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Visual attention in spatial cueing and visual search

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To characterize internal processes of an observer conducting perceptual tasks, we developed an observer model that combines the perceptual template model (PTM), the attention mechanisms in the PTM framework (Lu & Doshier, 1998), and uncertainty of signal detection theory (Green & Swets, 1966). The model was evaluated with a visual search experiment conducted in a range of external noise, signal contrast, and target-distractor similarity conditions. In each trial, eight Gabor patches were shown in each of two brief intervals, with one target at a different orientation from the distractors in one of the presentations. Subjects were precued to a subset of the stimuli (1, 2, 4, or 8) and asked to report (a) which interval contained the target and (b) where the target was. Individual roles of uncertainty and of attention in visual search were investigated by comparing models with and without an attention component. The results showed that decision uncertainty alone was sufficient to account for the set-size effect, even in conditions with high target-distractor similarity. Our theoretical model and empirical results provide a coherent picture regarding how visual information is selected and processed during feature search.

Moran & Desimone, 1985; Reynolds, Chelazzi, & Desimone, 1999), and brain imaging (Brefczynski & DeYoe, 1999; Gandhi, Heeger, & Boynton, 1999; Kanwisher & Wojciulik, 2000; Kastner, De Weerd, Desimone, & Ungerleider, 1998; Martínez et al., 1999; Somers, Dale, Seiffert, & Tootell, 1999; Watanabe et al., 1998) since the 1970s (Carrasco, 2011; Itti, Rees, & Tsotsos, 2005). It has been shown that selective attention can improve performance accuracy or response time relative to unattended locations or unattended objects (e.g., Bashinski & Bacharach, 1980; Downing, 1988; Duncan, 1984; Han, Doshier, & Lu, 2003; Nissen, 1985; Posner, 1978, 1980; Shiffrin & Czerwinski, 1988; Sperling & Doshier, 1986), although performance in attended and unattended conditions is identical in some circumstances (Doshier & Lu, 2000a; Lu & Doshier, 2000; Shiu & Pashler, 1994; Solomon, 2004). More recent work has suggested that the effects of attention on accuracy and response time may be dissociable (Smith, Ratcliff, & Wolfgang, 2004; Smith & Ratcliff, 2009).

Historically, researchers have argued that selective attention facilitates sensory analysis by strengthening the sensory representation (Cheal, Lyon, & Gottlob, 1994; Corbetta, Miezin, Shulman, & Petersen, 1993; Mangun, Hillyard, & Luck, 1993), redistributes limited capacity mental processing resources required for stimulus analysis (Bonnell & Miller, 1994; Henderson, 1996; Henderson & Macquistan, 1993; Palmer, Ames, & Lindsey, 1993; Shiffrin, 1988), and/or eliminates irrelevant stimuli (Shiu & Pashler, 1994) or masks (Enns & Lollo, 1997). These intuitive or verbal analyses of the potential roles of selective attention prompted us to develop the external noise paradigm and the perceptual

Introduction

Selective attention to a location in space or to an object has been a central topic in cognitive psychology (Chun & Potter, 1995; Posner, 1980; Posner & Cohen, 1984; Sperling & Melchner, 1978; Treisman & Gelade, 1980; Wolfe, 1994), physiology (Desimone & Duncan, 1995; Haenny, Maunsell, & Schiller, 1988;

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template model (PTM) theoretical framework in an attempt to generate a systematic and quantitative analysis of the mechanisms of attention (Lu & Doshier, 1998). The paradigm adds systematically increasing amounts of external noise to the visual stimulus and observes the effect on a perceptual task in attended and unattended conditions. Effects of attention are identified as one of three mechanisms: as improved filtering of external noise or distractors through retuning of perceptual templates (external noise exclusion), as amplification of the stimulus or reduction of internal noise sources (stimulus enhancement), or as reduction of contrast gain control (internal multiplicative noise reduction) or a mixture of these three mechanisms.

In a series of studies using the external noise method in spatial cueing, we showed that external noise exclusion was a major mechanism of spatial attention in complex multilocation displays with either central or peripheral location cues (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 2000), while stimulus enhancement was associated primarily with peripheral cueing of location (Lu & Doshier, 2000). Additionally, the magnitude of external noise exclusion increases with display size—external noise exclusion played little role in two-location displays, but it drastically reduced contrast thresholds in eight-location displays (Doshier & Lu, 2000a). Carrasco and colleagues (Carrasco, Penpeci-Talgar, & Eckstein, 2000; Yeshurun & Carrasco, 1998) have also shown attention improvements in noiseless displays with peripheral precues (but see Gould, Wolfgang, & Smith, 2007, for discussions on decision uncertainty). Moreover, that spatial attention excludes unwanted information has been consistently demonstrated at the neuronal level in monkey single-cell recording from V2 (Luck, Chelazzi, Hillyard, & Desimone, 1997; Reynolds et al., 1999), V4 (Haenny et al., 1988; Luck et al., 1997; Moran & Desimone, 1985; Reynolds et al., 1999; Spitzer, Desimone, & Moran, 1988), Inferior Temporal Cortex (IT) (Moran & Desimone, 1985), and Middle Temporal (MT) and Medial Superior Temporal (MST) areas (Treue & Andersen, 1996; for a review, see Desimone & Duncan, 1995) and at the neural population level by functional imaging (Kastner et al., 1998; Kastner & Ungerleider, 2000; Lu, Li, Tjan, Doshier, & Chu, 2011). Evidence for stimulus enhancement has also been documented in V4 (Reynolds, Pasternak, & Desimone, 2000; Williford & Maunsell, 2006) and MT (Martínez-Trujillo & Treue, 2002) in single-unit recordings and a host of visual areas in functional MRI (Brefczynski & DeYoe, 1999; Gandhi et al., 1999; Kanwisher & Wojciulik, 2000; Kastner et al., 1998; Li, Lu, Tjan, Doshier, & Chu, 2008; Martínez et al., 1999; Somers et al., 1999; Watanabe et al., 1998).

The previous PTM studies on mechanisms of attention have been mostly carried out using the spatial cueing paradigm with stimuli and perceptual tasks in

which the templates for distinct targets were essentially nonoverlapping or orthogonal (but see Doshier & Lu, 2013; Hetley, Doshier, & Lu, 2014; Liu, Doshier, & Lu, 2009, which tested the PTM for similar discriminations based on an elaboration of the PTM for nonorthogonal judgments in Jeon, Lu, & Doshier, 2009). In the current study, we extended the investigation of visual search to test regimes that require the discrimination of very similar, nonorthogonal targets and to high-contrast regimes typical of most of the classical attention studies (Posner, Nissen, & Ogden, 1978; Sperling & Weichselgartner, 1995; Treisman & Gelade, 1980) and to visual search.

Visual search for a target among distractor elements (e.g., finding a particular object among many others) is one of those classical tasks that have been typically studied in a high-contrast, supra(detection) threshold regime often with similar stimuli. In visual psychophysics, investigations have focused on the set-size effect in visual search accuracy (see Palmer, Verghese, & Pavel, 2000, for a review). Although spatial attention is known to integrate several features into an object in visual search (Treisman & Gelade, 1980), for high-contrast, near-orthogonal stimuli without external noise or masks, limited-capacity attention processes (i.e., improved perceptual quality on the attended location) have not been found. Instead, apparent set-size effects generally reflect statistical uncertainty in unlimited-capacity signal detection models (Davis, Shikano, Peterson, & Keyes Michel, 2003; Eckstein, Thomas, Palmer, & Shimozaki, 2000; Fahle, 1991; McLean, Palmer, & Loftus, 1997; Palmer, 1994; Palmer et al., 1993, 2000; Verghese & Stone, 1995), with more difficult conjunction searches being associated with a complex decision structure (Eckstein, 1998). Although it has been generally accepted that the set-size effects in visual search can be accounted for by spatial uncertainty in most cases, some researchers have suggested that spatial attention may also play a role in addition to spatial uncertainty in visual search in some stimulus conditions or tasks (Baldassi & Burr, 2000; Morgan, Ward, & Castet, 1998; Pöder, 1999; Rosenholtz, 2001).

We hypothesize that the conditions under which an effect of spatially cued attention is substantial should correspond to the circumstances in which attention effects over and above uncertainty should occur in visual search. Our analysis of the stimuli used in visual search studies in the literature suggests that many of the classical visual search experiments have been carried out using stimulus conditions (high contrast, zero external noise, moderate or low similarity), where attention effects on perception are least likely to be found.

In the present study, we extended the original single-channel and single-location PTM model to develop a new model of perception and spatial attention that consists of a multichannel and multilocation sensory front end and a decision structure with both feature and location uncertainties, as well as conducted a visual

search experiment in a range of external noise and contrast conditions for low and high template overlap (target-distractor similarity). The new comprehensive model allowed us to predict the effects of attention limits that operate in visual search in the entire stimulus space (external noise, contrast, target precision) in both cueing and visual search paradigms, as well as evaluate the contributions of stimulus enhancement, external noise exclusion, multiplicative noise reduction, and decision uncertainty in those paradigms. The empirical study may help us align the results from spatial cueing and visual search.

The multichannel and multilocation perceptual template model

In this section, we describe the development of the multichannel and multilocation perceptual template model for visual search and derive signature performance patterns of a number of attention mechanisms. We focus on an odd-ball visual search task in which the observer is presented with two brief displays and required to decide which one of the two displays contained a target. The two displays contain an equal number of items, one with only distractors (Gabors at the same orientation) and the other with a target that is oriented differently from the distractors. The orientation of the target varies from trial to trial: Sometimes it is similar to that of the distractors, but other times it is very dissimilar.

Our development extends the original PTM (Lu & Doshier, 1998, 1999; see Lu & Doshier, 2008, for review) in several important ways. First, the new model considers detection or discrimination in the feature domain as well as the contrast domain. The original PTM and other observer models (e.g., the linear amplifier model; Pelli, 1985) focused on the contrast domain, while subsequent extensions of the PTM were developed to account for feature similarity (Doshier & Lu, 2013; Hetley et al., 2014; Jeon et al., 2009), and the current development expands on this approach (see below).

Second, all previous models are single-channel models that assume each input stimulus activates only one perceptual template. In contrast, the new model considers the existence of multiple templates and concurrent activation by a single input. This allows us to model target-unknown situations. Finally, by incorporating spatial uncertainty calculations, the model can explain observers' performance in visual search tasks. In the following sections, we start with the original PTM and describe the stochastic version of the new model. An analytical version of the model is described in the [Appendix](#).

The original PTM

In the original (single channel) PTM (Figure 1; Lu & Doshier, 1998, 1999, 2008), each input stimulus—either with the signal embedded or not—is first processed by a perceptual template that has certain selectivity for stimulus characteristics but is broad enough to allow external noise to affect performance. This model was designed to account for performance in detection tasks or discrimination tasks with orthogonal or near-orthogonal stimuli. The output of the template exhibits a certain magnitude of internal activation if the stimulus matches the template or no activation if the stimulus does not. It is then processed by a nonlinear transducer function (Legge & Foley, 1980; Nachmias, 1981; Nachmias & Sansbury, 1974) and corrupted by multiplicative and additive internal noises. While the magnitude of the multiplicative internal noise is proportional to the total energy of the stimulus, the magnitude of the additive internal noise is independent of stimulus energy. In a two-interval alternative forced-choice (2iAFC) detection task, two input stimuli, one with signal present and the other without, are processed by the PTM, resulting in two noisy internal representations. If the magnitude of the representation of the signal present stimulus is greater, the observer makes a correct response. If the magnitude of the representation of the signal present stimulus is less, the observer makes an incorrect response. The behavior of the observer can be modeled with the probability distributions of the internal representations of the signal-present and signal-absent stimuli. The observer will have better performance when the two distributions are separated to a greater extent (large mean difference and/or small variances), often associated with high-stimulus contrasts, low levels of external noise, and/or small internal noises.

