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Reduced radiation losses in a channeled-beam x-ray laser by Bragg reflection coupling

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The effects of radiation losses and atomic motion in a distributed feedback induced by Bragg reflections in an electron-beam-channeling x-ray laser are investigated. Standing-wave fields with nodes in the atomic sites are generated in this cavity-mirror structure in single crystals, thereby reducing the losses located close to the atomic sites. An explicit expression for the low-threshold gain is derived which depends on the absorption, temperature, and on the order of Bragg reflection. It is noted that diffraction from several sets of atomic planes which satisfy the Bragg condition simultaneously may further reduce the threshold gain. These distributed-feedback schemes have possible application in reducing beam high-current requirements by many orders of magnitude.

I. INTRODUCTION

A relativistic electron beam propagating through planar or axial channels in a crystal free of imperfections may populate bound transverse-energy eigenstates.[1] Spontaneous dipolar transitions between these discrete eigenstates have been shown experimentally to yield narrow-width, highly polarized, and intense x-ray radiation which is strongly forward peaked.[2] One of the important issues in the possibility of using the channeling mechanism as a coherent x-ray source depends on future progress in creating sufficient gain from induced emission. this paper is related to the issue of identifying an efficient scheme for gain optimization in crystal channeling. Previous estimates suggest that in a one-passage amplification scheme even modest gains may require currents of the order of MA/cm^2 for energies near 10 MeV.[3-5] The aim should be to suggest a mechanism to reduce this high-current requirement by many others of magnitude, thereby bringing one aspect of the channeling x-ray laser closer to experimental reach.

An efficient scheme to significantly reduce the gain requirements for a channeling x-ray laser was proposed based on the concept of a distributed-feedback (DFB) laser which is supplied by multiple Bragg reflections of the radiation.[6] This scheme was very useful for atomic emitters in the optical range[7] and was extended later on to the x-ray range.[8] The advantages in using DFB lasers include the intrinsic compactness and high degree of spectral selectivity available without the need for cavity mirrors. The channeling DFB concept is favorable due to the possibility of radiation tunability. By adjusting the electron-beam energy the Doppler up-shifted radiation can be tuned onto a line in the DFB-mode spectrum near the Bragg-reflection frequency.[6] In Ref. 6 the threshold-gain condition for a DFB x-ray laser was obtained taking into account only reflections and neglecting the radiation losses in the crystal. The main loss mechanism is the photoelectric absorption by tightly bound electrons located close to the atomic sites in the crystal. Furthermore, the atomic motion in the crystal was ignored, which may influence the absorption and the reflections of the radiation.

This paper considers the channeling DFB scheme including the effects of absorption and atomic motion on the threshold-gain condition and spectral selectivity. We find that the formation of a standing-wave field with nodes on atomic sites, where absorption takes place, reduces drastically the effect of absorption. This effect is related to the Borrmann anomalous-transmission effect where standing-wave generation makes x-ray losses small.[9-12] The effects of atomic motion on increasing absorption and reducing reflections are considered. This effect is due to the zero-point motion at low temperatures and due to the thermal motion at higher temperatures relative to the Debye temperature. The atomic-motion effect can be expressed in terms of a Debye-Waller factor.[10,13,14] This effect limits the applicability of the DFB scheme to temperatures that are very low compared to the Debye temperature. We further consider the effect of the order of Bragg reflections on the threshold-gain condition. In spite of the limitations introduced by the radiation losses the DFB mechanism does reduce drastically the high-current requirements. However, the main threshold condition is dictated by the absorption.

It is pointed out that it is possible to further reduce the threshold gain by diffraction from several sets of atomic planes which satisfy the Bragg condition simultaneously.[15,16] In this case standing waves are generated in several directions relative to an atomic site, generating a larger nodal region in the radiation field and reducing the effects of radiation losses.

