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A Bird's-Eye View of Research Practices in Mathematical Cognition, Learning, and Instruction: Reimagining the Status Quo

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Abstract

Research on mathematical cognition, learning, and instruction (MCLI) often takes cognition as its point of departure, and considers instruction at a later point in the research cycle. In this paper, we call for psychologists who study MCLI to reflect on the "status quo" of their research practices and to consider making instruction an earlier and more central aspect of their work. We encourage scholars of MCLI (1) to consider the needs of educators and schools when selecting research questions and developing interventions; (2) to compose research teams that are diverse in the personal, disciplinary, and occupational backgrounds of team members; (3) to make efforts to broaden participation in research and to conduct research in authentic settings; and (4) to communicate research in ways that are accessible to practitioners and to the general public. We argue that a more central consideration of instruction will lead to shifts that make research on MCLI more theoretically valuable, more actionable for educators, and more relevant to pressing societal challenges.

Many psychologists who work on issues related to mathematics learning use the catchphrase: *"from cognition to instruction."* When applied to mathematical cognition and instruction, this catchphrase suggests a sequential ordering in scientific investigations, with cognition as the point of departure. Many psychologists who work in this area (including many members of the current author team) initially center their work around questions about mathematical cognition and how it develops. At a later point in the research process (or as a side issue), they may consider how knowledge about cognition can be used to support mathematical learning, for example, by guiding the design of instructional activities and curricular materials. Thus, for psychologists who work in the field of mathematical cognition, learning, and instruction (henceforth, *MCLI*), research has a typical directionality, often beginning with cognition and subsequently considering instruction.

In this paper, our broad goal is to consider how this directionality has shaped the "status quo" of research practices for psychologists who study mathematical cognition and learning. Our views are informed by specific studies and programs of research; however, our aim is not to review the research on mathematical cognition and instructional interventions. Instead, we contemplate the implications of proceeding *from* mathematical cognition *to* mathematics instruction by taking a bird's-eye view of typical research practices in the field. We consider what this directionality implies for the field, and we consider the implications for our own research practices and those of other scholars who study MCLI.

We consider four aspects of research practices in the field: how psychologists who study MCLI (1) formulate research questions, (2) compose research teams, (3) select research participants and settings, and (4) communicate the results and products of completed research. Within each section, we seek to characterize and reflect on the "status quo" for psychologists who conduct research on MCLI, and we consider ways to move beyond that status quo. Of course, there are many historical and practical factors that have shaped traditional and current practices in the field, and these practices have yielded many important findings and informed many successful interventions. Our aim is not to encourage scholars in

the area to abandon current practices, but rather to spur them to reflect on those practices, to consider how those practices align with their goals, and to consider shifts in their practices, as appropriate. We also acknowledge that the status quo for scholars who approach this intersection of fields from other disciplinary perspectives (e.g., from mathematics education) is likely to be different from that described here, because they start from other disciplinary assumptions and goals.

To preview our primary themes, we argue that a focus on research that proceeds *from* cognition *to* instruction can be limiting and that a more central consideration of instruction may lead to valuable shifts in the research—particularly at this moment in time, when educational systems are facing serious challenges due to the aftermath of pandemic-related school closures, and due to long-standing and increasing inequities in learning opportunities and outcomes. In this paper, we encourage psychologists who study MCLI (1) to consider the needs of educators and schools when selecting research questions and targeting issues for intervention; (2) to compose research teams that are diverse in the disciplinary, occupational, and personal backgrounds of team members; (3) to make efforts to broaden participation in research and to conduct research in authentic settings; and (4) to communicate research in ways that are accessible to practitioners and the general public.

A bird's-eye view of research practices in mathematical cognition, learning and instruction

Figure 1 depicts a birds-eye view of a landscape. In the foreground of the image are some well-plowed fields. Behind those fields, there is a river and some thick forest, which might be difficult to get to or to move around in. Off in the distance are some mountains, which would surely be challenging to climb. In examining the field's research practices, we use this landscape as an instructive analogy. For each of the categories of research practices that we consider, some practices are familiar and easy to navigate, like the well-plowed fields. These are the "status quo"—the typical practices that are straightforward and commonly employed. Other research practices are a bit "farther off" in the sense that they are more challenging to navigate, like the thick forest. And still others are even more distant and more difficult to navigate—one might say "aspirational"—like the mountains in the distance. These

practices go beyond the status quo, are often challenging to implement, and are employed by relatively few scholars in the area.

Figure 1. A bird's-eye view of a landscape. The figure depicts a bird looking out over a landscape. In the foreground are some well-plowed fields. Behind the fields, there is a river and some thick forest, and in the distance are mountains. This image is used as the basis for an analogy throughout the paper, as described in the text. Original pencil drawing by Petrit Alibali; converted to line drawing by Janet Trembley.

In the sections that follow, we take this bird's-eye view of common research practices for psychologists who study MCLI. We consider which practices are like well-plowed fields, and which are more like the thick forest or the distant mountains. Our broad goal is to encourage scholars of MCLI to

consider adjusting some of their research practices in ways that are akin to moving towards the forest, and possibly, the mountains.

Selecting research questions and targeting issues for intervention

We start by considering how psychologists who study MCLI typically select their research questions. As noted at the outset, most scholars in this area (including many members of the author team) tend to follow the sequence "*from* cognition *to* instruction". That is, they often start with research questions that focus on some aspect of mathematical cognition or development. When they discover new things about how people reason or acquire knowledge about mathematics, they may then build on those ideas to generate potential interventions aimed at enhancing students' mathematical thinking and learning.

Research questions selected in this way are like well-plowed fields, in the sense that they center familiar topics and issues that are already fairly well understood. These research questions might derive from what is known about cognition and learning in specific mathematical content areas or questions derived from theories of cognition, learning, or cognitive development, more generally.

Status quo: Research questions that address cognition in specific mathematical content areas

Many psychologists who study MCLI investigate cognition and learning in specific mathematical content areas, such as in arithmetic, fractions, algebra, or geometry. These scholars then sometimes generate approaches to interventions based on the knowledge they gain about cognition and learning in those content areas.

As one example, many psychologists who study MCLI investigate how children and adults understand fractions. It is now well established, for example, that knowledge of natural number magnitudes can interfere with judgements of fraction magnitudes (e.g., Ni & Zhou, 2005), that people's sensitivity to non-symbolic ratios is related to their understanding of symbolic fractions (e.g., Matthews et al., 2016; Park & Matthews, 2021), and that understanding of operations on whole numbers can support understanding of operations on fractions (e.g., Sidney & Alibali, 2017). Scholars of MCLI have sought to

develop interventions for fraction learning that build on these findings about fraction understanding and how it develops (e.g., Dyson et al., 2020).

Along similar lines, many psychologists who study MCLI have investigated children's understanding of the equal sign as an indicator of mathematical equivalence. This body of research has revealed that many elementary students view the equal sign as meaning "put the answer" or "perform the given operations on the given numbers" (McNeil & Alibali, 2005), that there are challenges in helping children progress to understanding the equal sign as signifying a relation between the expressions on either side of the symbol (McNeil, 2014; McNeil et al., 2017), and that a relational understanding of the equal sign is associated with greater success in later mathematics, including in algebra (Hornburg et al., 2022; Knuth et al., 2006; Matthews & Fuchs, 2020). Intervention studies that build on these findings seek to prevent children from developing an entrenched view of the equal sign as signaling the answer (Davenport et al., 2023; McNeil et al., 2015, 2019).

Status quo: Research questions derived from general theories of cognition, learning, and development

Another way of developing research questions involves building from more general theories of cognition, learning, or cognitive development—theories that apply, not only in the domain of mathematical cognition, but more broadly as well. Psychologists who study MCLI using this approach select research questions that address general theoretical issues about learning and development as they apply specifically in mathematics, and these scholars sometimes generate ideas and research questions about potential interventions that are grounded in these theoretical perspectives.

As one example, some psychologists who study MCLI address research questions about connecting concrete and abstract representations. One general, cross-cutting principle in cognitive development is that people often learn things first through action, and then gradually build more abstract forms of thinking (Bruner, 1966; Piaget, 1970; see also Sfard & Linchevski, 1994). Scholars have developed and tested interventions that support learners in making connections at each point along this bridge, in a sequence termed "concreteness fading" or "concrete-representational-abstract" (e.g., Donovan

& Fyfe, 2022; McNeil & Fyfe, 2012). The general issue of connecting concrete and abstract representations is like a well-plowed field, where much is known, but where there is still important work to be done (Fyfe et al., 2014; Fyfe & Nathan, 2019).

