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TECHNICAL REPORT

A space charge compensation model for positive DC ion beams

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ABSTRACT: In this paper, we revisit and extend a formula to predict the compensation of space charge in positive DC ion beams of non-relativistic energy, as they are for example found in the injector beam lines of heavy ion accelerator facilities.The original formula was presented in 1975 by Igor Gabovich et al. and takes into account the de-compensation through Coulomb collisions of the primary beam ions and the compensating electrons. We extend its usability to arbitrary (positive) charge states of the ions and non-quasineutral beams.The resulting formula compares well with measurements using a retarding field analyzer and a multi-species generalization of it was incorporated into beam transport simulations using the particle-in-cell code WARP.

KEYWORDS: Ion sources (positive ions, negative ions, electron cyclotron resonance (ECR), electron beam (EBIS)); Accelerator modelling and simulations (multi-particle dynamics; single-particle dynamics); Beam dynamics

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Contents

1 Introduction

Space charge and space charge compensation (neutralization) are important factors in the transport of medium to high intensity low energy ion beams. Space charge arises from the charges of the beam ions themselves and the resulting electric self field acts defocusing on the beam (the magnetic self field can be neglected for non-relativistic ion beams). Simulation codes can take into account space charge either through a linear analytical model (as for example presented in Reiser [1]) or by treating the beam fully three-dimensional through particle-in-cell (PIC) methods (e.g. OPAL [2] or WARP [3, 4]). On the other hand, as the beam travels along the beam line, the beam ions interact with the residual gas mainly through two processes:

$$A^{q+} + X^0 \rightarrow A^{(q-1)+} + X^+ \quad \text{(charge-exchange)}$$
$$A^{q+} + X^0 \rightarrow A^{q+} + X^+ + e^- \quad \text{(ionization)}$$

where A is a beam ion and X a residual gas molecule [5]. The resulting slow secondary ions are expelled by the beam, while the electrons are trapped in the potential well. The to first order stationary electrons are lowering the beam potential which decreases the defocusing effect of the self electric field. If the beam is approximated as a uniform cylindrical charge distribution, the radial potential distribution can readily be written down as:

$$\phi(r) = \begin{cases} \Delta\phi \left(1 + 2\ln\frac{r_p}{r_b} - \frac{r^2}{r_b^2}\right) & \text{for } r \leq r_b \\ \Delta\phi \;\; 2\ln\frac{r_p}{r} & \text{for } r_b \leq r \leq r_p \end{cases} \tag{1.1}$$

with

$$\Delta\phi = \frac{I_b}{4\pi\epsilon_0 v_b}. \tag{1.2}$$

Here, I_b is the beam current in electrical Amperes (A), ϵ_0 the vacuum permittivity ($\approx 8.854 \cdot 10^{-12}$ F/m), v_b the beam velocity in m/s, r_b the beam radius in m, and r_p the radius of a grounded pipe sorrounding the beam. In the simplest model, space charge compensation can be expressed in form of a space charge compensation factor f_e, modifying the total beam current in equation (1.2):

$$\Delta\phi = \frac{I_b \cdot (1 - f_e)}{4\pi\epsilon_0 v_b}. \tag{1.3}$$

In 1975 Gabovich, Katsubo, and Soloshenko published a paper presenting a formula to predict f_e taking into account decompensation by Coulomb interaction of the beam ions with the compensating electrons [6]. In 1977 Gabovich published a detailed review article on the processes involved in compensation and decompensation of positive and negative high intensity ion beams considering dynamic- and self-decompensation of the ion beams as well as collective processes in the beam plasma [7]. In the following years Soloshenko presented several papers on the same topics [8–10]. Their work compared well to measurements of high intensity proton [11] and H_2^+ [6] beams, which can be considered highly compensated. However, comparison of simulated and measured emittances as well as measurements of the ion beam potential in the Low Energy Beam Transport (LEBT) lines of Electron Cyclotron Resonance Ion Sources (ECRIS) suggested beams far from fully compensated [12, 13]. Hence one of the major assumptions in Gabovich's work no longer holds: quasi-neutrality. In addition, all derivations were performed for one singly charged ion species (e.g.: protons, H_2^+, He^+). In ECRIS, a multitude of ion species is produced at the time and most often a highly charged ion is the desired beam ion. In this paper the formula presented by Gabovich et al. for self decompensation by Coulomb collisions of the beam ions with the compensating electrons will be extended so it can be applied to ECRIS beam lines. To establish the framework for the manuscript, the original formula by Gabovich et al. is re-derived for an arbitrary charge-state q in section 2. SI units are used throughout the paper for clarity and consistency. In section 3 a simple generalization is made from quasi-neutrality to a non-neutral beam plasma. This formula can be used to estimate the space charge compensation factor attainable for a typical ECRIS beam, not taking into account dynamic decompensation due to beam instabilities and the more complex collective effects (plasma oscillations) as described by Gabovich [7]. However it will provide us with a best case scenario under optimal conditions. The results of the extended formula are compared to measurements in section 6. In the presented publication, we shall restrict ourselves to positive DC ion beams of low energy ($20 - 70 \cdot q$ keV, q being the charge state of the ions) and medium intensity (100 eμA - 10 emA) as they are provided by ECRIS like SuSI [14, 15] and VENUS [16, 17], and off-resonance microwave sources like the Low-Energy Demonstration Accelerator (LEDA) injector source [18, 19] and Versatile Ion Source (VIS) [20].

