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A Futures Market for Demand Responsive Travel Pricing

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16. Abstract

Dynamic toll pricing based on demand can increase transportation revenue while also incentivizing travelers to avoid peak traffic periods. However, given the unpredictable nature of traffic, travelers lack the information necessary to accurately predict congestion, so dynamic pricing has minimal effect on demand. Dynamic toll pricing also poses equity concerns for those who lack other travel options. This research explores a potential remedy to these concerns by using a simple "futures market" pricing mechanism in which travelers can lock in a toll price for expected trips by prepaying for future tolls, with the future price increasing as more travelers book an overlapping time slot. This approach encourages travelers to avoid driving during the peak periods when pricing increases toward capacity or to purchase trips in advance when the price remains low or discounted, thus using infrastructure capacity more efficiently. Travelers that do not prepurchase their trip are subject to the real-time market price, which is determined by dynamic congestion pricing. This futures-market mechanism can augment existing toll collection technologies and provide travelers with sufficient pricing information and purchasing options to preplan their travel and avoid excessive prices.

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Disclaimer

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Executive Summary

Executive Summary

Despite the proven operational and revenue benefits of dynamic toll pricing, dynamic tolls are seldom implemented in the United States, largely due to public disdain for tolls and the historic success and reliance on the gas tax–funded federal Highway Trust Fund (HTF). However, nearly three decades of unadjusted inflation, coupled with ever-increasing vehicle fuel efficiency, has eroded the HTF's solvency, sparking a renewed interest in alternative revenue streams.

The idea of dynamic toll pricing confronts three critical challenges:

- Low elasticity of demand, yielding a limited congestion mitigating effect
- Public disdain due to price risk
- Equity concerns due to lack of travel planning options

This research explores a potential remedy through a futures market-based, toll-pricing mechanism. The proposed concept is simple: Travelers can lock in their toll price by prepaying for future tolls, with the future price increasing as more travelers book an overlapping time slot. This approach encourages travelers to avoid driving during the peak periods when pricing increases toward capacity or to purchase trips in advance when the price remains low or discounted. Travelers that do not prepurchase their trip are subject to the real-time market price, or spot price, determined by dynamic congestion pricing.

The operational process of booking, density estimation, and then pricing is continually updated until the horizon of the future trip time or some cutoff period is reached. At this point, the sale of future trips in the "futures market" is closed to booking, and additional trips must be paid at the spot price based on the actual observed traffic density. It is possible—and likely—that there will be a difference between the futures price and the spot price. This difference is the motivating force behind real futures markets—for buyers to try to beat the market price, and for sellers to create stability of demand in exchange for a discount by locking buyers into a contract.

A basic booking interface could be a website or app where users can prepay for their trip during a specified time window. A similar concept exists in other ticketed transportation modes with discrete capacity, such as airlines and rail travel, but have yet to be implemented in more atomized modes, such as highways and transit. However, overcoming this challenge using density estimation, the system can then be easily integrated with existing toll collection and payment systems where the prepurchased trips are exchanged as a toll or fare waiver.

As an example, this research explores the possibility of augmenting an existing fixed bridge toll with a futures-based dynamic bridge toll. A methodology is provided to set the futures price based on expected future trip density. A sensitivity analysis of delay and revenue is conducted by varying the upper and lower price limits,

P_{max} and P_{min}, in relationship to the preexisting fixed price, P_{fixed}. In addition, a further sensitivity analysis is conducted by varying elasticity, ε, to see how elasticity assumptions can affect revenue and delay.

Figure 1 illustrates the break-even points between dynamic and fixed pricing. The y-axis is the ratio of the upper price limit, and the x-axis is the ratio of the lower price limit. The results show that congestion can be mitigated as long as $P_{max} > P_{min}$, which makes intuitive sense because any amount of price differential (i.e., dynamic pricing) will yield delay improvements. However, this does not guarantee a revenue gain. To avoid revenue loss, the upper pricing limit must be approximately double the amount lost from P_{min} compared to the fixed toll, or $P_{max} \approx P_{fixed}\left(2 - \frac{P_{min}}{P_{fixed}}\right)$. However, this varies depending on the elasticity, which causes the revenue break-even line to curve slightly upward, increasing the upper price limit, P_{max}, that is required to offset the discounted trips. In contrast to the revenue break-even point, the delay break-even point (between the yellow and red regions) appears unchanged by elasticity.

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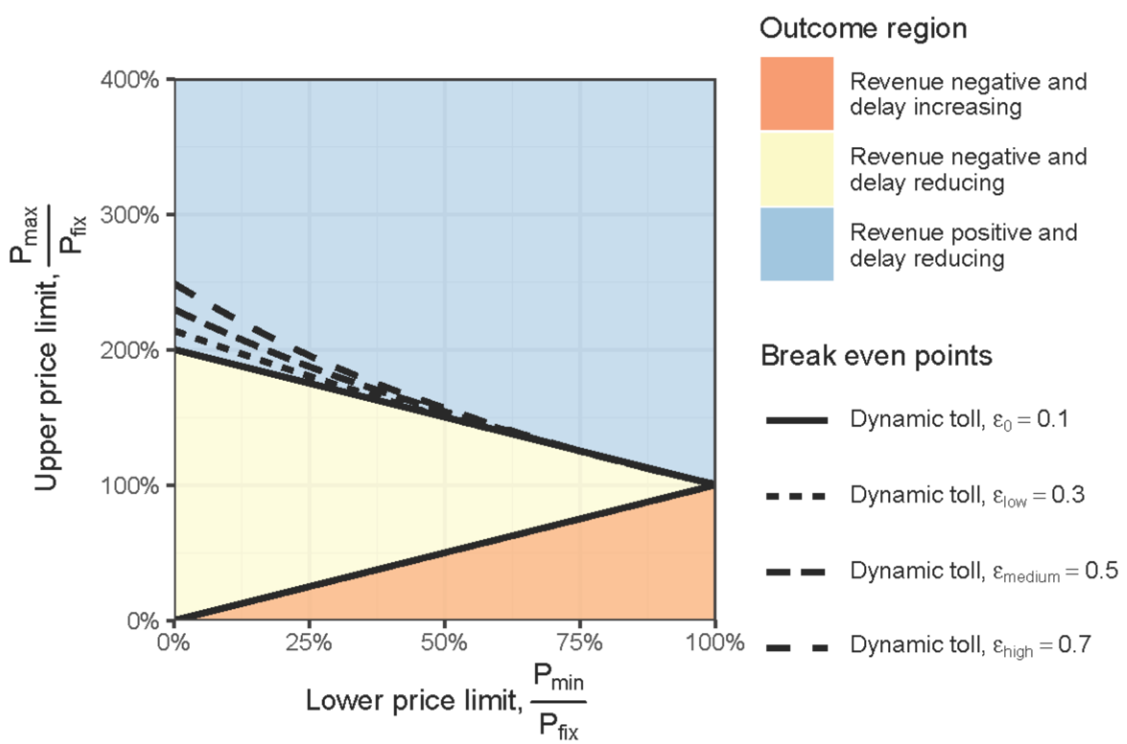


Figure 1. Break-even points between dynamic and fixed pricing with constant elasticity of $\varepsilon = 0.3$

An attractive feature of the futures market concept is that it can be developed independently and implemented as an incremental improvement of an existing toll system. An existing dynamic toll system can easily be augmented with a futures market to further improve demand optimization with just software changes. A fixed toll system can use a futures market to introduce dynamic pricing as an "opt-in" program, incentivizing users with discounts for booking travel during periods of low demand. Infrastructure investment would be minimal, especially if electronic tolling is already in place. In this case, a futures market would be a soft way to introduce dynamic pricing in an otherwise politically hostile environment.

Contents

Introduction

Despite the proven operational and revenue benefits of dynamic toll pricing, dynamic tolls are seldom implemented in the United States, largely due to public disdain for tolls and the historic success and reliance on the gas tax–funded federal Highway Trust Fund (HTF). However, nearly three decades of unadjusted inflation, coupled with ever-increasing vehicle fuel efficiency, has eroded the HTF's solvency, sparking a renewed interest in alternative revenue streams.

While a myriad of intersecting factors affect public acceptance, equity concerns, and the operational limitations of dynamic tolls, one root factor is the price and travel time uncertainty. Given the unpredictable nature of traffic congestion, the public disdain for dynamic tolls is understandable because regular travelers are exposed to both price risk and travel time risk (*1*). For lower-income travelers who tend to have stricter time constraints and commute longer distances, a dynamic toll presents a regressive tax (*2*), posing a major equity concern if no reasonable travel alternative exists. Lastly, automobile travel is notoriously inelastic, dampening the congestion mitigating benefits achievable through dynamic pricing. To summarize, there are three critical challenges with dynamic toll pricing:

- Low elasticity of demand, yielding a limited congestion mitigating effect
- Public disdain due to price risk
- Equity concerns due to lack of travel planning options

This research explores a potential remedy to these critical concerns through a futures market–based toll-pricing mechanism. The proposed concept enables travelers to lock in their toll price by prepaying for future tolls, with the future price increasing as more travelers book an overlapping time slot. This approach not only incentivizes travelers to preemptively avoid peak congested periods, but offers an opportunity for regular commuters to compare prices and minimize price risk of unexpected congestion by purchasing tolls in advance. Operationally, the proposed mechanism potentially yields greater congestion mitigation benefits by effectively increasing price elasticity of demand. To evaluate the effectiveness of such a system, this research conducts a thorough sensitivity analysis of elasticity and pricing constraints to explore possible system outcomes for reducing delay and collecting revenue.