As another example, psychologists who study MCLI often address research questions about how learners acquire conceptual understanding in mathematics. This cross-cutting issue has been the focus of a broad swath of research in cognitive and developmental psychology, and the field has a strong base of scientific knowledge about concept learning that spans a range of mathematical and scientific topics. This body of research has yielded important data about the sorts of activities that can help learners build conceptual understanding, such as studying worked examples, generating self-explanations, and using manipulatives (e.g., Barbieri et al., 2023; Donovan & Alibali, 2022; Rittle-Johnson, 2017). Scholars have sought to design instructional interventions for mathematics learning that draw on this knowledge (e.g., Booth et al., 2015; Star et al., 2015). This general issue is like a (somewhat) well-plowed field—much is known, but there is still work to be done, especially in regard to how what is known about concepts in general applies in mathematics.

To recap, for many psychologists who study MCLI, research progresses *from* cognition *to* instruction. Scholars tend to select research questions—and eventually, targets for intervention—either by building on existing knowledge about mathematical cognition, development, and learning, or by building on general theoretical accounts of cognition, development and learning. In many cases, the questions that scholars select are the burning theoretical questions that propelled those scholars to enter the field (e.g., where do new problem-solving strategies come from? how do symbols, such as language, influence the construction of knowledge?)—and for these scholars, addressing these basic questions is at the heart of their work. This basic work may inspire interventions to support mathematical cognition and learning, but developing interventions is rarely the initial impetus for the work.

In this context, it is also important to acknowledge that some research does proceed in the other direction—*from* instruction *to* cognition. Indeed, educators' identification of specific challenges that

mathematics learners face can spark research on the cognitive and instructional bases for those challenges. In our experience, however, this terrain is less well-plowed for scholars of MCLI, especially those who are trained as psychologists. There are multiple potential reasons for this state of affairs. First, psychologists who study MCLI are often unaware of the challenges that educators face and that educators wish to address—a point that we consider below. Second, there are broad disciplinary differences in the goals of research. Many educators who develop and test interventions focus on evaluating the interventions' outcomes, whereas many psychologists prioritize understanding the mechanisms that underlie the interventions' effects. These differing goals lead to different choices regarding research questions and targets for intervention. We turn next to the role of educational considerations in selecting and formulating research questions.

Moving beyond the status quo: Additional factors to consider in selecting research questions and targeting issues for intervention

Thus far, we have considered cognition and instruction in abstract terms, largely divorced from the many issues that face the educational system at large. Many believe, however, that this moment in historical time is a critical moment for education. Dr. Adam Gamoran, a prominent sociologist of education and the current director of the William T. Grant foundation, described the present moment in the United States in these terms:

> The history of education in the US is punctuated with exigent moments: points in time when the politics of the era intersect with the reality of schools in ways that demand action. … We have [now] arrived at one of these exigent moments: COVID-19 has introduced massive disruptions to learning, further increased inequality for learners, and resulted in unprecedented challenges for educators. These events—coupled with a new reckoning with America's legacy of racism—have challenged education systems to adopt, revise, rescind and reconsider policies of all kinds (Gamoran, 2022).

This exigent moment has many potential implications for the study of mathematical cognition, learning and instruction. We argue that this moment calls for changes in the "status quo" of research practices, including changes in how psychologists who study MCLI select their research questions.

One way that scholars may respond to this moment in time is by reflecting on how they select and formulate research questions. Psychologists who study MCLI have rarely been called upon to consider the immediate needs of learners, educators, or schools—but these needs have become ever more pressing in recent years. The COVID-19 pandemic disrupted learning and placed intense pressures on education systems at all levels. Many learners and their families experienced trauma, including illness, loss of family members and other loved ones, and serious financial and personal stress. Many learners experienced challenges in learning due to substantial time away from school and to systems ill-prepared to offer opportunities for virtual learning on a broad scale. These challenges manifested in different ways for different social groups, exacerbating pre-existing inequities and introducing new and complex challenges for learners, teachers, schools, and communities.

Education systems in the United States reflect and perpetuate the inequities across social groups that characterize American society (e.g., Ladson-Billings & Tate, 1995; Martin, 2019), and they may also have the potential to attenuate those inequities (Downey & Condron, 2016). These general principles apply not only to inequities in school funding and the composition of the labor force, but also to inequities in learning opportunities and learning outcomes. Many of these inequities have been exacerbated by the COVID-19 pandemic; for example, recent data from the United States National Assessment of Educational Progress (the NAEP, often called the "Nation's Report Card") indicates that racial and income-based gaps in test scores have widened during the pandemic, especially in mathematics (U.S. Department of Education, 2022). These findings underscore the great need for policy changes and interventions to address these inequities, and these findings should compel scholars of MCLI to consider how their work may contribute to addressing these issues. Of course, given psychology's general focus on individuals, most psychologists who study MCLI do not focus on interventions with curricula, school districts, or broader (state or national) education policy, but they instead tend to consider interventions that involve students, teachers and other educators, and caregivers. Although interventions at broader levels could undoubtedly have large impacts, we argue that interventions that involve students, educators,

and caregivers—those levels most likely to be addressed by scholars of MCLI—are also important to consider. Further, research on MCLI can involve developing or adapting curricula, which can have a large potential impact.

Given the challenges of this moment in time, we encourage psychologists who study MCLI to reflect on how they select research questions and how they decide what kinds of interventions to develop and test. In our view, the needs of learners, teachers, and schools should inform the choices that scholars make regarding what research questions to investigate and what interventions to develop—perhaps especially because of the consequences of the COVID-19 pandemic school closures, but also as a regular practice. If scholars with expertise in cognition and development turn their attention to the pressing needs of learners, educators, and schools, this might contribute to greater progress in addressing questions that have broad societal impacts. For example, how have disruptions to families' lives and disruptions in typical learning experiences affected students' readiness to learn, their abilities to manage their attention, and the stability and accessibility of the knowledge structures that they had built before the pandemic? Psychologists who study cognition and cognitive development have specific expertise that can contribute to addressing these questions—though addressing these questions may require them to move away from the well-plowed fields and into the forest or the mountains.

Research is needed on how children's developing mathematical knowledge was affected by disruptions to schooling, such as those caused by the move to online instruction during the early days of the pandemic, by challenges that arose due to changes in learners' daily routines and, in many cases, by experiences of trauma. During the pandemic and the associated school closures, many students felt disconnected from school, and many had difficulty sustaining attention to remote lessons and completing assigned work. Many teachers were unable to cover grade-level appropriate material fully and effectively (Namkung et al., 2022). These disruptions and challenges led to declines in academic achievement, gaps in knowledge, and shifts in attitudes towards school (Cingel et al., 2022; Engzell et al., 2021; Office for Civil Rights, U.S. Department of Education, 2021). Research is needed to more fully evaluate how these

issues have led to variations in paths of mathematical learning and development. Research is also needed on interventions to address gaps in knowledge and decreases in students' engagement in school. Some psychologists who study MCLI are already engaged in developing interventions to evaluate and address students' "unfinished learning." For example, some teams are collaborating with schools and community partners to connect large-scale tutoring programs to research on learning, and other teams are developing technology-based ways to assess student knowledge and to individualize instruction.

Remote instruction continues for some learners, and a remote learning environment is favorable for many learners, as it can provide increased accessibility features and flexibility. However, remote learning is also challenging or detrimental for some learners (Darling-Aduana et al., 2022; Fisher et al., 2022). There are pressing questions regarding the impact of remote instruction on mathematics learning and on attitudes towards mathematics. Thus, understanding and improving remote instruction is another arena in which the expertise of psychologists who study MCLI is greatly needed.

The pandemic conditions also brought certain mathematical skills to the forefront of people's daily lives. People encountered an influx of data of all sorts, and they needed to interpret and make sense of data on a regular basis—for example, to evaluate the risks of different settings and activities, and to make decisions about whether to get vaccinations or to wear masks in various settings. Appropriately evaluating health risks often requires considering, not only the *number* of cases, but also the *rate* of cases in the population, which requires reasoning with mathematical ratios, which is notoriously challenging for many people. Research is needed to understand how best to support learners in reasoning appropriately about case rates and other forms of data, and some scholars of MCLI have sought to develop interventions to respond to this need (Thompson et al., 2021).