In Sec. II we present the DFB x-ray laser model including absorption but neglecting the atomic motion in the crystal. The inclusion of atomic motion is considered in Sec. III. In Sec. IV we obtained the threshold and selectivity conditions. Numerical results and discussion are presented in Sec. V.

II. THE DFB X-RAY LASER MODEL

We characterize the set of channeling transverse eigenstates in the x direction as a two-level system with states

$|1\rangle$ and $|2\rangle$, where W and $\hbar\omega_0=\varepsilon_2-\varepsilon_1$ are the population and energy differences, respectively. The directions of beam channeling and Bragg reflections are taken in the z direction. The Doppler up-shifted electromagnetic wave frequency $\omega=\omega_0/(1-v/c)\simeq 2\gamma^2\omega_0$ in the forward direction is chosen to closely match the n-order Bragg frequency, $\omega\sim n\omega_B$, where v is the channeling-electron speed, $\omega_B=\pi c/a$ and a is the the periodic reflection-plane spacing. Typically $\hbar\omega_0$ is a few electron volts in the laboratory frame so that for the relativistic factor γ on the order of 20, $\hbar\omega$ is on the order of several keV. Consequently, the channeling-electron energy may be tuned to satisfy the Bragg-reflection condition and induce distributed feedback in the channeling crystal.

The behavior of the electric $\mathbf{E}$ field of the electromagnetic wave and the polarization $\mathbf{P}$ of the beam electrons are related by the Maxwell's wave equation:

$$\frac{\partial^2}{\partial z^2}\mathbf{E}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\mathbf{P}+c\nabla\times\mathbf{M}+\mathbf{J}\right], \quad (1)$$

where $\mathbf{M}=\mathbf{P}\times\mathbf{v}/c$ is the magnetization due to the beam electrons and transverse-field effects are not considered.[4] The induced current $\mathbf{J}=\mathbf{J}_{\rm osc}+\mathbf{J}_{\rm los}$, where $\mathbf{J}_{\rm osc}$ is the oscillatory part and $\mathbf{J}_{\rm los}$ is the dissipative current. We approximate $\mathbf{J}_{\rm osc}=n_e e\mathbf{v}_e$ and $(\partial/\partial t)\mathbf{J}_{\rm osc}=e^2 n_e\mathbf{E}/m_e$, where n_e is the spatially modulated atomic-electron density.[6,8] The current $\mathbf{J}_{\rm osc}$ provides coupling between the forward- and backward-propagating waves and is mainly due to the outer cloud of atomic electrons. The current $\mathbf{J}_{\rm los}=\sigma\mathbf{E}$, where σ is the modulated dissipative conductivity, and represents the photoelectric absorption of the radiation by the tightly bound electrons close to the atomic sites.[10] The induced current $\mathbf{J}$ in the right-hand side of Eq. (1) can be represented as

$$\frac{\partial}{\partial t}\mathbf{J}=\frac{c}{2\pi}\left[\omega K\mathbf{E}+\mu\frac{\partial\mathbf{E}}{\partial t}\right], \quad (2)$$

where $K=2\pi e^2 n_e/cm\omega$ is the reflection function, $\mu=2\pi\sigma/c$ is the absorption function, and an average is carried out over the transverse direction $\rho=(x,y)$. In this section the atomic motion is neglected so that K and μ are periodic functions in the z direction.

The electric $\mathbf{E}$ and polarization $\mathbf{P}$ fields are taken in the x direction and are defined in term of forward- and backward-traveling waves,

$$E(z,t)=\varepsilon_+(z,t)e^{-i\omega(t-z/c)}+\varepsilon_-(z,t)e^{-i\omega(t+z/c)}+\text{c.c.}, \quad (3a)$$

$$P(z,t)=P_+(z,t)e^{-i\omega(t-z/c)}+P_-(z,t)e^{-i\omega(t+z/c)}+\text{c.c.}, \quad (3b)$$

where $\varepsilon_\pm$ and $P_\pm$ are slowly varying envelope fields. Inserting Eqs. (3a), (3b), and (2) in Eq. (1) we obtain

