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Journal
Adaptive Optics for Extremely Large Telescopes 4 – Conference Proceedings, 1(1)

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Publication Date
2015

DOI
10.20353/K3T4CP1131628

Tip-tilt modelling and control for GeMS: a performance comparison of identification techniques

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ABSTRACT

Perturbation modelling and vibration compensation are key issues to reach better performance in adaptive optics systems for VLTs and future ELTs. In this context, an LQG controller is to be implemented for the tip-tilt loop of the multi-conjugate adaptive optics instrument GeMS. The performance of several models, associated with their identification techniques are considered for tip-tilt disturbance.

1. INTRODUCTION

The Multi-Conjugate Adaptive Optics system GeMS, at Gemini South Observatory (La Serena, Chile), is strongly affected by vibrations in the telescope environment. With the Gemini South Adaptive Optics team, we are implementing in GeMS Real Time Controller (RTC), an LQG controller with vibration filtering in order to improve GeMS tip-tilt loop performance. LQG control relies on a stochastic disturbance model in state-space form, *i.e.* a shaping filter with a Gaussian white noise as input. The improvement brought by this kind of control strategy has been studied for GeMS and presented in this conference.[1] The study is based on a model identification technique developed by Meimon *et al.*,[2] successfully tested on CANARY[3] and implemented on SPHERE[4] for tip-tilt modelling. We propose here to compare several model identification techniques including this one, in order to select the most appropriate strategy for GeMS tip-tilt loop control.

We compare four different methods:

- an Extended Kalman Filter (EKF), which identifies an Auto-Regressive (AR) model of order 30;
- a Prediction Error Minimization (PEM), which identifies a full AR Moving Average (ARMA) model of order 30;
- the method used on CANARY,[3] on SPHERE,[4] and in Leboulleux *et al*,[1] that we call AR2, and which identifies iteratively a sum of AR models of order 2;
- a method that we call cPEM[5] (for cascaded Prediction Error Minimizations) which identifies in cascade several ARMA models of order 2.

In this proceeding, we first introduce the four model identification techniques. Then, we explain how the performance of each model identification technique is computed and we evaluate the performance using 70 GeMS data sets in replay-mode (*i.e.* using pseudo open-loop data to simulate an AO loop) simulation. Finally, we discuss the results and the perspectives of this work.

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2. MODELS AND IDENTIFICATION METHODS

LQG control requires an underlying model of the perturbation. For closed-loop AO with a two-frame delay, this model is defined by the state-space model:

$$
\begin{aligned}
x_{k+1} &= Ax_k + \Gamma v_k, &(1)\\
\phi_k &= C_\phi x_k, &(2)\\
y_k &= Cx_k - DNu_{k-2} + w_k, &(3)
\end{aligned}
$$

where ϕ is the disturbance phase, x is the so-called state vector, the matrix C_ϕ extracts ϕ from x, (A, Γ) models the disturbance dynamics, with a Gaussian white noise v as input, y is the wavefront sensor (WFS) measurement vector, C models the measurement process, with an additive Gaussian white noise w, D is the WFS matrix and N is the deformable mirror (DM) influence matrix. LQG control uses a Kalman filter to predict the perturbation, and then projects the estimated perturbation onto the DM to compute commands u. According to equations (1)-(3), the associated Kalman filter is defined by:

$$
\begin{aligned}
\widehat{x}_{k+1|k} &= (A - LC)\,\widehat{x}_{k|k-1} + Ly_k, &(4)\\
\widehat{y}_{k|k-1} &= C\widehat{x}_{k|k-1}, &(5)
\end{aligned}
$$

where $\widehat{x}$ is the estimated state vector and L is the Kalman gain. In many cases we can define a model of the perturbations from only physical considerations as, e.g., the Fried parameter r_0, or the wind velocity norm. The systems installed on the largest telescopes are also significantly affected by vibrations. These vibrations can evolve during the observation, for example because of the telescope tracking. During telescope operation, the disturbance model has thus to be updated to account for the evolution of the atmospheric turbulence and vibrations. Model identification methods make use of WFS measurements, every 30 second as so, to account for changes. This way, the LQG controller will reject efficiently the identified disturbances.

