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Tactical Portfolio Construction

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Yue Chen

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ABSTRACT OF THE THESIS

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Yue Chen

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Professor Rick Paik Schoenberg, Chair

Modern portfolio theory says that unsystematic risk can always be diversified away. Although it is unrealistic to achieve perfect diversification, people developed different strategies trying to find an optimal portfolio. In this paper, issues of how to make a diversified portfolio are discussed. Six models of tactical portfolio construction strategies are used to make the optimal portfolios using the historical data from Jan. 2007 to Dec. 2009. From the out-of-sample test by market data from Jan. 2010 to Dec. 2011, the classic Markowitz portfolio has the highest wealth added. By comparing the portfolio evaluation parameters, Sharpe ratio suits an individual investor most. And Bayesian portfolio with informative prior scores highest by Sharpe ratio regardless its sensitivity to the accurate market prediction. The paper also demonstrates the application of statistical software R and the “stockPortfolio” package in the stock investment field.

The thesis of Yue Chen is approved.

Nicolas Christou

Qing Zhou

Rick Paik Schoenberg, Committee Chair

University of California, Los Angeles

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To my beloved parents,

who are always my strongest support and faith.

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Chapter 1 Introduction

Modern portfolio theory says that the expected return for a given amount of portfolio risk can be maximized or equivalently the risk for a given level of expected return can be minimized by carefully choosing the proportions of various assets. [1] This theory is based on the research work of Harry Markowitz about the risk-reduction benefits of diversification in early 1950s. [2] Using the standard deviation of a portfolio return as the measure of risk, he pointed out that combining risky securities into a portfolio reduces the risk because of diversification unless the returns of the risky assets are perfectly positively correlated. Later in 1960s, Treynor, Sharpe, Mossin and Lintner independently extended this work into modern portfolio theory (MPT). [3-6] MPT results in equilibrium expected returns for securities and portfolios that are a linear function of each security's or portfolio's market risk, which is the risk that cannot be reduced by diversification. During the normal market condition periods, portfolio diversification works fine while during periods of market crash diversification produces less benefit because the correlations tend to increase in such period.

One measure of the portfolio diversification is diversification ratio. It is calculated as the ratio of the risk of an equally weighted portfolio of n securities to the risk of a single security selected at random from the n securities. While the diversification ratio provides a quick measure of the potential benefits of diversification, an equally weighted portfolio is not necessarily the portfolio that provides the greatest reduction in risk. Besides that, Barnea and Logue argue that R^2 from the market model regression is a measure of diversification. And a study from Cresson also confirms R^2 is an unambiguous, objective and easily calculated measure of portfolio and mutual fund diversification. [7]

One important question in portfolio management is how many stocks make a diversified portfolio. Although Evans and Archer asserted 10 or 15 different assets maximize the benefits from naive diversification, Statman's study shows at least 30 stocks for a borrowing investor and 40 stocks for a lending investor must be included in a well-diversified portfolio.[8, 9] The study considers a stock portfolio with equal proportions and the variance of it can be expressed as follows.

$$\sigma_p^2 = \frac{1}{n}\left(\overline{\sigma_i}^2 - \overline{\sigma_{ij}}\right) + \overline{\sigma_{ij}}$$

where

$$\sigma_p^2 = variance\ of\ portfolio\ returns$$

$$n = number\ of\ stocks\ in\ the\ portfolio$$

$$\overline{\sigma_i}^2 = average\ variance\ of\ returns\ on\ stock\ i$$

$$\overline{\sigma_{ij}} = average\ covariance\ between\ returns\ on\ stocks\ i\ and\ j$$

To maximize the utility of the investor, the diversification should be increased as long as the marginal benefits exceed the marginal costs and will stop at a level when they are equal. Statman continued point out that the benefits of diversification are in risk reduction and the costs are transaction cost. Upon all the assumptions in the study, to use a 500-stock portfolio as benchmark is reasonable but to treat the cost of equally weighted portfolio identical to that of a value weighted portfolio is questionable. Regarding to the concept, equal-weighted portfolio has equal dollar amounts invested in each stock while the value-weighted portfolio has the same proportion of each stock as that of the security's market capitalization proportion in the portfolio.

One concern with an equal-weighted index is that the weights placed on the returns of the securities of smaller capitalization firms are greater than their proportions of the overall market value of the portfolio stocks. On the other hand, a value-weighted portfolio does not need to be adjusted when a stock splits or pays a stock dividend because the market capitalization does not change, which is not the case for an equal-weighted portfolio due to a more frequently rebalancing. So the transaction cost of an equal-weighted portfolio is higher than that of a value-weighted portfolio in general.

The cost of diversification is mainly the transaction cost. Individuals in different tax brackets, with different income needs and having any unique circumstances should be treated specifically. Nevertheless, Amihud and Mendelson also consider the cost of illiquidity and point out that stocks with higher spread have higher trading cost and should require a higher expected return. [10] Furthermore, they conclude the expected return is an increasing and concave function of the bid-ask spread by the empirical evidence from NYSE stock returns over the 1961-80 period. But once the investor holds a diversified portfolio, he has little to do with the cost of illiquidity. As a result, the longer holding period, the lower amortized transaction cost per unit of time. Hence, low-spread stocks will tend to be held in equilibrium by short-term investors.

Another argument is that leading to the efficient diversification requires the knowledge of accurate estimate of the risks and returns of individual stocks. And the lack of accurate information regarding future returns and risks of individual stocks pushes these investors toward naïve diversification. [11] Samuelson showed that if returns are identically distributed, buying equal amounts of each stock is optimal. [12] Another study by DeMiguel, Garlappi and Uppal indicates that among the performance of portfolio constructed by 14 optimal portfolio strategies, none is consistently better than the equal weighted portfolio in terms of Sharpe ratio, certaintyequivalent return, or turnover. And they conclude that the gain from optimal diversification is more than offset by out of sample estimation error. [13] Meanwhile, unless all the stocks are included in the portfolio, there always exists some diversifiable risk in a portfolio even in a so-called well-diversified portfolio.

Hence, Wit points out that instead of focusing on the number of stocks needed for a well-diversified portfolio, the consequence of holding portfolios of given numbers of stocks is more practically important. [14] So the right question to ask is: what is the excess risk of a portfolio of 20 stocks? And what excess return is required to compensate for that excess risk? According to his study, the required excess return of any imperfectly diversified portfolio is a function of the equity risk premium and the average correlation between stock returns in the portfolio. The average correlation coefficient determines the rate of risk reduction and the level of maximum risk reduction.

There are three major steps in the portfolio management process [15]:

1. Planning step. Analyze the investor's risk tolerance, return objectives, time horizon, tax exposure, liquidity needs, income needs, and any unique circumstances or investor preferences.

   The analysis results in an investment policy statement (IPS) that details the investor's investment objectives and constraints. I will also specify an objective benchmark against which the success of the portfolio management process will be measured. IPS should be updated periodically and anytime the investor's objectives or constrains change significantly.

2. Execution step. Analyze the risk and return characteristics of various asset classes to determine how funds will be allocated to the various asset types, including cash, fixed-income securities, publicly traded equities, hedge funds, private equity, real estate, as well as commodities and other real assets. Once the asset allocations are determined, portfolio managers may attempt to identify the most attractive security combination within the asset class.
3. Feedback step. Over time, investor circumstances will change, risk and return characteristics of asset classes will change and the actual weights of the assets in the portfolio will change with asset prices. So the portfolio manager must monitor these changes and rebalance the portfolio periodically in response, adjusting the allocations to the various asset classes back to their desired percentages. The manager must also measure portfolio performance and evaluate it relative to the return on the benchmark portfolio identified in the IPS.

