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# EFFECTS ON SEA LEVEL DUE TO CHANGES IN THE EARTH'S ROTATION

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# Effects on Sea Level due to Changes in the Earth's Rotation*

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Abstract. The decrease in the earth's angular velocity, $\omega$, over the last 100 million years has had an effect on sea level. Comparison of the ellipticity of the earth, as calculated from artificial-satellite observations, and the ellipticity of a hydrostatic model gives upper limits on the changes in sea level. These limits are 60 meters at the poles and -30 meters at the equator.

Introduction. Observations by Eardley [1964] suggest that sea level has risen in the polar regions and fallen near the equator. The change, which has occurred during the last 100 million years, amounts to about 180 meters near the equator and probably more than 180 meters near the poles. Eardley advanced the hypothesis that this variation was caused by a change in $\omega$, the earth's rate of rotation.

We first show, using simple models, that the changes in sea level would have been insignificant if the entire earth were in hydrostatic equilibrium (h. e.) during the last 100 million years. Hence any significant change in sea level requires a deviation from h. e.

Knowledge of the present deviation of the earth from h. e. [see Caputo, 1965] allows us to calculate the maximum possible effect of a change in $\omega$ on sea level. As the effect is not large enough to explain Eardley's data, we must rule out his hypothesis.

An earth in h. e. Let the earth, apart from the oceans, be represented by a homogeneous fluid, A, with density $\rho$. Let the oceans be represented by a thin layer of another homogeneous fluid, B, covering fluid A, and with density $\rho$. (This is a very simple model.) The entire system is rotating with angular velocity $\omega$. Jeffreys [1959] has shown that the surface of such a system contains no harmonics other than that representing the ellipticity. The outside surface of fluid B is then given by

$$r = a(1 + \epsilon S_2)$$

where $S_2 = \frac{1}{3} - \sin^2\phi$, $\phi$ is the latitude, $\epsilon$ is the ellipticity, and $a$ is the mean radius to first order. Inside the system the total potential, which consists of the gravitational potential plus the centrifugal potential, is

$$\psi = \frac{4}{3}\pi G\rho a^3\left(\frac{3a^2 - r^2}{2a^3} + \frac{3}{5}\frac{r^2\epsilon}{a^3} S_2\right) + \frac{1}{3}\omega^2 r^2 + \frac{1}{2}\omega^2 r^2 S_2$$

where $G$ is the universal gravitational constant. It follows now that all surfaces of constant potential are of the form

$$r = a' (1 + \epsilon S_2)$$

where $a'$ is the mean radius of the surface. In particular the above is the equation for the surface of fluid A. It also follows that

$$\epsilon = \frac{5}{4}\frac{\omega^2}{(4/3)\pi G\rho}.$$

At the equator the thickness of fluid B, which is the depth of the ocean, is

$$\lambda = (a - a')(1 + \frac{\epsilon}{3}).$$

The change in $\lambda$ resulting from a change in $\omega$ is given by

$$\frac{d\lambda}{d\omega} = \frac{1}{3}(a - a')\frac{d\epsilon}{d\omega} = \frac{1}{3}(a - a')(2\epsilon)\frac{1}{\omega}.$$

We have assumed incompressibility, which requires the mean radii $a$ and $a'$ to remain constant. Then the change in sea level over a given time interval $\Delta T$ is given by

$$\Delta\lambda = \frac{1}{3}(a - a')(2\epsilon)\frac{(d\omega/dT)}{\omega}\Delta T$$

where $T$ is the time.

Taking $a - a'$, the depth of the ocean where $\sin^2\phi = \frac{1}{3}$, as 5 km; $\epsilon$ as 1/300; $\Delta T$ as 100 million years and $(d\omega/dT)/\omega$ as $-2\times 10^{-10}$/year [Munk and MacDonald, 1960], we have

$$\Delta\lambda = 0.2 \text{ meter.}$$

Hence the change in sea level over the last 100 million years would have been insignificant if the earth were in h. e. during that time. We find that allowing the densities of the fluids A and B to differ does not alter this statement.