2 Re-derivation of the Gabovich formula in SI units

The energy necessary for the secondary electrons to leave the potential created by the beam is given by:

$$\left(\frac{d\mathrm{E}}{dt}\right)_{\mathrm{out}} = L \int_0^{r_b} 2\pi r \,\mathrm{d}r \int_0^{e\varphi(r)} f(\mathrm{E})\,(e\varphi(r) - \mathrm{E})\,\mathrm{dE} \tag{2.1}$$

with L the length of the beam, r_b the radius, $\varphi(r)$ the potential at radius r, and $f(\mathrm{E})$ the secondary electron energy distribution. On the other hand, a fast beam ($v_b > v_e$) will transfer energy mostly through Coulomb collisions. In simple classical Coulomb collisions, the rate of transfer of energy per unit length for one beam particle of energy $E_{\text{kin}} = m_b v_b^2/2$ to stationary target particles can be written as

$$\frac{\mathrm{dE}}{\mathrm{d}\ell} = n_t \left(\frac{Q_b Q_t}{4\pi\epsilon_0}\right)^2 \frac{4\pi}{m_t v_b^2} \cdot \ln\left|\frac{b_{\max}}{b_{\min}}\right| \tag{2.2}$$

which, for electrons as target species becomes

$$\frac{\mathrm{dE}}{\mathrm{d}\ell} = n_e \frac{q^2 e^4}{4\pi\epsilon_0^2} \frac{1}{m_e v_b^2} \cdot \ln\left|\frac{b_{\max}}{b_{\min}}\right|$$

where $b_{\min}$ and $b_{\max}$ are appropriate lower and upper limits of the impact parameter, and q the charge state of the primary ion beam. We can now write the energy transfer to the electrons per unit time for the sum of all beam particles $N_b = n_b r_b^2 \pi L$ with velocity v_b as

$$\left(\frac{dE}{dt}\right)_{\text{in}} = \frac{n_b n_e q^2 e^4 r_b^2 L}{4 m_e \epsilon_0^2 v_b} \cdot \ln\left|\frac{b_{\max}}{b_{\min}}\right| . \tag{2.3}$$

Since the main contribution to the process comes from small angle scattering, a good choice for $b_{\min}$ is b_{90}, the impact parameter, where the incident particle is scattered by 90°. b_{90} for ion-electron collisions is given by

$$b_{90} = \frac{-qe^2}{4\pi\epsilon_0} \frac{1}{m_r v_b^2} \tag{2.4}$$

with m_r the reduced mass, which for ion-electron collisions is $m_r \approx m_e$. The secondary electrons are not bound to nuclei and essentially form a low density electron plasma. A good value for $b_{\max}$ is then

$$b_{\max} = \frac{v_b}{\omega_{pe}} = \frac{v_b}{e} \cdot \sqrt{\frac{\epsilon_0 m_e}{n_e}} \tag{2.5}$$

with ω_{pe} the electron plasma frequency. We expect the effectiveness of the collisions to fall off because of dielectric effects for $b > b_{\max} = v_b/\omega_{pe}$. Substituting expressions (2.4) and (2.5) for $b_{\min}$ and $b_{\max}$, respectively, we obtain

$$\tilde{\mathcal{L}} = \ln\left(4\pi\epsilon_0^{3/2} \frac{m_e^{3/2} v_b^3}{q e^3 n_e^{1/2}}\right) \tag{2.6}$$

where we call $\tilde{\mathcal{L}}$ a Coulomb logarithm. Hence

$$\left(\frac{dE}{dt}\right)_{\text{in}} = \frac{n_b n_e q^2 e^4 r_b^2 L \tilde{\mathcal{L}}}{4 m_e \epsilon_0^2 v_b} . \tag{2.7}$$

In steady-state, expressions (2.1) and (2.7) have to be equal in order to conserve energy:

$$L \int_0^{r_b} 2\pi r \,\mathrm{d}r \int_0^{e\varphi(r)} f(\mathrm{E})\,(e\varphi(r) - \mathrm{E})\ \mathrm{dE} = \frac{n_b n_e q^2 e^4 r_b^2 L \tilde{\mathcal{L}}}{4 m_e \epsilon_0^2 v_b}$$

or:

$$\int_0^{r_b} r \,\mathrm{d}r \int_0^{e\varphi(r)} f(\mathrm{E})\,(e\varphi(r) - \mathrm{E})\ \mathrm{dE} = \frac{n_b n_e q^2 e^4 r_b^2 \tilde{\mathcal{L}}}{8\pi m_e \epsilon_0^2 v_b} . \tag{2.8}$$

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Let us now solve the integrals on the left hand side. At this point, Soloshenko et al. [9] and Gabovich [7] make an approximation for $f(\mathrm{E})$:

$$f(\mathrm{E}) \propto \frac{1}{(e\Phi_i + \mathrm{E})^2} \tag{2.9}$$

with $e\Phi_i$ the ionization energy of the gas molecules. They determine the proportionality constant by requiring:

$$\int_0^\infty f(\mathrm{E}) d\mathrm{E} = \frac{\partial n_e}{\partial t} = n_b v_b n_0 \sigma_e$$

with n_0 the neutral gas density, and σ_e the electron creation cross -section. This expression can be readily solved to yield

$$f(\mathrm{E}) = \frac{n_b v_b n_0 \sigma_e \cdot e\Phi_i}{(e\Phi_i + \mathrm{E})^2}. \tag{2.10}$$

And the energy integral in equation (2.8) can be solved to:

$$\begin{aligned} \int_0^{e\varphi(r)} f(\mathrm{E})\,(e\varphi(r) - \mathrm{E})\ d\mathrm{E} &= \eta \cdot \int_0^{e\varphi(r)} \frac{e\varphi(r) - \mathrm{E}}{(e\Phi_i + \mathrm{E})^2}\ d\mathrm{E} \\ &= \eta \cdot \left[\frac{\varphi(r)^2 - \Phi_i^2}{\Phi_i(\Phi_i + \varphi(r))} + 1 - \ln\left(1 + \frac{\varphi(r)}{\Phi_i}\right)\right] \\ &= \eta \cdot \left[\frac{\varphi(r) - \Phi_i}{\Phi_i} + 1 - \ln\left(1 + \frac{\varphi(r)}{\Phi_i}\right)\right] \\ &= \eta \cdot \left[\frac{\varphi(r)}{\Phi_i} - \ln\left(1 + \frac{\varphi(r)}{\Phi_i}\right)\right] \end{aligned} \tag{2.11}$$

where η was introduced for convenience as

$$\eta = n_b v_b n_0 \sigma_e \cdot e\Phi_i$$

η is considered constant in r and E in our simplified view of the problem. The potential $\varphi(r)$ inside a homogeneously charged cylinder inside a beam pipe of radius R is given by equation (1.1):

$$\varphi(r) = \frac{I_b}{4\pi\epsilon_0 v_b} \cdot \left(1 + 2\ln\frac{r_p}{r_b} - \frac{r^2}{r_b^2}\right)$$

with I_b the beam current and ϵ_0 the vacuum permittivity. Computing $\Delta\varphi = \varphi(r_b) - \varphi(0)$ yields

$$\Delta\varphi = \frac{I_b}{4\pi\epsilon_0 v_b}.$$

Since, for our considerations, we are only interested in $\Delta\varphi$, which is independent of the beam pipe radius, we can set $r_p = r_b$, and

$$\varphi(r) = \Delta\varphi \cdot \left(1 - \frac{r^2}{r_b^2}\right). \tag{2.12}$$

