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Evaluating the coherence of Take-the-best in structured environments

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Abstract

Heuristic decision-making models, like Take-the-best, rely on environmental regularities. They conduct a limited search, and ignore available information, by assuming there is structure in the decision-making environment. Take-the-best relies on at least two regularities: *diminishing returns*, which says that information found earlier in search is more important than information found later; and *correlated information*, which says that information found early in search is predictive of information found later. We develop new approaches to determining search orders, and to measuring cue discriminability, that make the reliance of Take-the-best on these regularities clear, and open to manipulation. We then demonstrate, in the well-studied German cities environment, and three new city environments, when and how these regularities support Take-the-best. To do this, we focus not on the accuracy of Take-the-best, as most previous studies have, but on a measure of its coherence as a decision-making process. In particular, we consider whether Take-the-best decisions, based on a single piece of information, can be justified because an exhaustive search for information is unlikely to yield a different decision. Using this measure, we show that when the two environmental regularities are present, the decisions made by limited search are unlikely to have changed after exhaustive search, but that both regularities are often necessary.

Keywords: Take-the-best, process coherence, environment structure, fast and frugal heuristics, diminishing returns.

1 Introduction

The 1992 Olympics was the first time professionals from the US NBA league were allowed to play in the men's basketball competition. The US "Dream Team", filled with stars like Michael Jordan, Magic Johnson, Larry Bird, Charles Barkley and Patrick Ewing, was one of the most dominant teams ever assembled for any sporting competition. Their closest game was a 117–85 victory in the final over Croatia, and head coach Chuck Daly never felt the need to call a timeout during the tournament.

Making predictions about the outcomes of sporting contests is notoriously difficult, but the Dream Team made some predictions easy. Imagine trying to predict whether or not the US would beat its first opponent in the tournament, Angola, and examining the players in each team, beginning with the starting five, and moving to the bench players. At some point early in the US list—maybe after Jordan, Johnson, Bird, Barkley and Ewing—there would be no need to look further. No matter who else was on the US roster, or the Angolan roster, the outcome is already clear. The Dream Team also made predictions easy during the course of games. With about 5 minutes to play in the first half against Angola, the US led 45 to 8. It was clear the US would win by a large margin, without needing to watch the rest of the game.

Both of these decisions about a US victory are *non-compensatory*, because not all of the available information is used, and so the ignored information cannot compensate for—that is, change the decision based on—the information that is used. The remaining player rosters are not examined, and the rest of the game is not watched. Yet the decision to forego further information seems sensible in both cases. It is not a reaction to limited time or cognitive resources, but a recognition of the nature of the environment in which decision-making is taking place. The first few US players are so good that there are no other players who could lead Angola to victory, and the first half performances of the teams are highly predictive of second half performances.

1.1 Fast and frugal heuristics

The adaptive value of non-compensatory decisions lies at the heart of the "fast and frugal" approach to modeling cognition (Gigerenzer & Goldstein, 1996; Gigerenzer, Todd, & the ABC Research Group, 1999). This approach has developed simple and effective heuristic models of human judgment and choice, built around two com-

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pelling ideas. The first idea is that decision-making needs to be *fast*, because the world is competitive. A basic reason for making decisions is to acquire resources, which are often scarce, and often contested. Making quick decisions usually offers a competitive advantage in these situations.

The second idea is that good decision-making mechanisms should be *frugal*, in the sense of being simple, because the world is changeable. A complicated decision making strategy will usually work well in the environment within which it was developed, but will often fail in new or altered environments. Decision making that over-fits, in this sense, is prone to failure when asked to generalize. Because real-world decision environments are continually changing, both quickly and slowly, and along many dimensions, simple decision making strategies that focus on the few stable features of the environment are likely to be the ones that succeed. Simplicity makes these decision strategies robust, just as a machine with few moving parts is unlikely to break.

As Gigerenzer et al. (1999) emphasize, neither of these motivating principles are about cognitive limitations. Fast and frugal heuristics are effective to the extent they seize on the opportunities presented by environmental regularities. They are rooted in properties of environments, not limits of memory, bounds on cognitive processing capabilities, or other internal constraints. What is important is that the heuristics mesh with their environment, allowing limited capacity cognitive processes to function effectively (Simon, 1956, 1990; Todd & Gigerenzer, 2003).

Two of the most important environmental regularities are highlighted in making predictions about the Dream Team. First, if the environment is searched in such a way that additional information provides *diminishing returns*, with less useful information being found as search progresses, it may be sensible to make an early decision. This is what happens when search of the basketball game starts with the most important information, examining the players in the starting five, before continuing to less important information, in the form of the bench players. Second, if there is *correlated information*, so that what is found early in search is predictive of what will be found later, it can also be sensible to make an early decision. This is what happens in watching the basketball game. The score in the first half of the game provides information that is likely to be highly predictive of the score in the second half of the game.

The basic idea of fast and frugal heuristics is that human decision-making exploits these sorts of possibilities. By assuming the environment has structure, non-compensatory decision processes can be used that are fast, robust and accurate.

1.2 Overview

In this paper, we study one of the most prominent fast and frugal heuristics—the Take-the-best model of decision-making—in terms of the diminishing returns and correlated information environmental regularities. To do this, we analyze the Take-the-best model in a way that allows its behavior under different assumptions about the environment to be studied. We also extend the way heuristics like Take-the-best have previously been justified, focusing not on an outcome (or correspondence) measure of their decision accuracy, but on a process (or coherence) measure of whether more exhaustive search changes the decisions suggested by limited search. Using the widely-considered German cities environment, and three new city environments, we show that there are good grounds for the limited search assumed by Take-the-best, but only when both the environmental regularities are satisfied.

2 Environmental regularities and Take-the-best

Take-the-best is perhaps the best developed fast and frugal heuristic (Gigerenzer & Goldstein, 1996; Gigerenzer et al., 1999). It chooses between two alternatives, each represented by a set of binary cues. Take-the-best says that people search these cues in a specific order, determined by a measure called cue validity, and terminate search as soon as a discriminating cue is found.

