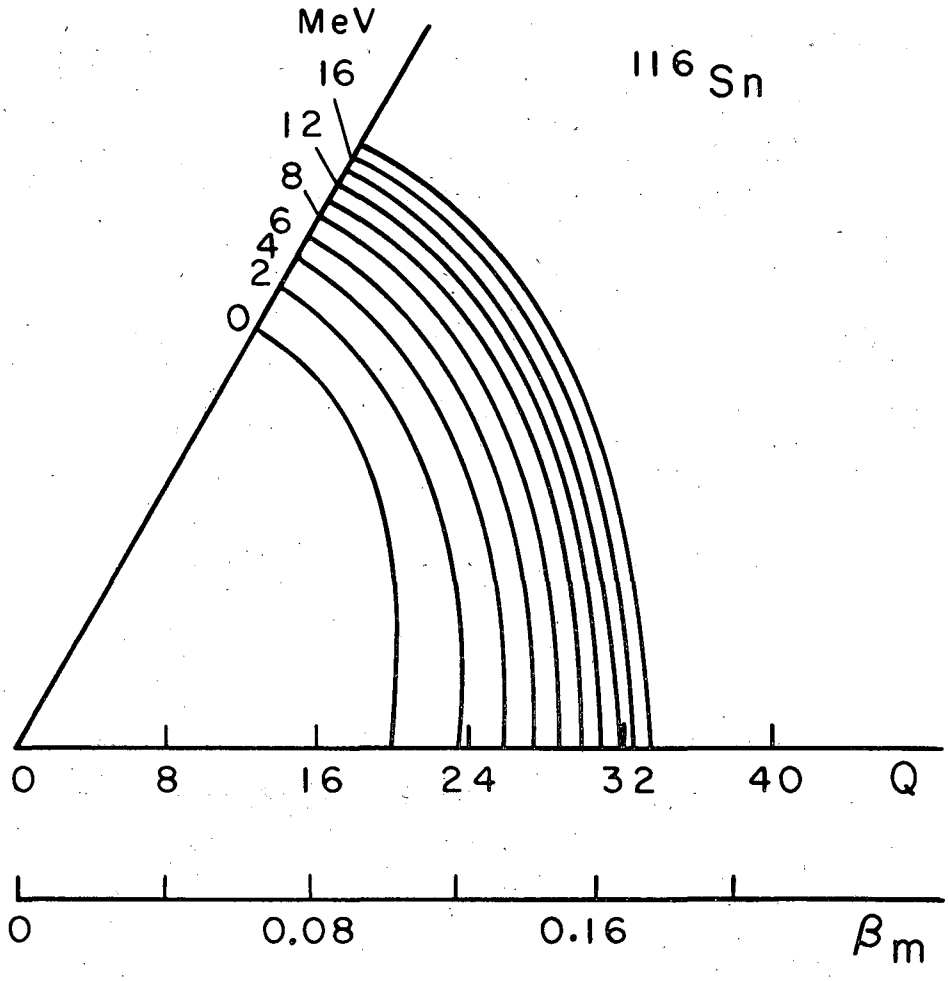


Fig. 16.