2.1 Extented Kalman Filter (EKF)

The EKF[5] method is a very standard method, which has to be tested in such a comparative study. This method requires an underlying model structure. Indeed, the parameters to identify are defined as a part of the extended state vector:

$$
x_k^e = \begin{pmatrix} x_k \\ \theta \end{pmatrix}, \tag{6}
$$

where x^e is the so-called extended state vector, and θ gathers all the parameters to identify. This method is usually running in parallel of the correction, in real time, adapting the model parameters to the evolution of the disturbances. In order to make the comparison consistent with the other methods, we decided to use the EKF in "batch" mode. This means that we use a part of the buffer (*i.e.* half of a tip-tilt data set of 8000 samples) to identify the parameters, and then we run a replay simulation with the second part of the buffer (see Section 3.2 for more details).

We have chosen an Auto-regressive (AR) model of order 30 for the EKF we implemented similarly to Kulcsar *et al.*[5] The perturbation is thus modelled by:

$$
\begin{aligned}
\phi_k &= a_1\phi_{k-1} + a_2\phi_{k-2} + \ldots + a_{30}\phi_{k-30} + v_k, &(7)\\
\theta &= (a_1\ a_2\ \ldots\ a_{30})^T, &(8)
\end{aligned}
$$

where $\{a_i\}_{i\in 1,\dots,30}$ are the AR30 model parameters. The AR30 structure have been chosen because of both its capacity of representation of complex phenomena and the simplicity of the model structure.

Fig. 1 represents the tip power spectral density (PSD) of the buffer ngs15108023248, acquired by GeMS in April 2015 and the PSD of the model identified by the EKF algorithm. We can see on Fig. 1 that high frequencies are badly modelled by this method. Another inconvenient of EKF is the lack of physical interpretation of the identified model. Indeed, identifying which parameter is related to a specific vibrations is not straightforward.

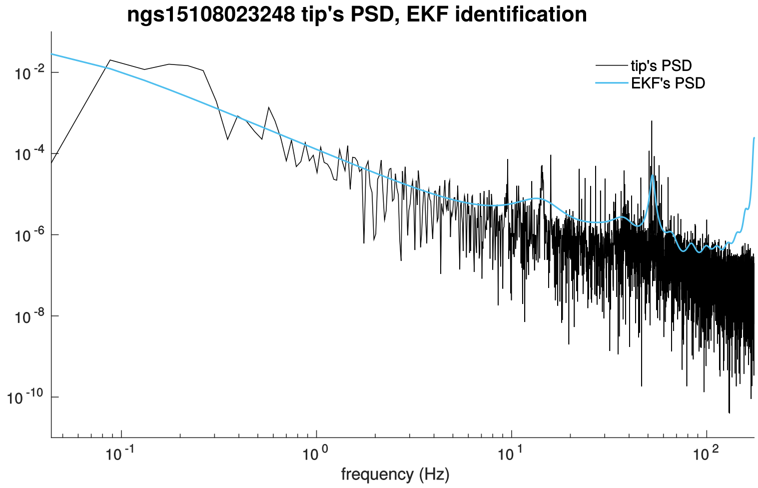

Figure 1: Buffer NGS15108023248, tip PSDs. Empirical tip PSD (black). EKF identified PSD (blue).

2.2 Prediction Error Minimization (PEM)

Prediction Error Minimization (PEM) model identification is particularly well adapted to the context of vibration modelling in adaptive optics. This technique does not need any predefined structure to work. Indeed, PEM methods identify directly from a set of measurement a full state space model of desired order. Matlab offers PEM model identification technique in its signal processing toolbox: pem.m. This function can be parametrized, choosing, e.g., the kind of optimization (Gradient, Gauss-Newton, Levenberg-Marquardt, non Linear Least Squares) or the initialization method. A complete description of PEM model identification method is given by Ljung.[6] The method consists in minimizing the distance:

$$J^{\mathrm{PEM}}(\theta) = \mathrm{argmin}_{\theta} \, \|y - \widehat{y}\|_2^2 \,. \tag{9}$$

PEM is particularly convenient because it identifies directly the Kalman filter matrices $(A - LC, C, L)$, and not only the model (A,C). Moreover, the unconstrained structure allows PEM models to represent finely the data, but the identified model cannot be related to the physical components of the disturbance, and much attention needs to be paid to the choice of the order to avoid over-parametrization. Fig. 1 represents the tip PSD of the buffer ngs15108023248, acquired by GeMS in April 2015 and the PSD of the model identified by the PEM (ARMA30) algorithm. This method appears to be the most accurate one regarding the PSD fit. However, its computation time is around 5 times longer than all the other techniques studied in this work.

