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UNIVERSITY OF CALIFORNIA  
RIVERSIDE

Measuring Risk and Uncertainty in Financial Markets

A Dissertation submitted in partial satisfaction  
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Najrin Khanom

August 2016

Dissertation Committee:

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The Dissertation of Najrin Khanom is approved:

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## ABSTRACT OF THE DISSERTATION

Measuring Risk and Uncertainty in Financial Markets

by

Najrin Khanom

Doctor of Philosophy, Graduate Program in Economics

University of California, Riverside, August 2016

Dr. Marcelle Chauvet, Co-Chairperson

Dr. Aman Ullah, Co-Chairperson

The theme of this dissertation is the risk and return modeling of financial time series. The dissertation is broadly divided into three chapters; the first chapter focuses on measuring risks and uncertainty in the U.S. stock market; the second on measuring risks of individual financial assets; and the last chapter on predicting stock return. The first chapter studies the movement of the S&P 500 index driven by uncertainty and fear that cannot be explained by economic fundamentals. A new measure of uncertainty is introduced, using the tone of news media coverage on the equity market and the economy; aggregate holding of safe financial assets; and volatility in S&P 500 options trading. Major contributions of this chapter include uncovering a significant non-linear relationship between uncertainty and changes in the business cycle. An increase in uncertainty is found to be associated with drastic but short-lived falls in stock prices; while economic fundamentals have a small but prolonged effect on the stock market prices. The second chapter proposes a new Value at Risk (VaR) and Expected Shortfall (ES) estimation procedure that involves estimating the variance of return using conditional semiparametric approach introduced by Mishra, Su and Ullah (2010). Thus, estimation of variance is independent from the assumed distribution. Monte Carlo simulations are used to compare the performance of these new estimates using normal, Student-t, laplace, ARCH, GARCH, and GJR GARCH distributions. VaR and ES for Amazon, SP500, Microsoft, Nasdaq, USD/GBP and USD/Yen are estimated and

the performance of each estimation method is further tested using a battery of tests. The third chapter explores whether non-parametric and semi parametric methods can reduce the bias in predictive regressions in the presence of high persistence in the predictive variables and non-linear relationship with the dependent variable. The predictive performance of the independent variables suggested in the literature to predict stock returns are re-evaluated in sample and out of sample using two step non-parametric and semi parametric models. Empirical RMSE are used to compare the proposed models with the historical average, OLS and non-parametric regression models.

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## **Introduction**

The line between Wall St. and Main St. has been becoming murkier and murkier. Paired together with the heightened globalization, decisions made in small board rooms at Wall St. might affect those sitting at a remote corner of the world. As evidenced by multiple occasions where a financial crisis was followed by a recession, stakes in the financial markets are no longer limited to investors and 401k holders. In order to avoid such crisis government and international bodies have placed regulation on financial institutions. News regarding the financial market has also grown from a page in the newspaper and a segment in tv nightly news, to dedicated financial newspapers and news channels. With the ease to invest and disinvest in financial assets, close market watchers attempt to forecast the movement of asset prices to either make profits or to avoid a loss. Predicting stock return and risk has long been pursued by academics and financial practitioner. This dissertation looks into both these risk and return predicting models.

While an individual investor's decision is not likely to sway the market in one direction, the same cannot be said when a large number of investors act the same way. Thus, investor's sentiment can potentially cause market movements. The investment decisions are based on the information set available to the investor, which includes the information regarding the firm, the economy, international economies and several other political and non-political events. If a stock's price is a function of the firms future stream of cashflow, the price should vary with new information regarding the firm's performance and the economy's performance (if sales are sensitive to the business cycle). However, fluctuations in the stock market are often attributed to non-fundamental factors and uncertainties that are not directly tied to the performance of the firm or the economy, and investors' behavior are categorized as panic or euphoria. The first chapter of this dissertation attempts to understand how much of the U.S. stock market's movement is driven by these non-fundamental factors and uncertainties. A new measure of uncertainty in the stock market is introduced, which is based on the tone of news, holding of safe financial assets and volatility in the options market. The uncertainty

index introduced is a measure of overall risk and panic in the U.S. stock market.

In the following chapter risk is devoted to the risk assessment of individual assets due to price movements. Two new models to measure tail risk are introduced, and the performance of the new models are evaluated and compared against popular models using empirical and simulated data.

Finally, the last chapter looks into the stock return prediction. Several variables have been put forward to have predictive power over stock returns either in theoretical models or with some empirical evidence. However, empirically there is no consensus whether these variables have predictive power or not. Rather the results are often sensitive to the econometric model of choice. The econometric models can further produce biased results due to the high persistence in the predictive variables in question. Apart from the high persistence the relationship between stock return and the predictive variable can also be misspecified in the model. Therefore, chapter three of this dissertation revisits this topic with two new methodologies to test the relationship between the stock returns and the popular predictive variables. The new methodologies exploit nonparametric and semi-parametric methods to avoid misspecification, and a two step method is used to accommodate for the high autocorrelation in the predictive variables.

## **1 Chapter 1: Role of Uncertainty and Fear in Stock Market Movements**

### **1.1 Introduction**

*“Market swings may be rooted in concerns about economic and corporate conditions, but sometimes volatility itself can feed investors’ anxiety.”*

- The New York Times (June 4, 2006)

*“What does matter is not what investors know but what they cannot know yet...”*

- President of Yardeni Research (August 12, 2007)

*“The big thing right now is panic”*

- The Wall Street Journal (November 20, 2007)

The quotes above are from some of many newspaper articles that relate stock market fluctuations to uncertainty. Uncertainty can arise from a number of factors including but not limited to the future outlook of the economy, forthcoming economic policy announcements, geopolitical risks and, as highlighted from the first quote, it can be accentuated from high stock market volatility itself. Pastor and Veronesi (2012; 2013) use a theoretical general equilibrium framework to show that periods of high uncertainty in the stock market are often associated with lower stock prices and higher levels of volatility, particularly during economic downturns. Uncertainty and investor sentiment are closely related, as fear may arise from bad news or from uncertainty. Measuring investor sentiment is gaining popularity among market watchers (Barberis, et al., 1998) <sup>1</sup>. Much like the third quote, fear, euphoria, hysteria, panic, overreaction, etc, are often used to explain various peaks and troughs of the stock market cycle (De Long et al., 1990; Daniel & Subrahmanyam, 1998). This is in contrast to traditional asset pricing models which are based on economic and firm specific fundamentals. Chen et al. (1983) and Hamilton & Lin (1996) have shown that stock return depends on the stage of the business cycle.

Statistical releases of economic and financial variables tied to economic fundamentals are expected to have an effect on the stock market (e.g. Chen et al., 1986; Pearce & Roley, 1985; Hardouvelis, 1987; Cutler et al., 1989, etc.). However, when there is uncertainty about the future, there are instances in which the stock market performs poorly despite fundamental variables indicating a strong economy. This can be the case, for instance, when there is a war looming in the horizon (geopolitical instability) or if there is uncertainty about announcements of fiscal or monetary policies. In addition, perceived risk in itself can affect expectations about the stock market. This chapter measures the impact of uncertainty and fear on stock market fluctuations that cannot be explained by economic fundamentals. Existing studies on stock market and uncertainty limit to one form of uncertainty. It could

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<sup>1</sup>CNNMoney and Bloomberg publish their own indices of Fear & Greed for their subscribers

be policy specific, such as monetary (Errunza & Hogan, 1998), fiscal (Sialm, 2006, Croce et al., 2012), defense, regulatory or overall government policy (Pastor & Veronesi, 2013; Baker et al., 2015), or uncertainty related to economic variables (Bansal et al., 2005, Anderson et al., 2009; Drechsler, 2012). This chapter studies the effect of an *overall* level of uncertainty on stock market fluctuation, for which a new measure of uncertainty is introduced. In addition, the model is controlled for economic fundamentals to account for the stage of the business cycle.

Interest in measuring and tracking investor sentiment and uncertainty have increased in the recent years. Due to the elusiveness of these concepts, creative methodologies have been used to measure them. For example, Bloom (2009) uses the implied volatility in stock return options trade volatility, Baker & Wurgler (2006) use equity market related variables, Arnold & Vrugt (2008) use dispersion in economic forecasts from participants in the Survey of Professional Forecasters. This chapter proposes a dynamic factor model to extract a latent proxy of uncertainty from the co-movement in stock returns with three sources that are expected to be correlated with the level of uncertainty. The consideration of several variables reduces the possibility of incorrectly interpreting a single series' idiosyncratic movement as changes in level of fear or uncertainty. The three sources considered include the tone used in newspaper articles to report news on equity markets and the economy, changes in holdings of safe financial assets, and the options traded volatility index, often referred to as the "fear gauge". High periods of uncertainty are expected to be associated with negative media coverage, investors hoarding their money in safe assets away from the equity market, and increased volatility in traded options.

There is a large number of events that might plausibly rattle the stock market, such as political elections, weak economy in Europe, monetary policy announcements, crash of China's stock market, among several others. It would be dimensionally prohibitive to add variables for each of the events. In order to capture them all, this chapter uses economic and equity market related news published in the top 10 U.S. newspapers, and performs textual analysis to build a negativity index based on the tone used in the articles. Similar indices have been

created in earlier work by Tetlock (2007) who uses the column “Abreast of the market” from the Wall Street Journal; and Baker, Bloom and Davis (2015), who use the number of newspaper articles mentioning words equivalent to the economy, uncertainty and policy. The scope of the negativity index in this chapter is much larger than previous related literature. The index utilizes more newspaper articles that investors might be exposed than Tetlock’s (2007) index and, unlike Baker, Bloom and Davis (2015) the articles used to build the index in this chapter are not only policy related but also include any article related to the economy or the equity market. Additionally, textual analysis designed specifically for economic and financial news is performed to understand the tone of the articles instead of counting the number of articles.

The news negativity index serves as a proxy for economic uncertainty that investors are exposed through the media. However, stock market participants may have their own sources of news that are not printed in the newspapers or are printed with a lag. Therefore, to incorporate the behavioral aspect of investors in the analysis, the uncertainty index also considers investors’ asset allocation. Investors have a broad range of financial assets with different degree of risk, which allows them to customize their portfolio according to the desired level of exposure. Apprehension regarding the equity market may cause investors to reallocate their investment to other safer and more liquid financial assets, such as T-bills and money market instruments (Beber et. al, 2009). Investors tend to hold on to more liquid and safe forms of assets when their expectations about the economy are grim. This is illustrated in Figures A.1 - A.4, which show how the composition of financial assets holdings of households and financial businesses’ have changed over the years. Finally, to build the uncertainty index a measure of expected volatility in the options traded in the S&P 500 (VIX) is also used, which is expected to rise with fear and uncertainty.

Unlike previous uncertainty measures, the index introduced in this chapter is a comprehensive one that includes all possible events that might cause disruption in the stock market, and it is not limited to a single economic or political source. The use of a dynamic factor model, which extracts the common movement in tone of newspapers, holding of safe financial



assets and volatility in options market, reduces error of incorrectly interpreting idiosyncratic changes in one of the variables as changes in the level of uncertainty. For instance, demand for holding safe financial assets might go up due to a rise in short term interest rates with no changes in the level of uncertainty in the market, however if one were to only use the changes in holding of safe financial assets as a proxy for the level of uncertainty, she would incorrectly conclude a rise in uncertainty.

The main goal of this chapter is to study fluctuations in the stock market due to uncertainty and fear that cannot be explained by economic fundamentals. However, the variables chosen to measure uncertainty may give rise to possible endogeneity. The release of weak fundamental variables may lead to news reporters writing grim articles, and investors holding more safer assets; causing the uncertainty index to rise. In this case, movements in the stock market is not only due to a rise in uncertainty but it can be a reaction to weak fundamentals. Therefore, the uncertainty factor is controlled for changes in short run economic fundamentals (business cycle). Characterizing the business cycle involves several challenges. First, most data related to economic performance are released at a low frequency and with lags. Second, data on leading or coincident series used to nowcast or forecast business cycles are released asynchronously and with different frequencies. This gives rise to issues of mixed frequency, missing data, and ragged edges. Finally, data available at the time of the study might not be the same received by investors, news reporters, or other stakeholders in real time. For instance, quarterly GDP growth rate is often revised as more information becomes available, thus results are sensitive to the time of the study. Extensive research has been pursued to nowcast the business cycle and GDP growth rate using real time data (Giannone et. al, 2008; Aruoba et. al., 2012; Barnett et. al., 2014). In order to accurately recreate the environment at which stock market participants found themselves in each point of time it is important to use real-time data vintages. This chapter uses Aruoba, Diebold and Scotti's (2009) dynamic factor model to capture the business cycle as it takes into account real time data, mixed frequency and lack of synchronicity with which economic data are released. Unlike Mariano & Murasawa (2003), Aruoba, Diebold and Scotti's (2009) model produces

nowcasts and forecasts of the business cycle at a higher frequency of weekly and daily data which is important to any analysis pertaining to the stock market.

chapter measuring uncertainty. Parametric and non-parametric regressions are then used to remove the effect of the business cycle on the uncertainty factor. The second specification involves extracting the business cycle factor first. Then the business cycle factor is introduced as an exogenous variable in a dynamic factor model used to extract the uncertainty factor. Both models indicate a cyclical component in the tone of newspapers and in the stock market return. While the linear, parametric model finds a negative insignificant relationship between uncertainty and the business cycle, the non-parametric model finds a non-linear statistically significant relationship between the two. High periods of uncertainty are associated with sharp jumps and falls in the business cycle. The chapter also finds that after adjusting for economic fundamentals, uncertainty in the stock market spikes before crucial policy announcements, during turmoil in influential foreign countries, wars, political elections, and when there is little consensus over key economic variables. An increase in uncertainty is found to be related with sharp falls in stock market prices and returns, although these effects are short-lived. On the other hand, economic fundamentals have a small but prolonged effect on stock market prices. The effect of economic fundamentals may be under-reported due to the long intervals with which economic data are released, as stock market participants may have already updated their expectations.

The chapter is structured as follows. Section 1.2 discusses the two proposed models and the state-space framework. Section 1.3 describes the data and the negativity index. Section 1.4 presents the empirical results, and section 1.5 concludes.

## **1.2 Methodology**

The objective is to isolate the movement in the stock market that is driven by uncertainty and not by the actual performance of the economy. Two set of observed variables are considered. The first group are variables that are susceptible to the level of uncertainty in the stock market and overall economy. The uncertainty variables under consideration

are also likely to be influenced by the business cycle. Therefore, to eliminate or control for the cyclical component a second group of variables that are fundamentally tied to the actual performance of the economy are also utilized. Two dynamic factor models cast in the state-space form are explored to estimate the uncertainty factor adjusted for business cycle. Alternative model specifications are used to verify the robustness of results.

### 1.2.1 Model 1

This model is estimated in three steps, the first step involves creating a dynamic factor that captures the comovement in variables that are susceptible to uncertainty, using the Kalman filter. The second step is nowcasting business cycle using only variables that are tied to the fundamentals of economic performance. To estimate the business cycle, Aruoba, Deibold and Scotti's (2009) mixed frequency dynamic factor model is applied. And the final step is to remove the fluctuations in the uncertainty factor that can be explained by the fundamentals. Parametric and non-parametric variations of final step are looked into to allow for both linear and non-linear relationship between the two factors.

#### Step 1: Estimating the uncertainty factor

The comovement in the uncertainty variables are extracted using the Kalman filter.  $y_{j,t}^u$  is a weekly uncertainty variable  $j$  at time  $t$ , where  $j = 1, 2 \dots r$ , and  $t = 1, 2 \dots \tau$ . The uncertainty variables are explained by both movements in the performance of the economy, and the uncertainty surrounding it.  $f_t^u$ , is the extracted factor;  $z_j^{uu}$  measures the responsiveness of  $y_{j,t}^u$  to the latent uncertainty factors, and  $\omega_{j,t}^u$  is the measurement shock.

$$y_{j,t}^u = z_j^{uu} f_t^u + \omega_{j,t}^u \quad (1.1)$$

Since, the uncertainty variables under consideration are available at a higher frequency, there is no issue of mixed frequency.

*Observation Equation*

$$\begin{aligned} \mathbf{y}_t^u &= \mathbf{H}^u \xi_t^u + \omega_t^u \\ \omega_t^u &\sim (0, R^u) \end{aligned} \quad (1.2)$$

$\mathbf{y}_t^u$  is a  $(r \times 1)$  vector of observed variables at time  $t$ , these economic and financial variables contain information about the performance of the economy. Since, it contains only observed values it is inundated with missing values.  $\mathbf{H}^u$  is a matrix of factor loadings and  $\xi_t^u$  is a vector containing  $f_t^u$  that captures the actual movements in the performance of the economy.  $f_t^u$  is assumed to evolve daily.  $\omega_t^u$  is vector of measurement shock.

$$\begin{aligned} \begin{bmatrix} \tilde{y}_{1,t}^u \\ \tilde{y}_{2,t}^u \\ \vdots \\ \tilde{y}_{r,t}^u \end{bmatrix}_{(r \times 1)} &= \begin{bmatrix} z_1^{uu} \\ z_2^{uu} \\ \vdots \\ z_k^{uu} \end{bmatrix}_{(r \times 1)} \begin{bmatrix} f_t^u \end{bmatrix} + \begin{bmatrix} \omega_{1,t}^u \\ \omega_{2,t}^u \\ \vdots \\ \omega_{k,t}^u \end{bmatrix}_{(r \times 1)} \\ \mathbf{R}^u &= \begin{bmatrix} \sigma_{\omega_1^u}^2 & 0 & \cdots \\ 0 & \sigma_{\omega_2^u}^2 & \\ \vdots & & \ddots \\ & & & \sigma_{\omega_r^u}^2 \end{bmatrix}_{(r \times r)} \end{aligned}$$

*Transition Equation*

$$\begin{aligned} \xi_{t+1}^u &= \mathbf{F}^u \xi_t^u + \nu_{t+1}^u \\ \mathbf{Q}^u &= E(\nu_t^u \nu_t^{u'}) \end{aligned} \quad (1.3)$$

The factors follow an AR(1) process, where future values of the factors at time  $t+1$ ,  $\xi_{t+1}^u$ , depend on the past through  $\xi_t^u$ .  $F^u$  is a  $(1 \times 1)$  scalar containing the autoregressive coefficients. And  $\nu_{t+1}^u$  is the transition shock.