To make clear the reliance of Take-the-best on the environmental regularity of diminishing returns, we develop a new approach to determining cue search orders. To make clear the reliance of Take-the-best on the environmental regularity of correlation information, we develop a refined measure of cue discriminability. Each of these theoretical developments is best described in concrete terms, and so we introduce them in terms of the German cities environment used in our later evaluation.

2.1 German cities and three new environments

The German cities environment is probably the most widely-used environment for studying fast and frugal heuristics. It represents 83 cities using 9 cues, and lists the population of each city, with the goal of using the cues to decide which of two cities has the larger population (Gigerenzer & Goldstein, 1996). For example, Hamburg is defined as having a soccer team in the Bundesliga, being the state capital, having been an exposition site, having an intercity train line, and having a university. Dortmund is defined similarly, except it is not a state capital, but is in the industrial belt. The goal of Take-the-best is to decide which of two cities, like Hamburg and Dortmund, has the larger population, based on their cues.

We also developed three other city environments to test the generality of our conclusions. The new environments are based on cities in Italy, the United Kingdom, and the United States, and use the same sorts of cues as the original German cities environment. All three new environments were collated in January 2012, and were completed before any of the analyses reported here were conducted. We report the basic details of these new environments, and the full datasets are are provided as supplementary materials along with this paper on the page for this issue: http://journal.sjdm.org/vol7.4.html.

For the Italian cities environment, we used the 149 cities with more than 50,000 people. We represented them in terms of 8 cues: whether they were the regional capital, whether they had a team in the Serie A soccer league, whether they had a team in the Serie B soccer league, whether they had a major rail station, whether they had an airport, whether they had a university, whether they were the national capital, and whether they were in the Po Valley (which we believe is commonly understood to be the most populous area of Italy).

For the US cities environment, we used the 74 cities with more than 250,000 people. We represented them in terms of 7 cues: whether they were the state capital, whether they had a major league sports team, whether they had a major airport, whether they had a metro system, whether they had been an exposition site, and whether they were the national capital.

For the UK cities environment, we used the 66 cities with more than 100,000 people. We represented them in terms of 6 cues: whether they were a national capital, whether they had a major airport, whether they had a team in the premier league, whether they had a rail station, whether they were the county capital, and whether they had a university.

2.2 Measures of the environments

It is instructive to examine the performance of some basic decision-making mechanisms, on all four environments. This allows benchmark accuracies on the well-studied German cities environment to be compared to the new environments. We calculated the accuracy of Take-the-best, naive Bayes, and profile memorization methods for each environment, averaged across all possible city pairs in each case. Naive Bayes basically combines the evidence for all cues to make a decision, but assumes each cue provides independent evidence. The profile memorization uses the probability that a city represented by one set of cues will have a larger population than another city represented by a different set of cues, choosing the city with higher probability for each pair. The results are shown in Table 1. It is clear that the relative performance of these methods observed in the German cities environment is approximately the same in the new environments. Both Take-the-best and naive Bayes perform extremely similarly, and profile memorization provides a natural upper-bound on achievable accuracy. It is also clear that the new environments are similar to the German cities in the absolute levels of accuracy they support, ranging from about 68% to about 75%.

Table 1: The accuracy of Take-the-best, naive Bayes and profile memorization decision-making methods, evaluated on all possible city pairs in the four environments.

	Take-the-best	Naive Bayes	Profile Memorization
German	74.0	74.0	80.0
Italian	68.5	68.9	70.1
United States	68.4	67.7	72.3
United Kingdom	75.2	75.2	78.3

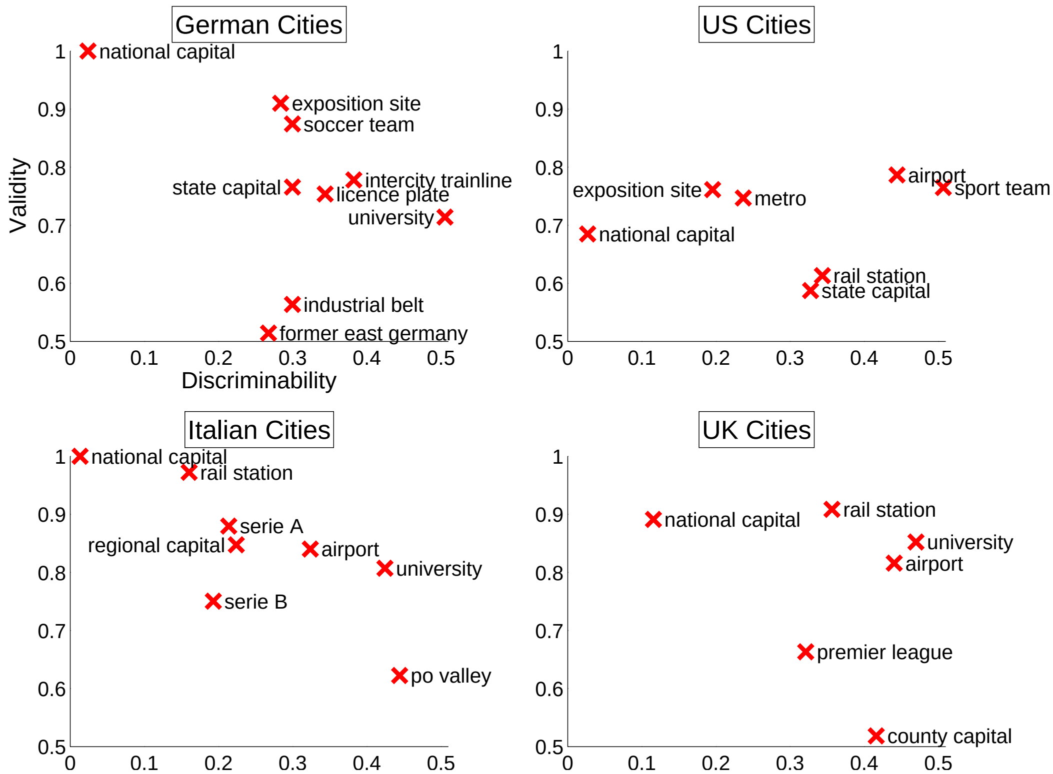
The two key measures associated with cues in our environments are their *discriminability*[1] and *validity*. Discriminability measures how often a cue discriminates between stimuli, as state capital and industrial belt discriminate between Hamburg and Dortmund. Validity measures how often a cue, given that it discriminates, belongs to the stimulus with the higher criterion value. Since Hamburg has more people than Dortmund, being a state capital is a valid cue for the comparison, but being in the industrial belt is not. Figure 1 shows the discriminability and validity of all of the cues in all four city environments. Discriminability is naturally bounded between 0 and 0.5, while validity is naturally bounded between 0.5 and 1. So, for example, in the German cities environment in the top-left panel of Figure 1, the national capital cue has low discriminability, because only Berlin has the cue, and so it only discriminates when Berlin is one of the cities in the paired comparison. But it has perfect validity because, when it does discriminate, it always gives the correct answer, since Berlin is Germany's most populous city. Meanwhile, the university cue has high discriminability, because about half the cities have universities and half do not. But it has a validity of about 0.7, because cities without universities are sometimes more populous than cities with universities.