$$e^{ikz}\left[\frac{\partial\varepsilon_+}{\partial z}+\frac{1}{c}\frac{\partial\varepsilon_+}{\partial t}+(\mu+iK)\varepsilon_+\right]+e^{-ikz}\left[-\frac{\partial\varepsilon_-}{\partial z}+\frac{1}{c}\frac{\partial\varepsilon_-}{\partial t}+(\mu+iK)\varepsilon_-\right]=\frac{2\pi i\omega}{c}[e^{ikz}(1-v/c)P_+ +e^{-ikz}(1+v/c)P_-], \quad (4)$$

where $k=\omega/c$ and second-order derivatives are ignored because $\partial^2\varepsilon_\pm/\partial t^2\ll\omega\partial\varepsilon/\partial t$ and $\partial^2\varepsilon_\pm/\partial z^2\ll k\partial\varepsilon/\partial z$.

Equation (4) must be supplemented by the equation for $P_\pm$ and is readily determined from a density-matrix approach obeying the Bloch equation[4,17]

$$\frac{\partial}{\partial t}P_\pm+v\frac{\partial}{\partial z}P_\pm=i\Delta_\pm P_\pm-i(1\mp v/c)d^2 n_b W\varepsilon_\pm/\hbar-\Gamma P_\pm, \quad (5)$$

where $d=e\langle 1|x|2\rangle$ is the electric dipole moment, n_b is the beam number density, Γ is the phenomenological damping constant related to the channeling coherence length v/Γ, $\Delta_\pm=\omega(1\mp v/c)-\omega_0$ is a detuning frequency, and v/c represents a magnetic dipole interaction correction. In the limit of short coherence v/Γ the left-hand side of Eq. (5) is small and Eq. (5) simplifies

$$P_\pm\simeq id n_b W(d\varepsilon_\pm/\hbar)(1\mp v/c)/(i\Delta_\pm-\Gamma).$$

Near resonance $\omega\sim 2\gamma^2\omega_0$ and $\Delta_+/\Gamma\ll 1$ giving

$$P_+=-id^2 n_b W\varepsilon_+(1-v/c)/\hbar\Gamma. \quad (6)$$

In this limit $\Delta_-\sim\omega$, $\Delta_-\gg\Gamma$, and in the case of low gain, P_- can be ignored in Eq. (4). We now define the scalar gain $g=2\pi\omega(d_1)^2 n_b W/\hbar c\Gamma$, where $d_1=d(1-v/c)$. Substituting Eq. (6) in Eq. (4) we obtain

$$e^{ikz}\left[\frac{\partial}{\partial z}\varepsilon_+ +\frac{1}{c}\frac{\partial}{\partial t}\varepsilon_+ +(\mu+iK)\varepsilon_+ -g_+\varepsilon_+\right]+e^{-ikz}\left[-\frac{\partial}{\partial z}\varepsilon_- +\frac{1}{c}\frac{\partial}{\partial t}\varepsilon_- +(\mu+iK)\varepsilon_- -g_-\varepsilon_-\right]=0, \quad (7)$$

where the forward gain factor $g_+=g$ and the backward gain factor $g_-=0$.

In the following we obtain the equation of motion for DFB x-ray laser by using the resonance parts of Eq. (7). Notice that K and μ are the periodic functions, i.e., $K(z)=K(z+a)$ and $\mu(z)=\mu(z+a)$. For a periodic function $f(z)=f(z+a)$ we can use the Fourier-series expansion

$$f(z)=\sum_{l=-\infty}^{\infty}f_l e^{2ilk_B z}, \quad (8)$$

where $k_B=\omega_B/c=\pi/a$ and

$$f_l=\frac{1}{a}\int_{-a/2}^{a/2}dz\, f(z)e^{-2ilk_Bz}\ . \tag{9}$$

We insert the Fourier expansion Eq. (8) for K and μ in Eq. (7). For the case the radiation frequency is close to the n-order Bragg-reflection condition $k\sim nk_B$ and ignoring highly oscillatory terms we obtain from Eq. (7)