In each of these critical areas (and others), psychologists who study MCLI have the potential to make important contributions—to understanding and ameliorating the impacts of learning under challenging conditions, to helping people build mathematical skills that are increasingly important in modern society (such as data interpretation), and to developing interventions that enhance learning and

make progress toward equity in learning outcomes. We encourage psychologists who study MCLI to consider the needs of learners, educators, schools, and society when selecting their research questions and when targeting ideas for interventions.

Some psychologists might question whether shifting towards research that addresses educational and societal needs could limit theoretical progress in understanding mathematical cognition and learning. Indeed, in some psychology departments, such research might be viewed by some as "too applied" or as not making a sufficient theoretical contribution for purposes of promotion and/or tenure evaluation. In contrast to this view, we believe that studying learning in a wide range of populations and in authentic instructional environments may lead to a deeper understanding of foundational learning processes and how they vary. Indeed, such research is critical for understanding "what works, under what conditions, and for whom" (Gutiérrez & Penuel, 2014, p. 22), and we believe that efforts to understand such variation will yield theoretical progress. As such, we believe that a focus on research that addresses educational and societal needs can inform theory and contribute to theoretical progress in ways that are valuable for the field (though we also acknowledge that the extent to which research reports are "theory-forward" or highlight theoretical contributions can also vary substantially).

In light of these considerations, it is important for early-career scholars to be mindful of the values and expectations of the institutions and departments within which they work, when deciding which research questions to pursue. Across career stages, scholars must regularly make decisions about how to apportion time and effort to different types of research questions and activities. Alignment with institutional values or expectations is one potentially relevant factor in these decisions, and the importance of this factor may vary over career stages.

In his influential work on the goals of scientific inquiry, Stokes (1997) argued that transformative basic science often emerges from use-inspired questions, and he cited Louis Pasteur's work in microbiology as an exemplar. In a similar vein, we encourage psychologists who study MCLI to consider "Pasteur's Quadrant"—that is, to consider "use-inspired basic research", which is motivated by the quest

for deep understanding, but that also explicitly addresses practical needs (see also Simplicio, et al., 2020). We do not mean to imply that such research is inherently more valuable than other forms of research (see Klahr, 2019, for discussion), but we believe that at this exigent moment, such research is greatly needed, on a wide range of topics.

Moving beyond the status quo: Understanding the needs of learners, teachers, and schools

This call for research that addresses broader educational and societal needs requires understanding those needs—both needs that have been exacerbated by the pandemic and the aftermath of school closures, and more long-standing needs. It may be straightforward to consider needs at a general level—but understanding the specific needs of learners and educators will require open communication between scholars and practitioners, including teachers, paraprofessionals, instructional coaches, guidance counselors, school social workers, principals, and school district personnel. For many psychologists who study MCLI, engaging in communication with these stakeholders is not part of the "status quo" of their everyday work. Bringing these voices into the process of selecting research questions and identifying targets for intervention will require that scholars move beyond well-plowed fields and into more challenging terrain. In many cases, scholars will need to enter the forests of communicating with school-based professionals who work directly with learners, such as teachers, paraprofessionals, instructional coaches, guidance counselors, and school social workers. In some cases, they may also need to climb the mountains of communicating with administrators and policy makers, such as principals and school district personnel.

It is widely observed that there is a disconnect between educational research and educational practice (Burkhardt & Schoenfeld, 2003; Ginsburg & Gorostiaga, 2001; Matthews et al., 2021). Although there are many potential sources for this disconnect, it is sometimes framed as an issue of scholars addressing questions that are not the questions that practitioners want to be answered (e.g., Farley-Ripple et al., 2018). In research that involves developing interventions, this disconnect can be likened to scholars developing interventions to address issues that practitioners do not view as priorities. This generates not

only a disconnect, but also missed opportunities for the field to have an impact, as there is more likely to be "buy-in" from teachers and schools if interventions focus on their top priorities. If psychologists who study MCLI wish to address research questions that are relevant to the needs of learners, educators, and schools, it is critical that there be channels of communication through which relevant questions can be identified, and their importance made clear. This may involve direct, informal communication with practitioners or more systematic inquiry into practitioners' ideas and needs—a step in the research process that psychologists who study MCLI seldom engage in, but that may bear fruit if it is included. Finding ways to identify and assess practitioners' needs can be challenging—and for many psychologists, this type of work requires navigating unfamiliar terrain, like the forests and mountains that can be seen from the bird's-eye view. At the same time, these types of interactions may highlight new research questions and open new opportunities for collaborating with practitioners.

As one example, in collaboration with a local school district, one member of our author team (Jay) recently interviewed a set of middle-school teachers, with the goal of considering teachers' views on students' conceptual understanding in algebra and algebra-readiness courses and teachers' opinions about how curricular materials could better support students' conceptual understanding. During these discussions, several of the teachers expressed their view that low arithmetic fluency was a critical barrier to learning advanced concepts and skills for many students. For example, teachers noted that students with low arithmetic fluency would sometimes get "stuck" on arithmetic operations when solving complex problems, and this seemed to prevent them from thinking deeply about underlying mathematical relationships in the problems. For Jay, these comments called to mind some observations from their previous, laboratory-based research (Jay & Alibali, in preparation), in which students sometimes had difficulties with equivalent fractions and algebraic transformations, and this seemed to prevent them from engaging deeply with the lesson content. Teachers' perspectives highlighted aspects of Jay's existing data that had previously not been a central focus, and these ideas are shaping the next steps in their research trajectory.

Conducting these interviews required Jay to move beyond the "status quo" of their previous research practices, and to communicate and engage with practitioners in ways that they had not done previously. This required preparatory work with the school district and new ways of engaging with practitioners. As such, these interviews represented a step away from well-plowed ways of selecting research questions based on previous work on mathematical cognition or on learning theories, and a step toward the more challenging forest of allowing teachers' ideas and needs to contribute to these questions.

As a second example, Shusterman and colleagues (2019) developed an early childhood numeracy curriculum based on recent work on early numeracy. The project was initially conceived as an effort to develop classroom materials for deaf and hard-of-hearing children who did not use sign language, but it gradually expanded to also target children at academic risk due to poverty. Through informal interactions with participating teachers, members of Shusterman's team learned that teachers were interested in having access to engaging and fun numeracy materials as well as information about how to integrate mathematics activities into their early childhood instruction. These impressions were confirmed in a qualitative interview study with the teachers, which was conducted by an undergraduate student in Shusterman's group as her senior honors thesis (Lo, 2014).

As these examples suggest, to understand the needs of teachers and schools, psychologists who study MCLI may need to engage with new methods that have historically been less widely used in psychology, including qualitative and mixed methods (Humphreys et al., 2021; Reischer & Cowan, 2020) and participatory research designs, which involve collaborations between researchers and community members (Chan et al., 2023). Psychologists may also face challenges in identifying potential sources of support for research that involves needs assessments. We encourage agencies and foundations that fund research to consider the potential value of such research when making funding decisions (see NASEM, 2022), and we encourage scholars to be creative in finding other ways to fund such work (e.g., via senior theses, research internship projects, or with resources from lab start-up funds or internal grants, as well as grants from foundations and federal agencies). In this regard, it is worth highlighting that in recent years,

federal agencies in the United States have begun to place greater weight on broader impacts of research and on dissemination plans that target practitioners and the public.

When researchers work with teachers to understand needs and identify areas of shared interest, teachers may share issues that arise in their classrooms for which there are already existing research-based practices. In such cases, researchers can connect teachers with existing research or research-based resources, and they may be able to support teachers in deciding whether and how to implement existing research-based practices in their classrooms.

Teachers may also be reluctant to implement research-based practices, because they may believe that implementing such practices will require substantial effort, or they may believe that such practices will not yield results in their classrooms, because their students are not like students who took part in the research. Some teachers may attempt to implement research-based practices in their classrooms and find that these practices do not work as they did in the published studies. At the same time, researchers may not appreciate the practical challenges that teachers face. More nuanced discussions are needed about how research findings may have differing implications for different subgroups of students, and about how implementation may need to be adjusted in different contexts or for different subgroups of students. Opportunities for communication between teachers and researchers may help to foster “common ground” on these issues. Such interactions may also highlight the inadequacy of dissemination efforts that do not address real-world, on-the-ground practical concerns. As such, these interactions may lead to identifying new research questions and new approaches to dissemination.