The PTM includes two forms of nonlinearity—transducer nonlinearity and multiplicative noise (or contrast gain-control). Both forms of nonlinearity make external noise nonadditive and nonindependent. All our experimental work related to the PTM indicated that both forms of nonlinearity were necessary to account for the data (see Lu & Doshier, 2008, for a review).

Lu and Doshier derived the signature performance patterns of three mechanisms of attention based on the PTM (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 1998, 1999, 2000): stimulus enhancement, external noise exclusion, and multiplicative noise reduction. With stimulus enhancement (Figure 2a), attention amplifies the gain of the perceptual template on the stimulus. This mechanism is mathematically equivalent to a reduction of internal additive noise. For stimuli in zero or low levels of external noise, it increases the separation between the distributions of the two internal representations and thus improves the observer's

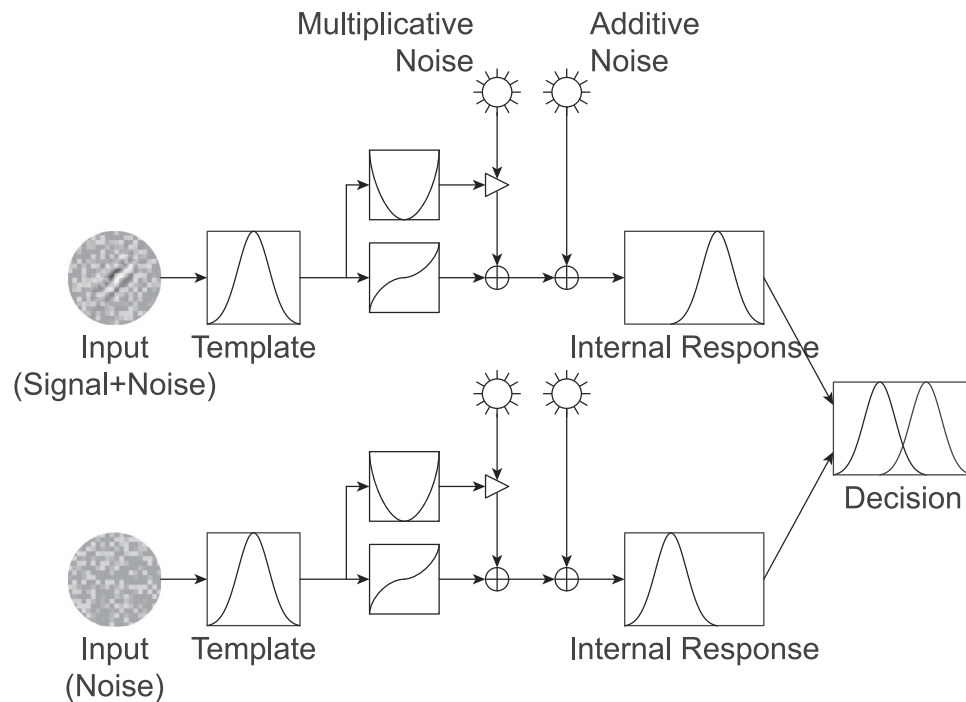


Figure 1. The original PTM model. Internal processes for detecting the target in a 2iAFC task.

performance. For stimulus in high levels of external noise, the mechanism is not effective. With external noise exclusion (Figure 2b), attention improves the filtering of external noise by changing the tuning curve of the perceptual template. Studies suggest that spatial attention excludes unwanted information by sharpening selectivity of the cellular signal (Haenny et al., 1988; Spitzer et al., 1988) and/or weighing the input from the attended region/object more heavily without changing cellular tuning characteristics (Desimone & Duncan, 1995; Luck et al., 1997; Moran & Desimone, 1985; Reynolds et al., 1999; Treue & Maunsell, 1996). For stimulus embedded in high levels of external noise, effective filtering of external noise reduces the variance of the internal representations. Although the distance of signal-present and signal-absent distributions is not changed, less noisy internal representations reduce the overlap between the two probability distributions and thus improve observers' performance. The mechanism is not effective in improving performance for stimuli in zero or low external noise. With multiplicative noise reduction (Figure 2c), attention reduces the variance of the internal representations in all external noise conditions. The mechanism improves performance in all external noise conditions. This original PTM was used to account for effects of spatial precueing in attention in a range of tasks and situations (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 1998, 1999, 2000).

The original PTM was subsequently extended to discrimination between similar (nonorthogonal) stimuli (Jeon et al., 2009). In this case, the magnitude

of activation in the template that mismatches the stimulus is positive because the stimuli are similar. The same basic PTM narrows the templates for experimental stimuli; this increases the differences in response to different stimuli, thereby increasing their discriminability. The patterns of response for the three mechanisms of attention are similar, except that both now show effects even at very high stimulus contrasts (at the asymptote of the psychometric functions) (see Doshier & Lu, 2015; Hetley et al., 2014). The extended PTM accounted for spatial precueing of attention that varied in external noise and in stimulus similarity (Doshier & Lu, 2015; Hetley et al., 2014). In these experiments, the assumption of the single-channel PTM of two templates matched to the stimuli (signal known) is engaged by collecting data for different similarity discriminations in different blocks.

In the current application, we extend the PTM model further to multiple locations and multiple templates operating simultaneously and apply it to a standard feature visual search task with different display sizes. This new multichannel, multilocation model differs from the original and extended PTM in that it takes inputs from multiple spatial locations (the search locations) and from multiple templates (target unknown). We examine whether any of the three attention mechanisms are at play in circumstances where visual spatial attention has been shown to be most impactful yet are rarely tested in visual search experiments. Alternatively, the effects of display size on performance in standard visual search

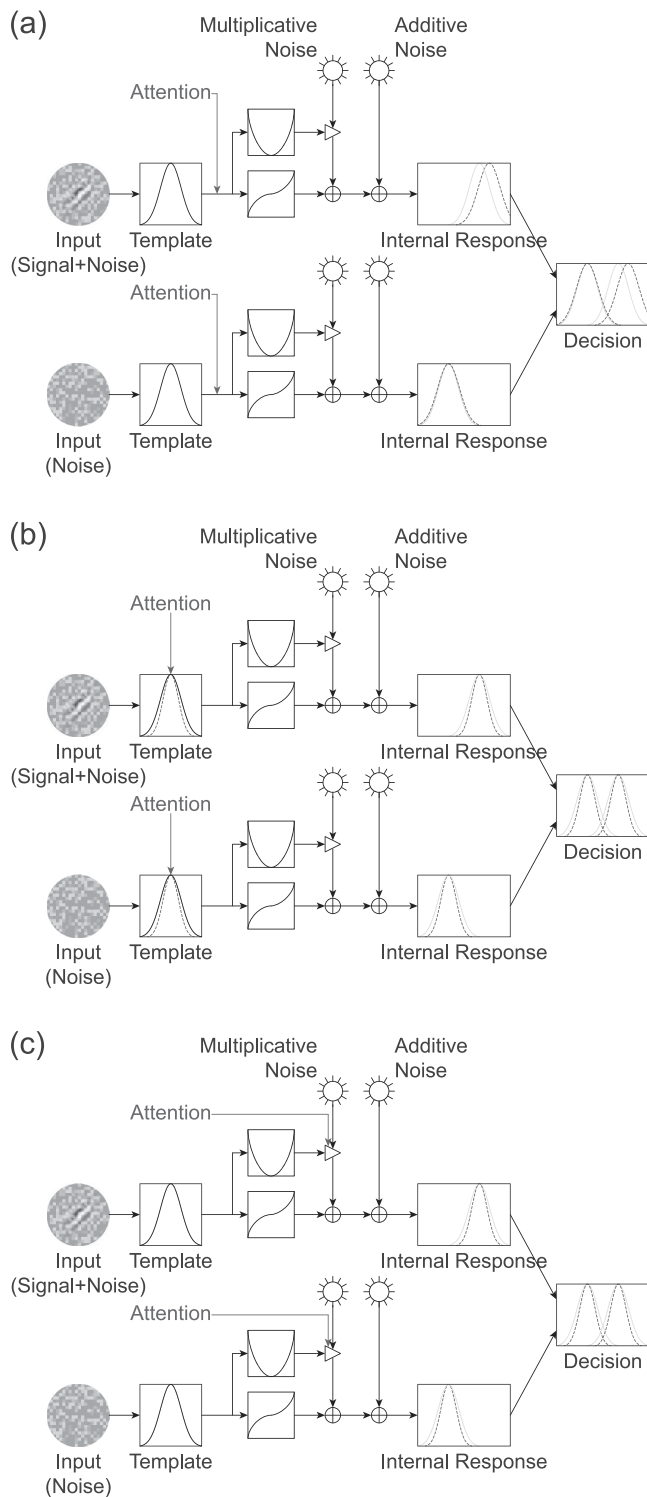


Figure 2. Mechanisms of attention in the PTM model. Internal processes for detecting target in a 2iAFC task. (a) Stimulus enhancement mechanism. (b) External noise exclusion mechanism. (c) Multiplicative noise reduction mechanism. In all diagrams, blue dashed lines show how attention changes internal processes.

may reflect simply irreducible effects of channel uncertainty.

Multiple channels

Physiological evidence indicates that neural receptors or visual detectors are selectively responsive to features of the visual input, such as orientation or spatial frequency. For example, a cell in the primary visual cortex responds best to stimuli within a specific range of orientations but less to others (De Valois, William Yund, & Hepler, 1982; Hubel & Wiesel, 1962, 1974). The firing rate of a neuron is strongest to the stimulus in its preferred orientation with some variance and decreases as the input orientation differs from the preferred orientation (Bradley, Skottun, Ohzawa, Sclar, & Freeman, 1987).

In the elaborated PTM model, we assume that the visual system has multiple orientation templates that are selectively tuned to six different orientations (i.e., 5°, 20°, 35°, 40°, 42.5°, and 45° counterclockwise from the horizontal). The model with multiple known channels for the simple feature stimuli (Gabor) used in the current study corresponds closely with known physiological properties of the visual system, although the assumption may not hold for more complex stimuli, and elaborated models such as those based on feature maps (Itti & Koch, 2000) may prove useful. A single input is processed through all the templates with different tuning profiles and produces different outputs from them. In each channel, the output of the template is processed in the same way as in the original PTM: It passes through the signal pathway and gain-control pathway. Figure 3 shows a schematic of the multiple-channel model.