Compressibility. We may now consider the actual earth. In order to account for the compressibility of the earth we may view the effect of a change in $\omega$ in the following way: The process of change is a shrinking of the earth toward its polar axis, followed by the changes associated with an incompressible earth. The shrinking corresponds to a small change in scale in the differences between absolute radii and is calculated in the next paragraph. Hence the equations for an incompressible earth can be applied.

The change in the height of the ocean's surface above the solid earth due to compressibility is given by the compression of the ocean's water itself due to the change in pressure at the surface of the earth. This change in height, at the equator, is given by d:

$$d = \frac{\rho\omega^2 a h^2}{k} \frac{\Delta\omega}{\omega}$$

where h is the depth of the ocean and k is the bulk modulus of water. With an average depth of 5 km, we obtain

$$d = 5 \text{ cm.}$$

As compression has its maximum effect on the equatorial radius, we have shown that compressibility is negligible.

Deviation from h. e. The earth is not in h. e. Recent satellite observations have provided accurate values for the ellipticity of the earth, independent of any assumption about the internal structure of the earth. These ellipticity values can be compared to the ellipticity of the hydrostatic model whose values of $\omega^2 a^3/GM$ and $p$ most closely approximate those observed. (Here $M$ is the mass of the earth, $p = J_2^0/H$, $J_2^0$ is the second term in the gravity potential, and $H$ is the precessional constant.) Values of $\epsilon^{-1}$ are given in Table 1.

We see that a discrepancy of about 0.5% exists between the two ellipticities. Thus the ellipticity of the solid earth differs by about 0.5% from its ellipticity were it in h. e., and certainly it differs at most by 0.5% from the ellipticity of the ocean's surface. This difference would allow a change in the sea level to have taken place at the equator in the amount

$$\Delta\lambda_{\text{equator}} = \frac{1}{3} a \Delta\epsilon = -30 \text{ meters} \qquad (1)$$

and at the poles

$$\Delta\lambda_{\text{poles}} = 60 \text{ meters} . \qquad (2)$$

We know of no known reason to suppose that 100 million years ago the ellipticity of the earth was less than the h. e. ellipticity for that time. A decrease in $\omega$ would result in an earth's ellipticity greater than the value for h. e. and would create stresses tending to force the earth to a smaller ellipticity, namely the h. e. value. The mechanism whereby the earth responds to stresses of this sort is little understood. If, however, we accept the assumption expressed by the first sentence of this paragraph, then (1) and (2) present valid upper limits on the rise of sea level at the poles and the fall of sea level at the equator. That is, sea level could not have fallen by more than 30 meters at the equator, although it could have risen an unknown amount.

In any case we see that a change in $\omega$ cannot cause a change in sea level large enough to explain Eardley's data.

One proviso must be added to the interpretation of ellipticities calculated from satellite data. The satellite instruments do not measure the ellipticity of the solid earth only; they measure the ellipticity of the whole earth. Hence the solid earth is slightly farther from h. e. than a satellite measurement might indicate, due to the effect of the oceans. We find, however, that such an effect would not be large enough to bring consistency with Eardley's data.

Conclusion. If the earth were in h. e. during the last 100 million years, the change in sea level due to a change in $\omega$ would have been insignificant. Calculations based on artificial-satellite observations have determined the possible deviation from h. e., which allows us to calculate the maximum possible change in sea level over the last 100 million years. The result is a maximum rise in sea level of 60 meters at the poles and a maximum fall of 30 meters at the equator. Eardley [1964] has observed that the fall at the equator is about 180 meters. A change in $\omega$, therefore, cannot explain his data.

Table 1. Values of $\epsilon^{-1}$.

Hydrostatic equilibrium assumed

| $\epsilon^{-1}$ | Reference |
|---|---|
| 300.0 | Henriksen [1960] |
| 299.7 | Jeffreys [1964] |
| 299.5 | Caputo [1965] (Method 4, Model 2) |

From the external gravity field

| $\epsilon^{-1}$ | Reference |
|---|---|
| 298.3‡ | Kozai [1961] |
| 298.3‡ | King-Hele et al. [1963] |
| 298.3 | Kaula [1963] |

‡ These authors provided values of $J_2^0$. Their determinations were the same as Kaula's within the accuracy we require.

Footnotes and References

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