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DECK MODEL FOR TRIPLE-REGGE COUPLINGS

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## II. THE MODEL

We are interested in calculating the double-differential cross section  $(d^2\sigma)/(ds'd|t|)$  for the inclusive experiment

$$a + b \rightarrow c + X,$$

where

$$s' = (p_a + p_b - p_c)^2,$$

$$t = (p_a - p_c)^2;$$

we also define

$$s = (p_a + p_b)^2.$$

The inclusive cross section may be obtained by summing over the exclusive ones. For the latter, we use a multiperipheral model of the type discussed in Ref. 3 and schematically represented in Fig. 1. Since we are interested in the region where  $s/s' \gg 1$  and  $t/s \ll 1$ , we allow the scattering represented by the leftmost blob of Fig. 1 to take place at high energies as well as low.<sup>4</sup>

As explained in Refs. 3 and 5 multiperipheral models yield a very simple result for the one particle inclusive cross section

$$\frac{d^2\sigma}{ds'd|t|} = \frac{\sum_i}{2\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \int ds_3 dp_c dk d^4k \delta(k^2 - s_3)$$

$$\times 2\lambda^{\frac{1}{2}}(s_3, \mu_\pi^2, m_b^2) \sigma_{\pi_i^b}^{\text{TOTAL}}(s_3) \frac{|A_{\pi_i^a \rightarrow \pi_i^c}(s_1, t)|^2}{(\mu_\pi^2 - u)^2}$$

$$\times \delta(t' - t) \delta(s' - s'') \delta^4(p_a + p_b - p_c - k - K) \quad (1)$$

$\sum_i$  indicates a sum over the three charge states of the pion,

$$dp_c = \frac{d^3p_c}{2(p_c^2 + m_c^2)^{\frac{1}{2}}} \frac{1}{(2\pi)^3} \quad \text{and similarly for } dk$$

$$s_1 = (p_c + k)^2$$

$$t' = (p_a - p_c)^2$$

$$u = (p_a - p_c - k)^2$$

$$s'' = (p_a + p_b - p_c)^2$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$

Formula (1) is represented schematically in Fig. 2. Depending on the quantum numbers of particles  $a$ ,  $b$ , and  $c$ , it may also be necessary to add contributions like the ones shown in Fig. 3. In our calculation of  $p + p \rightarrow p + X$  (see Sec. III), such terms have to be included.

The reader will recognize in Fig. 2 an extended Deck model.<sup>7</sup>

In previous applications of such a model the total cross section on the right was restricted to a given resonance (e.g.,  $\Delta$  or  $\rho$ ), in our case we use the total cross section for any energy.

The detailed structure of the multiperipheral chain is never used. The only feature required is the capacity for generating a realistic total cross section. The details could be much more complicated than the simple model of Ref. 3.

If the limit  $s' \rightarrow \infty$ ,  $s/s' \rightarrow \infty$  is taken, one obtains,<sup>10</sup> assuming that high-energy scattering is well described by Regge poles:

$$\frac{d^2\sigma}{ds'd|t|} = \frac{1}{16\pi s^2} \sum_{i,j,k} \beta_{aci}(t) \beta_{acj}(t) \zeta_i(t) \zeta_j^*(t)$$

$$\chi \left( \frac{s}{s'} \right)^{\alpha_i(t)+\alpha_j(t)} s'^{\alpha_k(0)} g_{ij,k}(t) \beta_{bbk}(0) \quad (2)$$

where

$$g_{ij,k}(t) = \frac{3}{16\pi^3} \int_{-\infty}^0 \frac{du}{(\mu_\pi^2 - u)^2} \left[ \lambda^{\frac{1}{2}}(\mu_\pi^2, u, t) \right]^{\alpha_i(t)+\alpha_j(t)}$$

$$\chi \beta_{\pi\pi i}(\mu_\pi^2, u, t) \beta_{\pi\pi j}(\mu_\pi^2, u, t) \beta_{\pi\pi k}(u, u, 0) \cdot \int_0^{(u/t)^{\frac{1}{2}} e^{-q}} dx x^{\alpha_k(0)}$$

$$\chi P_{\alpha_i(t)+\alpha_j(t)}(z); \quad (3)$$

$$z = \frac{\text{ch } q - (t/u)^{\frac{1}{2}} x}{\text{sh } q} \quad \text{ch } q = \frac{\mu_\pi^2 - u - t}{2(ut)^{\frac{1}{2}}};$$

$\beta$  denotes a factorized Regge residue,  $\zeta$  a signature factor. In formula (3), the fact that one of the pion masses is not  $\mu_\pi^2$  has been exhibited explicitly. We will have more to say about this later.

The normalization of the residues is such that the contributions of a pole  $i$  to the  $(a,b)$  total cross section and to the elastic differential cross section are:

$$\sigma_{i,ab}^{\text{Total}}(s) = \frac{1}{\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \beta_{aai}(0) \beta_{bbi}(0) s^{\alpha_i(0)}$$

$$\frac{d\sigma_{i,ab}(s,t)}{dt} = \frac{1}{16\pi\lambda(s, m_a^2, m_b^2)} |\beta_{aai}(t) \beta_{bbi}(t) \zeta_i(t)|^2.$$

The factor 3 in formula (3) reflects the three charge states of the pion. It is correct if the trajectories  $i$  and  $j$  have the quantum numbers of the vacuum.<sup>16</sup>

In our calculation of  $p + p \rightarrow p + X$  by means of formula (1), we have used the experimentally observed  $(\pi, p)$  total cross sections together with isospin invariance. For the elastic amplitude  $A_{\pi N \rightarrow \pi N}$ , we have adopted the following simplifying prescription,

$A_{\pi N \rightarrow \pi N}(s, t) = A_{\pi N \rightarrow \pi N}(s, 0) e^{\gamma t}$ . This form is fairly accurate for small  $t$ , even in the resonance region, when  $\gamma$  is taken to be  $\approx 4(\text{GeV}/c)^{-2}$ . For  $A_{\pi N \rightarrow \pi N}(s, 0)$  we used the tabulation of Ref. 17.