It is also possible that the needs teachers find most pressing may not relate directly to MCLI, or may intersect with scholarship on MCLI only in tangential or incidental ways (e.g., concerns about classroom management may make it difficult for teachers to engage in certain types of pedagogical practices). If teachers’ concerns are centered on an intersection of MCLI with an area outside of MCLI, it may be fruitful to expand the research team to include collaborators with other types of expertise, and this may also lead to identifying new research questions. Of course, some concerns may fall clearly outside

the scope of the research team's relevant knowledge or expertise, and in these cases, referring to other experts is most appropriate.

If researchers wish to allow the needs of learners, teachers, schools, and communities to inform their research questions, this will require new ways of engaging and interacting with individuals in each of these groups. This process may be more or less challenging, depending on the composition of research teams—an issue we turn to next.

Composing research teams

Another dimension of the "status quo" of research in mathematical cognition and instruction has to do with the composition of research teams. Like other complex social groups, research teams can be viewed as multi-layered, nested ecological systems, as in Bronfenbrenner's (1977; Bronfenbrenner & Morris, 2006) bioecological model. Depending on the setting where the research team is based (i.e., the type of university, research center, or other institution), the team may include many individuals in different roles, including team leaders (or principal investigators), research staff, postdoctoral fellows, graduate students, and undergraduate assistants. At the center of the system are direct interactions among members of the team, both one-on-one and in group settings. Moving out to broader layers, teams typically work within academic departments or research centers—often in higher education settings, such as colleges or universities—that have specific (though not always clearly articulated) values and expectations about research. These institutions are situated within local contexts that may have specific needs related to learning and education. They also operate within a national and political context that places importance on particular types of research and that offers opportunities for research funding through channels such as federal agencies and private foundations.

At each of these levels, there are considerations of resources, power, hierarchy, and opportunities —including opportunities for learning, career advancement, funding, and platforms for communication. These considerations have implications for how research teams are composed and how they operate. For example, these considerations may encourage teams that fit a "mold" viewed as desirable by funding

agencies, and they reward teams that address research questions that are valued by those funding agencies.

Given these factors, it is no surprise that large research teams are more common at some types of institutions than others (e.g., in the United States, they are more common at Research-I [R1] institutions, which are institutions that grant many doctoral degrees and that have high research expenditures). However, it is also possible to have productive research teams at other types of institutions. For example, the research conducted by Shusterman and colleagues (2019), described above, was conducted at a small liberal arts college with a team made up almost entirely of undergraduate students (i.e., with no graduate students or postdoctoral fellows). In fact, Shusterman and colleagues argue that "undergraduates [can] and should be marshaled as a central resource for spreading research-based materials through classrooms", in no small part because undergraduates' presence in classrooms "unravel[s] the status hierarchy that puts science above teaching" (Shusterman et al., 2019, p. 29).

Status quo: Individual and disciplinary considerations in composing research teams

Although scientific work is slowly becoming more interdisciplinary, it seems fair to say that, at the present time, many research teams that investigate MCLI are made up primarily of psychologists—and depending on the specific focus, typically cognitive, developmental, and educational psychologists. It is also common for teams to include experts in methodology or statistics, either as consultants or as core members. Research teams with this type of disciplinary expertise are often very good at addressing the "*from* cognition" side of the "from cognition to instruction" progression. For psychologists who study MCLI, teams of this sort are like well-plowed fields—familiar and easy to bring together.

In addition to being relatively homogeneous in terms of disciplinary perspectives, many research teams in academic settings are also homogeneous in terms of the personal backgrounds, lived experiences, and demographic characteristics of their team members. This status quo reflects the broader field of academic psychology, which in terms of commonly considered demographic characteristics (such as gender, race, and ethnicity) has a higher proportion of white and female individuals than the general

population (Lin et al., 2020). With respect to factors less frequently considered (e.g., socioeconomic status, disability, immigration status, sexual orientation, trans or non-binary identities), data are less available, but similar over- and/or under-representation of individuals from some groups is likely. Although many universities and academic departments are engaging in efforts to increase diversity, progress has been slow, and the professoriate and the graduate student body in psychology—as well as many research teams working on MCLI—remain relatively homogeneous.

Moving beyond the status quo: Purposeful composition of research teams that are diverse along multiple dimensions

There is abundant evidence that diverse teams are more creative and innovative than more homogeneous teams, although there are also many potential moderating factors (see Hundschell et al., 2022, for a review). Diverse teams are also likely to be more effective at addressing pressing challenges, as more diverse perspectives on "wicked" problems (Rittel & Webber, 1973) are likely to yield more novel ideas and better solutions. There are many dimensions of diversity that are relevant to research in MCLI, including diversity in team members' personal backgrounds, disciplinary perspectives, and occupational roles.

Teams that include individuals with diverse personal backgrounds have a larger set of perspectives on which to draw in their work. Individuals with different life experiences may "see" different aspects of complex and challenging situations, so they may foreground different research questions. For example, a research team might be more likely to explore the particular complexities of mathematics learning for students in night school or in schools with a large proportion of emergent bilinguals if a member of the research team has had similar experiences. Team members with different backgrounds may identify unique resources or "funds of knowledge" (Civil, 2016; Cunningham & Gomez, 2021; Moll et al., 1992) that hold promise for supporting learning in a given context, content area, or population of learners.

Teams that include individuals with diverse disciplinary backgrounds also have a larger set of theoretical perspectives and a broader set of methodological tools on which to draw when formulating and addressing research questions. As one example, two members of the author team (Vest and Alibali) have collaborated with experts in human-computer interaction to develop computer-based tutors for students learning to solve algebraic equations (Nagashima et al., 2022; Vest et al., 2022), and this collaborative work brought new methodological tools into their work flow—such as examining micro-interactions in log data, as a means to understand why a specific intervention technique was (or was not) effective. As a second example, another member of the author team (Rodrigues) is a special education researcher who has collaborated extensively with cognitive psychologists to design mathematics interventions. She has contributed her expertise in special education to design adaptations to a mathematics intervention—one that demonstrated effectiveness broadly for elementary students but did not specifically focus on disability (Yu et al., 2022)—to make it more accessible and to embed evidence-based practices for supporting students with or at risk for mathematics difficulty and/or disability.

Teams that include both psychologists and mathematics education researchers may have unique benefits for research in MCLI, as individuals with training in these distinct disciplinary traditions often focus on different aspects of learning tasks and environments. In broad strokes, for example, relative to mathematics education research, psychological research tends to focus more on basic cognitive processes, such as working memory and inhibition, and tends to focus less on authentic tasks and ecologically valid settings (see Alibali & Knuth, 2018 for discussion). Further, mathematics education research often focuses on deeply interrogating the mathematical constructs under study (Matthews, 2024). Research that brings these perspectives together can speak to scholars in both communities, thereby enhancing the impact of the work, and making each discipline's research base more widely available.

The same general principles apply to teams that include individuals in diverse *occupational* roles, such as scientists, teachers, school district staff, and community partners. Individuals in different occupational roles view research problems from different perspectives. For example, teachers may notice

or emphasize aspects of instructional activities and contexts that developmental psychologists may not. Occupational roles also shape people's deeply held values and goals. Is the broad purpose of the collaborative work to identify a causal mechanism of learning, or to develop a technique to help students be more successful in school mathematics? Both can be simultaneously true—and different goals can be more or less central for different members of a diverse team.

Teachers have a unique and central role in many aspects of learners' developing mathematical cognition, and as such, teachers—and former teachers—have unique potential to contribute to research on MCLI (Matthews et al., 2021). Teachers' experiences with learners' thinking, engagement, and attitudes positions them to offer novel insights about a wide range of topics, including, but not limited to (1) the structure of students' knowledge, (2) relations between distinct aspects of knowledge, (3) relevant misconceptions or overly narrow conceptions, and (4) insights into potential reasons why learners struggle. Teachers' insights can inform the selection of research questions that are relevant to classroom practice and can guide inquiry towards tasks and methods that are useful in classroom settings. Further, teachers' experience with assessment can be informative in efforts to measure important constructs. Individual teachers' experiences vary substantially, and their perspectives may depend on their specific experiences and on the context in which they currently practice or formerly practiced. Teachers from the community in which research is conducted may have critical insights into locally relevant history, policies, and practices—another dimension of their potential contributions as part of a diverse research team. Integrating teachers into research teams may also open opportunities for implementing research and collecting data on a wider scale.