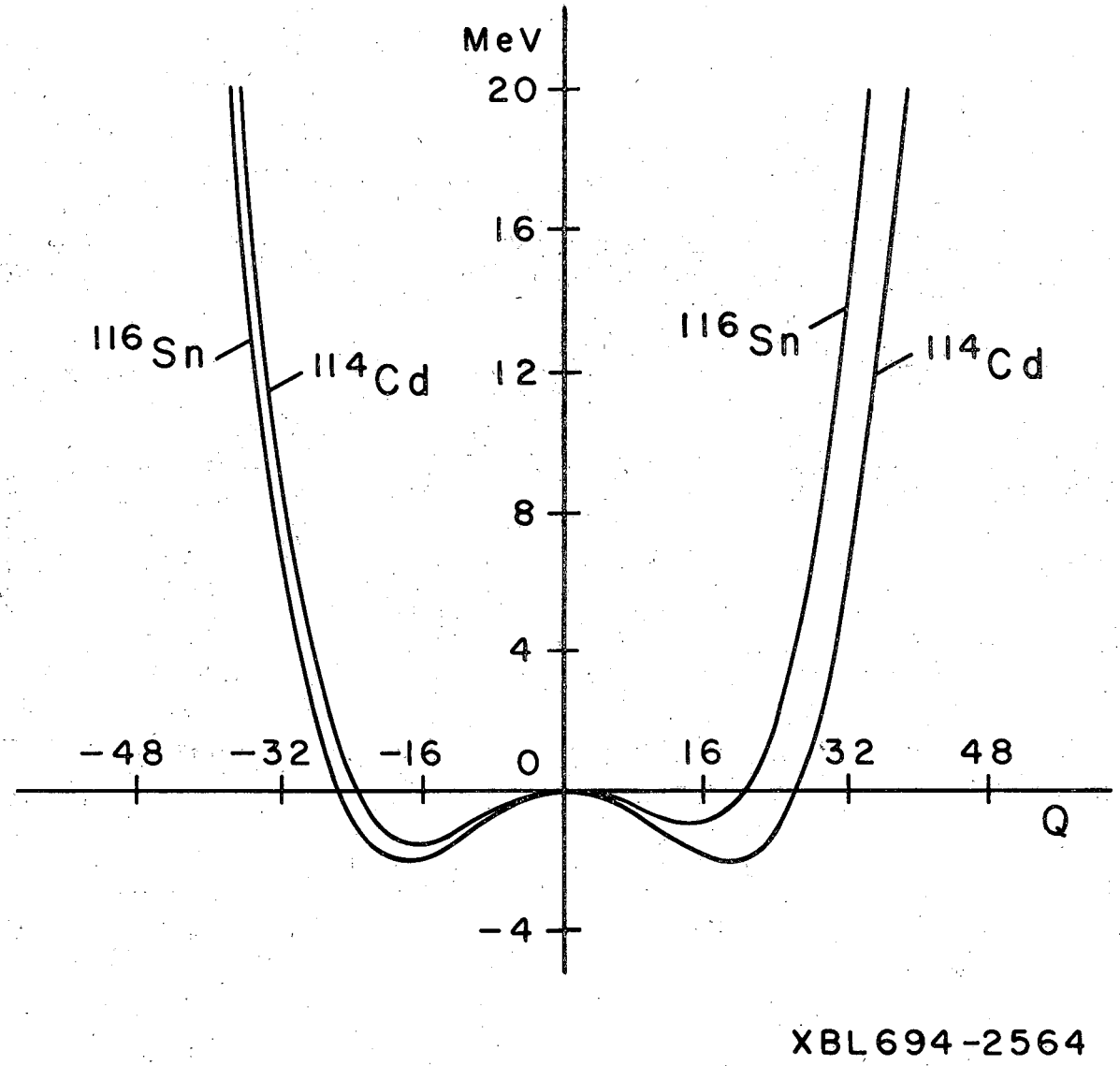

MeV
20
16
12
8
4
-4
-48
-32
-16
0
16
32
48
Q
116Sn
114Cd
116Sn
114Cd
XBL694-2564

Fig. 17.

