LBL-32580
UC-414

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

Presented at the Twenty-fifth International Conference on High Energy Accelerators, Hamburg, Germany, July 20–24, 1992, and to be published in the Proceedings

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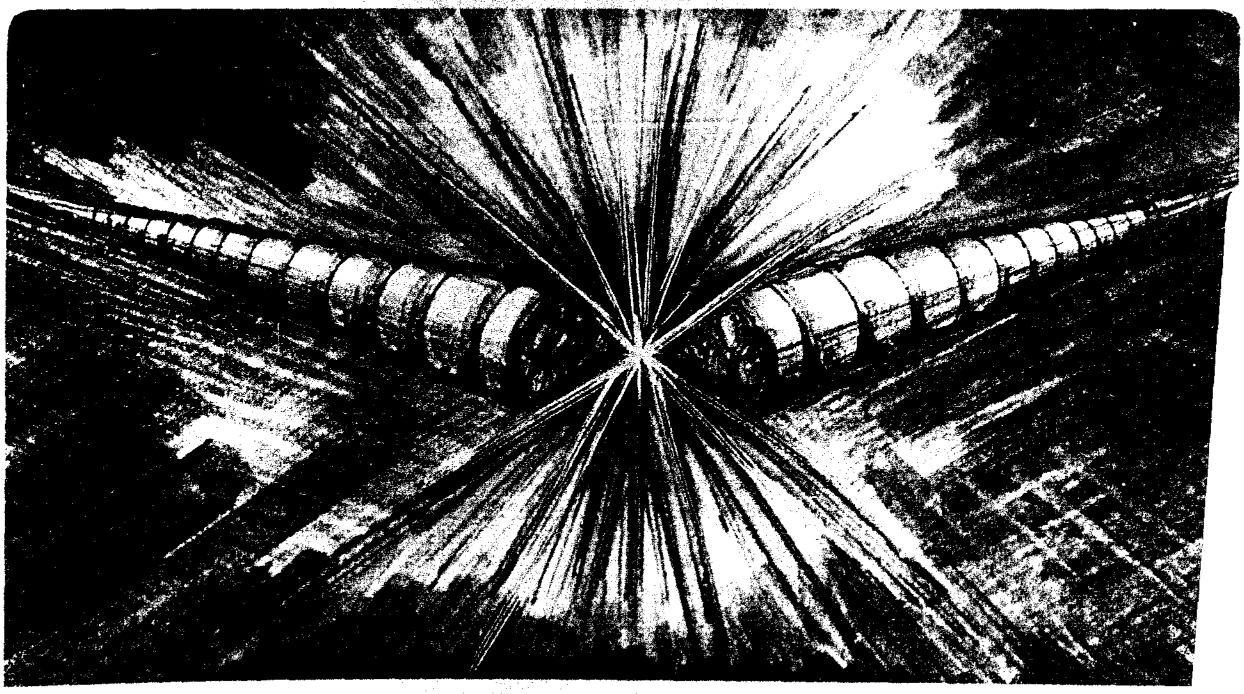
July 1992



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

LBL-32580
ESG-208

COMMON ANALYSIS OF THE RELATIVISTIC KLYSTRON AND THE STANDING-WAVE FREE-ELECTRON LASER TWO-BEAM ACCELERATOR

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ABSTRACT

This paper summarizes a new formalism which makes the analysis and understanding of both the relativistic klystron (RK) and the standing-wave free-electron laser (SWFEL) two-beam accelerator (TBA) available to a wide audience of accelerator physicists. A "coupling impedance" for both the RK and SWFEL is introduced, which can include realistic cavity features, such as beam and vacuum ports, in a simple manner. The RK and SWFEL macroparticle equations, which govern the energy and phase evolution of successive bunches in the beam, are of identical form, differing only by multiplicative factors. The analysis allows, for the first time, a relative comparison of the RF and SWFEL TBAs.

1. INTRODUCTION

The context and motivation for this work is the Two-Beam Accelerator (TBA) concept [1,2,3], which is, in essence, a high efficiency power converter, extracting energy from a low energy high-current electron "drive" beam and depositing it in a high energy electron or positron beam. In a TBA a drive beam of kiloampere current, bunched at centimeter wavelengths, passes through a periodic array of wiggler magnets, which extract the beam energy through a Relativistic Klystron (RK) or a Free-Electron Laser (FEL); at the same time, the beam passes through induction cells which replenish the beam energy, as seen in Fig. 1.

The TBA configuration of present interest, the Standing-Wave Free-Electron Laser TBA (SWFEL/TBA), has grown out of a number of theoretical and conceptual refinements, including considerations of microwave extraction and phase and amplitude control [4,5].