Background

Transportation agencies must constantly balance between providing sufficient capacity for the peak period while minimizing expenditure. Wasteful over-building is cost prohibitive as well as highly inefficient, leaving the majority off-peak periods underutilized. Moreover, experience over the past half century has shown that incremental highway expansion without demand controls can lead to a vicious cycle of induced demand in which agencies perpetually increase capacity as users inevitably fill any available excess capacity (*3*). More recently, transportation agencies are increasingly relying on tolls to raise revenue and to mitigate congestion. However, conventional fixed tolls fail to target the congested periods and merely apply a uniform downward disincentive on travel (see Figure 2). This situation is especially true when no travel alternative (e.g., transit) exists (*4*). Sufficiently high travel prices (e.g., tolls or gas prices) can reduce travel demand, but they do not necessarily encourage efficient use of infrastructure. It is also economically problematic to suppress overall travel demand because it can dampen economic productivity rather than increase efficiency.

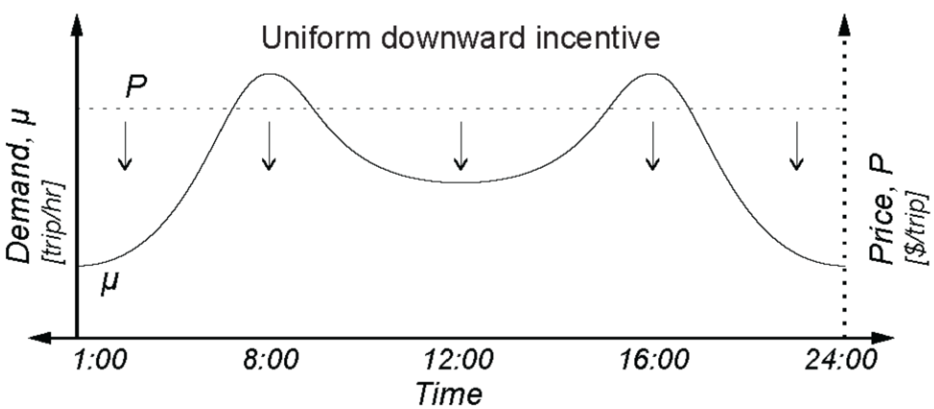


Figure 2. Effect of a fixed toll pricing incentive

One solution to this problem is to dynamically adjust pricing based on demand to help discourage vehicle travel during congested periods and shift travel to other modes or to off-peak periods (see Figure 3). Dynamic tolls have long been proposed as a solution to provide more targeted congestion mitigating incentives (*5*, *6*, *7*). Although similar schemes have been used in other transportation sectors (e.g., air and rail travel), the highly atomized, short-term, and high-frequency nature of highway and transit use presents a formidable technical challenge. However, with the advent of electric toll collection (e.g., FastTrak and EZ Pass), the remaining obstacles to dynamic tolls are largely political (*8*). Understandably, travelers do not want to pay more for travel beyond the sunk cost of fuel and automobile ownership, nor do they want to be subjected to unpredictable congestion-related costs that they have limited control over. In addition, dynamic pricing creates equity concerns if it extorts travelers who have no other reasonable alternative travel options (e.g., transit) (*2*, *4*, *9*).

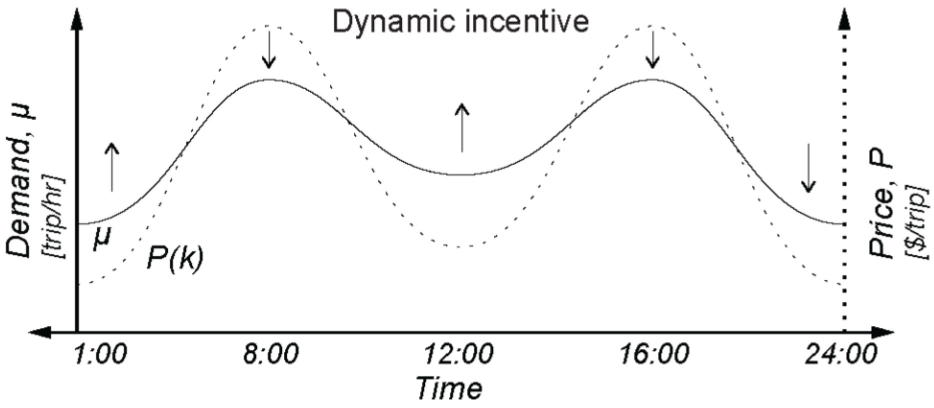


Figure 3. Effect of a dynamic toll pricing incentive

A fundamental issue with dynamic pricing is uncertainty and lack of information. For example, if travelers have no information about future travel cost or time, they are unlikely to adjust their behavior. Thus, price changes will have little effect on demand in the short term. Dynamic pricing depends on a feedback loop whereby a change in price results in some change in demand. To economists, this is called price elasticity of demand, where elasticity reflects the sensitivity of price to demand as a proportional change in demand to a change in price (*10*). Automobile travel is relatively inelastic (*11*) due to a variety of exogenous factors, such as housing location choice, personal investment in an automobile, land-use context, or lack of travel alternatives (*12*).

In practice, travelers often base their travel decisions on a combination of current traffic conditions and anecdotal evidence of past experiences, speculating future traffic conditions. However, this is hardly efficient and works only in relatively steady-state conditions. As a result, price elasticity of automobile travel demand remains relatively weak compared to other modes (e.g., rail or air travel). The effectiveness of dynamic pricing is dampened by travelers' inability to avoid congestion. Price elasticity can also vary due to departure and arrival time inflexibility (*13*, *14*, *15*).

Looking beyond automobile transport, insight can be drawn from industries with more mature dynamic pricing systems. In the era of airline price deregulation in the 1980s, the first airlines to adopt dynamic pricing were able to out-compete their less-efficient rivals (*16*). To date, most modern airlines rely on sophisticated dynamic pricing systems of some kind. More recently, it has been shown that airline ticket elasticity increased with the proliferation of telecommunications and online ticket sales (*17*), demonstrating the correlation of increased prior information and increased elasticity.

Recently, more modern concepts are being proposed, such as mobility as a service (MaaS) or transportation as a service (TaaS). In these frameworks, automated "travel brokers" provide integrated multimodal trip packages to travelers (*18*, *19*, *20*, *21*). However, the highly fragmented transportation industry makes implementing an integrated pricing and booking system extremely challenging, leaving MaaS and TaaS with little traction outside of academic literature (*22*). However, some large technology companies have implemented MaaS-like multimodal payment integration features in their platforms (e.g., integrated mobile payment with transit and ride-hailing services), potentially laying the groundwork for MaaS and TaaS in the future.

To increase the efficiency and social impact of dynamic pricing systems, a growing body of literature is proposing heavy-handed, market-based approaches to congestion pricing, such as “travel credits” or “travel permits” (*23*). Primitive systems that permit vehicles to travel only at certain times or locations, such as odd-even rationing or peak-hour permits, have existed for decades. Odd-even rationing was first implemented in the United States during the 1979 oil crisis, during which vehicles were permitted for use on alternating odd-even days of the month corresponding to the last digit of their license plates. More recently, some governments have instituted a targeted rationing scheme, such as the Singapore Area Licensing Scheme where drivers are required to purchase permits to enter the central city during specific times (*24*). This licensing scheme successfully operated from 1975 until 1998 when Singapore replaced it with the Electronic Road Pricing (ERP) system, which automatically charges drivers depending on both time and location (*25*). The ERP system enables the transportation agency to optimize infrastructure utilization throughout its entire road network, not just a single link or zone.

Singapore also famously set a quota on the number of automobiles that can be registered. Creating scarcity, the bidding-based system has resulted in extremely high costs of up to US$7,000–$44,000 ($10,000–S$60,000 in Singapore dollars) to own and operate a vehicle, which is approximately 12 times the median monthly household income. Although this system is highly effective at reducing the number of automobiles, scarcity of resources in any scheme can lead to severe equity imbalances if alternatives do not exist. Fortunately, transit options are plentiful in the dense city-state of Singapore, but that is not the case elsewhere, as in the United States. To mitigate this, “travel credit” schemes in which all travelers are issued travel credits have been proposed. This approach not only provides all users an equal quantity of travel, but also serves as a “cap and trade” model, essentially crediting people for not contributing to congestion or pollution (*9*). Unused credits can then be redeemed in some form, such as conversion to currency or a tax offset. To further combat inequity, credits can be allocated based on a variety of models (*26*, *27*), such as rewarding carpooling (*28*, *27*) or using alternative modes such as transit or bicycling (*29*).

A further market-based evolution of the travel credit scheme is to enable travelers to “sell” their trip rights to the highest bidder, achieving a similar goal without the need for governments to convert credits (*30*, *31*, *32*, *33*). In a fully realized system, a network-based travel credit scheme can optimize travel based on network capacity in the entire network (*34*, *35*, *36*, *37*, *33*). However, such a system would be difficult to implement due to public resistance in most western democracies.

In contrast to ideas like travel credits and travel permits, this research proposes an opt-in futures market to augment existing deployments of conventional revenue collection technologies (e.g., automated tolls and electronic fares). The proposed futures mechanism is not necessarily a radical new concept, nor is it a technologically driven paradigm shift, but rather a promising concept to overcome the political barriers of road pricing while improving price elasticity and addressing equity concerns. The objective is to enhance pricing information transmission and purchasing options in a way that incentivizes positive behaviors. The purpose of this research is to explore potential outcomes of such a concept by conducting a sensitivity analysis of price limits and elasticity.

