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# Theory of short-wavelength lasing from channeled projectiles: Nondegenerate dipole transitions

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A theory is developed for lasing action from ensembles of relativistically or subrelativistically propagating emitters whose motion is bound in the direction(s) transverse to the direction of propagation. These include relativistic electrons and positrons channeled in crystals or other hollow-channel structures, as well as fast ions wherein a bound electron is perturbed by the crystal potential or laser light. Apart from planar-channeled positrons in crystals, the confining potentials for all other emitters in this category are strongly anharmonic. Therefore, their spectral dipolar transitions are nondegenerate, each involving a different pair of nearly discrete levels of the confining potential. This implies that stimulated emission from such systems can exhibit coherence in the Glauber sense. The theoretical framework presented here consists of Heisenberg equations which have the Maxwell-Bloch form with modifications resulting from the high velocity of the emitters. Steady-state semiclassical solutions of these equations are obtained. It is shown that previous approaches, based on the assumption that the cross section for stimulated emission is uniform throughout the system, do not account for the spatial variation of the polarization at high velocities. As a result, these approaches do not yield the correct gain coefficient whenever the characteristic lengths for the dephasing of the dipole oscillation and for emission amplification are comparable. The latter conditions are realizable in structures composed of channels much wider than in crystals. Lasing schemes are investigated and the prospects for achieving gain in these schemes at wavelengths below 100 Å are discussed.

## I. INTRODUCTION

Many types of stimulated short-wavelength (vacuum uv and x-ray) radiation sources that have been suggested over the years employ beams of high-velocity (relativistic or subrelativistic) emitters. The appeal of such sources lies in the fact that the emission is continuously Doppler shifted towards shorter wavelengths and becomes progressively more collimated about the beam direction as the beam velocity is increased.

Such sources may be grouped into two categories.

(a) *Free-electron lasers* (FEL's), encompassing structures wherein a spatially periodic perturbation acts either on the electron beam (as in magnetic undulators,[1–3] light wigglers,[4–6] and crystals serving as coherent bremsstrahlung amplifiers)[7] or on the emitted radiation (due to periodic changes in the refraction index, e.g., in superlattices acting as transition-radiation devices[8,9]). The emission features in FEL's are determined by transfer of discrete momentum (wiggler quanta or reciprocal lattice vectors) to the structure from the emitter or the field. Generally, many quantum states of the emitting electron are involved in the dynamics, which is governed by equations of the Raman-Nath or Mathieu type.[10,11] As a result, short-wavelength FEL radiation is, in principle, not coherent (in the Glauber sense),[12,13] exhibiting non-Poissonian photon statistics. In the quantum regime of operation, quantum recoil (which is responsible for gain) hampers coherence and, during the start-up stage, many-particle effects lead to thermal statistics.[12] In the classical steady-state regime, too, small deviations from coherence are predicted.[13]

(b) *Transverse confinement emitters* (TCE's), i.e., emitters whose motion is bound in the direction(s) transverse to the direction of propagation. These include relativistic electrons and positrons channeled in crystals[14–16] or other hollow structures,[17] as well as fast ions wherein a bound electron is perturbed by the crystal potential[18,19] or laser light.[20] The emission wavelengths are determined by the nearly discrete spectrum of their transverse-energy states and by the Doppler shift due to their longitudinal motion. Planar channeled positrons in crystals are the only TCE's whose nearly harmonic transverse confining potential requires the inclusion of many states in the dynamical description. All other emitters in this category are confined by strongly anharmonic potentials and therefore their dynamics involves directly only the upper and lower nearly discrete states of a particular spectral (nondegenerate) transition. Hence, these systems can, in principle, exhibit true lasing (coherent stimulated emission).

 

Furthermore, the spectral efficiency, narrow spectral width, and high degree of polarization achievable in x-ray emission from TCE's (Refs. 14 and 15) prompt their consideration as short-wavelength lasing sources.

The physics of the various gain regimes in FEL's has been studied extensively.[2–4,10–13] In contrast, all calculations of gain in TCE's (Refs. 14, 17, and 21–23) have followed the procedure of Beloshitskii and Kųmakhov (BK) (Ref. 24) who obtained the stimulated emission rate in a dipolar transition between two transverse-energy states by the Fermi golden rule, on treating the field as a first-order perturbation and assuming a Lorentzian emission line shape. It will be shown that this approach does not allow fully for the time and space variation of the radiation intensity from high-velocity dipolar emitters and is valid only when the dephasing (coherence) length $L_2$ for the dipole oscillation is much shorter than the effective length for emission amplification $L_a$. Notably, the BK approach does not yield the correct gain at current densities exceeding $\mathrm{MA/cm^2}$ in structures with considerably wider channels than crystals,[17] which are promising hosts of x-ray lasing.

We present here a comprehensive theoretical framework for stimulated emission from TCE's. It consists of Heisenberg equations which, for two-level TCE's, assume the Maxwell-Bloch form,[25,26] but with modifications resulting from the high velocity ($v \sim c$) of the emitters (Sec. II). In this paper we investigate only the steady-state semiclassical solutions of these equations (Sec. III). The BK results are retrieved in the limit of "strong dephasing" $L_a \gg L_2$ (Sec. IV). The opposite "weak-dephasing" limit $L_a \ll L_2$ yields a drastically different dependence of the gain coefficient on current density, emission wavelength, $v$ and $L_2$ (Sec. V). Numerical estimates of short-wavelength lasing feasibility are given for various types of TCE's and the essential features of the gain expressions for these systems are summarized in the Discussion (Sec. VI).