Information integration: Integration of multiple channels

The visual system combines outputs from all channels. Since the number of internal responses is equal to the number of templates, the visual system needs to integrate all responses to build a single percept of the stimulus. The most common integration rule in visual psychophysics is the maximum rule (Graham, 1989; Nolte & Jaarsma, 1967): If the target identity is known to the observer (e.g., searching for a target with fixed orientation among distractors with various orientations), the system compares all the internal responses and chooses the template with the maximum output to make its response; On the other hand, if the target identity is not known but the distractor identity is known (e.g., searching for a target with any orientation among distractors with a single orientation), the system could use a variation of the max rule—the maximum of differences—to decide whether the input is a distractor

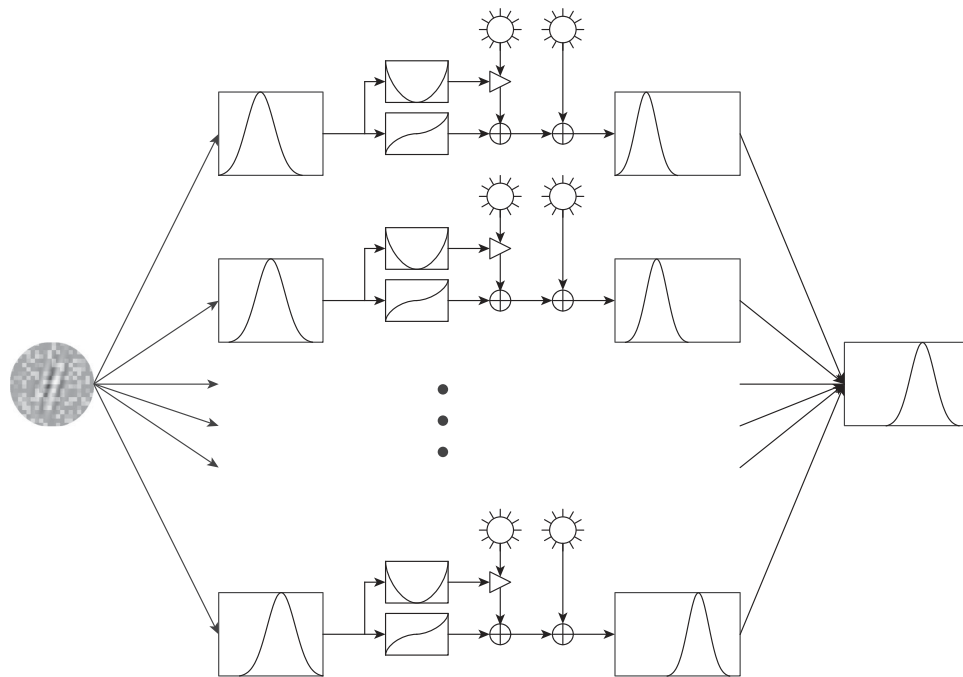


Figure 3. Multiple channels and integration of channel information. A single input is independently processed in multiple channels. Different channels have perceptual templates with different tuning profiles. Internal responses from the channels are integrated by the “max of difference” rule.

or not. Since we aimed to model internal processes during visual search in which the target identity (i.e., orientation) is unknown, the model assumes that the system computes the differences of responses between the distractor-prefering template (i.e., template tuned to 45°) and templates tuned to all possible target orientations (as responses in the five other templates in the model) and uses the maximum of these differences to make a decision. As the maximum of the differences (between the activations in the distractor template and any target template) is greater, the observer is more likely to report target presence. For a low-precision stimulus (a target obviously different from distractors), for example, responses would be strongest in the channel with its template matched to the target but very small in the channel with its template matched to the distractor. Greater response differences of the distractor channel from other channels therefore indicate higher probabilities of being a target, not a distractor. In contrast, for a high-precision stimulus such as a slightly tilted target, the response difference would be very small or even negative because the target and distractor templates overlap a considerable extent. This makes the max difference small, so that the task is difficult.

Information integration: Integration of multiple locations

In the next step, the model integrates information from all items in a display and chooses the stimulus

with the maximum amount of evidence favoring a target to decide the target location. Figure 4 shows a full description for set-size 2 display with two intervals. Integration of information from multiple locations with the maximum rule has well accounted for the set-size effects in psychophysical studies supporting the uncertainty model (Eckstein et al., 2000; Graham, Kramer, & Yager, 1987; Palmer et al., 2000; Shaw, 1982, 1984; Vergheese & Nakayama, 1994; Vergheese & Stone, 1995). In a 2iAFC task, the observer could make a correct response for detecting the target interval in two different ways: (a) The integrated response to a target is greater than all integrated responses in the target-absent interval, and (b) the integrated response to a distractor in the target interval is greater than all responses in the target-absent interval by chance. A correct response in detecting both target interval and target location could be made only in the first case.

Model predictions

Figures 5 and 6 show signature performance patterns for three mechanisms of attention, stimulus enhancement, external noise exclusion, and multiplicative noise reduction, in visual search. Each panel shows the probability of making a correct response (pc) as a function of target-distractor angular difference for different set-sizes in both target interval and location identification tasks (Figure 5) and in target interval identification (Figure 6). Compared to the case

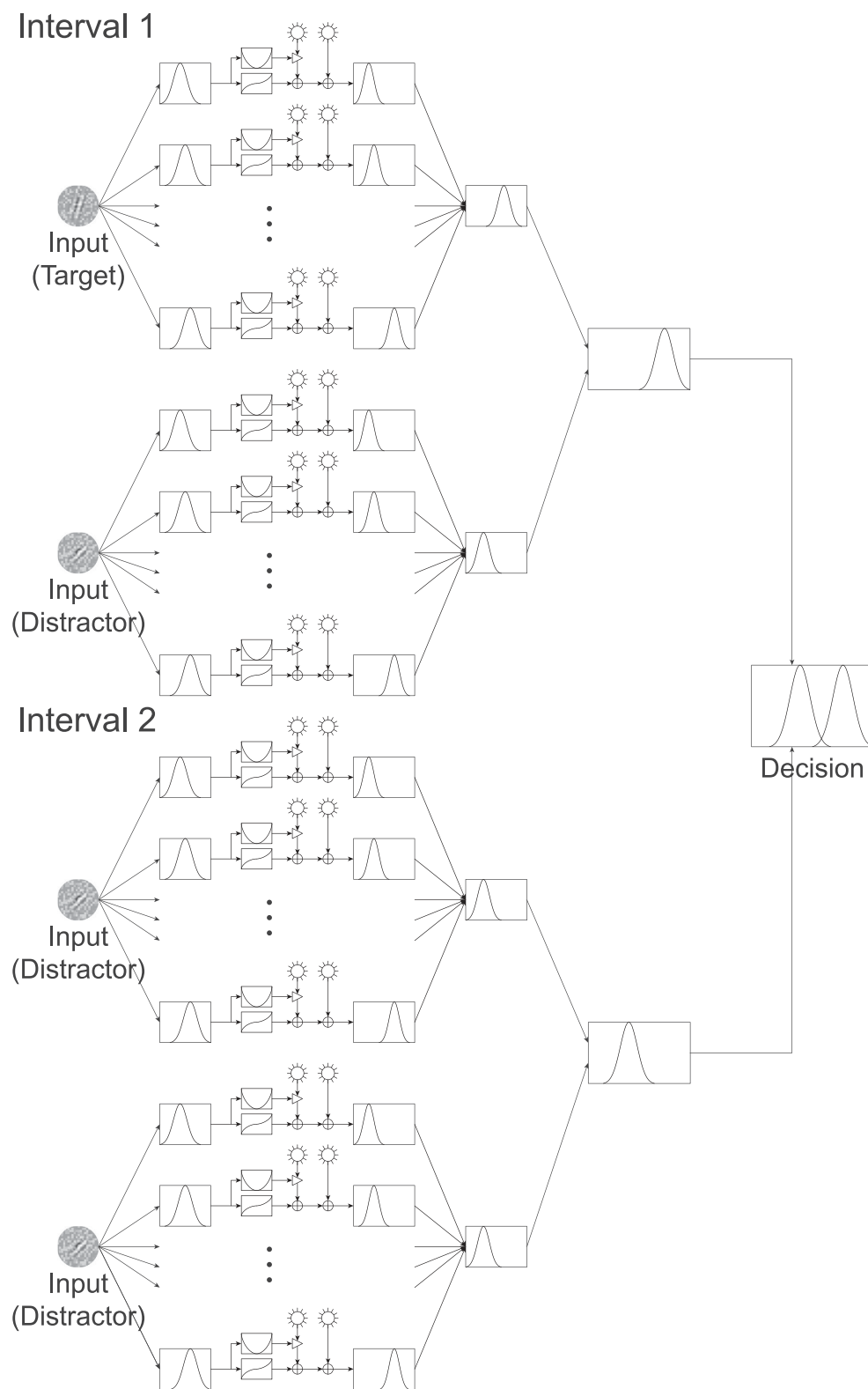
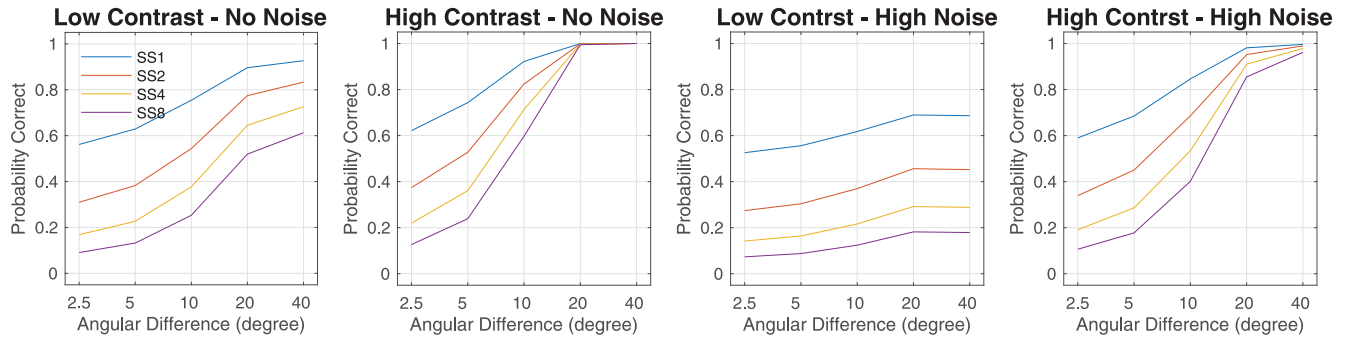
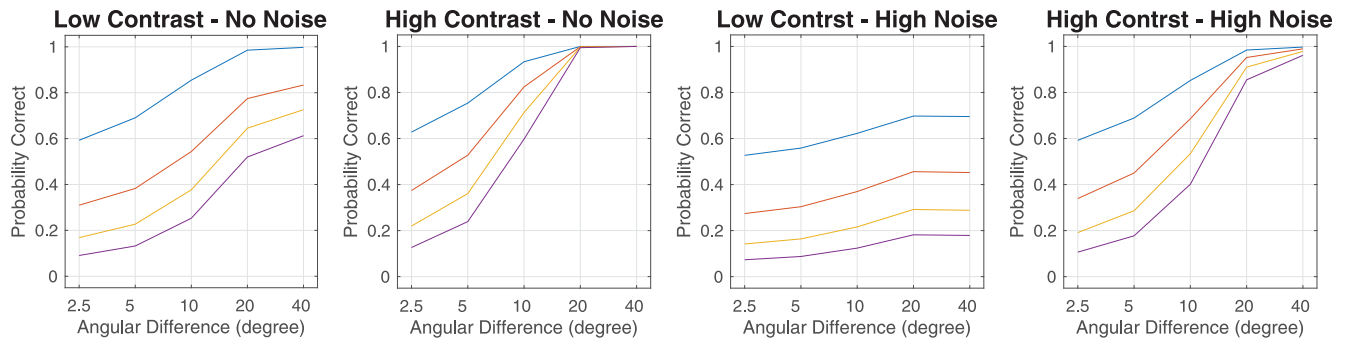


Figure 4. The multichannel, multilocation PTM for visual search with set-size 2 in a 2iAFC task. The target appears at Location 1 in Interval 1.