Now we can carry out the radial integral in equation (2.8):

$$\begin{aligned}
\int_0^{r_b} r\cdot\left[\frac{\varphi(r)}{\Phi_i}-\ln\left(1+\frac{\varphi(r)}{\Phi_i}\right)\right]\,\mathrm{d}r &= \int_0^{r_b} r\cdot\left\{\frac{\Delta\varphi}{\Phi_i}\left(1-\frac{r^2}{r_b^2}\right)-\ln\left[1+\frac{\Delta\varphi}{\Phi_i}\left(1-\frac{r^2}{r_b^2}\right)\right]\right\}\,\mathrm{d}r \\
&= \frac{\Delta\varphi}{\Phi_i}\left(\frac{r_b^2}{2}-\frac{r_b^2}{4}\right)-\int_0^{r_b} r\cdot\ln\left[1+\frac{\Delta\varphi}{\Phi_i}\left(1-\frac{r^2}{r_b^2}\right)\right]\,\mathrm{d}r \\
&= \frac{\Delta\varphi}{\Phi_i}\frac{r_b^2}{4}-\frac{\Phi_i}{\Delta\varphi}\frac{r_b^2}{2}\left[-\frac{\Delta\varphi}{\Phi_i}+\left(1+\frac{\Delta\varphi}{\Phi_i}\right)\ln\left(1+\frac{\Delta\varphi}{\Phi_i}\right)\right] \\
&= \frac{r_b^2}{2}\left[\frac{\Delta\varphi}{2\Phi_i}+1-\left(1+\frac{\Phi_i}{\Delta\varphi}\right)\ln\left(1+\frac{\Delta\varphi}{\Phi_i}\right)\right]. \qquad (2.13)
\end{aligned}$$

Expanding the logarithm for $\Delta\varphi/\Phi_i < 1$ yields

$$\ln\left(1+\frac{\Delta\varphi}{\Phi_i}\right)=\frac{\Delta\varphi}{\Phi_i}-\frac{1}{2}\left(\frac{\Delta\varphi}{\Phi_i}\right)^2+\frac{1}{3}\left(\frac{\Delta\varphi}{\Phi_i}\right)^3+\mathcal{O}(4).$$

Substituting into the solution of equation (2.13), carrying out the multiplication and dropping the terms of order 3 and higher gives

$$\frac{r_b^2}{2}\left[\frac{\Delta\varphi}{2\Phi_i}+1-\left(1+\frac{\Phi_i}{\Delta\varphi}\right)\cdot\left(\frac{\Delta\varphi}{\Phi_i}-\frac{1}{2}\left(\frac{\Delta\varphi}{\Phi_i}\right)^2+\frac{1}{3}\left(\frac{\Delta\varphi}{\Phi_i}\right)^3+\mathcal{O}(4)\right)\right]\approx\frac{r_b^2}{12}\frac{(\Delta\varphi)^2}{\Phi_i^2}.$$

And thus

$$\begin{aligned}
\int_0^{r_b} r\,\mathrm{d}r\int_0^{e\varphi(r)} f(\mathrm{E})\,(e\varphi(r)-\mathrm{E})\;\mathrm{dE} &= \eta\cdot\frac{r_b^2}{12}\frac{(\Delta\varphi)^2}{\Phi_i^2} \\
&= n_b v_b n_0\sigma_e e\Phi_i\cdot\frac{r_b^2}{12}\frac{(\Delta\varphi)^2}{\Phi_i^2} \\
&= n_b v_b n_0\sigma_e e\cdot\frac{r_b^2}{12}\frac{(\Delta\varphi)^2}{\Phi_i}. \qquad (2.14)
\end{aligned}$$

Then, equation (2.8) becomes

$$(\Delta\varphi)^2=\frac{3q^2\Phi_i n_e e^3\tilde{\mathcal{L}}}{2\pi m_e v_b^2 n_0\sigma_e\epsilon_0^2}. \qquad (2.15)$$

The only unknown quantity now is the electron density. Here, Gabovich and Soloshenko use quasi-neutrality:

$$n_e=q\cdot n_b+n_i \qquad (2.16)$$

and the secondary ion balance equation (number of secondary ions created in the beam is equal to the number of ions leaving the beam per unit time):

$$2r_b\pi n_i\bar{v}_i=r_b^2\pi n_b v_b n_0\sigma_i \qquad (2.17)$$

with $\bar{v}_i$ the average secondary ion velocity. Thus the electron density is

$$n_e=q\cdot n_b+\frac{n_b v_b n_0\sigma_i r_b}{2\bar{v}_i}. \qquad (2.18)$$

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We can now rewrite (2.15):

$$(\Delta\varphi)^2 = \frac{3q^2\Phi_i e^3\tilde{\mathcal{L}}}{2\pi m_e v_b^2 n_0\sigma_e\epsilon_0^2}\cdot\left(q\cdot n_b + \frac{n_b v_b n_0\sigma_i r_b}{2\bar{v}_i}\right)$$
$$= \frac{3q^2\Phi_i n_b e^3\tilde{\mathcal{L}}}{2\pi m_e v_b^2\epsilon_0^2}\cdot\left(\frac{q}{n_0\sigma_e} + \frac{v_b\sigma_i r_b}{2\bar{v}_i\sigma_e}\right). \tag{2.19}$$