While, the factors depend on their individual past values, the fundamental factor, also depends on past values of the uncertainty variable.

$$f_{t+1}^u = \phi^{uu} f_t^u + \nu_{t+1}^u \quad (1.4)$$

$$\begin{aligned} \begin{bmatrix} f_t^u \end{bmatrix}_{(1 \times 1)} &= \begin{bmatrix} \phi^{uu} \end{bmatrix}_{(1 \times 1)} + \begin{bmatrix} f_t^u \end{bmatrix}_{(1 \times 1)} + \begin{bmatrix} \nu_{t+1}^u \end{bmatrix}_{(1 \times 1)} \\ \mathbf{Q}^u &= \begin{bmatrix} \sigma_{\nu^u}^2 \end{bmatrix} \end{aligned}$$

## Step 2: Estimating the business cycle

$y_{i,t}^f$  is a weekly fundamental variable  $i$  at time  $t$  where,  $i = 1, 2 \dots k$  and  $t = 1, 2 \dots \tau$ . The fundamental variables are only explained by movement in the latent variable capturing actual state of the economy,  $f_t^f$ , and  $z_i^{ff}$  is the sensitivity of  $y_{i,t}^f$  to the business cycle. And  $\omega_{i,t}^f$  captures the idiosyncratic movement of  $y_{i,t}^f$  not explained by the business cycle.

$$y_{i,t}^f = z_i^{ff} f_t^f + \omega_{i,t}^f \quad (1.5)$$

Information about all variables are not always available daily, although they are evolving daily or continuously. Moreover, variables of interest often vary in the frequency with which they are released, posing a challenge to deal with mixed frequency.  $\tilde{y}_{i,t}^f$  is  $y_{i,t}^f$  observed in a daily or lower frequency. If analysis are to be carried on a daily basis it gives rise to a large number of missing values. Care has to be taken to deal with both the missing values and differences in stock and flow variables. If  $y_{i,t}^f$  is a stock variable then when it is observed it is a snapshot of the level at that day independent of the frequency with which it is observed.

$$\tilde{y}_{i,t}^f = \begin{cases} y_{i,t}^f = z_i^{ff} f_t^f + \omega_{i,t}^f & \text{if } y_{i,t}^f \text{ is observed} \\ NA & \text{if } y_{i,t}^f \text{ is not observed} \end{cases} \quad (1.6)$$

However, if  $y_{i,t}^f$  is a flow variable released with a lower frequency than daily, then the  $\tilde{y}_{i,t}^f$  is the sum of the all the last  $D_i$   $y_{i,t}^f$  till the last observed one.  $D_i$  is the number of days in the observation period.

$$\tilde{y}_{i,t}^f = \begin{cases} \sum_{p=0}^{D_i-1} y_{i,t-p}^f & \text{if } y_{i,t}^f \text{ is observed} \\ NA & \text{if } y_{i,t}^f \text{ is not observed} \end{cases} \quad (1.7)$$

## State-Space representation

*Observation Equation*

$$\begin{aligned} \mathbf{y}_t^f &= \mathbf{H}^f \xi_t^f + \omega_t^f \\ \omega_t^f &\sim (\mathbf{0}, \mathbf{R}^f) \end{aligned} \quad (1.8)$$

$\mathbf{y}_t^f$  is a  $(k \times 1)$  vector of observed variables at time  $t$ , these economic and financial variables contain information about the performance of the economy. Since, it contains only observed values it is inundated with missing values.  $\mathbf{H}^f$  is a matrix of factor loadings and  $\xi_t^f$  is a vector containing  $f_t^f$  that captures the actual movements in the performance of the economy.  $f_t^f$  is assumed to evolve daily.  $\omega_t^f$  is vector of measurement shock.

$$\begin{aligned} \begin{bmatrix} \tilde{y}_{1,t}^f \\ \tilde{y}_{2,t}^f \\ \vdots \\ \tilde{y}_{k,t}^f \end{bmatrix}_{(k \times 1)} &= \begin{bmatrix} z_1^{ff} \\ z_2^{ff} \\ \vdots \\ z_k^{ff} \end{bmatrix}_{(k \times 2)} \begin{bmatrix} f_t^f \end{bmatrix} + \begin{bmatrix} \omega_{1,t}^f \\ \omega_{2,t}^f \\ \vdots \\ \omega_{k,t}^f \end{bmatrix}_{(k \times 1)} \\ \mathbf{R}^f &= \begin{bmatrix} \sigma_{\omega_1^f}^2 & 0 & \cdots \\ 0 & \sigma_{\omega_2^f}^2 & \\ \vdots & & \ddots \\ & & & \sigma_{\omega_k^f}^2 \end{bmatrix}_{(k \times k)} \end{aligned}$$

*Transition Equation*

$$\begin{aligned} \xi_{t+1}^f &= \mathbf{F}^f \xi_t^f + \nu_{t+1}^f \\ \mathbf{Q}^f &= E(\nu_t^f \nu_t^{f'}) \end{aligned} \quad (1.9)$$

The factors follow an AR(1) process, where future values of the factors at time  $t+1$ ,  $\xi_{t+1}^f$ , depend on the past through  $\xi_t^f$ .  $F^f$  is a  $(1 \times 1)$  scalar containing the autoregressive coefficients. And  $\nu_{t+1}^f$  is the transition shock.

While, the factors depend on their individual past values, the fundamental factor, also depends on past values of the uncertainty variable.

$$f_{t+1}^f = \phi^{ff} f_t^f + \nu_{t+1}^f \quad (1.10)$$

$$\begin{aligned} \begin{bmatrix} f_t^f \end{bmatrix}_{(1 \times 1)} &= \begin{bmatrix} \phi^{ff} \end{bmatrix}_{(1 \times 1)} + \begin{bmatrix} f_t^f \end{bmatrix}_{(1 \times 1)} + \begin{bmatrix} \nu_{t+1}^f \end{bmatrix}_{(1 \times 1)} \\ \mathbf{Q}^f &= \begin{bmatrix} \sigma_{\nu^f}^2 \end{bmatrix} \end{aligned}$$

**Dealing with missing values** The latent state variables are extracted using the Kalman filter and smoother. If some elements of  $\mathbf{y}_t^f$  are missing and only  $N^* < k$  are observed then a weighted vector  $\mathbf{y}_t^*$  is used instead.  $\mathbf{W}_t$ , is the  $(k \times k)$  weight matrix, with rows identical to those of an identity matrix,  $I_k$ , for corresponding observed elements of  $\mathbf{y}_t^f$ , and zero otherwise. Similarly, the vector for measurement shocks and factor loading matrix are also transformed using the weight matrix<sup>2</sup>. The parameters are optimized by maximizing the log likelihood.

$$\begin{aligned} \mathbf{y}_t^* &= \mathbf{H}^* \zeta_t^f + \omega_t^* \\ \mathbf{y}_t^* &= \mathbf{W}_t \mathbf{y}_t^f, \quad \omega_t^* = \mathbf{W}_t \omega_t^f, \quad \mathbf{H}^* = \mathbf{W}_t \mathbf{H}^f \end{aligned}$$

**Step 3: Removing the movement in uncertainty factor explained by the business cycle**

**Linear regression** The uncertainty factor is regressed on the business cycle factor, and the residuals of the regression is the fluctuation in the stock market that cannot be explained by economic factor.  $\hat{\alpha}$  and  $\hat{\beta}$  are OLS estimators and  $f_{t,OLS}^{u*}$  is the uncertainty factor adjusted for the cyclical component.

$$\begin{aligned} f_t^u &= \alpha + \beta_1 f_t^f + u_t \\ f_t^{u*} &= f_t^u - \hat{\alpha} - \hat{\beta}_1 f_t^f \end{aligned}$$

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<sup>2</sup>See Aruoba, Diebold and Scotti (2009) for more details on the estimation

**Non-parametric regression** In case the relationship between the two factors are non-linear, a non-parametric local linear regression is performed to control for the business cycle.  $\hat{m}(f_t^f)$  is the non-parametric estimator,  $h$  is the bandwidth, and  $K$  is a smoothing kernel.

$$\begin{aligned}
 f_t^u &= m(f_t^f) + u_t \\
 \hat{m}(f_t^f) &= \frac{\sum_{t'=1}^T f_{t'}^u K \left\{ (f_{t'}^f - f_t^f)/h \right\}}{\sum_{t'=1}^T K \left\{ (f_{t'}^f - f_t^f)/h \right\}} \\
 f_{t,NP}^{u*} &= f_t^u - \hat{m}(f_t^f)
 \end{aligned} \tag{1.11}$$

### 1.2.2 Model 2

This model is estimated in two steps, the first step involves estimating the business cycle, the same method applied in step 2 of Model 1. The second step is to captures the comovement in uncertainty variables, using a dynamic factor, and unlike Model 1 the business cycle is controlled for by introducing it as an exogenous variable in the dynamic factor filtration.

#### Step 1: Estimating the business cycle

Same as Step 2 in Model 1.

#### Step 2: Estimating the uncertainty factor

The comovement in the uncertainty variables are extracted using the Kalman filter.  $y_{j,t}^u$  is a weekly uncertainty variable  $j$  at time  $t$ , where  $j = 1, 2 \dots r$ , and  $t = 1, 2 \dots \tau$ . The uncertainty variables are explained by both movements in the performance of the economy, and the uncertainty surrounding it.

$f_t^u$ , is the extracted factor;  $z_j^{uu}$  measures the responsiveness of  $y_{j,t}^u$  to the latent uncertainty factors.  $f_t^f$  is the business cycle factor estimated in the previous step, it is added here as an exogenous variable to control for expectations explained by the fundamentals.

$$y_{j,t}^u = +a_{j,t}^{uf} f_t^f + z_j^{uu} f_t^u + \omega_{j,t}^u \tag{1.12}$$

Since, the uncertainty variables under consideration are available at a higher frequency, there is no issue of mixed frequency.



*Observation Equation*

$$\mathbf{y}_t^u = \mathbf{A}f_t^f + \mathbf{H}^u\xi_t^u + \omega_t^u \quad (1.13)$$

$$\omega_t^u \sim (0, R^u)$$

$\mathbf{y}_t^u$  is a  $(r \times 1)$  vector of observed variables at time  $t$ , these economic and financial variables contain information about the performance of the economy.  $\mathbf{H}^u$  is a matrix of factor loadings and  $\xi_t^u$  is a vector containing  $f_t^u$  that captures the actual movements in the performance of the economy.  $f_t^u$  is assumed to evolve daily.  $\omega_t^u$  is vector of measurement shock.

$$\begin{bmatrix} \tilde{y}_{1,t}^u \\ \tilde{y}_{2,t}^u \\ \vdots \\ \tilde{y}_{r,t}^u \end{bmatrix}_{(r \times 1)} = \begin{bmatrix} a_1^{uf} \\ a_2^{uf} \\ \vdots \\ a_k^{uf} \end{bmatrix}_{(r \times 1)} \begin{bmatrix} f_t^f \end{bmatrix} + \begin{bmatrix} z_1^{uu} \\ z_2^{uu} \\ \vdots \\ z_k^{uu} \end{bmatrix}_{(r \times 1)} \begin{bmatrix} f_t^u \end{bmatrix} + \begin{bmatrix} \omega_{1,t}^u \\ \omega_{2,t}^u \\ \vdots \\ \omega_{k,t}^u \end{bmatrix}_{(r \times 1)}$$

$$\mathbf{R}^u = \begin{bmatrix} \sigma_{\omega_1^u}^2 & 0 & \cdots \\ 0 & \sigma_{\omega_2^u}^2 & \\ \vdots & & \ddots \\ & & & \sigma_{\omega_r^u}^2 \end{bmatrix}_{(r \times r)}$$

The transition equation is the same as model 1.

### 1.3 Data, Selection of Variables, and Negativity Index

#### 1.3.1 Uncertainty Variables

To create a factor that captures uncertainty about the future, two channels are used, the first is how media portrays the state of the economy to be, and the second is the investors asset allocation decisions. This period covered is 26<sup>th</sup> January 1998 to 26<sup>th</sup> January 2015, primarily due to the availability of data of some variables.

### *Financial Assets*

For asset allocation decisions changes in aggregate holdings of financial assets are used to capture changes in asset allocation of investors. When investor sentiments are bearish about the stock market, they would reduce their exposure to the stock market and invest in safer assets. Figure A.5 illustrate how investment in various assets changed during the dot.com bubble and the Great recession, The shaded regions mark the NBER recession dates and the lines mark the beginning of a bear market<sup>3</sup>. Often times changes in asset allocation are due to expectations of the economy formed from news, announcements or data released by the government, or/and forecasts from professional forecasters, available to all. However, institutional investors or savvy individual investors could have their own forecasting models or source of news inaccessible to the mass, that they use to make their own asset allocation decision. Therefore, changes in holding of financial assets will include changes due to information available to all and information available to a few investors.

Investors tend to hold on to more liquid and safe forms of assets when their expectations about the economy are grim. Figure A.1 and A.3 show the how the composition of household's and financial businesses' financial assets have change over the years. Changes in the major assets, such as corporate equity and time and savings deposits are visible, the changes in assets with smaller shares are difficult to read, despite that changes can be seen in the holding of money market mutual funds (MMMFs) during both the crisis. Movement in assets are more evident in Figure A.2 and A.4<sup>4</sup> which shows the the changes in aggregate holdings of each financial assets by households and financial businesses, respectively. It can be seen what assets investors opt for when they are faced with a crisis. For financial businesses MMMFs, agency and GSE backed securities, Treasury securities, checkable deposits and currency, and time and saving deposits have gone up during both the recessions. Choice of assets to include in non-fundamental factor is motivated by the movement in these assets.

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<sup>3</sup>Dated following Chauvet and Potter(2001)

<sup>4</sup>Data for these figures are from the Federal Reserve Board's Statistical release, Z1: Financial Accounts of the United States. The release is issued quarterly since 2009, prior to that it was issued annually. It contains detailed accounts of flow of funds, levels of holdings and balance sheet of households and different types of businesses.

The Federal Reserve Bank of New York publishes weekly data on Primary Dealer Statistics every Thursday, which includes the net positions (long positions-short positions) and dollar amount of total transactions of in several government securities, such as T-bills and other agency backed securities, conducted by primary dealers starting from January 28th 1998. Weekly data is collected by the NY Fed for the week ending every Monday. Data on money market mutual fund data, demand deposit and other is collected from the Federal Reserve Board that issues its Statistical Release H.6, "Money Stock Measures" every Thursday issues. The data is on the two monetary aggregates M1 and M2. Table 6 provides the retail and institutionally money market holdings, not seasonally adjusted, going back to 4th February 1980.

### *News*

Multiple studies (Tetlock 2007, Tetlock 2011) have shown that tone of news can influence the investors expectations beyond what the fundamental economic variables or forecasts say. An indicator for the tone of newspapers articles about the economy and the equity market is created as a proxy for state of affairs in each point in time.

Only newspaper articles related to economy or equity market are considered. It is beyond the scope of this chapter to analyze all the US newspapers that are in print, also it is assumed that most local newspapers have limited readership to influence enough investors. Therefore, only the top 10 newspapers ranked by their circulation are considered, Table A.1 presents the newspapers titles with their number of subscribers and online presence<sup>5</sup>. Over 110,000 articles collected from Factiva are analyzed<sup>6</sup>.

Keeping the subscribers interest in mind, publisher's decide the location of an article within a newspaper. Articles in different pages and sections of the newspapers are likely to vary in their perceived importance, and in the frequency with which they are read. Articles about the economy published on the front page are likely to have a greater impact than those

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<sup>5</sup>Source: Alliance for Audited Media, a private company providing its members information about readership, circulation, subscriber demographics, and digital activity metrics for more than 2,800 of North America's leading publishers via the Media Intelligence Center's deep database

<sup>6</sup>The number is restricted as only 100 articles can be downloaded at a time from Factiva, to retrieve every additional 100 articles the users has to input a captcha. Full articles with lead paragraphs and indexing are downloaded and appended in one text file to begin performing textual analysis.

buried in the middle of the newspaper. Therefore, only articles published on the first page, the business section or specific economy/stock market related columns in the newspaper are considered to narrow down the articles with the highest impact. The articles are filtered further to remove those that are irrelevant such as advertorials, company profile, etc. A detailed download criteria along with justification is given in Table A.2 for USA Today as an example.

The articles are extracted from Factiva, where each article is indexed with a number of categories, such as the source, publishing date, author’s name, page number, section, subject, headline, lead paragraph, main text, column name among many other. Only articles under the predefined subject<sup>7</sup> named “Equity Market” and “Economy”, that appeared on the front page or in the business section are retrieved, after excluding articles from all regions besides U.S.

### ***Textual Analysis***

A negativity index is build to mimic the overall tone used to report news about the economy and stock market. The negativity index measures the net proportion of negative words used after adjusting for the proportion of positive words, in all the articles published at day  $t$  reporting about performance of the economy and key economic variables. Frequently used stopwords, such as prepositions, conjunctions and pronouns that rarely add to the semantics are removed from the total number of words, to get more effective measures of the index and to uninundate the articles with unnecessary words. The list of stopwords is primarily that of MYSQL with minor addtitions and modifications, to accommodate for different ways of writing the same word. For the list of positive and negative words, Harvard’s Psychology Dictionary IV’s “Positiv” and “Negativ” lists are used, respectively. The lists are adjusted for economy and financial market specific words, that might have an opposite or ambiguous connotation than the category they are specified in. Words that have multiple appearances are also removed from the lists to avoid double counting.

$$\text{Negativity Index}_t = \text{Proportion of Negative Words}_t - \text{Proportion of Positive Words}_t$$

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<sup>7</sup>The subjects of the articles are categorized by Dow Jones Intelligent Indexing<sup>TM</sup> which follows the standard indexing of IPTC.

$$\text{Proportion of Negative Words}_t = \frac{\text{Number of negative words used at day } t}{\text{Number of words used at day } t - \text{Number of Stopwords}}$$

$$\text{Proportion of Positive Words}_t = \frac{\text{Number of positive words used at day } t}{\text{Number of words used at day } t - \text{Number of Stopwords}}$$

Since the objective of the negativity index is to measure the newspapers outlook for the economy along with how the sentences are being framed, it is imperative that any such index can reflect whether the articles are reporting good or bad news about the markets. To achieve this two lists of key economic and financial variables and terms are created. One list includes positive economic variables such as GDP growth and investment, an increase in these variables are considered good news; while increase in negative economic variables such as unemployment which are included in the other list, are considered bad news. If a positive economic keyword is preceded or followed by any word synonymous to increase, it is counted as positive word(s), similarly if it is synonymous to decrease, it is counted as negative word(s). The negative economic keywords are counted analogously. The list of words synonymous increase and decrease, are primarily from Harvard's Psychology Dictionary IV's "Inceas" and "Decreas" lists, with some additions of popular choice of words used in relation with economic and financial variables.

The counts of positive and negative words are also corrected for negation. For instance, if a sentence reads "GDP is not growing" will be considered as bad news<sup>8</sup>. List of words expressing negation is from Harvard's Psychology Dictionary IV's "Negate" with some additions. A Python script is written to perform textual analysis. For each day 4 negativity indices are created, one for the headlines, one for the lead paragraphs, one for the text and the one for all combined. The program creates an excel file with the count for positive words, negative words, positive economic keywords, negative economic keywords, stopwords and total words in the concatenated article for each category. The daily counts are summed to covert the data to weekly. Graph for the four positivity indices of economy related articles in WSJ are given below, the shaded region highlights the NBER Recession dates. The rise in negativity in articles are most pronounced before the great recession as shown in Figure A.6.

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<sup>8</sup>Double negative and sarcasm is not detected

### 1.3.2 Fundamental Variables

To accurately capture the latent business cycle co-movement in variables that are theoretically justified and empirically proven to be indicators of economic performance have to be used. Following the ADS index (Aruoba *et. al*, 2009) that has shown great success in estimating the business cycle movement, this chapter uses the daily yield curve (difference in yield between the 10 year and 3-month Treasury security), weekly initial jobless claims for unemployment, monthly manufacturing order, monthly non-farm employment payroll, monthly industrial production, monthly real personal income less transfers, and monthly trade sales. Data on Treasury securities is from Board of Governors of the Federal Reserve System (US), Initial Claims [ICSA] from US. Employment and Training Administration, ISM Manufacturing: PMI Composite Index© [NAPM] from Institute for Supply Management, and real time data for industrial production, non-farm employment payroll, real personal income less transfers and real GDP are available from the Federal Reserve bank of Philadelphia. Figure A.7 presents the weekly business cycle factor.

## 1.4 Results

The uncertainty factor is estimated under the specifications, of Model 1 and 2, the results are presented in Table 1.1 and 1.2, respectively. For model 1 the uncertainty factor is first estimated without making any adjustments for the business cycle<sup>9</sup>. From Table 1.1 it can be seen that the uncertainty factor created moves closely with the negativity index of news media coverage, retail money market holding, VIX and stock market return, whereas net position in T-bills of dealers and S&P 500 volume rarely move with the uncertainty factor. Subsequently, S&P 500 volume and net T-bills position of dealers are dropped from the estimation of the uncertainty factor, which barely changes the factors, but lead to a more parsimonious model. Adding too many variables for the estimation might result in capturing the noise specific to the current data that might not be there in some other time frame,

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<sup>9</sup>Similar analysis have been performed that are not reported in this chapter, using institutional and total money market fund, changes in demand deposit, holdings of agency backed securities, T-bill transactions,

moreover they involve estimating more unknown parameters. One plausible explanation for lower correlation with T-bills but not with money market funds is that money market instruments are more accessible to investors and are often used as a placeholder for money during portfolio restructuring, whereas T-bills serve a number of purposes, investors may take a long position to diversify, hedge, or take a short position to finance investment in riskier assets. Casually observing the net position of primary dealers in Figure A.6, fall in the dealer's net long position are mostly after or during the bear market, that is investors are holding T-bills after the market has started collapsing.

The uncertainty factors are then adjusted for changes in the business cycle index using OLS and non-parametric local linear regressions. The parametric model finds a negative but statistically insignificant linear effect of the business cycle on uncertainty, and the factors before and after the adjustment remain almost identical. The non-parametric model on the other hand, finds a statistically significant non-linear relationship between the business cycle and the uncertainty factor. The upper left of Figure 1.1 presents the parametric (red) and the nonparametric estimates (blue) of the corresponding regression functions. According to the non-parametric model uncertainty rises with sharp jumps and falls in the business cycle. There are few blue and red dots at the edges of the graph representing the handful of observations in the sample where there is an extreme changes in the business cycle over a week. Therefore, the errors are larger in two extremes of changes in business cycle, this is illustrated in the top right graph in Figure 1.1, which presents the fitted values of nonparametric estimation with their error bands in vertical dotted lines. The bottom two graphs present the gradients of the non-parametric estimation and the associated variability bounds. The slope is sensitive to size of expansion and contraction in the business cycle. Sharp economic contractions are met with more increase in uncertainty than subtle contractions. The responsiveness of uncertainty also increases with the magnitude of positive changes in the business cycle.