Off-shell corrections to the  $\pi N$  elastic amplitude and  $\pi N$  total cross section have been introduced to provide the necessary cut-off in the integrals over the virtual pion mass. We have used a form factor of the type suggested by the ABFST integral equation:<sup>3</sup>

$$A_{\pi N}(s, t, u) = A_{\pi N}(s, t, u = \mu_\pi^2) \left[ \frac{u_0}{u_0 - \frac{1}{2}(\mu_\pi^2 + u - \frac{t}{2})} \right]^{(\bar{\alpha}+1)}$$

$$\sigma_{\pi N}(s, u) = \sigma_{\pi N}(s, u = \mu_\pi^2) \left[ \frac{u_0}{u_0 - u} \right]^{\bar{\alpha}+1}.$$

The ABFST model suggests that the asymptotic behavior of the residue  $\beta_i$  of the pole  $i$  is  $(-u)^{\alpha_i+1}$  as  $-u \rightarrow \infty$ . Since, for low energies, we do not use a Regge parametrization of  $\sigma_{\text{Total}}$  or of  $A_{\pi N \rightarrow \pi N}$ , we cannot incorporate this result in a simple manner. Motivated by the perturbative approach of Refs. (4) and (10) we have taken an average intercept  $\bar{\alpha} = 0.7$ .

The form of the off-shell corrections given above has been shown to provide an adequate fit to numerical solutions of the ABFST equation even in the low-virtual-mass region if  $u_0 \approx 1 \text{ GeV}^2$ .<sup>15</sup> We have experimented with other cutoff procedures, such as "reggeizing" the pion, and found that the results do not change significantly. Thus our model contains almost no free parameters.

### III. RESULTS AT INTERMEDIATE AND HIGH ENERGIES

Figure 4 shows a comparison of the predictions of the model for the process  $pp \rightarrow pX$  with the experimental results of Anderson et al.<sup>6</sup> The following aspects of the experimental data are satisfactorily reproduced:

(i) Energy dependence: one of the striking features of the data of Ref. 6 is a rapid decrease of the average cross section with increasing beam momentum. This effect, indicative of the weakness of diffractive excitation in this energy range, is well reproduced by the model.

(ii) Missing mass dependence: Given that we cannot expect to reproduce the full resonance structure, we consider our results satisfactory. The calculations yield bumps in the missing mass at the appropriate locations<sup>11</sup> but with insufficient strength.

(iii) Absolute normalization: This point is sensitive to the cutoff procedure adopted for the integration over the momentum transfer  $u$  in formulas (1) and (2). This is the problem of off mass-shell corrections mentioned in Sec. II. It may be seen from Fig. 4 that our normalization is better for high missing masses, where resonances are not expected to be important.

Another interesting result from our calculations is the importance, at intermediate energies and small  $t$ , of pion exchange, in the sense of Fig. 3b. For high missing masses, this mechanism accounts for about 50% of the cross section at  $p_{\text{lab}} = 20 \text{ GeV}/c$  and  $t \approx -0.04 (\text{GeV}/c)^2$ . The importance of diagram 3b has also been recognized recently by other authors.<sup>8,9</sup>





$$\begin{aligned} \xi_{ij,k}^{(0)} &\propto \frac{1}{16\pi^3} \frac{3}{\alpha_k^{(0)} + 1} \int_{-\infty}^0 du (-u)^{\alpha_i^{(0)} + \alpha_j^{(0)} - 1} \\ &\times \beta_{\pi\pi i}^{(0,0,0)} \beta_{\pi\pi j}^{(0,0,0)} \beta_{\pi\pi k}^{(0,0,0)} \left( \frac{u_0}{u_0 - \frac{u}{2}} \right)^{\alpha_i^{(0)} + \alpha_j^{(0)} + 2} \\ &\times \left( \frac{u_0}{u_0 - u} \right)^{\alpha_k^{(0)} + 1}. \quad (4') \end{aligned}$$

In (4') we have exhibited the explicit form of the off-shell corrections.

The influence both of the trajectory intercept and of the magnitude of the couplings of the trajectories to the  $\pi\pi$  system is exhibited in (4) and (4'). Note that we use the appropriate values for the trajectory intercepts in the form factors. Since we have singled out a specific set of trajectories we do not have to use an average intercept as we did in Sec. II.

The effect of the trajectory intercept can be traced back to kinematical limitations on the minimum momentum transfer  $u$  between the two blobs of Fig. 3 when  $s_1$  and  $s_3$  are large.

The reader may have noticed that the only secondary trajectory included in Table I is the  $P'$ . As already discussed in footnote 16, the  $\rho$  and  $A_2$  trajectories are not important for the  $p + p \rightarrow p + X$  and  $\pi p \rightarrow X + p$  experiments in the kinematical regions that we have been considering. However, the absence of the  $\omega$  coupling is more serious. So long as we restrict ourselves to  $\pi$  exchange, the  $\omega$  decouples. This circumstance can be interpreted either as an interesting consequence of the  $\pi$ -exchange model or as a disturbing

violation of the cherished, and empirically well supported, notion of exchange degeneracy. Note that, if degeneracy is assumed, the extraction of  $\xi_{PP,P}$  from experimental fits becomes easier because nondiagonal terms of the form  $(PP',k)$  are approximately cancelled by terms with  $P'$  replaced by  $\omega$ .

V. CONCLUSIONS

The existence of the pion pole in connected parts together with the pole factorization theorem implies that the triple-Pomeranchon vertex cannot vanish.<sup>20</sup> The question is then whether the contribution to the cross section from the region of phase space dominated by the pion pole is a significant fraction of the total. We believe that the results of Sec. III show that the pion pole has something to do with the observed cross sections. We have not attempted to fit the data but rather to show that the gross features could be reproduced with a very simple model. Section IV, however, shows that the triple-Pomeranchon vertex is so small as to be almost unobservable except at extremely high energies.