Finally, we use the non-relativistic kinetic energy of the primary beam

$$qeU_0 = \frac{m_b v_b^2}{2},$$

with U_0 the source voltage, to replace v_b^2 and obtain the formula presented in [9] (but in SI units and for an arbitrary charge-state q of the primary ion beam):

$$(\Delta\varphi_{\text{neut}})^2 = 3\mathcal{L}\frac{m_b}{m_e}\frac{\Phi_i}{U_0}\frac{n_b q e^2}{(4\pi\epsilon_0)^2}\left(\frac{q}{n_0\sigma_e} + \frac{v_b\sigma_i r_b}{2\bar{v}_i\sigma_e}\right). \tag{2.20}$$

Note that we have absorbed a factor 4π into the definition of $\mathcal{L}$ ($\mathcal{L} = 4\pi\tilde{\mathcal{L}}$) to be consistent with [9]. We also use equation (2.18) to replace the electron density in the Coulomb logarithm (equation (2.6)). We now have an explicit expression for $\Delta\varphi$ in terms of quantities either experimentally accessible or calculable by theoretical models. The neutralization factor f_e can now be obtained by substituting the (partially) neutralized $\Delta\varphi_{\text{neut}}$ (equation (2.20)) into equation (1.3):

$$f_e = 1 - \frac{\Delta\varphi_{\text{neut}}}{\Delta\varphi_{\text{full}}} \tag{2.21}$$

where (with $\beta c = v_b$ the beam velocity):

$$\Delta\varphi_{\text{full}} = \frac{I_b}{4\pi\epsilon_0 v_b}. \tag{2.22}$$

3 Extension of the Gabovich formula to lower compensation factors

One of the drawbacks of the result presented in section 2 is the assumption of quasi-neutrality which naturally limits the usefulness of the formula to highly compensated beams. To remedy this, we propose a simple modification to equation (2.16) to reflect the fact that the electron density is only a fraction of the combined primary and secondary ion densities (corresponding to the level of neutralization) we can write it as:

$$n_e = fe\cdot(q\cdot n_b + n_i). \tag{3.1}$$

This only adds a factor $\sqrt{f_e}$ to the definition of $\Delta\varphi_{\text{neut}}$ and the new equation for calculation of the compensation factor is:

$$f_e = 1 - \sqrt{f_e}\cdot\frac{\Delta\varphi_{\text{neut}}}{\Delta\varphi_{\text{full}}} \tag{3.2}$$

where equation (2.20) was used:

$$\Delta\varphi_{\text{neut}} = \sqrt{3\mathcal{L}\cdot\frac{m_b}{m_e}\cdot\frac{\Phi_i}{U_0}\frac{n_b q e^2}{(4\pi\epsilon_0)^2}\left(\frac{q}{n_0\sigma_e} + \frac{v_b\sigma_i r_b}{2\bar{v}_i\sigma_e}\right)}.$$

Substituting

$$\chi = \frac{\Delta\varphi_{\text{neut}}}{\Delta\varphi_{\text{full}}}.$$

Equation (3.2) can easily be solved for f_e:

$$f_e = 1 + \frac{\chi^2}{2} - \frac{\chi}{2}\sqrt{\chi^2 + 4}. \tag{3.3}$$

4 Discussion

Let us now examine a few of the properties of the formula for f_e. Substituting $n_b = I_b/(r_b^2 \pi v_b q e)$ in equation (2.20) yields:

$$(\Delta\varphi_{\text{neut}})^2 = 3\mathcal{L}\frac{m_b}{m_e}\frac{\Phi_i}{U_0}\frac{I_b e}{(4\pi\epsilon_0)^2\pi}\left(\frac{q}{n_0\sigma_e v_b r_b^2} + \frac{\sigma_i}{2\bar{v}_i\sigma_e r_b}\right).$$

One can see that the space charge compensation factor f_e mainly depends on three variables (aside from fixed parameters like the cross-sections, beam energy, and residual gas composition):

- Beam current I_b. Trend: rising f_e with increasing beam current. This is not immediately visible, but becomes clear when one takes into account that $\Delta\phi_{\text{full}}$ also includes I_b (cf. equation (2.22)).
- Beam radius r_b. Trend: rising f_e with increasing beam size.
- Residual gas density n_0. Trend: rising f_e with increasing pressure.

In the LEBT of typical ECR ion sources, the currents are usually < 5 mA, and the pressure is usually $< 10^{-6}$ Torr to prevent beam loss due to charge-exchange. The formula then predicts space charge compensation as low as 5% and as high as 80% depending on the three parameters (this compares well with experiments, as is shown in section 6). Additionally, the calculated value can vary considerably locally along the LEBT due to the change in beam radius, beam losses on apertures, and the changing pressure profile. Consequently, these factors should be taken into account when trying to accurately simulate the beam transport (cf. section 7).

5 Cross-sections

One of the main sources of uncertainty in the prediction using the presented formula are the cross sections for electron production (σ_e) and ion production (σ_i). As mentioned in the introduction, the two main processes are charge-exchange and ionization:

$$A^{q+} + X^0 \longrightarrow A^{(q-1)+} + X^+ \quad \text{(charge-exchange)}$$
$$A^{q+} + X^0 \longrightarrow A^{q+} + X^+ + e^- \quad \text{(ionization)}$$

where A is a beam ion and X a residual gas molecule. As can be seen, charge-exchange contributes to σ_i and ionization to both σ_i and σ_e, thus:

$$\sigma_e \approx \sigma_{\text{ionization}}$$
$$\sigma_i \approx \sigma_{\text{ionization}} + \sigma_{\text{charge-exchange}}$$