In this study, I mainly focus on the stock portfolio construction using different strategies and compare the performance among them, which is part of the execution step. The strategies considered here are classic Markowitz model, single index model, multi-group model, equally weighted model, Bayesian portfolio selection including noninformative prior approach and informative prior approach. To compare the performance of these strategies, three measurements will be introduced and compared. Based on the historical stock return data, the portfolios will be constructed, the performance of which in the following years will be compared according to a suitable measurement.

Chapter 2 Portfolio Construction

2.1 Data source

In this research work, 25 stocks from 5 sectors are randomly chosen for the investigation purpose. They are:

1) Technology sector: Apple Inc., IBM, Intel Corp., Microsoft Corp. and Advanced Micro Devices Inc.
2) Basic material sector: Goldcorp Inc., Northern Dynasty Minerals Ltd., Potash Corp., Silver Standard Resources Inc. and Southern Copper Corp.
3) Financial sector: Citigroup Inc., HSBC, Morgan Stanley, American Express Company, Bank of America Corp.
4) Consumer/Non-cyclical sector: Coca-Cola Company, PepsiCo Inc., Kraft Foods Inc., Farmer Brothers Co. and Revlon Inc.
5) Healthcare sector: Johnson & Johnson, Delcath Systems Inc., Pfizer Inc., Abbott Laboratories and Exelixis Inc.

The corresponding tickers of each stock are tabulated as shown in the table 2.1.

Table 2.1 Summary of tickers

Sectors	Tickers				
Technology	AAPL	IBM	INTC	MSFT	AMD
Basic Material	GG	NAK	POT	SSRI	SCCO
Financial	C	HBC	MS	AXP	BAC

Consumer/Non-cyclical	KO	PEP	KFT	FARM	REV
Healthcare	JNJ	DCTH	PFE	ABT	EXEL

Besides these 25 stocks, S&P 500 index is used as the proxy for the market with SPY as the ticker.

The stock price data is publicly available from the market. Here, "stockPortfolio" package [16] in R is used to get the monthly returns online from Yahoo Finance.

Short sale is allowed in the portfolio construction, which means people can borrow the stocks from a third party and sell it now. [17] The risk-free rate is assumed to be 0.01%.

2.2 Sample period

The monthly returns of stocks over January 2007 to December 2009 period are adopted for the portfolio management study. So the total data size is 25 stocks by 36 months. The statistic summary of the data is in Table 2.2. The bottom row of the table is the performance of the market taking S&P500 as a proxy.

Table 2.2 Statistic summary of data for January 2007 to December 2009

Ticker	Average Return	Risk (standard deviation)
AAPL	0.03477482	0.3933738
IBM	0.011662064	0.3027398
INTC	0.005383034	0.3457537
MSFT	0.00478515	0.3242568
AMD	0.008878985	0.5395223
GG	0.024246635	0.3545828
NAK	0.016087643	0.4694901
POT	0.033242773	0.4169626
SSRI	0.008159839	0.4999892
SCCO	0.027776833	0.432173

C	-0.044253577	0.5880713
HBC	-0.00251393	0.3969022
MS	-0.011851639	0.4275879
AXP	0.005456479	0.5014196
BAC	-0.00559196	0.5639677
KO	0.009271932	0.2855292
PEP	0.001888528	0.2589532
KFT	-0.001404211	0.2756372
FARM	0.004713309	0.2784546
REV	0.047264495	0.5665698
JNJ	0.002627083	0.2716501
DCTH	0.028311109	0.4057474
PFE	-0.003508194	0.3054432
ABT	0.004122694	0.1905348
EXEL	0.007865948	0.4748137
SPY	-0.003713634	0.3366569

2.3 Portfolio efficient frontier

In Markowitz's 1952 paper [18], he introduced the efficient frontier concept. Among all the combination of portfolios, the investor would like to select the portfolios that are efficient, which have minimum risk given return or more and maximum return given risk or less. In the "stockPortfolio" package, "portPossCurve" function can be used to get the portfolio possibilities curve. Only the part beyond the minimum risk point is the efficient frontier. All the portfolios that locate at right side of the curve are attainable. The efficient frontier and a cloud of possible portfolios are shown as follows.

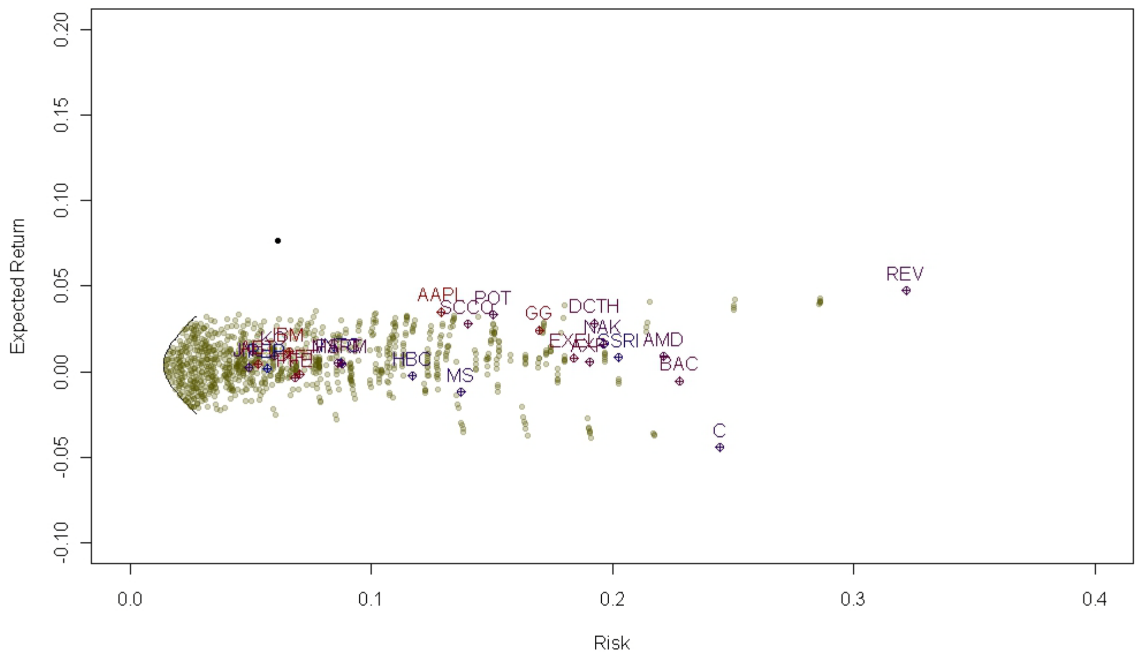

Figure 2.1Efficient frontier and cloud of possible portfolios for January 2007 to December 2009 period.

From this plot, we could see that all the individual stocks are located in the cloud of possible portfolios. But investing on individual stock is not wise because none of them is on the efficient frontier. The optimal portfolio should be on the efficient frontier. If we know the maximum risk that the investor can tolerant, then we could find the highest expected return point at this given risk on the efficient frontier. So any point on the efficient frontier is an optimal portfolio if people invest all money on stocks.

In reality, people can also put their money in the bank to earn risk-free rate or borrow money from bank at some cost. If the risk-free borrowing and lending are combined with stock investment, how to make the decision of how much should be invested in risk-free rate and how much should be invested in risky stock portfolio? Another concept should be introduced, which is capital allocation line (CAL).

As introduced in Sharpe's 1964 paper [19], if one invest on a risky stock portfolio and risk-free rate, then his return and risk relationship can be expressed as a line shown below. In his paper, Sharpe switched the meaning of x and y axis, in which x axis means return and y axis means risk. As we can see P is the risk-free rate since it locates on x axis. The shadow part represents the cloud of attainable stock portfolio while portfolio A, B, Y and Q are all on the efficient frontier and portfolio X is not efficient. If one choose to invest between P and A, then points on the line PA are attainable if some money is loaned at the risk-free rate and the rest placed in A. Similarly, by borrowing at the risk-free rate and investing in A, the segment of AY and beyond Y can be achieved. However, portfolio A is not the optimal. Among all the CALs, investing along line PZ generates the highest return at the expense of the same risk. So line PZ is the optimal CAL and portfolio Q is the optimal portfolio. In modern portfolio theory, a simple assumption that investors have homogeneous expectation is made. And portfolio Q is also called the market portfolio since all the wise investor will only invest in portfolio Q.