The dimensionless parameter<sup>10</sup>  $\eta_{PPP}$  that determines the displacement of  $\alpha_p(0)$  from 1 is, assuming  $\alpha'_p = 0.5 \text{ (GeV/c)}^{-2}$

$$\eta_{PPP} = \frac{g_{PP,P}^2(0)}{16\pi 2\alpha'_p(0)} \approx 5 \times 10^{-4}.^{12}$$

ACKNOWLEDGMENTS

I am grateful to Mordechai Bishari, Chun Fai Chan, Geoffrey Chew, and Martin Einhorn for many valuable discussions. Professor Chew suggested this investigation and provided indispensable guidance and advice.

## FOOTNOTES AND REFERENCES

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  2. For a survey of the early literature see R. D. Peccei and A. Pignotti, Phys. Rev. Letters 26, 1076 (1971).
  3. D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1972); L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1972).
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  7. R. T. Deck, Phys. Rev. Letters 13, 169 (1964).
  8. Successful calculations of inclusive cross sections from a multi-peripheral model have also been reported recently by F. Duimio and G. Marchesini, Phys. Letters 37B, 427 (1971).
  9. M. Bishari, Phys. Letters 38B, 510 (1972).
  10. H. D. I. Abarbanel et al., Phys. Rev. Letters 26, 675 (1971) and Princeton University report (April 1971) to be published in Ann. Phys. (New York).
  11. Given the success of Deck models in describing diffractively produced resonances, this is not very surprising. For more details on this subject see Ref. 13.

12. The authors of Ref. 10 obtained a much bigger value because
  - (i) They assumed exchange of the full pseudoscalar meson octet instead of just the  $\pi$  triplet. This gives in  $\eta_{PPP}$  an extra factor of  $(8/3)^2 \sim 7$ ; (ii) They were working in a perturbative scheme in which  $\alpha_p(0) \approx 0.7$ . According to the discussion of Sec. IV this will make  $\eta_{PPP}$  larger; (iii) The  $\pi\pi$  asymptotic total cross section then used was about a factor of 2 larger than what is expected from factorization. Given that  $\eta_{PPP} \propto (\sigma_{\pi\pi}^{\text{TOTAL}})^3$  an additional factor  $\approx 8$  is introduced.
13. E. L. Berger, Phys. Rev. 179, 1567 (1969) and references therein.
14. V. D. Barger and D. B. Cline, Phenomenological Theories of High Energy Scattering (W. A. Benjamin Inc., New York, 1969).
15. C. F. Chan and B. R. Webber, Solution of the ABFSST Equation with a Resonance Kernel, Lawrence Berkeley Laboratory Report LBL-341 September 1971, to be published in Phys. Rev. D. I am indebted to Chun Fai Chan for valuable discussions of this question.
16. We have in mind  $p + p \rightarrow p + X$  experiments. The  $\rho$  and  $A_2$  trajectories are known empirically to couple weakly to the  $\bar{N}N$  system at small  $t$ . The  $\omega$  does not couple to  $\pi\pi$  due to G-parity conservation.
17. G. Höhler and R. Strauss, Tables of Pion-Nucleon Forward Amplitudes, University of Karlsruhe Report, October 1971.
18. Y. M. Antipov et al., Missing-mass Spectra Observed in  $\pi^- p \rightarrow p + X^-$  at 25 and 40 GeV/c, February 1972 (submitted to Phys. Letters).
19. Since we were interested in the averaging properties of the triple-Regge expansion for low values of the missing mass, we

have used the variable  $v = s' - m_b^2 - t$  instead of  $s'$  in formula (2). This is consistent with what is usually done when extrapolating asymptotic expansions for two-body reactions to low values of the energy. When  $s' \gg m_b + t$  the variables coincide. I am indebted to Stephen Ellis and Tony Sanda for suggesting the use of this variable.

20. G. F. Chew, private communication, Lawrence Berkeley Laboratory, Summer 1971.

Table I. Triple-Regge vertices.<sup>a</sup>

$t$ (GeV/c) <sup>2</sup>	PP,P	PP',P	P'P',P	PP,P'	PP',P'	P'P',P'
0	0.142	0.299	0.745	0.236	0.488	1.21
-0.05	0.135	0.293	0.734	0.244	0.523	1.31
-0.10	0.127	0.285	0.734	0.247	0.547	1.40
-0.15	0.118	0.275	0.733	0.246	0.562	1.48
-0.20	0.109	0.263	0.731	0.240	0.569	1.55
-0.25	0.099	0.251	0.726	0.231	0.567	1.61
-0.30	0.091	0.237	0.719	0.219	0.559	1.65
-0.35	0.082	0.223	0.709	0.206	0.546	1.68

<sup>a</sup> Values of the triple-Regge vertices as a function of  $t$ , assuming that  $\beta_P(t)$  varies as  $e^{1.5t}$  and that  $\beta_{P'}(t)$  is a constant. The magnitude of the coupling of the  $P$  to the  $\pi\pi$  system has been obtained via factorization assuming that the asymptotic  $\pi N$  and  $\pi\pi$  cross sections are 23 mb and 38 mb respectively. The  $P'$  was assumed to couple to the  $\pi\pi$  system with the same strength as the  $P$ . This is empirically well supported (see for example, Ref. 14). The couplings given above are to be used with an expansion like Eq. (2). The scale parameter in the form factor has been taken to be  $u_0 = 1 \text{ GeV}^2$ .