After non-parametric adjustments are made to remove the business cycle element in the uncertainty factor, the correlation between news negativity index falls. There is a cyclical

Table 1.1: Model 1 (Correlation with Uncertainty Factor)

	W/o Adjusting for B.C.			Linear Regression			Non-Parametric		
Negativity index	0.37	0.37	0.37	0.37	0.37	0.37	0.26	0.26	0.26
Retail Money Market	0.44	0.45	0.45	0.44	0.44	0.44	0.44	0.44	0.44
T-Bill Net Positions	0.06	0.06	-	0.06	0.06	-	0.05	0.05	-
VIX	0.82	0.82	0.82	0.82	0.82	0.83	0.81	0.81	0.82
S&P 500 Volume	-0.04	-	-	-0.03	-	-	-0.04	-	-
S&P 500 Return	-0.83	-0.82	-0.82	-0.83	-0.82	-0.82	-0.81	-0.81	-0.80
$\beta_{OLS}$				-0.48	-0.48	-0.48			
p-values				0.16	0.12	0.11			
Median Gradient							-1.60	-1.60	-1.60
p-values							0.00	0.00	0.00

Table 1.2: Model 2 (Correlation with Uncertainty Factor)

	W/o Adjusting for B.C.			Model 2		
Negativity index	0.366	0.367	0.368	0.236	0.237	0.238
Retail Money Fund Holdings	0.443	0.445	0.445	0.461	0.462	0.462
T-Bill Net Positions	0.059	0.059	-	0.058	0.058	-
VIX	0.821	0.823	0.826	0.839	0.840	0.843
Stock Market Volume	-0.035	-	-	-0.027	-	-
S&P 500 Return	-0.827	-0.824	0.822	-0.804	-0.802	-0.800
Log Likelihood	-5749	-5306	-4864	-5708	-5266	-4824

component in the tone used by the media, hard economic times are followed with harsh headlines and articles. The correlation with stock returns also fall slightly, however, correlation with retail money fund and VIX are hardly altered. Although VIX and retail money fund can be cyclical it is plausible that the log first difference of these variables are not. Similar results are also found in Model 2, that controls for the economic fundamentals during the estimation of the uncertainty factor, as shown in Table 1.2 that compares the uncertainty factor before and after adjustments. Table 1.3 presents the corresponding factor loadings with the uncertainty factor (and the coefficients of the business cycle index). News and



Figure 1.1: Fitted values and gradients of nonparametric regression, with variability bounds

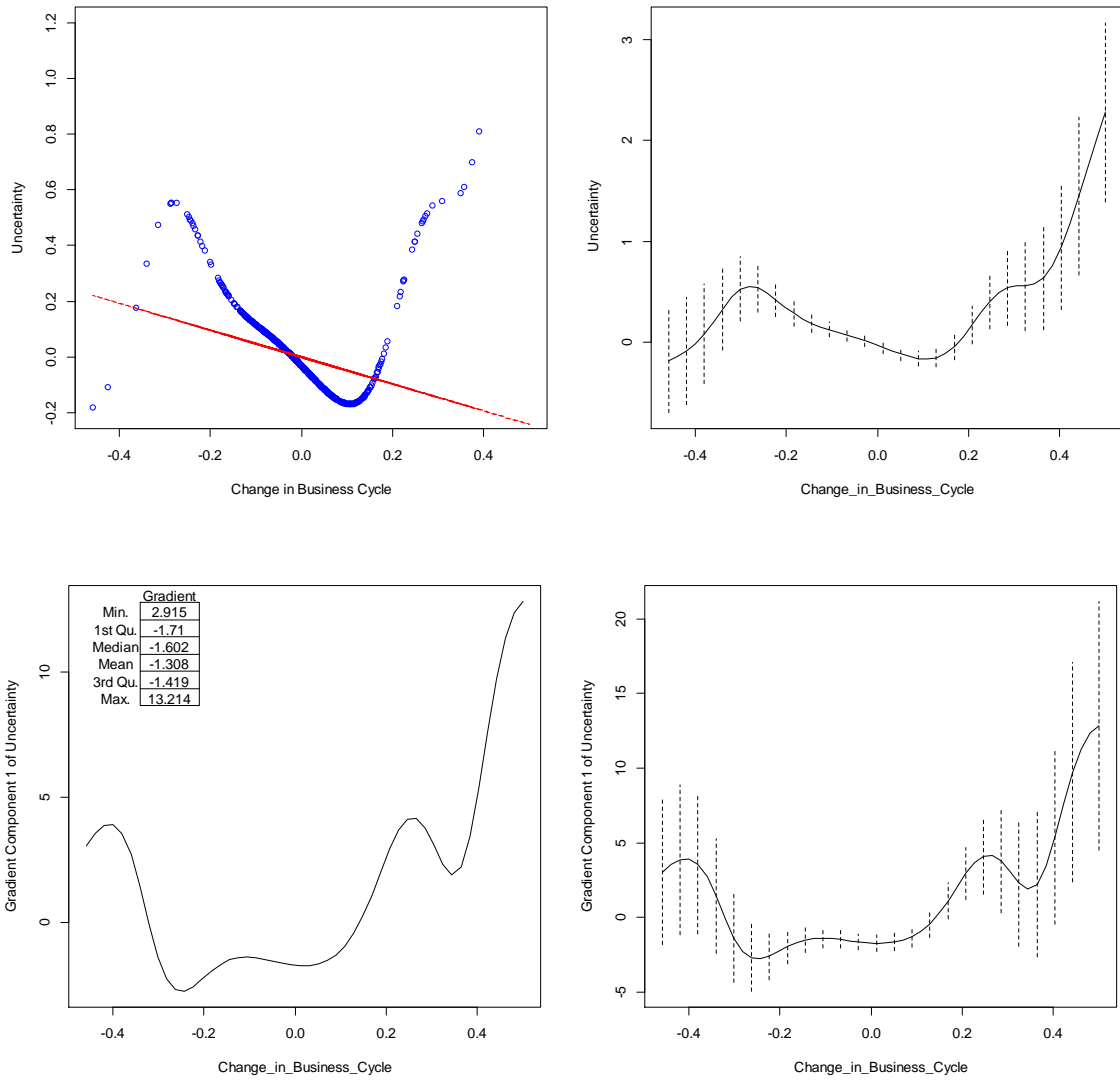


Table 1.3: Uncertainty Factor Loadings (Business Cycle Index Coefficient)

	W/o Adjusting for B.C.			Model 2		
Negativity index	0.225	0.226	0.227	0.102 (0.263)	0.102 (0.263)	0.104 (0.263)
Retail Money Market	0.331	0.333	0.333	0.348 (0.067)	0.349 (0.067)	0.349 (0.067)
T-Bill Net Positions	0.042	0.042		0.042 (-0.016)	0.042 (-0.016)	
VIX	0.593	0.595	0.597	0.612 (0.007)	0.614 (0.007)	0.616 (0.007)
Stock Market Volume	-0.024			-0.018 (-0.016)	-	-
S&P 500 Return	-0.634	-0.632	-0.631	-0.622 (0.184)	-0.620 (0.184)	-0.619 (0.184)

stock returns are the only variables with non-negligible coefficients for the business cycle index, indicating the cyclicity in the two variables. The uncertainty index moves closely with the news negativity index, retail money market holding, VIX and stock return.

Uncertainty in the stock market rises with the threat of war and public security, presidential elections, fiscal budgetary policies, anticipation of federal interest rate hikes, poor economic performance in influential foreign countries, lack of consensus about the direction in which key economic variables will move. There is also heightened fear before and during recessions and government failure. Uncertainty rises before close elections, Li & Born (2006) also find a rise in stock market volatility during tight major elections.

The newspaper negativity index used thus far takes the entire newspapers into consideration. In case, headlines or lead paragraphs have a stronger impact on uncertainty. Three additional negativity indices are created by performing textual analysis the headline, the body of text and the lead paragraph. All produce similar results however, the news negativity index for the entire article has the strongest correlation with the factor.

Figure 1.2: Uncertainty factor for Model 2 using 4 observed variables

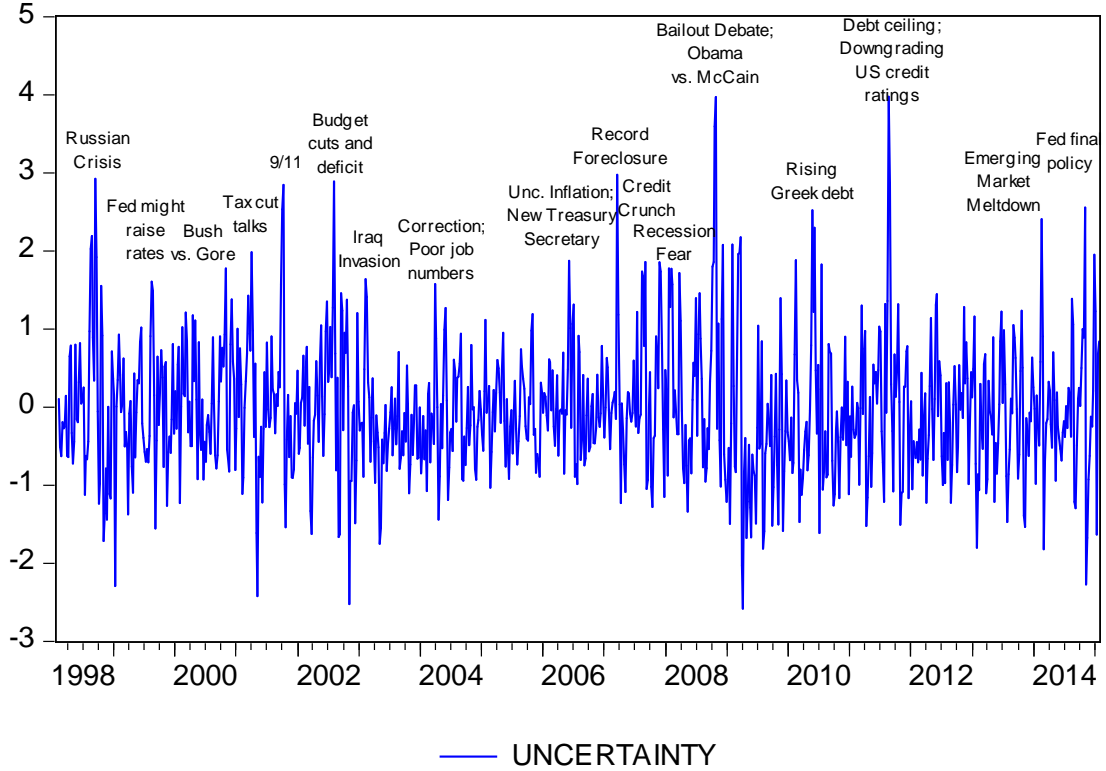
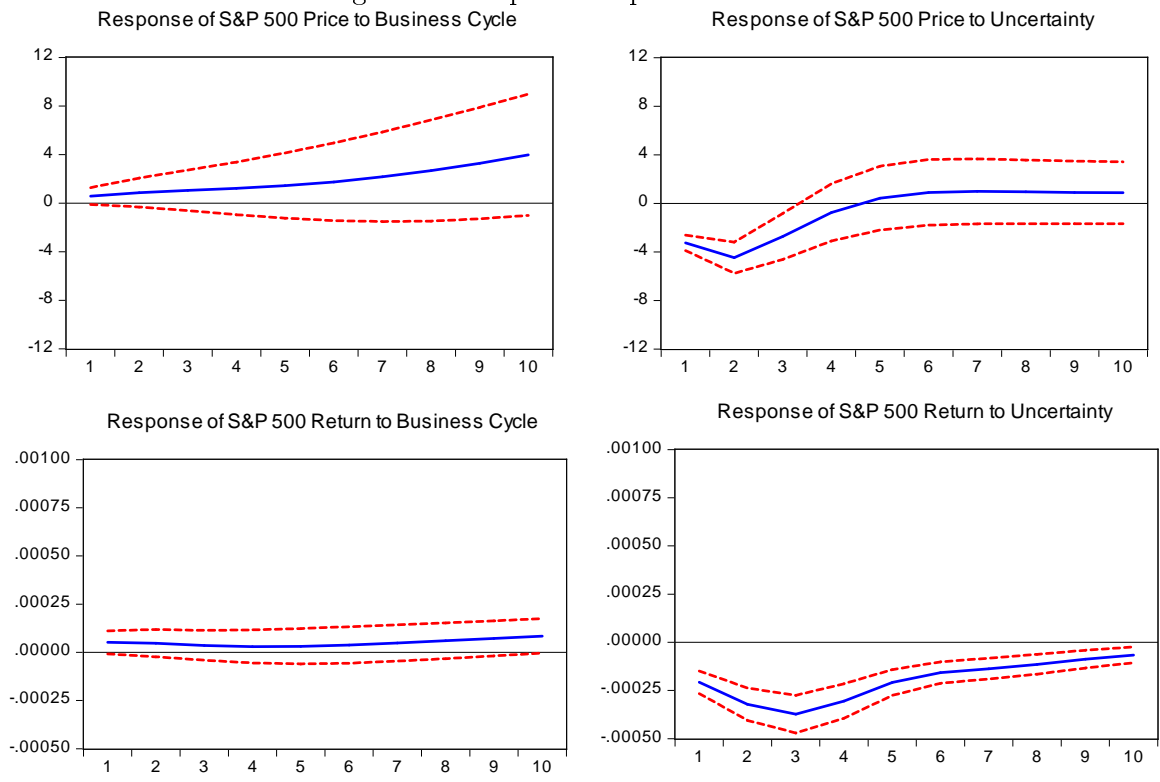


Table 1.4: Correlation of Negativity Indices with Uncertainty Factor

	W/o adjusting for B.C.				Model 2			
Negativity Index	0.37	-	-		0.24			
Lead Paragraph Negativity	-	0.26	-		-	0.24		
Text Negativity Index	-	-	0.32		-		0.22	
Headlines Negativity Index	-	-	-	-0.31	-			0.21
Retail Money Market	0.45	0.44	0.44	0.43	0.46	0.45	0.46	0.45
VIX	0.83	0.83	0.82	0.82	0.84	0.84	0.84	0.84
S&P 500 Return	-0.82	-0.83	-0.83	-0.83	-0.80	-0.81	-0.81	-0.81

Figure 1.3: Impulse Response Functions



In Figure 1.3 impulse response functions generated under a VAR framework show that increase in uncertainty is met with a large, negative but short-lived effect on both stock prices and return, while the fundamentals have a small, positive, long lived effect on the stock market prices. Thus, uncertainty that is not rooted from fundamental can cause stock market corrections or pull backs in the stock market. This is consistent with Antonakakis, Chatziantoniou & Filis (2013) who find increased policy uncertainty reduces stock returns. Poor economic fundamentals can however, have a prolonged effect on the market. Data used to build the business cycle index are often released with a month delay, within the month the stock market participants may have already gathered the information and updated expectations. Results may therefore reflect a small movement in the stock market due the the business cycle.

## 1.5 Conclusion

An overall measure of uncertainty and fear surrounding the stock market is introduced using the comovements in S&P 500 stock returns, media coverage of negative news, changes in aggregate holding of safe financial assets, and implied volatility in the trading of options of companies in S&P 500. In order to, decouple the influence of the economic fundamentals and uncertainty in the stock market, the uncertainty factor created is controlled for the business cycle. Aruoba, Diebold & Scotti's (2009) high frequency business cycle index is used, which accommodates for missing values, mixed frequency and lack of asynchronicity with which economic variables are released. The uncertainty factor is controlled for the business cycle using two alternative models.

A linear model finds the business cycle factor has a negative statistically insignificant effect on uncertainty, while a non-parametric regression indicates, a significant non-linear relationship between uncertainty factor and the business cycle index. That is uncertainty increases proportionately with both expansion and contraction of the economy, the more drastic the change is the higher is uncertainty. After controlling for the business cycle using the non-parametric method the correlation between the news negativity index and the uncertainty

factor falls, as the cyclical component of news are no longer correlated. The correlation with the stock returns also fall slightly, while the correlation with the other variables remain almost unaltered or fall slightly. Similar result is also obtained from the second method, that controls for the business cycle during the estimation of the uncertainty factor. Results indicate that news and stock returns have a cyclical component that are removed during the estimation of the uncertainty factor. T-bill holdings and S&P 500 volume contributed very little to the estimation of the uncertainty. One plausible reason for such low correlation could be the diverse roles T-bills perform in an investor's portfolio, it could be used for hedging, borrowing, diversifying, etc. Also a change in T-bills are usually noticed after a stock market crash and not concurrently. Stock volume similarly, could be higher due to both over optimistic and pessimistic view of the market. Unsurprisingly, retail money holding, and VIX are highly correlated with the factor. VIX itself is a volatility/fear measure which is often inversely related with stock returns, and retail money market instruments provide investors a liquid an accessible way to hold money, for precautionary measures or during reallocation of investments.

In case, headlines or lead paragraphs have a stronger impact on uncertainty. Three additional negativity indices are created by performing textual analysis the headline, the body of text and the lead paragraph. All produce similar results however, the news negativity index for the entire article has the strongest correlation with the factor.

A large, negative but short-lived effect of uncertainty on both stock prices and return is found. The fundamentals on the other hand have a short, positive, long lived effect of the business cycle on the stock market price and return. Thus, uncertainty that is not rooted from fundamental factors can cause stock market corrections or pull backs, which are financial downturns that are short lived. Poor economic fundamentals can however, have a prolonged effect on the market. Data used to build the business cycle index are often released with a month delay, within the month the stock market participants may have already gathered the information and updated expectations. This results in a small movement in the stock market due the the business cycle.

Future research endeavors include building similar uncertainty index for firm specific analysis; and a one step state-space framework for estimating both the business cycle and uncertainty factor is also worth looking into.

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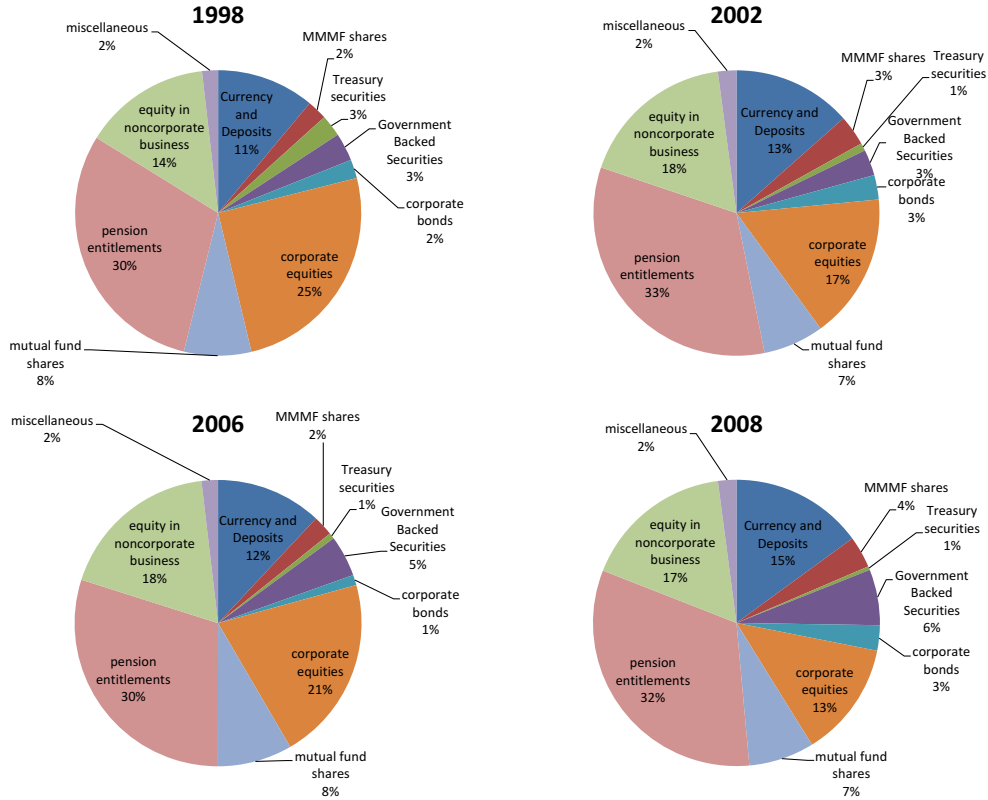
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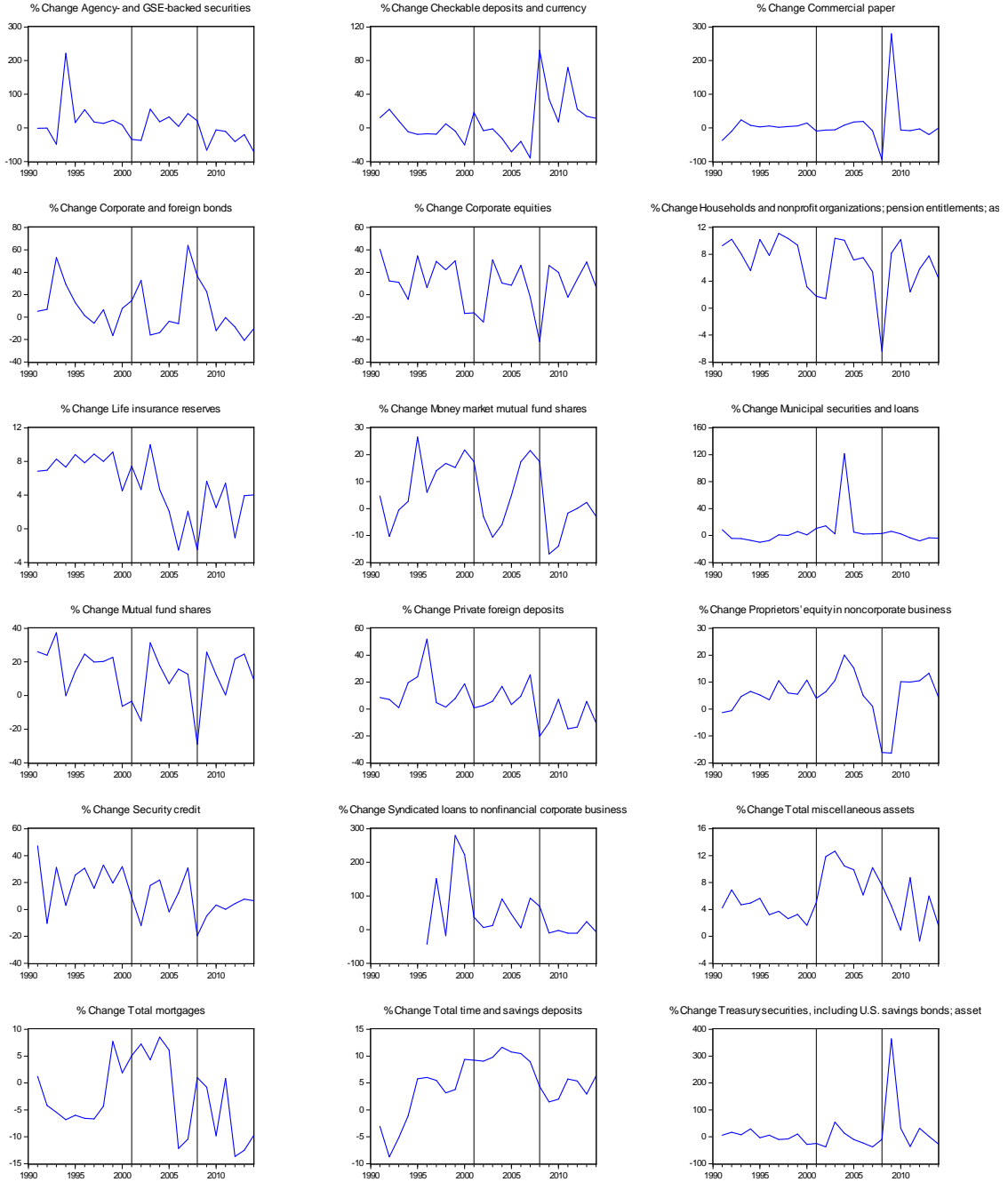
## A Appendix