Conceptual Framework

The basic concept for the system is shown in Figure 4, where travel demand (e.g., on transit, over a bridge, along a corridor, or in a zone) can be divided into discrete travel windows (e.g., between 8:00–8:15) when there is a finite quantity of capacity available (e.g., a road capacity of 2,000 vehicles per hour per lane). Travelers then have the option to prepurchase a planned trip from the available capacity, with prices increasing as the number of booked trips in that spatio-temporal window increase. This approach encourages travelers to avoid traveling during those peak periods as the price increases toward capacity or to purchase trips in advance while the price remains low or discounted. Travelers that do not prepurchase their trip are subject to the real-time market price, or spot price, determined by dynamic congestion pricing.

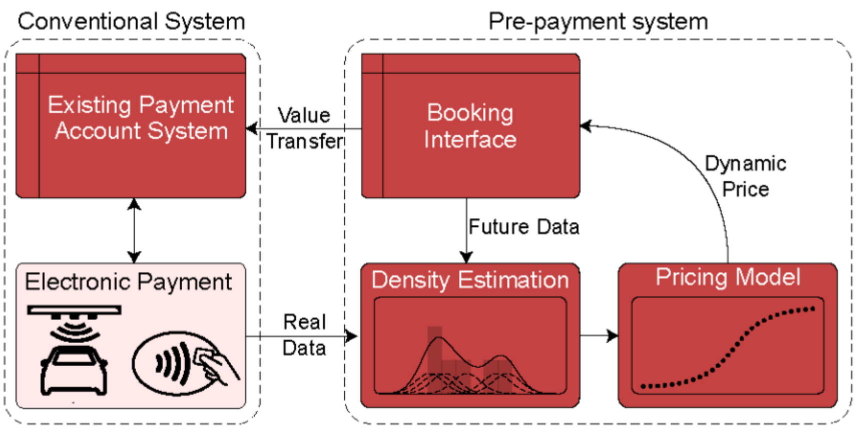


Figure 4. Conceptual system model

The process of booking, density estimation, and then pricing is continually updated until the horizon of the future trip time or some cutoff period is reached. At this point, the sale of future trips in the futures market is closed to booking, and trips must be paid at the spot price based on the actual observed traffic density. It is possible—and likely—that there will be a difference between the futures price and the spot price. This difference is the motivating force behind real futures markets: for buyers to try to beat the market price, and for sellers to create stability of demand in exchange for a discount by locking buyers into a contract. In this case with transportation, as opposed to commodities like oil and grain, there are two possible outcomes for price differences:

- **Futures rate < Actual rate.** This outcome occurs when too few people book trips, having limited congestion mitigating effects until sufficient market penetration is reached. The prebooked travelers would yield a large discount against the conventional toll, thus incentivizing more travelers to adopt the booking system, eventually correcting this difference and reducing congestion.

- **Futures rate > Actual rate.** Although highly unlikely but still possible, this outcome occurs when too many travelers fail to meet their target window or did not show up at all. The travelers would incur a loss on their booked trips, thus incentivizing travelers to be punctual.

A basic booking interface could be a website or app where users prepay for their trip during a specified time window. The same concept exists in other ticketed transportation modes, such as airlines and rail travel. The system can then be easily integrated with existing toll collection and payment systems where the prepurchased trips are exchanged as a toll or fare waiver.

Trip Booking

The overall objective is to increase prices when too many travelers try to occupy the same time and space in a transportation system. One approach is to discretize time into convenient time slots, such as 1-, 5-, or 15-minute intervals. Each time slot becomes tokenized for purchase, with the price of each time slot varying as demand approaches capacity, as shown in Figure 5. When a traveler books their desired time window, they effectively purchase a sequential bundle of time slots. For example, a time window from 8:00 a.m. to 9:00 a.m. could have 60 slots at a 1-minute duration or 4 slots at a 15-minute duration, depending on the time discretization.

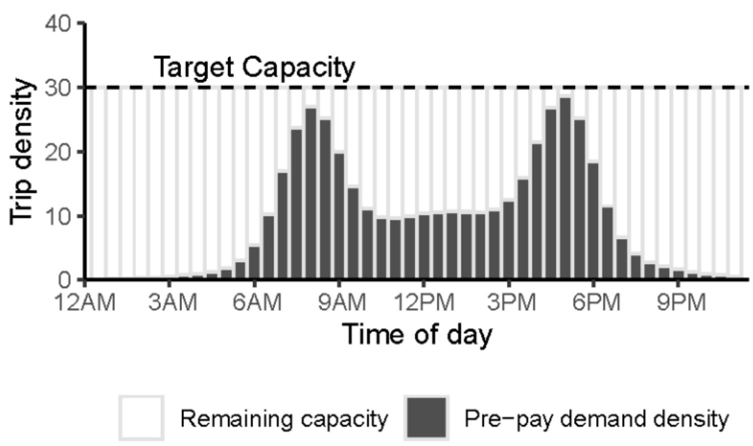


Figure 5. Capacity across discretized time

The total purchase price is the sum of token prices across that time window. From this, a variety of pricing policies could be implemented. For example, travelers pay only for the actual time slot that they use and are reimbursed for the remainder. Alternatively, the total cost of a typical time window (e.g., 30 minutes) could be calibrated to have a nominal total cost comparable to a typical toll.

A potential problem with rectangular windows is safety concerns from psychological effects on drivers attempting to avoid missing the window. For example, a driver who is running late and risks missing the window would be financially motivated to drive aggressively to avoid losing the discount. One approach to avoid this undesirable outcome is to allow drivers to purchase non-rectangular (e.g., Hann, Truncated-Gaussian) time windows with some target arrival time and some width as illustrated in Figure 6. In this scheme, a window function can provide a slowly diminishing discount the further the traveler arrives from their booked arrival time. The price could be calculated as a convex combination of the booked and spot prices:

$$\mathrm{P} = \varphi(\mathrm{t}_a - \mathrm{t}_b)\,\mathrm{P}_{booked} + (1 - \varphi(\mathrm{t}_a - \mathrm{t}_b))\,\mathrm{P}_{spot}$$

where φ is the price ratio formulated as a zero-mean Gaussian distribution, t_b is the booked arrival time, and t_a is the actual arrival time.

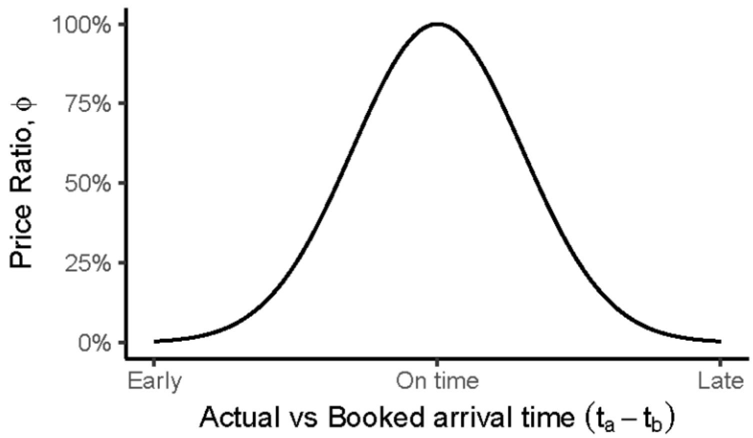


Figure 6. Smooth price window function

Trip Density

A notion of trip density is introduced to convert the demand captured by the number of prebooked trips into a smoothed representation of "presence" on the transportation facility. This notion of trip density is used to determine prices, and it is better if it is not wildly discontinuous. One solution is to smooth the discrete trips using Kernel Density Estimation (KDE), converting the discrete values into a continuous function of expected trip densities. There are a variety of KDE kernel types, but a common approach is to use a Gaussian normal curve to smooth the arrivals:

$$\hat{k}(t) = \frac{1}{N}\sum_{i=1}^{N} K_h(t - t_i) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{t - t_i}{h}\right)$$

where $\hat{k}(t)$ is the estimated density function for time t, t_i is the booked arrival time of the i^{th} customer, K is the non-negative kernel function (e.g., Gaussian normal curve), h is a smoothing bandwidth parameter that must be greater than 0, and N is the total number of prebooked trips. KDE essentially functions by cumulatively applying a continuous density function K at each finite sample point, providing a smoothed probability distribution, as illustrated in Figure 7.

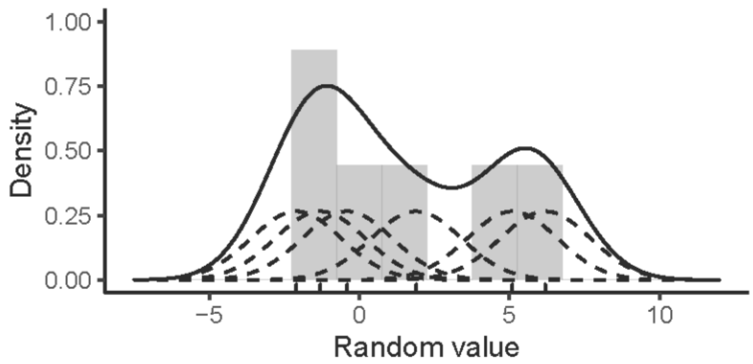


Figure 7. Kernel density estimation using a Gaussian function

The modular nature of KDE is useful in this context, especially if the arrival distribution is complex, so that a more suitable kernel can then be utilized. For example, Figure 7 demonstrates KDE using a normal distribution as the kernel. If arrival times are skewed (e.g., drivers are more often early than late or vice versa), a log-normal function could be utilized.