2.3 Sum of order 2 Auto-Regressive models (AR2)

This model identification method[2] consists in identifying directly from the disturbance PSD, first the turbulence and then iteratively N_{vib} vibrations, all as AR models of order 2. Each time an AR2 is identified, its PSD is subtracted from the disturbance PSD. The algorithm identifies the matrix A (see eq. (1)), the shaping white noise covariance matrix Σ_v, and the measurement noise level σ_w. We can thus compute the Kalman gain (by resolving the associated discrete algebraic Riccati equation).

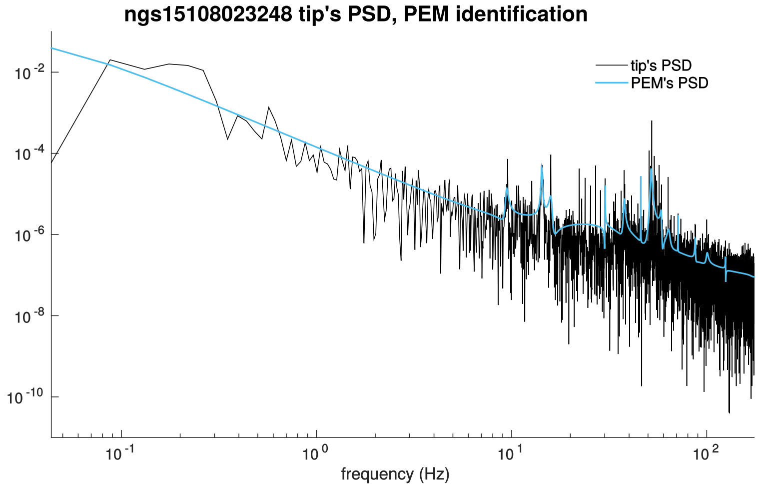

Figure 2: Buffer NGS15108023248, tip PSD. Empirical tip PSD (black). PEM identified PSD (blue).

Fig. 3 represents the tip PSD of the buffer ngs15108023248, acquired on GeMS in April 2015 and the PSD of the model identified by the AR2 algorithm. The identified PSD appears to be less accurate than with the order 30 PEM model in Section 2.2 as expected due to a more constrained model structure. The main benefits of AR2 simple model structure is that the relation between AR2 parameters and the physical components of the disturbance is straightforward[73], [8]

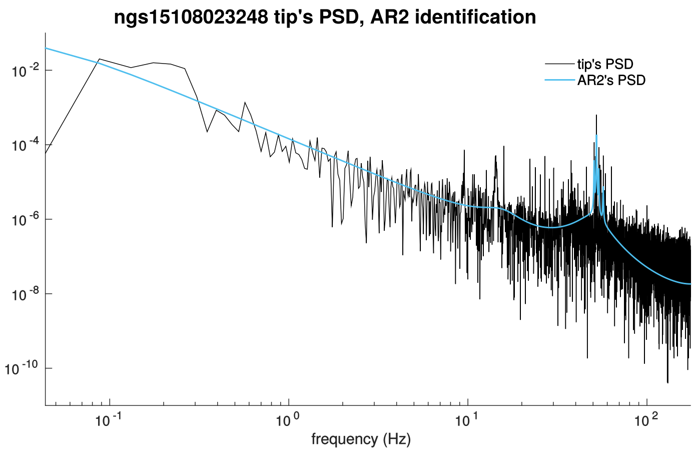

Figure 3: Buffer NGS15108023248, tip PSD. Empirical tip PSD (black). AR2 identified PSD (blue).