Figure A.1: Share of Financial Assets of Households



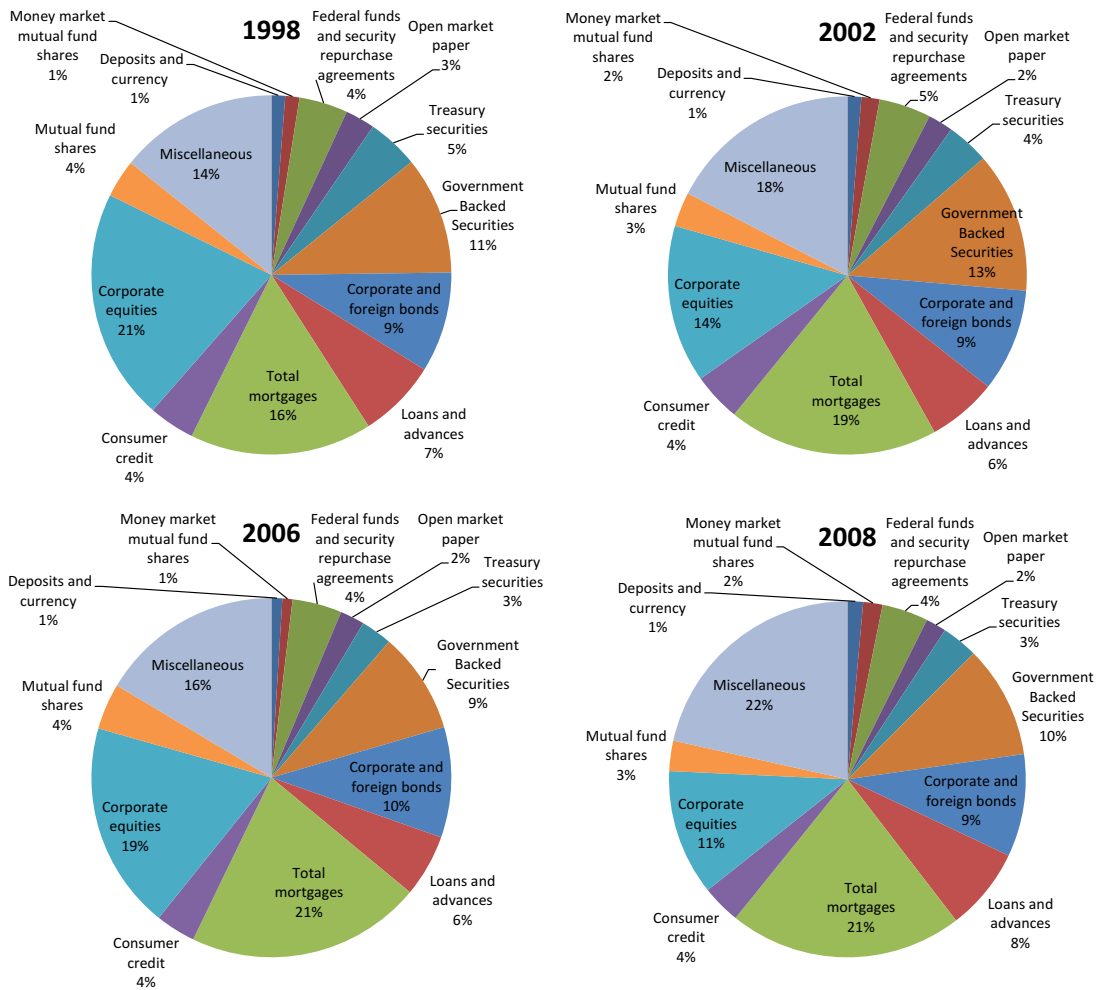
Source: Federal Reserve Board's Statistical release, Z1: Financial Accounts of the United States

Figure A.2: Change in Level of Holdings in Financial Assets of Households (with shaded NBER Recessions)



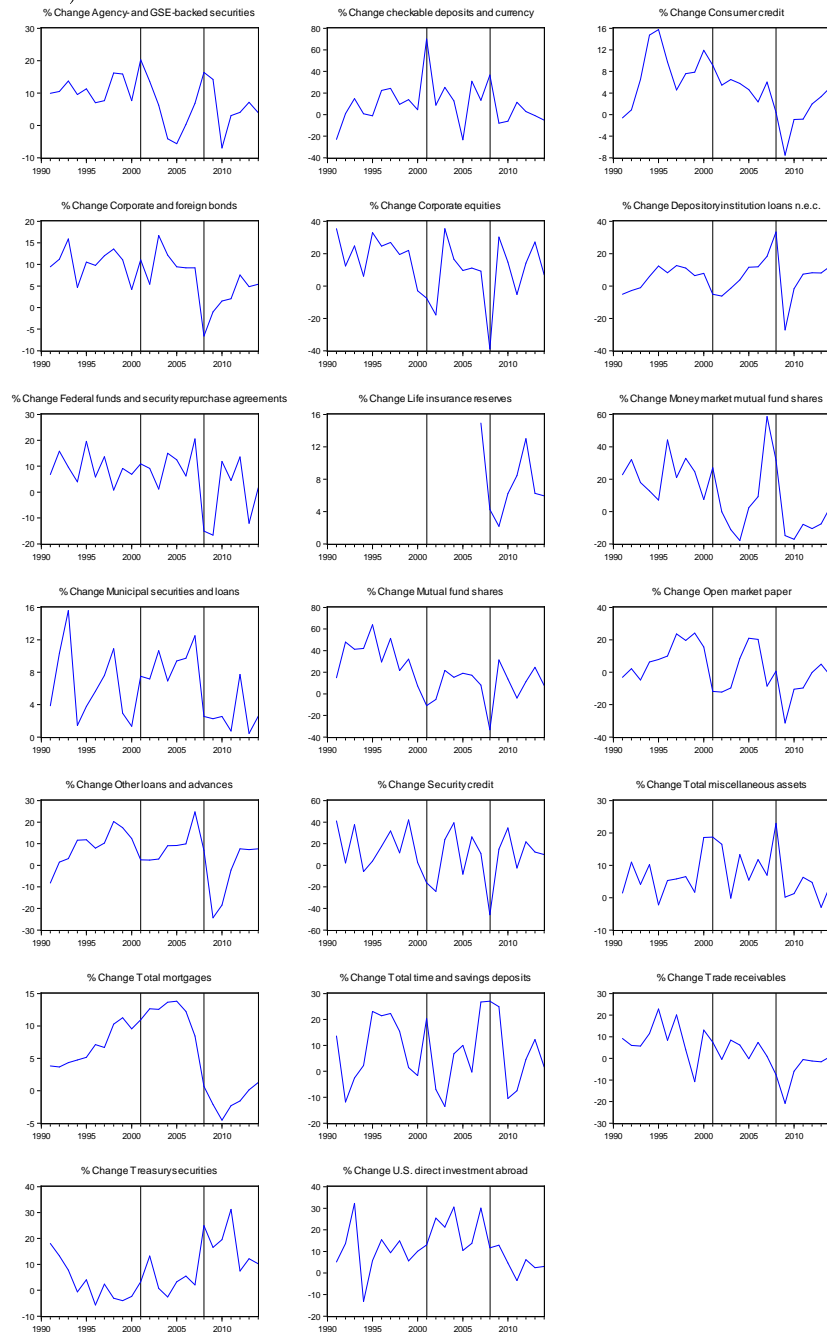
Federal Reserve Board's Statistical release, Z1: Financial Accounts of the United States

Figure A.3: Share of Financial Assets of Financial Institutions



Federal Reserve Board's Statistical release, Z1: Financial Accounts of the United States

Figure A.4: Change in Level of Holdings in Financial Assets of Households (with shaded NBER Recessions)



Federal Reserve Board's Statistical release, Z1: Financial Accounts of the United States

Figure A.5: Level of Holdings or Transactions of Financial Assets (with shaded bear markets and NBER Recession)

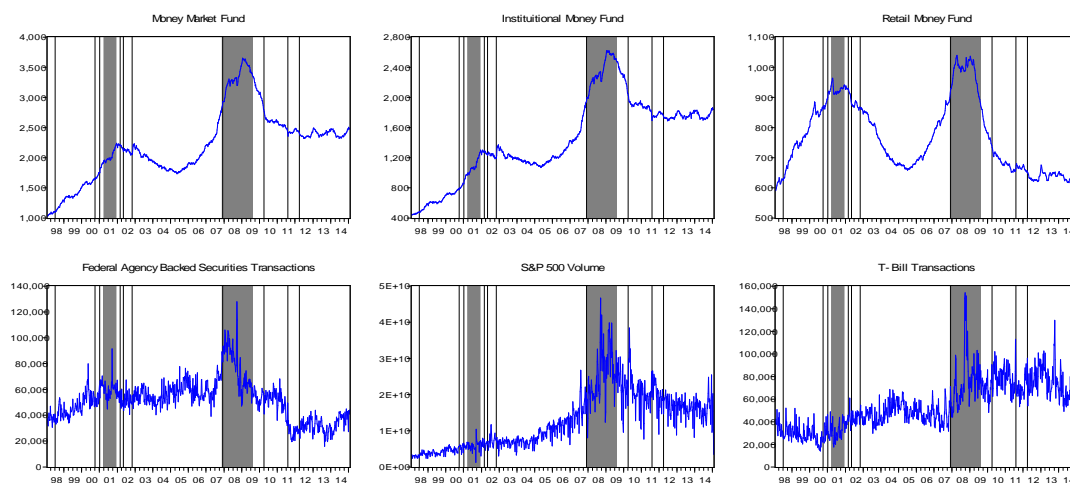


Table A.1: Newspaper Circulation

Newspaper	Circulation	Digital Edition (Branded Edition)
The Wall Street Journal	2,378,827	898,102
The New York Times	1,865,318	1,133,923
USA Today	1,674,306	249,900
Los Angeles Times	653,868	177,720 (43,275)
Daily News of New York	516,165	155,706
New York Post	500,521	200,571
The Washington Post	474,767	42,313 (1,305)
Chicago Sun-Times	470,548	77,660 (208,087)
The Denver Post	416,676	192,805 (10,041)

Source: Alliance for Audited Media

Table A.2: Download Criteria for Articles in USA Today

Table A.2 Download Criteria for Articles in USA Today		
	Download Criteria	Explanation
For USA Today	Page=01 or Section=Money	Business section of USA Today, "Money"
Date	01/26/1998 to 01/26/2015	
Source	USA Today	
Subject	Economy, Equity Market	
	Letters	Most letters to the editors are to express resentment towards past articles
	People Profiles	Career moves of public figures
	Reviews	Reviews of books about the economy, financial sector or finances of a corporation
excluding	Country Profiles and Trade/External Payments	Difficult to distinguish the tone used for different countries
	Personal Finance	Advice on mortgages, debt and saving habits.
	Corrected Items	Corrections of previously published articles, might no longer be relevant to the readers
	Advertorials, Calendar of Events, Headline Listings, Obituaries, Personal Announcements,	Self explanatory

Figure A.6: News Negativity indices (with shaded bear markets)

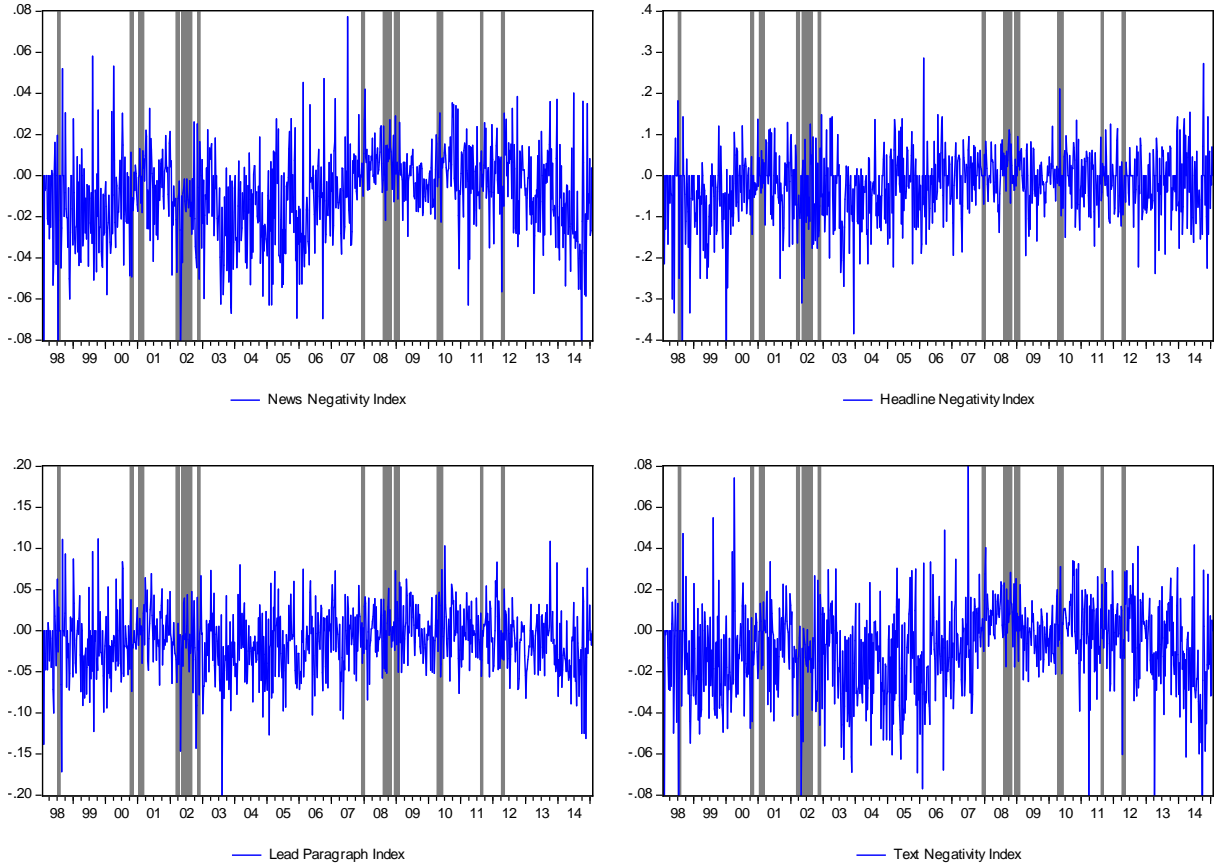
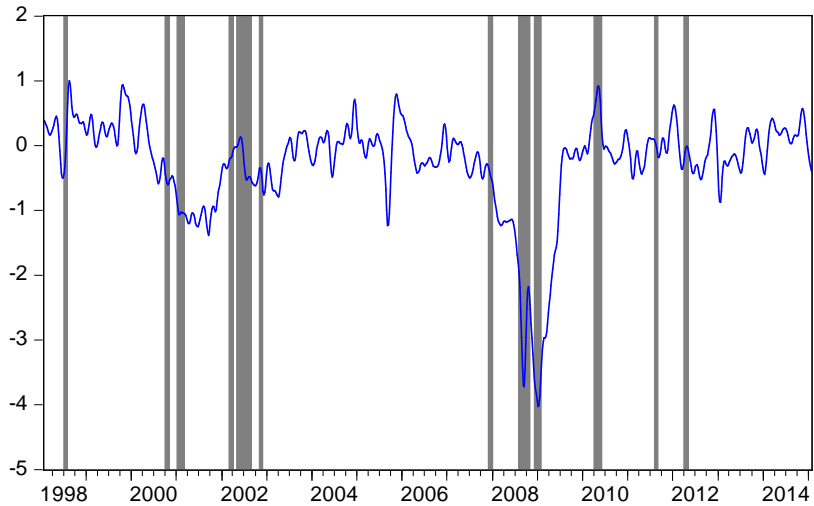


Figure A.7: Business Cycle Index (with shaded bear markets)





## **2 Chapter 2: Estimating Value-at-Risk and Expected Shortfall using Semi-parametric Conditional Variance**

### **2.1 Introduction**

Turmoil in financial markets such as those experienced during the recent financial crisis, dot.com bubble, Asian financial crisis and October 1987, have caused catastrophic losses to investors and institutions holding large portfolios of financial assets. Well documented cases of Orange County and Procter & Gamble Co. exhibit that even in the absence of a financial crisis immense losses can be incurred by making risky investments without necessary precautions. These events have greatly emphasized the need for regulation and management of risk. Effective quantitative risk measurement is considered as the primary means of mitigating such financial risks.

In finance literature, risk is broadly categorized as credit risk, operational risk, liquidity risk and market risk. Credit risk focuses on the borrowers' inability to adhere to payment obligations; liquidity risk on the firm's inability to fund short term needs; and operational risk on errors in internal processes. Market risk, primarily focuses on the adverse movements in market factors that may reduce the value of the firm's investments. In light of the growing sizes of investment portfolios held by financial institutions the need to quantify their risk exposure has become a crucial task for regulators and internal risk managers. One of the most prominent measures to quantify market risk is Value-at-Risk (VaR). Introduced first in the early 1990s in the financial industry to manage assets and minimize risk, its simplicity and usefulness quickly made it a popular analytical tool among risk managers, regulators and academicians. Conceptually, VaR for a given probability, is the maximum loss in a portfolio over a specified time horizon. Statistically, it is an extreme quantile, usually 5% or 1%, of the profit and loss distribution of the portfolio. A single monetary number or proportion incorporates information about the exposure of trading activities to fluctuations in the market factors, and summarizes several bad outcomes succinctly. So much so that European and American banks are required to set aside a portion of their capital as specified by their VaR to cover unanticipated losses from adverse market movements.