0 0 0 0 3 8 0 1 9 9 6

## FIGURE CAPTIONS

- Fig. 1. Schematic representation of the exclusive matrix element.
- Fig. 2. Schematic representation of formula (1). If the detected particle is a pion other diagrams having a pion coming not from the leftmost blob may have to be considered. The contribution of these other terms is not important very close to the kinematical boundary.
- Fig. 3. Other terms that may have to be included depending on the quantum numbers of particles  $a$ ,  $b$ , and  $c$ .
- Fig. 4. Comparison of our results with the data of Anderson et al. The diamonds ( $\diamond$ ) correspond to the experimental values, the smooth curve is our calculation and the broken line is what a triple-Regge expansion with the couplings of Table I predicts. In 4 (a-d) we have plotted the same quantity as have the authors of Ref. 6 for "fixed"  $t$  and varying energy (see the relevant footnotes in Ref. 6).
- Fig. 5. Comparison of our results with the  $t$  dependence of the data of Ref. 6 at three different values of the missing mass: (a),  $(s')^{\frac{1}{2}} = 1.4$  GeV; (b),  $(s')^{\frac{1}{2}} = 1.7$  GeV; (c),  $(s')^{\frac{1}{2}} = 1.9$  GeV. The normalization of our results has been adjusted to coincide with the data at the lowest value of  $t$ .
- Fig. 6. Expectations at higher energies. We plot  $\ln[s'(d^2\sigma)/(ds'dt)]$  versus  $\ln s'$  at  $t = -0.04$  (GeV/c)<sup>2</sup>. The triple-Pomeron component, if sufficiently strong, would be evident as an almost flat, energy independent, section of the curve. The flat broken line at the bottom of the graph is the contribution of the triple Pomeron. The

curvature at small  $s'$  is due to the use of the variable  $v = s' - m_b^2 - t$  instead of  $s'$ . See footnote 19. Continuous lines are the result of calculating with the complete model; broken lines correspond to the triple-Regge expansions. The reaction is  $p + p \rightarrow p + X$ .

- Fig. 7. The experimental results of the CERN-IHEP boson missing-mass spectrometer (continuous curves) compared with the predictions of a triple-Regge expansion with the vertices given in Table I. We have taken  $t = -0.25$  (GeV/c)<sup>2</sup>. The experimental  $|t|$  varies between  $0.17$  (GeV/c)<sup>2</sup> and  $0.35$  (GeV/c)<sup>2</sup>.