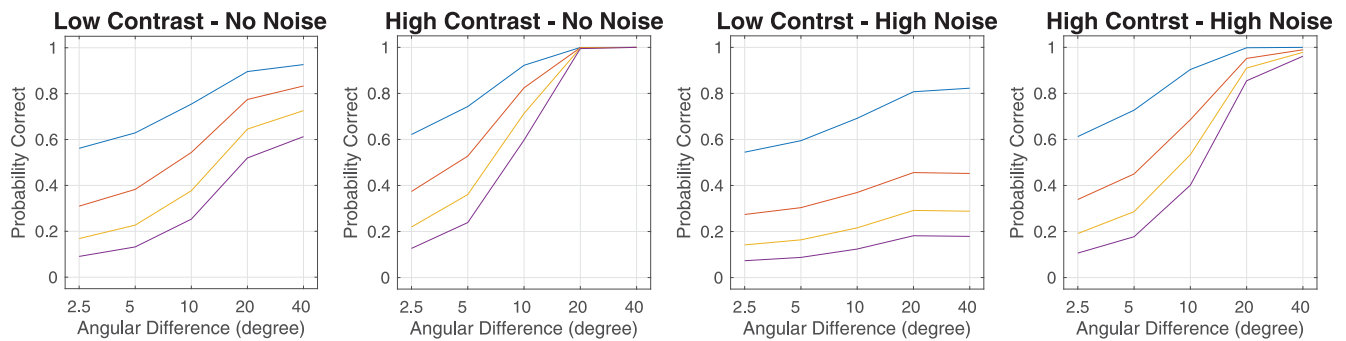
No Attention



Stimulus Enhancement



External Noise Exclusion



Multiplicative Noise Reduction

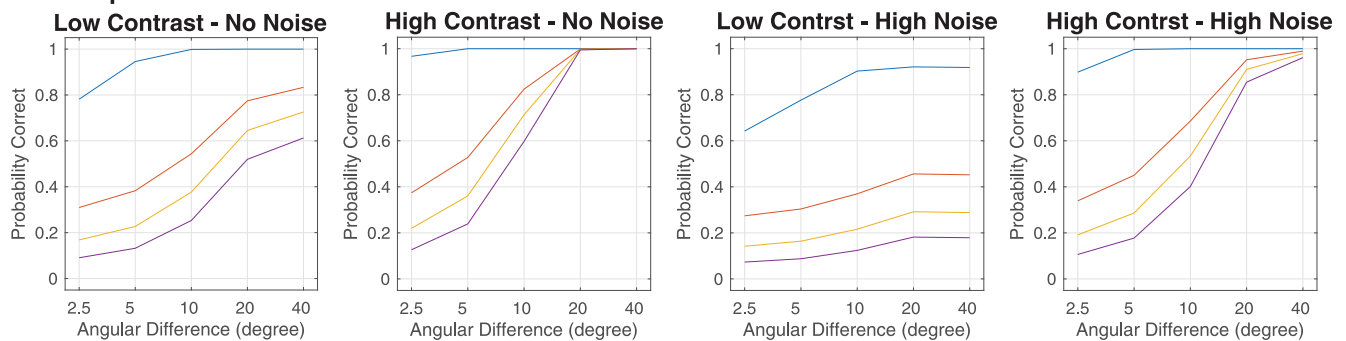
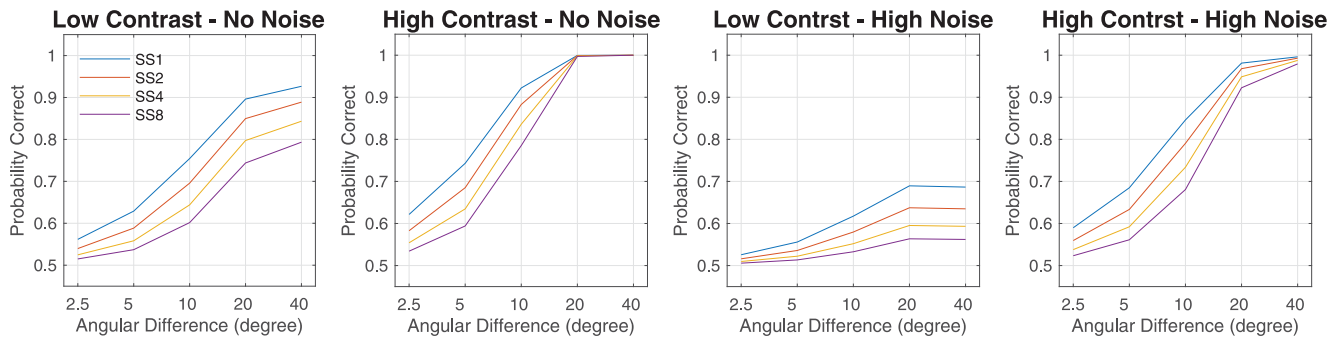
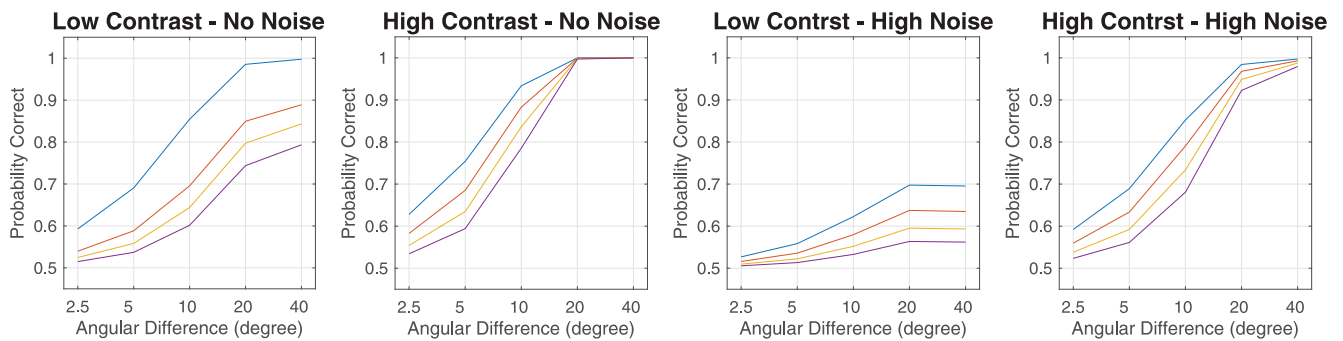


Figure 5. Model predictions for three attention mechanisms. Each panel shows the probability of making a correct response (p_c) as a function of target-distractor angular difference for different set-sizes in both target interval and location identification tasks. No attention effects, stimulus enhancement, external noise exclusion, and multiplicative noise reduction mechanisms are illustrated in each row.

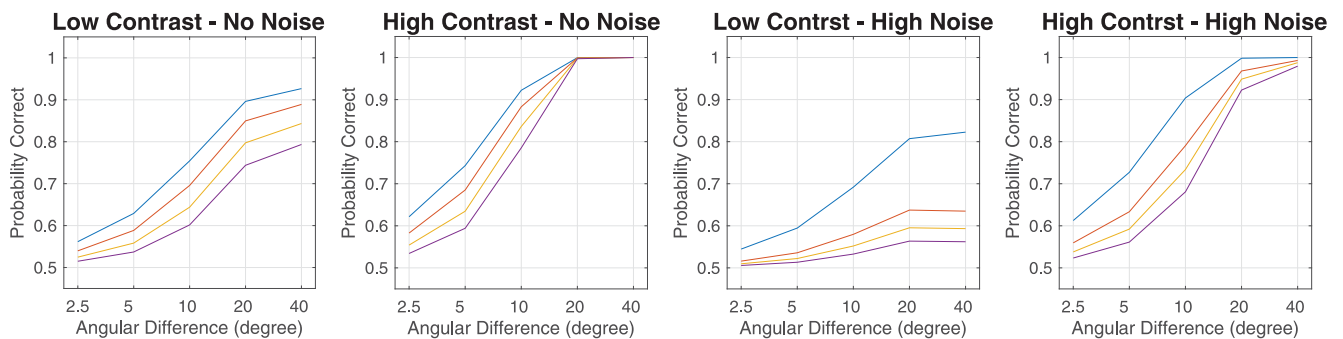
No Attention



Stimulus Enhancement



External Noise Exclusion



Multiplicative Noise Reduction

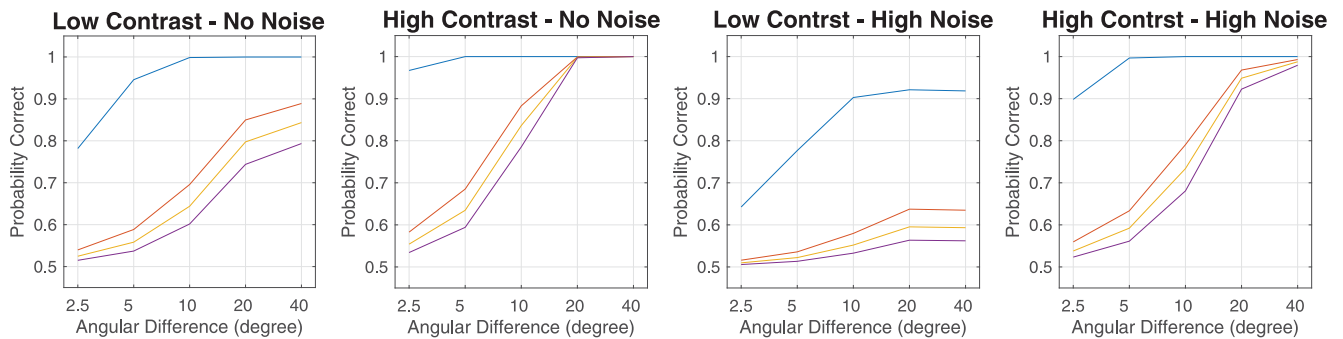


Figure 6. Model predictions for three attention mechanisms. Each panel shows the probability of making a correct response (p_c) as a function of target-distractor angular difference for different set-sizes in target interval identification. No attention effects, stimulus enhancement, external noise exclusion, and multiplicative noise reduction mechanisms are illustrated in each row.

in which there is no attention effect in visual search (first row), stimulus enhancement significantly increases pc only for small set-sizes in low signal contrast conditions (second row), and external noise exclusion increases pc only for small set-sizes in high external noise conditions (third row). Multiplicative noise reduction mechanism increases pc for small set-sizes in all signal and external noise conditions (fourth row).

Experiment

It is not possible to cover the entire stimulus space in a single experimental study. Here, we conducted a visual search experiment in three of the most important stimulus regimes based on the PTM analysis: (a) low-contrast signal without external noise, (b) high-contrast signal without external noise, and (c) high-contrast signal with high external noise. We used a relevant set-size manipulation (Palmer et al., 1993) to control sensory processes such as lateral masking (Palmer et al., 1993). In this relevant set-size manipulation, stimuli are presented in all locations in all trials, but the relevant locations (set-size) for visual search are marked by precues.

