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THE REACTION $\pi^+p \to \rho^+p$ AT 2.67 GeV/c; A STUDY OF ISOSCALAR EXCHANGES*

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Abstract

Results are presented of a study of the reaction $\pi^+p \to \rho^+p$ at 2.67 GeV/c incident π^+ momentum. The differential cross section and spin density matrix elements are shown, and the contributions due to given spin-parity exchanges are isolated. By further combining these results with those of a similar π^- experiment, the $I_t = 0$ component of each series is separated. The ω ($I = 0$, $J^{PG} = 1^{--}$) exchange contribution shows a pronounced dip at $-t = 0.4$ $(GeV/c)^2$. Evidence is presented for the exchange of a state H ($I = 0$, $J^{PG} = 1^{+-}$).

This letter reports the results of a high statistics hydrogen bubble chamber experiment to study the reaction

$$\pi^+ p \to \rho^+ p \tag{1}$$

at an incident π^+ momentum of 2.67 GeV/c, giving the experimental parameters pertinent to current exchange models.[1]

A sample of some 9500 events of the reaction $\pi^+ p \to \pi^+ p \pi^0$ has been obtained from a 300,000 picture exposure of the 25" hydrogen bubble chamber at the Bevatron. The events were measured on the Flying Spot Digitizer, and in addition to conventional kinematic fitting, the automatically obtained ionization information has been used to aid in the separation of hypotheses.[2]

The Dalitz plot of the events assigned to π^0 production shows that this reaction is dominated by the quasi-two-body final states $\rho^+ p$, $\Delta^{++}\pi^0$, and $\Delta^+\pi^+$, and also exhibits smaller amounts of final states involving higher mass $N^{*\prime}$s and diffractive dissociation of the proton.

A maximum likelihood fit has been made to determine the amounts and parameters of each of the several identifiable processes.[3] The events have been fitted with a distribution of the form

$$D(m,\theta,\phi) = \Sigma\, b_k(m) \quad w_k(\theta,\phi) + \Phi$$

where for the k^{th} process, $b_k(m)$ is a Breit Wigner line shape, of given orbital angular momentum, for which the mass and width were allowed to vary for the major processes, and $w_k(\theta,\phi)$ is the s-channel helicity decay distribution whose parameters (the spin density matrix elements) were allowed to vary for the major processes.[4] Φ is a constant representing phase space, and m, θ, and ϕ are the appropriate invariant masses and decay angles of the event. The amount of constant "phase space" background, Φ, is found by this fit to be zero. Some 50% of the events are attributed to ρ^+ production. This corresponds to a total cross section for ρ^+ production of 1.40 ± 0.08 mb.

To determine the physical parameters as a function of four-momentum transfer to the proton, t, the domain of t was first subdivided into "coarse" intervals and a fit using all identified processes (from the fit to the full domain) was made for each interval. Any process not contributing in a given interval was excluded from the distribution for further fits within that interval. The intervals were then subdivided into finer bins and another fit performed. The results for the differential production cross section of ρ^+ and its spin density matrix elements are presented in Figs. 1a-c and 2a. The density matrix elements in the t-channel helicity (or Gottfried-Jackson) frame were calculated from the moments of all events (in a given t-bin) weighted according to their probability of resulting from ρ^+ production as found by the maximum likelihood fit (in the s-channel). The values obtained are shown in Fig. 1d-f.

In interpreting these results, certain relationships between the quantum numbers of the exchanged system and the t-channel helicity state of the ρ^+ should be borne in mind, viz., from conservation of angular momentum and parity,[5]

unnatural parity, zero spin exchange populates helicity zero only;

natural parity exchange populates helicities $\pm$ 1 only;

unnatural parity, non-zero spin exchange populates all helicity states.

Note in particular that non-zero values of $\mathrm{Re}\rho^{t}_{10}$ imply either the presence of unnatural parity non-zero spin exchange (i.e. minimum $J^P = 1^+$), or an interference between pseudo-scalar and natural parity exchange. (This latter possibility however vanishes asymptotically with s.)

