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Analyses of p-p elastic scattering data[1-8] in the region 3-23 GeV/c and $\pi^{\pm}$-p data[7] near 10 GeV/c have provided strong evidence for the existence of a repulsive real part of the (spin-independent) nuclear forward scattering amplitude at these momenta. In both instances the ratio of real to imaginary parts of the forward amplitude, $\alpha(0) = \frac{\text{Re}A_n(0)}{\text{Im}A_n(0)}$, is approximately -0.25. The possibility of spin-dependence in the p-p case provides an alternative explanation,[2] but the π-p result has no such alternative explanation since the spin-flip amplitude goes to zero in the forward direction.[9]

Spin-independent models proposed[10,11] to describe the data have employed the basic assumption that the elastic scattering is produced by the strongly absorptive processes at high energy, resulting in a scattering amplitude that is either completely imaginary or has, at least, a much larger imaginary than real part. A recent treatment[12] develops an impact-parameter expansion for the amplitude and assumes, for p-p scattering, a repulsive real contribution from small impact parameters. Support for this claim is provided by dispersion-relations calculations,[13,14] which have given the proper magnitude and sign of $\alpha(0)$. In neither case, however, is suggestion made as to the origin of the $\text{Re}A_n$ with a sign corresponding to a repulsive potential.

The purpose of this note is to point out that both the necessity for, and the sign of the $\text{Re}A_n$ can result from purely absorptive processes. Consider the partial-wave expansion for the nuclear scattering amplitude,[15]

$$A_n(\theta) = \sum_\ell (2\ell+1) \frac{(\eta_\ell - 1)}{2ik} P_\ell(\theta) \ , \tag{1}$$

so that

$$\frac{\mathrm{Re}A_n(0)}{\mathrm{Im}A_n(0)} = \alpha(0) = \frac{\sum_\ell (2\ell+1)\ \mathrm{Im}\eta_\ell}{\sum_\ell (2\ell+1)(1-\mathrm{Re}\eta_\ell)} \ . \tag{2}$$

we write

$$\eta_\ell = |\eta_\ell| \exp(2i\delta_\ell) \tag{3}$$

so that $|\eta_\ell|$, the (absolute) amplitude of the outgoing ℓ^{th} partial wave, and δ_ℓ, the phase shift, are real quantities. Since $0 \leq |\eta_\ell| \leq$, with $|\eta_\ell| = 1$ corresponding to no absorption and $|\eta_\ell| = 0$, to complete absorption of that partial wave, it follows that $\mathrm{Im}A_n(0) \geq 0$. Thus $\alpha(0) < 0$ can result only from the condition that $\mathrm{Re}A_n(0) < 0$, which from (2) and (3) requires that

$$-\frac{\pi}{2} \leq \delta_\ell \leq 0 \tag{4}$$

for a sufficient number of partial waves.[16] In particular, one expects (4) to be necessary for values of ℓ near $L = kR$, with R defined as the absorption radius,[17,18] because of the $(2\ell+1)$ weighting factor in (2). Also, in instances of strong absorption $|\eta_\ell|$ may approach zero for $\ell < (L - \frac{\Delta\ell}{2})$ with $\Delta\ell \ll L$, where $\Delta\ell$ is the interval in ℓ-space over which the transition from no absorption to maximum absorption takes place. Condition (4) has been met in cases involving the scattering from nuclei of strongly absorbed "particles" such as deuterons, alpha particles and heavier ions at non-relativistic energies, and we see no apparent reason for the explanation of this result not to apply in the relativistic region. The "repulsive" effect of absorption can be seen by examining complex-potential model (CPM) analyses[19,20] of elastic scattering data; and this concept has been developed qualitatively for application in parameterized phase-shift (PPS) analyses[21] of such data. Specifically, in CPM analyses numerical solution of the (radial) Schrödinger equation, with the nuclear interaction represented by a complex potential