Methods

Observers

Three observers, the first author and two naive observers with no prior experience in psychophysical experiments, participated in the study. All observers had normal or corrected-to-normal vision. The study protocol was approved by the institutional review board at the University of Southern California.

Apparatus

The experiment was conducted on an IBM PC compatible computer, running MATLAB (Mathworks, Natick, MA, USA) with Psychtoolbox (Brainard, 1997; Pelli & Zhang, 1991) and EyeLink Toolbox (Cornelissen, Peters, & Palmer, 2002) extensions. The stimuli were displayed on a Hewlett-Packard 19-in. monitor with a 100-Hz refresh rate and a 1,024 × 786-pixel resolution. A special circuit (Li, Lu, Xu, Jin, & Zhou, 2003) was used to generate a monochromatic signal with a high grayscale resolution (> 12.5 bits). Gray levels were linearized using a psychophysical procedure such that available contrasts ranged from –100% to 100% (Lu & Doshier, 2013). All displays were viewed binocularly with natural pupil at a viewing distance of approximately 97 cm in dim light. A chinrest was used for observers to maintain their head position and fixation throughout the experiment.

Observer eye movement was recorded using an SR Research Desktop-Mount EyeLink 1000 system (SR Research, Osgoode, ON, Canada) with a sampling rate of 1,000 Hz. The eye tracker was placed below the monitor, 60 cm from the observer's dominant eye.

Stimuli

The target and distractors were approximately equal spaced on an imaginary circle with a radius of 5° from the fixation point. The position of each item was randomly jittered around its original location on the circle by randomly changing its vertical and horizontal positions independently within 0.5° of visual angle in each trial.

Each individual stimulus was an elliptical Gaussian-windowed sinusoidal grating (Gabor). The global orientation of each stimulus was aligned to its local orientation using elliptical Gabors with its profile described by

$$L(x, y) = L_0 \left[1.0 + c \sin(2\pi f(x \cos \theta \pm y \sin \theta)) \times \exp\left(-\frac{(2x')^2 + y'^2}{2\sigma^2}\right) \right], \quad (1)$$

where $(x', y') = (x \cos \theta + y \sin \theta, -x \sin \theta + y \cos \theta)$, θ is the orientation of the Gabor, c is the signal contrast, and the background luminance L_0 was set in the middle of the dynamic range of the display ($L_{min} = 1 \text{ cd/m}^2$; $L_{max} = 42 \text{ cd/m}^2$) (Hu, 2015). The orientation of the distractors was 45° from the horizontal, and the target was tilted to 2.5°, 5°, 10°, 20°, and 40° counterclockwise from the orientation of distractors. The Gabors were rendered on a 60 × 60-pixel grid, extending 1° × 1° of visual angle.

In a given trial, external noise images were made of 2 × 2-pixel elements (0.03° × 0.03°) with jointly independent, identically distributed randomly generated contrasts. The contrast of each noise element was drawn independently from a Gaussian distribution with a mean of 0 and a standard deviation of 0.33. Because the maximum achievable contrast is 1.0, a sample with a standard deviation of 0.33 conforms reasonably well to a Gaussian distribution.

Each display always included eight stimuli. To control sensory effects (e.g., lateral masking) on the set-size effects, we manipulated the relevant set-size using central precues to indicate possible target locations, instead of manipulating the number of displayed stimuli (Palmer, Ames, & Lindsey, 1993). The precues consisted of lines extending 1.5° from the center of the display to the locations of the relevant stimuli (on the imaginary circle). The thickness of all lines was 0.017°. The relevant set-size manipulation is illustrated in Figure 7a.

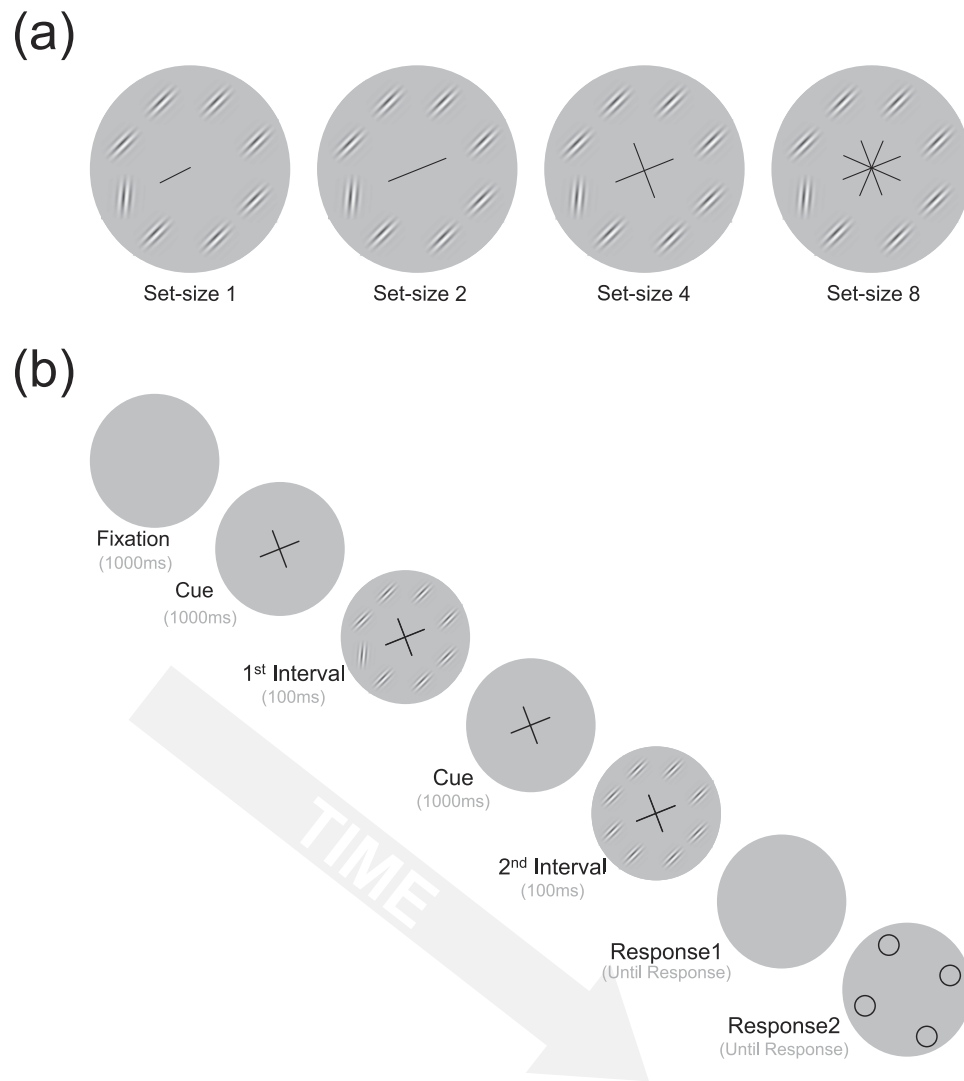


Figure 7. (a) Relevant set-size conditions and (b) an example of the trial sequence of the psychophysical experiment.

Design

We tested four relevant set-sizes (one, two, four, and eight items). The distractors were tilted 45° from vertical. The target differed from the distractors by one of five angular differences (2.5° , 5° , 10° , 20° , and 40°). There were three external noise \times contrast conditions: low contrast/no noise ($c = .25$, $N_{ext} = 0$), high contrast/no noise ($c = 1$, $N_{ext} = 0$), and high contrast/high noise ($c = 1$, $N_{ext} = .33$). Thus, the total number of tested conditions was 60 ($= 4$ set-sizes $\times 5$ angular differences $\times 3$ noise-signal contrasts). All conditions were intermixed in each session. All observers ran 20 sessions of 300 trials, for a total of 6,000 trials (100 trials per experimental condition).

Procedure

An example trial sequence is illustrated in Figure 7b. Each trial began with the observer fixating on a small

dot for 1,000 ms, followed by a central precue (1,000 ms) indicating potential target locations, the first stimulus display (100 ms), an interstimulus interval (1,000 ms), the second stimulus display (100 ms), and two response displays. The stimulus sequence consisted of five stimulus frames of 20 ms each: external noise, signal, external noise, signal, and external noise. In the first response display, observers were asked first to identify which interval contained the target with a keyboard. Auditory feedback was provided after each incorrect target interval response. Then, in the second response display, placeholders (circles with a 1° radius) appeared at all possible target locations. Observers were required to indicate the target location with a mouse click on one of the placeholders. Auditory feedback was also provided after each incorrect location response.

Observers were instructed to maintain fixation throughout each trial. To discourage eye movements, gaze position of the dominant eye was tracked. If the

eye position deviated more than 1.5° of visual angle from the fixation, a low-tone beep was used to inform the observer, the trial was discarded, and the condition was repeated in a trial randomly placed within the remaining trials of the session. Eye-tracker calibration was performed in the beginning of each session, with drift correction in the beginning of each block (30 trials). Each experimental session consisted of 10 blocks and lasted about 60 min.

Results

Observers successfully maintained fixation in most trials. They broke fixation in only on average 4.5% of trials (7.5%, 3.8%, and 2.1% for observers BH, JB, and KL, respectively), and these trials were repeated within the block.

We calculated the fraction of trials in which the observer made correct responses for the target interval (detection task) and for both target interval and location (both tasks). A repeated-measures analysis of variance was conducted on probability correct (pc) to examine effects of set-size and target-distractor orientation difference. For the detection task, performance significantly decreased as set-size increased ($F(3, 6) = 647.53, p < 0.001$) and increased as target-distractor orientation difference increased ($F(4, 8) = 26.43, p < 0.001$). The interaction between set-size and target-distractor difference was also significant ($F(12, 24) = 9.85, p < 0.001$). All main effects and the interaction were significant for percent correct in “both tasks” ($F(3, 6) = 214.11, p < 0.001$ for set-size; $F(4, 8) = 40.49, p < 0.001$ for target-distractor difference; and $F(12, 24) = 109.48, p < 0.001$ for interaction). If the location errors were made due to spatial uncertainty alone, probability of choosing locations should be the same over all distractor locations in incorrect trials. As shown in Figure 8, incorrect responses of the location task were not focused on locations next to the target. The probability of choosing the adjacent locations (i.e., ± 1) was 0.068 and that of choosing far locations (i.e., $\pm 2, \pm 3$, and ± 4) was 0.061 (0.061 vs. 0.050 for BH; 0.053 vs. 0.067 for JB; 0.089 vs. 0.064 for KL).

