Title

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Coherent structures in stably stratified plane Couette flow

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Abstract

A large body of recent work in wall-bounded, *unstratified* shear flow has been dedicated to finding Exact Coherent Structures (ECS) (e.g. see the review Kawahara et al. (2012)). These are fully non-linear, invariably unstable, exact solutions which help understand how turbulence arises. The growing consensus is that the existence of these solutions is a necessary (but not sufficient) condition for turbulent dynamics to be possible in parameter space. Plane Couette flow - a fluid sheared between two parallel, differentially moving plates - is an exemplar of this in that the flow is linearly stable for all Reynolds numbers Re yet turbulence can occur at a finite Re beyond that where ECS disconnected from the basic sheared state start to exist. In this presentation we add extra physics in the form of stable stratification (gravity perpendicular to the plates) to plane Couette flow to explore this hypothesis further and to investigate transition in a stably stratified shear flow.

1 Introduction

Mathematically, adding stratification introduces two further control parameters to the plane Couette flow problem, the bulk Richardson number Ri_b and the Prandtl number Pr. Fixing Pr (here to 1), means that the boundary between laminar flow and turbulent transition is no longer essentially a point on the Reynolds number line but now a *line* in the 2D Ri-Re plane which plausibly extends to infinite Re for some $Ri_b = Ri_b(Re)$ bounded away from 0. Using a small domain for our computations (2π long and π wide where the plate separation is 2), we characterise this laminar-turbulent boundary at least for low Re and identify important ECS via edge tracking which appear to organise the transition process. The latter is demonstrated by using a new approach based on nonlinear optimal growth (Kerswell et al., 2014). This technique has recently been used to explore how the 'minimal seed' - the initial condition of minimum energy that can reach the turbulent state - is modified by stratification (Eaves and Caulfield, 2015). Here, we show how the approach can more generally used beyond the laminar-turbulent boundary to illustrate bursting events associated with the presence of ECS.

The Boussinesq approximation, $\rho = \rho_0 + \Delta\rho$, where $\Delta\rho << \rho_0$, is employed to obtain the nondimensional equations of the stratified plane Couette flow (pCf). Using the following scales,

$$\mathbf{u} = U\mathbf{u}^*, \qquad t = \frac{h}{U}t^*, \qquad y = h\,y^*, \qquad \rho = \rho_0 \,+\, \rho^*\Delta\rho, \qquad p = \rho_0 U^2 p^* - \rho_0\, g\, \hat{\mathbf{y}},$$

where $\mathbf{u} = (u, v, w)$ is the velocity field, $2U$ is the velocity differential across the plates (separated by a distance $2h$, see figure 1), κ is the thermal diffusivity, ρ_0 is the reference density, p is the pressure and ν is the kinematic viscosity. The nondimensional governing equations (after dropping the *) are,

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p - Ri_b\,\rho\,\hat{\mathbf{y}} + \frac{1}{Re}\nabla^2\mathbf{u}, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u}\cdot\nabla\rho = \frac{1}{Re\,Pr}\nabla^2\rho, \tag{3}$$

where:

$$Ri_b := \frac{\Delta\rho\, g\, h}{\rho_0\, U^2}, \qquad Re := \frac{U\,h}{\nu}, \qquad Pr := \frac{\nu}{\kappa}. \tag{4}$$

Stable stratification is maintained by imposing a constant upper wall density which is smaller than that at the bottom-wall, see figure 1.

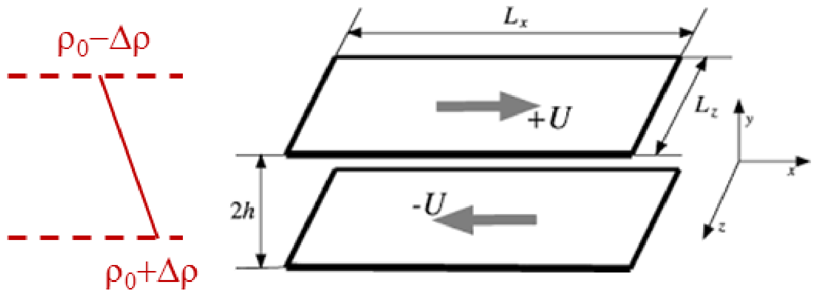

Figure 1: Plane Couette flow configuration with stable strafication imposed at upper and lower walls.

The imposed boundary conditions are

$$u(x,\pm1,z,t)) = \pm1 \quad \& \quad \rho(x,\pm1,z,t) = \mp1, \tag{5}$$

and periodic boundary conditions over $(x,z)\in[0,L_x]\times[0,L_z]$.

The steady 1D solution of this system, (1) - (3), is given by,

$$\mathbf{u} = y\,\hat{\mathbf{x}} \quad \& \quad \rho = -y. \tag{6}$$

We work with (possibly large) disturbances away from this basic state

$$\mathbf{u} = y\,\hat{\mathbf{x}} + \mathbf{u}'(x,y,z,t), \qquad \rho = y + \rho'(x,y,z,t). \tag{7}$$

2 Stratified ECS and Continuations in Ri_b

Figure 2 shows the (Re, Ri_b) parameter space for stratified pCf in a geometry $[L_x, L_z] = [2\pi, 2\pi]$ for $Pr = 1$. When $Ri_b < 0$ (unstable stratification), the system corresponds to

Rayleigh-Benard convection with shear, where the Rayleigh number is $Ra = -Ri_b\, Re^2\, Pr$. For this case, it has been proven that the linear stability of the rolls aligned to the flow are unaffected by the shear (Kelly, 1994).