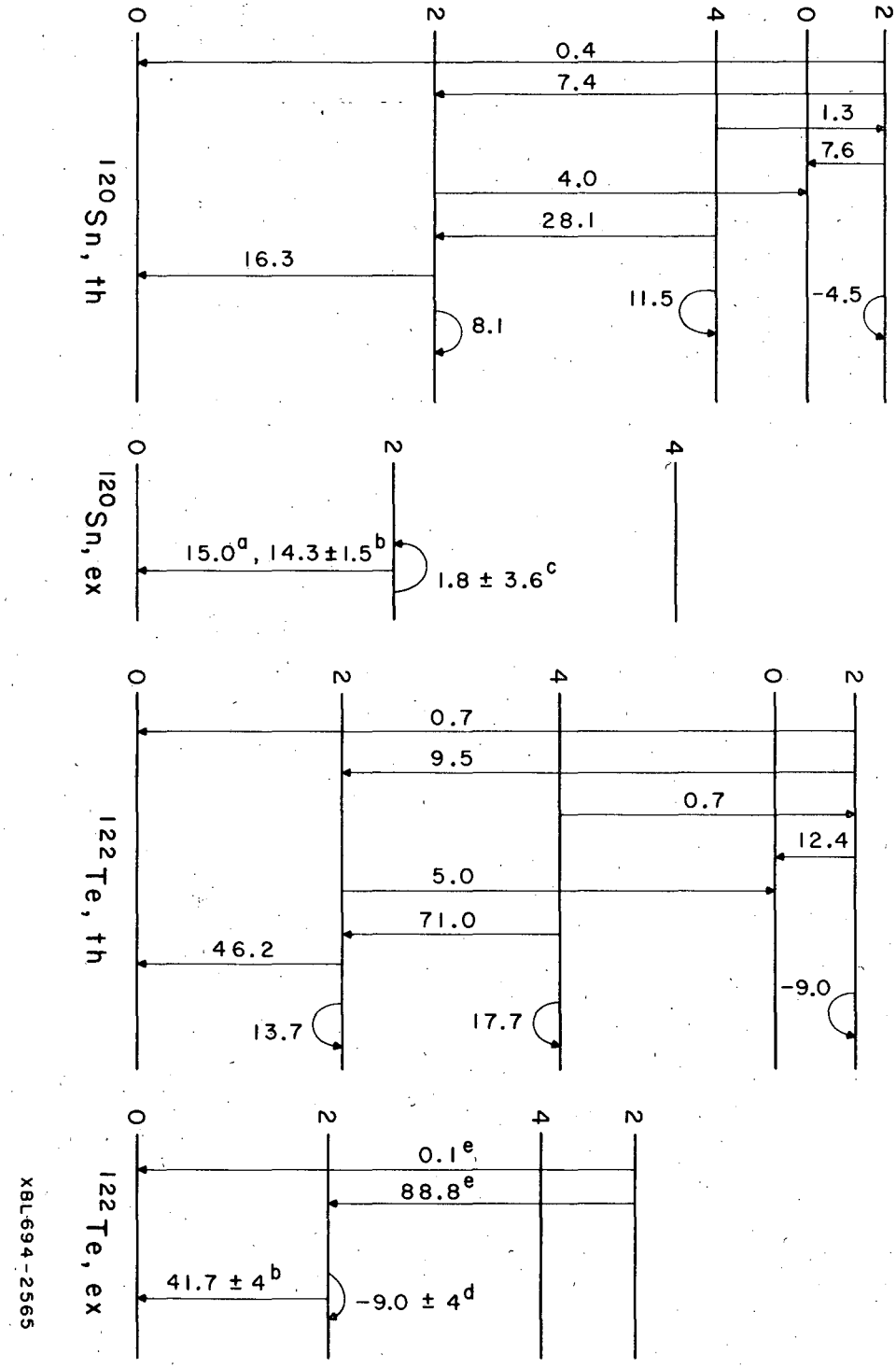

0
2
4
0
2
0.4
7.4
1.3
7.6
4.0
28.1
16.3
11.5
-4.5
8.1
120Sn, th
0
2
4
15.0a, 14.3±1.5b
1.8 ± 3.6c
120Sn, ex
0
2
4
0
2
0.7
9.5
0.7
12.4
5.0
71.0
46.2
-9.0
13.7
17.7
122Te, th
0
2
4
2
0.1e
88.8e
41.7 ± 4b
-9.0 ± 4d
122Te, ex
XBL694-2565

Fig. 18.