2.3 Balancing discriminability and validity

Take-the-best assumes that cues are searched in decreasing order of their validity. This corresponds to a strong

[1]We use "discriminability" as an exact synonym for the term "discrimination rate" that is also used in the literature.

Figure 1: Discriminability and validity of the cues in all four of the city environments.



assumption about the environmental regularity of diminishing returns, because it guarantees that those discriminating cues with the most information are found earliest.

It might be, however, that this assumption is too strong, and other search orders are used. One line of work that considers other search orders is presented by Todd and Dieckmann (2005). These authors develop and evaluate a number of heuristic methods for learning and adapting search orders. Another line of work considers ways of defining search orders that do not depend entirely on cue validity. In particular, cue discriminability is assumed to influence the order of search.

Discriminability focuses on finding information. Validity focuses on information being highly important if it is found. Clearly, both are important, and it seems reasonable that both might influence search. Evidence from experimental investigations of search rules (e.g., Newell, Rakow, Weston, & Shanks, 2004; Rakow, Newell, Fayers, & Hersby, 2005) finds both discriminability and validity can be relevant to search, with individual differences and task constraints influencing how they combine to determine the order of search (e.g., Martignon & Hoffrage, 1999). Several authors (e.g., Martignon & Hoffrage, 1999; Newell et al., 2004) consider a "success measure" that combines validity and discriminability so that cues are ordered in terms of their ability to make accurate decisions in isolation from all other cues.

Another way to formalize how discriminability and validity jointly influence search was introduced recently by Lee and Newell (2011). In their approach, the cue search order is determined by giving a weight w to the validity of each cue, the remaining weight $1 - w$ to discriminability. The search order is then based on ordering cues according to the sum of these two weighted components. Setting $w = 1$ therefore gives a validity based search, as for the original Take-the-best heuristic. Setting $w = 0$ gives a discriminability based search, and intermediate values balance both measures in determining search order. This approach is well suited to our goals, since it defines a natural continuum from validity-based to discriminability-based search, and so allows the adherence of search to the regularity of diminishing returns to be manipulated.

2.4 Extending discriminability to capture correlation

Take-the-best is consistent with the environmental regularity of correlated information. By employing a one-reason approach to search termination, it assumes that information from future cues will not present contradictory or compensatory evidence. This is again a strong assumption. There is empirical evidence that people sometimes search in non-compensatory ways, but extend their consideration beyond the first discriminating cue. People sometimes look for two or three or more reasons to make a decision, even if they do not search exhaustively (e.g., Dhami, 2003).

A less extreme way to capture the idea of correlated information in a heuristic like Take-the-best is to refine the cue discriminability measure. Correlated environments are those where the information provided by one cue is consistent with information provided by other cues. One way to measure this correlation is to break the traditional measure of discriminability into two parts. These parts measure whether or not a cue is consistent with other cues, in terms of the stimulus it favors when it discriminates. We call the consistent part the positive discrimination rate d^+, and the inconsistent part the negative discrimination rate d^-.

Because they are new theoretical measures, we need to explain how d^+ and d^- are calculated, and this requires a bit more formality. If the kth cue discriminates in favor of stimulus A over stimulus B, the definition of cue validity means that the log-odds evidence it provides in favor of stimulus A is $\log\left(v_k / \left(1 - v_k\right)\right)$ (Bergert & Nosofsky, 2007; Lee & Cummins, 2004). It is natural to express evidence on the log-odds scale, because this allows the evidence contributed by different cues to be summed (e.g., Cover & Thomas, 2006).[2]

The evidence that all of the cues *except* the kth cue provide in favor of stimulus A or stimulus B can be formed by such a sum. We write all of the cues except the kth that discriminate in favor of stimulus A as $k' \in A$, and similarly write all of the cues except the kth that discriminate in favor of stimulus B as $k' \in B$. Then the total evidence is

$$t_{k'} = \sum_{k' \in A} \log \frac{v_{k'}}{1 - v_{k'}} - \sum_{k' \in B} \log \frac{v_{k'}}{1 - v_{k'}}.$$

This total $t_{k'}$ will be positive if the other cues overall provide evidence in favor of stimulus A, and negative if they favor stimulus B. The positive discrimination rate d^+ for the kth is then the proportion of times, over all stimulus pairs, when the kth cue discriminates, that it favors the same alternative as that favored by the remaining cues, as calculated from the sign of $t_{k'}$. The negative discrimination rate d^- is the proportion of times the alternative favored by the kth cue is different from that favored by the other cues over all stimulus pairs. The positive and negative discrimination rates partition[3] the traditional discrimination rate d, with $d = d^+ + d^-$. Intuitively, d_k^+ measure how often the kth cue discriminates in favor of the same alternative as that favored by the other cues.

Table 2 shows the cue validity v, discriminability d, positive discriminability d^+, and negative discriminability d^- for all of the cues in each of our city environments. The validity and discriminability measures are those already presented in Figure 1. The refinement of the traditional unitary discriminability measure into positive and negative discriminabilities shows that most cues have much greater positive discriminability. This means that they tend to discriminate in favor of the alternative favored by the other cues.