Pricing

After a smooth and stable value of future traffic density $\hat{k}(t)$ is calculated, it can be easily input into a pricing function to calculate an estimated future price for any travel window. Although it is theoretically feasible to float the price in a bid-based system, this is impractical for drivers at any reasonable scale and is susceptible to severe market fluctuations given the instability of traffic flow. For example, when traffic flow comes to a near standstill, there is little to stop the price from increasing toward infinity. Alternatively, a more stable price could be determined using an engineered pricing function calibrated to match the limited “supply” of roadway capacity with demand. This would provide both a stable and efficient pricing system. Unlike most traditional ticketed modes (e.g., planes and trains), there is not a discrete number of spaces available on highways. Instead, there are steady speed conditions that approach an optimal point of traffic flow where maximum throughput (vehicles per hour) is achieved as density increases but begins to collapse into congestion when exceeded.

As opposed to using traffic speed or traffic flow as the target measure, traffic density provides an ideal dependent variable for pricing. While the objective in certain cases (i.e., roadways) is to maintain smooth traffic flow and speeds, a dynamic pricing function based directly on traffic speed or flow would fail to capture the underlying infrastructure utilization. Traffic speed is not a reliable measure of utilization because speed will remain relatively constant (i.e., near the speed limit) across a range of traffic densities. Speed only begins to fall when traffic reaches a critical density and conditions deteriorate into congestion. This makes it difficult to set prices based on speed, because pricing would be flat below this critical point and then suddenly jump when congestion occurs. Moreover, traffic flow is also an unsuitable measure because it has a U-shape form where uncongested high-speed and low-density conditions can achieve the same throughput in vehicles per hour as a congested low-speed and high-density situation. This makes it difficult for a pricing function to distinguish between under- and over-saturated traffic conditions. Thus, basing the pricing on density provides a reliable measure of infrastructure utilization across a range of traffic conditions.

The pricing function itself could be a simple linear or monotonically increasing function, but it may be beneficial for political reasons to set upper and lower price constraints on a dynamic pricing system. Rather than using a piece-wise function, the smooth and bounded pricing function shown in Figure 8 uses an S-shaped sigmoid as a function of trip density k (e.g., traffic density in veh/km/hr).

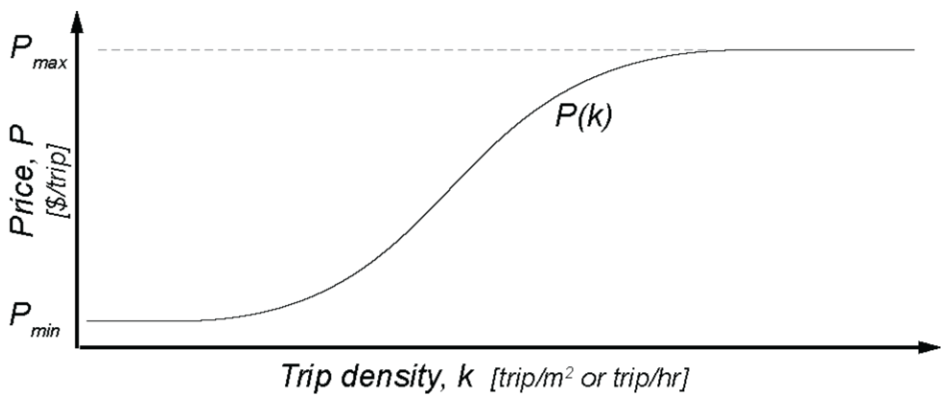


Figure 8. Pricing sigmoid function

The sigmoidal price function provides natural parameters for price minimum P_{min} and maximum P_{max} boundaries and can be expressed as:

$$P(k) = P_{min} + \frac{P_{max} - P_{min}}{1 + e^{\alpha - \beta k}}$$

where α and β are tuning parameters to specify the horizontal shift and “steepness” of the function, respectively.

The sigmoid function provides the purchase price, but this price can also vary depending on the actual arrival time of drivers. If drivers fail to meet their booked time window, it would be unfair to other punctual drivers to still provide the full discount. This however, can be softened through the use of the aforementioned arrival time pricing window function shown in Figure 6.

Methodology

Although there are dual objectives for dynamic toll pricing—revenue generation and delay reduction— the goal of this study is not to formulate bi-criteria optimization but to explore and evaluate possible outcomes through simulation. Through simulated parameter exploration, a sensitivity analysis is conducted on the proposed framework by varying the lower price limit, P_{min}, upper price limit, P_{max}, and price elasticity of demand, ε. The analysis is conducted in three parts:

- A fixed-parameter example case
- Exploration of how varying the upper and lower price limits affect total delay and revenue
- Exploration of how changes in elasticity affect total delay and revenue

The purpose is to explore how different upper and lower price limits can affect revenue and delay outcomes. Moreover, the importance of exploring price elasticity is to evaluate whether increasing the price elasticity through a prepay futures market could enhance revenue generation and delay reduction benefits of dynamic tolling. The following subsections describe the price elasticity of demand and the macroscopic traffic flow model used for simulation.

Price Elasticity of Demand

To simulate the resulting demand shift from pricing, the simple microeconomic principle of price elasticity of demand is utilized. Elasticity, ε, reflects the demand sensitivity to price, that is the percent change in demand resulting from a percent change in price. Elasticities of $\varepsilon < 1$ are considered "inelastic," resulting in a proportionally smaller change in demand from a change in price. Elasticities of $\varepsilon > 1$ are considered "elastic," resulting in a greater proportional change in demand from a change in price. Lastly, elasticities of $\varepsilon = 1$ are considered "unit elastic" when the change is proportionally equal. Price elasticity of demand in transportation tends to be fairly inelastic, typically with values of just 0.10–0.20 for shorter-term price changes and 0.20–0.80 for longer-term price changes. Long-term price changes are considered to take place over periods greater than two years (*12*).

Although temporal variation in transportation elasticity is due to a variety of complex socioeconomic and behavioral factors, a simple explanation is that it takes time for people to adjust their behavior depending on conditions in their lives (*38*, *15*). For example, housing choice and automobile ownership are long-term decisions. If fuel prices rise, people cannot instantly move homes or purchase a more fuel-efficient car. However, the objective of the proposed travel pricing futures market is to increase elasticity by giving travelers some of the advantages typically associated with longer-term travel behavior. That is, to be able to compare prices well in advance rather than speculating future traffic conditions based on anecdotal evidence. For this simulation, a constant isoelastic price-demand function is used, which can be expressed as:

$$\Delta\mu = e^{-\varepsilon \Delta P} - 1$$

where $\Delta\mu$ is the percent change in demand, and ΔP is the percent change in price, with the function centered on the origin, as shown in Figure 9. Because an improvement in price elasticity with a futures market is unknown, several different elasticities are explored in the following simulation. A range of elasticity points are used: a very low elasticity of $\varepsilon_0 = 0.1$ to represent a conventional dynamic toll with no futures market mechanism, and then three increasing elasticities of $\varepsilon_{low} = 0.3$, $\varepsilon_{medium} = 0.5$, and $\varepsilon_{high} = 0.7$ are used to represent dynamic pricing with a futures market mechanism.

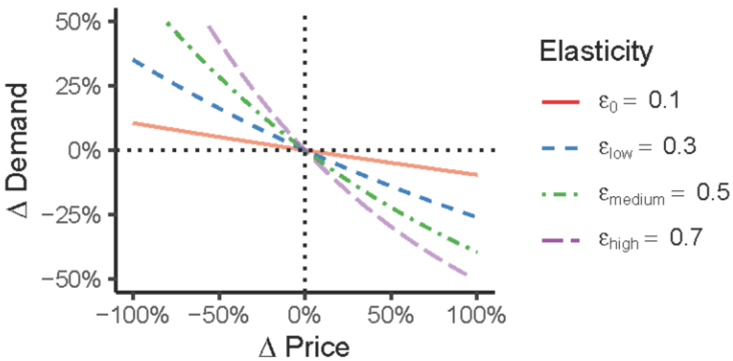
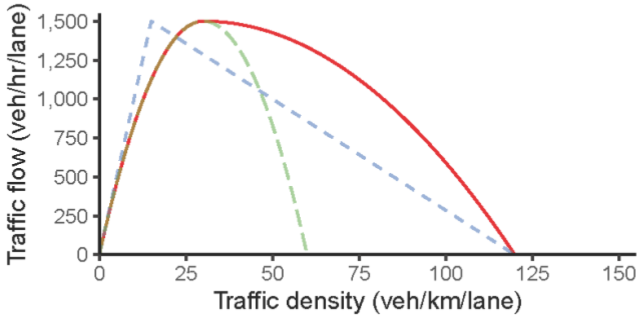


Figure 9. Price elasticity

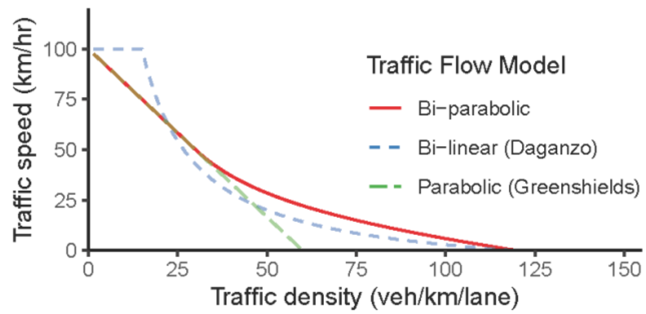
Traffic Flow

Congestion that impacts the traffic flow across a facility, such as a bridge, can be characterized by the fundamental diagram. Two common classical models that require no additional calibration parameters are Greenshields's parabolic function (*39*) and Daganzo's simple bilinear model (*40*), as shown in Figure 10.