Team members' diverse personal backgrounds, disciplinary perspectives, and occupational roles can also prompt discussions and reflection about core scientific values, goals, and ideals. For example, one member of our author team (Vest) noted that in interdisciplinary collaborations, he often felt "pulled between the specificity and causal power of carefully controlled experiments and the complexity and

authenticity of in-class data collection"—reflecting sets of values that he learned to appreciate and articulate in new ways through his work on a diverse team.

Building a team that is diverse along multiple dimensions can be challenging, as it requires moving beyond the well-plowed fields of collaborating with others who are similar to oneself, to the more challenging forests of finding ways to connect with and build working relationships with others who "run in different circles", view research problems through different lenses, or embrace different values. Substantial efforts and new ways of engaging may be needed to establish these relationships and develop fruitful patterns of working together. As a mundane example, when teams include individuals with diverse occupational roles, even simple tasks such as finding a meeting time can be challenging. At a deeper level, when teams include individuals with different backgrounds, values, and goals, it may take more time and more discussion to come to decisions about what research questions to address and how and where to address them.

Composing a diverse research team is only one step towards creating a generative and productive research environment. Team leaders should work with their teams to develop practices and policies to guide their collaborative work and to ensure that all members of the team have voice, agency, and a sense of belonging and contribution. In this regard, it is worth bearing in mind that team members do not need to agree on every detail of a project in order to be able to work together effectively (Alibali & Knuth, 2018). Compromises can allow projects to address multiple goals and, potentially, multiple related research questions.

Research participants and settings

The "status quo" of MCLI research also extends to the *participants* who take part in the research and the *settings* in which the research is conducted. These aspects of research practices have implications for the nature and timeline of the work and the potential generalizability of research findings across populations and settings.

Status quo: Homogeneous cultural settings, convenience samples and laboratory settings

As in other subfields of psychology, the majority of MCLI research is conducted in cultural settings that have come to be known as "WEIRD", an acronym for Western, Educated, Industrialized, Rich, and Democratic (Henrich et al., 2010). Further, within these settings, a large subset of the research is conducted with convenience samples of participants, such as undergraduate students in Introductory Psychology participant pools or children in private schools that are located close to large research universities. Such samples often do not reflect the substantial diversity that is present in the broader societies from which they are drawn, and consequently, reliance on convenience samples leads to an overrepresentation of white and high-SES participants in research (Doebel & Frank, 2023).

A large subset of MCLI research is also conducted in laboratory settings. Such settings enable researchers to control extraneous factors that may influence the cognitive and learning processes under study, such as variations in physical surroundings, noise level, interruptions, and other factors. However, such settings do not generally reflect the variability and complexity of the environments in which mathematical cognition and learning typically occur. Of course, some strands of cognitive and learning science research are conducted in authentic settings, including research that investigates how best to design such environments to optimize learning and cognition (e.g., design-based research; Brown, 1992; Collins et al., 2004). However, it seems fair to say that the majority of research on MCLI—especially research conducted by teams led by psychologists—is conducted in laboratory settings, typically on university campuses, rather than in authentic educational settings.

The "status quo" practices are often justified on the assumption that "cognition is cognition"—that is to say, "the basic architecture of memory and the brain are the same whether you are five or 55 years old, learning about polynomials or poetry" (Perry, 2022)—and, one might argue, whether individuals come from Western or non-Western cultural settings, or whether learners are engaging in mathematics in a noisy classroom or in a quiet laboratory. Indeed, this argument makes some sense with respect to questions about fundamental principles of learning (e.g., the principle that mastery of a skill or concept often requires repetition and practice spaced over time), but it makes much less sense when it

comes to addressing questions about learning in specific populations of learners and in specific contexts and settings.

Of course, there are many research questions that can be answered effectively in the well-plowed fields of convenience samples and laboratory settings in WEIRD cultural settings. However, there are many reasons for moving beyond them, as well. Most importantly, research that addresses the needs of educators and schools often requires samples that are representative of the populations of students that educators and schools serve, and it also requires understanding performance and learning in settings that are like those in which students learn. Further, research in diverse cultural contexts can highlight aspects of common practices that might otherwise go unnoticed and that can raise new possibilities for interventions.

Convenience samples are by definition, (relatively) "easy" to obtain, but they often do not reflect the characteristics of the learners to whom researchers would like to generalize—in terms of age, race and ethnicity, socioeconomic status, educational background, and other factors. For example, many psychologists who study MCLI (especially those who are interested in middle and high school mathematics learning) initially test out research ideas with adult participants. Adults frequently struggle with the same types of mathematical content as younger learners, but adults are easier to study than children for many reasons: they can provide their own informed consent, they can schedule participation on their own, and they can organize their own transportation to the research site. For researchers who work at institutions that have participant pools associated with psychology or other courses, research with adults is also often "free" in the sense that participants receive extra credit points in psychology courses as compensation for their participation rather than monetary compensation. These practical factors push many scholars to rely on convenience samples; however, there is no guarantee that findings based on convenience samples will generalize to populations with different demographic characteristics. The same issue also applies to null results—an intervention that yields a null effect in a convenience sample (e.g.,

with adults in a university participant pool) might yield a positive result in a more representative sample of learners (e.g., with younger students in a classroom).

Some scholars have sought to develop statistical methods to evaluate the similarity between a study sample and the target inference population. For example, Tipton (2014) developed a generalizability index that can be used both to evaluate when a sample is a good reflection of the target population (on a set of selected covariates) and to determine how well reweighting methods (i.e., propensity score matching) could be used to estimate the average treatment effect in the target inference population. This method was used in a recent study of links between high school students' expectations for success in mathematics and their mathematics progress, with a specific focus on how these links varied depending on identity groups (gender, SES), schools' peer culture, and students' perceptions of classroom gender stereotyping (Carroll et al., 2023). The investigators started with a nationally representative sample (drawn from the National Study of Learning Mindsets) and then identified several subsamples for focused analyses. They used Tipton's (2014) generalizability index to establish that the analytic subsamples appropriately reflected the broader population. Although such methods for assessing the alignment between samples and target populations are powerful, they are still rarely used in psychological studies, and their use in education research is growing slowly.

Like research with convenience samples, research in laboratory settings is often relatively "easy" to carry out, as researchers do not need to engage with schools or other organizations in order to obtain permission to work in those settings. However, there is no guarantee that findings based on learners' performance in laboratory settings will generalize to classrooms or to other more authentic settings. Further, there are currently no viable ways to quantify or statistically equate similarities and differences across settings.

As an illustrative example of limits on generalizability across samples and settings, consider a recent series of studies on the instructional technique of *person-presentation* in instruction about mathematical problem solving. Person-presentation involves linking a particular problem-solving

approach to an individual who uses that approach. The initial study of person-presentation was conducted in a laboratory setting with a convenience sample of college undergraduates. Riggs and colleagues (2015) compared learning and transfer of a new problem-solving strategy that was either linked to a specific person (e.g., "Juan's strategy") or presented without linking it to a person. Participants showed greater learning and transfer in the condition *without* person-presentation, suggesting that person-presentation was harmful for learning and transfer. These findings sparked the interest of a different team of researchers who had been developing curricular materials for high school algebra that involved worked examples of problem-solving strategies. This team developed comparable sets of instructional materials that did and did not use person-presentation (i.e., that linked worked examples of strategies to fictitious students or that simply used strategy labels), and they tested these materials with five algebra teachers and their students during a multi-week unit on equation solving (Loehr et al., 2020). In this study of ninth-grade students in real algebra classes, no differences between conditions were observed!

There could be many reasons why person-presentation was harmful for adults in the laboratory, but there was no evidence of harmful effects for younger students in a classroom setting (see Loehr et al., 2020 for discussion). For present purposes, the key point is that findings observed with convenience samples in controlled laboratory settings do not always generalize to other populations and authentic educational settings. Notably, there are similar concerns regarding null effects observed in convenience samples or in laboratory settings. Interventions that might help students in classrooms might never make it out of the laboratory if they do not influence the learning of undergraduate students drawn from university participant pools.

This example also underscores the fact that findings about mathematical cognition and learning may vary across samples and across settings. For the continuing development of the field of MCLI, it will be crucial to increase understanding of "what works for whom, and in what settings"—that is, to understand the heterogeneity of effects (see Bryan et al., 2021). To address this goal, scholars in the field need to study key phenomena in different participant samples and in different settings, ideally in studies

that are guided by theories about how and why patterns of performance and learning may vary. The field also needs studies with sufficient power to allow for analyses of heterogeneity of effects within samples. As the body of relevant evidence grows, the field will also need additional efforts to aggregate findings across studies, as in meta-analyses and other research syntheses (e.g., Barbieri et al., 2023; Carbonneau et al., 2013).