2.4 Cascaded order 2 PEM models (cPEM)

cPEM[5]) consists in identifying in cascade ARMA models of order 2 (called cell). We assume with this method that any part of the disturbance (turbulence, vibrations, ...) can be modelled with a full order 2 ARMA model. This assumption is less constrained than using AR2 models (see Section 2.3), which should therefore lead to a better performance. The algorithm is described by *Kulscar et al.*[5] We parametrize the pem.m Matlab function in order to use a Levenberg-Marquardt optimization, which gives the best results in our conditions, and the first ARMA cell is initialized with N4SID, a sub-space identification method.[9]

Fig. 4 represents the tip PSD of the buffer ngs15108023248, acquired with GeMS in April 2015 and the PSD of the model identified by the cPEM algorithm. As expected, the structure of the model identified with cPEM algorithm allows to fit more accurately the PSD than the AR2 method. The cascaded order 2 model structure is more relevant of the physical representation of the perturbations, but the mathematical relation between the 4 ARMA parameters and the physical quantities is not direct. The identified model is still less complex than an ARMA model of order 30, and therefore faster to compute.

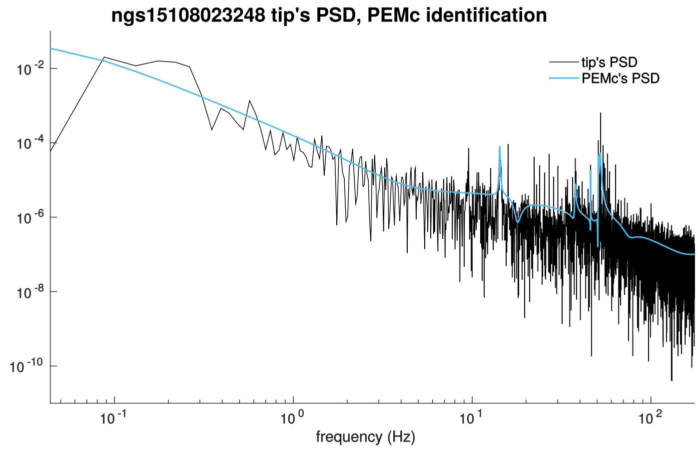

Figure 4: Buffer NGS15108023248, tip PSD. Empirical tip PSD (black). cPEM identified PSD (blue).

3. PERFORMANCE ASSESSMENT AND COMPARISONS

3.1 GeMS data sets

In order to perform this model identification algorithm comparison, we studied 70 data sets provided by Gemini South Observatory, acquired with GeMS during the 2015, April observations. These data sets have been acquired with a sampling frequency varying from 150 Hz to 700 Hz, and each is made of 8000 temporal samples. GeMS can process tip-tilt measurement with up to 3 Natural Guide Star (NGS). The 3 NGS tip-tilt measurements are then averaged to form a single tip-tilt measurement in the telescope pupil. Several vibrations affect GeMs performance, and particularly a strong one at 37 Hz.[1]

Four data sets lead to unstable behaviors in replay-mode simulations with the AR2 method. These data sets have thus been removed from the study. The comparison is thus based on the remaining 66 data sets to allow performance computation for this method. Fig. 5 shows a typical residual tip (ngs15108023248) provided by Gemini, and Fig. 6 shows the pseudo open-loop (POL) tip computed from ngs15108023248 (residual tip + tip command, see Fig. 7).

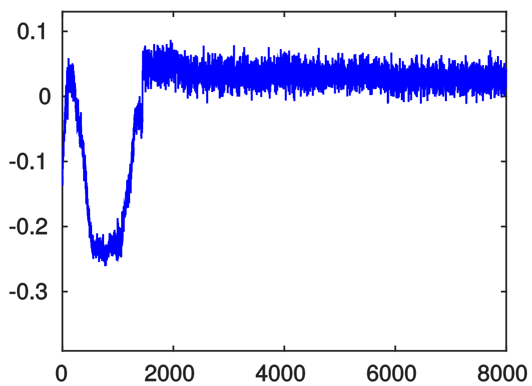

Figure 5: ngs15108023248 residual tip (registered by GeMS).

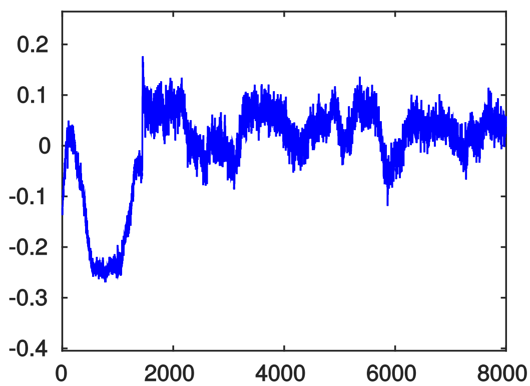

Figure 6: Pseudo open-loop tip (computed from GeMS ngs15108023248 data set).