As large banks are intertwined with each other and the economy, collapse of one bank can potentially translate to the collapse of other banks and the vitality of the economy. To avoid such a predicament and to protect private investors, tighter regulations are placed on banks and financial institutions. The Basel Committee of the Bank of International Settlement (BIS) has also selected VaR as the benchmark for risk measurement in their Capital Adequacy Directive (Basel Committee, 1996; 2006; 2010). As per their guidelines banks and financial institution's must have sufficient risk capital to cover 99% of losses on trading portfolios from market risks<sup>10</sup>. Banks can use internal VaR model to comply with the regulatory capital requirement. A wide selection of alternative methodologies, that produce varying VaR estimates, are available for financial practitioners to choose from; see Duffie and Pan (1997), and Jorion (2001) for details on applications. This exposes the risk managers to model risk, the risk of selecting an inefficient model. Incorrect estimation of the underlying risk might cause banks to violate the regulations and suffer losses or to hold unnecessarily high levels of risk capital, that could have been used for more lucrative projects. Therefore, it is important to verify the accuracy of the model.

The poor performance of several VaR models to estimate the tail risk during the recent financial crisis, ignited the need for more informative and coherent risk measure, such as Expected Shortfall (ES) (Acerbi & Tasche, 2002a; 2002b). ES is the expected size of loss of a financial investment, given the loss is at least as large as a specific quantile such the VaR. What was predominantly a tool of the actuaries, is now a commonly used risk measure among financial risk managers, as an alternative of VaR. Artzner *et al.* (1999) argues that a coherent risk measure should have four attributes, namely monotonicity, positive homogeneity, translation invariance and subadditivity<sup>11</sup>. While ES satisfies all the four conditions to be a coherent risk measure VaR violates subadditivity, i.e. the risk of a portfolio is larger than the sum of risk of individual components. Artzner *et al.* (1999) point out that this may pose concern if banks were to set aside VaR for each assets individually. Moreover,

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<sup>10</sup>For internal risk minimization purposes managers can determine risk capital for different confidence level and holding period

<sup>11</sup>Artzner *et al.* (2002) extends it further for multi-period risk estimation

VaR, doesn't say anything about the size of a loss to expect when it exceeds VaR, only how often to expect violations. Taking these into consideration regulatory boards and BIS have been encouraging the use of ES to estimate the capital requirements for financial firms and banks (Basel Committee, 2016).

Like any risk modeling, market risk is encapsulated in probability theory; here return is the random variable whose outcomes have associated probabilities. Although, the true probability distribution is not known, past realization of return provide some tangibility. The core of the challenge lies in specifying the probability distribution that will be used to explain the extreme quantiles of the assets' returns. As the lowest return are used for the estimation of VaR, it is critical that the probability distribution fits the tail closely if not the entire distribution. A financial practitioner has to make several critical decisions, the first of which is to decide whether to estimate VaR as a quantile of the unconditional or conditional return distribution. Unconditional models assume returns to be stationary and *i.i.d*, that is not affected by time shift. Conditional models incorporates history of market environment and risk factors such as past volatility till time  $t$ , to estimate VaR for a future period  $t+h$ . As market factors fluctuate overtime, market risk may vary accordingly. It is well established that exceptionally good and bad days are followed by increased market fluctuations, heightening market risk (Duffie & Singleton, 2003; Engle & Manganelli, 2004). To obtain reliable forecasts of asset prices and risk, it might therefore be beneficial for risk managers to use conditional models that use a time series setting to capture change over time. Both unconditional and conditional models have their own merits, while unconditional models are fairly easy to implement and has some intuitive appeal; conditional models are more likely to react to market movements promptly (McNeil & Frey, 2000; Alexander & Sheedy, 2008).

Unconditional approach of VaR estimation mostly involves finding a parametric distribution to fit the fat tails usually found in financial series, popular choices include Gaussain, t-distribution,  $\alpha$ -stable and extreme value theory. Efficiency of the model relies on how accurately the distributions are specified. A poor fit in the lower tail due to model mis-

specification may result in underestimation of risk. On these grounds it is evident why non-parametric estimation of distributions have been gaining momentum in the VaR front. Misspecification bias is eliminated as non-parametric approaches do not require the user to specify the functional form of the distribution. However, reliance on the empirical data of past return dramatically increases. The most straightforward unconditional VaR estimation that does not require the user to specify the functional form is the historical simulation, also known as the empirical VaR; as the name suggests it is the upper threshold of the lowest 1% or 5% returns. Kernel based unconditional non-parametric approaches involve finding the extreme quantiles of the data after fitting a continuous kernel. Since, these models rely on the data heavily they work best for measuring quantiles that are closer to the center where there are more observations; the extremes tails have very few observations. Moreover it is difficult to predict a loss greater than those in the past. Unconditional models also have a large reaction time to crisis, a long string of bad events have to happen before the distribution changes in the tails, meanwhile huge losses will be incurred by then. There is also strong empirical support that financial time series are heteroscedastic (Pagan, 1996), this violates any *i.i.d* assumption. This has led researchers to pursue conditional models, which take the volatility clustering into account and are more responsive to risk.

Most conditional models assume the distribution of returns belong to a location-scale family, and VaR is estimated using the quantiles of standardized return distribution. Conditional models therefore, require the estimation of the first two moments and the quantile for the standardized return series. Differences among the models mainly revolve around the estimation of the conditional variance, while the conditional mean is assumed to be zero under the efficient market hypothesis; or assumed to follow an ARMA structure. Traditionally, to capture heteroscedasticity found in financial series GARCH models that assumed returns to be conditionally normal were proposed. However, stock returns are known for being leptokurtic and asymmetric, leading these models to produce poor estimates (Danielsson & de Vries, 1997). To overcome this an influx of alternate ARCH-GARCH type models have been proposed in the parametric arena, where the underlying distribution of the standardized return

is assumed to follow a different parametric distribution; see Poon and Granger (2003) for an overview of volatility models used in the finance literature. Conditional parametric models are the most efficient when they are correctly specified but vulnerable to severe misspecification bias. Bias can stem from two sources, first in defining the relationship between future volatility and current volatility and the second in specifying the underlying distribution of standardized return. Both the conditional variance and the distribution of the standardized return can be estimated non-parametrically to eliminate such bias in parametric models, such non-parametric models include Cai (2002), Cai and Xu (2008), Chen and Tang (2005), among others. However, in case of extreme events non-parametric estimation which heavily relies on data might not be able to adequately foresee losses that haven't been experienced before. Therefore, in this chapter a semiparametric estimation of the conditional variance following Misha, Su and Ullah (2010) and a non-parametric estimation of the standardized return quantile is proposed to estimate the VaR and Expected shortfall. The semiparametric conditional volatility estimator reduces to that of the parametric model when the parametric model is correctly specified, and in cases where the parametric model is not correctly specified the estimator can be adjusted with a non-parametric volatility estimator of the standardized residuals. Section 2.2 introduces the new VaR and ES estimators, and describe some of the most popular unconditional and conditional methods; followed by empirical results and simulation results in section 2.3 and 2.4, respectively; and section 2.5 concludes.

## 2.2 Estimation

VaR and ES can be expressed in monetary terms as the value of the investment that could be potentially lost. They can also be expressed as return, for instance a -0.10 VaR can be interpreted as a minimum of 10% of the initial investment could be lost in the worst 5% of scenarios. Since, return is universal for any size of investment in the same asset, in this chapter VaR and ES is expressed in terms, of return. Therefore, for a confidence level of  $(1 - p)$ , the  $VaR_{p,t}$  for the future period  $t$ , of an investment with a holding period of  $\tau$  is expressed as the  $p^{th}$  - quantile of return distribution of the investment at time  $t$ .

Let the random variable  $r_t$  be the return at time  $t$ . Similarly,  $ES_{p,t}$  for the confidence level  $(1 - p)$  is the expected return of the investment, which are lower than the specified  $VaR_{p,t}$ .

$$P(r_t \leq VaR_{p,t}) = p \quad a.s$$

$$ES_{p,t} = E[r_t | r_t < VaR_{p,t}]$$

Depending on the specification VaR models can be broadly categorized as unconditional and conditional models. This section will specify some of the most popular parametric and non-parametric models within each of these categories, and introduce the new semiparametric conditional volatility VaR model.

### 2.2.1 Unconditional models

Unconditional VaR models assume returns of all periods to be identically distributed, and not affected by past returns. Unconditional models solely differ in their specification of return's distribution,  $\mathbf{F}(\cdot)$ .  $\mathbf{F}(\cdot)$  can be assumed to be a known distribution such as Gaussian for which the analytical form for the pdf is known, or the probability distribution is assumed to be similar to the the historically observed past returns. Once the cdf of returns  $F(\cdot)$ , is specified it can be inverted to obtain the desired quantile. VaR acts as an upper threshold for the lowest returns, such that the probability that return will be smaller than the specified VaR is at most  $p$ .

$$VaR = \sup\{r \in \mathbb{R} : F(r) \leq p\}$$

#### Gaussian

Returns on the investment are assumed to follow a normal distribution,  $r \sim N(\mu_t, \sigma_t^2)$ . Since the entire distribution can be explained by the first two moments, the estimation of VaR and ES depend on the estimation of mean, variance and the left tail critical value at level  $p$  of the standard normal distribution,  $z_p$ .

$$VaR_{p,t, Gaussian} = \mu_t + \sigma_t z_p \tag{2.1}$$

$$ES_{p,t, Gaussian} = \mu_t - \frac{f(z_p)}{p} \sigma_t \quad (2.2)$$

where,  $f(\cdot)$  is the pdf of a standard normal distribution and the mean and variance are estimated by:  $\hat{\mu}_t = \frac{1}{t} \sum_{i=1}^t r_i$  and  $\hat{\sigma}_t^2 = \frac{1}{t-1} \sum_{i=1}^t (r_i - \hat{\mu}_t)^2$ . If the marginal distribution of returns are truly normal and *i.i.d.* this would be the ideal model to estimate the VaR and ES. However, financial returns are mostly non-normal, as exhibited from the high Kurtosis and skewness shown in Table 2.1. Parametric models that can accommodate for thicker-tails tend to do better in fitting the empirical distribution of return than normal. Moreover, non-parametric models that are free from misspecification bias are commonly sought to estimate the distribution of the returns. Two such unconditional VaR estimations are discussed below.

### Historical Simulation (HS)

HS or the empirical model is one of the most straight forward methods to calculate VaR, where the past returns,  $\{r_i\}_{i=1}^t$  are used to non-parametrically estimate the marginal distribution of returns. The  $p^{th}$  quantile,  $Q_p(\cdot)$  of the ordered past returns  $\{r_i^*\}_{i=1}^t$ , where  $r_1^* \leq r_2^* \leq r_3^* \dots \leq r_t^*$  is used as an estimate of the VaR. The empirical CDF of returns,  $\mathbf{F}_t(\cdot)$ , is estimated as a step function, VaR as an inverse of the CDF; and ES as an average of the returns lower than the corresponding VaR. The estimations are shown in equations (2.3), (2.4) and (2.5), respectively.

$$\mathbf{F}_t(r) = \frac{1}{t} \sum_{i=1}^t \mathbf{I}(r_i \leq r) \quad (2.3)$$

$$VaR_{p,t, HS} = Q_p(r_t^*) = \mathbf{F}_t^{-1}(p) \quad (2.4)$$

$$ES_{p,t, HS} = \frac{1}{p} \sum_{i=1}^t \mathbf{I}(r_i \leq VaR_{p,t, HS}) * r_i \quad (2.5)$$

$$\begin{cases} \mathbf{I}(A) = 1 & \text{if event } A \text{ is true} \\ \mathbf{I}(A) = 0 & \text{if event } A \text{ is not true} \end{cases}$$

The theoretical underpinning of HS is the Glivenko-Cantelli theorem, which states that if the sample size of an *i.i.d.* random variable is large enough, the sample empirical CDF will converge to that of the population.

$$\lim_{t \rightarrow \infty} \sup |\mathbf{F}_t(r) - \mathbf{F}(r)| = 0 \quad a.s.$$

A large number of past return is however needed to reach a reliable estimate. Moreover, risk estimates are bound by those observed in the past, extraordinary loss that hasn't been experienced before cannot be predicted. Alternative HS methods have been proposed over time that are not discussed in this chapter for brevity, other historical simulation models include Hull and White (1998), Barone-Adesi *et al.* (2002), and Barone-Adesi (2008).

### Kernel Smoothing (KS)

Kernel smoothing<sup>12</sup> also estimates the return density non-parametrically using finite past returns,  $\{r_i\}_{i=1}^t$ . Unlike HS, kernel smoothing can obtain VaR estimates that are smaller than the smallest past return. And while, HS uses a step function which is not differentiable, KS uses a symmetric, continuous Kernel,  $K(\cdot)$  to obtain a smooth empirical distribution function,  $\hat{F}(\cdot)$ <sup>13</sup>. A wide range of Kernel functions are at the user's disposal to choose from, Normal, Epanechnikov, Triangular, Rectangular, Cosine are among the most frequently used Kernel functions. A bandwidth,  $h$ , also has to be chosen to decide on the degree of smoothness of the estimated density.

$$\hat{F}_t(r) = \frac{1}{th} \sum_{i=1}^t A\left(\frac{r - r_i}{h}\right)$$

$$A(r) = \int_{-\infty}^r K(u) du$$

Unlike the Kernel function, the choice of bandwidth can affect the quality of estimation of the density. There is a vast literature on bandwidth selection as oversmoothing results in

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<sup>12</sup>referred also as unconditional non-parametric

<sup>13</sup>Unlike KS, HS can only assign density estimates for points with realized returns.



larger bias between the estimate density and true density; while undersmoothing results in larger variance. Popular bandwidth selection methods include Silverman’s Rule of Thumb, Plug-in-method and Cross-validation<sup>14</sup>.

The VaR is estimated as an inverse of the distribution function.

$$VaR_{p,t,KS} = Q_p(r_t) = \mathbf{F}_t^{-1}(p) \quad (2.6)$$

In the recent years there has been a growing interest in non-parametric estimation of expected shortfall (Scaillet, 2004; Chen, 2008; Yu *et al.*, 2010). This chapter follows Scaillet’s (2004) ES estimation because it allows for strong mixing in the data, commonly found in financial data, the estimation is shown in (2.7).

$$ES_{p,t,KS} = \left(\frac{1}{thp}\right) \sum_{i=1}^t r_i A\left(\frac{VaR_{p,t,KS} - r_i}{h}\right) \quad (2.7)$$

Although kernel smoothing is free from assumptions about the distribution and fits the empirical distribution better than HS, like all these unconditional models discussed above, it does not account for serial dependence and volatility clustering commonly found in financial data. For small finite samples and large confidence levels, there are very few realized observations to infer precise tails estimates.

## 2.2.2 Conditional Models

Most conditional models assume returns to be in a location-scale family. This reduces the VaR estimation to that of the conditional mean and variance. There are several ways the conditional mean can be estimated, however, most conditional VaR model’s key variation lies in how the conditional volatility is estimated.

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<sup>14</sup>See Pagan & Ullah (1999) for a discussion on bandwidth selection

$$\begin{cases} r_t = \mu_t + \sigma_t \epsilon_t \\ \epsilon_t \sim m.d.s(0, 1) \text{ with conditional CDF } F_t(\cdot) \end{cases}$$

$\epsilon_t$  is the martingale difference sequence (m.d.s),  $E(\epsilon_t | I_{t-1}) = 0$  a.s

The conditional mean and variance are estimated using the information set available at time  $t$ ,  $I_{t-1}$  which usually includes the past returns.

$$\mu_t = \mu_t(I_{t-1})$$

$$\sigma_t^2 = \sigma_t^2(I_{t-1})$$

$$F_t(\cdot) = F_t(\cdot | I_{t-1})$$

Value at risk therefore can be estimated succinctly by (2.8).

$$VaR_{p,t} = \mu_t + \sigma_t q_{p,t} \tag{2.8}$$

where,  $q_{p,t} = q_{p,t}(I_{t-1}, p)$  is the  $p$ -th quantile of  $F_t(\epsilon_t)$ .

### **ARCH/GARCH**

It has been long known that volatility clustering is present in financial time series, but it was the introduction of the (generalized) autoregressive conditional heteroscedasticity models, (G)ARCH in the 1980's (Engle, 1982; Bollerslev, 1986) that popularized incorporating the conditional variance to estimate returns. The ARCH model uses the past, squared, and de-meaned return to estimate the conditional variance,  $\sigma_t^2$  as shown in (2.9). While the GARCH model is further extended by including the past conditional variance, as shown in (2.10).

$$\hat{\sigma}_t^2 = \hat{\alpha} + \hat{\beta} (r_{t-1} - \hat{\mu}_t)^2 \tag{2.9}$$

$$\hat{\sigma}_t^2 = \hat{\alpha} + \hat{\beta} (r_{t-1} - \hat{\mu}_t)^2 + \gamma(\hat{\sigma}_{t-1}^2) \quad (2.10)$$

Under the assumption of normality in the error term, the VaR and expected shortfall can be estimated as the Gaussian models in (2.1) and (2.2), replacing the unconditional mean,  $\mu_t$ , and standard deviation,  $\sigma_t$  with their conditional counterparts.

### Conditional Nonparametric

In order to avoid misspecification bias in the estimation of the conditional variance, it can be estimated non-parametrically. Härdle and Tsybakov (1997) propose a non-parametric estimation of  $\mathbf{E}(r_t^2|I_{t-1})$  and  $\mathbf{E}(r_t|I_{t-1})^2$ , and then taking the difference of the two to estimate the conditional variance. Fan and Yao (1998) also propose a two-step procedure, but first estimating the conditional mean,  $\mu_t$ , and then using the residuals to estimate the conditional variance, both using local linear estimation. The estimation of the non-parametric conditional variance estimator of Fan and Yao (1998),  $\sigma_{t,CNP}^2$ , is illustrated in (2.11), where,  $K$  is a smooth Kernel and  $h$  is the bandwidth or the smoothing parameter.

$$\hat{\sigma}_{t,CNP}^2 = \hat{m}(r_{t-1} - \hat{\mu}_{t-1}) = \frac{\sum_{t'=2}^T K(r_{t'} - \hat{\mu}_{t'})^2 \{ \{ (r_{t'-1} - \hat{\mu}_{t'-1})^2 - (r_{t-1} - \hat{\mu}_{t-1})^2 \} / h \}}{\sum_{t'=2}^T K \{ \{ (r_{t'-1} - \hat{\mu}_{t'-1})^2 - (r_{t-1} - \hat{\mu}_{t-1})^2 \} / h \}} \quad (2.11)$$

The conditional mean and variance are further used to estimate the VaR and the ES as shown in (2.12) and (2.13), where,  $Q_p(\cdot)$ , is the  $p^{th}$  quantile estimated using Kernel smoothing.

$$VaR_{p,t,CNP} = \hat{\mu}_{t,CNP} + Q_p(r_t - \mu_t) * \hat{\sigma}_{t,CNP} \quad (2.12)$$

$$ES_{p,t,CNP} = \left( \frac{1}{thp} \right) \sum_{i=1}^t r_i A \left( \frac{VaR_{p,t,CNP} - r_i}{h} \right) \quad (2.13)$$

Further extensions of the Fan and Yao's (1998) method have been put forward, some notable ones include Ziegelmann's (2002) local exponential estimator for the conditional variance to ensure nonnegativity.

### Conditional Semiparametric

Mishra, Su and Ullah (2010) introduces a multiplicative, semiparametric estimation (SP) of the conditional variance that improves upon Ziegelmann's (2002) estimator. The SP method first applies a parametric model to estimate the volatility in the series,  $\hat{\sigma}_{P,t}$ , and then uses the standardized residuals of the parametric estimation,  $\hat{\epsilon}_{p,t}$ , to capture the remaining volatility using a non-parametric local linear or exponential method. The SP estimator is a product of the parametric,  $\hat{\sigma}_{P,t}^2$ , and non-parametric,  $\hat{\sigma}_{NP,t}^2$  variance estimators, as described in (2.14). The estimation of the VaR and ES using the SP estimator follows the same methods as the conditional non-parametric ones illustrated in (2.12) and (2.13), respectively, by replacing the  $\hat{\sigma}_{t,CNP}^2$ , and  $VaR_{p,t,CNP}$ , with their SP counterparts.

$$\hat{\epsilon}_{p,t} = r_{t-1} - \hat{\mu}_{t-1} / \hat{\sigma}_{P,t}$$

$$\hat{\sigma}_{NP,t}^2 = \hat{m}_1(r_{t-1} - \hat{\mu}_{t-1})$$

(2.14)

$$\hat{m}_1(r_{t-1} - \hat{\mu}_{t-1}) = \frac{\sum_{t'=2}^T K(\hat{\epsilon}_{p,t'})^2 \{ (r_{t'-1} - \hat{\mu}_{t'-1})^2 - (r_{t-1} - \hat{\mu}_{t-1})^2 \} / h}{\sum_{t'=2}^T K \{ (r_{t'-1} - \hat{\mu}_{t'-1})^2 - (r_{t-1} - \hat{\mu}_{t-1})^2 \} / h}$$

$$\hat{\sigma}_{SP,t}^2 = \hat{\sigma}_{P,t}^2 * \hat{\sigma}_{NP,t}^2$$

The SP estimator improves upon both parametric and non-parametric models. In case of misspecified parametric estimator which is inconsistent with the true variance, the SP may still remain as a consistent estimator. When compared to Ziegelmann's (2002) non-parametric estimator, the SP estimator performs better in terms of bias reduction, provided the parametric model specified captures some features of the true variance. Unlike Ziegelmann's estimator, the SP estimator can be applied to infinite dimensional information set, which can be described by finite conditioning variables, see Mishra, Su and Ullah (2010).