Moving beyond the status quo: Broadening participation in research

Many scholars in MCLI are currently making efforts to move beyond the well-plowed fields of convenience samples, to address learning in samples more representative of the populations to which they wish to generalize. These efforts often require new practices that aim to broaden participation in research, including new approaches to participant recruitment and data collection.

Broadening participation in research will also require addressing some of the common barriers to participation, including language barriers, transportation challenges, and most importantly, lack of trust. Some barriers to research participation can be addressed with additional resources, for example, by hiring staff who can communicate with families in their home languages or by covering the cost of participants' transportation to the research site. Other barriers may be addressed by "going where people go"—that is, conducting research or recruiting participants in sites where children and families go in the course of their everyday lives, such as schools, libraries, community centers, and even grocery stores (e.g., Hanner et al., 2019). Such efforts may also require new channels of communication for reaching prospective participants and families such as social media. Recruiting participants via these approaches will also often require more resources than recruiting convenience samples. Therefore, in planning research and in pursuing research funding, scholars who wish to study more representative samples will usually need to budget a larger share of funds and to devote a large share of staff time to these efforts. Researchers should also attend to paying participants an appropriate wage for their participation. We encourage funding agencies to support requests (e.g., in grant budgets) for resources to support recruitment of diverse participants, including both participant payments and staff time devoted to recruitment efforts.

The lack of trust in research that many teachers, school administrators, and potential participants experience deserves special consideration. In many cases, this lack of trust is related to researchers' past and current failures to appropriately consider participants' needs and realities. Researchers sometimes treat participants (including students, teachers, and school staff) in a transactional or extractive manner, rather than seeking to establish meaningful relationships built on genuine interest and care. This approach cements perceptions of researchers as distant intellectuals, more interested in fame and funding than on the relevance or meaningfulness of research.

Building trust with prospective participants can be challenging and can require new ways of engaging with participants and with the community—moving away from well-plowed terrain to more challenging forests or even mountains. One simple (forest) way to start is to follow up with research participants by sharing findings or research products—a courtesy that many researchers frequently neglect. Another way is to "give back" by sharing knowledge and perspectives in ways that educators or school districts value—for example, by presenting a research summary at a staff meeting, contributing to a workshop on an in-service day, or serving on a district external research committee. A more demanding way (perhaps a mountain) is to be present in and to make contributions to the community, such as through sustained volunteer partnerships in which members of research teams engage in service that directly benefits communities, while also building relationships. For example, one member of the author team (McNeil) expects all members of her research group to do a "practicum" in which they serve as a teacher, tutor, or mentor for children in an educational or community setting. This experience enhances research group members' training by providing them with direct experience in authentic learning contexts, and it also helps build relationships with schools and in the community. Of course, building trust in these ways requires investments of time and of heart—investments that can have a large impact over the long term.

Moving beyond the status quo: Conducting research in authentic settings

Research that addresses the needs of educators and schools will also require understanding learning in settings that are like the settings in which students learn, including classrooms, tutorial

sessions, and online instruction, as well as informal settings such as museums and online games. It is common for scholars of MCLI to highlight *implications* of their work for instruction, but studies that take place *in* authentic instructional settings are much less common. Given the many constraints that apply in real-world instructional settings—including resource constraints, time constraints, curricular constraints, and teachers' perceptions of the usability of tasks and materials—it can be challenging to conduct research in such settings. For example, conducting a study in classrooms often requires many preparatory steps, including obtaining approval to conduct research in the district, obtaining approval to conduct the research in specific schools, recruiting teachers to participate (either in delivering the intervention, or arranging class time for the researchers to implement the intervention), and obtaining consent from students (if needed, depending on the nature of the research and its relation to regular classroom practices). For researchers who typically work in laboratory settings, these practices represent more challenging terrain–in some cases, forests, and in other cases, mountains, depending on the difficulty involved.

Although addressing these additional challenges can be both time-consuming and resource-intensive, in our view, there can be great value in conducting research in authentic settings. Such research can be part of a cycle that proceeds from the laboratory to the classroom and back again (Klahr & Li, 2005; Toth et al., 2000), raising new research questions and addressing real-world implications related to implementation of interventions, sequencing of activities, and variability in instructional conditions. For example, as noted above, person-presentation of strategies in a laboratory setting had negative effects on adults' generalization of a strategy for solving a specific type of algebra problem (Riggs et al., 2015), but subsequent research yielded *no* evidence that person-presentation was harmful for younger students' learning of problem-solving strategies in real algebra classrooms (Loehr et al., 2020). It is possible that other aspects of the provided instruction—specifically, engaging in comparison and discussion of multiple strategies—counteracted the negative effects of person-presentation (see Loehr et al., 2020, for discussion). Thus, this classroom research sparked a new question for further study in the laboratory.

Moving beyond the status quo: Research in diverse cultural settings

Conducting research on MCLI in diverse cultural settings presents a different sort of challenge to the status quo—but a challenge that we believe is well worth engaging, as such work has great potential to yield progress on both basic and applied questions. There are substantial variations in instructional practices across cultures, and there is a long tradition of educational research comparing such practices, including work that takes a cognitive perspective (e.g., Ding, 2021; Richland et al., 2007; Schleppenbach et al., 2007; Stigler, 1999). There are also important variations in how mathematical ideas are expressed in different languages (e.g., Miller et al., 1995; Miura et al., 1993, 1999; Moeller et al., 2015). These variations in educational and linguistic practices, and the associated variations in performance and learning, can be a source of insights about processes of mathematical cognition and about potential interventions (e.g., Paik & Mix, 2003).

As one example, two members of our author team (Meng and Alibali) have recently studied how mathematics teachers connect ideas during their instruction, as part of a broader effort to understand how different instructional practices can support learners' acquisition of conceptual understanding. Previous work on this issue had been based on samples of lessons drawn from teachers in the United States, and this decision led to a focus on how teachers communicate links (e.g., in linking episodes that frequently include both gesture and in speech; Alibali et al., 2014). However, recent work based on a sample of lessons from classroom teachers in China (Meng et al., 2023) revealed that Chinese teachers frequently offered part of a link statement in the form of a question and invited their students (sometimes individually, sometimes as a group) to complete the statement—thus, many links were not stated by the teacher, but were instead constructed collaboratively. This teaching practice was extremely common in the Chinese lessons, but had been observed very infrequently in U.S. lessons (Alibali et al., 2024). This cultural difference in instructional practices—along with the well-documented advantages in conceptual understanding for students in China compared to the U.S.—suggests some possible approaches to intervention.

Importantly, the team for this study included individuals who work in and who were educated in the cultural setting being studied. It was also an interdisciplinary team—including a mathematics educator (Ding) as well as several psychologists (Meng, Yeo, and Alibali). The team drew on multiple literatures from psychology and education to frame the study, and the work included both quantitative analyses of the frequency of different types of linking episodes and qualitative analyses of classroom discourse. Engaging in this collaborative work required all members of the research team to step beyond the well-plowed fields of their typical disciplinary practices and to enter more challenging forests.

Communicating research

If research on MCLI is to have an impact on issues that face educators, schools, and the general public, those stakeholders must have access to that research, and they must be able to see the value of that research. Above, we emphasized the importance of bringing these stakeholders' perspectives into the research process and including them as members of research teams, so they can contribute to formulating research questions and to shaping the work. In this section, we consider the process of communicating findings in a more general way, in an effort to reach individuals who are not directly engaged in research. Psychologists who study MCLI often disseminate their work through channels that are largely inaccessible to these stakeholders, such as in articles in scientific journals and presentations to academic societies. These modes of dissemination are effective for communicating with other researchers, but they are less effective for communicating findings to practitioners, for highlighting the potential value of research findings for practice, and for encouraging practitioners to draw on research in their practice.

Research on instructional interventions is potentially useful for educators, because such research can yield readily usable lesson materials. Basic research on mathematical cognition can also be valuable for educators, because information about students' thinking can inform instructional practices in numerous ways (e.g., Carpenter et al., 1989). As one example, if teachers are aware that elementary students often interpret the equal sign to mean "put the answer", they may be likely to recognize this misconception when it arises in classroom contexts. They may also be more inclined to address this

misconception directly in their instruction (e.g., by talking about the meaning of the symbol or by increasing the variety of positions in which they present the equal sign in equations), and they may make other efforts to support students' understanding of equivalence and the equal sign. In brief, both research on instructional interventions and foundational research on mathematical cognition can be of value to practitioners.