3.2 Performance evaluation

We compare the performance of each model identification technique from replay-mode simulations. For each data set, we first compute the pseudo open-loop (POL) measurements (as GeMS is working in closed loop) by compensating the residual tip/tilt with the associated command values, also recorded in the data set. Then, we separate each POL sequence into 2 batches of 4000 samples each. We use the first batch to identify the disturbance models and then we use the second batch to evaluate the performance of the LQG controllers derived from the identified models. Fig. 7 summarizes this procedure.

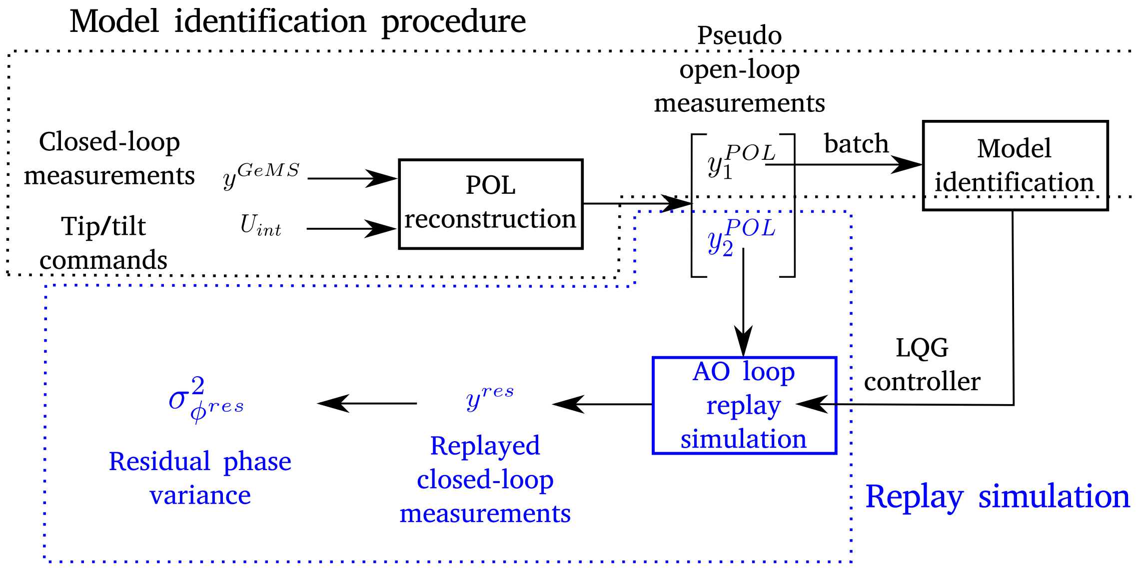

Figure 7: Performance analysis procedure.

3.3 Models and identification performance comparison

The performance of each method, evaluated for the 66 data sets described in Section 3.1, is represented in Fig. 8. Performance is displayed in residual RMS values expressed in on-sky angle (milli-arcseconds), and is compared to the current operational control strategy: an integral controller (black curve). Solid lines give the performance of each method for each GeMS data set whereas dotted lines give the average performance over all data sets. We note first that LQG leads to better results than integral controller, whatever the model identification method. This is consistent with the results in Leboulleux *et al.*[1] study. This is also in line with Sivo *et al.*,[3] because LQG is efficiently rejecting vibrations thanks to the identification strategy meanwhile the integral controller may be completely impacted by the vibrations in its overshoot area.

Note first that the most constrained model leads to the worst performance. Indeed, EKF model identification algorithm provides an AR30 model, which corresponds to a quite sparse matrix. On the other hand, the unconstrained PEM method leads to the best performance. This is expected as a higher number of parameters in the disturbance model can be related to a better modelling precision. However, cPEM and AR2 methods are quite close to PEM method, and require around 5 times less computing resources than PEM. Moreover, their structure is easier to relate to the physical components of the disturbance phase (mainly turbulence and vibrations). Comparing only cPEM and AR2 methods, we can see that cPEM is better than AR2 for 75% of the data sets, with a performance gap of 10%. For the other 25%, AR2 is only better by 5%. Moreover, as addressed in Section 3.1, we observed some stability issues in replay-mode simulations with AR2 method, for 4 data sets whereas the other methods were working correctly. This is mostly due to the discrepancy between the empirical PSD of the first 4000 samples and of the last part of the buffer where the data do not follow the estimated model. Indeed, even if a strict theoretical stability condition is verified in the AR2 algorithm, instability can occur if the model is too far from the data.