$$V_n(r) = - [V(r) + i\, W(r)] ,$$

has shown, beside the obvious absorptive result that $|\eta_\ell| \leq 1$, that $\delta_\ell < 0$ for strongly absorbed partial waves. Since this occurs even in the presence of the attractive real potential, which alone would result in $\delta_\ell > 0$, the absorptive repulsion is clearly indicated. Recent analyses[22-24] of 11-12 and 15-MeV elastic deuteron scattering data provide interesting examples of cases for which $\alpha(0) < 0$. Figure 1 shows $|\eta_\ell|$ and δ_ℓ resulting from CPM analysis[22] of 11.8-MeV d + Sn data. Substitution of these values into Eq. (2) results in

$$\alpha(0) \simeq -0.20 \quad .$$

Since the machine calculations are not distinguished by their transparency, we make the following argument: consider spin-independent s-wave scattering, for which the radial wave function outside the interaction region is

$$u_0(r) = r\, R_0(r) = \frac{i}{2k} \left[\exp(-ikr) - \eta_0 \exp(+ikr) \right] = u_0^{(-)}(r) + u_0^{(+)}(r) \qquad (5)$$

and take the ingoing wave incident on a purely absorptive region with the boundary $r = R$. Then

$$\begin{aligned} k &= k & r &> R \\ k &= k + i\kappa & r &< R \end{aligned} \qquad (6)$$

For simplicity we take κ constant. Then, from the requirement that $u_0^{(-)}$ and $du_0^{(-)}/dr$ be continuous across the boundary R, one finds, using the imaginary part of $u_0^{(-)}$, that

$$\tan kR - \frac{\kappa}{k} = \tan (kR + \delta_0) \tag{7}$$

An equivalent relation is found by using the real part of $u_0^{(-)}$. Since $\frac{\kappa}{k} > 0$, it follows from (7) that $\delta_0 \leq 0$. This negative phase shift results simply from $u_0^{(-)}(r)$ external to R being "pushed out" of the absorptive region because of the increase in slope in the internal region, which is produced by the attenuation factor $\exp[\kappa(r-R)]$. The same reasoning applies to $u_0^{(+)}(r)$ and Eq. (7) is valid again for the phase shift of the outgoing wave; hence, relative to the ingoing wave, the outgoing wave has been phase shifted by $2\delta_0$. From (7) one sees that

$$\begin{aligned} \tan \delta_0 &= -\frac{\kappa}{k} && \text{for } kR = n\pi \\ \delta_0 &= 0 && \text{for } kR = (n + 1/2)\pi \ . \end{aligned} \tag{8}$$

This is a consequence of the sharp boundary at $r = R$ with an internal region of constant absorption. If one takes, more realistically, a finite interval, ΔR, over which $\kappa(r)$ goes from zero to κ, the limits (8) on δ_0 would not be reached. Clearly $\lim\limits_{k\to\infty} \frac{\kappa}{k} = 0$, but the ever increasing number of inelastic channels opening as k increases may keep $\frac{\kappa}{k}$; hence δ_0, finite at the energies available to date.

It seems clear that the discussion presented here directly applies for all partial waves. For values of ℓ not small compared to kR one uses, of course, the actual ingoing and outgoing waves rather than their asymptotic forms.

Physically, one knows that $\delta_\ell \to 0$ for $\ell > (L + \frac{\Delta\ell}{2})$. Since this general reasoning is based entirely on the effect of absorptive processes on the scattering amplitude and does not invoke the concept of a potential, its application to high energy scattering seems justified.

One can make phenomenological PPS analyses of the high energy data, employing a parameterization of $\delta\ell$ that gives an ℓ-dependence of the form indicated in Fig. 1. Such analyses could provide useful descriptions of the energy-dependence of such quantities as absorption radii, surface thicknesses and opacities. The results of such analyses, which are presently underway, will be reported elsewhere.

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FIGURE CAPTION

Figure 1. The ℓ-dependence of $|\eta_\ell|$ and δ_ℓ from CPM analysis of 11.8 MeV d + Sn elastic scattering data, reference 22.