By making appropriate linear combinations of the density matrix elements, it is possible to isolate, to order 1/s, the contribution to the cross section of the exchange of a given spin-parity series as follows:(6)

$\rho_{00}\ d\sigma/dt$ measures the meson helicity non-flip unnatural parity exchange (2)

$(\rho_{11}-\rho_{1-1})\ d\sigma/dt$ measures the meson helicity flip unnatural parity exchange (3)

$(\rho_{11}+\rho_{1-1})\ d\sigma/dt$ measures the natural parity exchange (necessarily meson helicity flip) (4)

Graphs of these products in the t-channel helicity frame are shown in Fig. 2d-f. Also shown in Fig. 2b,c are the product (2) and the product $\mathrm{Re}\rho_{10}\ d\sigma/dt$ in the s-channel helicity frame.

These results show a number of striking features:

(i) The differential cross section (Fig. 2a) has a sharp forward peak, a dip at $-t \simeq 0.4$, a broader second maximum, and a second minimum at $-t \simeq 1.6$.(7)

(ii) The natural parity exchange contribution (product (4), Fig. 2d) shows a turnover in the very forward direction, a pronounced dip at $-t \simeq 0.4$, and a suggestion of a dip at $-t \simeq 1.6$. The minimum quantum numbers for this contribution should be represented by

$\omega(I = 0, J^{PG} = 1^{--})$ and A_2 $(I = 1, J^{PG} = 2^{+-})$.

(iii) The non-flip unnatural exchange contribution (product (2), Fig. 2e) falls off smoothly from a strong forward peak, showing no evidence of any structure below $-t \simeq 1.0$. On the other hand, ρ^t_{oo} (Fig. 1d) drops from a value of about 0.7 in the forward direction, rises to about 0.9 at $-t \simeq 0.4$, and then decreases smoothly with increasing momentum transfer. This shows that unnatural parity non-flip exchange accounts for some 90% of the cross section at the first dip in $d\sigma/dt$. The pion and a 1^+ object, e.g. A_1 or H, should represent the minimum quantum numbers for this contribution.

(iv) The helicity flip unnatural parity contribution (product (3), Fig. 2f) shows a sharp forward spike, and some discontinuity at $-t \simeq 1.6$. Again, A_1 or H should represent the minimum quantum numbers for this contribution.

(v) In both s- and t-channel helicity frames, $\mathrm{Re}\rho_{10}$ has significantly non-zero values. In the t-channel (Fig. 1f) it starts at negative values near $-t = 0$ and rises to zero at $-t \simeq 0.65$. As noted above, these non-zero values of $\mathrm{Re}\rho^t_{10}$ imply either the presence of non-zero spin unnatural parity exchanges (including both flip and non-flip couplings) or an interference, to a higher order in $1/s$, between pseudo-scalar and natural parity exchange. This observed behavior of $\mathrm{Re}\rho^t_{10}$ is essentially identical to that observed in ρ^- production,[8,9] and since in particular the sign is the same it indicates that the interference is among isoscalar exchanges.

In the s-channel (Fig. 1c) $\mathrm{Re}\rho^s_{10}$ reaches its maximum allowed value near $-t \simeq 0.3$, then drops to zero at $-t \simeq 0.65$. The product $\mathrm{Re}\rho^s_{10}\, d\sigma/dt$, (Fig. 2c) also first reaches zero at $-t \simeq 0.65$.

(vi) The product $\rho^s_{00}\, d\sigma/dt$ (Fig. 2b) drops very sharply from a forward peak, then remains approximately constant to beyond $-t = 1$. Strong cut models of this reaction have predicted a dip in this quantity at $-t \simeq 0.5$ and a corresponding zero in $\mathrm{Re}\rho^s_{10}\, d\sigma/dt$. In WSNZ models this structure would be expected closer to $-t = 1$, but the introduction of weak cuts could modify this prediction.

Additional information may be obtained by combining these results with those of a similar high statistics bubble chamber experiment using a beam of negative pions at 2.77 GeV/c in which the reactions

$$\pi^- p \rightarrow \rho^- p \qquad (5)$$

$$\pi^- p \rightarrow \rho^o n \qquad (6)$$

were studied.[8] Appropriate linear combinations of the differential cross sections and the products (2) - (4) for ρ^+, ρ^- and ρ^o production serve to separate the isospin components of that quantity.[10] Since the $I_t = 1$ component of the amplitude changes sign under charge conjugation while the $I_t = 0$ component does not, the isoscalar, T_o, and isovector, T_1, t-channel amplitudes appear in the combinations $T_o \mp T_1$ for $\rho^\pm$ production and $\sqrt{2}\, T_1$ for ρ^o production.