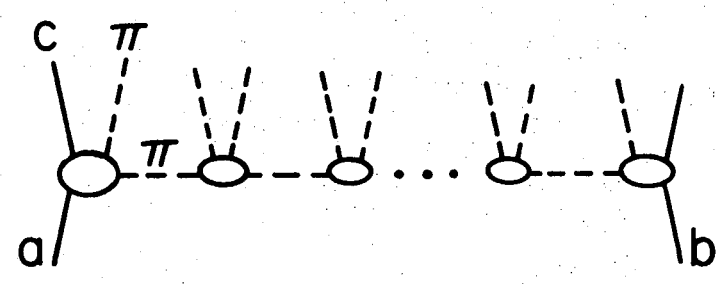


Fig. 1

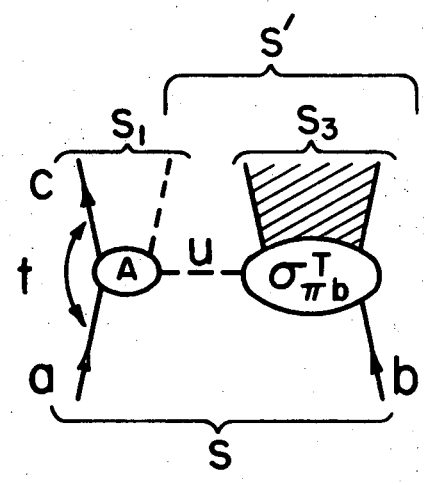


Fig. 2

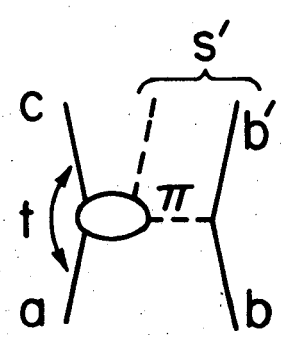


Fig. 3a

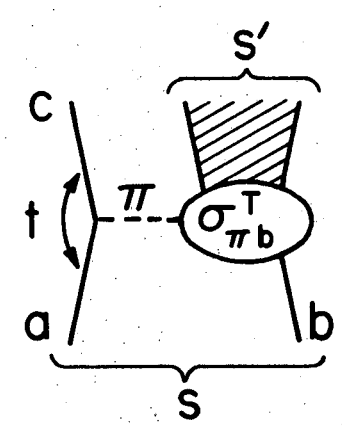
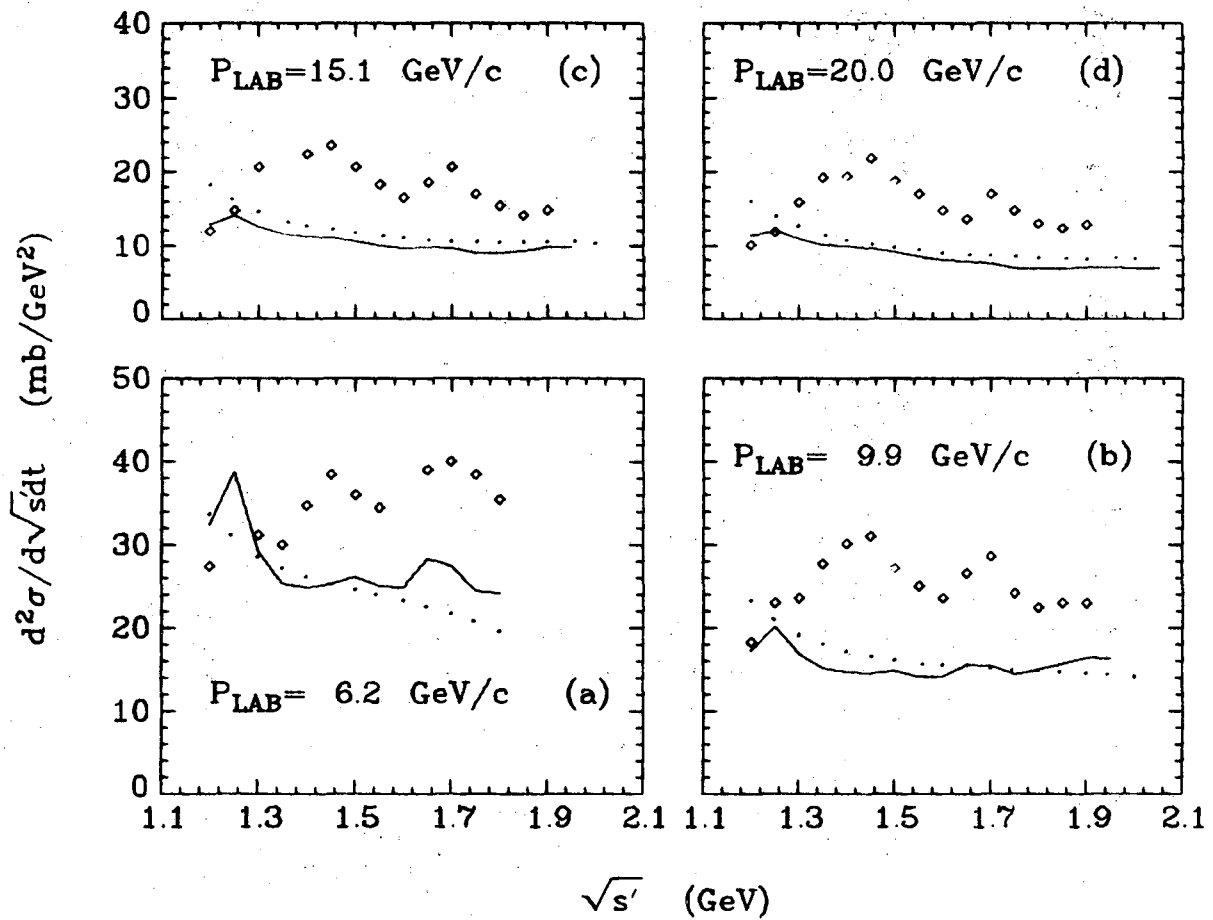
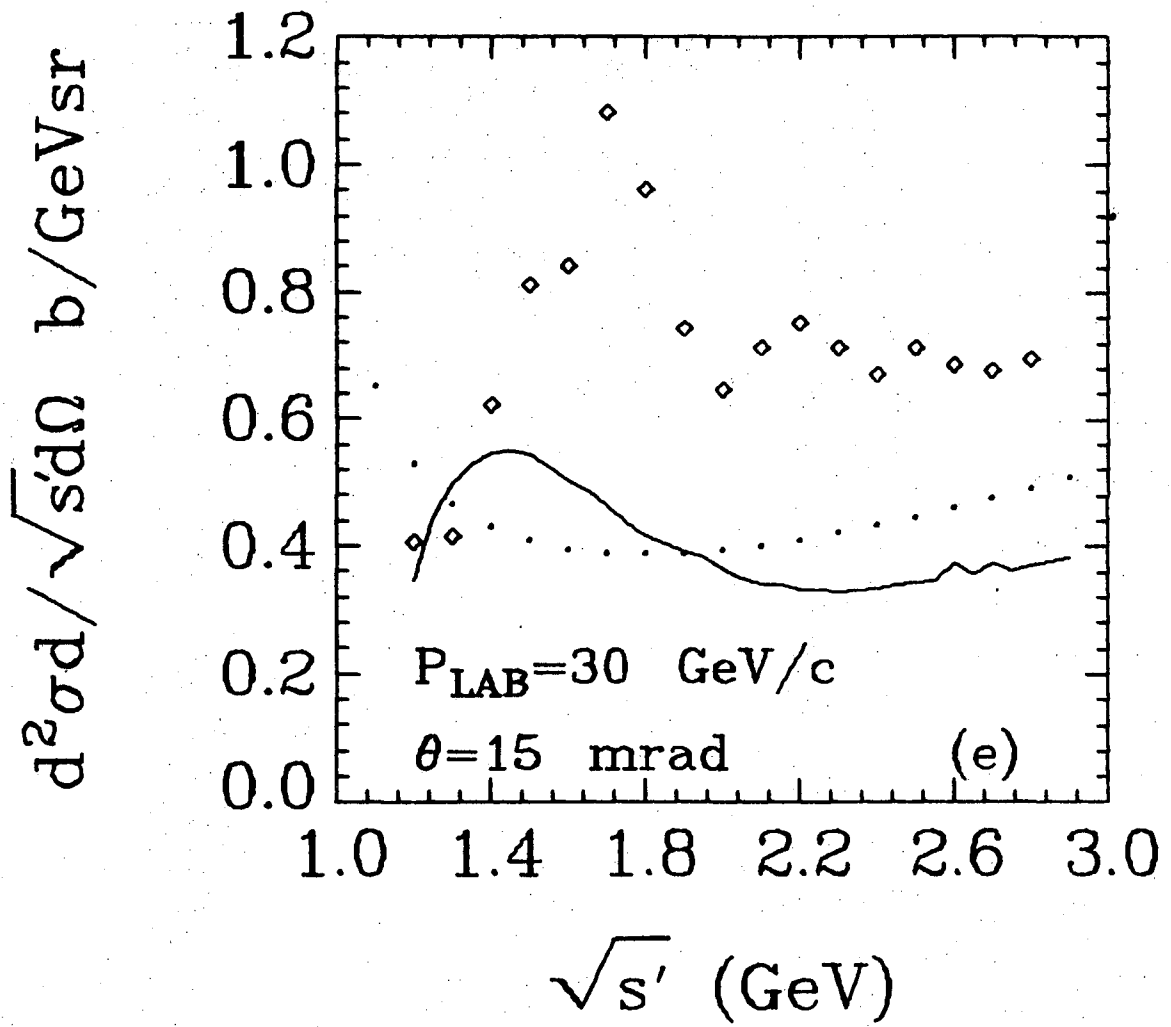