## II. MAXWELL-BLOCH EQUATIONS FOR RELATIVISTIC DIPOLAR EMITTERS

Relativistic electrons and positrons with energy $E = mc^2\gamma$ propagating in a potential $V(\mathbf{r})$ of the structure are described by the Dirac bispinors

$$\Psi(\mathbf{r}) = \begin{bmatrix} \phi(\mathbf{r}) \\ \chi(\mathbf{r}) \end{bmatrix} \tag{1a}$$

satisfying[27]

$$\begin{aligned} c(\boldsymbol{\sigma}\cdot\mathbf{p})\phi &= [E + mc^2 - V(\mathbf{r})]\chi \, , \\ c(\boldsymbol{\sigma}\cdot\mathbf{p})\chi &= [E - mc^2 - V(\mathbf{r})]\phi \, . \end{aligned} \tag{1b}$$

Here $\sigma$ is the Pauli spin matrix and $\mathbf{p}$ the momentum operator. On expressing $\chi$ in terms of $\phi$, then neglecting $V^2\phi$ and the spin-orbit term

$$(E + mc^2)^{-1}\hbar c^2(\boldsymbol{\sigma}\cdot\nabla V)(\boldsymbol{\sigma}\cdot\mathbf{p})\phi$$

compared to $EV\phi$, one obtains[28]

$$\Psi(\mathbf{r}) \simeq \left[ \frac{\gamma+1}{2\gamma} \right]^{1/2} \begin{bmatrix} u\psi(\mathbf{r}) \\ \dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})u\psi(\mathbf{r})}{mc(\gamma+1)} \end{bmatrix} . \tag{2a}$$

Here $u$ is a constant ($\mathbf{r}$-independent) spinor, as implied by the neglect of spin-orbit coupling, and $\psi(\mathbf{r})$ satisfies the Schrödinger equation for a particle with mass $m\gamma$

$$H_0\psi = (\hbar^2 k_0^2/2m\gamma)\psi, \quad H_0 = p^2/2m\gamma + V(\mathbf{r}) \, , \tag{2b}$$

$\hbar k_0 = mc(\gamma^2 - 1)^{1/2}$ being the momentum of the particle on entering the structure.

Relativistic particles channeled in a structure propagate at small angles to the longitudinal direction $z$, which is either the symmetry axis of an axial channel or an axis nearly parallel to the direction of the beam incidence $\mathbf{k}_0$ in a planar channel $y$-$z$. In the first case, the propagation of the channeled particle is confined by the potential $V(\mathbf{r})$ in the transverse plane $\mathbf{r}_\perp = (x,y)$ while in the second case the direction of transverse confinement is $\mathbf{r}_\perp = \mathbf{x}$.

The treatment of channeled particles commonly invokes the continuum approximation,[29] wherein $V(\mathbf{r})$ is replaced by its average over $z$ (in an axial channel) or $y$ and $z$ (in a planar channel), i.e., the propagation along the channel is taken to be completely free. In this approximation the Hamiltonian (2b) can be separated into longitudinal and transverse parts

$$H_0 = p_z^2/2m\gamma + H_{0\perp}, \quad H_{0\perp} = p_\perp^2/2m\gamma + V(\mathbf{r}_\perp) \, , \tag{3a}$$

thereby allowing us to write $\psi(\mathbf{r})$ as a superposition of waves

$$\psi_{\mathbf{k}_0,n}(\mathbf{r}) = w_{\mathbf{k}_0,n}(\mathbf{r}_\perp)e^{ik_n z} \tag{3b}$$

with nearly constant amplitudes $c_n$. Here $w_{\mathbf{k}_0,n}(\mathbf{r}_\perp)$ is an eigenfunction of the $n$th state of $H_{0\perp}$, corresponding to the transverse energy eigenvalue $E_{\perp n}$.

The longitudinal velocity $v_{zn}$ of the $n$th wave is determined by

$$\begin{aligned} v_{zn} &= \langle n \,|\, p_z \,|\, n \rangle / m\gamma \\ &= \hbar k_n / m\gamma = c(1 - 1/\gamma^2 - 2E_{\perp n}/mc^2\gamma)^{1/2} \, . \end{aligned} \tag{4a}$$

At $\gamma \gg 1$, the relative dispersion of longitudinal velocities about the mean velocity $v_z$

$$v_z \equiv \beta_z c \simeq c(1 - 1/2\gamma^2 - \langle E_\perp \rangle / mc^2\gamma) \tag{4b}$$

($\langle E_\perp \rangle$ being the mean transverse energy) is small:

$$|\, v_{zn} - v_z \,| / v_z \simeq |\, E_{\perp n} - \langle E_\perp \rangle \,| / mc^2\gamma \ll 1 \, . \tag{4c}$$

Hence, the many waves $\psi_{\mathbf{k}_{0,\mathbf{n}}}(\mathbf{r})$ describing the particle propagation at such energies form a narrow quasiclassical wave packet with a group velocity $v_z$ which performs an undulatory motion in the $\mathbf{r}_\perp$ direction(s) under the influence of $H_{0\perp}$.[30] Therefore, the particles in the beam can be *labeled* by their mean classical-like positions $z(t) = z(0) + v_z t$. This provides an analogy between relativistic channeled particles and atoms whose center of mass moves classically at a velocity $v_z$. We shall therefore denote also the coordinate of an optically active electron in a free or channeled ion relative to the ionic center of mass by $\mathbf{r}_\perp$.

Deviations from the uniform motion along $z$, which result from the longitudinal variation of $V(\mathbf{r})$, will not be considered. We are only concerned with dipole transitions between transverse energy states $n$, giving rise to channeling radiation (CR). The peak frequencies of CR in crystals[14,15] are typically 2 orders of magnitude lower than those of the radiation emitted (at the same $\gamma$) due to the longitudinal periodic variation of the crystal potential,[16,31] which is analogous to FEL emission.

The stimulated emission of radiation from relativistic electrons or positrons interacting with an electromagnetic vector potential $\mathbf{A}(\mathbf{r})$ (a classical or quantal field) can be described in two ways.

(a) The Dirac interaction Hamiltonian[27]

$$-e\boldsymbol{\alpha}\cdot\mathbf{A}=-e\begin{bmatrix}0 & \boldsymbol{\sigma}\cdot\mathbf{A}\\ \boldsymbol{\sigma}\cdot\mathbf{A} & 0\end{bmatrix}$$

is treated as a perturbation causing transitions between the eigenstates (2a). On labeling the initial and final eigenstates by $I$ and $F$, and using the operator identity[27]

$$(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b})=\mathbf{a}\cdot\mathbf{b}+i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}) ,$$

one finds, in the Coulomb gauge $\nabla\cdot\mathbf{A}=0$,

$$\begin{aligned}\langle F\,|\,(-e\boldsymbol{\alpha}\cdot\mathbf{A})\,|\,I\rangle\\ &=[(1+1/\gamma_F)/(1+1/\gamma_I)]^{1/2}\\ &\times\{\langle u_F\psi_F\,|\,[(-e\mathbf{A}\cdot\mathbf{p}/mc\gamma_I)(1+\tfrac{1}{2}R_q)-(e\hbar\boldsymbol{\sigma}\cdot\mathbf{B}/2mc\gamma_I)(1+R_q)+ieR_q\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{p})/2mc\gamma_I]\,|\,u_I\psi_I\rangle\}\ .\end{aligned} \tag{5a}$$

Here $R_q=(\gamma_I-\gamma_F)/(\gamma_F+1)$ is the emission quantum-recoil factor.[16] It gives rise to the term $\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{p})$ and changes the magnitude of the $\mathbf{A}\cdot\mathbf{p}$ term, which is responsible for CR transitions, as well as of the spin-magnetic term $\boldsymbol{\sigma}\cdot\mathbf{B}$, where $\mathbf{B}=\nabla\times\mathbf{A}$.