3 Evaluating Take-the-best

Previous evaluations of Take-the-best have focused on the accuracy of the decisions it makes. Early work demonstrated that Take-the-best was impressively accurate, both in absolute terms, and relative to the performance of various alternative and benchmark statistical decision-making methods (e.g., Gigerenzer & Goldstein, 1996). The general finding is that, despite its very limited search, Take-the-best often matches, and sometimes exceeds, the accuracy of benchmark statistical methods that use all of the available cue information, such as the naive Bayes method reported in Table 1.

Subsequent analyses provide a mixture of formal and simulation results for understanding what properties of the environment and decision-making situation affected the performance of Take-the-best and other methods (e.g., Hogarth & Karelaia, 2007; Katsikopoulos & Martignon, 2006; Martignon & Hoffrage, 2002), and many studies are reviewed by Katsikopoulos (2011). These analyses are quite different in the methods they use, and the assumptions about issues like the precision with which cues and their properties are known.

At a general level, however, these previous studies identify the regularities of diminishing returns and correlated information structure in environments as key determinants of the success of simple heuristics like Take-the-best. A strong relationship between validity-based search and accurate decision-making is widely observed, supporting the need for diminishing returns. The need for correlated information is made very explicit in, for exam-

[2] Unfortunately, some previous research has added the cue validities themselves to combine evidence, which has no information theoretic justification, and has likely led to errors in the experimental design and the interpretation of results in earlier work (e.g., Gigerenzer & Goldstein, 1996; Rieskamp & Otto, 2006). A reviewer asked for a concrete example, and so we note that the "weighted linear model" considered by Gigerenzer and Goldstein (1996, p. 654) is miscalculated. Our implementation of the WADD method that combines evidence across all cues to choose between cities, as used earlier to characterize the four environments, applies the correct method.

[3] This means that the extremely rare case where the other cues favor neither alternative is distributed equally to the positive and negative discrimination rates.

Table 2: The cue validity v, discriminability d, positive discriminability d^+, and negative discriminability d^- for the cues in the four city environments.

German cities

	National capital	Exposition site	Soccer team	Intercity trainline	State capital	Licence plate	University	Industrial belt	Former East Germany
v	1.000	0.910	0.875	0.778	0.766	0.754	0.71	0.564	0.514
d	0.024	0.284	0.300	0.383	0.300	0.344	0.505	0.300	0.267
d^+	0.022	0.260	0.217	0.298	0.225	0.268	0.370	0.110	0.143
d^-	0.002	0.024	0.083	0.084	0.075	0.076	0.136	0.189	0.125

Italian cities

	Regional capital	Serie A	Rail station	Airport	National capital	University	Serie B	Po valley
v	0.847	0.880	0.972	0.840	1.000	0.807	0.751	0.622
d	0.224	0.214	0.160	0.324	0.013	0.425	0.193	0.444
d^+	0.207	0.195	0.157	0.278	0.013	0.368	0.134	0.268
d^-	0.018	0.019	0.004	0.045	0.000	0.056	0.059	0.176

US cities

	State capital	Sport team	Airport	Metro	Rail station	Exposition site	National capital
v	0.588	0.765	0.787	0.747	0.613	0.761	0.685
d	0.328	0.507	0.444	0.237	0.344	0.196	0.027
d^+	0.182	0.446	0.374	0.217	0.271	0.160	0.026
d^-	0.145	0.061	0.071	0.020	0.072	0.035	0.001

UK cities

	National capital	Airport	Premier league	Rail station	County capital	University
v	0.891	0.816	0.663	0.909	0.519	0.852
d	0.116	0.441	0.321	0.357	0.416	0.470
d^+	0.106	0.355	0.213	0.319	0.293	0.409
d^-	0.010	0.085	0.108	0.037	0.124	0.061

ple, the work of Karelaia (2006), who evaluates a heuristic that requires two discriminating cues that agree before a decision is made.

3.1 Evaluating process coherence

Previous evaluations of Take-the-best have emphasized accuracy as a measure of performance. This reasonable approach to evaluation, focused on the assessment of external outcomes. A natural complementary approach, however, is to measure the internal coherence of a decision process. This is the approach we adopt, and makes our evaluation of environmental regularities and Take-the-best different from previous analyses that have focused on accuracy.

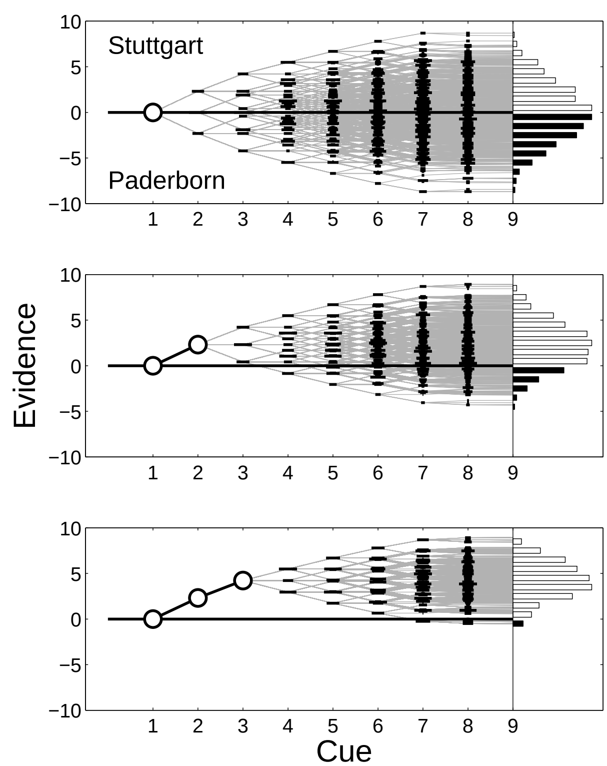
As a simple example to make the key distinction between process and outcome clear, imagine a friend has three marbles, tells you they were drawn from one of two urns, and asks you to decide which urn they were drawn from. Two of the three marbles drawn by your friend are red, and one is blue. The first urn contains 20 red marbles and 10 blue ones. The second urn contains 20 blue marbles and 10 red ones. It seems clear the best decision is to choose the first urn, because that inference follows from the available information. But, it is possible, of course, that your friend actually drew the marbles from the second urn, and just happened to draw two reds and a blue. That situation would make choosing the first urn incorrect, in the sense that the decision would not match the true state of affairs.