$$\frac{\partial}{\partial z}\varepsilon_+ +\frac{1}{c}\frac{\partial\varepsilon_+}{\partial t}+(\mu_0+iK_0)\varepsilon_+ +(\mu_n+iK_n)\varepsilon_-e^{-2i(k-nk_B)z}=g_+\varepsilon_+\ , \tag{10}$$

$$-\frac{\partial}{\partial z}\varepsilon_- +\frac{1}{c}\frac{\partial\varepsilon_-}{\partial t}+(\mu_0+iK_0)\varepsilon_- +(\mu_{-n}+iK_{-n})\varepsilon_+e^{2i(k-nk_B)z}=g_-\varepsilon_-\ , \tag{11}$$

where to obtain Eq. (10) and Eq. (11) we divide Eq. (7) by $\exp(ikz)$ and $\exp(-ikz)$, respectively, and keep only the resonance terms. Finally we redefine $\varepsilon_\pm$ as $\varepsilon_\pm\exp[\mp i(k-nk_B)z]$ and get

$$\frac{\partial}{\partial z}\varepsilon_+ +\frac{1}{c}\frac{\partial\varepsilon_+}{\partial t}-(g_+-i\delta-\mu_0-iK_0)\varepsilon_+ +(\mu_n+iK_n)\varepsilon_-=0\ , \tag{12}$$

$$-\frac{\partial}{\partial z}\varepsilon_- +\frac{1}{c}\frac{\partial\varepsilon_-}{\partial t}-(g_--i\delta-\mu_0-iK_0)\varepsilon_- +(\mu_n^*+iK_n^*)\varepsilon_+=0\ , \tag{13}$$

where $\delta=nk_B-k$ is the detuning from the n-order Bragg reflection. We have assumed that K and μ are real functions, i.e., $K_{-m}=K_m^*$ and $\mu_{-m}=\mu_m^*$. Equations (12) and (13) are a coupled set of equations of motion for the DFB x-ray laser. In Sec. III we include the average effect of the atomic motion in the crystal on Eqs. (12) and (13).

III. ATOMIC MOTION EFFECTS ON THE DFB EQUATIONS

To include the atomic motion effects in an approximate manner we introduce the reflection function K and absorption function μ as an ensemble average over a set of realizations of atomic displacements,

$$K(z)=\langle\hat{K}(z)\rangle,\quad \mu(z)=\langle\hat{\mu}(z)\rangle\ ,$$

where $\hat{K}(z)$ and $\hat{\mu}(z)$ are the reflection and absorption functions, respectively, for a given realization averaged over the transverse direction $\boldsymbol{\rho}$. To carry out the averaging procedure on $\hat{K}$ and $\hat{\mu}$ we consider a general function $f(z)$ such that $f(z)=\langle\hat{f}(z)\rangle$, where $f(z)$ is a periodic function $f(z)=f(z+a)$. We can write $\hat{f}$ as

$$\hat{f}(z)=\sum_j\{V(\mathbf{r}-\mathbf{R}_j-\mathbf{U}_j)\}_\perp\ , \tag{14}$$

where $V(\mathbf{r}-\mathbf{R}_j-\mathbf{U}_j)$ is the contribution at $\mathbf{r}=(\boldsymbol{\rho},z)$ of the atom located close to site $\mathbf{R}_j$ with an atomic displacement $\mathbf{U}_j$. Here $\{V\}_\perp$ is an average over the transverse direction. Applying a Fourier-series expansion and averaging over the transverse direction we obtain

$$\hat{f}(z)=\sum_j\sum_{k_z}V(k_z)e^{ik_z(z-R_j^{(z)}-U_j^{(z)})}\ , \tag{15}$$

where $R_j^{(z)}$ and $U_j^{(z)}$ are the z components of $\mathbf{R}_j$ and $\mathbf{U}_j$, respectively. Here k_z is the z component of the Fourier-series expansion.