(b) The vector potential $\mathbf{A}$ is incorporated into the Dirac equation by making the transformation $\mathbf{p}\rightarrow\mathbf{p}-e\mathbf{A}/c$ in (1b). Repeating the steps leading to (2a) one finds, instead of (2b), that $\psi(\mathbf{r})$ is now governed in the Coulomb gauge by the Hamiltonian

$$H=H_0+H_{\rm int}=(\mathbf{p}-e\mathbf{A}/c)^2/2m\gamma-e\hbar\boldsymbol{\sigma}\cdot\mathbf{B}/2mc\gamma+V\ . \tag{5b}$$

In what follows we neglect in (5a) the quantum-recoil factor $R_q$, which is small compared to 1 for CR from electrons and positrons with energies below 1 GeV.[14,15] Then (5a) together with the matrix elements of $(e\boldsymbol{\alpha}\cdot\mathbf{A})^2$ become the same as the matrix elements of $H_{\rm int}$ in (5b). In this energy range CR emission is dipolar.[14,15] This means that, when calculating the matrix elements of (5b), one neglects contributions $O(\mathbf{q}_\perp\cdot\mathbf{r}_\perp)$ in the $\mathbf{A}\cdot\mathbf{p}$ term and $O(\mathbf{q}_\perp\cdot\mathbf{r}_\perp)^2$ in the $\mathbf{A}\cdot\mathbf{p}_z$ term, $\mathbf{q}_\perp$ being the transverse component of the photon wave vector. The same limits hold for soft-x-ray emission from ions with kinetic energies anywhere above $\sim100$ keV. We can omit the $\boldsymbol{\sigma}\cdot\mathbf{B}$ term from (5b) in the dipolar range, since this term does not contribute to CR electric dipole transitions. Then $H_{\rm int}$ becomes identical to the minimal-coupling nonrelativistic Hamiltonian for a particle with mass $m\gamma$.[27]

We intend to obtain the equivalent of the atomic Maxwell-Bloch equations[25,26] for stimulated dipolar CR, using the aforementioned analogy between an ensemble of moving atoms and a beam of channeled particles. To this end, $H_{\rm int}$ must be rewritten in the multipolar form, i.e., as the interaction of particles' polarization with the electric field of the radiation.

This is achieved conveniently by the Healy transformation,[32] which consists in writing (5b) in terms of the canonical transform of $\mathbf{A}(\mathbf{r})$ and $\mathbf{p}$, thus saving the trouble of transforming the eigenfunctions as well. This transformation allows us to write the Hamiltonian of the entire beam-field system, on labeling the particles in the beam by index $i$ and keeping the leading terms in the multipolar expansion in the transverse coordinates,[32,33]

$$\begin{aligned}\sum_i\left[\left[\mathbf{p}_i-\frac{e}{c}\mathbf{A}(\mathbf{r}\simeq\mathbf{z}_i(t))\right]^2\Big/2m\gamma+V(\mathbf{r}_{\perp i})\right]+H_{\rm field}\\ =\sum_i H_{0i}+H_{\rm int}^{\rm mult}+\frac{1}{8\pi}\int\{[\mathbf{E}(\mathbf{r})]^2+[\mathbf{B}(\mathbf{r})]^2\}d\mathbf{r}\ ,\\ H_{\rm int}^{\rm mult}\simeq-\int d\mathbf{r}[\mathbf{P}(\mathbf{r})\cdot\mathbf{E}(\mathbf{r})+\mathbf{M}_R\cdot\mathbf{B}(\mathbf{r})]\ .\end{aligned} \tag{6a}$$

Here $\mathbf{E}(\mathbf{r})$ is the transformed electric field (to be precise, this should be the electric displacement $\mathbf{D}$, but we assume the effective dielectric constant to be $\sim1$). The other operators in (6a) are

$$\mathbf{P}(\mathbf{r},t)=e\sum_i\mathbf{r}_{\perp i}(t)\delta(\mathbf{r}-\mathbf{z}_i(t))\ , \tag{6b}$$

the electric polarization operator, and

$$\mathbf{M}_R(\mathbf{r},t)=\tfrac{1}{2}[\mathbf{P}\times(\mathbf{p}_z/mc\gamma)-(\mathbf{p}_z/mc\gamma)\times\mathbf{P}]\ , \tag{6c}$$

the Röntgen magnetization, caused by the longitudinal motion. Classically, this magnetization results from the Lorentz force acting on the fast particles.

We consider specifically a single-mode "traveling-wave" radiation at a frequency $\omega_{\hat{q}}=qc$, propagating along a unit vector $\hat{\mathbf{q}}$ at small angle to the beam direction $z$ (the generalization to the multimode case is exactly as in standard treatments[26]). All dipoles will be taken to be parallel to each other and perpendicular to $z$. This assumption holds for planar channeled particles or a linearly polarized near-resonant signal field.