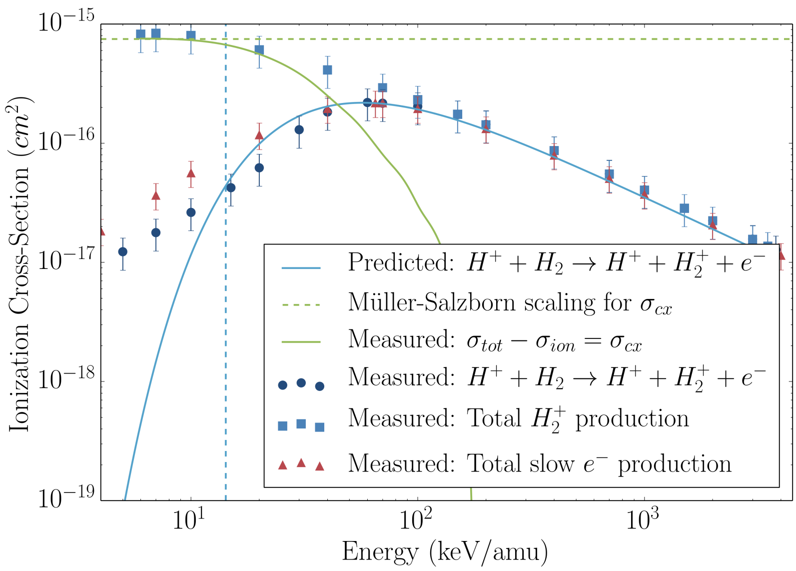

Figure 1. Relevant cross-sections for production of slow H_2^+ ions and electrons. The vertical dashed line denotes the lower limit of applicability of the prediction by Kaganovich et al. [25]. The experimental data denoted as "Measured" are from [23], while the solid green line is a fit to the difference of measured total H_2^+ ion production cross-section and measured $\sigma_{\text{ionization}}$. It can be seen that the measured cross-sections agree well with the fit down to the limit. Below, the fit underestimates the cross-section. It can also be seen, that the cross-section for the production of secondary electrons is a bit higher than the cross-section for ionization. This is due to other processes contributing as well.

Data on both processes is sparse and usually comes with large error-bars. Data for proton, helium and lithium beams in the desired range of 10 keV to 100 keV can be found for gaseous hydrogen, helium and lithium targets [21–23], because the cross-sections are interesting for fusion research. For sufficiently low projectile energies ($<$ 20 keV/amu), charge-exchange can be predicted with the Müller-Salzborn scaling law [24] and dominates over ionization in this regime (see for example figure 1). For other projectiles and targets, cross-section measurements exist at significantly higher energies. Based on classical and quantum theories, several models have been developed and compared to the existing measurements. A good summary of previous efforts was given by I.D.Kaganovich et al. in several papers between 2003 and 2005 [25–27]. He presents a scaling law that seems to fit the data very well. Unfortunately, for the energy regime of typical ECRIS low energy beam transport systems the situation is problematic, as the combination of lower extraction voltage (10-30 kV) and higher mass-to-charge ratio leads to significantly lower projectile energy in keV/amu. (cf. figure 2). If the velocity of the incident particle becomes too low, the interaction time of projectile and target electron increases enough for tunneling effects to become an important factor in the ionization process and the Kaganovich fit underestimates the cross-section (this can be seen in figure 1 for the proton case). The difficulty becomes even more clear in figure 2, where the ionization cross-sections for argon and oxygen impinging on N_2 (cf. section 6 on experimental verification) are plottet versus energy. The lower limit of applicability (as given in [26]) is marked with a dot. This is far higher than the projectile energies during the measurements (vertical lines). On the positive side, in this energy regime the Müller-Salzborn scaling law is a good ap-

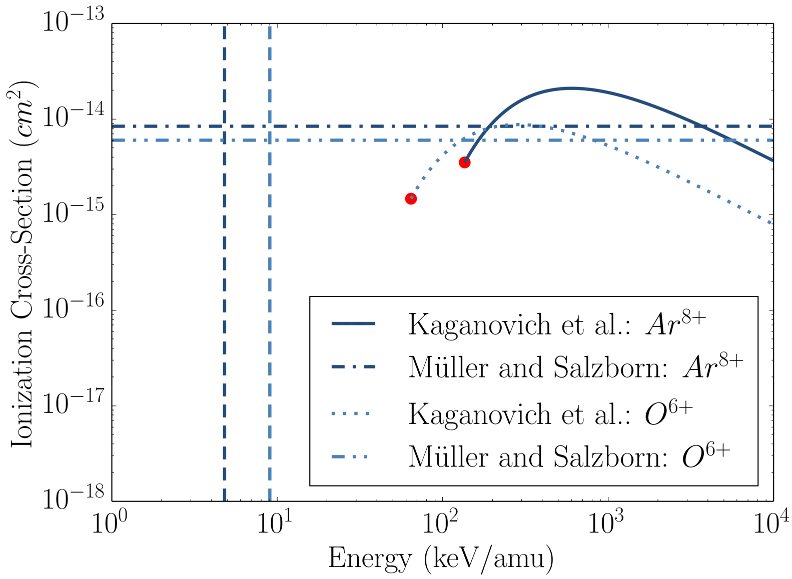

Figure 2. Prediction of ionization and charge exchange for argon and oxygen impinging on N_2. The curves are the prediction using the scaling law of Kaganovich et al. [25] with the lower cut-off denoted by a dot. The horizontal lines are the prediction using Müller-Salzborn fit [24]. The vertical lines are the projectile energies for a 4.8 keV/amu Ar^{8+} beam (left) and a 9 keV/amu O^{6+} beam (right) used during experiments reported on in section 6.

proximation for σ_{cx} (which clearly dominates over $\sigma_{\text{ionization}}$. Comparing with RFA measurements (cf. next section) and with the saturation current of the RFA used in the measurements (which directly depends on $\sigma_{\text{ionization}} + \sigma_{\text{cx}}$ [11]), the authors found the value for the ionization cross-section $\sigma_{\text{ionization}}$ at the lower limit of applicability (red dot) a reasonable first order approximation of the cross-section at the respective projectile energies in the measurements. This is, of course a very crude approximation and a better fit is desirable.