In this situation, there are two sorts of measures of the decision you made, supporting different choices. Choosing the first urn is the right thing to do from a process- or coherence-based notion of correctness, because it follows rationally from the available information. Choosing the second urn is the right thing to do from an outcome- or correspondence-based notion of correctness, because it is gives an accurate answer in terms of what your friend actually did.

This distinction between the assessment of process and outcome is an important one in the decision sciences (e.g., Simon, 1976), sometimes presented as a distinction between correspondence and coherence (Dunwoody, 2009; Hammond, 2007). While people cannot always make accurate decisions, they can always follow effective decision processes, and it seems important to assess both aspects of decision-making (e.g., Lee, 2006).

For non-compensatory search, one measure of a reasonable decision process is that it terminates once the current decision is unlikely to change. If it becomes clear that further search is unlikely to change the current decision, it is sensible to stop searching. Regardless of the accuracy of that decision, further search will not change the outcome and, in that sense, the decision is internally coherent. Indeed, the definition of non-compensatory encourages a measure like this, since it captures the idea that additional information cannot change the overall decision of current information. So, our assessment of Take-the-best focuses not on whether it makes accurate decisions, but on whether it stops when further search is unlikely to change the current decision.

Figure 2: Three stages of search in deciding whether Stuttgart or Paderborn has the larger population, and the projection of evidence tallies at each stage. The top panel corresponds to the case where the first cue has been examined, and does not discriminate between the cities. The middle panel corresponds to the case where the second cue has been examined, and discriminates in favor of Stuttgart. The bottom panel corresponds to the case where the third cue has also been examined, and also discriminates in favor of Stuttgart. In each panel, the validity and discriminability measures for the cues that have not been searched are used to project possible outcomes, shown by gray lines. These projected evidence totals result in final distributions in favor of Stuttgart, shown by the white histogram, and in favor of Paderborn, shown by the black histogram, to the right of each panel.



3.2 Proportion of extra cues measure

Figure 2 shows three potential stages of search in comparing Stuttgart to Paderborn. In the first stage, in the top panel, only the first cue, national capital, has been examined, and did not distinguish between the cities. This means the total evidence shown by the white dot is zero. Even without searched cues providing information, however, knowledge of the validities and discriminabilities of the remaining cues can be used to project possible future evidence totals. The discriminability of each cue gives the probability that it will provide evidence. The validity provides the magnitude of that potential evidence. Using this information, the possible sequences of total evidence that could be observed as all the cues are searched can be determined. These are shown by the gray lines, with the horizontal black bars showing the total mass at each possible evidence total after each cue has been examined. The distribution of final possible evidence totals, after the ninth cue has been examined, is shown by the histogram on the far right of the panel, with those totals favoring Stuttgart in white and those favoring Paderborn in black. In the top panel, the final evidence tallies are the same for both Stuttgart and Paderborn, because no discriminating cues have yet been found to provide evidence in favor of one city or the other,

The middle panel in Figure 2 corresponds to the case where the second cue, exposition site, has been examined, and found to favor Stuttgart. The evidence for Stuttgart, again shown by the white dot, is now the log odds validity of that cue. As before, the discriminabilities and validities of the remaining cues can be used to project the distribution of possible future evidence totals. The distribution of possible final totals now shows that Stuttgart is more likely to be favored than Paderborn.

The bottom panel in Figure 2 corresponds to the case where the third cue, soccer team, has been examined, and also favors Stuttgart. Now the projected final evidence tallies overwhelmingly would lead to the final decision being Stuttgart. This corresponds to the rationale for a process coherence measure of limited search. At this stage in search, with two cues found favoring Stuttgart, almost no sequence of remaining cue information could change that decision.

Using an analysis like that presented in Figure 2, in which each successive stage of search is considered, it is possible to quantify the extent of search justified by a process coherence measure. We formalize this in terms of the Proportion of Extra Cues (PEC) measure, used previously by Newell and Lee (2009). The PEC ranges from 0 to 1, measuring the proportion of cues beyond the first discriminating cue that must be searched to make the probability of exhaustive search changing the current decision smaller than some fixed threshold. For example, if the threshold is 5%, then, based on Figure 2, deciding between Stuttgart and Paderborn requires one cue (the third cue, soccer league) beyond the first discriminating cue (the second cue, exposition site) to make the probability of the final decision changing to Paderborn sufficiently small (i.e., the final evidence tallies on which the decision is based favor Stuttgart in white over Paderborn in black sufficiently strongly). Thus the PEC is 1/7, since one extra cue must be searched out of a possible 7.

3.3 Manipulating environmental assumptions

When PEC=0, the first discriminating cue terminates search, as in Take-the-best, and when PEC=1, search exhausts all of the cues, as in naive Bayes. For intermediate values, the PEC quantifies how much search beyond the first discriminating cue must be conducted, so that the available evidence is very unlikely to be over-turned by exhaustive search of all the cues. This PEC measure can be calculated for any environment, using any search order for the cues, and any measure of discriminability. The four examples in Figure 3 demonstrate the effect of changing search order and the measure of discriminability, continuing to use Stuttgart and Paderborn as a concrete example.

In the top left panel of Figure 3, a search order based entirely on validity is used, as in Figure 2. The first cue, national capital, does not discriminate the cities, but the second cue, exposition site, does, and favors Stuttgart. The top right panel of Figure 3 shows the analysis for the same search order, but using positive and negative discrimination rates. Now, if the current evidence favors one alternative, the probabilities of future discrimination in favor or against that alternative follow the positive and negative rates, rather than being evenly split between the two possibilities. Because positive discrimination rates are usually larger the distribution of final evidence tallies gives greater support the currently favored alternative. This is clear in the top right panel, through an upward shift in the evidence path, and in the distribution of final tallies. There is now only a very small probability of exhaustive search changing the decision from Stuttgart to Paderborn.