The positive and negative frequency parts of the operators are then expressed as

$$\mathbf{P}^{\pm}=\mathscr{P}^{\pm}e^{\pm i(\omega_{\mathrm{q}}t-\mathbf{q}\cdot\mathbf{z})}\,,$$
$$\mathbf{E}^{\pm}=\mathscr{E}^{\pm}e^{\mp i(\omega_{\mathrm{q}}t-\mathbf{q}\cdot\mathbf{z})}\,. \tag{7a}$$

This representation can be used to rewrite (6a) more explicitly, using $\mathbf{B}^{\pm}=\hat{\mathbf{q}}\times\mathbf{E}^{\pm}$ (as implied by the Maxwell equations) and setting $\mathbf{p}_z/mc\gamma=\boldsymbol{\beta}_z$. The latter step is consistent with neglecting the longitudinal velocity dispersion about $v_z$ [cf. (4)] and dropping

$$[p_z,P]=-i\hbar\partial P/\partial z\sim\hbar q_z P$$

as compared to $Pp_z\sim\hbar k_0 P$, in the limit of small quantum recoil. Taking all this into account and making the rotating-wave approximation (discarding the antiresonant terms $\mathscr{P}^{\pm}\cdot\mathscr{E}^{\mp}$) we can rewrite (6a) as[34]

$$(H_{\mathrm{int}}^{\mathrm{mult}})_{\mathrm{q}}=-\sum_{s=\pm}\int d\mathbf{r}[\mathscr{P}^{(s)}\cdot\mathscr{E}_{\mathrm{q}}^{(s)}+(\mathscr{P}^{(s)}\times\boldsymbol{\beta}_z)(\hat{\mathbf{q}}\times\mathscr{E}^{(s)})]$$
$$=-\sum_{s=\pm}\int d\mathbf{r}[(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\mathscr{P}^{(s)}\cdot\mathscr{E}^{(s)}+(\mathscr{P}^{(s)}\cdot\hat{\mathbf{q}})(\mathscr{E}_{\mathrm{q}}^{(s)}\cdot\boldsymbol{\beta}_z)]\,. \tag{7b}$$

We choose the field $\mathscr{E}$ to be perpendicular to $z$. Then the last term in (7b) drops out and we find that, as compared to the low-velocity limit, the field-dipole interaction is reduced by the inverse Doppler shift $(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})$.[34]

We are now in a position to write the Heisenberg equations of motion describing the emission due to a dipole transition between two transverse energy levels $e$ and $g$ [associated with the wave functions $w_e(\mathbf{r}_\perp)$ and $w_g(\mathbf{r}_\perp)$]—cf. Eq. (3b). This description holds for singlet transitions in ions, electron CR in crystals, and radiation from particles channeled in other hollow-channel structures. We follow the well-known derivation of Bloch equations for two-level systems,[26] using (3a) and (7) instead of the standard small-velocity Hamiltonian. These equations are written in terms of the population inversion operator $W$ and the positive and negative frequency parts of the polarization $\mathbf{P}^{\pm}$:

$$\frac{d\mathbf{P}^{\pm}}{dt}=\frac{i}{\hbar}\left[\sum_i (H_{0\perp})_i+H_{\mathrm{int}}^{\mathrm{mult}},\mathbf{P}^{\pm}\right] \tag{8a}$$

and likewise for $W$, where

$$W=2\sum_i\sigma_{zi}\delta(\mathbf{r}-\mathbf{z}_i(t))\,,$$
$$\mathbf{P}^{\pm}=\sum_i(\boldsymbol{\mu}_{eg})_i\sigma_i^{\mp}\delta(\mathbf{r}-\mathbf{z}_i(t))\,. \tag{8b}$$

Here $\sigma_{zi},\sigma_i^{\mp}$ are Pauli matrices in the $|e\rangle,|g\rangle$ space and $\boldsymbol{\mu}_{eg}$ is the transition dipole moment. On allowing for the transition linewidth $T_2^{-1}$, the inversion relaxation time $T_W$, and the effective rate of pumping $\Lambda$, the Bloch equations assume the form

$$\left[\frac{\partial}{\partial t}+c\beta_z\frac{\partial}{\partial z}\right]\mathscr{P}^{\pm}=(\pm i\Delta\omega_{\hat{\mathrm{q}}}-1/T_2)\mathscr{P}^{\pm}\pm\frac{i\mu_{eg}^2}{\hbar}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\mathscr{E}^{\mp}W\,, \tag{9a}$$

$$\left[\frac{\partial}{\partial t}+c\beta_z\frac{\partial}{\partial z}\right]W=\frac{i}{\hbar}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})(\mathscr{P}^{+}\cdot\mathscr{E}^{+}-\mathscr{P}^{-}\cdot\mathscr{E}^{-})+\Lambda\rho-W/T_W\,. \tag{9b}$$

Here $\rho$ is the beam density and

$$\Delta\omega_{\hat{\mathrm{q}}}=\omega_{eg}-\omega_{\hat{\mathrm{q}}}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\,, \tag{10}$$

$\hbar\omega_{eg}$ being the transverse energy difference $E_{\perp e}-E_{\perp g}$. The convectional part of the derivative $c\beta_z\partial/\partial z$ expresses the parametric dependence of the operators on $z(t)$ [cf. (8)], consistent with setting $p_z/mc\gamma=\beta_z$ [cf. (6c)—(7b)].

The Bloch equations are supplemented by the Maxwell (Helmholtz) wave equation[25]

$$\left[(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times)+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{E}=-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\left[\frac{\partial\mathbf{P}}{\partial t}+c\boldsymbol{\nabla}\times(\mathbf{P}\times\boldsymbol{\beta}_z)\right]\,, \tag{11}$$

the last term on the right-hand side being the Röntgen (motional) magnetization current [cf. (6)]. We apply the slowly varying envelope approximation:

$$|\partial^2\mathscr{E}^{\pm}/\partial t^2|\ll\omega_{\hat{\mathrm{q}}}|\partial\mathscr{E}^{\pm}/\partial t|\,,$$
$$|\partial^2\mathscr{E}^{\pm}/\partial z^2|\ll q\,|\partial\mathscr{E}^{\pm}/\partial z|\,, \tag{12}$$

and likewise for $\mathscr{P}^{\pm}$. This approximation is tantamount to the Markovian assumption that $\mathbf{P}$ responds instantaneously to changes in $\mathbf{E}$, thus neglecting retardation effects which play no role in lasing.[35,36] Then, on accounting for field damping by introducing the mean free path $2c\delta^{-1}$ for photon loss (via absorption and scattering), (11) reduces to

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}+\delta/2\right]\mathscr{E}^{\pm}=\pm i2\pi\omega_{\hat{\mathrm{q}}}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\mathscr{P}^{\mp}\,. \tag{13}$$

The motional modifications in these Maxwell-Bloch equations essentially amount to the reduction of the low-velocity field-dipole coupling by the inverse Doppler factor $1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}}$ due to Röntgen magnetization (which is ignored in standard laser theory[26]). Within the relativistic forward cone[14] (of angular width $\sim1/\gamma$) $1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}}\sim1/2\gamma^2$, and this reduction is therefore strong for $\gamma\gg1$. The inverse Doppler factor in (13) [arising from the combined effect of polarization and magnetization currents in (11)] implies that the field envelope is affected only by changes in the polarization envelope on the time scale of

$$[\omega_{\hat{q}}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})]^{-1}\sim\omega_{eg}^{-1}$$

or the corresponding spatial scale $\sim c\beta_z\omega_{eg}^{-1}$. As shown below, the spatial part of the total time derivative $c\beta_z\partial/\partial z$ in (9) can strongly affect the gain, whereas in conventional (gas laser) theory[37] it only Doppler shifts the resonance frequency.