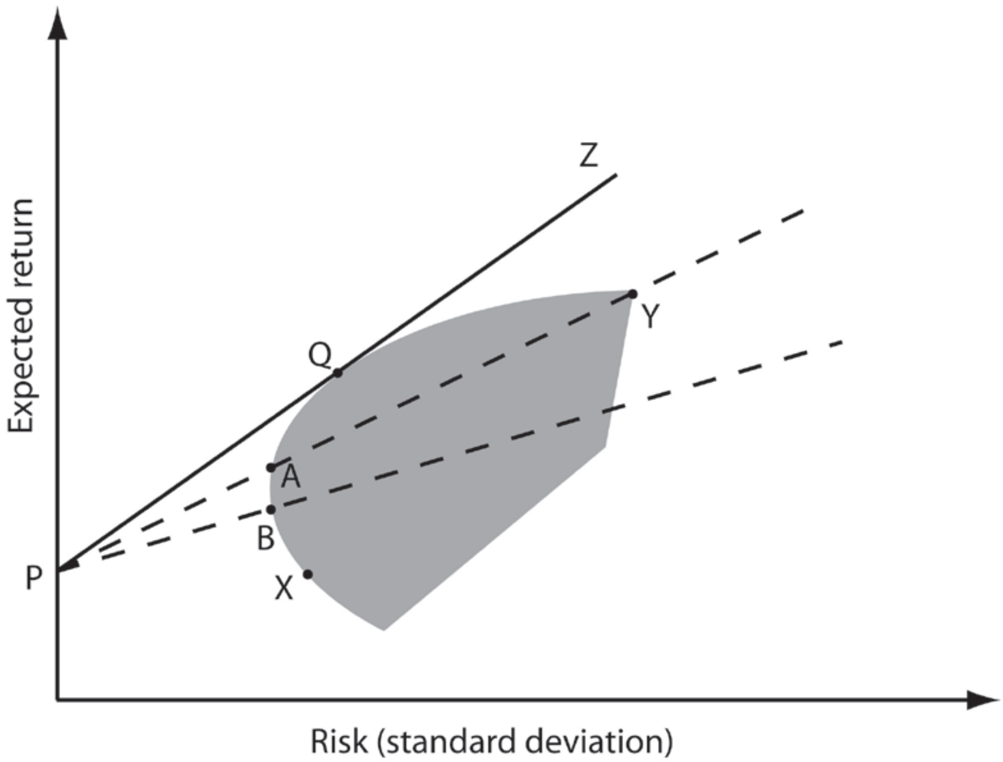

Figure 2.2 Capital allocation line (CAL)

So the task here is to use different models to find the market portfolio. Because we construct portfolio using the historical data and measure the performance in the next period, the key point is how people expect the future return and the relationship between stocks, which is the main underlying difference between these models.

Assume the risk-free lending and borrowing exists with the rate 0.01% and short sales allowed. The following models are considered to construct the optimal portfolios.

2.4 Classic Markowitz model

Markowitz is the father of modern portfolio theory. He proved the fundamental theorem of mean variance portfolio theory that is holding constant variance maximize expected return, and holding constant expected return minimize variance. The classic Markowitz model considers only the mean return and variance of return of a portfolio from historical data, ignoring the other moments such as skewness or other more realistic descriptions of the distribution of return. [11] The optimal portfolio lies on optimal CAL which can be expressed as [19]

$$\overline{R_p} = R_f + (\frac{\overline{R_A} - R_f}{\sigma_A})\sigma_p$$

The solution is to find the point of tangency of this line to the efficient frontier as shown in Fig. 2.2, which maximize the slope of the line

$$\max \theta = \frac{\overline{R_p} - R_f}{\sigma_p}$$

subject to the constaint

$$\sum x_i = 1$$

In this model, it only considers the return distributions over a single period. In addition, the correlations between all pairs of assets being considered need to be estimated. One major problem is when it applied to multi-period the estimation of variance covariance matrix has uncertainty. So people developed different models to estimate the correlation coefficients as the input of classic Markowitz model, such as single index model and multi-group model.

2.5 Single index model

This model employs a market proxy and considers all the fluctuations of stocks are correlated with the market while the residual fluctuations between stocks are uncorrelated. That means the individual stocks are related through the market. Another important characteristic of this model is that the number of estimates required is reduced and the accuracy of portfolio optimization is increased. [11]

Using single index model [20], the return of a stock can be expressed as a linear function of market return.

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

where R_{it} is the return of stock i at time t and R_{mt} is the return of the market at time t. The assumptions of this model is

$$E(\epsilon_i) = 0, var(\epsilon_i) = \sigma_{\epsilon_i}^2, cov(R_m, \epsilon_i) = 0, var(R_m) = \sigma_m^2, E(R_m) = \overline{R_m}$$

One can estimate the variance covariance matrix using the historical data and input it to the classic Markowitz model. Or one can use a simple ranking algorithm developed by Elton [21].

The calculation of optimal portfolio by the simple ranking algorithm has three steps.

1) Rank the securities based on the excess return to beta ratio

$$\frac{\overline{R_i} - R_f}{\beta_i}$$

2) Calculate the cut-off point C*. Those for which the excess return to beta is greater than the cut-off point C* will be held long. Otherwise, it will be held short.

$$C^* = \frac{\sigma_m^2 \sum_{j=1}^{N} \frac{(\overline{R_j} - R_f)\beta_j}{\sigma_{\epsilon_j}^2}}{1 + \sigma_m^2 \sum_{j=1}^{N} \frac{{\beta_j}^2}{\sigma_{\epsilon_j}^2}}$$

3) The proportion of the funds invested in each stock can be calculated by

$$z_i = \frac{\beta_i}{\sigma_{\epsilon_i}^2}\left(\frac{\overline{R_i} - R_f}{\beta_i} - C^*\right)$$

$$x_i = \frac{z_i}{\sum_{i=1}^{N} z_i}$$

The advantage of single index model is that it dramatically reduced the parameters needed in the model when the number of stocks is large, because it considers the correlation between any pair of stocks through the market proxy reducing the correlation need to be estimated. However, single index may have limited power in explaining the reality. So people explored multi-index model by extracting index using factor analysis or principal components analysis [22-25] which is out of the scope of this study.

2.6 Multi-group model

Elton pointed out that the main reasons why portfolio theory has not been implemented are the difficulty in estimating the input data necessary, the time and cost to solve the optimization problem, and the difficulty of educating portfolio managers. [21] He said that using the constant correlation between stocks can simplify the process by employing one number to estimate all the pair-wise correlation. However, doing this seems arbitrary since companies in different industries may differ so much from companies within one industry. So he developed a more complex model, multi-group model, which groups companies by the industry they belong to. [26]. Assume the correlations for all pairs of stocks within group are the same ρ_{ii} and between groups are the same ρ_{ij} .

So if use two groups and six stocks for example, the correlation matrix looks like

$$\rho = \begin{pmatrix} 1 & \rho_{11} & \rho_{11} & \rho_{12} & \rho_{12} & \rho_{12} \\ \rho_{11} & 1 & \rho_{11} & \rho_{12} & \rho_{12} & \rho_{12} \\ \rho_{11} & \rho_{11} & 1 & \rho_{12} & \rho_{12} & \rho_{12} \\ \rho_{21} & \rho_{21} & \rho_{21} & 1 & \rho_{22} & \rho_{22} \\ \rho_{21} & \rho_{21} & \rho_{21} & \rho_{22} & 1 & \rho_{22} \\ \rho_{21} & \rho_{21} & \rho_{21} & \rho_{22} & \rho_{22} & 1 \end{pmatrix}$$

As long as we get the variance covariance matrix estimate, we can use it as the input to solve the tangency to the efficient frontier using the classic Markowitz model as described in Section 2.4.