Fig. 3b



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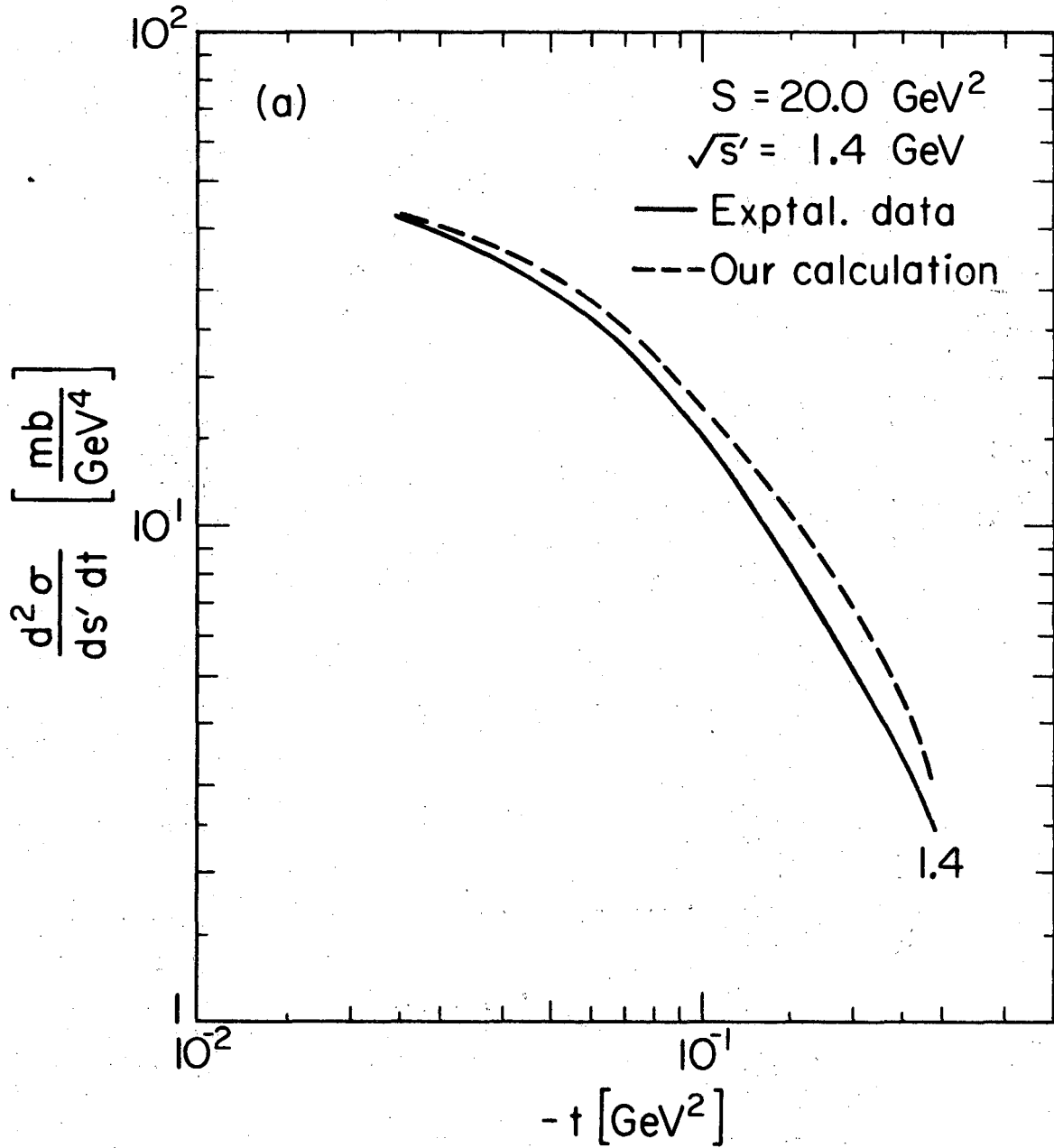
Fig. 4a - d