The set of operator Maxwell-Bloch equations (9) and (13) is capable of describing the buildup of coherent stimulated emission starting from incoherent spontaneous emission and ending at the regime above lasing threshold.[26] In this paper, however, we are interested only in estimating the gain and the conditions for attaining the lasing threshold. For such purposes it suffices to consider the semiclassical steady-state solutions of these equations.[37]

## III. STEADY-STATE SMALL SIGNAL GAIN

Our aim is to determine the conditions under which a nondegenerate transition $|e\rangle\rightarrow|g\rangle$ of the fast projectile can amplify a single-mode traveling wave. The considered setup is either a single-pass amplifier of an injected signal or a self-sustained oscillator of a field mode selected by a closed-path Bragg resonator. Such resonators for x-ray radiation can be constructed of multilayer mirrors[5,38] or single crystals diffracting x rays at 90° with losses of few percent.[39]

We assume steady-state operation, i.e., the dropping of all partial time derivatives in (9) and (13). On eliminating $\mathscr{P}^{\pm}$ from (13), the reduced amplitude

$$b^{\pm}(z)=e^{\delta z/2c}\mathscr{E}^{\pm}(z)/\mathscr{E}^{\pm}(z=0) \tag{14}$$

is then found to obey

$$\frac{\partial^2 b^{\pm}}{\partial z^2}+\frac{T_2^{-1}\pm i\Delta\omega_{\hat{q}}}{c\beta_z}\frac{\partial b^{\pm}}{\partial z}-\frac{[2\pi(\omega_{eg}-\Delta\omega_{\hat{q}})(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\rho_W\mu_{eg}^2]b^{\pm}}{\hbar c^2\beta_z}=0\ . \tag{15}$$

The inverted population density $\rho_W\equiv\langle W\rangle$ in (15) is nearly unaffected by $\mathscr{E}$ in the small-signal regime.[37] It is therefore found from (9b) to be

$$\rho_W=\Lambda T_W\rho+e^{-z/cT_W}[\rho_W(0)-\Lambda T_W\rho]\ . \tag{16}$$

In order to achieve optimal conditions for gain, constant (spatially uniform) inversion must be maintained by pumping. We therefore choose

$$\rho_W(0)=\rho_W(z)=\Lambda T_W\rho\ . \tag{17}$$

The pumping agent can be a high-power laser at an optical or infrared frequency $\omega_p=\omega_{e'g'}/[1-\beta_z\cos\theta)]$, so that $\Lambda\propto[\boldsymbol{\mu}_{e'g'}\cdot\mathbf{E}(\omega_p)/\hbar]^2$. Here $e',g'$ are determined by the pumping scheme (cf. Secs. IV and V) and do not necessarily coincide with the levels $e,g$ of the lasing transition. For a given $g'\rightarrow e'$ transition, the lowest pumping frequency obtains for $\theta=\pi$, i.e., a pumping laser beam counterpropagating to the channeled beam. The conversion of optical or infrared laser light into short-wavelength radiation from channeled relativistic particles has been suggested previously by Pantell.[40] The process he considered was Compton scattering, in which both absorption and emission occur between the same levels $e$ and $g$, whereas here pumping is part of a more general lasing scheme.

On assuming constant values (independent of $z$) for $\rho_W$ and $\Delta\omega_{\hat{q}}$ (the validity of the latter assumption is discussed in Secs. IV and V) the solution of (15) corresponding to gain (growth with $z$) becomes

$$b^{\pm}(z)=e^{G^{\pm}z}\ ,$$
$$G^{\pm}=-\tfrac{1}{2}L_2^{\pm}+\tfrac{1}{2}[1/(L_2^{\pm})^2+4/L_a^2]^{1/2}\ . \tag{18}$$

Here

$$L_2^{\pm}=c\beta_z(1/T_2\pm i\Delta\omega_{\hat{q}})^{-1} \tag{19a}$$

is the "dephasing length" and

$$L_a=\left[\frac{\hbar c^2\beta_z}{2\pi(\omega_{eg}-\Delta\omega_{\hat{q}})(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\rho_W\mu_{eg}^2}\right]^{1/2} \tag{19b}$$

is the "amplification length."

In what follows, the gain condition

$$G\equiv\tfrac{1}{2}(G_{+}+G_{-})>\delta/2c\ , \tag{20}$$

where $2G$ is the gain for the intensity $b^{+}b^{-}$, will be investigated for various TCE's describable by this model. We shall distinguish between the regimes of strong dephasing ($L_a\gg L_2$) and weak dephasing ($L_a\lesssim L_2$). It should be noted that in the limit $\beta_z\rightarrow 0$ of conventional laser theory only the strong-dephasing regime is possible, since $L_a\propto\beta_z^{1/2}$ whereas $L_2\propto\beta_z$. Hence, beam propagation effects associated with the spatial derivative in (9a) are essential for the existence of weak dephasing.

## IV. THE STRONG-DEPHASING REGIME

### A. General features

In the limit $L_a\gg L_2$, Eq. (18) is approximately reduced to

$$G=\tfrac{1}{2}(G^{+}+G^{-})\simeq(L_2^{+}+L_2^{-})/2L_a^2\equiv G_{\rm BK}=\frac{\pi\omega_{eg}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\rho_W\mu_{eg}^2}{\hbar cT_2(T_2^{-2}+\Delta\omega_{\hat{q}}^2)}\ . \tag{21}$$

This is exactly the result of Beloshitskii and Kumakhov (BK) (Ref. 24, note that their result is the gain for intensity, i.e., $2G_{\rm BK}$).