6 Experimental verification

The original formula by Gabovich et al. was compared to measurements with the LEDA injector source by Ferdinand et al. [11] and showed good agreement for a 50 -130 mA proton beam at 75 keV (space-charge compensation was $> 90\%$, so quasi neutrality was justified). The cross-sections displayed in figure 1 were used. Similar measurements were done using the same source but extracting significantly lower beam current to benchmark the extended formula [28, 29]. Good agreement was found as well. Following these encouraging results, the formalism was applied to more complex systems such as ECRIS transport systems. Measurements of ion beam space charge compensation were performed at the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University (MSU) in the low energy beam transport line of the LEDA injector source [18, 19] and the Superconducting Source for Ions (SuSI) [14, 15] using a Retarding Field Analyzer (RFA). Details about the setup and the measurements are reported elsewhere [28–30] and shall only be summarized briefly here: by using the RFA to measure the energy distribution of secondary ions from beam-residual gas interaction, the beam potential distribution can be calculated.

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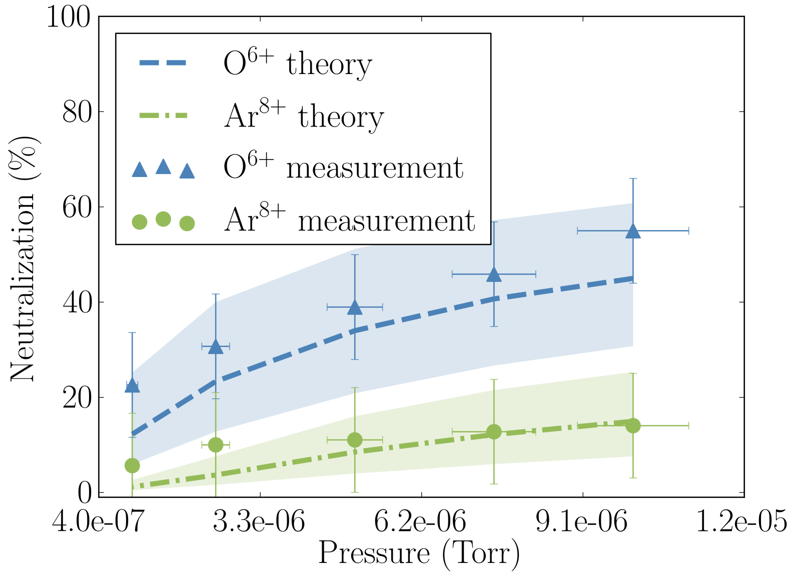

Figure 3. Neutralization factors of a 200 eμA Ar^{8+} and a 700 eμA O^{6+} beam at different vacuum pressures in the LEBT of the SuSI source at NSCL. The dashed lines indicate the model description with the shaded areas being the uncertainty due to the not well known cross sections. From [30].

Comparing the measured beam potential with the theoretical value of an uncompensated beam, f_e is obtained. The results from the LEDA injector source compared well with earlier measurements by Ferdinand et al. [11] and with the theoretical predictions. The results from the measurements in the SuSI LEBT agreed reasonably as well (an example for Ar^{8+} and O^{6+} is shown in figure 3). The rather large error bars on both measurement and theoretical prediction are due to uncertainties in beam current, residual gas pressure and ion/electron production cross-sections.

7 Incorporation into computer simulations

The good agreement between the model and the measurements presented in the previous section led to the idea of including the formalism into beam transport simulations of the SuSI LEBT using the particle-in-cell code WARP [3, 4] in 2D "XY-Slice" mode. Under the over-simplified assumption that all beam species in a multispecies beam have the same radius, generalization of equation (3.3) for multiple ion species is straight-forward. At each time-step of the simulation, an algorithm then gathers the average 2-RMS beam radius and each species' beam current, velocity, mass, and charge state. From these and additional user inputs (residual gas pressure, σ_i, σ_e), f_e is calculated using the above-mentioned multispecies generalization of the Gabovich formula to be used as scaling factor for the beam current in the subsequent time-step. First results of this technique have been presented elsewhere [28–30]. In this paper, the argon beam used during the measurements shown in figure 3 (4.5 mA current, extracted at 24 kV, with Ar^{3+} to Ar^{12+} measureably present in the beam) is chosen as an example for the simulation technique . The desired ion (Ar^{8+}) was transported to the location of the neutralization monitor, passing the following beam line elements: SuSI (ion source), EL (einzel lens at -20 kV), Dipole (90°bending magnet), C1-C5 (collimators and slits),

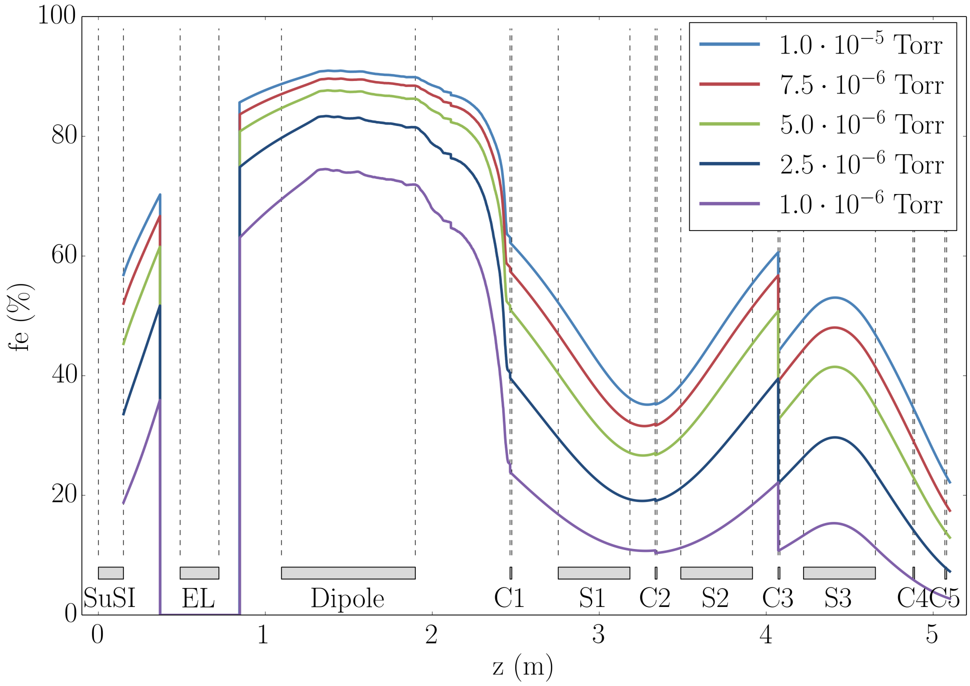

Figure 4. Space charge compensation factors along the beam line for five different beam line pressures calculated by WARP during the simulation of the SuSI LEBT. The lowest f_e corresponds to the lowest pressure and f_e increases with higher pressure. The relevant beam line elements are: SuSI (ion source), EL (einzel lens), Dipole (90°bending magnet), C1-C5 (collimators and slits), S1-3 (solenoid magnets). Note that inside the einzel lens, f_e was forced to 0 to account for loss of the compensating electrons due to the lens' electrostatic field. Abrupt drops in f_e are usually at collimator positions ("C") where the beam current can drop fast.