Elton showed that although this model is a simplification of problem but it is efficient in both time and cost.

2.7 Equally weighted model

Victor, Lorenzo and Raman in their 2007 paper said that equally weighted portfolio outperforms the portfolios by 14 models in Sharpe ratio. [13] The reason for this might be the estimation error on the input to the model used.

Using this strategy, the proportion of each stock is just $\frac{1}{N}$ where N is the number of stocks without any optimization or estimation. Victor, Lorenzo and Raman also mentioned that this strategy can be thought of imposing a restriction that expected returns are proportional to total risk when estimating the expected return and variance covariance matrix. [13]

2.8 Bayesian model with noninformative prior

All the models above do not involve the preference of the investor. If investor has his own prediction on the movement of stocks, then Bayesian model is preferred, in which prediction can be described by prior distribution. The return of stocks can be considered as multivariate normal distribution given expectation **μ** and variance covariance matrix ∑ [27].

$$R_t|\mu,\Sigma \sim MN(\mu,\Sigma)$$

$$p(R_t|\mu,\Sigma) = (2\pi)^{-\frac{K}{2}}(\det(\Sigma))^{-\frac{1}{2}}\exp\left(-\frac{1}{2}(R_t-\mu)^T\Sigma^{-1}(R_t-\mu)\right)$$

where t is the time spot and K is the stock number.

If the prior distribution of **μ** and ∑ is given p(**μ**, ∑), then the posterior prediction for **μ** and ∑ is

$$p(\mu,\Sigma|R_t) \propto p(R_t|\mu,\Sigma)p(\mu,\Sigma)$$

$$p(\mu,\Sigma|D_t) \propto (\det(\Sigma))^{-\frac{t}{2}}\exp\left(-\frac{1}{2}\sum_{i=1}^{t}(R_i-\mu)^T\Sigma^{-1}(R_i-\mu)\right)p(\mu,\Sigma)$$

where $D_t = (R_1, R_2, \dots, R_t)$. So the predictive distribution of returns is given by

$$p(R_{t+1}|D_t) = \int p(R_{t+1},\mu,\Sigma|D_t)\,d\mu d\Sigma = \int p(R_{t+1}|\mu,\Sigma)p(\mu,\Sigma|D_t)d\mu d\Sigma$$

$$\propto \int p(R_{t+1}|\mu,\Sigma)(\det(\Sigma))^{-\frac{t}{2}}\exp\left(-\frac{1}{2}\sum_{i=1}^{t}(R_i-\mu)^T\Sigma^{-1}(R_i-\mu)\right)p(\mu,\Sigma)d\mu d\Sigma$$

The noninformative prior can be chosen as

$$p(\mu, \Sigma) \propto (\det(\Sigma))^{-\frac{K+1}{2}}$$

So the predictive distribution of future returns is multivariate t with

$$p(R_{t+1}|D_t) \propto \left|(t-1)\hat{\Sigma} + \frac{t}{t+1}(R_{t+1} - \bar{R})(R_{t+1} - \bar{R})'\right|^{-\frac{K+1}{2}}$$

where $\bar{R} = \frac{1}{t}\sum_{i=1}^{t} R_i$ and $\hat{\Sigma} = \frac{1}{t-1}\sum_{i=1}^{t}(R_i - \bar{R})(R_i - \bar{R})'$. Therefore, the expectation of the return is unchanged $E(R_{t+1}|D_t) = \bar{R}$ and the variance covariance matrix is [28]

$$\Sigma_p = var(R_{t+1}|D_t) = (1 + \frac{1}{t})\frac{t-1}{t-K-2}\hat{\Sigma}$$

Because Σ_p is proportional to the standard maximum likelihood estimator $\hat{\Sigma}$ and the return expectation is unchanged, the resulted portfolio composition is the same as the result obtained using the classic Markowitz model in Section 2.4.

2.9 Bayesian model with informative prior

Noninformative prior does not put any preference on the stocks while informative prior can be chosen that indicates the investor's own prediction. Then in this hierarchical model the prior can be specified by the conditional structure

$$\mu|\Sigma \sim N(\mu_0, \Sigma)$$

$$\Sigma \sim Wishart^{-1}(\nu, S)$$

The prior hyper-parameters (μ_0, ν, S) determine the portfolio manager's strength of belief in the expected return and variance covariance matrix. Under this prior, the predictive expected return and variance covariance matrix is given by [27]

$$E(R_{t+1}|D_t) = \frac{(t \times \bar{R} + \mu_0)}{t}$$

$$\Sigma_p = var(R_{t+1}|D_t) = C\left(\hat{\Sigma} + S + \mu_0\mu_0^T + t\bar{R}\bar{R}^T - tE(R_{t+1}|D_t)E(R_{t+1}|D_t)^T\right)$$

$$C = (1 + \frac{1}{t})\frac{\nu + t}{(\nu + t - 2)(t - K - 2)}$$

Then using $E(R_{t+1}|D_t)$ and Σ_p, the optimal portfolio can be found by method in Section 2.4. The choice of the hyper-parameters is quite subjective. If the investor has correct perfect prediction, then the resultant portfolio will outperform the market definitely. Otherwise, it depends on how well the investor can predict. In this study, two cases will be considered. One is to use the next period data and to predict perfectly. The other is to predict poorly, using identity matrix for variance covariance and historical average return as the expected return. And two portfolios will be compared.

2.10 Summary of portfolio composition

The summary of the portfolio composition using above models is shown in table 2.3. Total wealth is assumed to be 1. And because short sales are allowed, some of the proportions can be negative. From the table we can see that the classic Markowitz model generates the most fluctuation in the portfolio composition with the highest proportion 0.8678 and the lowest proportion -0.9174. As contrast, the equally weighted model generates the least fluctuation while other models are in between. It is because the covariance between paired stocks average out in the single index model and multi-group model. And in Bayesian models, the prior for variance covariance matrix is unlikely to have extreme values. Intuitively, it is unlikely to put all the money on one stock or to completely exclude one stock when people know little about it.

Table 2.3 Portfolio composition by different models

	x_classic	x_sim	x_mgm	x_equal	x_bayes_ noninfo	x_bayes_i nfo_1	x_bayes_i nfo_2
AAPL	0.8678	0.5672	0.5188	0.0400	0.8678	0.1780	0.4211
IBM	0.7004	0.5947	0.5437	0.0400	0.7004	0.0579	0.5416
INTC	-0.7624	-0.0138	-0.0209	0.0400	-0.7624	0.0249	-0.0911
MSFT	0.0756	0.0039	-0.0438	0.0400	0.0756	0.0218	-0.1662
AMD	0.0077	-0.0147	-0.0388	0.0400	0.0077	0.0359	-0.0868
GG	0.6657	0.1457	0.0382	0.0400	0.6657	0.1218	0.3558
NAK	-0.0652	0.0540	-0.0860	0.0400	-0.0652	0.0755	-0.0112
POT	0.0012	0.2941	0.2415	0.0400	0.0012	0.1644	0.1125
SSRI	-0.6178	0.0004	-0.1624	0.0400	-0.6178	0.0286	-0.3932
SCCO	0.1666	0.3333	0.1980	0.0400	0.1666	0.1397	0.1089
C	-0.3812	-0.3531	-0.2784	0.0400	-0.3812	-0.2339	-0.2834
HBC	-0.5110	-0.2224	-0.1470	0.0400	-0.5110	-0.0195	-0.3330
MS	-0.4445	-0.2624	-0.2761	0.0400	-0.4445	-0.0709	-0.3027
AXP	0.1534	-0.0627	-0.0055	0.0400	0.1534	0.0203	0.2408
BAC	0.2354	-0.1194	-0.0792	0.0400	0.2354	-0.0293	0.1793
KO	0.0140	0.5199	0.6206	0.0400	0.0140	0.0464	0.5753
PEP	-0.9174	-0.0896	0.0787	0.0400	-0.9174	0.0079	-0.7339
KFT	0.5265	-0.2421	-0.1380	0.0400	0.5265	-0.0097	0.1943
FARM	0.0198	0.0394	0.1152	0.0400	0.0198	0.0248	0.0097
REV	-0.0329	0.0806	0.1098	0.0400	-0.0329	0.2276	-0.0276
JNJ	-0.3101	-0.0963	-0.0076	0.0400	-0.3101	0.0115	-0.3581
DCTH	0.1776	0.1325	0.1296	0.0400	0.1776	0.1410	0.0925
PFE	0.4325	-0.4977	-0.4109	0.0400	0.4325	-0.0202	0.0725
ABT	0.6072	0.2084	0.1160	0.0400	0.6072	0.0205	0.7356
EXEL	0.3912	0.0003	-0.0155	0.0400	0.3912	0.0349	0.1473