The expression $G_{\rm BK}$ has been obtained on the assumption that the detuning $\Delta\omega_{\hat{q}}$ is independent of $z$. In order to maximize $G_{\rm BK}$, the resonance condition $T_2^{-1}\gtrsim|\Delta\omega_{\hat{q}}|$ must be satisfied. In reality, however, if the detuning is zero at the entrance to the structure $[\Delta\omega_{\hat{q}}(z=0)=0]$, it will exceed $T_2^{-1}$ at $z\geq l$, after energy-loss effects have reduced $\beta_z$ sufficiently. Using (10), it is easy to show that $l$ is given by the relation

$$T_2^{-1}=\omega_{eg}(0)\left[\frac{1-\boldsymbol{\beta}_z(l)\cdot\hat{\mathbf{q}}}{1-\boldsymbol{\beta}_z(0)\cdot\hat{\mathbf{q}}}-[\gamma(0)/\gamma(z)]^{\kappa}\right], \tag{22}$$

where we have set $\omega_{eg}\propto\gamma^{-\kappa}$: $\kappa=0$ for ions, $\frac{1}{2}>\kappa>\frac{1}{3}$ for CR in crystals, and $\kappa=1$ for CR in structures with wide cylindrical channels (cf. Sec. V). It is clear that only if

$$l\gg G_{\rm BK}^{-1} \tag{23}$$

can the assumption $\Delta\omega_{\hat{q}}=$const be justified in the derivation of (21). Whenever (23) is violated, (15) is solved upon dropping the second derivative (on account of the strong-dephasing condition) while allowing for the $z$ dependence of $\Delta\omega_{\hat{q}}$. This yields

$$|b(z)|=\exp\left[\int_0^z G_{\rm BK}(z')dz'\right], \tag{24}$$

which can be evaluated on substituting $\Delta\omega_{\hat{q}}(z)$ for a given type of TCE and energy-loss behavior.

### B. Electrons channeled in crystals

The typical relevant parameters for CR from electrons channeled in crystals with energies in the range between a few MeV and a few hundred MeV (above which the dipolar approximation breaks down)[14] are $\hbar\omega_{eg}\lesssim\gamma^{-1/2}50$ eV, $\mu_{eg}\lesssim3\times10^{-18}$ esu cm, and $c\beta_zT_2\lesssim1$ $\mu$m. These parameters imply that the strong-dephasing regime $L_2^2\ll L_a^2/4$ prevails up to enormous beam densities exceeding $10^{18}$ cm$^{-3}$ (or $10^{10}$ A/cm$^2$) for short-wavelength ($\lambda<10$ Å) electron CR in crystals in the considered electron energy range. Such densities are well above the crystal damage threshold, hence one can safely ignore any deviations from strong dephasing in this system.

The pumping scheme which is appropriate for planar channeled electrons must be aimed at maintaining maximal inversion in the strongest transition[15,21] $e\to g$, where the lowest level (band) is designated by $g$. This can be achieved by (a) choosing a beam incidence angle and divergence at which $N_e>N_g>N_{e'}$ (here $N_i$ are the fractional populations of the respective levels, $e'$ being a higher level than $e$), (b) pumping the $g\to e'$ transition by a high-power optical or infrared laser at a rate $w_{g\to e'}\gtrsim w_{e\to g}$. The relevant rate equations[26] yield, at steady state [cf. (16) and (17)],

$$\rho_W/\rho\simeq(w_{g\to e'}N_g+w_{e'\to e}N_{e'})T_W\equiv\Lambda T_W\ . \tag{25}$$

Pumping at a rate $w_{g\to e'}\gtrsim T_W^{-1}$ allows us to achieve $\rho_W/\rho\gtrsim\frac{1}{2}$.

In the x-ray region[41] $c/\delta\lesssim0.1$ cm within the crystal. This value together with the parameters quoted above imply that the gain condition (20) is satisfied only for $\rho_W\gtrsim10^{16}\gamma^{1/2}/(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})$ cm$^{-3}$ on resonance [cf. (10)], setting $\omega_{eg}\propto\gamma^{-1/2}$. For $\lambda\lesssim100$ Å this amounts to current densities above $10^8$ A/cm$^2$. The possibilities to enhance the quoted parameters so as to reduce the threshold current are rather limited. Thus, for given $\gamma$ and $\hat{\mathbf{q}}$ (which determine $\omega_{\hat{q}}$), $T_2$ (and consequently $G_{\rm BK}$) can only be increased by a factor of 2 or so by cooling the crystal down to cryogenic temperatures.[42] However, Ohtsuki's interesting argument[23] implies that photon diffraction in bent crystals may increase the x-ray absorption length $c/\delta$ by as much as $10^2$. This could lower the lasing threshold to $\sim$MA/cm$^2$ in such systems.

The estimates above, which imply that x-ray lasing in crystals without bending may require current densities above $10^8$ A/cm$^2$, have been challenged as being too optimistic[17] in a paper claiming that the length of the resonance region is $l\sim10^{-5}$ cm. This figure is, in fact, a result of a severe underestimate of $T_2^{-1}$. We find, using (22), that for nearly forward emission

$$l\simeq\frac{2\gamma^{1/2}(0)L_{\rm rad}}{15\omega_{eg}(\gamma=1)T_2}\gtrsim\gamma^{1/2}(0)L_{\rm rad}/500\ , \tag{26}$$

where $L_{\rm rad}/5=\ln(183Z_{\rm crys}^{-1/3})/5$ (cm) ($Z_{\rm crys}$ being the crystal atomic number) is the effective path for radiative energy loss (due to incoherent bremsstrahlung) of the channeled electron. It is shorter roughly by a factor of 5 than the corresponding path $L_{\rm rad}$ for a randomly incident electron.[14] Hence, $l\gtrsim\gamma^{1/2}(0)10^{-2}$ cm is plausible for low $Z_{\rm crys}$. This value is at least $10^3$ times larger than the cited estimate.[17] It is seen that for $\gamma(0)\lesssim10$. when the condition (23) is not satisfied, (24) must be used. Even then, however, the loss of resonance should not hamper the conditions for gain in crystals thinner than 0.1 cm.