Hence, the $I_t = 0$ component of a quantity X is given by

$$X_o(t) = \frac{1}{2}\,[X^+(t) + X^-(t) - X^o(t)] \tag{7}$$

and the $I_t = 1$ component by

$$X_1(t) = \frac{1}{2}\,X^o\,(t)$$

where X^+, X^-, and X^o are the corresponding quantities for the reactions (1), (5), and (6), respectively. Similarly, the relative phase, δ, between the isoscalar and isovector exchange components is given by

$$\cos\,\delta\,(t) = \frac{X^-(t) - X^+(t)}{4\sqrt{X_o(t)}\,\sqrt{X_1(t)}}$$

The $I_t = 0$ contributions to the differential cross section and the products (2) - (4) are shown in Fig. 3a-d. The cosines of the relative isoscalar and isovector phases for the differential cross section and the products (2) and (3) are shown in Fig. 3e-g. The density matrix element $\mathrm{Re}\rho^t_{10}\big|_{I\,=\,0}$, defined as $X_o(\mathrm{Re}\rho^t_{10}\;d\sigma/dt)/X_o(d\sigma/dt)$ (X_o given by Eq. (7)), is plotted in Fig. 3h.

In computing the results of Fig. 3, a small correction ($\approx$ 7%) has been applied to the overall normalization of the π^- experiment, to account for the slight difference in beam momentum of the two experiments. This correction was taken from a power law fit to all available data for this reaction.(11) The sensitivity of these results to the normalization was checked by varying the relative normalization of the two experiments by ± 20%. (Both experiments are within less than 20% of the power law fits for their respective charges.(12))

In no case did the points plotted in Fig. 3a-d change by more than one standard deviation.

Some salient features of these results are:

(i) The $I_t = 0$ differential cross section (Fig. 3a) shows structure similar to that observed for natural parity exchange above, especially the sharp dip at $-t \approx 0.4$. Such a dip in $X_o(d\sigma/dt)$ has been reported previously.[10, 1, 13]

(ii) The $I_t = 0$ natural parity contribution (Fig. 3b) shows this same dip structure (and in fact is consistent with zero at $-t \simeq 0.4$) indicating the source of this structure as a system having the quantum numbers of the ω-meson. In a WSNZ Regge model this would fix the zero of the ω trajectory at $-t = 0.4 \pm 0.05$. Together with the ω mass this implies a trajectory $\alpha_\omega(t) = 0.4 + t$, lying somewhat lower than the commonly accepted ρ trajectory.

(iii) The $I_t = 0$ non-flip unnatural exchange contribution (Fig. 3c) shows a strong forward peak, representing about one-half the total forward peak in $I_t = 0$. This must be explained by the presence of an effective exchange having minimum quantum numbers $I = 0$, $J^{PG} = 1^{+-}$, e.g. the H-meson or a π-ρ Regge-Regge cut. Likewise the $I_t = 0$ component of $\mathrm{Re}\rho^t_{10}$ (Fig. 3h) can be non-zero only if such a state is present. The presence of such a state would also be sufficient (but not necessary) to account for the observed behavior of $\mathrm{Re}\rho^t_{10}$ for the ρ^+ data alone (compare Fig. 1f and Fig. 3h). The $I_t = 0$ helicity flip unnatural exchange component (Fig. 3d), which to order 1/s measures this same state, is non-zero in the same region of momentum transfer.

(iv) The cosine of the relative phase between isoscalar and isovector exchanges can serve to indicate exchange degeneracy in a given spin-parity series, since this cosine will be zero for exchange degenerate states. The converse statement is not generally true, but is essentially true in particular cases.[(14)] For the natural series, representing ω and A_2, this cosine (Fig. 3f) is consistently non-zero (negative). For the unnatural series, representing predominantly π and H, this cosine (Fig. 3g) is consistent with zero in the region of significant H signal, an indication that the H is exchange degenerate with the pion.

In summary, the maximum likelihood technique used here appears to be quite satisfactory for extracting the quasi-two-body parameters of ρ^+ production from the three-body final state. The quality of the data has permitted a detailed decomposition of the cross section into its t-channel spin-parity contributions. The natural parity contribution has features generally associated with ω-exchange, and this is confirmed by separation of the $I_t = 0$ component. The unnatural parity contribution shows features generally associated with pion exchange, but non-zero values of $\mathrm{Re}\rho^t_{10}$ indicate the presence also of non-zero spin unnatural parity exchanges. This is confirmed by separation of the $I_t = 0$ component, where a strong signal is seen in $X_o(\rho_{oo}\, d\sigma/dt)$. This can be explained only by the presence of a state of minimum effective quantum numbers H ($I = 0$, $J^{PG} = 1^{+-}$).