In the SWFEL [6] power is produced in a series of uncoupled cavities (the rf is cut off between the cavities), each of which is of order one wiggler oscillation in length. The FEL thus operates as a standing-wave device. The propagating beam provides the only coupling between the cavities. Numerical studies [7,8] have examined the phase sensitivity and longitudinal particle stability in the standing wave FEL in some detail.

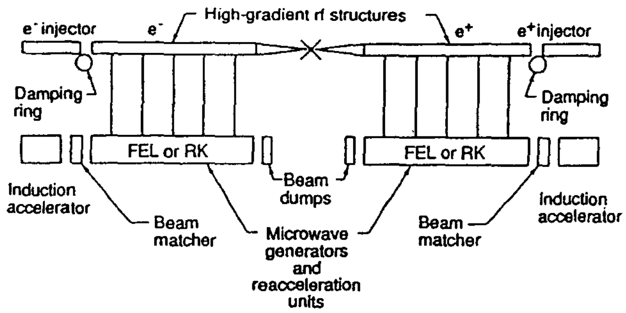

Fig. 1. A schematic of the structure of a TBA. The cavities can either be those of a relativistic klystron (RK), or those of a Standing-Wave FEL (SWFEL) in which case there is a wiggler magnetic field passing through the cavities.

In the above theoretical work, the extraction units were taken as FELs. Alternatively, of course, it is possible to consider a Relativistic Klystron TBA (RK/TBA). This approach has been developed by the CERN Group [9]. Because it has been demonstrated experimentally that high power can be extracted from an RK [10], as well as from an FEL [11], both of these approaches are attractive. In fact, the standing wave FEL has many similarities to the relativistic klystron, the main differences between them being that (1) the FEL produces power through the coupling of the transverse wiggle oscillation with the transverse electric field, while the klystron couples the longitudinal component of the electric field, and (2) the FEL resonance condition precludes using ultra-high energies to produce rf power, but allows coupling to modes (of highly overmoded cavities) with phase velocity greater than light, while the klystron interaction does not similarly limit the choice of drive beam energy. Until now, no serious comparisons of these two approaches has been made. In fact, not even the formal framework in which such comparisons can be made has been developed. It is the purpose of this paper to set down such a framework.

* Supported by the U.S. Department of Energy, Division of Nuclear and High Energy Physics, under contract No. DE-FG02-91ER40648.
** Supported by the Japan Society for the Promotion of Science, the U.S. National Science Foundation and the National Laboratory for High Energy Physics (KEK).
*** Supported by the U.S. Department of Energy, Division of Nuclear and High Energy Physics, under contract No. DE-AC-03-SF-00098.

A full description of this work has recently been presented; here, for the convenience of the reader, we reproduce the first part of Section 2 of that paper [12].

2. FORMALISM

We derive equations describing the coupling of beam electrons to cavity modes. First, we decompose the vector potential in the Lorentz gauge,

$$\vec{A} = \frac{mc^2}{e} \sum_{\alpha} q_\alpha(t) \vec{a}_\alpha(\vec{r}), \qquad (1)$$

where α is the mode index, q_α is the dimensionless mode amplitude and $\vec{a}_\alpha$ gives the spatial dependence of the mode. The electron mass is m, the speed of light is c and the electron charge is $-e$. The mode normalization is

$$\int_V d^3 r' \, \vec{a}_\alpha(\vec{r}) \bullet \vec{a}^*_\alpha(\vec{r}) = V \qquad (2)$$

with V the cavity volume.

Maxwell's equations reduce to the well-known form

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_\alpha}{Q_\alpha} \frac{\partial}{\partial t} + \omega_\alpha^2 \right) q_\alpha(t) = \frac{4\pi e}{mc} \frac{1}{V} \int d^3 r' \vec{J}(\vec{r}', t) \bullet \vec{\alpha}^*_\alpha(\vec{r}'), \qquad (3)$$

where the integral is over the cavity volume. We consider the interaction of the beam with a single cavity mode with a very high Q, and make an eikonal approximation, $q(t) = \Re\{ b \, e^{i\varphi} e^{-i\omega t} \}$, where the phase φ and the amplitude b vary slowly on the time scale of the mode period. In terms of b, the energy stored per unit length is

$$U = \frac{1}{8\pi} \left[\frac{\omega^2 h w}{c^2} \right] \frac{m^2 c^4 b^2}{e^2}, \qquad (4)$$

where h and w are the height and width of the cavity.