In order to find the ensemble average of $\hat{f}$ we use the average relation for a harmonic crystal[18]

$$\langle e^{ik_zU_j^{(z)}}\rangle=\exp[-k_z^2\langle(U_j^{(z)})^2\rangle/2] \tag{16}$$

and obtain

$$f(z)=\sum_j\sum_{l=-\infty}^{\infty}V(k_l)e^{-k_l^2\langle(U_j^{(z)})^2\rangle/2}e^{ik_l(z-R_j^{(z)})}\ , \tag{17}$$

where in Eq. (17) we use the fact that for $f(z)$ periodic the only contribution is for $k_z=k_l\equiv 2lk_B$ and l is an integer.

To find the ensemble average $\langle(U_j^{(z)})^2\rangle$ we use the phonon representation for $\mathbf{U}_j$. For simplicity we consider a monatomic crystal of atomic mass M and with N unit cells,[18]

$$\mathbf{U}_j=\frac{1}{\sqrt{N}}\sum_{\mathbf{q},s}\left[\frac{\hbar}{2M\omega_{\mathbf{q}s}}\right]^{1/2}(C_{\mathbf{q}s}+C_{-\mathbf{q}s})\boldsymbol{\epsilon}_s(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{R}_j}\ , \tag{18}$$

where $C_{\mathbf{q}s}$ ($C_{\mathbf{q}s}^\dagger$) is the creation (annihilation) operator of a phonon with momentum $\mathbf{q}$, frequency $\omega_{\mathbf{q}s}$, polarization vector $\boldsymbol{\epsilon}_s(\mathbf{q})$, and $\boldsymbol{\epsilon}_s(\mathbf{q})=\boldsymbol{\epsilon}_s^*(-\mathbf{q})$. The sum s in Eq. (18) is over all the possible phonons bands. Using the ensemble-average relations

$$\langle C_{\mathbf{q}s}^+C_{\mathbf{q}s}\rangle=n_{\mathbf{q}s},\quad \langle C_{\mathbf{q}s}C_{\mathbf{q}s}^+\rangle=n_{\mathbf{q}s}+1,\quad \langle C_{\mathbf{q}s}C_{\mathbf{q}s}\rangle=0\ ,$$

where $n_{\mathbf{q}s}$ is the occupation number of the phonon $\mathbf{q}s$ and depend on the temperature of the system, we obtain

$$\langle(U_j^{(z)})^2\rangle=\frac{1}{N}\sum_{\mathbf{q},s}\left[\frac{\hbar}{2M\omega_{\mathbf{q}s}}\right](2n_{\mathbf{q}s}+1)[\boldsymbol{\epsilon}_s(\mathbf{q})\cdot\hat{\mathbf{z}}]^2\ . \tag{19}$$

Equation (19) is independent of the specific location j.

We define the Debye-Waller factor[18] as $W_l=k_l^2\langle(U_j^{(z)})^2\rangle/2$ and use Eq. (19) to obtain

$$W_l=\frac{2(lk_B)^2}{N}\sum_{\mathbf{q},s}\left[\frac{\hbar}{2M\omega_{\mathbf{q}s}}\right](2n_{\mathbf{q}s}+1)[\boldsymbol{\epsilon}_s(\mathbf{q})\cdot\hat{\mathbf{z}}]^2\ . \tag{20}$$

From Eqs. (19) and (20) we can rewrite Eq. (17) as a series expansion,

$$f(z)=\sum_{l=-\infty}^{\infty}(f_le^{-W_l})e^{2ilk_Bz}\ , \tag{21}$$

where

$$f_l=\sum_j V(2lk_B)e^{-2ilk_BR_j^{(z)}}\ .$$

Comparing Eqs. (21) and (8) we find that the inclusion of the atomic motion is by replacing f_l by a Debye-Waller-dependent term $f_l\exp(-W_l)$.