XBL 725-138

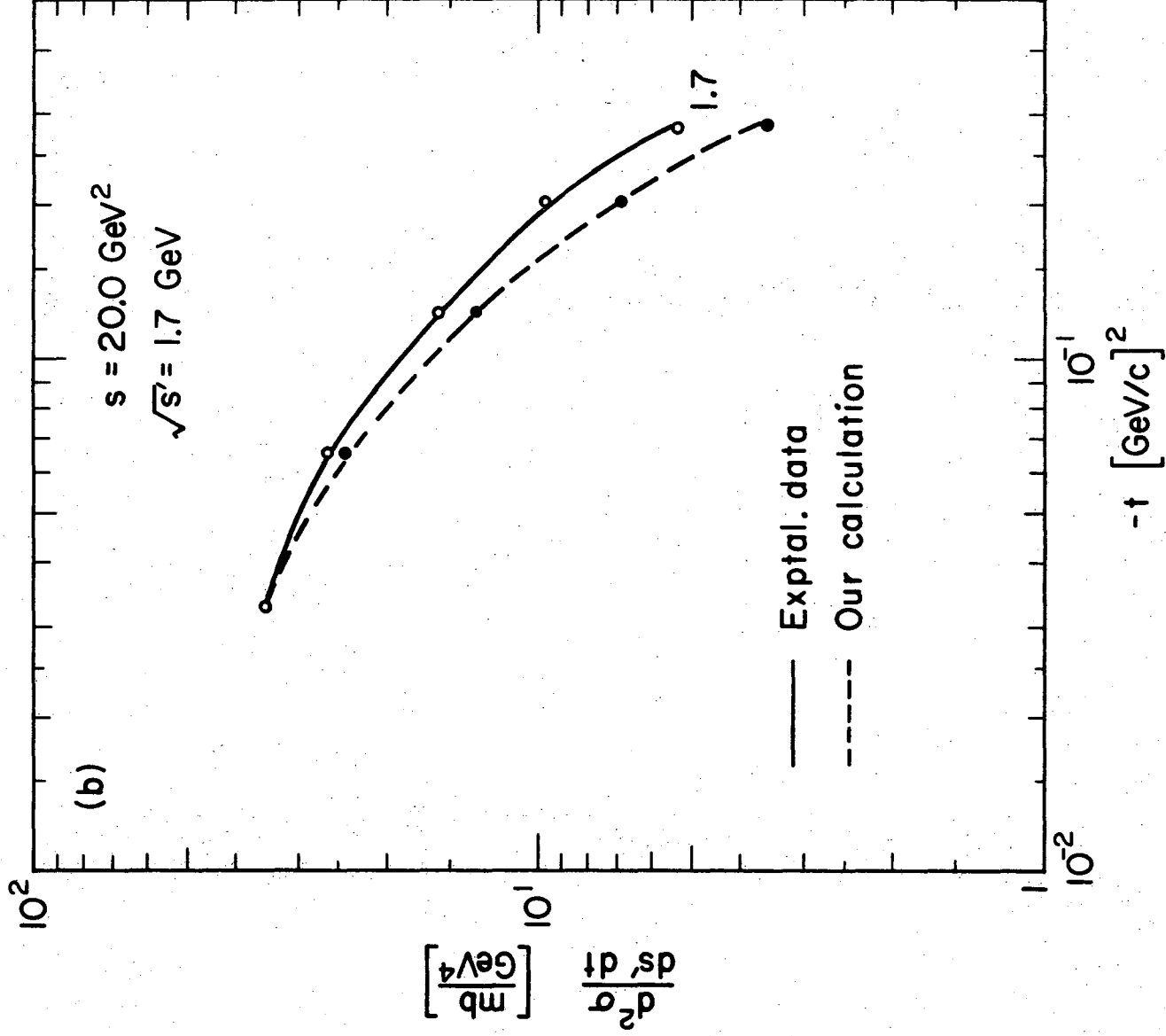
Fig. 4e





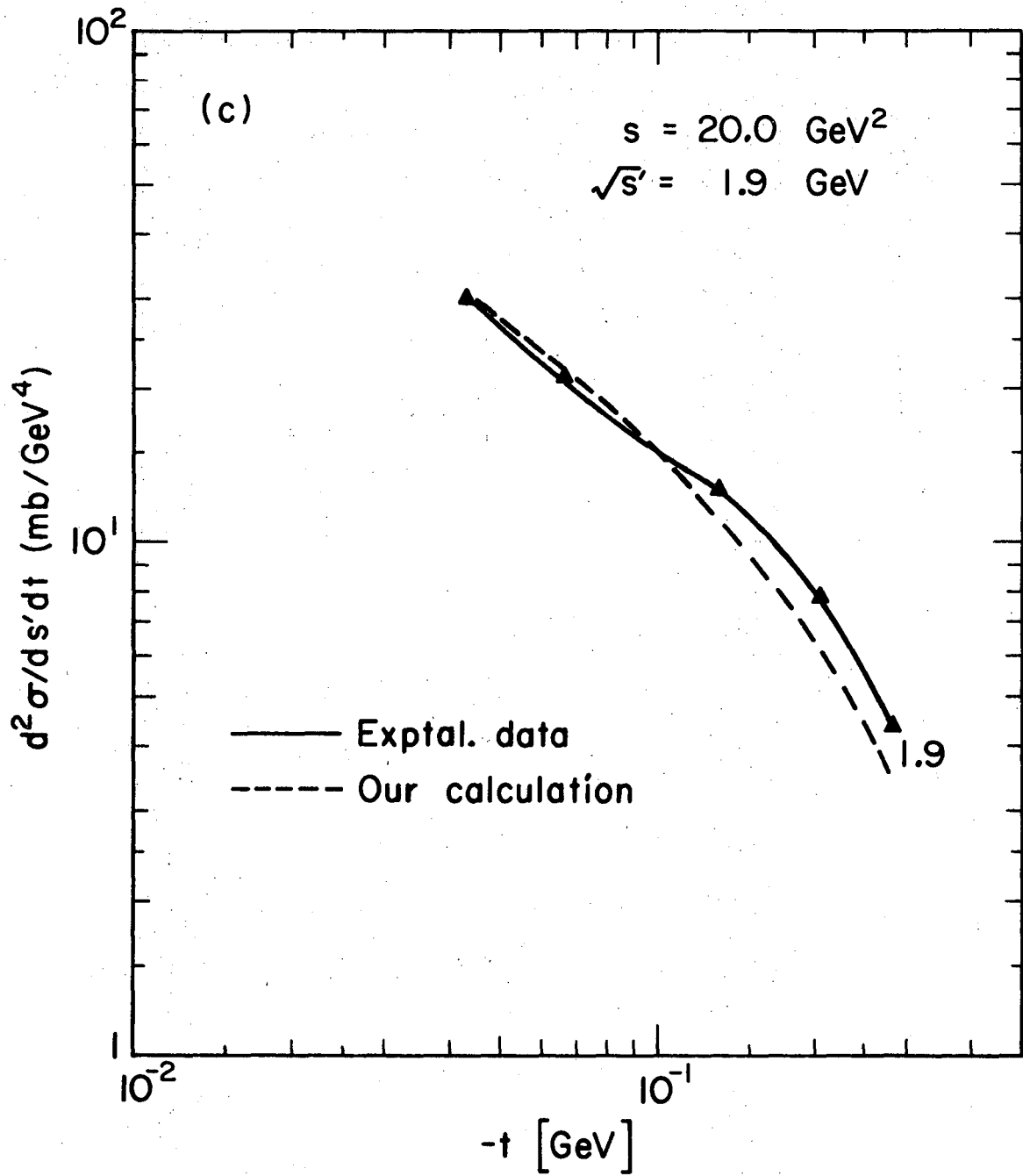
XBL724-2840

Fig. 5a



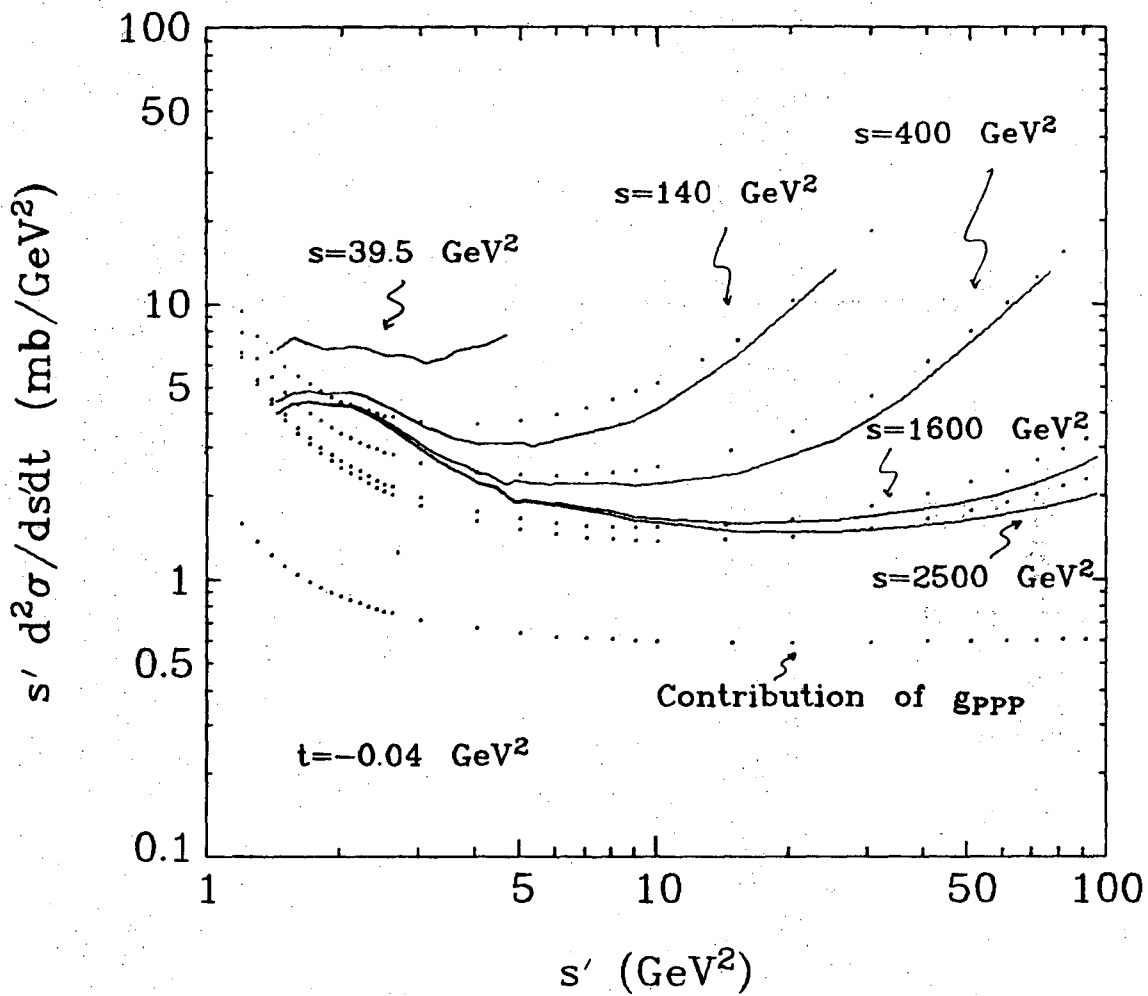
XBL 724-2841

Fig. 5b