Fig. 2.3 illustrates all the portfolios on the return-risk plot. Only the classic Markowitz portfolio locates on the efficient frontier. It is because the efficient frontier is constructed using the historical data that exactly used as input by classic Markowitz model. Some assumptions made by other models drive the expected return or variance covariance matrix as inputs away from the historical value, which makes the portfolio away from the tangency

of the efficient frontier. But this does not mean the portfolios are not efficient. Whether the model is useful or not should be judged using out-of-sample data and by some other parameters discussed in Chapter 3.

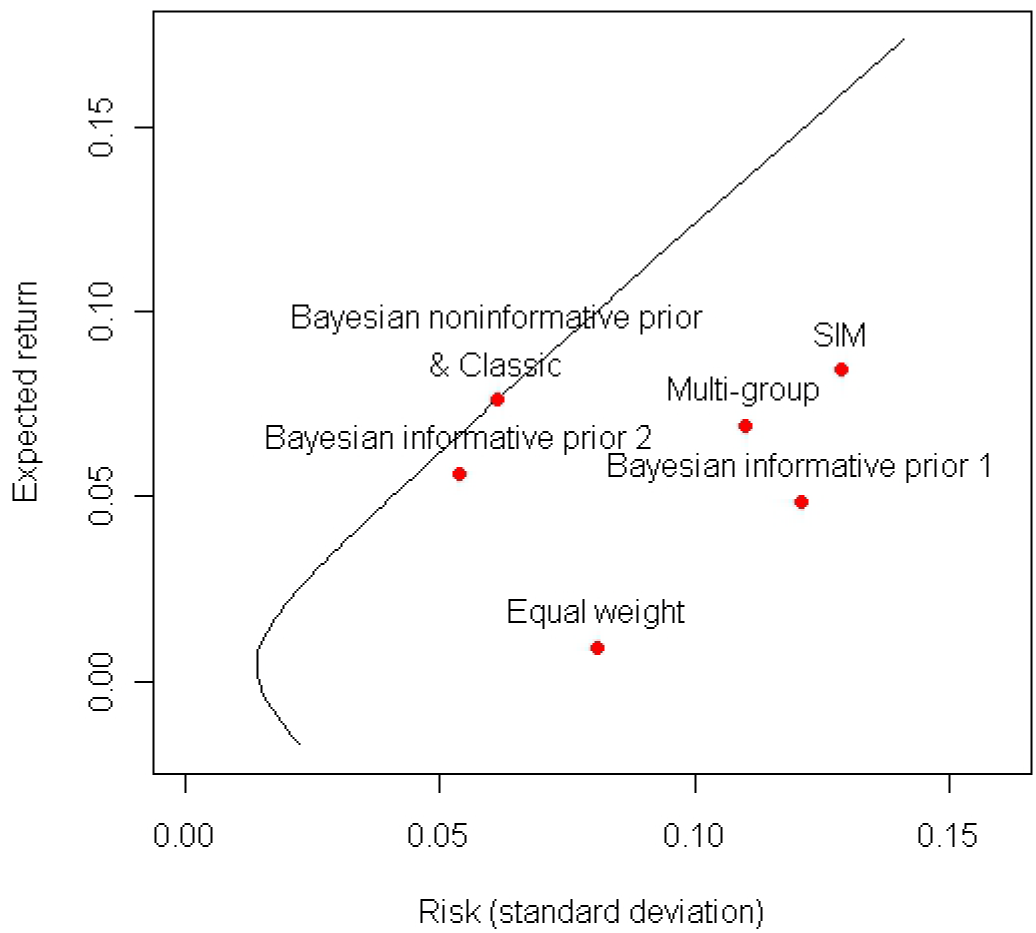

Figure 2.3 Illustration of all the constructed portfolios on return-risk plot

The expected return and portfolio risk as standard deviation is shown in table 2.4.

Table 2.4 Expected return and risk of portfolios

	risk	return
classic	0.0617873	0.0768051
sim	0.1291018	0.0846229
mgm	0.1101773	0.0691142
equal	0.0808054	0.0086957
bayes_noninfo	0.0617873	0.0768051
bayes_info_1	0.1211359	0.0484694
bayes_info_2	0.053708	0.0562133

So far we have construct 7 portfolios, in which the composition of the classic Markowitz portfolio and the Bayesian noninformative prior portfolio are exactly the same. So we will ignore the Bayesian noninformative prior portfolio in the following chapter. In the next chapter, we will take out-of-sample data to test which portfolio gives better performace.

Chapter 3 Out-of-Sample Study

3.1 Study objective

Out-of-sample study has been performed to compare the risk-adjusted returns of portfolios constructed in Chapter 2. The study is conducted by employing monthly return data from January 2010 to December 2011. The purpose of the study is to investigate which portfolio construction strategy outperforms others and whether it is useful for tactical asset allocation. The statistic summary of the data in this period can be found in Table 3.1.

Table 3.1 Statistic summary of data for January 2010 to December 2011

Ticker	Average Return	Risk (standard deviation)
AAPL	0.02971956	0.2759133
IBM	0.016622104	0.2025124
INTC	0.012459457	0.2796206
MSFT	-0.002451406	0.3008658
AMD	-0.014620128	0.4519047
GG	0.010352792	0.2327229
NAK	0.012097976	0.5648561
POT	0.013352818	0.3182764
SSRI	-0.007615403	0.4082401
SCCO	0.008694919	0.4130049
C	-0.004166464	0.3921553
HBC	-0.011724494	0.2968206
MS	-0.020152666	0.4235802
AXP	0.010201432	0.3019058
BAC	-0.034652186	0.3754697
KO	0.01176165	0.1864108
PEP	0.00690118	0.1824112
KFT	0.016995595	0.183904
FARM	-0.028279946	0.3159098
REV	0.005654706	0.4145067
JNJ	0.004473482	0.1790721

DCTH	0.019769931	0.5413247
PFE	0.01222696	0.2242739
ABT	0.005505359	0.1921184
EXEL	0.006703688	0.5521267
SPY	0.007656358	0.2894599

3.2 Portfolio performance measurement methods

To measure the performance of portfolio, people usually have three sets of tools. They are Treynor ratio, Sharpe ratio and Jensen's alpha [29]. All these methods combine the risk and return into a single value to judge the overall performance besides each is slightly different from the other.

Treynor ratio considers the excess return regarding the relative risk of the portfolio to the market. The formula is

$$Treynor\ Ratio = \frac{R_{portfolio} - R_{free}}{\beta_{portfolio}}$$

The higher the Treynor ratio, the better the performance is. However, because this measure only uses systematic risk, it pre-assumes that the investor already has an adequately diversified portfolio and unsystematic risk is not considered.