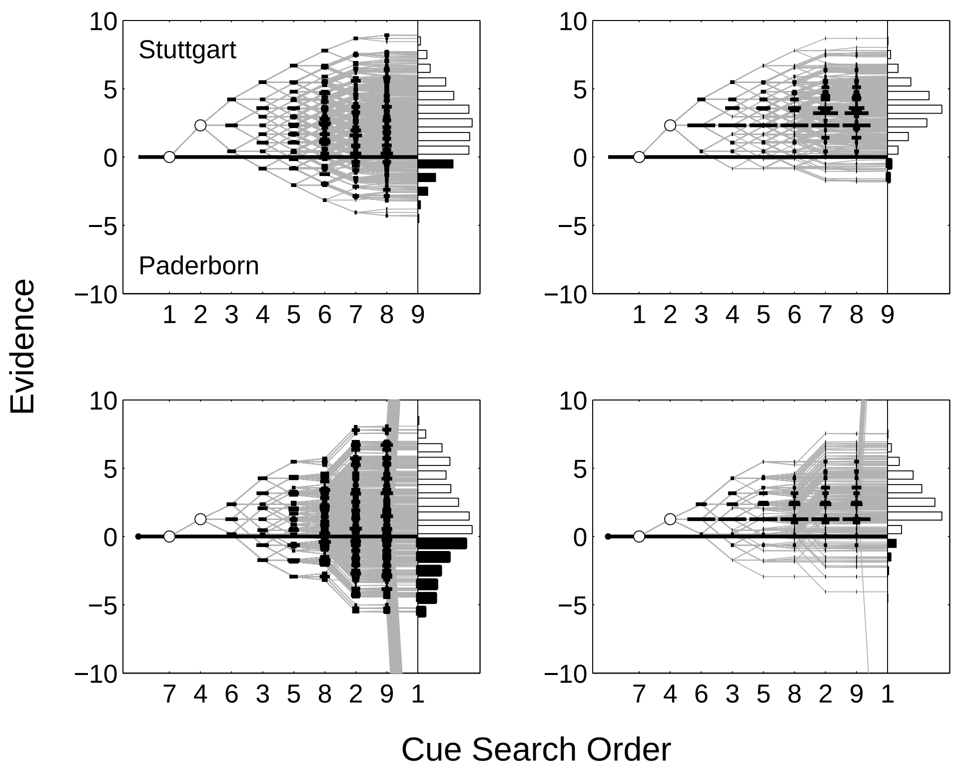
The bottom two panels of Figure 3 show the same two analyses, but for a different search order. This is a search order based only on discriminability, starting with the most discriminating university cue and moving to the least discriminating national capital cue. As before, the first cue does not discriminate between Stuttgart and Paderborn, but the second intercity trainline cue does, and favors Stuttgart. The bottom left panel of Figure 3 shows the evidence paths that follow, based on this search order, using the traditional single measure of discriminability. The distribution of final tallies shows there is a strong probability of the decision being changed to Paderborn after exhaustive search.

The bottom right panel of Figure 3 considers the same search order, but uses the positive and negative discrimination rates. The distribution of final tallies now shows a much lower probability of Paderborn being favored by exhaustive search.

The four examples in Figure 3 show two key trends, corresponding to two environmental regularities. One trend is made by the top versus bottom panels. The top panels show the results of search based on validity, which builds into decision-making an assumption of diminishing returns. These diminishing returns are visually clear from the convex nature of the evidence paths, as later cues provide successively less evidence. This means that early decisions are less likely to be over-turned by later evidence, because the later evidence is less compelling.

The bottom row of panels shows the change when a

Figure 3: Evidence paths, and distributions of final tallies, for a comparison of Stuttgart and Paderborn where the first discriminating cue favors Stuttgart. Each panel shows by gray lines the possible evidence paths for future cues, culminating in a distribution of final evidence tallies. The final tallies agreeing with the current decision are shown in white, while those corresponding to the alternative decision are shown in black. All four panels consider the case where two cues have been searched, and the current evidence favors choosing Stuttgart. Panels in the top row corresponds to validity-based search, while those in the bottom row corresponds to discriminability-based search. Panels in the left column correspond to using traditional discriminability to assess evidence, while panels in the right column correspond to using positive and negative discriminabilities.



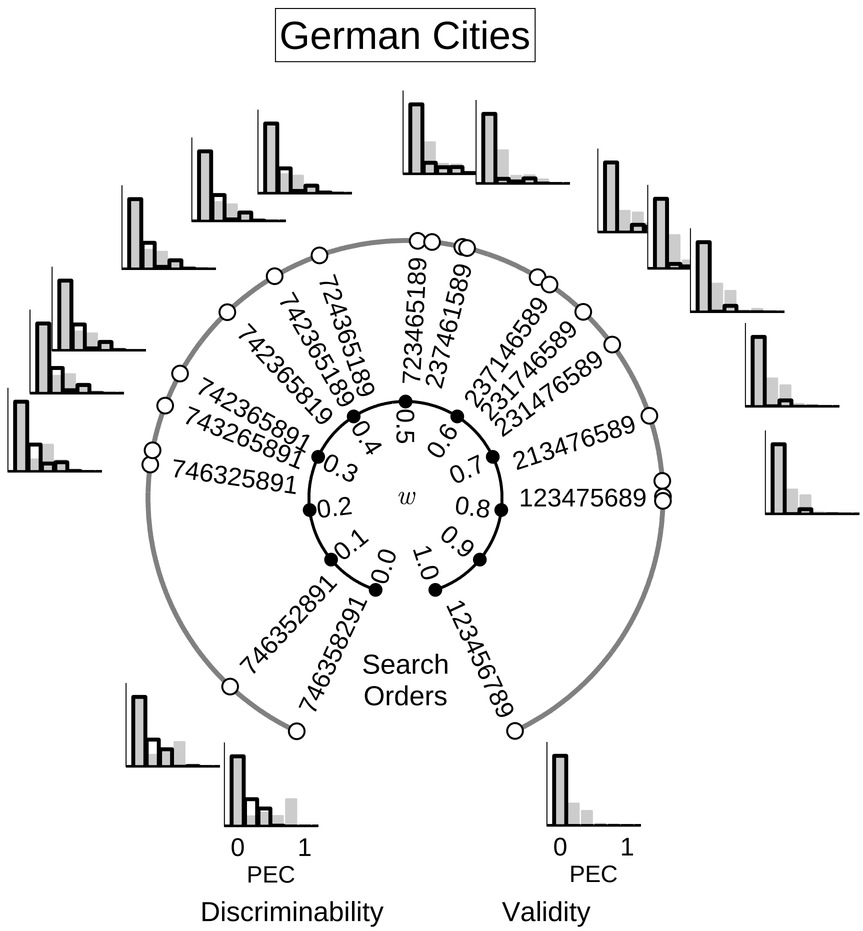
different search strategy, based on discriminability in this case, is used. Now later cues can provide strong evidence, the evidence paths can move quickly toward one alternative or another late in search, and the distribution of final evidence tallies covers both choices. Basing search on cue validity corresponds to assuming diminishing returns, and provides grounds for non-compensatory decision-making. When this assumption is not made, early decisions can be over-turned by later evidence.