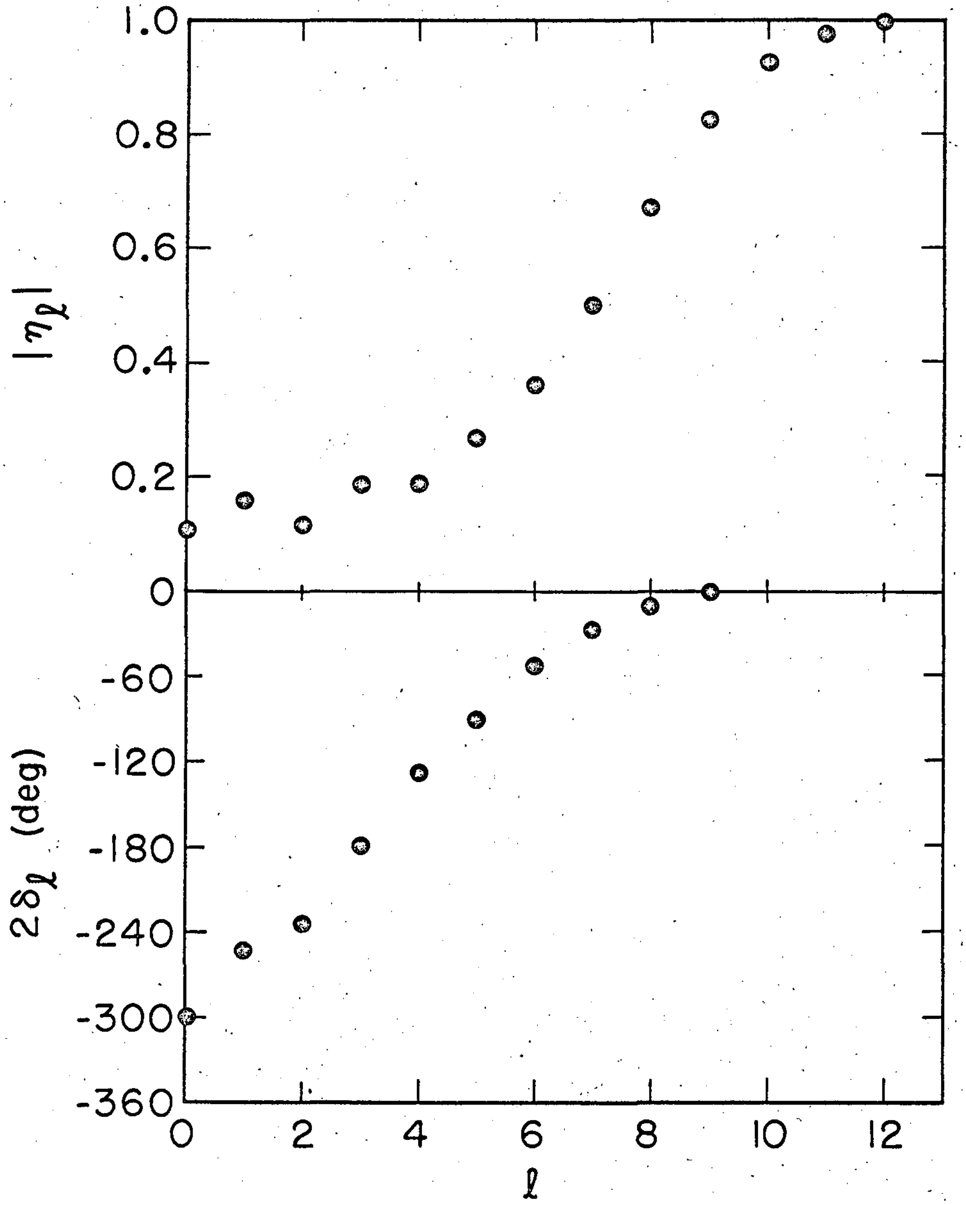

Fig. 1 MUB-5306