Sharpe ratio is similar to Treynor ratio except that the risk measure uses the absolute risk of the portfolio, the standard deviation. It can be expressed as

$$Sharpe\ Ratio = \frac{R_{portfolio} - R_{free}}{\sigma_{portfolio}}$$

As the same as Treynor ratio, the portfolio performance is better with higher Sharpe ratio.

Jensen’s alpha calculates the excess return that a portfolio generates over its expected return. It measures how much a portfolio’s return is above average return adjusted for market risk. It can be expressed as

$$Jensen's\ Alpha = R_{portfolio} - R_{free} - \beta_{portfolio} \times (R_{market} - R_{free})$$

The higher the value, the better the risk adjusted return is. Like the Treynor ratio, Jensen’s alpha uses beta, a systematic risk, and thus assumes the portfolio is already adequately diversified.

The three performance measures may yield substantially different performance rankings. So it really depends if we want to find an optimal measurement to evaluate portfolio performance. Generally speaking, to evaluate an entire portfolio held by an investor, the Sharpe ratio is appropriate. To evaluate securities or portfolios for possible inclusion in a broader or "master" portfolio, either the Treynor ratio or Jensen's alpha is appropriate. The Treynor ratio and Jensen's alpha are really very similar; the only difference being that the Treynor ratio standardizes everything, including any excess return, relative to beta.

3.3 Portfolio performance

During out-of-sample study period January 2010 to December 2011, the monthly returns of the constructed portfolios are shown in Fig. 3.1. The S&P500 monthly return is also plotted on the figure to represent the market return.

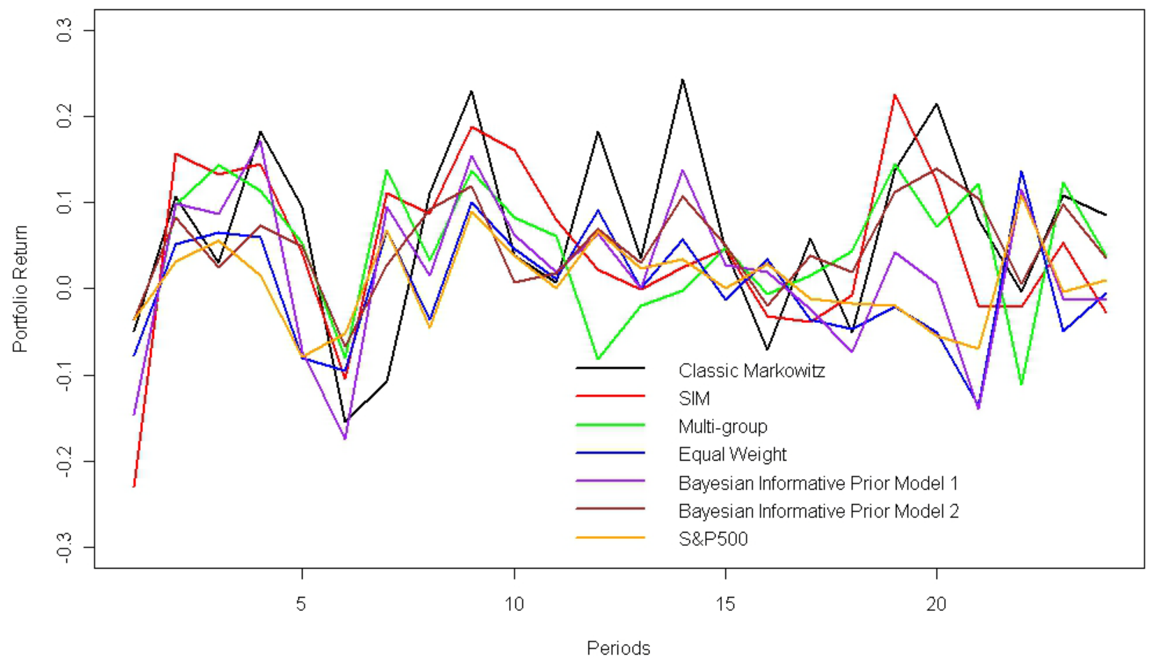

Figure 3.1 Monthly return of portfolios for January 2010 to December 2011 period

The portfolio return is quite fluctuated with the market that we cannot tell which one has higher return. If a two-year holding period is assumed, we could exam how much wealth will be built up for these portfolios. The relative wealth added as a percentage of initial wealth is calculated and illustrated in Fig. 3.2.

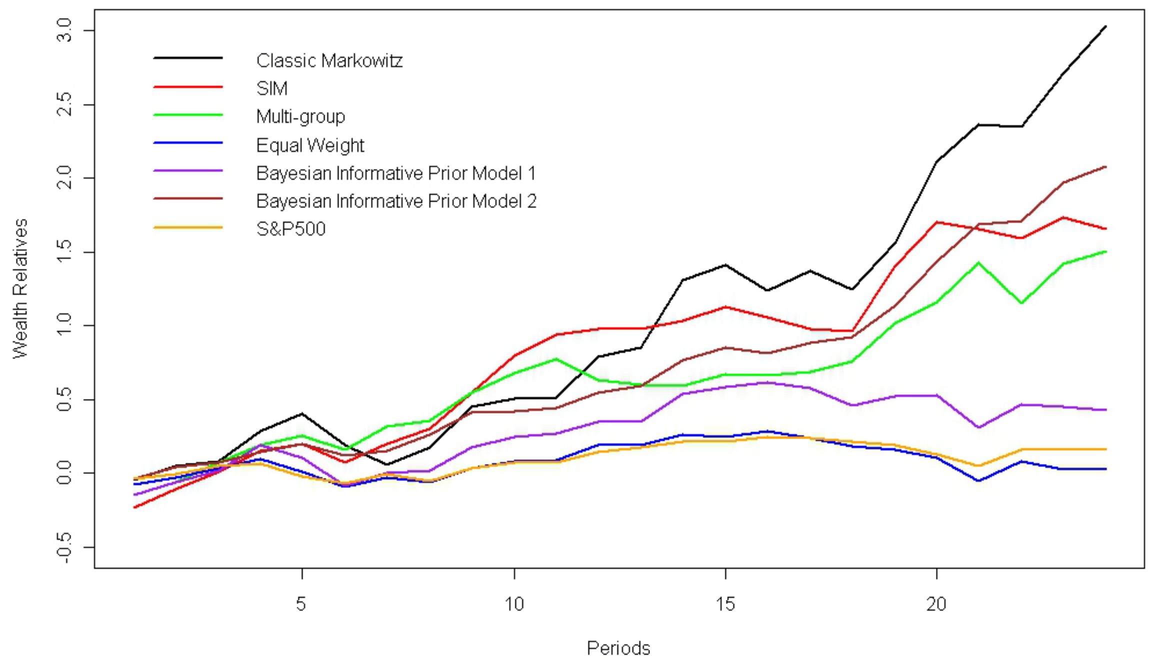

Figure 3.2 Relative wealth added as percentage of initial wealth

Although the wealth fluctuates, the overall trend is wealth increasing during this period. If one want to invest following the market, his wealth will only increase 17% of the initial wealth. The equally weighted portfolio performs even worse. It only increases 2% of the initial wealth. The highest wealth added is from classic Markowitz portfolio, which adds 3 times more wealth to the initial. Bayesian informative prior model 2 adds 2 times wealth. The next tier is SIM portfolio and Multi-group portfolio with 1.7 and 1.5 times wealth added respectively. Following is Bayesian informative prior model 1 adding 43% wealth to the initial.

Fig. 3.3, Fig. 3.4 and Fig. 3.5 shows the distribution for Treynor Ratio, Sharpe Ratio and Jensen's Alpha respectively.