### 2.3 Empirical Results

The unconditional and conditional VaR and ES models discussed in section 2.2 are applied to real financial data series to compare their performances. A wide range of assets are used

Table 2.1: Summary Statistics of Daily Asset Return

	Mean	Std. Dev	Skewness	Kurtosis
BAC	0.0003	0.0269	-0.3627	29.730
MSFT	0.0007	0.0221	-0.7099	18.906
WMT	0.0005	0.0174	-0.0178	7.150
S&P500	0.0003	0.0118	-1.2914	30.980
NASDAQ	0.0003	0.0146	-0.2270	10.514
USD/YEN	0.0000	0.0069	0.3951	7.259
USD/GBP	0.0000	0.006	-0.3020	7.049

starting from stock indices, stocks of a bank, stocks in the technology sector to prominent currencies. The specific assets are of Bank of America (BAC), Microsoft (MSFT), Walmart (WMT), S&P 500, NASDAQ, US Dollar to Japanese YEN (USD/YEN), and US Dollar to British Pound (USD/GBP). The descriptive statistics of log-differenced daily returns of the financial assets, spanning from March-11-1987 to February-2-2015 are given in Table 2.1. The skewness and Kurtosis values indicates that the asset returns are starkly different from a normal distribution.

Regulations require banks and financial institutions to hold reserves based on their VaR and ES models. On one hand if a bank's VaR model repeatedly under-predicts the actual loss, it would violate the regulations. On the other hand, a conservative VaR model would hold excess reserves than required, that could have been invested for higher returns. Given the trade off, a desirable  $VaR_p$  estimator's proportion of violations,  $[r_t < VaR_{p,t}]$ , would not be statistically different from  $p$ . The Kupiec test (1995) is a two sided likelihood ratio test, where under the null, the proportion of violations/exceedances<sup>15</sup> is equal to  $p$ .

The VaR models are evaluated using the aforementioned financial series on the basis of the Kupiec test. Table 2.2 and Table 2.3 presents  $p$ -values for the Kupiec tests' for  $p = 5\%$  and  $p = 1\%$ , respectively. The  $VaR_p$  for each series are calculated on a rolling window of 250, for 7270 data points. The first 250 are dropped for estimation, leaving 7020 VaR to be

<sup>15</sup>Realized return is lower than the estimated  $VaR_{p,t}$

Table 2.2:  $VaR_{.05}$  Kupiec test for Empirical Data

P-values for $VaR_{.05}$ Kupiec test (Actual Exceed). Expected Exceedances = 351							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
WMT	0.01 (307)	0.00 (614)	0.01 (304)	0.03 (392)	0.00 (287)	0.02 (311)	0.00 (298)
MSFT	0.00 (278)	0.00 (672)	0.00 (280)	<b>0.09</b> <b>(383)</b>	0.00 (263)	0.00 (275)	0.00 (263)
BAC	<b>0.22</b> <b>(332)</b>	0.00 (646)	<b>0.31</b> <b>(336)</b>	0.03 (394)	0.00 (297)	<b>0.08</b> <b>(323)</b>	0.00 (307)
YEN/USD	<b>0.83</b> <b>(347)</b>	0.00 (830)	<b>0.41</b> <b>(366)</b>	<b>0.23</b> <b>(373)</b>	0.00 (301)	<b>0.06</b> <b>(317)</b>	0.04 (314)
GBP/USD	0.02 (393)	0.00 (713)	0.03 (392)	0.04 (388)	0.04 (313)	<b>0.30</b> <b>(332)</b>	<b>0.36</b> <b>(334)</b>
S&P 500	<b>0.61</b> <b>(334)</b>	0.00 (702)	<b>0.30</b> <b>(362)</b>	<b>0.21</b> <b>(366)</b>	0.00 (286)	<b>0.05</b> <b>(309)</b>	<b>0.37</b> <b>(327)</b>
NASDAQ	<b>0.21</b> <b>(375)</b>	0.00 (759)	0.01 (401)	0.03 (391)	0.04 (316)	<b>0.33</b> <b>(334)</b>	0.04 (315)

calculated for each series. The value in each of the parenthesis in Table 2.2 and 2.3 represents the actual number of violations/exceedances observed when  $VaR_p$  of the corresponding row is estimated using the method of the corresponding column. The expected number of violations for a correctly estimated a  $VaR_{.05}$  model with a sample size of 7020 is about 351 violations. Bold typeface indicates  $p$ -values larger than 5%, and that the test fails to reject the null that the proportion of violations are significantly different from  $p$ . At the 5% level it can be seen from table 2.2 that in almost all cases the conditional non-parametric model produces proportion of violations that are not statistically different from 5%. The Gaussian method, GARCH and the Empirical (Historical simulation) also produce desirable number of violations in some of the cases. The unconditional non parametric and conditional semi-parametric models are conservative in terms of estimating the risk, resulting in fewer violations. Large estimates of the VaR results in fewer violation, thereby rejecting the null of the Kupiec Test. In contrast to the  $VaR_{.05}$  cases in the  $VaR_{.01}$  estimation semiparametric model performs better in capturing the 1% of violations. In all the cases the parametric models have a large number of violations. Non-parametric methods in general perform better, particularly the unconditional estimator.

Table 2.3:  $VaR_{0.01}$  Kupiec Test for Empirical Data

P-values for $VaR_{0.01}$ Kupiec test (Actual Exceed). Expected Exceedances = 70							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
WMT	0.00 (104)	0.00 (395)	0.00 (108)	0.00 (96)	<b>0.60</b> <b>(66)</b>	<b>0.05</b> <b>(87)</b>	<b>0.21</b> <b>(81)</b>
MSFT	0.00 (100)	0.00 (452)	0.00 (100)	0.00 (103)	<b>0.06</b> (55)	<b>0.51</b> <b>(76)</b>	<b>0.94</b> <b>(71)</b>
BAC	0.00 (125)	0.00 (425)	0.00 (118)	0.00 (100)	<b>0.90</b> <b>(72)</b>	<b>0.10</b> <b>(85)</b>	<b>0.55</b> <b>(76)</b>
YEN/USD	0.00 (151)	0.00 (589)	0.00 (143)	0.00 (103)	<b>0.73</b> <b>(73)</b>	<b>0.20</b> (81)	<b>0.73</b> <b>(73)</b>
GBP/USD	0.00 (142)	0.00 (461)	0.00 (134)	0.00 (101)	<b>0.91</b> <b>(71)</b>	0.00 (97)	<b>0.09</b> <b>(85)</b>
S&P 500	0.00 (137)	0.00 (490)	0.00 (144)	0.00 (101)	<b>0.60</b> <b>(73)</b>	0.00 (94)	0.01 (91)
NASDAQ	0.00 (147)	0.00 (547)	0.00 (140)	0.00 (109)	<b>0.58</b> <b>(75)</b>	0.00 (96)	0.02 (90)

While the Kupiec test, tests whether the number of violations or exceedances are within the expected amount, it does not take into consideration the pattern of these violations. If there is a pattern in the violations, this indicates the VaR model's inadequacy to capture it. Repeated violations may also have severe consequences, this would imply that the banks have to deplete their reserves to meet one shortfall only to find themselves in the same position the next day. This may lead to liquidity shortage, or even make a bank collapse like those experienced during the last financial crisis. Therefore, it is of paramount importance that the VaR model can avoid such repeated violations. Taking this issue into consideration Christoffersen and Pelletier (2004) test whether the violations are independent of each other, using the duration between two concurrent violation. More specifically, if the violations are independent of each other, the duration between them should also be independent, or have no memory. Christoffersen and Pelletier (2004) argues that since exponential is the only memory free continuous random distribution, under the null the violations are independent of each other and the duration between them follows an exponential distribution. The

Table 2.4:  $VaR_{0.05}$  Duration Based Test for Empirical Data

P-values for $VaR_{0.05}$ Duration Based test (Weibull) <sup>16</sup>							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
WMT	0.00 (0.83)	0.00 (0.80)	<b>0.98</b> <b>(0.70)</b>	0.00 (0.86)	0.00 (0.81)	0.00 (0.85)	<b>0.58</b> <b>(0.98)</b>
MSFT	0.00 (0.81)	0.00 (0.76)	<b>0.22</b> <b>(0.94)</b>	0.00 (0.83)	0.00 (0.81)	0.00 (0.82)	<b>0.06</b> <b>(0.92)</b>
BAC	0.00 (0.76)	0.00 (0.77)	0.04 (0.91)	0.00 (0.79)	0.00 (0.74)	0.00 (0.76)	<b>0.05</b> <b>(0.92)</b>
YEN/USD	0.00 (0.88)	0.00 (0.83)	<b>0.59</b> <b>(0.97)</b>	0.00 (0.90)	0.00 (0.87)	0.00 (0.88)	<b>0.74</b> <b>(0.98)</b>
GBP/USD	0.00 (0.83)	0.00 (0.85)	<b>0.62</b> <b>(0.98)</b>	0.00 (0.86)	0.00 (0.81)	0.00 (0.82)	<b>0.71</b> <b>(0.98)</b>
S&P 500	0.00 (0.76)	0.00 (0.79)	0.02 (0.91)	0.00 (0.79)	0.00 (0.75)	0.00 (0.76)	0.03 (0.91)
NASDAQ	0.00 (0.75)	0.00 (0.83)	<b>0.17</b> <b>(0.95)</b>	0.00 (0.76)	0.00 (0.75)	0.00 (0.78)	<b>0.07</b> <b>(0.93)</b>

exponential being a special case of the Weibull distribution, where the Weibull parameter,  $b$ , is 1, the null can be also be expressed as  $b=1$ , against the two sided alternative.

$$\begin{cases} H_0 & f(D, p, 1) = p \exp(-pD) \\ H_a & f(D, p, b) = p^b b D^{b-1} \exp(-(pD)^b) \end{cases}$$

Christoffersen and Pelletier (2004) duration test is applied to test whether the violations are independent of each other. Table 2.4 and 2.5 presents the  $p$ -values of the Christoffersen and Pelletier (2004) test with the Weibull estimate in the parenthesis, for  $VaR_{0.05}$  and  $VaR_{0.01}$ , respectively. Despite, having proportion of violations close to 5%, for Gaussian, Historical Simulation and Conditional Non-parametric,  $VaR_{0.05}$ , the violations are not independent of each other. Only GARCH and the semiparametric estimators provided violations without a recognizable pattern in most cases. Similar results were obtained from the  $VaR_{0.01}$  test, where the semiparametric performed even better than the GARCH model.



Table 2.5:  $VaR_{0.01}$  Duration Based Test for Empirical

P-values for $VaR_{0.01}$ Duration Based test							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
WMT	0.00 (0.69)	0.00 (0.65)	0.02 (0.83)	0.00 (0.68)	0.00 (0.69)	0.00 (0.72)	<b>0.14</b> <b>(0.87)</b>
MSFT	0.00 (0.74)	0.00 (0.65)	<b>0.43</b> <b>(0.94)</b>	0.00 (0.77)	0.07 (0.82)	0.03 (0.82)	<b>0.81</b> <b>(0.97)</b>
BAC	0.00 (0.66)	0.00 (0.68)	<b>0.57</b> <b>(0.96)</b>	0.00 (0.69)	0.00 (0.71)	0.00 (0.74)	<b>0.74</b> <b>(0.97)</b>
YEN/USD	0.00 (0.77)	0.00 (0.72)	<b>0.09</b> <b>(0.89)</b>	0.00 (0.76)	0.00 (0.79)	0.00 (0.78)	<b>0.63</b> <b>(1.04)</b>
GBP/USD	0.00 (0.77)	0.00 (0.70)	0.01 (0.83)	0.00 (0.79)	0.00 (0.76)	0.00 (0.73)	<b>0.17</b> <b>(0.89)</b>
S&P 500	0.00 (0.65)	0.00 (0.70)	0.04 (0.87)	0.00 (0.66)	0.00 (0.61)	0.00 (0.64)	0.00 (0.77)
NASDAQ	0.00 (0.61)	0.00 (0.72)	0.00 (0.83)	0.00 (0.63)	0.00 (0.58)	0.00 (0.60)	0.00 (0.77)

The McNeil and Frey test (2000) are also used to test the  $ES_{0.025}$ . The Basel committee has been gearing to change the regulations to require banks to hold reserves equivalent to the  $ES_{0.025}$  instead of the  $Var_{0.01}$ . Therefore the 2.5<sup>th</sup> percentile is used for the expected shortfall. The McNeil and Frey (2000) test, tests whether the mean of the standardized residuals of the violations are equal to zero; against the alternative that it is less than zero.

$$H_0 : E\left(\frac{r_t - ES_{t,p}}{\sigma_t} \mid r_t < VaR_{p,t}\right) = 0$$

$$H_a : E\left(\frac{r_t - ES_{t,p}}{\sigma_t} \mid r_t < VaR_{p,t}\right) < 0$$

Table 2.6 presents the  $p$ -values of the McNeil and Frey test (2000), with the bootstrapped  $p$ -values in parenthesis. In all the cases studied both conditional non-parametric and conditional semi-parametric models produces expected shortfall estimates for which the mean of excess violation are not significantly different from zero. The  $p$ -values are also higher for the semi-parametric ES than for its conditional counterparts, in most cases. Besides historical simulation in all other models studied the mean of excess violations are less than zero.

Table 2.6:  $ES_{0.025}$  McNeil and Frey Test for Empirical Data

P-values (boot-p-values) McNeil and Frey test for $ES_{0.025}$							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
WMT	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	<b>0.06</b> <b>(0.13)</b>	0.00 (0.00)	<b>0.95</b> <b>(0.99)</b>	<b>0.93</b> <b>(0.98)</b>
MSFT	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.02)	0.00 (0.00)	<b>0.09</b> <b>(0.16)</b>	<b>0.61</b> <b>(0.58)</b>
BAC	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.08 (0.15)	0.00 (0.00)	<b>0.77</b> <b>(0.72)</b>	<b>0.97</b> <b>(0.92)</b>
YEN/USD	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	<b>0.06</b> <b>(0.13)</b>	0.00 (0.00)	<b>0.75</b> <b>(0.68)</b>	<b>0.90</b> <b>(0.82)</b>
GBP/USD	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.02 <b>(0.07)</b>	0.00 (0.00)	<b>1.00</b> <b>(0.99)</b>	<b>0.99</b> <b>(0.99)</b>
S&P 500	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.21 (0.30)	0.00 (0.00)	<b>0.94</b> <b>(0.87)</b>	<b>0.99</b> <b>(0.99)</b>
NASDAQ	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	<b>0.11</b> <b>(0.20)</b>	0.00 (0.00)	<b>0.99</b> <b>(0.98)</b>	<b>0.99</b> <b>(0.99)</b>

Most models studied in this chapter can produce  $VaR_{0.05}$  estimates that are greater than the realized returns in 5% of the cases. The unconditional non-parametric and conditional semiparametric  $VaR_{0.05}$  are however very conservative and has violations in less than 5% of the cases. This might be a desirable feature for regulators and investors who use the VaR measures for personal risk assessment, and would prefer to have as few violations as possible. Banks on the other hand that are trying to hold the smallest reserve that would allow them to abide by the regulations, might not find a conservative VaR desirable as it implies holding larger reserves than required by law. The Christoffersen and Pelletier (2004) test reveals that only the GARCH and the conditional semiparametric models'  $VaR_{0.05}$  estimates produce violations that are not dependent on each other. Repeated violations may have severe consequences for the financial asset holder.

At the 1% level the proportion of violations of the parametric and historical simulation models are significantly greater than 1%. Although the proportions of violations of the conditional semiparametric and unconditional non-parametric models'  $VaR_{0.01}$  estimates

are statistically close to 1%, only the violations from the conditional semiparametric are not dependent on each other. As most regulators require banks to report the  $VaR_{.01}$ , this is also empirically more relevant.

## 2.4 Simulation

The performance of the VaR and ES models also are evaluated in a controlled setting using the same tests in the previous section, but where the true data generating process (DGP) is known. 50 samples of size 7000 are drawn<sup>17</sup> from six alternate DGPs, three unconditional distributions and three conditional. The unconditional DGPs include Gaussian, Student- $t$  and Laplace distributions. The remaining three DGPs are from the GARCH family, namely ARCH (1), GARCH (1,1) described in (2.9) and (2.10); and Golsten *et al.* (1993) GJR GARCH, given in (2.15). The unconditional mean for all the DGPs are set to 0.0003 and the unconditional standard deviation to 0.00118, similar to S&P 500's sample statistics for the period March-11-1987 to February-2-2015. The parameters for the conditional model are set using the 'rugarch' package in R, to fit the sample statistics of the daily S&P 500 return series.

$$\hat{\sigma}_t^2 = \hat{\alpha} + \hat{\beta} (r_{t-1} - \hat{\mu}_t)^2 + \gamma(\hat{\sigma}_{t-1}^2) + \delta (r_{t-1} - \hat{\mu}_t)^2 I(r_{t-1} - \hat{\mu}_t \leq 0) \quad (2.15)$$

VaR and ES are estimated for each of the simulated samples, the estimates are evaluated using the Kupiec test (1995), Christoffersen and Pelletier's duration based test (2004) and McNeil and Frey's (2000) test. Table 2.7 and 2.8 presents the median *p-values* for the Kupiec test with the median number of violations and expected number of violations in the parenthesis. Each rows represents a DGP and each column the VaR estimation model used to estimate VaR, bold typeface indicates *p-values* larger than 5%, and that the test fails to reject the null that the proportion of violations are significantly different from 5% or 1%, with a confidence interval of 95%. Similar to the empirical results at the 5% level the non-parametric and semi-parametric models have fewer violations than expected, but

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<sup>17</sup>Monte Carlo Simulations have also been performed for 10.000 replications of sample size 100, and 100 replications of sample size 600. The sample sizes were too small to draw any meaningful comparison.

Table 2.7:  $VaR_{0.05}$  Kupiec Test for Simulated Data

Median P-values for $VaR_{0.05}$ Kupiec test (Actual Exceed). Expected Exceedances=337							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
Normal	<b>0.45</b> ( <b>323</b> )	0.00 (796)	<b>0.58</b> ( <b>328</b> )	<b>0.36</b> ( <b>354</b> )	0.00 (281)	0.01 (293)	0.02 (296)
Student-t	<b>0.20</b> ( <b>315</b> )	0.00 (889)	<b>0.15</b> ( <b>312</b> )	<b>0.08</b> ( <b>369</b> )	0.00 (277)	0.00 (280)	0.00 (283)
Laplace	0.00 (241)	0.00 (451)	0.00 (241)	<b>0.12</b> ( <b>366</b> )	<b>0.11</b> ( <b>309</b> )	<b>0.12</b> ( <b>310</b> )	<b>0.27</b> ( <b>318</b> )
ARCH	0.00 (214)	<b>0.07</b> ( <b>370</b> )	0.00 (234)	0.00 (234)	0.00 (237)	0.00 (247)	0.00 (270)
GARCH	0.00 (188)	0.00 (401)	<b>0.13</b> ( <b>365</b> )	0.00 (390)	0.00 (211)	0.00 (260)	0.00 (238)
GJR	0.00 (206)	0.00 (401)	0.03 (382)	0.00 (243)	0.00 (238)	0.04 (275)	<b>0.19</b> ( <b>361</b> )

at the 1% level the the conditional non-parametric and semi-parametric methods are better able to produce the expected number of violations.

The median results for the Christoffersen and Pelletier (2004) test are presented at table 2.9 and 2.10. In most cases the models produce violations that are independent of each other, this is not surprising as the DGPs are well behaved with no structural breaks. The McNeil and Frey (2000) test results presented in table 2.11 on the other hand clearly demonstrates that under all studied distributions the semiparametric model's predicted ES estimates are the closest to the observed mean of violations. The conditional non-parametric can produce such close estimates only under conditional DGPs.

The non-parametric and semiparametric models overestimate the risk at the 5% and has fewer violations than expected, in the 1% case this is no longer observed and the conditional non-parametric and semiparametric models produces the expected number of violations. The realized deviations from the predicted ES is also the smallest under the conditional semiparametric model than other models.