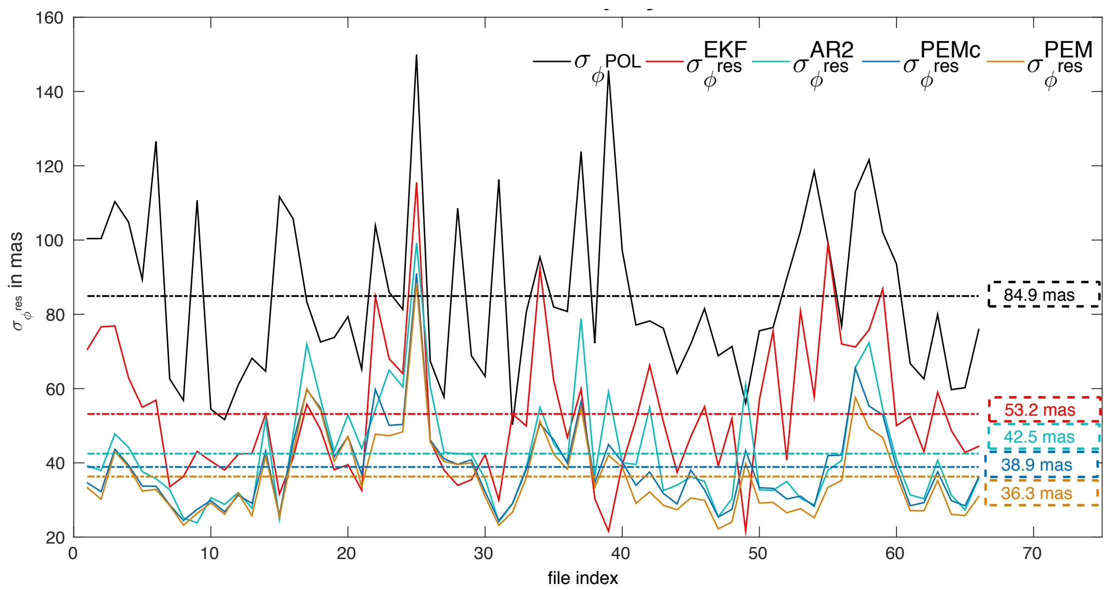

Figure 8: Performance for each model and identification method and each data set.

4. CONCLUSION AND PERSPECTIVES

Leboulleux *et al.*[1] show that LQG control with vibration mitigation can improve significantly the performance of the multi-conjugated AO system GeMS. We have shown here that other model choices and identification strategies can impact the performance brought by LQG. In this paper, four models and their identification techniques are studied, leading to the selection of 3 techniques: AR2,[?] already tested on other systems,[3,4] a new technique, which seems to be an interesting option called cPEM,[5] and the PEM method which gave the best results. Indeed, both AR2 and cPEM have performance almost as good as PEM (the most complex algorithm and model), but are significantly faster. cPEM strategy seems more appealing than AR2 because we did not observe any stability issue, and the average performance is slightly better (3.6 mas improvement in average).

LQG control has been integrated in GeMS RTC in November 2015, with a parallelized structure, insuring quite fast computing time. The first on-sky experimentation is planed for 2016 for the 3 best techniques (PEM, cPEM and AR2).

Acknowledgements

Based on observations obtained at the Gemini Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc., under a cooperative agreement with the NSF on behalf of the Gemini partnership: the National Science Foundation (United States), the National Research Council (Canada), CONICYT (Chile), the Australian Research Council (Australia), Ministério da Ciência, Tecnologia e Inovação (Brazil) and Ministerio de Ciencia, Tecnología e Innovación Productiva (Argentina).

Rémy Juvénal and Caroline Kulcsár would like to thank the Gemini South Observatory staff for its welcome and its availability during the November 2015 RTC preparation.

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