We will consider two cases: (1) coupling to a TE mode through the transverse current induced by a magnetic wiggler (FEL) and (2) coupling to a TM mode through the axial current (RK). In each case the coupling depends on the phase $\psi = \varphi + \theta$ of an electron's motion relative to the phase of the cavity fields. Here the phase θ is a particle variable. For an FEL this phase is given by

$$\theta = (k_w + k_z) z - \omega t \qquad (5)$$

where k_w is the wiggler wavenumber and k_z is the axial wavenumber for the forward-going component of the cavity mode. For a steady-state klystron this phase is

$$\theta = k_z z - \omega t - \theta_r, \qquad (6)$$

where we have introduced the phase θ_r, that of a reference particle. Typically klystrons operate with $k_z = 0$, in a nearly single mode cavity. The SWFEL, on the other hand, operates in a highly overmoded cavity.

An important distinction between the SWFEL and the RK is that Eq. (5) defines a synchronous energy in term of the system parameters, while Eq. (6), for the RK, only relates the phase of a particle to a reference phase and does not define a synchronous energy. The RK, therefore can be operated at any energy (even GeV energies are possible), whereas the SWFEL requires a low (of order ten MeV) energy for resonance at microwave frequencies with reasonable wiggler parameters.

In terms of these variables the field equations in a given cavity may be written as

$$\frac{\partial}{\partial s} b e^{i\varphi} = ic \frac{1}{\eta} \frac{r}{Q} \frac{I}{I_A} \langle e^{-i\theta} \rangle, \qquad (7)$$

where $s = v_z t - z$, with v_z the beam velocity. I is the average beam current, $I_A = mc^3/e \sim 17$kA, and the brackets indicate an average over a beam slice. The factor η depends on the kind of coupling. For an RK $\eta = 2$, while for an FEL, $\eta = a_w / 2\gamma$, with γ the Lorentz factor, and a_w the wiggler parameter.

The shunt impedance per unit length r is given by [6]

$$\frac{r}{Q} = \frac{4\pi}{VL\omega} \left| \int_{-L/2}^{+L/2} dz \frac{\vec{v}(z)}{v_z} \bullet \vec{a}(z) \exp\left(-\frac{i\omega z}{v_z} \right) \right|^2, \qquad (8)$$

where L is the cavity length. The SWFEL typically operates in the TE_{01p} mode of a rectangular cavity of width w and height h, so that

$$\frac{r}{Q} = \frac{Z_0}{8\pi} \frac{\lambda}{hw} \left(\frac{a_w}{\gamma} \right)^2 \left(\frac{\sin\chi}{\chi} \right)^2 \qquad (9)$$

where $Z_0 = 4\pi/c$ (377Ω in MKS), λ is the free-space wavelength and $\chi = (\omega L / v_z - p\pi - k_w L)/2$ is the effective transit angle. For an RK operating in the TM_{m1p} mode,

$$\frac{r}{Q} = \frac{Z_0}{4\pi} \frac{\lambda}{hw} \left\{ \frac{k_x^2 + k_y^2}{k_x^2 + k_y^2 + k_z^2} \right\} \left(\frac{\sin\chi}{\chi} \right)^2 \qquad (10)$$

where the transit angle is $\chi = (p\pi + \omega L / v_z)/2$. The coupling in the SWFEL is from the interaction of the wiggling velocity imparted to the beam by the wiggler and the transverse field of a TE mode, while the RK generates a shunt impedance from the axial coupling of the beam to the z-component of the electric field of a TM mode.

To complete the formulation, equations are required for the particle motion. It is convenient to linearize about the reference energy, so that the dynamical variables are θ and $\delta\gamma = \gamma - \gamma_r$, where γ_r is the resonant γ in the case of the FEL, or in the case of an RK, a reference γ. The phase evolution is found from Eqs. (5) and (6), so that [6]

$$\frac{d\theta}{dz} \approx 2 \kappa \frac{\delta\gamma}{\gamma_r}, \qquad (11)$$

$$\frac{d\delta\gamma}{dz} = -\eta \frac{\omega}{c} b \sin(\theta + \varphi) - \frac{eE_z}{mc^2}. \qquad (12)$$

The constant $\kappa = \omega(1 + a_w^2/2)/2c\gamma^2$ for an FEL, while $\kappa = \omega/2c\gamma^2$ for the RK. Equations (7), (11), and (12) describe the self-consistent evolution of the beam and the cavity fields. *The SWFEL and RK are distinguished only through the values of η, κ and r/Q.*

3. SENSITIVITIES

The sensitivities of the RK/TBA and an SWFEL/TBA can now be compared using the above results. Details may be found in Ref. [12], where we have noted that the dependence upon current error, ΔI, is not excessive, nor is it very different for the RK and the SWFEL. This source of sensitivity must, and can, be controlled in either device.