S1-3 (solenoid magnets). In figure 4 the space charge compensation factor along the LEBT is shown for five different pressures inside the beam line. The horizontal beam envelopes of the argon beam at 10^{-6} Torr can be seen in figure 5. Of course, all simulation settings were identical to the settings during the experiment. Overall, the results of these preliminary simulations using the extended Gabovich formula for space charge compensation are promising. The trends seen in the measurements with a retarding field analyzer at C5 are reproduced and the beam profiles at C4 compare well with slit scans performed during the experiments for argon (round profile) as well as oxygen beams (triangular profile). However, it should be noted that in order to obtain the correct beam sizes at C4, an additional reduction of f_e to 80% of the Gabovich prediction was necessary inside the dipole. It is not surprising that the formula is less accurate inside a dipole as it does not account for the magnetic field and there is a large deviation from cylindrical symmetry. This is certainly an area of development. Further improvements could include the generalization of the formula for multi-species with different radii, and incorporating realistic pressure profiles along the beam line.

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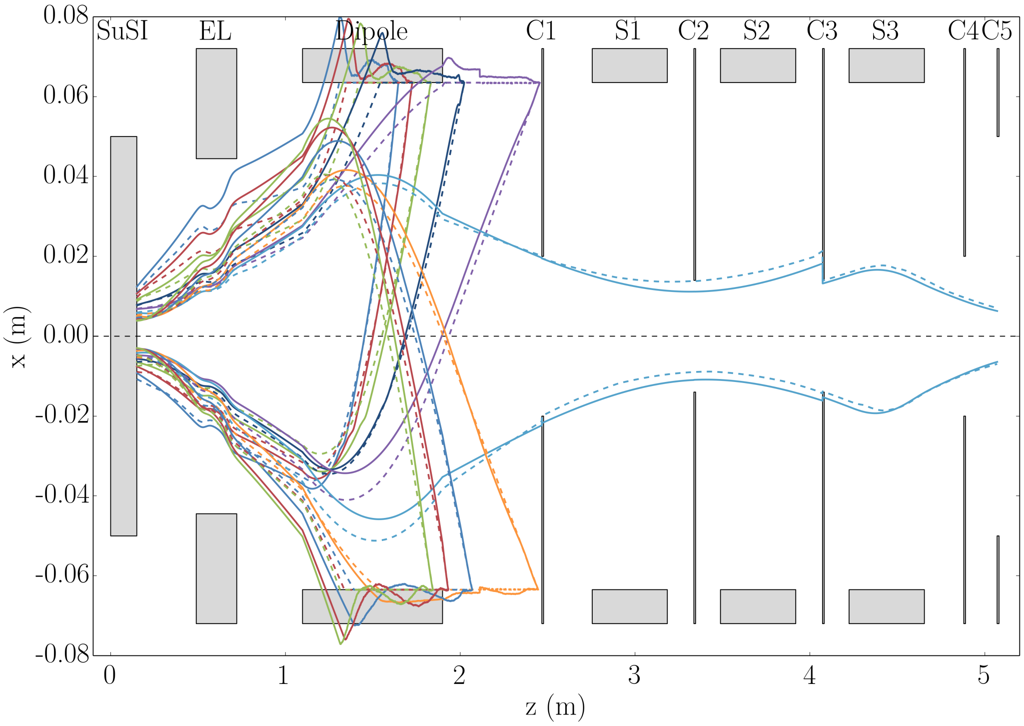

Figure 5. Horizontal beam envelopes of the SuSI LEBT simulation of an argon beam going through a beam line with base pressure of $1.0 \cdot 10^{-6}$ Torr. Ar^{8+} (light blue) is selected using the dipole magnet and a set of slits. The solid lines represent the 2-rms beam envelopes and the dashed lines the maximum beam envelopes. A large percentage of the beam current is lost at this point and the consequence is a drop in f_e which can be observed in figure 4.

8 Conclusion

In this paper we revisited and extended a formula for the prediction of the space charge compensation factor f_e in DC ion beams first presented by Gabovich et al. in 1975. The formula was re-derived in more detail than previously presented, highlighting the various assumptions and simplifications that were used. In addition, a simple generalization of the formula to non-neutral beam plasmas ($f_e \leq 90\%$) was proposed. This formula does not take into account beam instabilities and collective effects that can further decrease the space charge compensation and is strictly speaking only valid for uniform round beams. However, the extended formula was compared to measurements conducted at the NSCL (MSU) and reasonable agreement was found between theory and experiment even for non-rotationally symmetric beams. The formula relies heavily on the cross-sections for electron and ion production through beam-residual gas interaction which are not well known and impose a large uncertainty on the prediction. Even within the rather large uncertainties, an interesting observation could be made about beams in the LEBT of the ECRIS SuSI at the NSCL: the measured values of f_e were consistently below 60% at the location of the measurement and sometimes as low as 0%. These results agree with the prediction by the extended Gabovich formula. According to the formula, the space charge compensation factor depends on three variables: the beam intensity, the beam radius, and the residual gas pressure in the beam line. In the

ECRIS regime, this leads to an f_e that can vary considerably along the LEBT due to the change in beam radius and beam losses on apertures as well as with the changing pressure profile. As was shown in section 7, the Gabovich formula can be implemented in beam transport codes like WARP to include a first order approximation of the space charge compensation factor into the simulations.

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References

[1] M. Reiser, *Theory and design of charged particle beams*, 2nd ed. Weinheim: Wiley-VCH, 2008.