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costly. The maximum reasonable correction, however, would not change the qualitative aspect of any result involving that bin.

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Figure Captions

Fig. 1. Spin density matrix elements of the ρ^+ produced in the reaction $\pi^+ p \to \rho^+ p$ at 2.67 GeV/c. The superscripts indicate the channel in which the ρ helicity is measured.

Fig. 2. a) Differential cross section for the reaction $\pi^+ p \to \rho^+ p$ at 2.67 GeV/c. b)-f) Products of the differential cross section with pertinent conbinations of spin density matrix elements. The combinations d)-f) isolate the natural parity, unnatural parity non-helicity-flip, and unnatural parity helicity-flip contributions respectively. The products b) and c) are relevant to certain absorption models.

Fig. 3. a)-d) Isoscalar components of the differential cross section and particular spin-parity contributions. e)-g) Cosine of the relative phase between the isoscalar and isovector components corresponding to a)-c). h) Isoscalar density matrix element $\mathrm{Re}\rho^t_{10}$ (compare Fig. 1f).

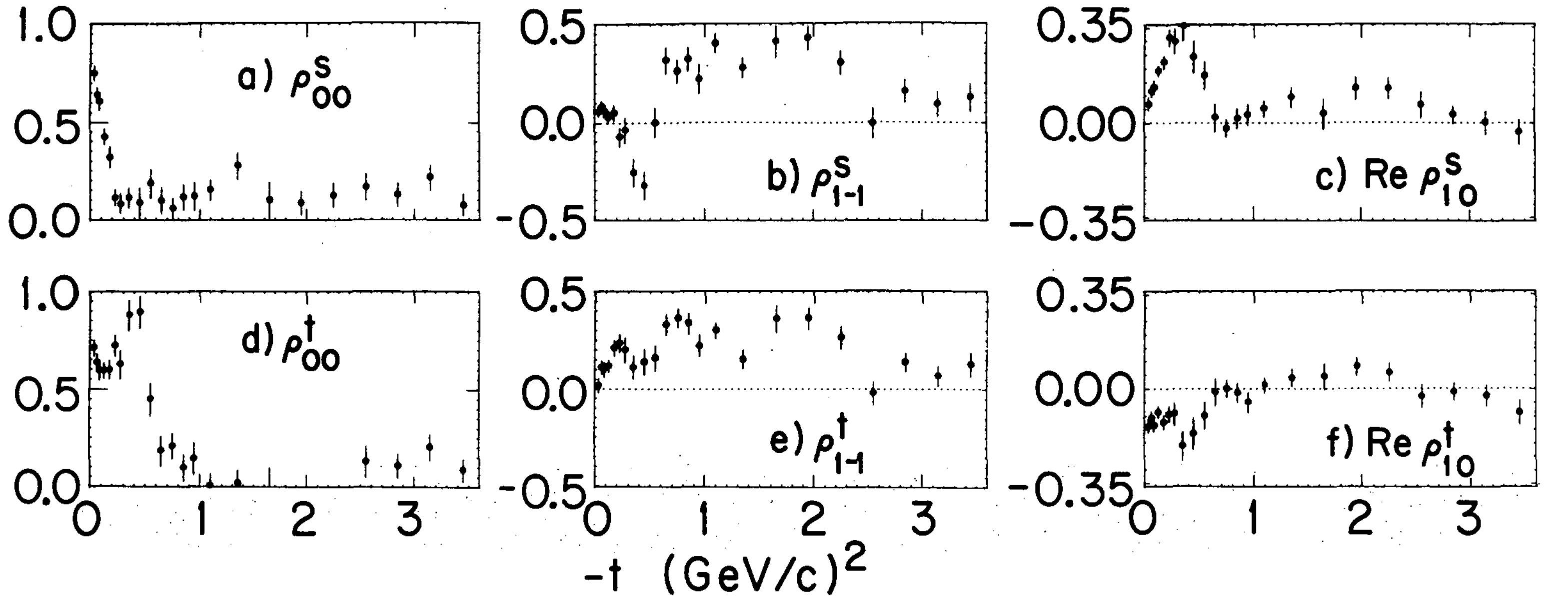

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Fig. 1

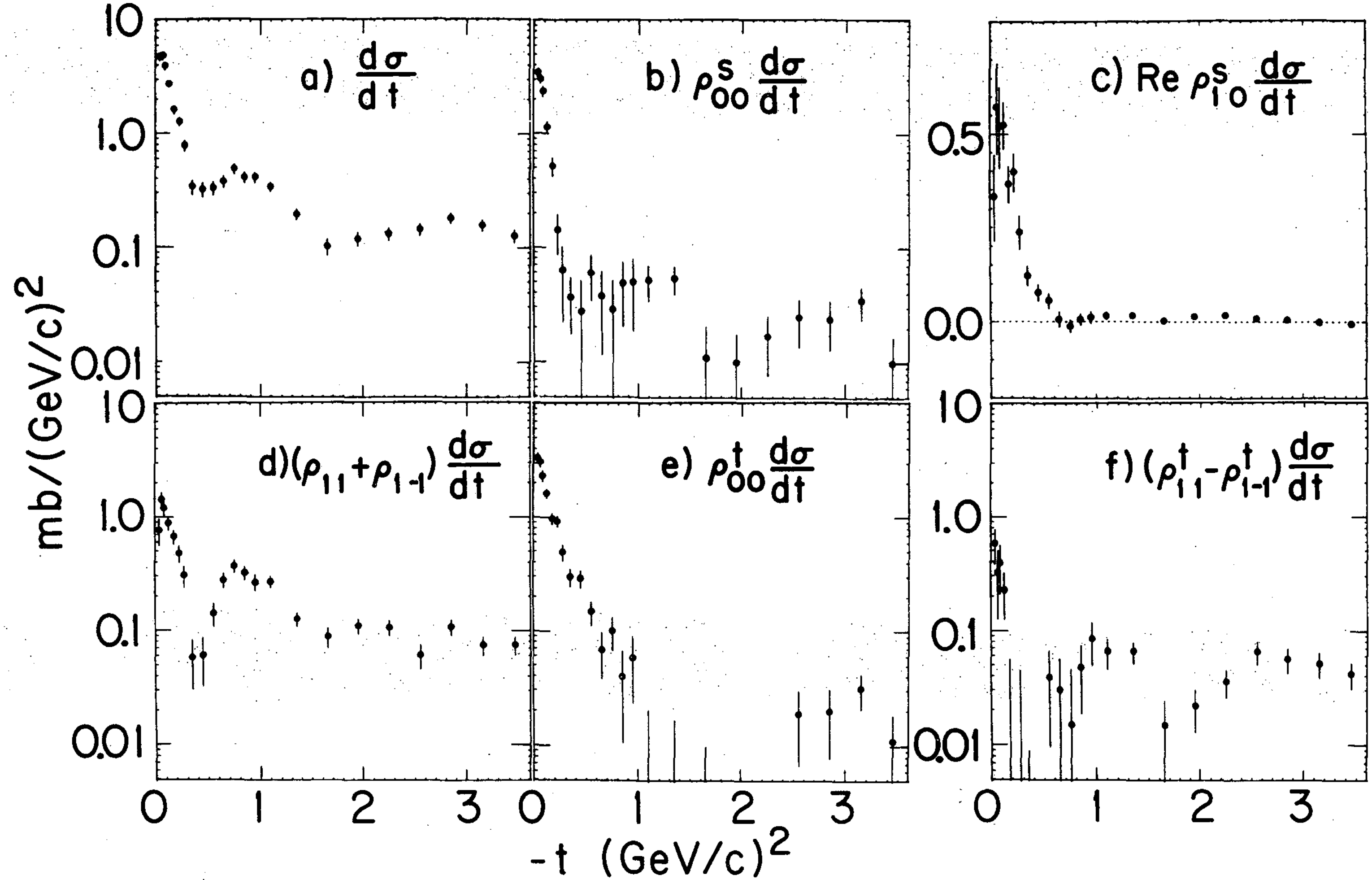

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Fig. 2