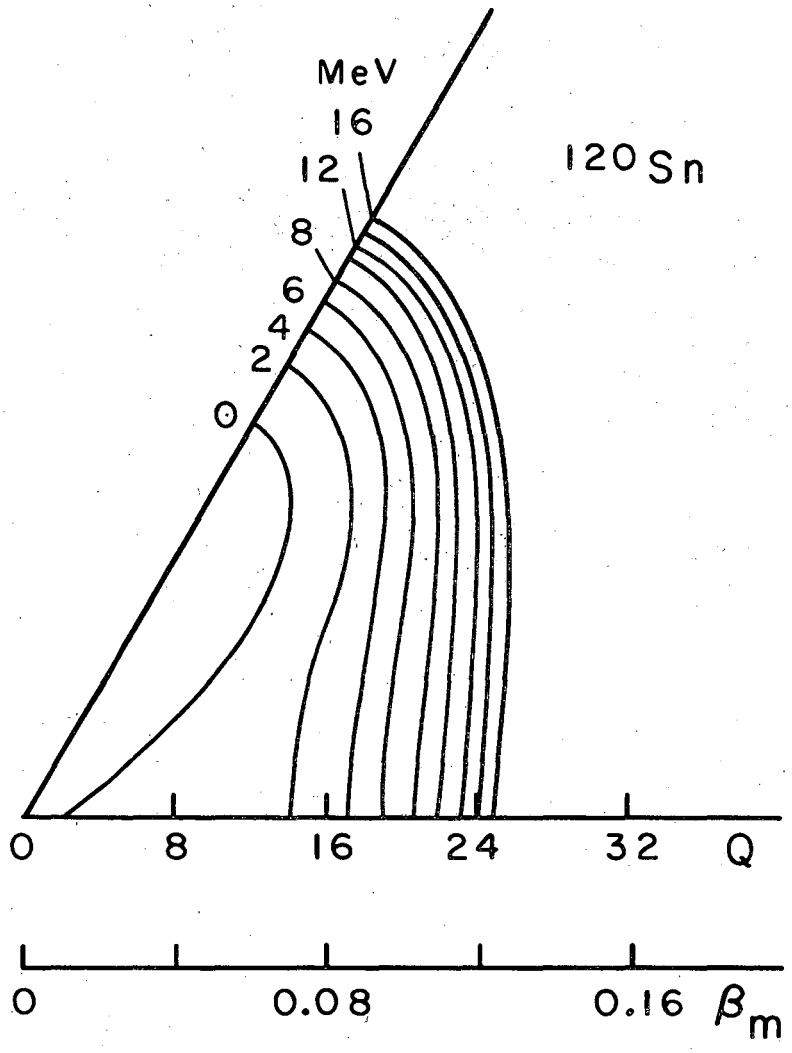


Fig. 19.

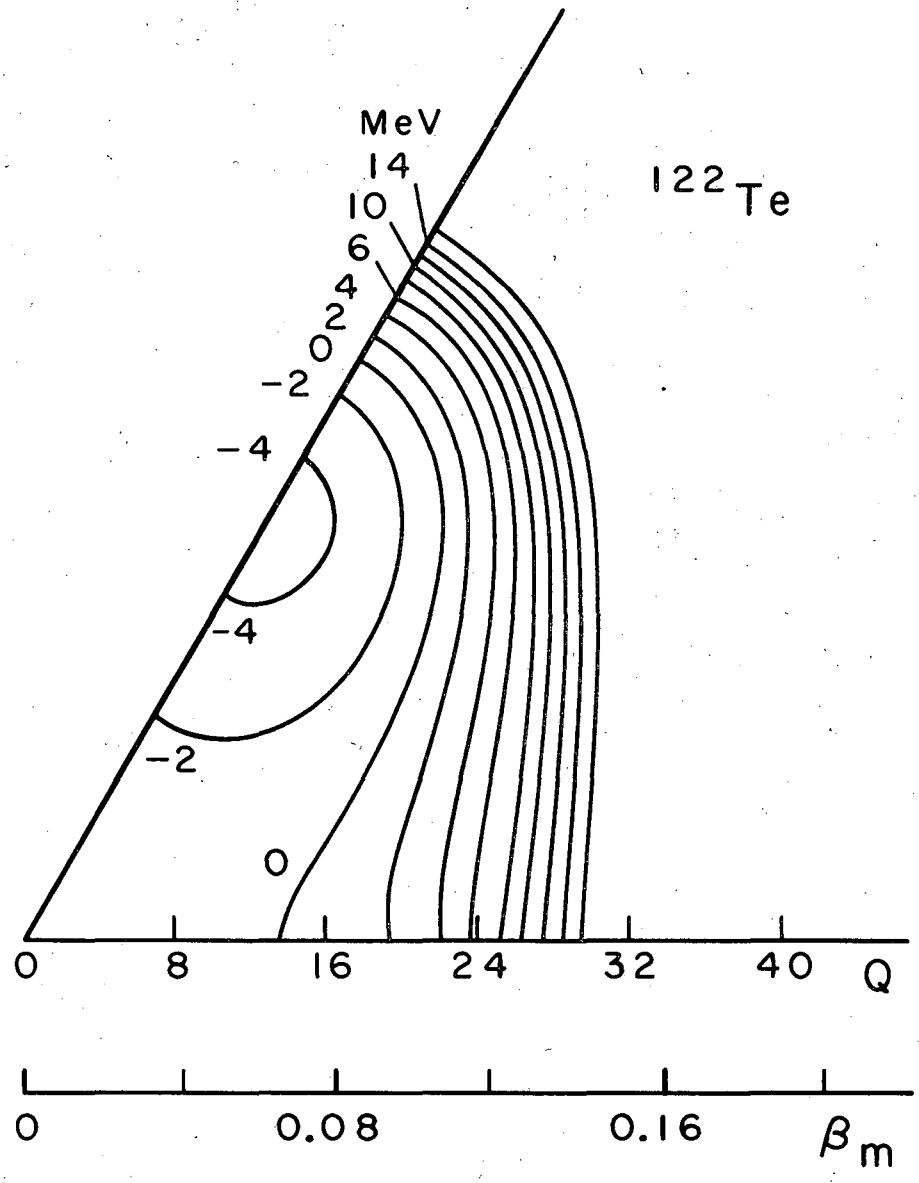

MeV
14
10
6
4
2
0
-2
-4
-4
-2
0
^{122}Te
0
8
16
24
32
40
Q
0
0.08
0.16
β_m
XBL694-2567

Fig. 20.