Greenshields's seminal function is elegantly simple, but its symmetric parabolic shape has since proven a poor fit in reality, particularly when critical density, k_c, is exceeded. In contrast, Daganzo's model provides a simple, parsimonious model with an asymmetric form. Its linearity assumes a constant traffic speed in the free-flow regime, and a constant backward wave speed in the congested regime. The constant free-flow speed not only ignores minor delay caused by a gradual slowing of traffic as density increases, but causes an abrupt transition at the critical density between the two regimes.



(a) Flow-density functions



(b) Speed-density functions

Figure 10. Fundamental diagrams for traffic flow

The bi-parabolic function employed in this study is a combination of both Greenshields's and Daganzo's models. It is not the most elegant or precise model, but it satisfies the needs for this simple simulation because it requires no additional calibration parameters and contains no abrupt transitions between free-flow and congested regimes. The modified piecewise function is composed of two different parabolic functions to provide the more realistic asymmetric form observed empirically, as illustrated in Figure 10a, while avoiding abrupt transitions, as shown in Figure 10b. The piecewise bi-parabolic function can be described as:

$$\mathrm{q(k)} = \begin{cases} k < k_c & v_f k\left(1 - \dfrac{k}{2k_c}\right) \\ k \geq k_c & \dfrac{k_c v_f}{2}\left(1 - \dfrac{(k - k_c)^2}{(k_j - k_c)^2}\right) \end{cases}$$

where k_c is the critical density when traffic flow is greatest, k_j is the jam density when traffic flow completely stops, and v_f is the free-flow traffic speed. For reference, critical density is approximately 20–40 veh/km, and jam density is about 100–150 veh/km.

The free-flow regime corresponds to $k < k_c$. The congested regime corresponds to $k \geq k_c$. For this study, the critical and jam densities, k_c and k_j, are chosen to be 30 veh/km and 120 veh/km, respectively.

Performance Measures

Although a variety of alternative performance measures exist in practice, such as vehicle miles traveled, energy consumption, greenhouse gas emission, etc., this study focuses more narrowly on the primary operational outcomes of revenue and delay. Revenue is simply calculated as the sum of the products of demand, μ_i, and price, P_i, per time increment, i.

$$\text{Revenue} = \textstyle\sum_i \mu_i P_i$$

Similar to revenue, operational performance can be evaluated in terms of delay. The total delay in each scheme can be compared as the percent change in aggregated total delay, $\Delta\%d$, calculated as:

$$\Delta\%d = \frac{\sum_i d_{2i} - \sum_i d_{1i}}{\sum_i d_{1i}} = \frac{\sum_i \frac{n_{2i}}{v_{2i}}(v_f - v_{2i})}{\sum_i \frac{n_{1i}}{v_{fi}}(v_f - v_{1i})}$$

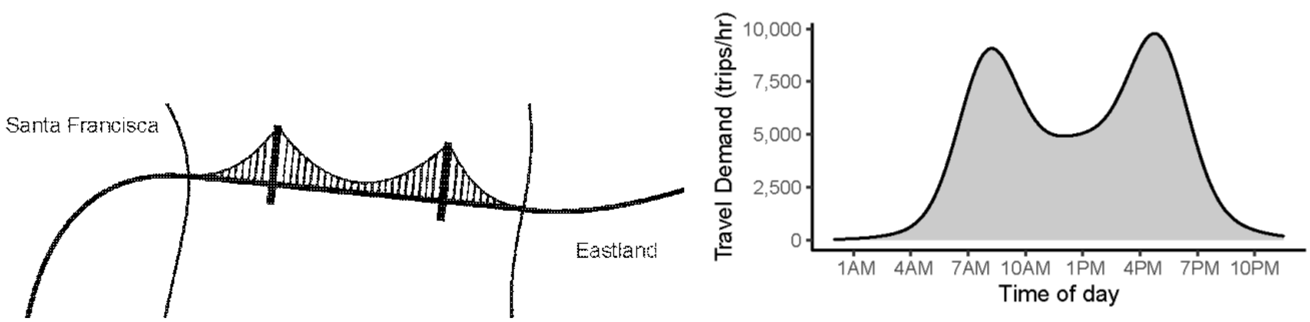
where delay is $d = n_i L(\frac{1}{v_i} - \frac{1}{v_f})$ for each time increment i. L is the distance traveled, which cancels out in percent change; n_{1i} and n_{2i} are the demand in number of trips, and v_{1i} and v_{2i} are the calculated traffic speeds for the fixed and dynamic tolls, respectively. The equation above effectively sums up the total delay experienced across the entire 24-hour period and calculates the percentage change between the fixed and dynamic tolling schemes.

Results

The results are organized as follows. First, a fixed-parameter example case is tested to demonstrate the model in a simple scenario. Next, a sensitivity analysis of delay and revenue is conducted by varying the upper and lower price limits. This illustrates how setting the upper and lower price limits can affect revenue and delay outcomes. Similarly, a further sensitivity analysis is conducted by varying elasticity to see how greater changes in elasticity can affect revenue and delay.

Simulation Case—Fixed Parameters

For contextual orientation, a simple case example provides context for discussion but also in selecting reasonable parameter ranges. As an example application, suppose there are two adjacent urban centers in a metropolitan region, as shown in Figure 11a, which are separated by a body of water. The two cities are connected by a bridge that carries 100,000 trips per day with an existing fixed toll. The daily travel demand has a distribution with two severe peaks, as shown in Figure 11b. For simplicity, assume that the traffic flow is balanced in each direction, and the bridge has three lanes in each direction.



(a) Scenario **(b) Demand distribution**

Figure 11. Toy simulation scenario

Suppose that the regional metropolitan transportation planning commission wants to alleviate congestion during the peak periods with a dynamic tolling system. A study determines that the price elasticity of demand for a conventional dynamic tolling system without a prepay futures market is $\varepsilon_0 = 0.1$, and that a futures market mechanism would boost elasticity to somewhere between $\varepsilon_{low} = 0.3$ on the low end, and $\varepsilon_{high} = 0.7$ on the high end. The community is willing to approve a dynamic toll system that is limited between a 50 percent discount for off-peak travel, and no more than a 200 percent surcharge above existing fixed prices during the peak period (see Table 1).

Table 1. Example of fixed pricing parameters

Parameter	Value	Description
P_{min}	50%	Minimum dynamic price ratio ${P_{min}}/{P_{fixed}}$
P_{max}	200%	Maximum dynamic price ratio ${P_{max}}/{P_{fixed}}$
α	7.0	Price calibration parameter
β	0.3	Price calibration parameter

Calibration parameter values of 7 and 0.3 are set for α and β, respectively, which provides a smooth S-shaped sigmoid function with the upper and lower limits approximately located near traffic densities of 0 and 30 veh/km, as shown in Figure 12. A pricing function in this case should align the upper price limit to incentivize demand to result in densities below the maximum desired values.

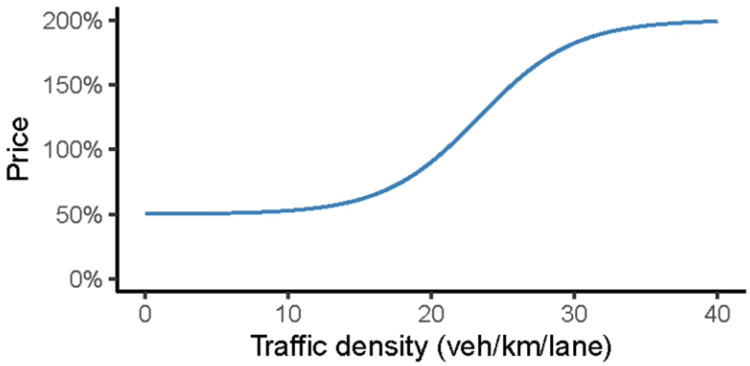


Figure 12. Pricing sigmoid function

Using the parameters in Table 1, traffic conditions can be simulated to compare results between a dynamic pricing scheme against a fixed toll. The price changes cause demand to shift from the peak to the off-peak, resulting in a decrease in congestion and an improvement in average travel speed, as shown in Figure 13. Although there is a travel speed improvement in all cases, the magnitude depends on the elasticity. The greater the elasticity, the greater the improvement.