Table 2.8:  $VaR_{0.01}$  Kupiec Test for Simulated Data

Median P-values for $VaR_{0.01}$ Kupiec test (Actual Exceed) Expected Exceedances =67							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
Normal	<b>0.26</b> <b>(61)</b>	0.00 (526)	<b>0.56</b> <b>(68)</b>	0.01 (91/67)	0.04 (51)	<b>0.38</b> <b>(60)</b>	<b>0.67</b> <b>(64)</b>
Student-t	<b>0.35</b> <b>(60)</b>	0.00 (647)	<b>0.42</b> <b>(61)</b>	0.00 (96)	0.01 (47)	<b>0.11</b> <b>(55)</b>	<b>0.15</b> <b>(56)</b>
Laplace	0.01 (89)	0.00 (292)	0.07 (83)	0.00 (93)	<b>0.58</b> <b>(63)</b>	<b>0.59</b> <b>(72)</b>	<b>0.85</b> <b>(66)</b>
ARCH	0.00 (125)	<b>0.85</b> <b>(66)</b>	<b>0.24</b> <b>(70)</b>	0.01 (90)	0.02 (87)	<b>0.58</b> <b>(64)</b>	<b>0.51</b> <b>(73)</b>
GARCH	0.00 (105)	0.00 (246)	<b>0.95</b> (68)	0.00 (93)	0.00 (98)	<b>0.14</b> <b>(80)</b>	<b>0.23</b> <b>(58)</b>
GJR	0.00 (104)	0.00 (245)	0.00 (127)	0.00 (94)	<b>0.67</b> <b>(66)</b>	0.00 (102)	<b>0.07</b> <b>(83)</b>

Table 2.9:  $VaR_{0.05}$  Duration Test for Simulated Data

Median P-values (Weibull) Duration Based test for $VaR_{0.05}$							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
Normal	<b>0.35</b> <b>(1.04)</b>	0.00 (0.85)	<b>0.08</b> <b>(1.08)</b>	<b>0.13</b> <b>(1.06)</b>	<b>0.25</b> <b>(1.06)</b>	<b>0.39</b> <b>(1.04)</b>	<b>0.34</b> <b>(1.05)</b>
Student-t	0.00 (1.14)	0.00 (0.87)	0.00 (1.17)	0.00 (1.14)	0.00 (1.19)	0.00 (1.18)	0.00 (1.24)
Laplace	<b>0.73</b> <b>(0.98)</b>	0.00 (0.81)	<b>0.22</b> <b>(0.94)</b>	<b>0.21</b> <b>(1.05)</b>	<b>0.34</b> <b>(1.04)</b>	<b>0.14</b> <b>(1.07)</b>	<b>0.24</b> <b>(1.05)</b>
ARCH	0.00 (0.88)	<b>0.83</b> <b>(0.99)</b>	<b>0.46</b> <b>(1.04)</b>	<b>0.43</b> <b>(1.03)</b>	<b>0.56</b> <b>(1.03)</b>	<b>0.27</b> <b>(0.95)</b>	<b>0.53</b> <b>(1.03)</b>
GARCH	<b>0.05</b> <b>(0.90)</b>	0.00 (0.90)	<b>0.50</b> <b>(0.97)</b>	<b>0.57</b> <b>(1.02)</b>	<b>0.62</b> <b>(0.97)</b>	<b>0.14</b> <b>(0.93)</b>	<b>0.94</b> <b>(1.00)</b>
GJR	<b>0.11</b> <b>(0.92)</b>	0.00 (0.89)	<b>0.71</b> <b>(0.99)</b>	<b>0.55</b> <b>(1.01)</b>	<b>0.13</b> <b>(0.93)</b>	0.03 (0.91)	<b>0.82</b> <b>(0.99)</b>

Table 2.10:  $VaR_{0.01}$  Duration Test for Simulated Data

Median P-values (Weibull) Duration Based test for $VaR_{0.01\%}$							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
Normal	<b>0.47</b> ( <b>0.98</b> )	0.00 (0.67)	<b>0.58</b> ( <b>1.04</b> )	<b>0.47</b> ( <b>1.06</b> )	<b>0.47</b> ( <b>1.07</b> )	<b>0.43</b> ( <b>1.00</b> )	<b>0.54</b> ( <b>1.05</b> )
Student-t	<b>0.55</b> ( <b>1.06</b> )	0.00 (0.72)	<b>0.26</b> ( <b>1.12</b> )	<b>0.63</b> ( <b>1.04</b> )	<b>0.71</b> ( <b>1.04</b> )	<b>0.47</b> ( <b>1.08</b> )	<b>0.61</b> ( <b>0.95</b> )
Laplace	<b>0.97</b> ( <b>1.00</b> )	0.00 (0.69)	<b>0.98</b> ( <b>1.00</b> )	<b>0.70</b> ( <b>0.97</b> )	<b>0.62</b> ( <b>1.05</b> )	<b>0.50</b> ( <b>1.07</b> )	<b>0.89</b> ( <b>0.99</b> )
ARCH	0.00 (0.81)	<b>0.87</b> ( <b>1.02</b> )	<b>0.59</b> ( <b>0.97</b> )	<b>0.09</b> ( <b>1.15</b> )	<b>0.39</b> ( <b>1.09</b> )	<b>0.75</b> ( <b>1.03</b> )	<b>0.52</b> ( <b>1.06</b> )
GARCH	<b>0.61</b> ( <b>0.96</b> )	0.00 (0.84)	<b>0.49</b> ( <b>0.95</b> )	<b>0.18</b> ( <b>0.90</b> )	<b>0.97</b> ( <b>1.00</b> )	<b>0.01</b> ( <b>0.81</b> )	<b>0.96</b> ( <b>1.00</b> )
GJR	<b>0.20</b> ( <b>0.90</b> )	0.00 (0.82)	<b>0.52</b> ( <b>1.01</b> )	<b>0.27</b> ( <b>0.92</b> )	<b>0.53</b> ( <b>0.97</b> )	0.02 (0.84)	<b>0.59</b> ( <b>0.97</b> )

Table 2.11:  $ES_{0.025}$  McNeil and Frey Test for Simulated Data

Median p-values (boot-p-values) McNeil and Frey test for $ES_{0.025}$							
	Normal	ARCH	GARCH	HS	Uncond. NP	Cond. NP	Cond. SP
Normal	<b>0.74</b> ( <b>0.70</b> )	0.00 (0.00)	<b>0.64</b> ( <b>0.62</b> )	<b>0.09</b> ( <b>0.17</b> )	0.00 (0.00)	0.00 (0.03)	0.03 ( <b>0.09</b> )
Student-t	<b>0.41</b> ( <b>0.44</b> )	0.00 (0.00)	<b>0.52</b> ( <b>0.52</b> )	<b>0.05</b> ( <b>0.12</b> )	0.00 (0.00)	0.00 (0.00)	<b>0.52</b> ( <b>0.47</b> )
Laplace	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	<b>0.09</b> ( <b>0.18</b> )	0.00 (0.00)	0.00 (0.01)	<b>0.39</b> ( <b>0.41</b> )
ARCH	0.00 (0.00)	<b>0.10</b> ( <b>0.18</b> )	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	<b>0.91</b> ( <b>0.83</b> )	<b>0.35</b> ( <b>0.40</b> )
GARCH	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.03 ( <b>0.08</b> )	<b>0.49</b> ( <b>0.52</b> )	<b>0.84</b> ( <b>0.76</b> )
GJR	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	<b>0.06</b> ( <b>0.13</b> )	<b>1.00</b> ( <b>1.00</b> )	<b>0.99</b> ( <b>0.96</b> )

## 2.5 Conclusion

A new value at risk and expected shortfall estimators are introduced in this chapter, based on Mishra, Su and Ullah's (2010) semiparametric, conditional variance estimator. The semiparametric variance is a multiplicative estimator of a parametric conditional variance estimator, and the non-parametric conditional variance of the parametric model's residuals. This allows the user to enjoy the perks of both the parametric and the non-parametric models. It eliminates the need to identify the true parametric model, and worry about misspecification. In addition, as long as the parametric model can pick up some features of the true volatility, the non-parametric estimation becomes less strenuous than a full non-parametric model, and producing less bias. Value at risk models that use conditional variance estimators are better equipped to pick up the volatility clustering in financial series. The new estimator's performance are empirically tested against other popular VaR models, at the 1% and 5% level, and ES at the 2.5% level. At the 5% level the semiparametric model has lower violations than expected. Although this would imply it would rarely not meet the regulatory requirements, the opportunity cost might be high for some investors. The violations produced by the semiparametric model also do not follow any recognizable pattern for both the 1% and 5% percentiles. The expected shortfall estimated by the semiparametric model are also closest to the observed mean of the violations, than all other models studied. Tests performed of simulated data generated from unconditional and conditional distributions reach similar conclusions. Thus, the semiparametric VaR model produces less violations that do not follow a pattern; upholding the regulatory requirements and better able to avoid catastrophic losses.

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## **3 Chapter 3: Bias Reduction in Predictive Regression using Nonparametrics**

### **3.1 Introduction**

Predicting equity premium is one of the most studied topics in the finance literature. Reliable forecasts of stock returns have the potential to influence asset allocation decisions of an investor. From an economic viewpoint, fluctuations in the financial market can provide insights to the fluctuations in the real economy. These among many reasons explain the plethora of papers attempting to predict stock returns.

Given the noisy nature of stock return a sizable portion of the series tend to remain unpredictable, however based on in-sample tests there now seems to be consensus among the financial economists that the series do contain a significant predictable component (Campbell, 2000). Preliminary work done in this area involved using OLS regression of returns on lagged instrument variables that have predictive power over stock returns. Variables that are most commonly used are short-term interest rates, the dividend yield, the book-to-market ratio, and the earnings-price ratio (e.g. Fama and French, 1988; Pontiff and Schall, 1998; Ang and Bekaert, 2007; Lettau and Van Nieuwerburgh, 2008). Using bivariate predictive regression Goyal & Welch (2008) show that these predicting variables perform poorly, in comparison with historical average excess stock return in out of sample forecasts. Campbell & Thompson (2008) on the other hand, using a priori knowledge about the regression parameters, impose sign restrictions on the regression parameters; and show that many predicting variables have better out of sample performance than historical average return.

The non-robust results of return predictability may stem from the econometric methods in hand (Lamoureux & Zhou, 1996). Using a linear model when the true data generation process is non-linear may seriously undermine forecasts. Chen & Hong (2009) point out that linear models might not be appropriate to capture the movements in stock return and suggest using non-parametric regressions, which can capture the linearities and non-linearities in the data without imposing parametric restrictions. Their findings also show that semi-parametric methods tend to perform better than non-parametric methods.

In addition to possibilities of model misspecification predictive regressions used to forecast excess returns are notoriously well known for producing biased estimates due to the high degree of persistence in the dependent variables. To correct for this bias many methods have been explored. Stambaugh (1999) uses the analytical expression of the bias in univariate linear, popularly known as Stambaugh's bias, and corrects the biased estimates accordingly. Amihud and Hurvich (2004) propose using an augmented regression. Zhu (2013) introduced Moving-block Jackknife estimator to reduce the bias further, this process works for both single and multiple regressors. Campbell and Thompson's (2008) sign restriction model is also an attempt to correct for this bias.

Bates & Granger's 1969 seminal paper where they show weighted average of forecasts from different models produces better forecast than an individual model, inspired many alternative forecast combination models. One of which is the complete subset regression (Elliott, Gargano, & Timmermann, 2013) where forecasts are weighted average of the forecasts from all possible combination of linear regression models for a fixed number of regressors in a set of predictive variables. Jin, Su, & Ullah (2013) also built combination forecast using non-parametric and semi parametric methods and block bootstrap, popularly known as bagging, where the forecasts are done using blocks of the data. These non-parametric models are further extended by Lee, Tu, & Ullah (2014) who incorporate sign restrictions in addition to bagging.

The analytical expression of bias derived by Stambaugh (1999) holds only when the dependent variable is stationary and under normality. Both stationarity of predictive variables and normality in error terms are strong assumptions in models of excess return (Roll, 2002). Torous, Valkanov and Yan (2004) find the presence of unit root in almost all commonly used predictive variables, within a 95% confidence interval. In pre 1926 and post 1994 data Torous, Valkanov, & Yan's (2004) tests indicate the presence of unit root in dividend yield and when dividend yield from those sub-periods are used to predict stock excess return, the predictive power is lost. Thus, the presence of unit root in predictive variables might explain why in certain cases they are found to have predictive power and not in other cases.

In this chapter two step non-parametric and semi parametric methods, which estimate the conditional mean and the residuals separately are used to predict excess stock return both in sample and out of sample. The empirical performance of the proposed models are compared with the historical mean model, simple OLS model, local constant and local linear non-parametric models, on the basis of the root mean squared (forecast) errors. Analysis is performed using Goyal and Welch's (2008) original data till 2005 and using the extended data till 2015<sup>18</sup>.

### 3.2 Literature Review

Prior to the late twentieth century the consensus in the finance literature was that excess stock returns were entirely unpredictable (Fama, 1970), attributing to the efficient market hypothesis. However, towards the end of the century, numerous studies came out that believed otherwise; several variables were found to have predictive power over excess stock return. Fama and French (1988b) and Poterba and Summers (1988) find that the statistical significance of their univariate model using only past returns improve greatly when predictive variables are added to the model. Among many economic variables that are found to have predictive powers, the most notable are short term interest rates (Fama E. S., 1977), yield spreads (Campbell J. Y., 1987), stock market volatility (Goetzmann & Santa-Clara, 2003), book-to-market ratios (Ponti and Schall, 1998), and price-earnings ratios (Lamont, 1998; Campbell and Shiller 1988), dividend-price ratio (Campbell and Shiller, 1988; Fama and French, 1988; Lettau and Van Nieuwerburgh, 2008).

Despite, evidence of predictability within in sample models, Bossaerts & Hillion (1999) and Goyal and Welch (2008) find the out of sample performance for these predictive variables to be poor. Goyal and Welch (2008) find the historical average return outperforms different predictive variables in terms of mean squared forecast error. Campbell and Thompson (2008) on the other hand, find that many of the variables in Goyal and Welch's (2008) study do indeed beat the historical average. Campbell and Thompson (2008) impose a sign restriction

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<sup>18</sup>Data is collected from Amit Goyal's website

on parameters of a linear forecasting model to reconcile the in-sample and out of sample performance of predictors.

Controversy surrounding the out of sample performance of the predictive variables cast doubt over the predictive ability of these variables. Whether, the contradicting results are due to model misspecification pose even serious concern. Therefore, Chen and Hong (2009) propose using non parametric and semi parametric models that impose no or very little parameter restrictions and are more capable of capturing linearities and nonlinearities in the data. According to Chen and Hong (2009) the restrictions imposed by Campbell and Thomson are ways of introducing non-linearity into the model, they too like the latter find predictive variables to outperform historical average in a non-parametric setting. Parametric and non-parametric forecast combination models also reach similar conclusion (Elliott et. al,2013; Jin et. al, 2013). Lee, Tu and Ullah (2014) use bootstrap aggregating and monotonicity constraints (sign restrictions) in a non-parametric setting and they too find predictive variables to outperform the historical average return, using second order stochastic dominance they also show that nonparametric and semiparametric models improve the statistical significance of predictive variables over their linear counterparts.

Another plausible reason of contradicting results on out-of sample predictive ability of variables noted as predictive variables in the literature is due to the non-stationarities in the explanatory variables. Roll (2002) argues that in the presence of rational expectation, if the innovations are identically and independently distributed then the expectation about a future quantity must follow a random walk. Stock prices are based on expectation about a future quantity, and explanatory variables like dividend yield and book to market ratio are in turn functions of stock prices. Thus, these explanatory variables must also follow a random walk. Unbalanced predictive regression of stationary stock return and non-stationary dividend yield may lead one to conclude that dividend yield has no predictive power. Given the poor power of unit root tests to distinguish between cases with near unit root and unit root Torous, Valkanov and Yan (2004) construct a confidence band to test the presence of unit root. Structural breaks might also be present in the data, for instance Fama and

French (2001) have pointed out a dramatic fall in the proportion of firms paying dividends in the late 1970s. If not careful these structural breaks might be incorrectly categorized as non-stationarity, therefore Torous, Valkanov and Yan's (2004) test also accommodates presence of structural breaks. Apart from the term spread prior to 1952 and dividend yield in the period 1926 to 1994, they find the presence of unit root in all popular predictive variables. Using international data Torous, Valkanov and Yan (2004) show that when dividend to price ratio is stationary it has predictive power and not when it is non-stationary. Therefore, due to the possibility of nonlinear relationship between excess stock return and predictive variables, and nonstationarities in the predictive variables this chapter proposes using nonparametric and semiparametric models.

### 3.3 Predictive Regressions and Biases

#### OLS

Preliminary studies use a linear regression to predict excess return using other financial variables and their lags, that tend to move with excess return, such a model is shown by (3.1), where,  $r_t$  is the excess return and  $x_{t-1}$  are lagged explanatory variables. The parameters of the simple OLS regression are estimated by (3.2), where, the  $t^{th}$  row of matrix  $X$  and vector  $R$  are  $(1, x_{t-1})$  and  $(r_t)$ , respectively, and the predicted return,  $\hat{r}_{t,OLS}$  is given by (3.3)

$$r_t = \alpha + \beta x_{t-1} + u_t \quad (3.1)$$

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = (X'X)^{-1}X'R \quad (3.2)$$

$$\hat{r}_{t,OLS} = \hat{\alpha} + \hat{\beta}x_{t-1} \quad (3.3)$$

OLS estimates are unbiased if all the information in  $x_{t-1}$  has been used to predict  $r_t$ . As most financial variables are highly persistent, there are information about the lags in  $x_{t-1}$

that are not independent of  $u_t$ . For instance if the predicting variable,  $x_{t-1}$ , follows an AR(1) process like (3), then  $E(x_{t-1}|u_t) \neq 0$ . If  $x_{t-1}$  is persistent the error terms in (3.1) and (3.3) are not independent of each other and can be expressed using (4), where  $\xi \neq 0$  and  $\varepsilon_t$  are *i.i.d.* errors that are independent of  $v_t$  and its lags. Thus, a simple OLS with autoregressive predicting variables will result in biased estimates.

$$x_t = \phi + \rho x_{t-1} + v_t \quad (3.4)$$

$$u_t = \xi v_t + \varepsilon_t \quad (3.5)$$

### Historical Average

Goyal and Welch (2008) compare the simple OLS predicted returns with the historical average (HA) returns shown in (3.6), the predicted returns are the average of the past realized returns.

$$\hat{r}_{t,HA} = \frac{1}{t-1} \sum_{i=1}^{t-1} r_i \quad (3.6)$$

### Stambaugh's bias

The difference between the OLS estimates of  $\hat{\beta}$  and  $\beta$  can be expressed using (3.7), where  $\bar{x}$  is the sample mean,  $\bar{x} = \sum_{t=1}^T x_t/T$ .

$$\hat{\beta} - \beta = \frac{\sum_{t=1}^T (x_{t-1} - \bar{x}) u_t}{\sum_{t=1}^T (x_{t-1} - \bar{x})^2} \quad (3.7)$$

Rearranging (3.3) to  $v_t = x_t - \phi - \rho x_{t-1}$ , and substituting  $E(u_t | v_t) = \xi v_t$  in (3.7) results in (3.8).

$$E(\hat{\beta}) - \beta = \xi E \left\{ \frac{\sum_{t=1}^T (x_{t-1} - \bar{x}) E(x_t | v_t)}{\sum_{t=1}^T (x_{t-1}^2 - \bar{x}^2)} - \rho \right\} \quad (3.8)$$



Using the OLS estimate of  $\hat{\rho}$  the bias of  $\hat{\beta}$  can be expressed as a function of the bias in  $\hat{\rho}$

$$E(\hat{\beta}) - \beta = \xi(E(\hat{\rho}) - \rho) \quad (3.9)$$

Marriott and Pope (1954) expressed the bias of  $\hat{\rho}$  in an AR(1) process under normality as follows:

$$E(\hat{\rho}) - \rho = (-(1 + 3\rho)/T + O(1/T^2)) \quad (3.10)$$

The bias of  $\hat{\beta}$  can thus be expressed as (3.11)

$$E(\hat{\beta}) - \beta = \xi(-(1 + 3\rho)/T + O(1/T^2)) \quad (3.11)$$

This is most popularly known as Stambaugh's bias and is used primarily to adjust the biased OLS estimates and the process itself is the plug-in method, where like the name suggests the bias is plugged into the OLS estimate. This is however, only applicable for univariate models with  $\rho < |1|$ . Kiviet and Phillips (2005) on the other hand, provide approximation for unit root case.