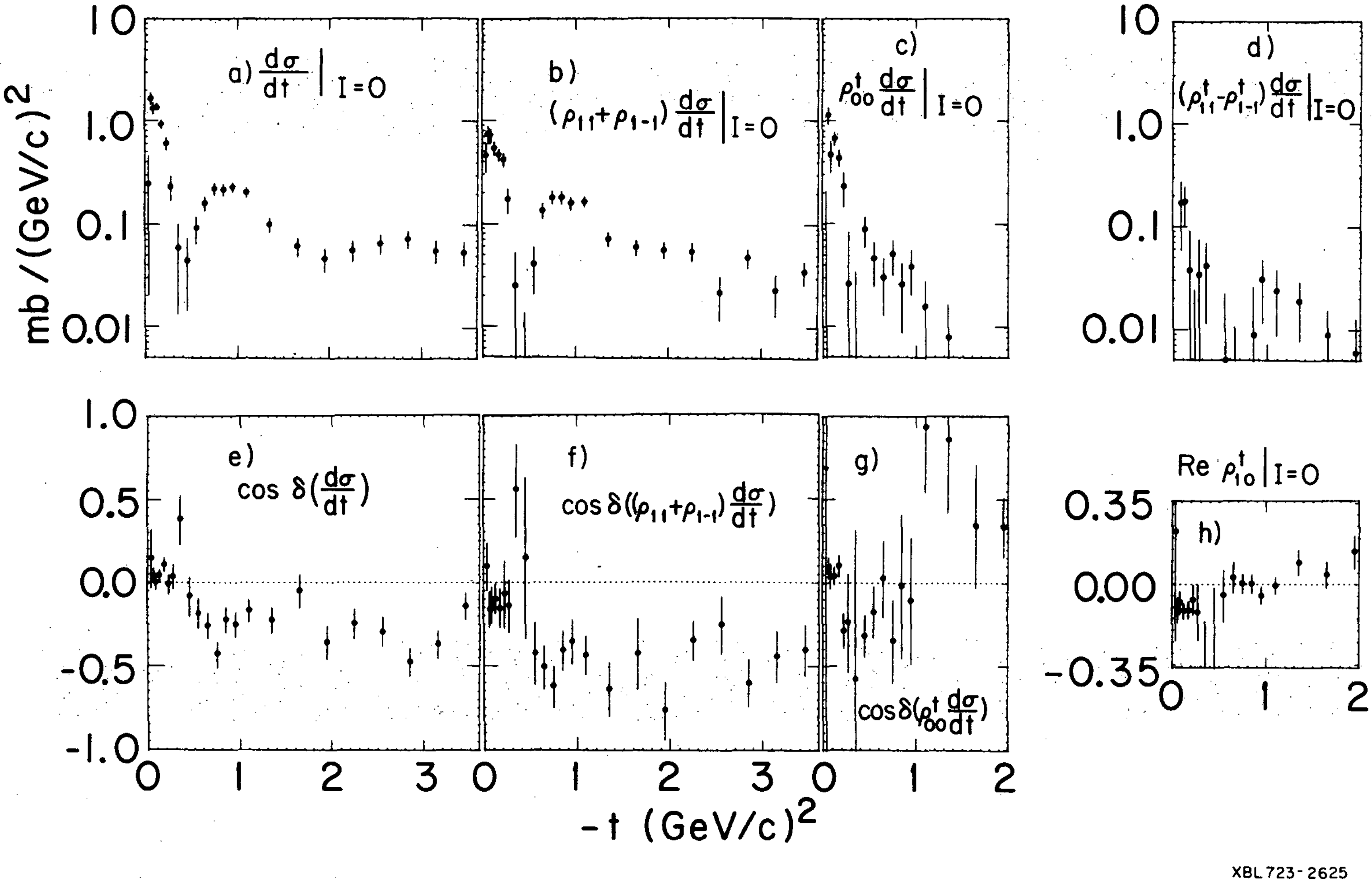
mb / (GeV/c)^2
10
1.0
0.1
0.01
a) $\frac{d\sigma}{dt}\Big|_{I=0}$
b) $(\rho_{11}+\rho_{1-1})\frac{d\sigma}{dt}\Big|_{I=0}$
c) $\rho^t_{00}\frac{d\sigma}{dt}\Big|_{I=0}$
d) $(\rho^t_{11}-\rho^t_{1-1})\frac{d\sigma}{dt}\Big|_{I=0}$
1.0
0.5
0.0
-0.5
-1.0
e) $\cos\delta(\frac{d\sigma}{dt})$
f) $\cos\delta((\rho_{11}+\rho_{1-1})\frac{d\sigma}{dt})$
g) $\cos\delta(\rho^t_{00}\frac{d\sigma}{dt})$
Re $\rho^t_{10}\Big|_{I=0}$
h)
0.35
0.00
-0.35
0 1 2 3
0 1 2 3
0 1 2
0 1 2
$-t$ (GeV/c)^2
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Fig. 3

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