Following the above ensemble averaging procedure for $\hat{K}$ and $\hat{\mu}$ the atomic-motion effect can be included in the equations of DFB x-ray laser, Eqs. (12) and (13), by replacing K_n and μ_n by $K_n \exp(-W_n)$ and $\mu_n \exp(-W_n)$, respectively,

$$\frac{\partial}{\partial z}\varepsilon_+ + \frac{1}{c}\frac{\partial}{\partial t}\varepsilon_+ - (g_+ - i\delta - \mu_0 - iK_0)\varepsilon_+ + e^{-W_n}(\mu_n + iK_n)\varepsilon_- = 0 , \quad (22)$$

$$-\frac{\partial}{\partial z}\varepsilon_- + \frac{1}{c}\frac{\partial}{\partial t}\varepsilon_- - (g_- - i\delta - \mu_0 - iK_0)\varepsilon_- + e^{-W_n}(\mu_n^* + iK_n^*)\varepsilon_+ = 0 , \quad (23)$$

where we use that $W_0 = 0$.

The inclusion of the atomic motion by a Debye-Waller factor was verified experimentally in anomalous transmission of x-rays by multiple diffraction (the Borrmann effect).[10,13] From Eq. (20) we find that even at zero temperature, where $n_{qs} = 0$, $W_l \neq 0$ and the zero-point motion influences Eqs. (22) and (23). In the Sec. IV, Eqs. (22) and (23) will be used to analyze the threshold conditions of DFB x-ray laser.

IV. THRESHOLD AND SELECTIVITY CONDITIONS

We now find the effect of absorption and atomic motion on the threshold gain and selected resonance frequency. The system at threshold is presented by the solution of Eqs. (22) and (23) at steady state,[6]

$$\frac{d}{dz}\varepsilon_+ - (g_+ - i\delta - \mu_0 - iK_0)\varepsilon_+ + e^{-W_n}(\mu_n + iK_n)\varepsilon_- = 0 , \quad (24)$$

$$-\frac{d}{dz}\varepsilon_- - (g_- - i\delta - \mu_0 - iK_0)\varepsilon_- + e^{-W_n}(\mu_n^* + iK_n^*)\varepsilon_+ = 0 , \quad (25)$$

where g is identified as a threshold gain and δ as the selected frequency.

The coupled waves Eqs. (24) and (25) describe the spatial variation of transmitted- and reflected-wave amplitudes in a beam-channeling DFB medium. For a slab of length L centered at $z = 0$, the accompanying boundary conditions read: $\varepsilon_+(-L/2) = \varepsilon_-(L/2) = 0$ and no external radiation sources are assumed. The corresponding eigenvalue solutions to Eqs. (24) and (25) for the case that K_n and μ_n are real numbers are found directly,

$$\varepsilon_+(z) = e^{gz/2}\sinh[\lambda(z + L/2)] , \quad (26)$$

$$\varepsilon_-(z) = \pm e^{gz/2}\sinh[\lambda(z - L/2)] , \quad (27)$$

where

$$\lambda = \left[\left(\frac{g}{2} - i\delta - iK_0 - \mu_0\right)^2 + e^{-2W_n}(K_n - i\mu_n)^2\right]^{1/2} \quad (28)$$

and the dispersion relation is

$$\left[\lambda - \frac{g}{2} + i\delta + iK_0 + \mu_0\right] + \left[\lambda - \frac{g}{2} - i\delta - iK_0 - \mu_0\right] \times e^{-2\lambda L} = 0 . \quad (29)$$

A formal solution of allowed resonance frequencies δ and threshold values g can be obtained by inserting Eqs. (26) and (27) in Eq. (24),

$$\lambda = \pm(iK_n + \mu_n)e^{-W_n}\sinh(\lambda L) , \quad (30)$$

$$\frac{g}{2} - i\delta = (iK_0 + \mu_0) \pm (iK_n + \mu_n)e^{-W_n}\cosh(\lambda L) . \quad (31)$$

Equation (30) determines λ. Substitution of λ into Eq. (31) and equating real and imaginary parts yields the allowed δ and g.