The dependence upon energy errors, $mc^2\Delta\gamma$, is much more severe and it is different for the two devices. In linear approximation it only affects φ (and not the amplitude b). Explicitly the jitter in γ, due to energy variation $\Delta\gamma$, is given by:

$$\begin{aligned}\Delta\varphi &= -\left(\frac{\omega}{c\gamma^2}\right)\left(\frac{\Delta\gamma}{\gamma}\right)\frac{\sin\Omega(0)z}{\Omega(0)}; \qquad RK,\\ \Delta\varphi &= -\frac{\omega}{c\gamma^2}\left(1 + a_w^2/2\right)\left(\frac{\Delta\gamma}{\gamma}\right)\frac{\sin\Omega(0)z}{\Omega(0)}; \qquad SWFEL,\end{aligned} \tag{13}$$

where $\Omega(0)$ is the initial synchrotron period.

We see that, as a rule of thumb, the RK is roughly two times less sensitive to energy errors at a rather low energy than is the SWFEL. However, we must remember that the RK will have more severe wake-field effects than the SWFEL since it necessarily consists of smaller structures.

On the other hand, it is possible to operate the RK at a very high energy since no resonance condition must be satisfied (as in the SWFEL). At large γ, the sensitivity to energy errors, $mc^2\Delta\gamma$, is very much less in the RK than in the SWFEL. Successful operation of an RK of high energy will, however, depend on acceleration of an intense bunched beam from a low energy, during which process phase errors may accumulate. Indeed, accelerating the drive beam of an RK to high energies, while maintaining its phase insensitivity is an important challenge for such a device, and remains to be analyzed. Given that there is no vast difference between the two approaches, the choice between them will probably be made on the basis of such issues as ease of construction, cost, and beam break-up limits (BBU).

4. CONCLUSIONS

The formalism which has been developed allows the input of a coupling impedance into the SWFEL and, therefore, the introduction of the features of a realistic cavity. We thus have the capability of employing coupling impedances obtained by various electrodynamic codes such as SUPERFISH or MAFIA. In short, we have put the analysis of the standing-wave free-electron laser on the same footing as that of the relativistic klystron. The result of application of the formalism to the study of sensitivities of the TBA is presented.

5. REFERENCES

1. Sessler, A.M., "The Free Electron Laser as a Power Source for a High Gradient Accelerating Structure", in *Laser Acceleration of Particles*, P.J. Channell ed., AIP Proceedings No. **91**, p. 163, New York, (1982).

2. Hopkins, D.B., Sessler, A.M., and Wurtele, J.S., "The Two-Beam Accelerator", Nucl. Instr. and Meth. in Phys. Res. **228**, 15 (1984).

3. Sternbach, E. and Sessler, A.M., "A Steady-State FEL: Particle Dynamics in the FEL Portion of a Two-Beam Accelerator", Nucl. Instr. and Methods in Phys. Res. **A250**, 464 (1986).

4. Sessler, A.M., Sternbach, E. and Wurtele, J.S., "A New Version of a Free Electron Laser Two-Beam Accelerator", Nucl. Instr. and Methods in Phys. Res. **B40/41**, 1064 (1989).

5. Sessler, A.M., Whittum, D.H. and Wurtele, J.S., "A Study of Phase Control in the FEL Two-Beam Accelerator", Proc. of the XIV Intern. Conf. on High Energy Accel., Tsukuba, Part. Accel. **31**, 69 (1990).

6. Sessler, A.M., et al., "Standing-Wave Free-Electron Laser Two-Beam Accelerator", Nucl. Instr. and Methods in Phys. Res. **A306**, 592 (1991).

7. Sharp, W.M., et al., "Simulation of a Standing-Wave Free-Electron Laser", Linear Accelerator Conf, Albuquerque, Los Alamos National Laboratory Report LA-12004-C, 656 (1990).

8. Sharp, W.M., et al., "Phase Stability of a Standing-Wave Free-Electron Laser Two-Beam Accelerator", Intense Microwave and Particle Beams II, O/E LASE' 91, Los Angeles, in Proc. Intern. Soc. Opt. Eng. (SPIE) (1991).

9. Van Der Meer, S., "The CLIC Approach to Linear Colliders", Particle Accelerators **30**, 127 (1990).

10. Allen, M.A., et al., "High-Gradient Electron Accelerator Powered by a Relativistic Klystron", Phys. Rev. Lett. **63**, 2472 (1989).

11. Orzechowski, T., et al., "High Efficiency Extraction of Microwave Radiation from a Tapered Wiggler Free Electron Laser", Phys. Rev. Lett. **57**, 2172 (1986).

12. Wurtele, J., Whittum, D. and Sessler, A., "Impedance-Based Analysis and Study of Phase Sensitivity in Slow-Wave Two-Beam Accelerators", to be published in the Proceedings of the Third Workshop on Advanced Accelerator Concepts, Pt. Jefferson, NY, June 14-20, 1992.