### C. Channeled ion beams

X-ray transitions $g'\to e'$ involving $K$- or $L$-shell electrons in fast channeled ions can be resonantly pumped by the longitudinally periodic crystal potential if the following resonance condition holds:[18]

$$\omega_{e'g'}=c\beta_z\gamma(2\pi n/d)\ . \tag{27}$$

Here $d$ is the lattice periodicity in the $z$ direction and $n$ the harmonic index. The pumping rate is proportional to the effective Rabi frequency squared, namely [Eq. (19) in Ref. 18],

$$\Lambda\propto(\mu_{e'g'}\omega_{e'g'}eZ_{\rm crys}/\hbar c\beta_z)^2=\left[\frac{\gamma\mu_{e'g'}neZ_{\rm crys}}{\hbar d^2}\right]^2. \tag{28}$$

This corresponds to pumping rates achievable with high-power lasers, allowing us to satisfy $\Lambda T_W\gtrsim1$.

The $L_2$ values characterizing transitions in channeled ions with $\beta_z\sim1$ are comparable to those of channeled positrons,[14,15,19] i.e., up to ten times larger than for electron CR. Since the $\mu_{eg}$ and $\omega_{eg}$ ($\sim$120 eV for the $1s2p\ ^1P-1s^2\ ^1S$ transition in $Be^+$) are comparable to those of electron CR, the x-ray lasing threshold for $\gamma\lesssim2$ and $1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}}\sim1$ should occur in this case above $\sim10^7$ A/cm$^2$, i.e., also at extremely high current densities.

This threshold can be lowered considerably if the lasing itself takes place downstream from the crystal, while the crystal is used only as a pumping agent for the ion beam. Choosing the crystal thickness to be $L\simeq c\beta_z\pi/\Lambda$, maximal inversion can be achieved and then kept constant outside the crystal for $z\lesssim c\beta_zT_W\sim c\beta_zT_1\lesssim10^{-2}$ cm, $T_1$ being the radiative lifetime. (Inversion can also be maintained, of course, by a high-power laser.) The beam density required to attain the lasing threshold in this case wherein[14] $T_2 \sim T_1 = 3\hbar c^3/4\mu_{eg}^2\omega_{eg}^3\gamma^2$ is, for $\gamma \sim (1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}}) \sim 1$, $\rho_W \gtrsim G_{\rm BK}(2\pi c/\omega_{eg})^2$. Assuming that outside the crystal $c/\delta$ may exceed 1–10 cm and taking $\lambda = \omega_{eg}/2\pi c \lesssim 100$ Å, this sets the threshold at $10^3$–$10^4$ A/cm$^2$.

## V. THE WEAK-DEPHASING REGIME

Whenever $L_a \lesssim L_2$, $G_{\rm BK}$ [Eq. (21)] is no longer an adequate estimate for the gain. In the *extreme* weak-dephasing case $L_a \ll L_2$, (18) yields

$$G \equiv G_a \simeq 1/L_a \simeq \frac{\mu_{eg}}{c}\left[\frac{\pi\rho_W(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\omega_{eg}}{\hbar\beta_z}\right]^{1/2} . \quad (29)$$

In contrast to $G_{\rm BK}$, $G_a$ is independent of $T_2$ and decreases more slowly with $\gamma$ or $(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})$. On the other hand, it scales more slowly with $\mu_{eg}$, $\omega_{eg}$, and $\rho_W$.

The weak-dephasing condition $L_a \lesssim L_2$ is equivalent to

$$\rho_W \gtrsim \hbar/8\pi\beta_z\omega_{eg}(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})\mu_{eg}^2T_2^2 . \quad (30)$$

Inequality (30) can be satisfied in systems wherein $\mu_{eg}$ and $T_2$ are substantially larger than in crystals. The threshold for lasing in such systems is lower also in the strong-dephasing regime. In order to achieve the latter aim, Vysotskii and Kuz'min[17] (who adopted the BK theoretical approach) proposed the use of positron beams channeled in zeolite crystals containing hollow channels with larger diameter than in crystals.

In a structure composed of cylindrical channels with diameter $R$ one has, for the fundamental transition $(n,m_A)\to(n-1,m_A+1)$ and $(n,m_A)\to(n,m_A-1)$ [$(n,m_A)$ being the radial and azimuthal quantum numbers],[43]

$$\omega_{\hat{\mathbf{q}}} = \omega_{eg}/(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}}) \simeq \pi^2\hbar n/[2mR^2\gamma(1-\boldsymbol{\beta}_z\cdot\hat{\mathbf{q}})] ,$$
$$\mu_{eg} \simeq 4eR/\pi^2, \quad n_{\max} \propto R\gamma^{1/2} . \quad (31a)$$

The contribution of inelastic interactions between the structure electrons (localized within the channel walls) and the particle to $T_2$ obeys[17]

$$(T_2)_{\rm inelas} \propto R^3\gamma/n . \quad (31b)$$

The relations above show that, in order to reduce the values of $\rho_W$ satisfying (30) for a given $\omega_{\hat{\mathbf{q}}}$, it is desirable to maximize $R$ and $\gamma$ and minimize $n$.

Since emission in such structures may well be confined to the nearly forward direction, we shall concentrate on $\omega_{\hat{\mathbf{q}}=\hat{\mathbf{z}}}$ in what follows. In order to obtain $\hbar\omega_z = \hbar\pi^2 n\gamma/mR^2 \gtrsim 100$ eV one must have $\gamma n \gtrsim 2500$ for $R = 40$ Å and $\gamma n \gtrsim 1.6\times 10^4$ for $R = 100$ Å. For $R \gtrsim 40$ Å the total $T_2$ (due to beam energy spread as well as inelastic scattering) may attain the value[17] $\sim T_1/6$, where $T_1 \propto \gamma R^4/n^3$ is the radiative lifetime for the considered transitions. Then (30) reduces to

$$\rho_W \gtrsim \pi^3\hbar^4 e^2\gamma n^5/c^6 m^5 R^8 . \quad (32)$$

Using $\gamma = 10$, $n = 150$, and $R \simeq 40$ Å (to obtain stimulated emission at $\lambda = 150$ Å) we then find that $\rho_W \gtrsim 10^{12}$ cm$^{-3}$ satisfies both the weak-dephasing condition (32) and the threshold condition $G(L_2 \simeq L_a) \gtrsim c/\delta$, assuming that $c/\delta \sim 1$ cm for such wide channels. Since the fractional population at $n = 150$ does not exceed a few percent, this is equivalent to current densities above MA/cm$^2$.