Approximate formulas can be obtained in the limit of strong reflections: $(K_n L)^2 \gg (gL)^2 + 1$ and $|\lambda L| \ll 1$. Upon expanding Eq. (30) in this limit and using the expression for λ we find for the first resonance that

$$\delta = -K_0 + [\mu_0^2 + e^{-2W_n}(K_n^2 - \mu_n^2)]^{1/2} , \quad (32)$$

and the threshold gain condition g_t is

$$g_t = 2\mu_0 + \frac{\left[\frac{6K_n e^{W_n}}{(K_n^2 + \mu_n^2)L^3} - 2e^{-2W_n}K_n\mu_n\right]}{[\mu_0^2 + e^{-2W_n}(K_n^2 - \mu_n^2)]^{1/2}} . \quad (33)$$

Equations (32) and (33) can be simplified for the typical case of stronger reflection compared to absorption: $K_n^2 \gg \mu_0^2, \mu_n^2$. For this case,

$$\delta = -K_0 + K_n e^{-W_n} , \quad (34)$$

$$g_t = \frac{6e^{2W_n}}{K_n^2 L^3} + 2(\mu_0 - e^{-W_n}\mu_n) . \quad (35)$$

The threshold gain in Eq. (35) includes two independent terms; the first term is due to reflection,[6] the second one is due to absorption. In a one-passage amplification system (with no reflections) the absorption is with the average absorption coefficient μ_0 which is large for x rays, $\mu_0 > 10\ \text{cm}^{-1}$.[11] In the Bragg-reflection coupling system, standing waves are generated with nodes on the atomic sites and the absorption, located mainly near the atomic sites, is strongly reduced. For this case at low temperature compare to the Debye temperature, where $\exp(-W_n) \sim 1$, μ_n is of the order of μ_0 and g_t due to absorption is strongly reduced compared to μ_0. As the temperature increases the atomic displacement and the absorption increases, the reflectivity of the atomic planes K_n is reduced to $K_n \exp(-W_n)$; thus the threshold-gain condition is increased. Section V is devoted to numerical results and discussion of the threshold-gain condition in Eq. (35).

V. DISCUSSION

We first consider the threshold-gain condition by neglecting the atomic motion $\exp(-W_n)=1$. From Eq. (35) we can write $g_t=6/(K_n^2L^3)+2(\mu_0-\mu_n)$. As a numerical example let us consider a periodic absorption function $\mu(z)=\mu(z+a)$ defined by

$$\mu(z)=\begin{cases}(a/b)\mu_0, & |z|\le b/2\\ 0, & b/2<|z|<(a-b)/2\end{cases}, \tag{36}$$

where b is the region of absorption around an atomic site located at $z=0$ and $b\ll a$. We use the Fourier expansion Eq. (8) to obtain

$$\mu_n=\mu_0\frac{\sin(n\pi b/a)}{n\pi b/a}\ . \tag{37}$$

From Eq. (37) as $b/a\to 0$, $\mu_n\to\mu_0$ for all n and the absorption $\mu_0-\mu_n$ is drastically reduced. Standing waves are generated with nodes on the atomic sites reducing the absorption located on the atomic sites and the threshold gain. As the order of Bragg reflection n increases, μ_n decreases relative to μ_0 and g_t due to absorption is increased. Thus the lowest threshold is obtained for the first-order Bragg reflections, $n=1$. From numerical calculation of the anomalous absorption in a germanium single crystal the value of $1-\mu_n/\mu_0$ can be of the order of 10^{-3}.[14] Thus, using Eq. (37) for $n=1$, $a=3$ Å gives $b=0.1$ Å and the absorption is mainly due to the tightly bound electrons located very close to the atomic sites.