The other key trend is shown by the left versus right panels in Figure 3. The left panels show final evidence tallies based on the traditional measure of discriminability. Those on the right show the tallies coming from using positive and negative discrimination rates. These measures allow the environmental regularity of correlated information to be incorporated in the analysis. In the panels on the right, once evidence favors a decision, future discriminating cues are generally expected also to favor that decision, because positive discriminability is greater than negative discriminability for most cues in the German cities environment. What is seen early in search is generally predictive of what is seen later, and so early decisions are unlikely to be over-turned. The presence of correlated information provides grounds for non-compensatory decision-making.

3.4 Results

The results of our analyses of the German cities environment are summarized by the "wheel" in Figure 4. The wheel rotates from the bottom left clockwise to the bottom right, increasing the value of w to move from pure discriminability based search when $w = 0$ to pure validity based search when $w = 1$. The inner rim of the wheel shows w increasing. The outer rim denotes by circular markers each time the increase in w leads to a change in the search order, and the orders themselves are listed, using the cue numbering from Figure 1.

Figure 4: Results of manipulating the search order by emphasizing validity or discriminability, and manipulating the measure of discriminability, on the proportion of cues beyond the first discriminating one that must be searched to reduce the probability of a changed decision below 5%. The inner rim shows the change in the w parameter that weights validity in determining the cue search order, ranging from strictly discriminability-based search at the bottom-left, to strictly validity-based search at the bottom right. The outer rim shows the change in patterns of the actual search orders by circular markers, and provides the details for a selected representative subset of these orders. The histograms for these selected order shows the distribution of the Proportion of Extra Cues (PEC) measure, over all possible questions, assuming both traditional discriminability (shaded gray) and positive and negative discriminability (unshaded).



The histograms outside the search orders in Figure 4 show the extent of search, using the criterion that search should continue while there is a greater than 5% chance of exhaustive search leading to a different decision.[4]

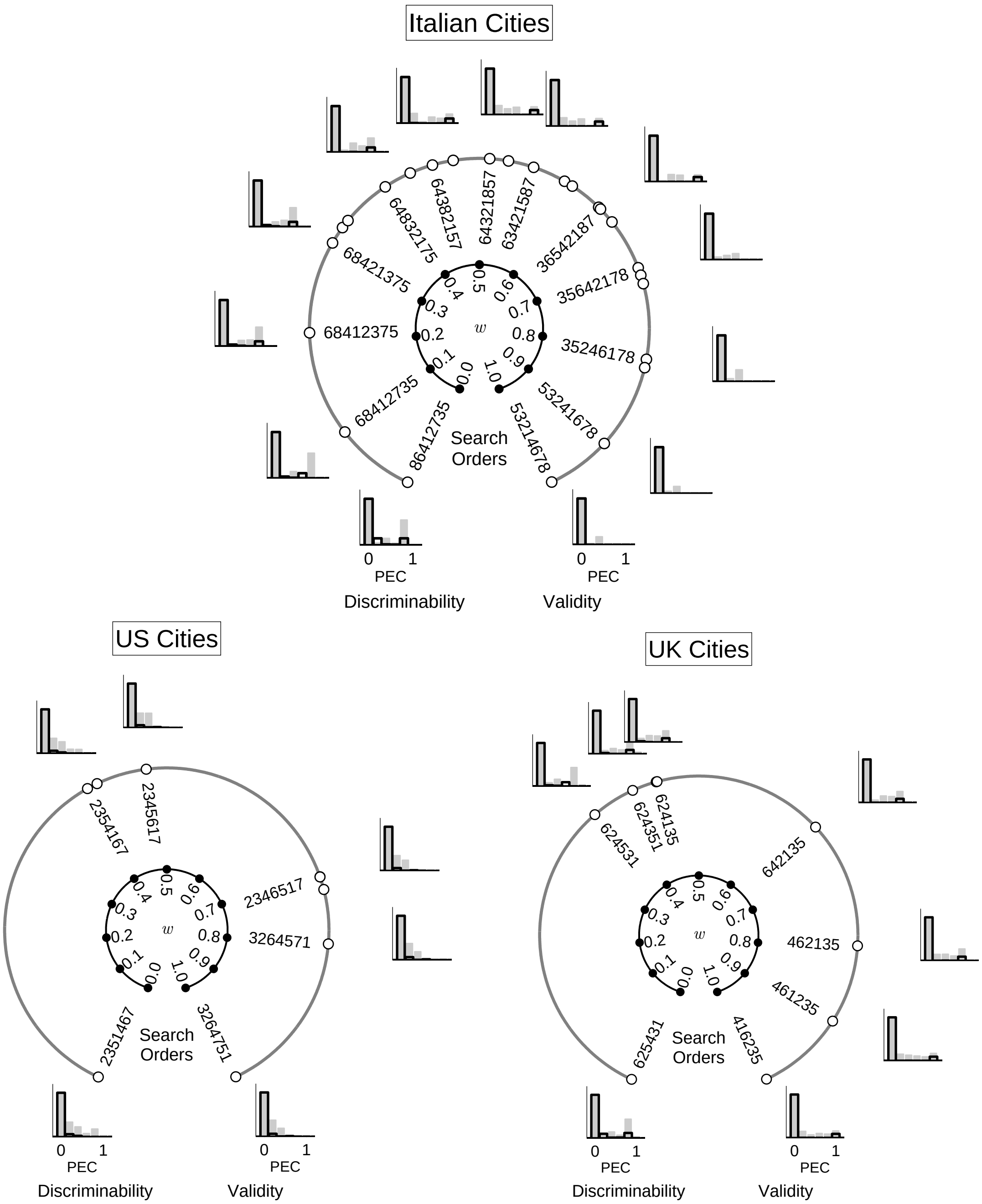
Two key results are clear. One is that, as the emphasis in search order shifts to validity, search becomes less extensive. In other words, as the assumption of diminishing returns is built into the search order, there are stronger grounds for stopping sooner. The other key result is that, when positive and negative discriminability are used, search becomes much less extensive. In other words, when the correlation of cues in the environment is considered, it often becomes clearer earlier that the likelihood of a decision being reversed is small, and the grounds for stopping earlier are again stronger.