Zeolite crystals, which were proposed by Vysotskii and Kuz'min as suitable hosts for x-ray lasing, are limited in practice[44] to $R \lesssim 7$ Å. Structures with $R$ values of tens of Å can be produced artificially, e.g., by ion beam etching of YAG plates which are highly resistive to damage by the channeled current (YAG represents yttrium aluminum garnet). In such a structure $T_2$ can be $O(T_1)$, if the spread in $R$ and in channel axis directions are small enough.

The pumping scheme for such structures differs from the one described for crystals. For beam incidence along the channel axis, $m_A = 0$ states are predominantly populated. The fractional populations of these states are[17] $N_n \propto \varepsilon_{n,0}^{-1}$. Here $\varepsilon_{n,m_A=0}$ are the transverse-energy eigenvalues, whose ordering is $\varepsilon_{n,0} > \varepsilon_{n-1,1} > \varepsilon_{n-1,0}$. This implies that in order to maintain spatially uniform inversion for the lasing transition $(n,0)\to(n-1,1)$, we can take advantage of the fact that at $z = 0$ $N_{n-1,0} > N_{n,0} > N_{n-1,1} = 0$ and pump the transition $(n-1,0)\to(n,0)$ by means of a high-power optical or infrared laser. Then $\rho_W$ satisfying (17) is[26]

$$\rho_W/\rho = \Lambda T_W \simeq w_{n-1,0\to n,0} T_W N_{n-1,0} , \quad (33)$$

where $T_W$ is the rate of decay from $(n,0)$ to $(n-1,1)$. Since $T_W \sim T_1$ for $R$ above a few tens of Å, it turns out that optimal pumping is achievable with reasonable optical power: taking $R \simeq 40$ Å, $\gamma \sim 100$, and $n \simeq 150$, we find that $\Lambda T_W \gtrsim N_{n-1,0}$ when $w_{n-1,0\to n,0} \gtrsim 10^9$ sec$^{-1}$.

## VI. DISCUSSION

On the theoretical side, the main interest in the gain expression derived here [Eq. (18)] is that for relativistic emitter velocities $(\beta_z \sim 1)$ it can differ drastically from that obtained by Beloshitskii and Kumakhov[24] using the Fermi golden rule for stimulated emission rate. It is interesting to compare these expressions with the general gain expression derived by Gover *et al.*[45] for quasifree emitters, which should apply to emitters of both the TCE and the FEL types (cf. the Introduction). The latter expression becomes in the quantum limit (i.e., when the emission and absorption lines are well separated)

$$G_{\rm quan} = \frac{\lambda^3\rho}{\hbar c\Delta_{\hat{\mathbf{q}}}}(dP_{\mathbf{q}\eta}/d\Omega) . \quad (34)$$

Here we have taken the filling factor to be one (i.e., the radiation beam is of the same width as the emitters beam). The Doppler-upshifted linewidth is denoted by $\Delta_{\hat{\mathbf{q}}}$ and $dP_{\mathbf{q}\eta}/d\Omega$ is the spontaneous emission power per projectile and unit solid angle in the amplified mode with polarization $\eta$ and wave vector $\mathbf{q}$. On substituting the expressions appropriate for CR,

$$\Delta_{\hat{q}} \simeq T_2^{-1}/(1-\boldsymbol{\beta}_z \cdot \hat{\mathbf{q}}) ,$$
$$\frac{dP_q(\phi=\pi/2)}{d\Omega} = \frac{\mu_{eg}^2 \omega_{eg}^4}{2\pi c^3 (1-\boldsymbol{\beta}_z \cdot \hat{\mathbf{q}})^3} \quad (35)$$

(where $\phi$ is the polar angle)[14] into (34), we find that $G_{\text{quan}} \sim G_{\text{BK}}$. This result should not come as a surprise, since the general formula of Gover *et al.* is based on the Fermi golden rule, like that of BK. Such a description tacitly assumes that the cross section for stimulated emission is uniform throughout the system. It cannot reproduce local self-consistent changes in the observables which, in the present case of two-level TCE's are embodied in the spatial variation of the polarization at $\beta_z \sim 1$ [Eq. (9a)].

We have demonstrated the possibility to observe deviations of the gain behavior from that of $G_{\text{BK}}$ (the strong-dephasing regime) at current densities ($\gtrsim$MA/cm$^2$) which are expected to cause no damage in structures composed of large-diameter ($\gtrsim 40$ Å) channels [Eq. (32)]. These deviations should be manifested in the scaling of gain with $\rho_W^{1/2}$ instead of $\rho_W$ in $G_{\text{BK}}$, and a weaker dependence on $T_2$ than in the latter.

The present work has elucidated the concept of resonant length [Eq. (22)] which is limited by energy loss of the projectile. For short resonance lengths, modifications of the gain behavior have been noted [Eq. (24)]. Another topic elaborated upon here has been schemes of pumping by a low-frequency high-power laser or the crystal potential (in the case of channeled ions). Such schemes allow us to maintain spatially uniform inversion [Eq. (17)], which is needed to optimize the gain.

The evaluation of prospects for short-wavelength lasing (at $\lambda \lesssim 100$ Å) by electrons or ions channeled in crystals requires the study of possibilities to increase drastically the x-ray absorption length, e.g., by using bent crystals, as well as ways to minimize damage to crystals from MA/cm$^2$ current densities (perhaps by fast beam scanning as suggested by BK, Ref. 24). Better possibilities for short-wavelength lasing are offered by ions in free space, downstream from the crystal in which they are channeled only for the purpose of pumping. However, the most promising hosts of such lasing seem to be artifically produced arrays of parallel, highly uniform cylindrical channels whose diameter $R$ is well above 10 Å. The increase in dipole moment and $T_2$ with $R$ [Eq. (31)] allows us, on the one hand, to lower the lasing threshold by several orders of magnitude and, on the other hand, to raise the damage threshold, as compared to crystals.

The realization of short-wavelength lasing in such systems would provide a source of tunable *coherent* radiation. The coherence properties of this radiation are dictated by the two-level character of the considered emitters. These properties differ from the non-Poissonian photon-statistical properties of FEL's that are candidates for short-wavelength operation.[12,13]

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