The threshold-gain condition due to reflection can be evaluated using the reflection function $K(z)$ with a modulated atomic electron density: $n_e(z)=n_0[1+\cos(2k_Bz)]$.[6] Applying the Fourier expansion, Eq. (8), for $K(z)$ we obtain $K_0=2\pi e^2n_0/cm\omega_B$ and $K_1=+K_0/2$. Typically K_0 is on the order of 10^4 cm^{-1} in a number of crystalline samples used in channeling studies, for example, silicon and diamond, where n_0 is approximately the crystal bound-electron density. For first-order Bragg reflections $n=1$ and $L=0.1$ cm the value of g_t due to reflection is 2×10^{-4} cm^{-1}.[6] In crystals with low atomic numbers, e.g., LiH, $\mu_0\sim10$ cm^{-1} and for $1-\mu_1/\mu_0\sim10^{-3}$ the value of g_t due to absorption is 10^{-2} cm^{-1}. Thus, the main contribution here to the threshold-gain condition is absorption. But as L decreases below 200 μm the reflection contribution to g_t exceeds the absorption one.

It is shown in Refs. 15 and 16 that by applying a diffraction scheme from several sets of atomic planes which satisfy the Bragg condition simultaneously, it is possible to reduce the absorption of the radiation. In this method, called the multibeam Borrmann effect, standing waves are generated in several directions relative to the atomic site, generating larger nodal regions in radiation fields and the absorption for some of the radiation modes can be reduced by a factor of 10^{-4}.[16] For this case and $\mu_0\sim10$ cm^{-1}, g_t due to absorption can be reduced to 10^{-3} cm^{-1}.

The inclusion of the atomic motion in Eq. (35) is through the Debye-Waller factor and is related to the average displacement U, where $U^2=\langle(U_j^{(z)})^2\rangle$ decreases with the increase in the Debye temperature T_D. For a crystal with a high Debye temperature W_n due to the zero-point motion can be as slow as $W_n\sim10^{-3}n^2$.[14] Thus, for $n=1$ and temperatures $T\ll T_D$ the zero-point motion does not change g_t appreciatively. But as the temperature increases above the Debye temperature the phonon occupation numbers in Eq. (20) introduce in W_1 a temperature dependence proportional to $1+2T/T_D$. This thermal motion has a strong effect on g_t due to absorption which takes the form of $2\mu_0[1-\exp(-W_n)]$ for $\mu_0\sim\mu_n$, and for large W_n the absorption is $2\mu_0$. The thermal motion reduces the reflectivity of the atomic planes to $K_n\exp(-W_n)$ and the total g_t is increased. Thus, in the DFB scheme of x-ray laser one should consider temperatures $T\ll T_D$ with $n=1$. Furthermore, by applying the multibeam Borrmann effect together with the DFB scheme the atomic-motion effect and the threshold-gain condition can be further reduced.

For the case the gain g is larger than the threshold gain g_t the radiation fields $\varepsilon_\pm$ increase with time as $\exp[(g-g_t)ct/2]$ in the linear range. Thus, an amplification factor $(g-g_t)ct/2\sim1$ is obtained for a beam-pulse duration of 50 ns, $L=0.1$ cm, and $\mu_0\sim10$ cm^{-1} for $g\sim10^{-2}$ cm^{-1} in the DFB scheme and by including multibeam Borrmann effect for $g\sim2\times10^{-3}$ cm^{-1}. These results should be compared to the gain $(g-\mu_0)L\sim1$ obtained in a one passage amplification, wherefore $L=0.1$ cm and $\mu_0\sim10$ cm^{-1}, $g\sim20$ cm^{-1}. Thus, in spite of the limitations on g_t due to the absorption, in terms of beam-current requirements the DFB mechanism in beam channeling has possible application in reducing current requirements by many orders of magnitude.

In the present paper we pointed out that the combined effects of DFB mechanism and multibeam Borrmann anomalous transmission can be useful in reducing radiation absorption. Threshold conditions for these combined effects for specific geometry of reflection planes require further study.

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