### **Non-parametric**

Instead of assuming the data generation process, like a linear model shown in (3.1) the local constant non-parametric model lets the functional form be expressed as  $m(x_{t-1})$  as shown in (3.12).

$$r_t = m(x_{t-1}) + u_t \quad (3.12)$$

For a discrete random  $x_{t-1}$  there are  $n^*$  observations in its neighborhood, let them be  $x$ ,  $m(x_{t-1})$  is the average of the  $r_t$ 's corresponding to the  $x$ 's (Pagan & Ullah, 1999).  $h$  is the window width that determines the size of the neighborhood of  $x_{t-1}$  that will be used to find  $m(x_{t-1})$ .

$$\hat{m} = \frac{\sum_{t=1}^T I(-.5 < \psi_{t-1} < .5)r_t}{\sum_{t=1}^T I(-.5 < \psi_{t-1} < .5)} \quad (3.13)$$

where,  $\psi_{t-1} = (x - x_{t-1})/h$ . A kernel function  $K$  can be used to smooth.

$$\hat{m} = \frac{\sum_{t=1}^T K(\psi_{t-1})r_t}{\sum_{t=1}^T K(\psi_{t-1})} \quad (3.14)$$

While Local constant minimizes  $\sum_{t=1}^T [r_t - m]^2 K(\psi_{t-1})$  with respect to  $m$ ; local linear minimizes  $\sum_{t=1}^T [r_t - m - (x_{t-1} - x)\beta]^2 K(\psi_{t-1})$ .

### Model 1: A two step semi-parametric model

Excess stock returns are predicted using a combination of linear and non-linear models. Any linear relationship between the excess stock return and the predictive variable is first captured using an OLS regression as (1). Any remaining non-linearities and the endogeneity between  $x_{t-1}$  and  $u_t$  are then addressed by non-parametrically estimating the residuals of (3.1),  $u_t$ , using the residuals of the AR(1) process of  $x_{t-1}$ ,  $v_t$ . After running the OLS regressions (3.1) and (3.3) the residuals are saved and used in the estimation shown in equation (3.15). The estimated values of  $\hat{u}_{t,SP} = m(\hat{v}_t)$  are then used to update equation (3.1). The predicted excess stock return,  $\hat{r}_{t,SP}$ , are a sum of the predicted excess return from the OLS model in (3.1) and the predicted residual in (3.15). The linear prediction is thus re-scaled for additional non-linearities.

$$u_t = m(v_t) + \varepsilon_t \quad (3.15)$$

$$\hat{r}_{t,SP} = \hat{\alpha}_{OLS} + \hat{\beta}_{OLS}x_{t-1} + \hat{u}_{t,SP} \quad (3.16)$$

### Model 2: A two step non-parametric model

A two step non-parametric model is similar to the previous model discussed, except (3.1) and (3.3) are replaced with non-parametric regressions. Step 1: Excess stock returns are regressed on the predictive variables using non-parametric regressions as in (3.17) and

the residuals,  $\hat{u}_{t,NP}$  are saved. Step 2: Residuals of a non-parametric AR (1) process of  $x_{t-1}$  described in (3.18) are saved. Step 3:  $\hat{u}_{t,NP}$  is regressed on  $\hat{v}_{t-1,NP}$ , non-parametrically as in (3.19). Step 4: Excess Stock return are predicted as the sum of the predicted values of (3.17) and (3.18). An across the board non-parametric model addresses not only any non-linear relationship between excess stock return and the predictive variable, but also any non-linear relationship the predictive variable may have with its own past.

$$r_{t,NP} = m(x_{t-1}) + u_{t,NP} \quad (3.17)$$

$$\hat{u}_{t,NP} = r_t - \hat{m}(x_{t-1})$$

$$x_{t,NP} = m_1(x_{t-1}) + v_{t,NP} \quad (3.18)$$

$$\hat{v}_{t,NP} = x_t - \hat{m}_1(x_{t-1})$$

$$\hat{u}_{t,NP} = m_2(\hat{v}_{t-1,NP}) + \epsilon_{t,NP} \quad (3.19)$$

$$\hat{r}_{t,NPP} = \hat{m}(x_{t-1}) + \hat{m}_2(\hat{v}_{t-1,NP}) \quad (3.20)$$

$$\hat{r}_{t,NPP} = \hat{r}_{t,NP} + \hat{u}_{t,NPP}$$

In the next section the predictive performance in sample and out of sample of the two proposed models are compared with the historical average, OLS and non-parametric regressions, for the predictive variables used in Goyal and Welch (2008) and Campbell and Thompson (2008).

### 3.4 Empirical Results

Annual S&P 500 Index return in excess of the risk free return are predicted using the past average, and the predictive variables used by Goyal and Welch (2008). The prediction methods studied include the historic average<sup>19</sup>, OLS regression model in (3.1), non-parametric regression (NP) as in (3.12), proposed two step semiparametric (two step SP) and nonparametric models (two step NP). Table 3.1 presents the In Sample (IS) root mean squared error

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<sup>19</sup>Predicted excess stock return = sample average of past returns

for the five models in predicting the S&P 500 excess return for the years 1872-2005<sup>20</sup>, both local constant (LC) and local linear (LL) regressions are used non-parametric regressions. Bold typeface in each row indicates the model with the lowest RMSE. Column 2 reports the start year of the sample, the end year for all samples is 2005. The one-lag autocorrelation of the independent variable,  $\rho$ , is also presented in column 3. Apart from long term yield, the two step semiparametric/nonparametric models perform just as well if not better than the OLS and non-parametric model. Overall the two-step non-parametric model has the most number of cases with the lowest RMSE. The historical average is beaten by the non-parametric methods in all cases. It is also to be noted that local linear regressions are relatively better in most cases than their local constant counterparts in predicting excess stock returns.

Similar to Table 3.1, the out of sample RMS(F)E of the aforementioned models are presented in Table 3.2. Rolling expanding window is used for estimation, with the first sample using 20 years or data. The estimated model is used to forecast the one year ahead excess S&P 500 return. The bold typeface here too indicates the model with the lowest RMSE for respective predictive variables. Like Goyal and Welch (2008), the historic model tends to beat the other models in out of sample analysis. In almost all cases a nonparametric or semiparametric model produces lower RMSE than the OLS model. In out of sample local constant regressions tend to produce lower forecast errors than corresponding local linear models.

In sample and out of sample performance of the models using the extended data till 2015 are presented in Table 3.3 and Table 3.4, respectively. In sample the historical models and OLS model are beaten by nonparametric and semiparametric methods. Local linear models outperform local constant in sample, while the opposite holds true out of sample. The two step non parametric model continues to dominate the compared models in terms of lower RMSE in sample. The models studied do not out-perform the historical average even in the extended period.

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<sup>20</sup>Start date for the samples may differ due to the availability of data of the predictive variables

Table 3.1: In sample RMSE for years 1872- 2005

		In Sample								
	Start	$\rho$	Hist.	OLS	Two Step SP		NP		Two Step NP	
					LC	LL	LC	LL	LC	LL
Default Yield Spr.	1920	0.80	0.186	0.186	0.186	0.176	0.186	0.171	0.186	<b>0.169</b>
Inflation	1920	0.58	0.186	0.186	0.186	0.186	0.186	0.186	0.186	0.186
Stock Variance	1886	0.69	0.180	0.181	0.171	<b>0.165</b>	0.177	0.176	0.177	0.172
Dividend Payout	1873	0.69	0.178	0.178	0.178	0.178	0.178	0.177	0.178	<b>0.166</b>
Long Term Yield	1920	0.96	0.186	0.185	0.184	0.183	0.186	0.185	<b>0.167</b>	0.183
Term Spread	1921	0.60	0.187	0.186	0.186	<b>0.184</b>	0.187	0.185	0.187	<b>0.184</b>
Treasury-bill rate	1921	0.89	0.186	<b>0.185</b>	<b>0.185</b>	<b>0.185</b>	0.187	<b>0.185</b>	0.186	<b>0.185</b>
Default ret. spr.	1927	-0.34	0.190	0.189	<b>0.188</b>	<b>0.188</b>	0.190	<b>0.188</b>	0.189	<b>0.188</b>
Dividend/Price	1873	0.86	0.178	0.176	<b>0.171</b>	<b>0.171</b>	0.173	0.174	0.173	<b>0.171</b>
Dividend Yield	1873	0.92	0.178	0.176	0.175	0.174	0.174	0.173	0.172	<b>0.171</b>
Long term return	1927	-0.08	0.190	0.188	0.188	<b>0.183</b>	0.188	0.188	0.188	<b>0.183</b>
Earning/price	1873	0.73	0.178	0.176	0.176	<b>0.175</b>	0.177	0.176	0.176	<b>0.175</b>
Book/market	1922	0.83	0.187	0.183	<b>0.162</b>	0.175	0.185	0.183	<b>0.160</b>	0.173
Investment/cap.	1948	0.72	0.159	<b>0.152</b>	<b>0.152</b>	<b>0.152</b>	0.154	<b>0.152</b>	0.154	<b>0.152</b>
Net equity exp	1928	0.46	0.189	0.177	0.177	<b>0.149</b>	0.171	0.168	0.171	0.164
Pct equity	1928	0.49	0.189	0.178	0.178	0.178	0.170	<b>0.169</b>	0.170	<b>0.169</b>
Consumption	1946	0.57	0.156	0.143	0.143	0.126	0.114	0.120	<b>0.103</b>	0.117
Dividend yield	1928	0.93	0.189	0.186	0.186	0.184	0.179	0.179	0.179	<b>0.178</b>
Earning/price	1928	0.78	0.189	0.184	0.184	0.174	0.185	0.176	<b>0.160</b>	0.166
Book/market	1928	0.83	0.189	0.183	<b>0.159</b>	0.183	0.185	0.183	0.160	0.183

Table 3.2: Out of sample RMSE for years 1872- 2005

	Start	$\rho$	Out of Sample							
			Hist.	OLS	Two Step SP		NP		Two Step NP	
					LC	LL	LC	LL	LC	LL
Default Yield Spr.	1920	0.80	<b>0.158</b>	0.160	0.162	0.240	<b>0.158</b>	0.164	0.159	0.181
Inflation	1920	0.58	<b>0.158</b>	0.160	<b>0.158</b>	0.160	0.182	0.232	0.182	0.239
Stock Variance	1886	0.69	<b>0.193</b>	0.216	0.237	0.655	0.205	0.212	0.208	0.213
Dividend Payout	1873	0.69	<b>0.185</b>	0.188	0.190	0.210	0.186	0.190	0.195	0.191
Long Term Yield	1920	0.96	<b>0.159</b>	0.164	0.169	0.221	0.167	0.211	0.174	0.214
Term Spread	1921	0.60	<b>0.158</b>	0.159	0.159	0.184	0.161	0.171	0.164	0.160
Treasury-bill rat	1921	0.89	<b>0.158</b>	0.160	0.167	0.165	0.160	0.168	0.163	0.177
Default ret. spr.	1927	-0.34	0.159	<b>0.159</b>	0.159	0.165	0.160	0.168	0.160	0.168
Dividend/Price	1873	0.86	<b>0.185</b>	0.186	0.186	0.187	0.187	0.190	0.188	0.191
Dividend Yield	1873	0.92	<b>0.185</b>	0.186	0.186	0.191	0.187	0.194	0.187	0.194
Long term return	1927	-0.08	<b>0.159</b>	0.164	0.164	0.169	0.161	0.168	0.162	0.166
Earning/price	1873	0.73	<b>0.185</b>	0.186	0.191	0.224	0.192	0.200	0.192	0.204
Book/market	1922	0.83	0.159	0.159	0.161	0.161	0.158	0.159	<b>0.156</b>	0.170
Investment/cap.	1948	0.72	0.166	<b>0.162</b>	<b>0.162</b>	0.165	0.165	<b>0.162</b>	0.165	<b>0.162</b>
Net equity exp	1928	0.46	0.162	0.165	0.165	0.188	0.161	0.377	<b>0.160</b>	0.375
Pct equity	1928	0.49	0.162	<b>0.158</b>	<b>0.158</b>	0.159	<b>0.158</b>	<b>0.158</b>	0.159	0.162
Consumption	1946	0.57	0.161	0.145	0.150	0.139	0.152	<b>0.136</b>	0.155	0.142
Dividend yield	1928	0.93	<b>0.162</b>	0.171	0.171	0.189	0.165	0.178	0.165	0.177
Earning/price	1928	0.78	0.162	0.158	0.158	<b>0.156</b>	0.160	0.159	0.164	0.160
Book/market	1928	0.83	<b>0.162</b>	0.174	0.173	0.180	0.164	0.191	0.164	0.189

Table 3.3: In sample RMSE for years 1872- 2015

		In Sample								
		Hist.	OLS	Two Step SP		NP		Two Step NP		
	Start	$\rho$			LC	LL	LC	LL	LC	LL
Default Yield Spr.	1920	0.80	0.186	0.185	0.185	<b>0.184</b>	0.186	0.171	0.185	0.171
Inflation	1920	0.58	0.186	0.186	0.186	0.185	0.186	0.186	<b>0.166</b>	0.185
Stock Variance	1886	0.69	0.181	0.181	0.171	<b>0.159</b>	0.181	0.180	0.181	0.170
Dividend Payout	1873	0.69	0.178	0.178	0.172	0.175	0.178	0.178	0.177	<b>0.168</b>
Long Term Yield	1920	0.96	0.186	0.185	0.185	0.184	0.183	0.184	<b>0.182</b>	<b>0.182</b>
Term Spread	1921	0.60	0.186	0.185	<b>0.184</b>	0.185	0.185	0.185	0.186	<b>0.184</b>
Treasury-bill rat	1921	0.89	0.186	<b>0.185</b>	0.170	0.171	0.186	<b>0.185</b>	0.186	0.171
Default ret. spr.	1927	-0.34	0.189	0.189	0.189	<b>0.186</b>	0.189	0.189	0.189	<b>0.186</b>
Dividend/Price	1873	0.86	0.178	0.177	0.173	<b>0.172</b>	0.178	0.177	0.178	0.174
Dividend Yield	1873	0.92	0.178	0.177	0.175	0.175	0.176	0.175	0.173	<b>0.172</b>
Long term return	1927	-0.08	0.189	0.188	0.188	<b>0.182</b>	0.189	0.188	0.189	0.188
Earning/price	1873	0.73	0.178	0.177	0.177	0.177	0.178	0.177	0.178	<b>0.175</b>
Book/market	1922	0.83	0.186	0.183	0.164	0.175	0.185	0.183	<b>0.161</b>	0.175
Investment/cap.	1948	0.72	0.162	<b>0.154</b>	<b>0.154</b>	<b>0.154</b>	<b>0.154</b>	<b>0.154</b>	<b>0.154</b>	<b>0.154</b>
Net equity exp	1928	0.46	0.188	0.180	0.177	0.162	0.178	0.155	0.178	<b>0.154</b>
Pct equity	1928	0.49	0.188	0.181	0.181	0.181	<b>0.173</b>	0.174	0.174	<b>0.173</b>
Consumption	1946	0.57	0.160	0.155	0.150	0.147	0.157	0.149	0.157	<b>0.144</b>
Dividend yield	1928	0.93	0.188	0.186	0.186	0.184	0.181	0.181	<b>0.168</b>	0.180
Earning/price	1928	0.78	0.188	0.186	0.186	0.186	0.187	0.185	<b>0.174</b>	0.179
Book/market	1928	0.83	0.188	0.183	<b>0.160</b>	0.175	0.185	0.183	0.167	0.174

Table 3.4: Out of sample RMSE for years 1872- 2015

	Start	$\rho$	Out of Sample							
			Hist.	OLS	Two Step SP		NP		Two Step NP	
					LC	LL	LC	LL	LC	LL
Default Yield Spr.	1920	0.80	<b>0.162</b>	0.163	0.165	0.235	<b>0.162</b>	0.166	<b>0.162</b>	0.182
Inflation	1920	0.58	0.163	0.164	<b>0.162</b>	0.164	0.179	0.229	0.179	0.235
Stock Variance	1886	0.69	<b>0.192</b>	0.213	0.220	0.270	0.212	0.281	0.214	0.285
Dividend Payout	1873	0.69	<b>0.186</b>	0.188	0.190	0.232	<b>0.186</b>	0.190	0.194	0.195
Long Term Yield	1920	0.96	<b>0.162</b>	0.166	0.171	0.217	0.168	0.211	0.180	0.216
Term Spread	1921	0.60	<b>0.162</b>	<b>0.162</b>	<b>0.162</b>	0.164	0.164	0.163	0.165	<b>0.162</b>
Treasury-bill rat	1921	0.89	<b>0.162</b>	0.163	0.167	0.183	0.163	0.169	0.166	0.173
Default ret. spr.	1927	-0.34	<b>0.163</b>	0.166	0.166	0.211	0.164	0.176	0.165	0.178
Dividend/Price	1873	0.86	<b>0.186</b>	0.186	0.187	0.187	0.188	0.191	0.189	0.192
Dividend Yield	1873	0.92	<b>0.186</b>	0.187	0.187	0.191	0.188	0.195	0.188	0.195
Long term return	1927	-0.08	<b>0.163</b>	0.168	0.170	0.175	0.165	0.172	0.166	0.170
Earning/price	1873	0.73	<b>0.186</b>	0.187	0.191	0.222	0.192	0.199	0.191	0.202
Book/market	1922	0.83	0.162	0.163	0.168	0.169	0.162	0.163	<b>0.161</b>	0.174
Investment/cap.	1948	0.72	0.171	0.164	0.164	0.168	0.167	0.164	0.167	<b>0.163</b>
Net equity exp	1928	0.46	<b>0.165</b>	0.174	0.191	0.194	0.192	0.188	0.191	0.201
Pct equity	1928	0.49	0.165	0.166	0.166	0.167	<b>0.164</b>	0.165	<b>0.164</b>	0.169
Consumption	1946	0.57	0.167	0.164	0.166	<b>0.161</b>	0.176	0.166	0.176	0.170
Dividend yield	1928	0.93	<b>0.165</b>	0.174	0.174	0.188	0.170	0.181	0.170	0.181
Earning/price	1928	0.78	<b>0.165</b>	<b>0.165</b>	<b>0.165</b>	0.611	0.166	0.187	0.168	0.188
Book/market	1928	0.83	<b>0.165</b>	0.177	0.174	0.175	0.167	0.189	0.167	0.185

While the predictive variables are able to produce better estimates than the historical average in sample, out of sample the predictive power is lost when OLS, non-parametric regression, two step semiparametric and non-parametric models are used, for most variables. Two-step non-parametric model outperforms the other models compared in IS analysis. Local linear models tend to do better IS compared to local constant, whereas OOS local constant produces lower forecast errors. Similar results are obtained using the extended data till 2015.



### 3.5 Conclusion

Predictability of stock return is an elusive subject, and whether certain variables have predictive power over stock return have yet to cease the interest of many academics and practitioners. The presence of high autocorrelation in the predictive variables and possible non-linearities in their relationship with stock return, further complicates the matter. In order to address the possible non-linearity and endogeneity between the residuals due to the persistent independent variables in the predictive regression, two step semiparametric and non-parametric methods are proposed, where the conditional mean and the residuals are estimated separately, and added to obtain the predicted excess stock return. Using Goyal and Welch's (2008) predictive variables, the proposed models particularly the two step non-parametric model, produces better estimates of the excess S&P 500 return in sample than the historical average and OLS regression. Out of sample however, the historical average continues to dominate OLS, non-parametric and the proposed models. Jin, Su, & Ullah (2013) and Lee, Tu, & Ullah (2014) have found that non-parametric and semiparametric bagging and sign restrictions however beat the historic model in out of sample. The asymptotic theory for the proposed two step models would be subject of future study, in addition to incorporating bagging and sign restrictions to the proposed models.

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## 4 Conclusion

A new uncertainty index is introduced which measure the overall level of uncertainty in the U.S. stock market. The index is further adjusted for business cycle shocks to capture the non-fundamental uncertainties. The uncertainty index rises prior to major fiscal and monetary policy announcements, FOMC meetings, and political elections; and during periods of heightened geopolitical risks. A non-linear relationship between the level of uncertainty in the stock market and the business cycle is uncovered, which indicates that uncertainty does not only rise when there is a negative shock to the business cycle but also when there are positive shocks to the business cycle. Additionally, distinct reactions of stock prices and returns to fundamental and non-fundamental shocks are observed. While fundamental shocks have a small but prolonged impact on stock prices, non-fundamental shocks have a large but short-lived impact.

New semiparametric Value at Risk and Expected Shortfall estimators are introduced. At the 5% level the semiparametric model has lower violations than expected, which is desirable for investors that want to avoid risk. Moreover, the 1% VaR reported by banks, produces statistically the correct number of violations, allowing banks to hold just enough reserves to comply with the regulations. The violations produced by the semiparametric model also do not follow any recognizable pattern, thus reducing the chances of bankruptcy or severe liquidity constraints due to repeated losses that are greater than the VaR estimates. The expected shortfall estimated by the semiparametric model are also close to the observed mean of the violations

In order to address the possible non-linear relationship between excess stock returns and its predictive financial variables, and potential endogeneity bias due to the high persistence in the predictive variables, two step semiparametric and non-parametric methods are proposed to predict excess stock return. The proposed models particularly the two step non-parametric model, produces better estimates in sample than the historical average and OLS . Out of sample the historical average continues to dominate. Future work done in this

area will include introducing bagging and sign restrictions to the proposed models, along with their asymptotic theory.