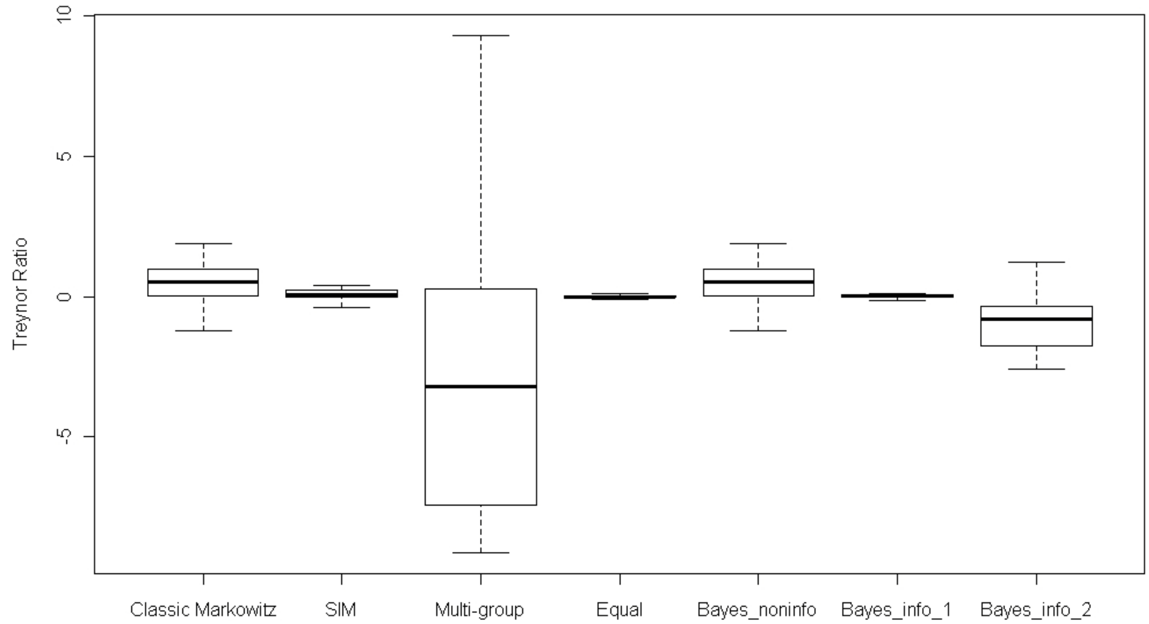

Figure 3.3 Distribution of Treynor Ratio

As we can see from Treynor ratio, the classic Markowitz portfolio has the highest mean value and the multi-group portfolio has the lowest value. And there exists extreme values in multi-group portfolio. One possible reason is the portfolio is not well diversified as required by the valid usage of Treynor ratio.

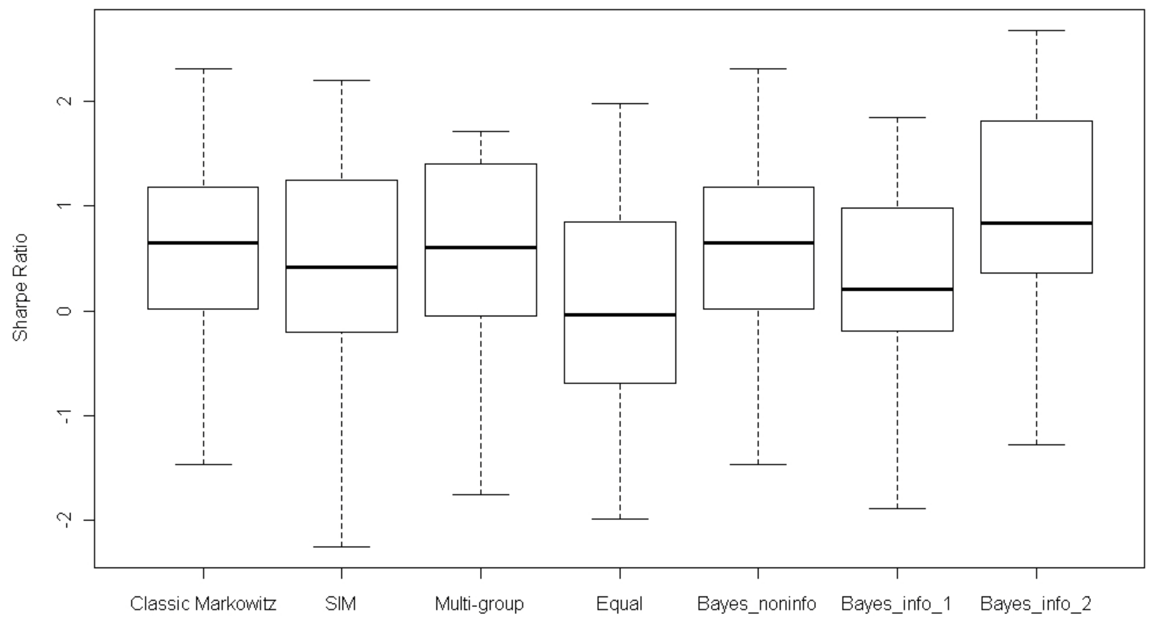

Figure 3.4 Distribution of Sharpe Ratio

By Sharpe ratio, the portfolio by Bayesian model with informative prior 2 gives highest value and the equally weighted portfolio has the lowest value. The mean values and ranges between different portfolios seem comparable and not extremely large or extremely small.

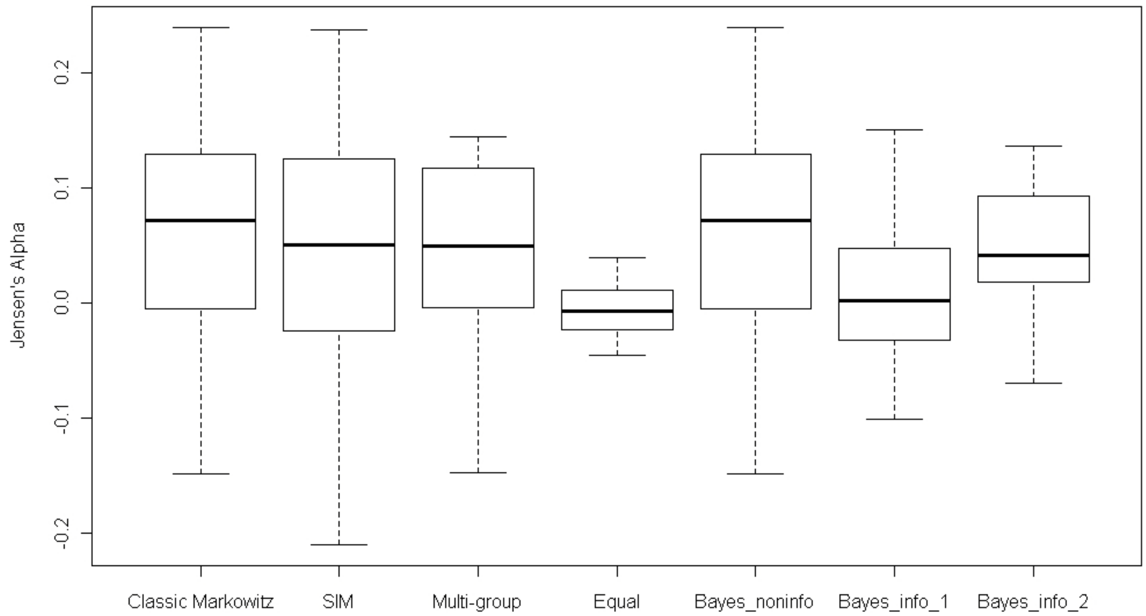

Figure 3.5 Distribution of Jensen's Alpha

Using Jensen's Alpha, we got similar ranking as that with Treynor ratio. It is because both of them use systematic risk as the compensation factor. But multi-group portfolio now is not the lowest ranking. It is because the beta of multi-group portfolio is almost zero which will enlarge the effect in the ratio form. And because Jensen's Alpha is measured by excess return form, not the ratio form, it has little extreme values.