The analytic model (Equation A16 in Appendix) was fitted to the probability of correct responses in identifying both target interval and location using a least squares estimation procedure. In the model, probability correct is a function of observer parameters (σ_m , the proportion constant of multiplicative internal noise; σ_a , the standard deviation of additive internal noise; β_{max} , the maximum gain of the template to the preferred stimulus; β_σ , the bandwidth of the template; γ , transducer nonlinearity) and stimulus parameters (σ_{ext} , the standard deviation of external noise; c , signal stimulus contrast; θ , target-distractor orientation difference; SS , set-size). In the model-fitting

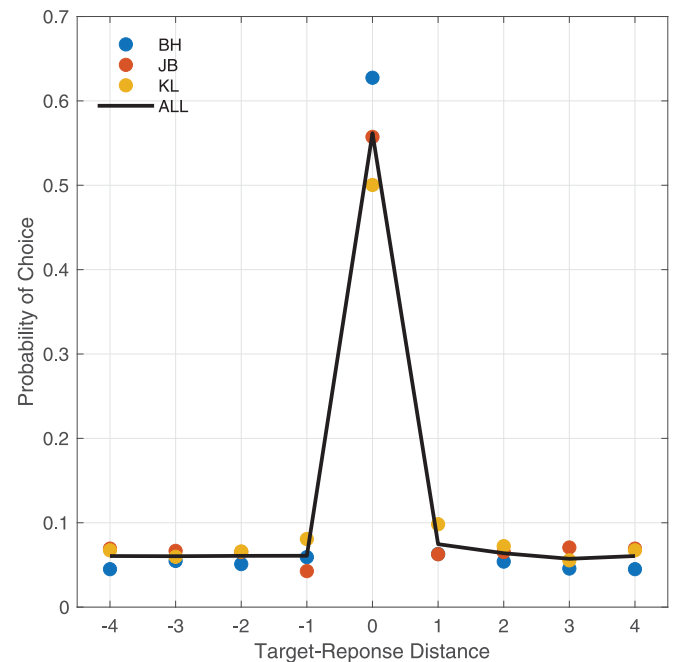


Figure 8. Probability of choosing locations in the set-size 8 condition as a function of target-response distance. A target-response distance of 0 indicates a correct response; responses at all the other target-response distances are incorrect in the target location task.

procedure, all observer parameters were the same for all stimulus conditions in the experiment and free to vary. The model was fitted to the aggregated data across observers as well as individual data separately. Model parameters were adjusted using a gradient descent method to minimize the error function, the sum of the squared differences between the predicted and observed probability correct. The best-fitting parameters are listed in Table 1, and the corresponding PTM model predictions are plotted in Figure 9 along with the data. Even without the operation of attention mechanism factors, psychophysical data were well accounted for by the model ($r^2 = .9809$), indicating the ability of a pure spatial uncertainty model in accounting for the bulk of the data.

The same fitting procedure was also carried out with the model with either one or both of the two primary attention mechanisms (stimulus enhancement and external noise exclusion) applied to the data in the set-size 1 condition. (There is no evidence in the current spatial attention literature using the PTM for the third, multiplicative noise mechanism.) In addition to observer parameters of the no-attention model, there are multipliers on additive internal noise (A_a) for the stimulus enhancement mechanism and on the maximum gain (A_f) and width ($A_{f\sigma}$) of the external noise filter for the external noise exclusion mechanism. These attention parameters were applied to fit models to only data in the set-size 1 condition. Thus, models

Observer	Model	β_{max}	β_{σ} (°)	γ	σ_m	σ_a	$A_{f(1)}$	$A_{f\sigma(1)}$	$A_{a(1)}$	r^2	F	p
All	NE + SE	8.36	14.09	1.06	0.0005	0.6947	0.97	1.00	1.00	0.9810	—	—
	NE	8.36	14.09	1.06	0.0005	0.6947	0.97	1.00	—	0.9810	0.00	0.986
	SE	8.42	14.00	1.06	0.0004	0.6993	—	—	1.00	0.9809	0.19	0.827
	No attention	7.84	14.07	1.06	0.0000	0.6510	—	—	—	0.9809	0.13	0.943
BH	NE + SE	6.34	10.99	1.00	0.0009	0.4505	1.00	1.00	1.00	0.9714	—	—
	NE	6.33	10.99	1.00	0.0009	0.4498	1.00	1.00	—	0.9714	0.01	0.935
	SE	6.34	11.01	1.00	0.0009	0.4493	—	—	1.00	0.9714	0.00	0.998
	No attention	6.30	10.99	1.00	0.0002	0.4476	—	—	—	0.9714	0.01	0.998
JB	NE + SE	2.90	15.25	1.36	0.0000	0.2423	0.87	1.00	1.00	0.9516	—	—
	NE	2.90	15.25	1.36	0.0000	0.2423	0.87	1.00	—	0.9516	0.00	1.000
	SE	2.77	14.71	1.32	0.0000	0.2363	—	—	1.00	0.9468	2.53	0.089
	No attention	2.77	14.71	1.32	0.0000	0.2363	—	—	—	0.9468	1.69	0.181
KL	NE + SE	5.94	15.09	1.00	0.0007	0.5007	1.00	1.00	1.00	0.9713	—	—
	NE	5.25	14.90	1.00	0.0009	0.4463	1.00	1.00	—	0.9712	0.26	0.615
	SE	5.94	15.09	1.00	0.0007	0.5007	—	—	1.00	0.9713	0.00	1.000
	No attention	5.54	14.39	1.00	0.0089	0.4748	1.00	1.00	1.00	0.9712	0.04	0.989

Table 1. Parameters of the best-fitting model and results of nested model tests (probability correct in detecting the target interval and location). *Notes:* NE = external noise exclusion; SE = stimulus enhancement; β_{max} = maximum gain of template; β_{σ} = bandwidth of template; γ = exponent of the nonlinear transducer function; σ_a = additive noise; σ_m = multiplicative noise.

with attention parameter(s) for set-size 1 and models without attention parameter(s) for other set-sizes were concurrently fitted for each attention mechanism. The attention parameters had constraints of $0 \leq A_a \leq 1$, $A_f \leq 1$, and $0 \leq A_{f\sigma} \leq 1$. The models with attention factors showed slightly better goodness of fit than the model without attention factors: $r^2 = .9809$, $.9810$, and $.9810$ for stimulus enhancement, external noise exclusion, and both mechanisms, respectively. However, these differences are quantitatively small.

To quantitatively evaluate contributions of the mechanisms of attention involved in visual search, we compared the goodness of fit of several models using the F test for nested models. For two models with k_{full} and $k_{reduced}$ parameters, the F statistic is defined as

$$F(d f_1, d f_2) = \frac{(r_{full}^2 - r_{reduced}^2) / d f_1}{(1 - r_{full}^2) / d f_2}, \quad (2)$$

where $d f_1 = k_{full} - k_{reduced}$, $d f_2 = N - k_{full}$, and N is the number of data points.

For the average data across all the subjects, we found that models with additional attention parameters did not significantly improve the goodness of fit over the model without any attention parameters ($F(2, 52) = 0.19$, $p = 0.827$ for external noise exclusion; $F(1, 52) = 0.00$, $p = 0.986$ for stimulus enhancement; and $F(3, 52) = 0.13$, $p = 0.943$ for both mechanisms). These results suggest that enhanced perceptual quality for the attended items is not a major component to explain search accuracy and that statistical uncertainty alone is sufficient to explain the observed set-size effects.

To investigate individual differences in attention effects, we also fit the model to each observer's data. The results were consistent with those from the average data. The model without any attention mechanism well accounted for the data (r^2 ranged from $.9468$ to $.9714$), and no attention mechanism was necessary to account for the data of any observer (see Table 1).

We also fitted the analytic model (Equation A17 in Appendix) to probability correct in detecting the target interval, ignoring performance in the localization task (Table 2 and Figure 10). The average data from the experiment were well accounted for by the model without any attention mechanism but solely location uncertainty ($r^2 = .9676$) and the models with attention mechanisms ($r^2 = .9676$ for all stimulus enhancement, external noise exclusion, and both mechanisms). Differences in the goodness of fit between the models with and without attention mechanisms were not statistically significant ($F(2, 52) = 0.00$, $p = 0.997$ for external noise exclusion; $F(1, 52) = 0.01$, $p = 0.909$ for stimulus enhancement; and $F(3, 52) = 0.00$, $p = 1.000$ for both mechanisms). The conclusion is also supported by individual data analysis (see Table 2). These results suggest that we can account for observers' performance in the visual search task without any spatial attention, with all differences between set sizes reflecting the statistical effects of location uncertainty.

Discussion

To characterize internal processes of an observer conducting visual search, we developed an observer

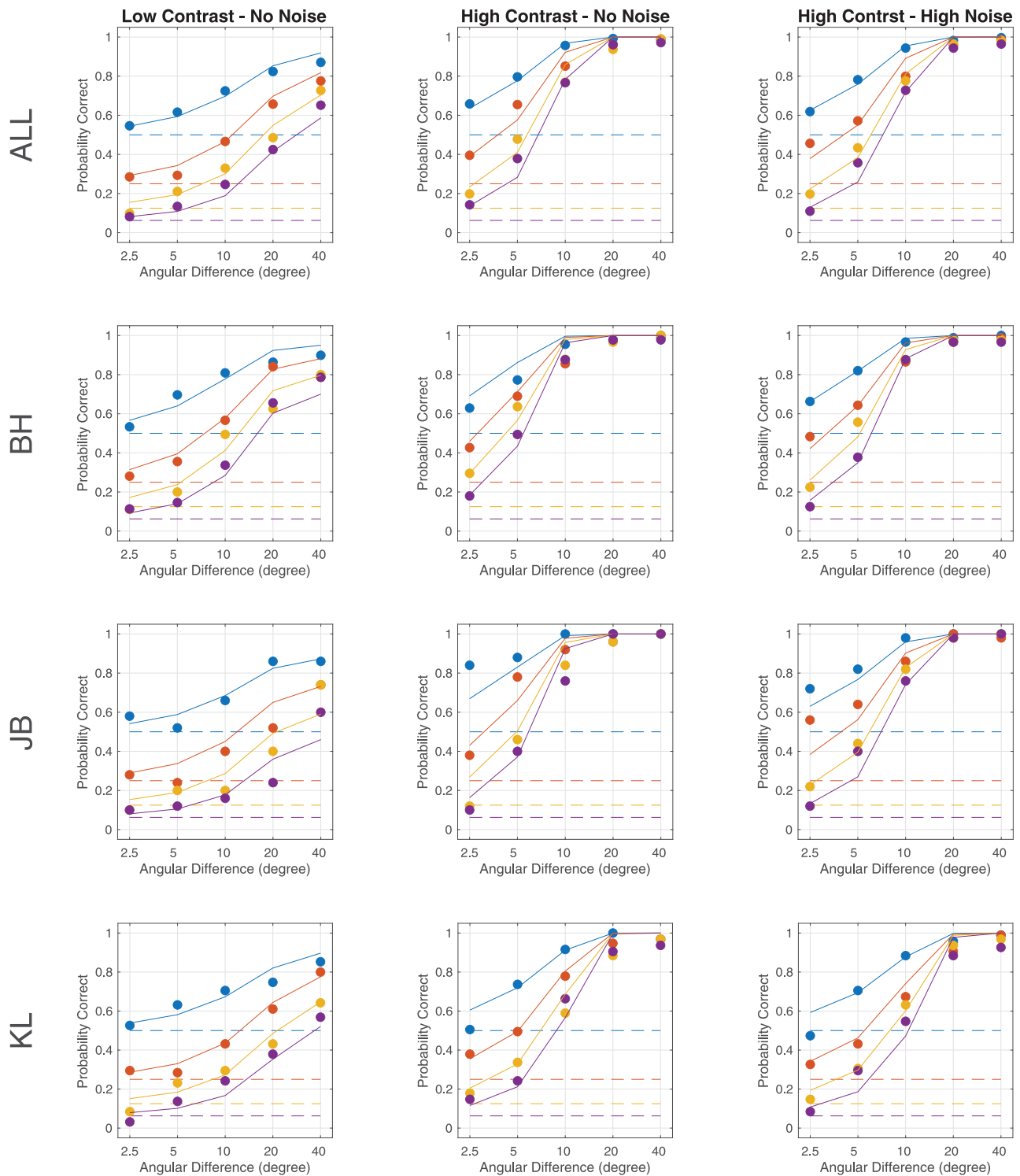


Figure 9. The best-fitting model. Probability correct in detecting the target interval and location. Curves represent the predictions of the model, and filled circles represent data from the psychophysical experiment (aggregated data in the first row and individual data in the following rows). Different colors represent set-size conditions (blue: SS1; red: SS2; yellow: SS4; purple: SS8). Dashed lines show chance levels for different set-size conditions (.5, .25, .125, and .0625 for set-sizes 1, 2, 4, and 8, respectively).