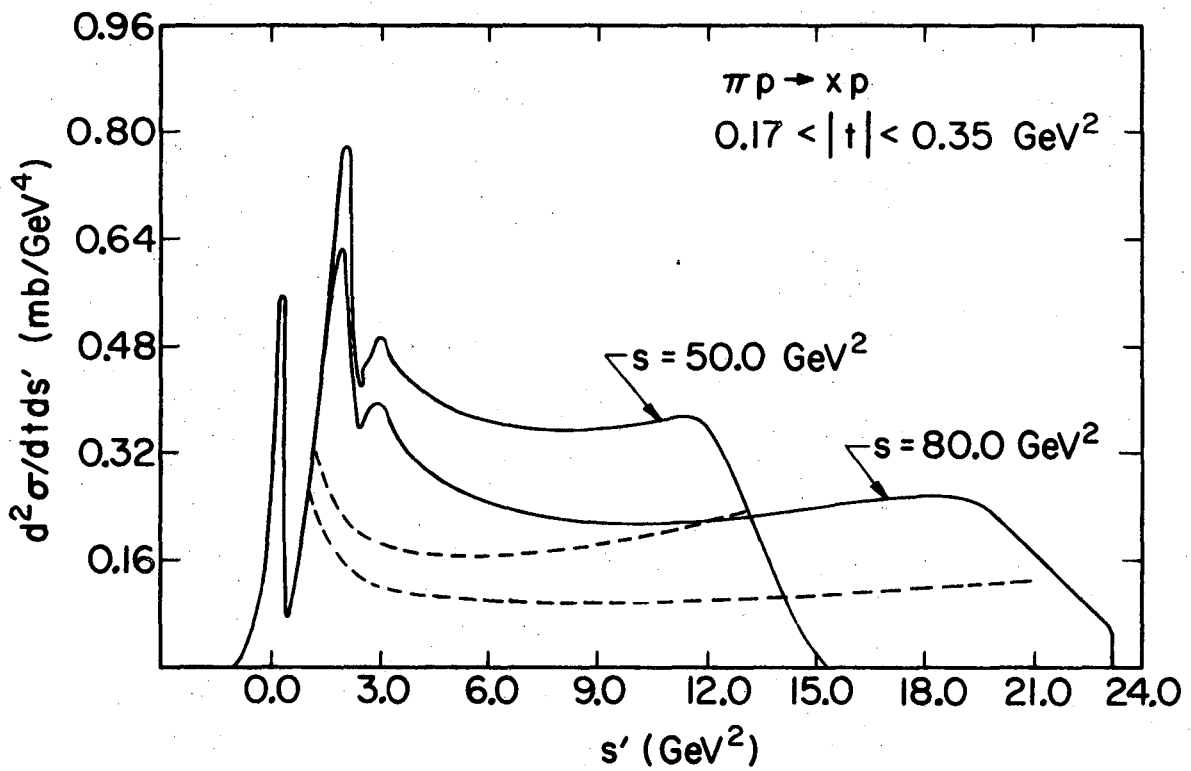
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Fig. 5c



XBL 725-139

Fig. 6



XBL724-2837

Fig. 7

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