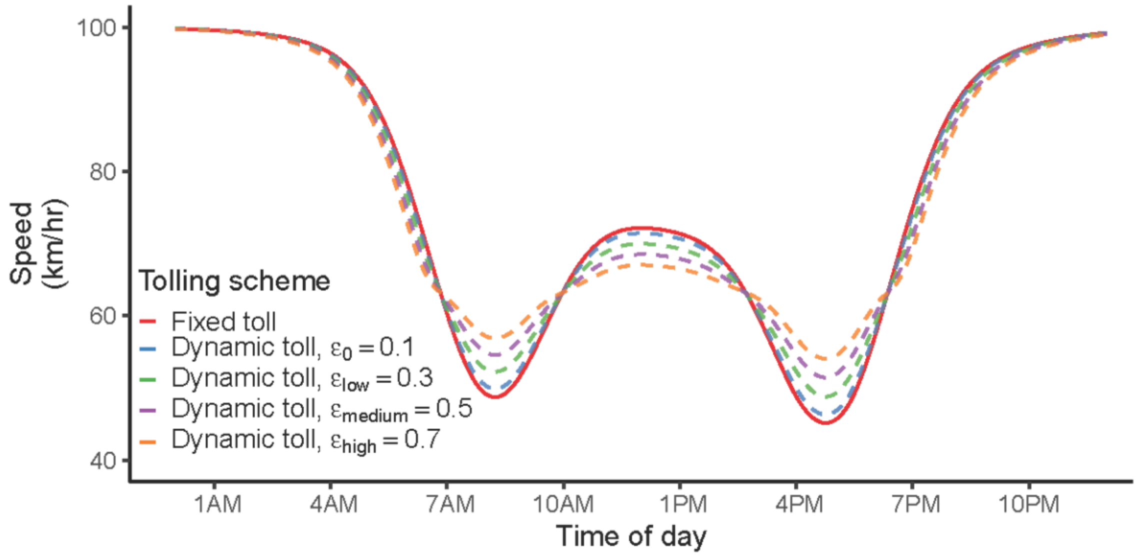


Figure 13. Average traffic speed by time of day

The elasticity will not only determine how sensitive travelers are to price changes, but will also affect the aggregated total revenue collected. A revenue increase from a dynamic pricing scheme compared to a fixed price is not guaranteed and depends on the parameters selected in the pricing function. Ultimately, the revenue collected from a dynamic pricing scheme primarily depends on the elasticity, ε, and the upper and lower limits of the pricing function, P_{max} and P_{min}. The net revenue resulting from the example actually yielded a slight increase in revenue in this case, as shown in Figure 14, despite offering a discounted P_{min} toll.

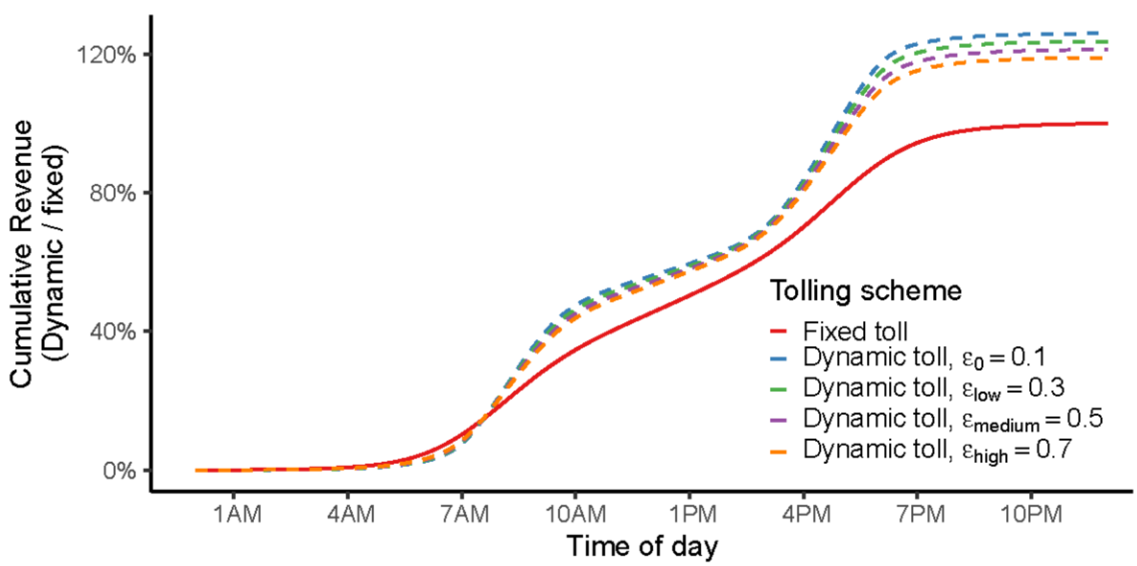


Figure 14. Cumulative revenue collection by time of day

This shows that a modest adjustment in price results in demand shift (i.e., commuters shifting behavior to avoid high tolls) without revenue loss. In this case, conventional dynamic pricing increased revenue by 26 percent over fixed tolls. However, as elasticity increases, this revenue gain is reduced slightly to 24 percent, 21 percent, and 19 percent for low, medium, and high elasticity futures market cases, respectively. The reason for this is that if travelers are more sensitive to price changes, fewer travelers are going to pay the higher prices and shift to cheaper off-peak times. There is a trade-off between improving congestion and increasing revenue, highlighting the critical nature of setting upper and lower price limits P_{max} and P_{min}.

Price Change Effects

To explore price and elasticity parameters, a simulation results matrix can be computed and plotted. The colorized results form the surfaces in Figure 15 and Figure 16, which show the percent change in revenue or delay achievable by varying the combination of P_{min} and P_{max} simultaneously with a fixed elasticity of $\varepsilon = 0.3$.

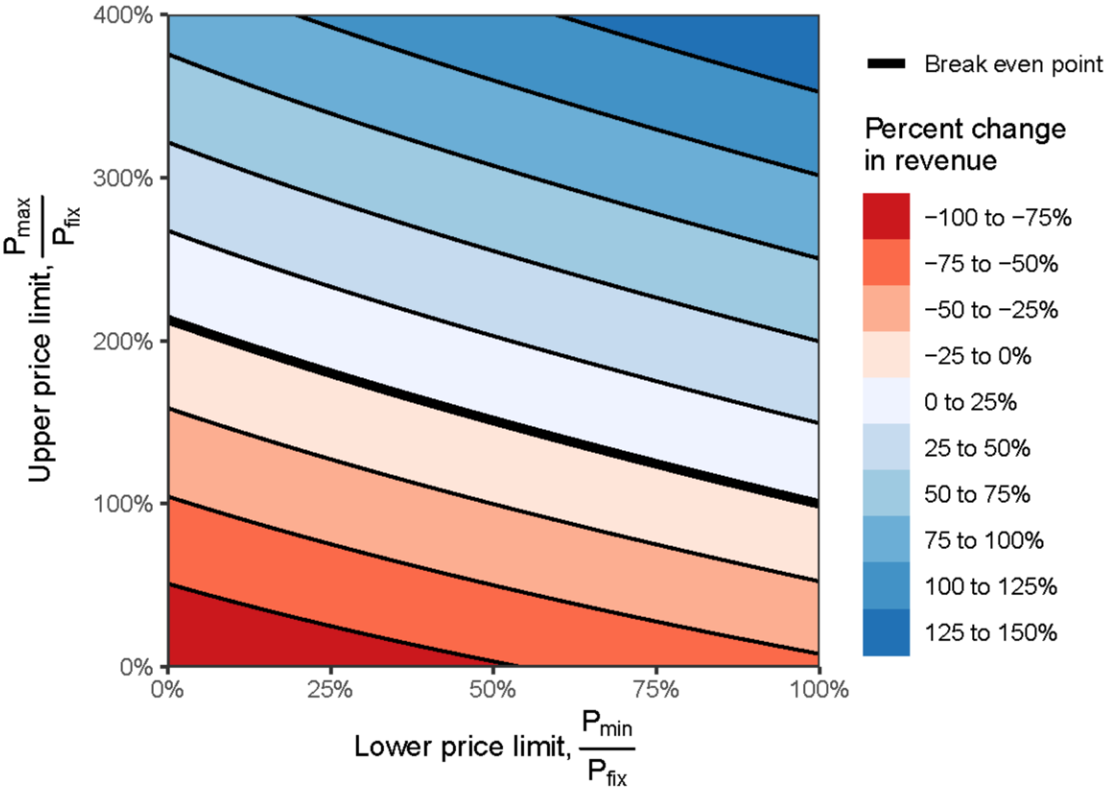


Figure 15. Revenue comparison between dynamic and fixed pricing varying P_{max} and P_{min} with constant elasticity of $\varepsilon = 0.3$