Observer	Model	β_{max}	B_{σ} (°)	γ	σ_m	σ_a	$A_{f(1)}$	$A_{f\sigma(1)}$	$A_{a(1)}$	r^2	F	p
All	NE + SE	5.76	13.37	1.10	0.0001	0.4793	1.00	1.00	1.00	0.9676	—	—
	NE	5.96	13.12	1.09	0.0007	0.4973	1.00	1.00	—	0.9676	0.01	0.909
	SE	5.95	13.14	1.09	0.0021	0.4968	—	—	1.00	0.9676	0.00	0.997
	No attention	5.98	13.11	1.09	0.0000	0.4982	—	—	—	0.9676	0.00	1.000
BH	NE + SE	6.25	11.12	1.00	0.0010	0.4472	1.00	0.99	1.00	0.9559	—	—
	NE	6.13	11.10	1.00	0.0001	0.4391	1.00	0.95	—	0.9559	0.04	0.844
	SE	6.02	11.06	1.00	0.0001	0.4315	—	—	1.00	0.9558	0.05	0.948
	No attention	6.02	11.06	1.00	0.0001	0.4316	—	—	—	0.9558	0.04	0.991
JB	NE + SE	3.49	22.08	1.50	0.0001	0.2587	0.74	1.00	1.00	0.9162	—	—
	NE	3.49	22.06	1.50	0.0001	0.2589	0.74	1.00	—	0.9162	0.00	1.000
	SE	4.07	24.07	1.55	0.0000	0.3085	—	—	0.59	0.9107	1.70	0.192
	No attention	3.47	18.63	1.42	0.0000	0.2706	—	—	—	0.8965	4.07	0.011
KL	NE + SE	5.73	14.13	1.11	0.0011	0.4816	1.00	1.00	1.00	0.9016	—	—
	NE	5.73	14.13	1.11	0.0011	0.4816	1.00	1.00	—	0.9016	0.00	1.000
	SE	6.15	13.86	1.09	0.0006	0.5188	—	—	1.00	0.9013	0.06	0.938
	No attention	6.14	13.86	1.09	0.0000	0.5179	—	—	—	0.9013	0.04	0.989

Table 2. Parameters of the best-fitting model and results of nested model tests (probability correct in detecting the target interval only).

model that combines the PTM, the attention mechanisms in the PTM framework (Lu & Doshier, 1998), and decision uncertainty in signal detection theory (Green, 1961; Green & Swets, 1966). Previous studies based on the PTM framework have focused on the role of spatially cued attention in altering the internal representation of sensory information while controlling structural uncertainty as much as possible (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 1998, 2000; Lu, Liu, & Doshier, 2000). In these studies, stimulus discrimination tasks, the location of the target was indicated by a response cue, and external noise exclusion was the primary mechanism (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 1998, 2000; Lu et al., 2000), a pattern that held for either low- or high-precision discrimination tasks (Hetley et al., 2014). The differences between the current visual search experiment and those attention experiments are considered below.

The current study differed from those explicit studies of spatial attention to ask whether the same mechanisms, which improved sensory coding and performance, also operated to distribute attention resources among the stimuli in different display sizes in standard visual search. It extends the previous investigations of the role of attention over and above location uncertainty (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 1998, 2000; Lu, Liu, & Doshier, 2000) or target similarity (Hetley et al., 2014) in those studies by examining tasks with varying precision in the target discrimination (stimulus unknown) and integrating the effects of stimulus contrast and external noise, factors by which the PTM accounts for psychophysical performance over a large range of

stimulus manipulations in visual search. That is, it explicitly considers both target variation (by assuming multiple overlapping templates to represent the oriented stimuli) and multiple locations. By assuming multiple overlapping templates for a single input and using the power of the PTM, the current model successfully characterized visual search performance in a wide range of conditions, including signal intensity, external noise level, discrimination precision, and target identity and location uncertainties with only five parameters.

The contributions of decision uncertainty and attention in a visual search task were evaluated with the model. We did not find any contribution of attention—or effects of different display sizes on the sensory quality of the stimulus representations. Target identity and location uncertainties were sufficient to account for the observed set-size effects in different stimulus regimes. The results are consistent with decision uncertainty-based theories of visual search (Davis et al., 2003; Eckstein, 1998; Eckstein et al., 2000; Fahle, 1991; McLean et al., 1997; Palmer, 1994; Palmer et al., 1993, 2000; Verghese & Stone, 1995). On the other hand, our results differ from the results in spatial cueing experiments (Doshier & Lu, 2000a, 2000b; Lu & Doshier, 2000) where strong attention effects were found in multiple location displays that involved four to eight stimuli in conditions that were apparently very similar to those of the current study.

The powerful role of attention in spatial cueing and lack of attention distribution on the sensory representation of individual stimuli in visual search surely reflect the different deployment of attention in the two cases and may reflect other paradigm differences as well. A two-alternative forced identification task with

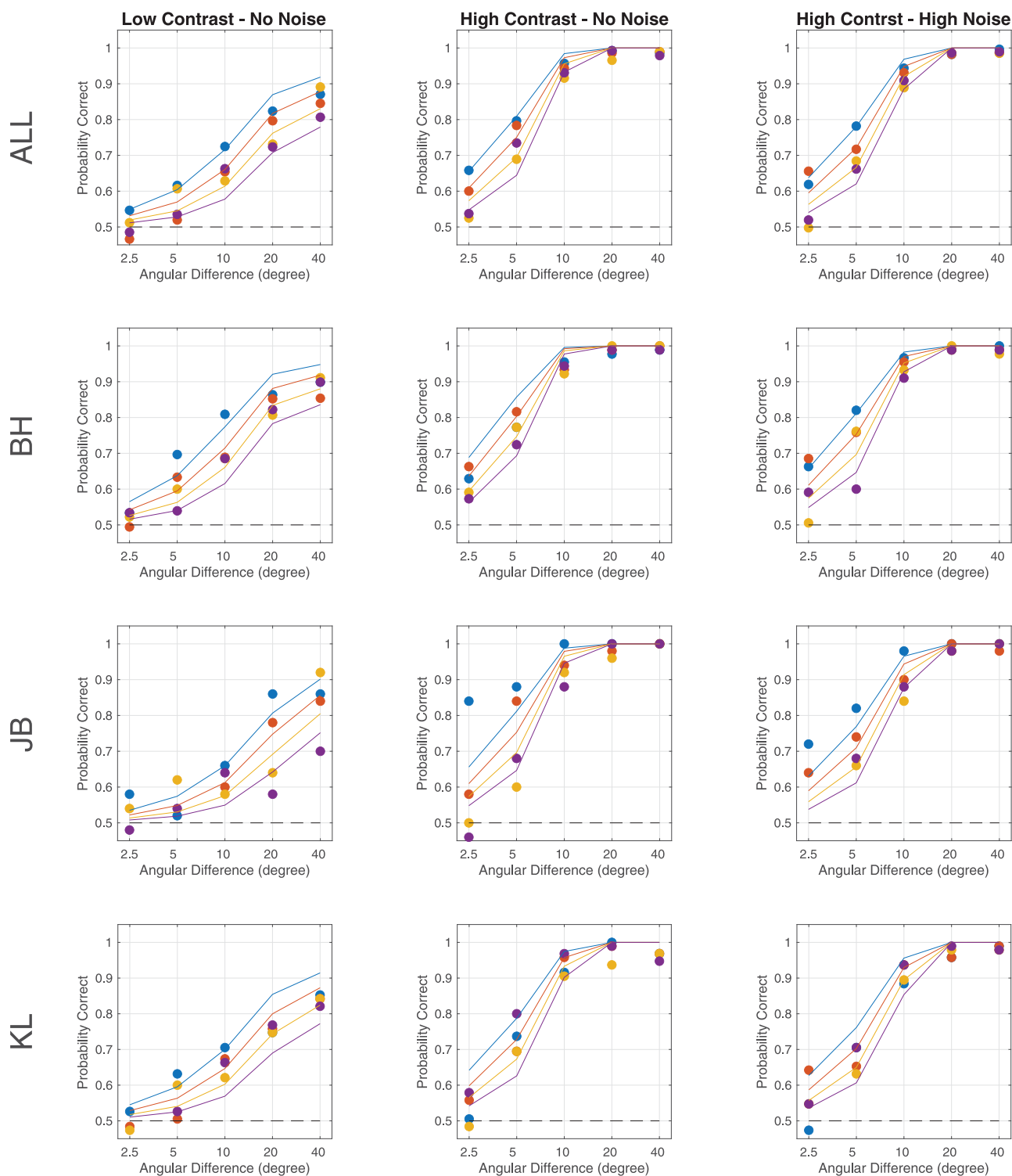


Figure 10. The best-fitting model. Probability correct in detecting the target interval only. Curves represent model predictions, and filled circles represent data from the psychophysical experiment (aggregated data in the first row and individual data in the following rows). Different colors represent set-size conditions (blue: SS1; red: SS2; yellow: SS4; purple: SS8). Black dashed lines show chance levels, .5, in all set-size conditions.

a single display with attention cues to a single item and elimination of location uncertainty by response cues was used in the spatial cueing studies (Doshier & Lu, 2000a, 2000b; Doshier, Liu, Blair, & Lu, 2004;

Lu & Doshier, 2000; Lu et al., 2000). In these, a valid precue (or, alternatively, a simultaneous valid cue) indicated a single stimulus to be attended and resulting in the same performance regardless of display size

(Doshier et al., 2004; Hetley et al., 2014). In contrast, a 2-interval forced-choice task with two displays was used in the current experiment, and the “attention” manipulation was effective set-size of the display. We chose this 2iAFC task to be compatible with most of the visual search studies looking for attention effects over and above the effects of location uncertainty in psychophysical tasks (especially those of Palmer). While distributing spatial attention across different numbers of locations might have led to a different quality of sensory representation, as in the spatial cueing experiments, an inducement to differential attention deployment may be limited by only small benefits from broad distribution in the largest two set-sizes. The typical spatial attention experiments use cues that are highly predictive of the target locations. Finally, the practice level of the observers might be another contributor to any differences. Large amounts of practice may eliminate attention effects in visual tasks (Doshier, Han, & Lu, 2010). Because of the factorial design, we had to collect 20 sessions of data from each observer. It is possible that we did not observe any significant attention effects because our observers had considerable practice. In sum, several factors may have led to the lack of attention effects over and above spatial uncertainty in the current visual search study, which tested some conditions associated with strong effects of spatially cued attention. Additional studies are necessary to reconcile the apparent inconsistency between lack of attention effects in visual search and the presence of attention effects in spatial cuing. Nevertheless, our theoretical model may serve as a framework to investigate the taxonomy of visual attention by identifying the circumstances in which attention limitations play a substantial role in performance or, conversely, in which focused attention improves performance.