Figure 5 shows the same set of results for the Italian, US and UK city environments.[5] The results for the Ital-

[4]The choice of 5% is obviously not principled, but is often a default choice in statistical inference. Using values near 5% lead to qualitatively similar results, and behave in the ways one would expect. As the criterion becomes more stringent, search is more extensive. Our goal is not to study or justify this criterion, but to show the impact of the environmental regularities for a reasonable criterion setting.

[5]For completeness, we also calculated the search order given by the success measure for combining discriminability and validity. For the US

Figure 5: Results of manipulating the search order by emphasizing validity or discriminability, and manipulating the measure of discriminability, on the proportion of cues beyond the first discriminating one that must be searched to reduce the probability of a changed decision below 5%. The Italian cities environment is shown at the top, and the US and UK cities environments are shown below.



ian cities are very similar to those found for the German cities. As validity-based search is used, and positive and negative discriminability are used, the PEC becomes zero for all city comparison. When either the search moves away from being based on validity (i.e., away from the bottom right panel), or the standard measure of discrim-

cities, the order is 2346517, which is shown in Figure 5. The success orders for the other environments are not among the sets we consider. But, their PEC distributions are all similar to those observed in Figures 4 and 5 for orders half-way between validity and discriminability.

inability is used (i.e., gray histograms), some city comparisons require more extensive search.

The results for the US and UK city environments show less variability. In general, the distribution of PEC measures shows many values near zero, but does not vary as the search order and assumptions about discriminability are manipulated. These findings are consistent with the idea of limited search, but are less interesting. With hindsight, it seems, in comparison to the German and Italian environments, that the US and UK city environments have fewer cues, and many fewer possible search orders. We suspect they are too impoverished to reveal the effects of environmental manipulation, and could usefully be enriched in the future. (But we did not want to revisit the nature of the environments having seen these results).

Overall, the results in Figures 4 and 5 provide support for Take-the-best as an effective decision-making heuristic. But the results also show the bounds on the support offered by a process- or coherence-based analysis. When one or other or both of the environmental assumptions are not met, especially in the German and Italian cities environments, search needs to be more extensive than the first discriminating cue. The gray histograms, when a correlated environment is not incorporated, always show search extending well beyond the first discriminating cues for many of the questions. The histograms not in the bottom-right corner, considering discriminability as part of the search order, and so not relying on diminishing returns, also show more extensive search, however discriminability is measured.

4 Discussion

In this paper, we have considered two key theoretical assumptions underpinning the Take-the-best heuristic for decision-making. These are both assumptions about the type of information structure of the environment. One is that the environment has diminishing returns, so that evidence found later in search is less important than evidence found earlier. The other is that the environment has correlated information, so that information found early in search is likely to be consistent with information found later.

Demonstrating that the grounds for Take-the-best rely on these assumptions requires the ability to manipulate the assumptions. We proposed a richer set of search orders, generalizing the validity search of Take-the-best, so that the assumption of diminishing returns could be manipulated. We also proposed dividing discriminability into positive and negative components, to capture the assumption of correlated information.

Our results, for the well-studied German cities environment, and for three new city environments, show that combining diminishing returns and correlated information provide grounds for non-compensatory decision-making. When these assumptions are met, the probabilities that exhaustive search would change an initial decision are very small. The first discriminating cue favors the same alternative as exhaustive search favors. The detailed results in Figures 4 and 5 show that both environmental regularities are important, and quantify their effect. The results also show, however, the bounds on the justification for limited search. In several of the environments we studied, when the basis of search moved away from validity, and so did not emphasize diminishing returns, more extensive search was needed. Similarly, if the correlation of information in the environment is not assumed, many more cues than the first discriminating one need to be examined to make it very unlikely a decision will change.

More general conclusions about the relative impact and usefulness of each assumption, and their interaction, require a much more extensive study of a broad range of environments. Unfortunately, many of the other environments studied in the fast and frugal literature (e.g., Czerlinski, Gigerenzer, & Goldstein, 1999) formed binary cues by taking median splits of continuous variables. By construction, this means all of the cues have discriminabilities of 0.5, and so create impoverished environments from the perspective of the current analyses. With appropriate environments, however, our findings give some theoretical tools and initial results to motivate that broader exploration.

Another line of future work is to extend Take-the-best as a decision-making heuristic, and evaluate these extensions. Our results present a detached analysis of when and why Take-the-best works in structured environments. But, the mechanism we used for manipulating search orders immediately gives a set of possible new decision-making heuristics (see Lee & Newell, 2011). The mechanism we developed for breaking discriminability into positive and negative components, to assess correlated information, does not immediately give rise to new heuristics. It does, however, give a theoretical opening for their development. For example, one sensible stopping rule would require a discriminating cue with a positive discrimination rate larger than the negative discrimination rate. This would correspond to stopping only when the discriminating cue was more likely than not in agreement with the information that would be provided by further search. Possibilities like these seem worth exploring.

What our results do show is when and why non-compensatory heuristics like Take-the-best are justified in stopping their search. When environments have diminishing returns, and when environments have correlated information, exhaustive search does not change decisions, and there are good process- or coherence-based grounds for limited search. Asked before the Angola game what he knew about the Angolan team, US player Charles Barkley replied: "All I know about Angola is Angola's in trouble." That was a non-compensatory opinion, unlikely to have been changed on the basis of further reflection. It proved to be accurate.

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