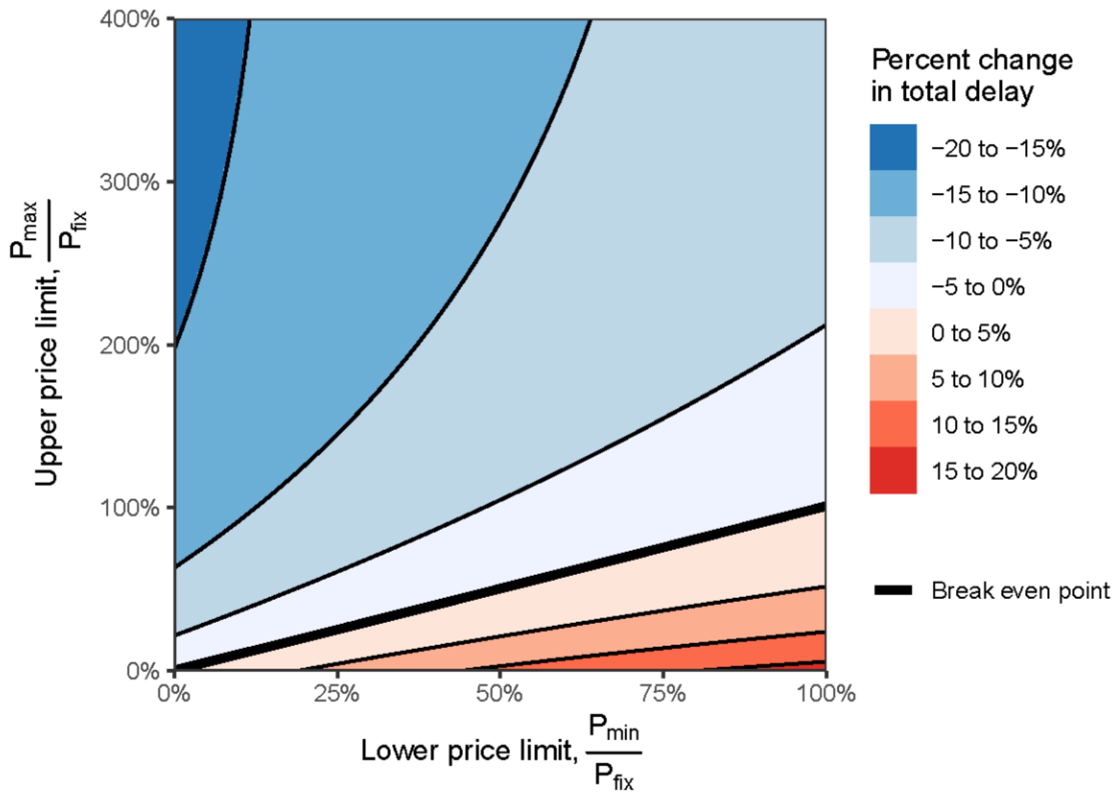


Figure 16. Delay comparison between dynamic and fixed pricing varying P_{max} and P_{min} with constant elasticity of $\varepsilon = 0.3$

Both Figure 15 and Figure 16 show that an appropriately balanced combination of lower and upper price limits must be chosen. For example, if the lower price limit is decreased, the upper price limit must be sufficiently raised to compensate for the lost discount revenue. This is a result of the relatively low elasticities (i.e., $\varepsilon < 1$), which means that a change in price will have a proportionally smaller impact on demand.

The dark solid lines in Figure 15 and Figure 16 indicate the break-even points where the upper and lower price limits balance out, resulting in 0 percent change in the objective of either revenue or delay. Extracting the break-even lines and combining the two plots, the general regions of comparative gains can be illustrated in Figure 17. The plot illustrates more clearly which regions provide improvements in both revenue and delay, only one, or neither.

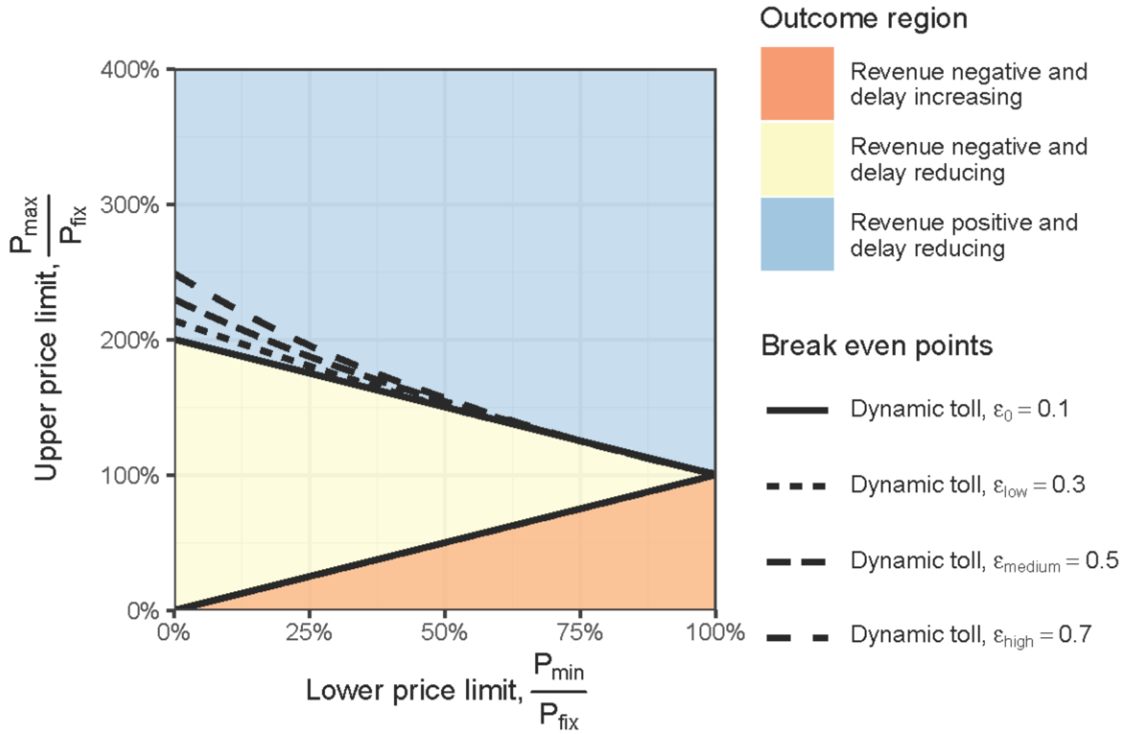


Figure 17. Break-even points between dynamic and fixed pricing with constant elasticity of $\varepsilon = 0.3$

It is apparent that congestion can be mitigated as long as $P_{max} > P_{min}$. This makes intuitive sense; any amount of price differential (i.e., dynamic pricing) will yield delay improvements. However, this does not guarantee a revenue gain. To avoid revenue loss, the upper pricing limit must be approximately double the amount lost from P_{min} compared to the fixed toll, or $P_{max} \approx P_{fixed}\left(2 - \frac{P_{min}}{P_{fixed}}\right)$. However, this varies depending on the elasticity, which causes the revenue break even line to curve slightly upward, increasing the upper price limit, P_{max}. In contrast to the revenue break-even point, the delay break-even point appears unchanged by elasticity. This is merely because the break-even point shifts proportionally with elasticity from this particular perspective shown in Figure 17. The following analysis explores how elasticity affects both revenue and delay in more detail.

Elasticity Change Effects

Exploring how elasticity effects the revenue and delay outcomes, a set of plots can be similarly created by varying the maximum price and elasticity, P_{max} and ε. For simplicity, the maximum lower price limit P_{min} of $0 was chosen as the extreme floor for dynamic pricing. While it is technically possible to go below $0, effectively offering a subsidy for off-peak times, for simplicity this theoretical scenario is not explored. As reflected in the break-even region plot, Figure 18 shows that as elasticity increases, an exponentially higher P_{max} is required to maintain revenue levels.

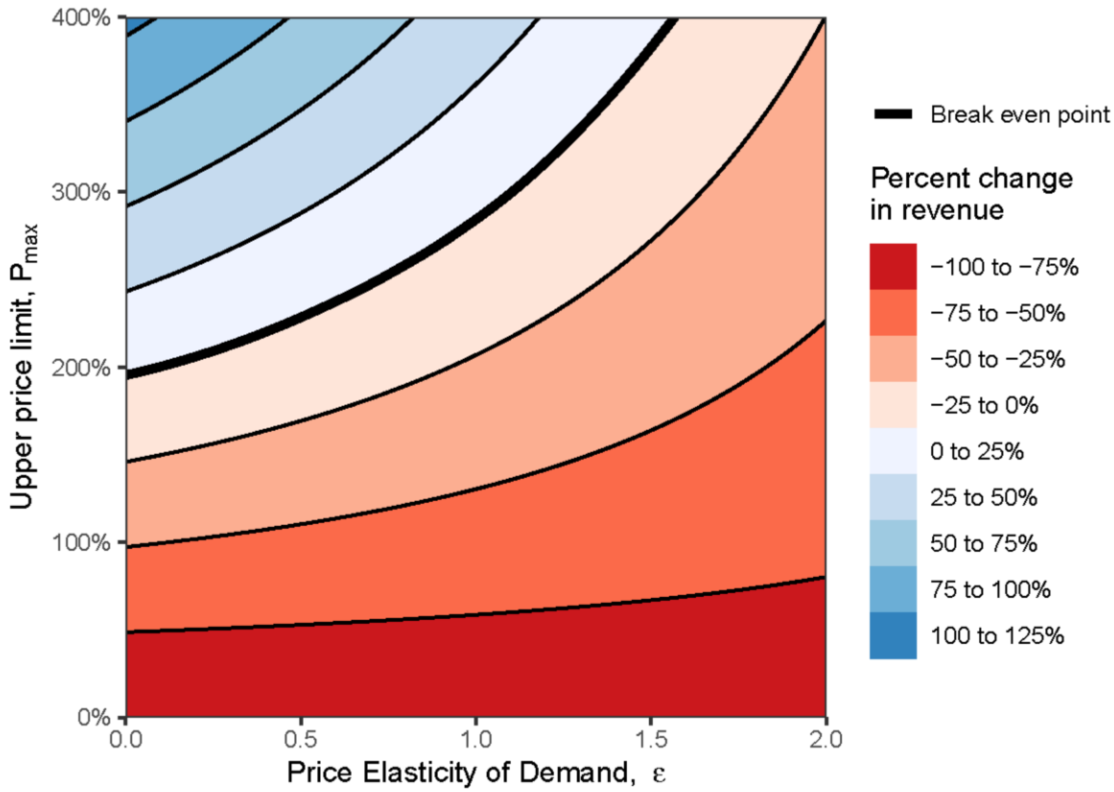


Figure 18. Revenue comparison between dynamic and fixed pricing varying elasticity ε and P_{max} with constant $P_{min} = 0$

Figure 19 illustrates changes in delay as the upper price limit and elasticity are varied. The maximum performance benefits appear to be saddled around an elasticity of 1.0 and a maximum price above 100 percent. The saddle shaped region is an interesting outcome, showing that there is diminishing returns as elasticity moves away from unit elasticity. In the most extreme case, where $\varepsilon > 1$ and $P_{max} > 100\%$, travelers shift demand such that overall traffic congestion can worsen. However, this is a very unrealistic scenario because transportation demand is generally very inelastic, especially with short-term price changes. In practice it is likely that the feasible range exists where $\varepsilon < 1$ and delay is reduced.