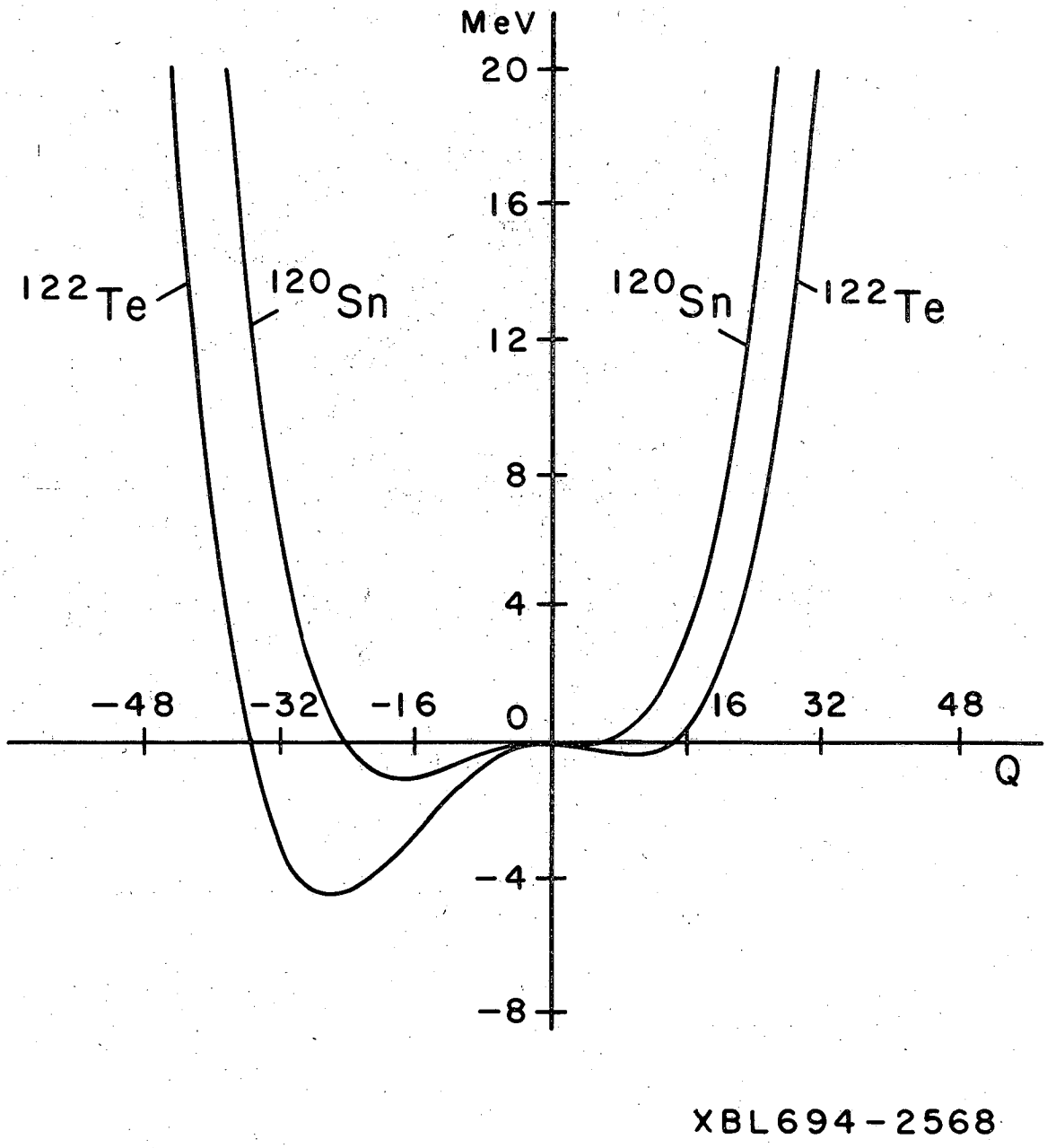

MeV
20
16
12
8
4
0
-4
-8
-48
-32
-16
16
32
48
Q
122Te
120Sn
120Sn
122Te
XBL694-2568

Fig. 21.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720