Status quo: Journal articles and presentations at academic conferences

For psychologists who study MCLI, the impact of their research is often measured at the institutional level in terms of the number of journal articles published, the number of presentations at scientific conferences, and the number of citations to these "products". These measures primarily reflect intellectual impact among communities of scholars rather than impact among wider audiences, including educators or the general public. Given the incentives to produce scholarly publications (e.g., to earn promotion and tenure), it is not surprising that scholars tend to focus on such outlets. However, for research on MCLI to be successful at addressing the needs of learners, educators, and schools, it must reach wider audiences—that is, it must reach practitioners.

Most practitioners have little time to seek out or consume research products. Further, many research products are inaccessible to practitioners for a wide range of reasons. Many practitioners do not have easy access to scientific journals, and many primary research reports require extensive background knowledge to understand. Of course, teachers may encounter research summaries and interpretations during professional development workshops, and such workshops *can* be effective at communicating research broadly. However, professional development workshops focused on mathematics learning tend to be infrequent (in part, because schools often prioritize professional development that is applicable to teachers across content areas and not limited to mathematics), and such workshops often do not incorporate current research. Further, many teachers do not attend such workshops due to cost and time constraints, the need to miss classroom time, and other logistical challenges. For all of these reasons, much research on MCLI does not reach teachers or influence practice in efficient ways.

Moreover, most intervention studies that are published in scientific journals are not designed to be maximally useful for teachers (Willingham & Daniel, 2021). Intervention studies conducted by psychologists who study MCLI often aim to provide empirical evidence that specific practices *can* lead to learning; therefore, reports of intervention studies typically present data on novel interventions compared to "business as usual" instruction or compared to the absence of intervention. Such studies seldom compare multiple potential interventions, a type of design that would yield more actionable information for teachers. Further, as discussed above, many intervention studies are conducted in laboratory settings, so their direct relevance to classroom practice is often unclear. Of course, intervention studies can be valuable even if the interventions under consideration are never tested in authentic settings, but effective interventions can yield the greatest value if they are actually implemented.

One benchmark for evaluating whether an intervention is successful is whether it is taken up by practitioners. Indeed, a recently-convened national panel of experts charged with considering the future of education research (the National Academies of Sciences, Engineering and Medicine's *Committee on the Future of Education Research at the Institute of Education Sciences*) highlighted the need for interventions that (1) address critical needs, (2) are effective, and importantly, (3) that will be taken up by teachers and schools (NASEM, 2022). These features are intertwined: for example, the likelihood that teachers will take up an intervention is presumably greater if they view it as addressing a critical need (see above) and if they view it as effective (based on whatever type of evidence they deem valuable). However, teachers and schools cannot take up interventions that they do not know about, so communicating through channels that go beyond scientific journals and academic presentations is essential.

Moving beyond the status quo: Communicating with practitioners and the public

For research on MCLI to influence educational practice more routinely, there will need to be changes in the "ecosystem" through which research findings reach practitioners and other school district staff—that is, in how knowledge gained from research can be "mobilized". Because teachers generally do

not have access to primary reports about research, they often obtain information about research in other ways, such as through practitioner journals, social media sites, blogs, podcasts, popular press articles, and trade books.

One pathway for communicating research results that does reach many practitioners is through publishing findings in practitioner journals (e.g., *Mathematics Teacher: Learning and Teaching PK-12,* the journal of the US-based National Council of Teachers of Mathematics, or *Mathematics Teaching*, the journal of the UK-based Association of Teachers of Mathematics). Papers in practitioner journals typically incorporate summaries of research findings, rather than primary reports of results. Such papers also often focus on how findings can be translated to practice, and they often include examples of student work and/or tasks that can be used in the classroom. In this regard, however, it is worth noting that some psychologists who study MCLI do not feel comfortable making direct recommendations for practice, and many research studies do not use classroom-appropriate tasks or gather student work that can be shared. For these reasons, many MCLI-focused articles in practitioner journals include teachers or former teachers as part of the author team, as teachers are in a better position to make recommendations about practice (e.g., Hornburg et al., 2021; Lange et al., 2014). Research teams that include teachers may be especially well suited for developing publications for these outlets.

Practitioner journals typically require that papers be short, written in accessible language, and free of research jargon. Given these requirements, writing for practitioner journals demands different skills than writing for academic audiences. Scholars who wish to publish in these outlets may need to be intentional about developing such skills—stepping away from their typical writing practices and into more challenging terrain.

Some other pathways for disseminating and communicating research specifically about learning and education have been developed in recent years. As one salient example, in 2002, the Institute of Education Sciences (the research arm of the U.S. Department of Education), launched the What Works Clearinghouse, a free online repository of information about education research. The What Works

Clearinghouse reviews research on education, evaluates the quality of evidence for claims about "what works", and provides several different types of research summaries, including (1) summaries of individual studies, (2) *intervention reports*, which synthesize and evaluate research on specific curricula, interventions or practices (e.g., *Cognitive Tutor® Algebra I (Secondary Mathematics)* [U.S. Department of Education, 2016], and (3) *practice guides*, which synthesize evidence-based recommendations for teachers in specific content areas and/or grade bands (e.g., *Improving Mathematical Problem Solving in Grades 4-8* [Woodward et al., 2012], *Developing Effective Fractions Instruction for Kindergarten through 8th Grade* [Siegler et al., 2014], *Teaching Strategies for Improving Algebra Knowledge in Middle and High School* [Star, et al. 2015], and *Teaching Mathematics to Young Children: A Practice Guide* [Frye, et al. 2013]). One goal of the What Works Clearinghouse is to provide educators with accessible evaluations of the *quality* of the evidence for specific practices, so they can know what research is trustworthy and should be used to inform their decision making. However, although the What Works Clearinghouse has existed for over 20 years, many educators are unaware of its existence or have never accessed the resources it offers. Time will tell if this clearinghouse becomes an effective means by which research can affect practice.

In recent years, many funding agencies (including the Institute for Education Sciences and the National Science Foundation) have placed increasing importance on the broader impacts of research in making funding decisions. Some broader impacts can be realized through communicating research in ways that are accessible and useful to the individuals—including educators, schools, and other organizations—who seek to put the research into practice (e.g., Moore, 2023; National Academies of Sciences, Engineering, and Medicine, 2022). As such, we encourage funding agencies to consider the full range of practitioner-focused research products when evaluating the "output" of funded research. Improving knowledge mobilization (Levin, 2013) through diverse, practitioner-focused research products is a key avenue for addressing the research-practice gap and increasing the likelihood that all students have access to high-quality mathematics practices.

Moving beyond the status quo: Visual communication strategies

One form of knowledge mobilization that has proven valuable for public-facing dissemination of research involves using visual communication strategies. An example is the *visual abstract*, a strategy that was first pioneered by the medical research community for sharing empirical research articles with a scientific audience on Twitter (Brownlee & Ibrahim, 2020). A visual abstract is a visual summary of an empirical journal article's textual abstract, and it has the same purpose as the traditional abstract: to convey an overview that helps readers determine whether to read the full paper (APA, 2020). One benefit of the visual abstract over purely textual information is that it takes advantage of people's ability to process visual representations quickly—indeed, more quickly than textual information (e.g., Thorpe et al., 1996).

Rodrigues (2021) proposed a novel adaptation of the visual abstract, known as a "key abstract" or translational visual abstract, for sharing education-focused research with broad audiences. A key abstract is a concise, infographic-type summary of a journal article or of takeaway messages, with visuals and minimal text and with alternative text available to support accessibility. Key abstracts thus provide a concise and clear summary of key research messages that members of a particular audience (such as mathematics teachers) can quickly and easily peruse to assess if the content may be relevant to their practice and if they are interested in learning more about the topic. Importantly, researchers can design and share key abstracts with particular audiences in mind; for example, if they wish to reach teachers, researchers can create a key abstract with accessible language (e.g., avoiding technical or academic jargon) and share the resource on social media platforms that have a large teacher following. Figure 2 presents an example of a key abstract designed for sharing a finding from an empirical study with elementary general education and special education teachers.