We have focused on feature search (e.g., orientation) in this study. Eckstein et al. (2000) showed that spatial uncertainty is sufficient to explain search performance in conjunction search without assuming limited capacity of attention. Itti and Koch (2000) developed a feature saliency model to account for covert attentional shift and overt eye movements in free-viewing natural scenes. The model includes multiple templates in multiple feature dimensions (e.g., color and orientation) and combines outputs of feature maps to produce a “saliency map” over space. We only focused on feature search with simple stimuli. A natural extension of our current development is to model visual search with more complex stimuli and conjunction search.

In this study, we developed a model to account for search accuracy in brief displays that limited eye movements. In our model as well as other signal-detection models, set-size effects in search accuracy can be a simple statistical consequence of integrating more sources of information or statistical

decision effects (Eckstein, 1998; Palmer, 1994; Palmer, Verghese, & Pavel, 2000; Shaw, 1982; Sperling & Doshier, 1986). Others measured response times in visual search using displays that remained on until response (e.g., Cave & Wolfe, 1990; Treisman, 1988). In this paradigm, observers’ eye movement is uncontrolled, so that results include unknown contributions of eye movements. Set-size effects in response times have been modeled with two-stage architectures in which attention is associated with a serial processing stage. These serial processing models include the feature integration theory (Treisman & Gelade, 1980), selective search models (Doshier, 1998; Egeth, Virzi, & Garbart, 1984), and guided search models (Cave & Wolfe, 1990; Wolfe, 1994). In addition, Smith, Ratcliff, and Wolfgang (2004) showed that effects of attention on accuracy and response time were dissociable. In their spatial cueing experiment, attention had a substantial effect on response time but not on accuracy for perceptually well-localized stimuli without backward masks. We believe it is possible to combine the multichannel, multilocation observer model developed in this study with response time models to account for response times or speed-accuracy trade-offs in various visual search tasks (e.g., Doshier, Han, & Lu, 2004, 2010; Purcell, Schall, Logan, & Palmeri, 2012), a goal that would require a substantial additional elaboration that we have not pursued here.

Keywords: attention, uncertainty, visual search, spatial cueing

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Appendix: The multichannel and multilocation analytical PTM

The PTM

The single-channel PTM is constructed with four components: the gain of the perceptual template (β) to the signal stimulus, the exponent of the nonlinear transducer function (γ), internal additive noise (σ_a), and coefficient of the multiplicative internal noise (σ_m) (see Lu & Doshier, 2008, for details). The magnitude of the response to a signal stimulus is expressed by $(\beta c)^\gamma$, where c is the contrast of the signal. The multiplicative internal noise follows a zero-mean Gaussian distribution with its standard deviation proportional to the total energy of the input stimuli (with proportional constant σ_m). The additive internal noise also follows a zero-mean Gaussian distribution but with a fixed standard deviation, σ_a .

In each trial, the internal response is the sum of outputs from the signal pathway and the internal and external noises. The final output of the system to a stimulus with contrast c is

$$(\beta c)^\gamma + g(0, \sigma_a) + g(0, \sigma_m)((\beta c)^\gamma + g(0, \sigma_{ext})^\gamma) + g(0, \sigma_{ext})^\gamma \quad (\text{A1})$$

where $g(a, b)$ is a random sample from a Gaussian distribution with a mean of a and standard deviation of b . The probability distribution of responses follows a Gaussian distribution with

$$\begin{aligned} \mu &= (\beta c)^\gamma \\ \sigma &= \sqrt{\sigma_{ext}^{2\gamma} + \sigma_m^2 \left((\beta c)^{2\gamma} + \sigma_{ext}^{2\gamma} \right) + \sigma_a^2} \quad (\text{A2}) \end{aligned}$$

If the input does not contain a signal stimulus, the system output to an external noise-only stimulus is

$$\begin{aligned} \mu &= 0 \\ \sigma &= \sqrt{\sigma_{ext}^{2\gamma} + \sigma_m^2 \sigma_{ext}^{2\gamma} + \sigma_a^2} \quad (\text{A3}) \end{aligned}$$

In a simple case of a 2-interval forced-choice task, observers' discriminability can be formulated as a

normalized distance between two distributions (Green & Swets, 1966):

$$d' = \frac{\mu_{signal} - \mu_{noise}}{\sqrt{\sigma_{signal}^2 + \sigma_{noise}^2}} \quad (\text{A4})$$

where μ_{signal} and σ_{signal} are the mean and standard deviation of the internal responses to signal + noise stimulus, and μ_{noise} and σ_{noise} are the mean and standard deviation of the internal responses to noise-only stimulus.

PTM with multiple channels

In the present extension of the original PTM, it is assumed that there are multiple perceptual templates. Each template is tuned to one of the potential input orientations (one distractor orientation and five possible target orientations in our experiment) and has a Gaussian tuning function with a bandwidth of β_σ and maximum gain, β_{max} , to the preferred stimulus. To reiterate multiple templates, β_i represents a template gain of the i th detector to an input stimulus:

$$\beta_i = \beta_{max} g(\Delta\theta_i, 0, \beta_\sigma) \quad (\text{A5})$$

where $\Delta\theta$ is the angular difference between a template's preferred orientation and the stimulus orientation. Since a bank of channels is located in a small spatial location, the variance of the multiplicative noise is proportional to the total energy from the outputs of all the templates. In sum, an output distribution in the i th channel is a Gaussian distribution with

$$\begin{aligned} \mu_i &= (\beta_i c)^\gamma \\ \sigma_i &= \sqrt{\sigma_{ext}^{2\gamma} + \sigma_m^2 \left(\left(\sum_i \beta_i c \right)^{2\gamma} + \sigma_{ext}^{2\gamma} \right) + \sigma_a^2} \quad (\text{A6}) \end{aligned}$$

Mechanisms of attention in PTM with multiple channels

In the model, attention may work in three different ways: (a) stimulus enhancement, which is mathematically equivalent to internal additive noise reduction in the PTM, is modeled by multiplying σ_a by A_a ; (b) external noise exclusion with increased weight of the input from the attended region/object without changing the tuning characteristics of the perceptual template is modeled by multiplying σ_{ext} with A_f ; and (c) external noise exclusion that changes the tuning bandwidth of the perceptual template is modeled by

multiplying β_σ with $A_{f\sigma}$. The output distribution of the i th channel for an attended location is a Gaussian distribution with

$$\begin{aligned} \beta_i &= \beta_{max} g(\Delta\theta, 0, \beta_\sigma A_{f\sigma}) \\ \mu_i &= (\beta_i c)^\gamma \\ \sigma_i &= \sqrt{\left(A_f \sigma_{ext} \right)^{2\gamma} + \sigma_m^2 \left(\left(\sum_i \beta_i c \right)^{2\gamma} + \left(A_f \sigma_{ext} \right)^{2\gamma} \right) + (A_a \sigma_a)^2} \quad (\text{A7}) \end{aligned}$$

Information integration: Integration of multiple channels

To compute the “max of differences” for a single input, the model computes (a) μ_{dist} and σ_{dist} representing the channel tuned to distractor orientation, (b) μ_i and σ_i of the response distributions in all nondistractor channels ($i = 1, 2, 3, \dots$), (c) distributions of the differences between the distractor channel and nondistractor channels, and (d) then the distribution of maximum difference across all nondistractor channels. To simplify the computation, we first computed the distribution (with μ_{max} and σ_{max}) of the maximum internal response across all nondistractor channels and then a distribution of differences (with μ_{diff} and σ_{diff}) between the max distribution and the distributions of the distractor channels. The two approaches are mathematically equivalent.

“Max” distribution

According to Clark (1961), the max of two random samples picked from two different Gaussian distributions with a correlation $\rho = 0$ could be approximated by a Gaussian distribution with mean and variance as

$$\begin{aligned} \mu_{max} &= \mu_2 G(\beta) + \mu_1 G(-\beta) + \alpha g(\beta) \\ \sigma_{max} &= \sqrt{\left(\mu_2^2 + \sigma_2^2 \right) G(\beta) + \left(\mu_1^2 + \sigma_1^2 \right) G(-\beta) + (\mu_1 + \mu_2) \alpha g(\beta) - \mu_{max}^2} \quad (\text{A8}) \end{aligned}$$

where $G(x)$ is a cumulative Gaussian distribution and

$$\begin{aligned} \alpha &= \sqrt{\sigma_1^2 + \sigma_2^2} \\ \beta &= \frac{\mu_2 - \mu_1}{\alpha} \quad (\text{A9}) \end{aligned}$$

For the max distribution from more than two distributions, the approach can be used recursively:

$$f_{\max} = \max(f_1, \max(f_2, \dots)) \quad (\text{A10})$$

where $\max()$ represents Clark's approach.

Max difference distribution

The distribution of differences (with μ_{diff} and σ_{diff}) between the max distribution (μ_{max} and σ_{max}) and the distractor channel distribution (μ_{dist} and σ_{dist}) is a Gaussian distribution with

$$\begin{aligned} \mu_{diff} &= \mu_{\max} - \mu_{dist} \\ \sigma_{diff} &= \sqrt{\sigma_{\max}^2 + \sigma_{dist}^2} \end{aligned} \quad (\text{A11})$$

In a set-size 1 trial of the 2iAFC task in which an observer reports which interval includes a target among the two intervals (i.e., target present and target absent), the model computes distributions of “max difference” for both target-present (μ_{diff_int1} and σ_{diff_int1}) and target-absent intervals (μ_{diff_int2} and σ_{diff_int2}).

Sensitivity and performance

Signal detection theory models d' and probability correct as

$$d' = \frac{\mu_{diff_int1} - \mu_{diff_int2}}{\sqrt{\frac{\sigma_{diff_int1}^2 + \sigma_{diff_int2}^2}{2}}} \quad (\text{A12})$$

$$pc = \int g(x - d')G(x) \quad (\text{A13})$$

Information integration: Integration of multiple locations

The model is extended to multiple locations (or stimuli) using the max rule. Observers might make a correct answer when internal response for target exceeds the max of all distractors in the other interval. According to signal detection theory, this probability could be formulated by

$$pc = \int g(x - d')G(x)^{2U+1} \quad (\text{A14})$$

where $U = SS - 1$. Another possibility of correct response in the 2iAFC detection task is the chance that max response to distractors is greater in target interval than in the other interval:

$$pc = \int U g(x)G(x)^{2U} g(x - d') \quad (\text{A15})$$

In the latter case, response is correct even though the observer does not correctly detect the target. For this reason, the formulations for detection correct and both correct are

$$pc = \int g(x - d')G(x)^{2U+1} + U g(x)G(x)^{2U} g(x - d') \quad (\text{A16})$$

$$pc = \int g(x - d')G(x)^{2U+1} \quad (\text{A17})$$