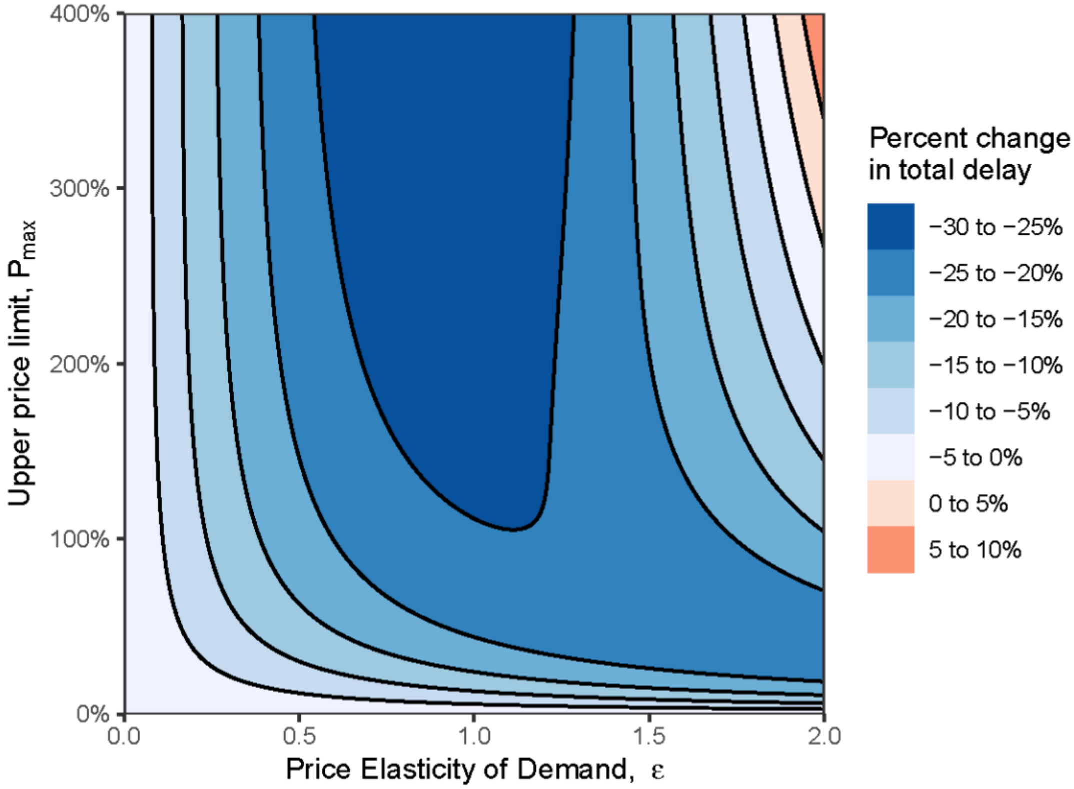


Figure 19. Delay comparison between dynamic and fixed pricing varying elasticity ε and P_{max} with constant $P_{min} = 0$

Discussion

An attractive feature of the futures market concept is that it can be developed independently and implemented as an incremental improvement of an existing toll system. An existing dynamic toll system can easily be augmented with a futures market to further improve demand optimization with only software changes necessary. A fixed toll system can use a futures market to introduce dynamic pricing as an opt-in program, incentivizing users with discounts for booking travel during periods of low demand. Infrastructure investment would be minimal, especially if electronic tolling is already in place. A futures market in this case would be a soft way to introduce dynamic pricing in an otherwise politically hostile environment. In this case, the futures market could result in revenue loss if the fixed toll, which is used as P_{max}, is not sufficiently high enough. The loss could be considered acceptable if improving operational performance is the priority. However, it is also possible that improved operations could yield an overall increase in demand throughput, offsetting lost revenue.

Although this report presents the futures concept in the context of a simple bridge toll, it can easily be expanded to other applications, such as corridor tolls, congestion pricing zones, transit fares, ride hailing, or applications beyond transportation. When creating a new market, care must be taken to ensure that the operation is smooth, stable, and equitable. The following subsections discuss several basic market safeguards and practical constraints.

Market Manipulators and Exploitation

If unused trips can be sold back to the system, there is potential for profiteering, which would cause undesirable price fluctuations in the system that do not correspond to physical infrastructure capacity and demand. To prevent users from inappropriately exploiting the system, restrictions should be set. For example, to prevent users from manipulating the market price, a practical restriction could be to limit users to purchase only one trip per time slot (otherwise it would be a travel impossibility) or to limit the number of purchases a user can make to ensure that a net profit cannot be made. Ensuring zero profit would also help avoid user tax complexities. Although restrictions might seem market prohibitive, the intent of the system is to promote efficient travel and revenue collection, not market capitalization.

Beyond individual profiteering, large agents, such as freight carriers or transportation network companies, could manipulate the market to for example, coordinate vehicle arrival to drive up the cost for their competitors. However, unlike other cases where this has occurred, such as with major airlines at airports, the more atomized nature of roadway users makes it more difficult to manipulate (*41*, *42*). Still, it is worthy of regulatory consideration.

Arrival Windows

While it is desirable to encourage punctuality, it is undesirable to cause drivers undue anxiety for missing their target arrival times. The system could be designed in such a way to allow for users to specify the desired width of their travel windows, offering greater discounts to more punctual travelers and proportionally lower discounts to those who are less punctual. More research is required to determine a window shape that provides an appropriate punctuality incentive.

Kernel Density Estimation

It is uncertain whether travelers are as likely to arrive early as they are to arrive late. If so, this can be modeled with a Gaussian normal curve. If travelers trend on being late but not early, a log-normal curve might be a better choice. Research is needed to model accurately the relative reliability of travelers, which is helpful for selecting an appropriate kernel for density estimation because this is the basis for accurate pricing.

Pricing Calibration

To avoid reaching outrageous prices and face a public outcry, the pricing function needs to be calibrated to match several criteria. Ideally, the upper bound is kept as low as possible to avoid public dissatisfaction but high enough to achieve the desired shift in travel behavior. The lower bound must also be kept low enough to encourage a shift in travel behavior but high enough to collect sufficient revenue. Further research is needed to measure elasticity variation in the long term as users become accustomed to the futures scheme. It is possible that the pricing requires regular tuning to adjust for evolving user perceptions.

Conclusions

This research evaluated revenue and operational benefits of a dynamic toll pricing "futures market" with a sensitivity analysis of price elasticity of demand and pricing constraints. The sensitivity analysis was conducted using a simple elasticity-based simulation model in an effort to explore functional form and parameter effects. Results show that:

- Dynamic pricing is beneficial within most elasticity ranges (i.e., approximately $\varepsilon < 1.5$ in absolute terms)
- Larger elasticity improves delay but reduces revenue.
- Dynamic pricing can improve revenue and delay within a feasible region of pricing boundaries.

This simple example showed strong potential improvements in both revenue and performance but relies on several simplifications and assumptions, the primary being that elasticity can vary depending on a variety of factors, such as time, purpose, and individual preferences and flexibility. This is important to consider, particularly with regard to transportation equity when transportation costs increase over a heterogeneous population.

Another important element not considered is the possibility of overall demand suppression from pricing because travel demand is assumed constant. While dynamic pricing is intended to more efficiently utilize infrastructure capacity, it is possible that a pre-existing fixed toll has suppressed demand overall. Replacing a fixed toll with a dynamic toll might actually cause an increase in overall trips due to the newly available peak-hour capacity and the reduced priced off-peak trips. This could cause both positive and negative outcomes with regards to revenue and delay, as well as other endogenous outcomes, such as emissions and fuel consumption.

While simple microeconomic elasticities may often be too theoretical for practical use, they do offer general insight for policy analysis. It is clear from this simulation that price elasticity of demand is critical to both congestion and revenue. A greater elasticity means travelers will more easily change their trip time, having a greater effect on delay, but it also means that more travelers will choose to travel at lower-priced times, thus decreasing total revenue. Intuitively, this means that high inelasticity is good from a revenue perspective but does little to mitigate congestion, and vice versa.

In practice, simple price elasticity of demand could be too crude for day-to-day optimization. A more refined approach might be to calibrate the parameters with a bounded bi-criterion objective that seeks to optimize for both revenue and/or congestion in a bounded region, such as that shown in Figure 17, but also accounting for variable elasticity. The pricing function itself could be modified from the symmetric sigmoid function to some custom optimized function using empirical data (e.g., using artificial intelligence to set prices to maximize the objective). The proposed booking system lends itself to such optimization, providing data of future demand to help best predict optimal prices, a feature unavailable in current dynamic pricing systems. Moreover, integration and revenue-sharing policies with other modes, such as public transit, could yield wider benefits.

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