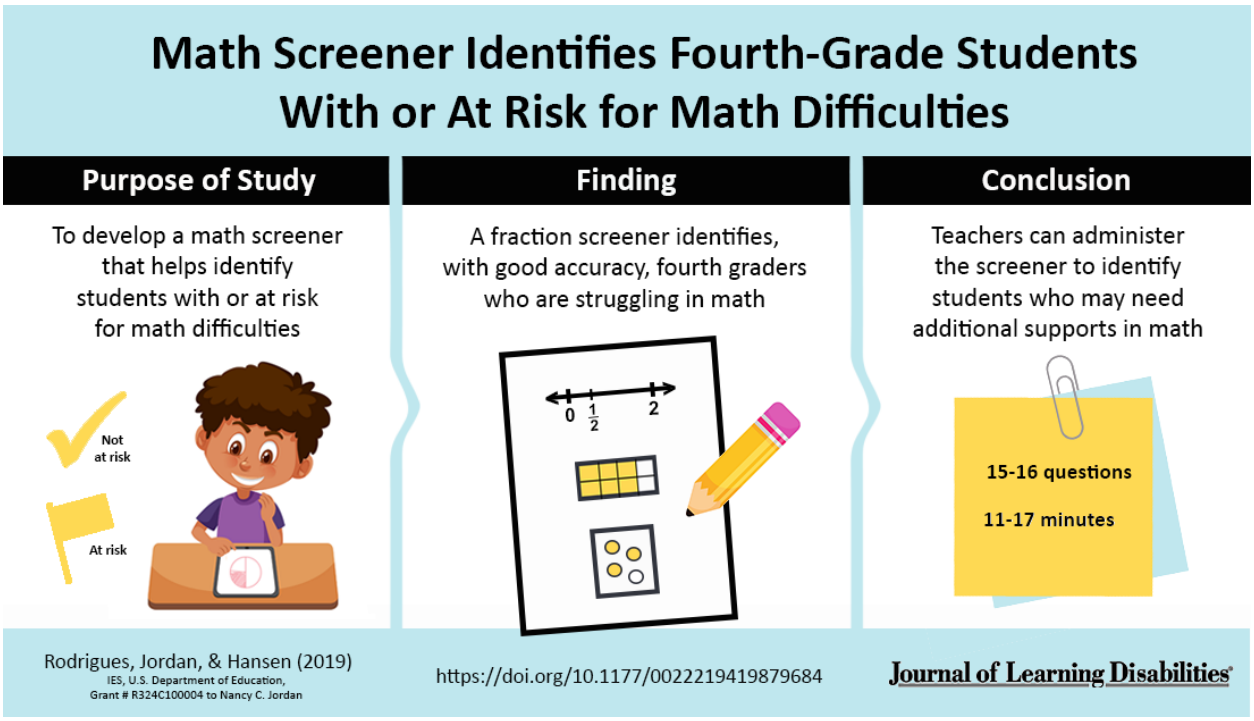

Figure 2. A sample key abstract designed for communicating research findings with a teacher audience (Rodrigues et al., 2019).

More generally, we suggest that efforts to communicate with practitioners and the public about research findings may be most effective if scholars focus on identifying outlets that are broadly accessible and on presenting findings in formats that are easy and fast for people to take in.

Moving beyond the status quo: Considerations of incentive structures and evaluations of scholars

In an academic system that prizes journal articles and scientific presentations as indicators of productivity, scholars—especially early-career scholars—may need to weigh advice or suggestions about alternative pathways for disseminating research in light of the indicators and criteria their employers use to evaluate productivity and impact. In our view, incentive structures in departments where MCLI researchers are employed should be expanded to place a higher value on practitioner-facing and public-facing products than is currently the case (although we acknowledge that there is substantial variability). Senior scholars who are in a position to evaluate early-career scholars can also highlight the value of

practitioner- and public-facing contributions, as well as the costs (in time and resources) that go into preparing them.

Academic units should also consider ways to evaluate the impact of research beyond traditional citation counts. For example, one recently-developed Web-based metric—the Altmetrics or "alternative metrics" score; Altmetric LLP, London, UK)—has led to fast and dynamic analyses of the "reach" of research beyond traditional bibliometrics (e.g., in online spaces such as social media platforms, blogs, articles in the mainstream media, and so on). Altmetrics provides "Altmetric Attention Scores" (AAS) for individual research publications, with higher scores indicating greater online attention, as aggregated from several sources. There are also other ways to document the "reach" of research products beyond citations of journal articles. For example, scholars who make their intervention materials publicly available can track how frequently those materials are downloaded, and they can ask practitioners to let them know if they use those materials. Some public-facing news sites track how often articles are shared on other social media sites. For example, a mathematical-cognition-focused article that was published on *The Conversation* in April 2020 (Thompson et al., 2020) has been shared on Facebook over 7000 times.

It is also important to note that the changes we have suggested in previous sections may also require a shift on what the field considers "research activity." Engaging with educators, schools, and communities takes time and effort, and in our view, these activities should be acknowledged and valued in tenure and promotion cases as research activity (rather than as service, as is often the case). These activities can be part of a broader effort to identify needs and opportunities within communities, and they can lay a foundation for more focused research projects that target those needs and opportunities. Focusing on particular groups or communities is sometimes devalued (e.g., labeled as "me"-search; Devendorf et al., 2023; Victor et al., 2022). In contrast to these views, we call for a greater appreciation of research approaches that allow researchers to identify needs and opportunities in specific communities or participant groups.

We wish to highlight specifically the importance of research with marginalized communities, including research that focuses on the strengths inherent in these communities, as well as the challenges, including environmental, social, and political challenges, faced by these communities. These assets and challenges affect learning and development in myriad ways, and a deeper understanding of these processes can yield theoretical progress, as well as new empirical findings and new approaches to assessment and instruction. As one example, recent work on mathematical problem solving in bilingual learners (e.g., DiNapoli & Morales, 2021) highlights the value of having multiple languages as a resource for mathematical thinking and collaboration (i.e., translanguaging, which involves flexibly using and blending multiple languages; Garcia & Wei, 2014). As another example, recent work on air pollution and cognition demonstrates that children from lower-income households and those from households with lower levels of parent education have greater exposure to pollutants in ambient air (Medrano et al., 2022). Further, patterns of associations with cognitive outcomes suggest that such exposure may have negative consequences for children's cognitive development, setting the stage for future work investigating the mechanisms by which pollution affects cognition (Medrano et al., 2022). As a third example, recent work on mathematics learning in Deaf children examined whether the timing of language exposure is related to a foundational mathematical skill, the ability to connect different representations of the same quantity. Children who were exposed to American Sign Language (ASL) from birth performed better than children who were first exposed to ASL later in childhood. In fact, they performed as well as hearing children who learned English from birth—building the case that early access to language is critical for early mathematical development (Walker et al., 2024). All three of these lines of work with learners from marginalized groups make both theoretical and empirical contributions to scientific understanding of the development of mathematical cognition, and they point to important hypotheses for future work.

Finally, some might see the shift away from deriving research questions primarily from general learning theories as producing knowledge that is atheoretical, and some might see the shift to focusing on specific communities and specific settings as too narrow or "niche". These factors may influence how the

work of scholars who use these approaches is evaluated or taken up. We encourage a shift of perspective, to recognize that research with schools and teachers, with marginalized communities, and with a wide range of methodologies can be highly valuable, for theory development, theory testing, and hypothesis generation, as well as for practical applications. We emphasize the importance of both identifying convergences in findings and understanding heterogeneity of effects across different samples, different settings, and studies that use different methodological approaches.

Conclusion

In this paper we have discussed ways that the status quo of research in MCLI—which typically involves starting from cognition and subsequently considering instruction—may be limiting and may make it difficult for scholars to contribute effectively to addressing the challenges that face society at this moment in time. To address these limitations, we have called for scholars of MCLI to reflect on the "status quo" of their research practices and to consider moving to more challenging terrain, specifically, by making considerations of instruction an earlier and more central aspect of their research. We have called for scholars of MCLI (1) to consider the needs of educators and schools when selecting research questions and developing interventions, (2) to compose research teams that are diverse in the disciplinary, occupational, and personal backgrounds of team members, (3) to make efforts to broaden participation in research and to conduct research in authentic settings, and (4) to communicate research in ways that are accessible to practitioners and to the general public. We recognize that making such shifts in typical practices can be like traversing an unknown forest or scaling a steep mountain, as such shifts involve new challenges and require new learning and new ways of deploying resources (including time and funding). However, we believe that the value of these efforts will be richly rewarding for the field. If scholars foreground consideration of instruction, research on MCLI will become more theoretically valuable, more actionable for educators, and more relevant to pressing societal challenges.

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