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IN A BOOTSTRAP MODEL

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ABSTRACT

It is argued on the basis of a simple bootstrap model of indefinitely rising Regge trajectories that all the trajectories asymptotically should rise linearly proportional to the energy with the same slope.

Recent experiments suggest that Regge trajectories rise indefinitely with increasing energy.[1] Although earlier dynamical calculations had qualitative success at low energy, it is not clear at present how to attack the problem of high-spin resonances at high energy when multiparticle channels become important. From the decay of high-spin particles, however, we observe that they couple most strongly to two-particle channels, with one or both of the decay particles being unstable. Thus one might be able to approximate the many-particle channels by a series of quasi-two-body channels. Qualitative encouragement for this idea has been obtained by various authors.[2] However, in the schemes so far considered the spin of only one of the two-channel particles is allowed to increase indefinitely.

Chew and Jones have proposed a simple model for the indefinitely rising Regge trajectory.[3] The essential idea is that a trajectory should be dominated in any particular energy region by the highest-spin channels whose thresholds lie in that region, the orbital angular momentum in the dominant channels thus remaining small no matter how high the total angular momentum may become.

It is instructive to examine the above model in non relativistic potential theory. If the potential $|V(r)| < \frac{N_0}{r^2}$ for all r (superpositions of Yukawa potentials belong to this category), one can argue heuristically that the maximum orbital angular momentum of a bound state at threshold is bounded by

$$\ell = -\frac{1}{2} + \sqrt{N_0} \quad ,$$

because, if $\ell > -\frac{1}{2} + \sqrt{N_0}$, the effective potential $\ell(\ell+1)/(r^2) + V(r)$ will be repulsive everywhere. Of course, the potential $V(r)$ depends on the external spin of the channel in question, so the number N_0 may also depend on the channel. However, there is no reason to believe that N_0 will increase indefinitely as the spin of the channel is increased. This point is important in the following discussion.

Let us consider a bootstrap system of three Regge trajectories: the pion trajectory α_π, the nucleon trajectory α_N, and the anti-nucleon trajectory $\alpha_{\bar{N}}$. We assume π to be a bound state of the $N\bar{N}$ system and N to be a bound state of the πN system. At high energy we conjecture a power behavior for the trajectories, i.e.,

$$\alpha_N = \alpha_{\bar{N}} = c_1 E^{a_1} , \qquad (1)$$

$$\alpha_\pi = c_2 E^{a_2} . \qquad (2)$$

Consider first the $N\bar{N}$ system; in the model described above, the maximum spin of an open channel at threshold is

$$\alpha_{max} = \max \{\alpha_N(xE) + \alpha_{\bar{N}}[(1 - x)E] \; ; \text{for } 0 \leqslant x \leqslant 1\} , \qquad (3)$$

if E is the threshold energy, a fraction x of which is carried by N and $1 - x$ by $\bar{N}$. Thus the total angular momentum of the highest output trajectory at this given energy is

$$\alpha_\pi(E) = \alpha_{max} + N_0 . \qquad (4)$$

Since N_0 remains finite, we have the two equations

$$\frac{d}{dx} \{c_1 x^{a_1} E^{a_1} + c_1 (1 - x)^{a_1} E^{a_1}\}_{x=x_{max}} = 0 , \qquad (5)$$

$$c_2 E^{a_2} = c_1 (x_{max})^{a_1} E^{a_1} + c_1 (1 - x_{max})^{a_1} E^{a_1} . \qquad (6)$$

From these equations we immediately deduce

$$a_1 = a_2 , \quad c_2 = 2c_1 (\tfrac{1}{2})^{a_1} . \qquad (7)$$

We do the same for the πN system and get two more equations similar to Eqs. (5) and (6). We can then conclude that

$$a_1 = a_2 = 1 , \quad c_1 = c_2 . \qquad (8)$$

In this model, therefore, all Regge trajectories will increase asymptotically proportional to E with the same proportionality constant.

We note that although we deduce our final result by requiring the output trajectory to be the bound state of the open channel with the highest spin, with the final solution the various quasi-two-body channels will have the same maximum total spin independent of the value of x defined in Eq. (3).

The above reasoning can obviously be applied to more complicated bootstrap systems. It is clear that our conclusion is always a self-consistent one in any bootstrap system. Furthermore, it is plausible that the more complicated the system, the more likely is our solution to be the "only" self-consistent one.

At present the Regge trajectories seem to increase linearly with increasing s $(=E^2)$. It is therefore extremely interesting to see whether the trajectories will start to curve at higher values of s, and whether all trajectories will look quite similar to each other.

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