3.4 Performance comparison

As an individual investor, one should look at Sharpe ratio to evaluate the best portfolio. According to the average Sharpe ratio for these portfolios, the ascending rank order is

Table 3.2 Sharpe ratio rank for different model portfolios

Bayesian informative prior model 2	Classic Markowitz model	Bayesian noninfomative model	Multi-group model	SIM	Bayesian informative prior model 1	Equally weighted model
0.945664	0.613759	0.613759	0.504530	0.453411	0.205229	0.044914

Hence, the Bayesian model with informative prior performs best and equally weighted model performs worst.

Chapter 4 Discussion and Conclusion

In this study, we first introduced and discussed different points of view of modern portfolio theory. Main issue is focused on how to judge a diversified portfolio. One group of people believes that there exists an optimal stock number making the marginal benefit equal the marginal cost of diversification. However, due to the different diversification strategies, the cost varies case by case. Thus, it is hard to achieve a universal consensus on the number of stocks that should be included. On the other hand, another group of people argue that instead of seeking the optimal stock number, one should ask the excess return to the current diversification level. And a good strategy is to maximize this excess return or to minimize the risk at a certain diversification level. Then we reviewed the portfolio management process and decided to focus on the execution step to construct portfolios by six strategies, which are classic Markowitz model, single index model, multi-group model, equally weighted model, Bayesian portfolio selection including noninformative prior approach and informative prior approach. The details of each strategy can be found in Chapter 2. Monthly return data from yahoo finance online are used. “stockPortfolio” package in R is used for construct the portfolios employing different models. As to the measurement of the portfolio performance, three parameters are defined, which are Treynor Ratio, Sharpe Ratio and Jensen’s Alpha. Among these parameters, Treynor Ratio and Jensen’s Alpha are using Beta as the risk measurement by assuming an already diversified portfolio and are suitable for evaluating mutual funds, portfolio manager and inclusion or exclusion of stock to the existing portfolio. Sharpe Ratio is suitable for individual investor since the standard deviation is used for the measurement of risk. Then assuming as an individual investor, according to the Sharpe Ratio measurement, portfolio constructed by Bayesian model with informative prior outperforms others, while equally weighted portfolio performs worst.

Although five industries are included, the choice of individual stock is random as a naïve investor. One can include more industries to future diversify the portfolio. Or one can vary the stock number based on one's own transaction cost consideration. The dataset is from monthly stock return. To include more information, one can instead use daily return data. One shortcoming of the study is that we did not consider a continuous rollover portfolio performance, which means the rebalance period and cost are not considered. But as long as we can get specific transaction cost, it can be easily included. The performance measurement is on a two-year period because after two years, the correlation and return information are usually changed and should be re-estimated. Moreover, based on IPS (investment purpose statement), the investment object and constraints should be updated which may cause the re-construct of the portfolio. In order to get outperformed portfolio, the estimate of expected return and variance covariance matrix is very important for Bayesian model with informative prior. In this study, we demonstrate the usage of different models in the portfolio construction and get the conclusion that Bayesian model with informative prior which is the accurate prediction of the future outperforms others. But this does not mean other models are useless. Obviously we could see all the portfolios outperform the market from relative wealth added. But because of the complexity and the ability to access the accurate data, people may choose different models by its own limitation.

Bibliography

[1] wikipedia, http://en.wikipedia.org/wiki/Modern_portfolio_theory

[2] Markowitz, H. (1952). "Portfolio Selection." The Journal of Finance 7(1): 77-91.

[3] Lintner, J. (1965). "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." The Review of Economics and Statistics 47(1): 13-37.

[4] Sharpe, W. F. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." The Journal of Finance 19(3): 425-442.

[5] Treynor, J. L. and F. Black (1973). "How to Use Security Analysis to Improve Portfolio Selection." The Journal of Business 46(1): 66-86.

[6] Mossin, J. (1968). "Optimal Multiperiod Portfolio Policies." The Journal of Business 41(2): 215-229.

[7] Cresson, J. E. (2002). "R2: A Market-Based Measure of Portfolio and Mutual Fund Diversification." Quarterly Journal of Business and Economics 41(3/4): 115-143.

[8] Evans, J. L. and S. H. Archer (1968). "DIVERSIFICATION AND THE REDUCTION OF DISPERSION: AN EMPIRICAL ANALYSIS*." The Journal of Finance 23(5): 761-767.

[9] Statman, M. (1987). "How Many Stocks Make a Diversified Portfolio?" The Journal of Financial and Quantitative Analysis 22(3): 353-363.

[10] Amihud, Y. and H. Mendelson (1986). "Liquidity and Stock Returns." Financial Analysts Journal 42(3): 43-48.

[11] Elton, E. J. and M. J. Gruber (1997). "Modern portfolio theory, 1950 to date." Journal of Banking & Finance 21(11–12): 1743-1759.

[12] Samuelson, P. A. (1967). "General Proof that Diversification Pays." Journal of Financial and Quantitative Analysis 2(01): 1-13.

[13] DeMiguel, V., L. Garlappi and R. Uppal (2009). "Optimal versus Naive Diversification: How Inefficient Is the 1/N Portfolio Strategy?" The Review of Financial Studies 22(5): 1915-1953.

[14] Wit, D. P. M. D. (1998). "Naive Diversification." Financial Analysts Journal 54(4): 95-100.

[15] investopedia, http://www.investopedia.com/exam-guide/cfa-level-1/portfolio-management/portfolio-management-process.asp#axzz24FwjUtm0

[16] Diez, D. and N. Christou (2012). "Package 'stockPortfolio' version 1.2 manual."

[17] wikipedia, http://en.wikipedia.org/wiki/Short_(finance)

[18] Markowitz, H. (1952). "Portfolio Selection." The Journal of Finance 7(1): 77-91.

[19] Sharpe, W. F. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." The Journal of Finance 19(3): 425-442.

[20] Sharpe, W. F. (1967). "A Linear Programming Algorithm for Mutual Fund Portfolio Selection." Management Science 13(7): 499-510.

[21] Elton, E. J., M. J. Gruber and M. W. Padberg (1976). "Simple Criteria for Optimal Portfolio Selection." The Journal of Finance 31(5): 1341-1357

[22] Roll, R. and S. A. Ross (1980). "An Empirical Investigation of the Arbitrage Pricing Theory." The Journal of Finance 35(5): 1073-1103.

[23] Dhrymes, P. J., I. Friend and N. B. Gultekin (1984). "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory." The Journal of Finance 39(2): 323-346.

[24] Brown, S. J. and M. I. Weinstein (1983). "A New Approach to Testing Asset Pricing Models: The Bilinear Paradigm." The Journal of Finance 38(3): 711-743.

[25] Cho, D. C., E. J. Elton and M. J. Gruber (1984). "On the Robustness of the Roll and Ross Arbitrage Pricing Theory." Journal of Financial and Quantitative Analysis 19(01): 1-10.

[26] Elton, E. J., M. J. Gruber and M. W. Padberg (1977). "Simple Rules for Optimal Portfolio Selection." The Journal of Finance 12(3): 329-345

[27] Polson, N. G. and B. V. Tew (2000). "Bayesian Portfolio Selection: An Empirical Analysis of the S&P 500 Index 1970-1996." Journal of Business & Economic Statistics 18(2): 164-173.

[28] Frost, P. A. and J. E. Savarino (1986). "An Empirical Bayes Approach to Efficient Portfolio Selection." The Journal of Financial and Quantitative Analysis 21(3):293-305

[29] investopedia, http://www.investopedia.com/articles/08/performance-measure.asp#axzz24FwjUtm0