[2] A. Adelmann et al., *The OPAL (Object Oriented Parallel Accelerator Library) Framework*, Paul Scherrer Institut, Tech. Rep. PSI-PR-08-02, (2008–2013).

[3] D.P. Grote, A. Friedman, J.-L. Vay and I. Haber, *The WARP code: Modeling high intensity ion beams*, *AIP Conf. Proc.* **749** (2005) 55.

[4] D.P. Grote, *WARP Manual*, (2000).

[5] I. Brown, Ed., *The physics and technology of ion sources*, first edition, John Wiley and Sons Inc., New York, U.S.A. (1989).

[6] M. Gabovich, L. Katsubo and I. Soloshenko, *Selfdecompensation of a stable quasineutral ion beam due to coulomb collisions*, *Fiz. Plazmy* **1** (1975) 304.

[7] M.D. Gabovich, *Ion-beam plasma and the propagation of intense compensated ion beams*, *Sov. Phys. Usp.* **20** (1977) 134.

[8] I.A. Soloshenko, *Physics of ion beam plasma and problems of intensive ion beam transportation (invited)*, *Rev. Sci. Instrum.* **67** (1996) 1646.

[9] I.A. Soloshenko, *Space charge compensation of technological ion beams*, in proceedings of the *XVIIIth International Symposium on Discharges and Electrical Insulation in Vacuum, ISDEIV*, volume **2**, IEEE, (1998) pg. 675.

[10] I.A. Soloshenko, *Transportation of intensive ion beams*, *Rev. Sci. Instrum.* **69** (1998) 1359.

[11] R. Ferdinand, J. Sherman, R.R. Stevens Jr. and T. Zaugg, *Space-charge neutralization measurement of a 75 keV, 130 mA hydrogen-ion beam*, in proceedings of the *Particle Accelerator Conference, 1997*, volume **3**, IEEE, (1997), pg. 2723.

[12] D.S. Todd, D. Leitner and M.M. Strohmeier, *Low energy beam diagnostics at the venus ecr ion source*, in proceedings of *BIW 2008*, 2008.

[13] D. Winklehner, D. Todd, J. Benitez, M. Strohmeier, D. Grote and D. Leitner, *Comparison of extraction and beam transport simulations with emittance measurements from the ECR ion source venus*, 2010 *JINST* **5** P12001.

[14] G. Machicoane et al., *Performance investigation of the nscl 18 ghz superconducting ecr ion source susi*, in proceedings of the *2009 Particle Accelerator Conference*, Vancouver, Canada, 2009.

[15] P.A. Zavodszky et al., *Design of SuSI — Superconducting source for ions at NSCL/MSU - I. The magnet system*, *AIP Conf. Proc.* **749** (2005) 131.

[16] M.A. Leitner, C.M. Lyneis, D.C. Wutte, C.E. Taylor and S.R. Abbot, *Construction of the superconducting ECR ion source Venus*, *Phys. Scripta* **2001** (2001) 171.

[17] D. Leitner et al., *Next generation ECR ion sources: First results of the superconducting 28 GHz ECRIS–VENUS*, *Nucl. Instrum. Meth.* **B 235** (2005) 486.

[18] T. Taylor and J.F. Mouris, *An advanced high-current low-emittance dc microwave proton source*, *Nucl. Instrum. Meth.* **A 336** (1993) 1.

[19] J. Sherman et al., *Development of a 110 mA, 75 KeV proton injector for high-current, cw linacs*, in proceedings of the *XVIII Linac Conference (LINAC96)*, Geneva, 1996.

[20] R. Miracoli et al, *Characterization of the versatile ion source and possible applications as injector for future projects*, *Rev. Sci. Instrum.* **83** (2012) 02A305.

[21] C. Barnett et al., *Atomic data for controlled fusion research*, Tech. Rep., Oak Ridge National Lab., TN U.S.A. (1977).

[22] R. Ito, T. Tabata, T. Shirai and R.A. Phaneuf, *Analytic cross sections for collisions of H, H2, He and Li atoms and ions colliding with atoms and molecules.III*, http://www-jt60.naka.jaea.go.jp/english/JEAMDL/code/95008.html, (1995).

[23] C. Barnett, H. Hunter, M. Kirkpatrick, I. Alvarez and R. Phaneuf, *Atomic data for fusion. Collisions of H, H2, He and Li atoms and ions with atoms and molecules*, http://www-cfadc.phy.ornl.gov/redbooks/redbooks.html, (1990).

[24] A. Müller and E. Salzborn, *Scaling of cross sections for multiple electron transfer to highly charged ions colliding with atoms and molecules*, *Phys. Lett.* **A 62** (1977) 391.

[25] I.D. Kaganovich, E.A. Startsev and R.C. Davidson, *Scaling cross sections for ion-atom impact ionization*, *Phys. Plasmas* **11** (2004) 1229.

[26] I.D. Kaganovich, E.A. Startsev and R.C. Davidson, *Scaling of cross sections for ion-atom impact ionization*, Technical Report TRN: US0303977.

[27] I.D. Kaganovich, E.A. Startsev and R.C. Davidson, *Comparison of quantum-mechanical and classical trajectory calculations of cross sections for ion-atom impact ionization of negative and positive ions for heavy-ion fusion applications*, *Phys. Rev.* **A 68** (2003) 022707.

[28] D. Winklehner, *Ion beam extraction from electron cyclotron resonance ion sources and the subsequent low energy beam transport*, Ph.D. Thesis, Michigan State University, 2013.

[29] D. Winklehner, D. Leitner, G. Machicoane, D. Cole and L. Tobos, *Space charge compensation measurements in the injector beam lines of the nscl coupled cyclotron facility*, in proceedings of the *20th International Conference on Cyclotrons and their Applications (CYC13)*, 2013.

[30] D. Winklehner, D. Leitner, D. Cole, G. Machicoane and L. Tobos, *Space-charge compensation measurements in electron cyclotron resonance ion source low energy beam transport lines with a retarding field analyzer*, *Rev. Sci. Instrum.* **85** (2014) 02A739.