When $Ri_b > 0$, stable stratification suppresses vertical motion, hence, a larger Re is needed in order to trigger and sustain turbulence. This gives rise to the turbulent-laminar boundary shown in figure 2. We arbitrarily considered an event as turbulent when turbulence is sustained for a time 1000 h/U.

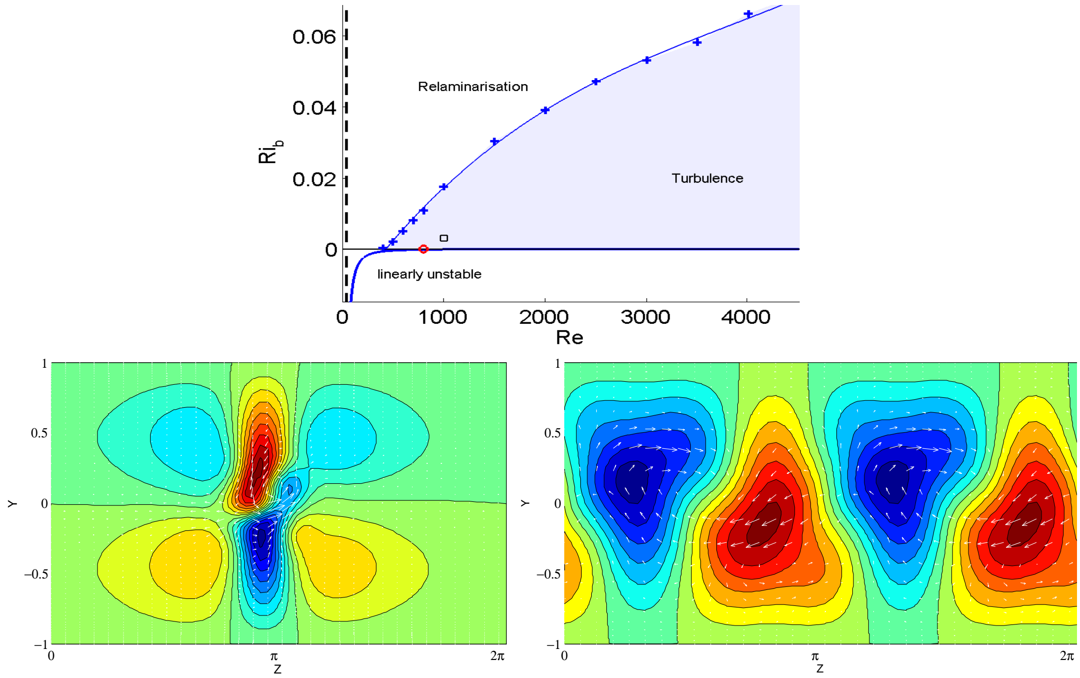

Figure 2: Top, (Re, Ri_b) plane for stratified pCf and turbulent-laminar boundary for long turbulent events (1000 h/U) in a domain size $[L_x, L_z] = [2\pi, 2\pi]$. Bottom , contours of yz cross-sections of streamwise component of perturbation velocity $\mathbf{u}'$ (white arrows indicate velocity field of yz plane). Bottom left stratified ECS, found as edge state by edge tracking at $Ri_b = 0.003$ and $Re = 1000$ (square in (Re, Ri_b) plane). Bottom right is the known unstratified solution found (Schneider et al., 2008) at $Ri_b = 0$ and $Re = 800$ (circle in (Re, Ri_b) plane).

The yz cross-sections in figure 2 show the streamwise component of perturbation velocity $\mathbf{u}'$ of two ECS found in a domain size $[L_x, L_z] = [2\pi, 2\pi]$ using the edge tracking algorithm (Itano and Toh, 2001, Skufca et al., 2006). These ECS are edge states computed in stratified and unstratified conditions at $Re = 1000$ and $Re = 800$, respectively. When performing branch continuation to move these ECS to higher stratification, both solutions reach a maximum Ri_b of $O(10^{-3})$, which is still far below the turbulent-laminar boundary which reaches $Ri_b \approx 1.8 \times 10^{-2}$ at $Re = 1000$.

When the width of the domain is reduced to $L_z = \pi$ we found a steady ECS as an edge state that can be tracked to higher Ri_b values, at least for relatively low Re. Figure 3 shows that for $Re = 300$, this ECS found at $Ri_b = 5.0 \times 10^{-3}$, hereafter *ECS1*, reaches its turning point at $Ri_b = 4.7 \times 10^{-2}$, whereas, the edge state originally found at $Ri_b = 0$ (Schneider et al., 2008), has its turning point at $Ri_b = 9.7 \times 10^{-3}$. Continuation to negative Ri_b is also computed and we observe that both solutions connect to 2D rolls solutions from Rayleigh-Benard convection.

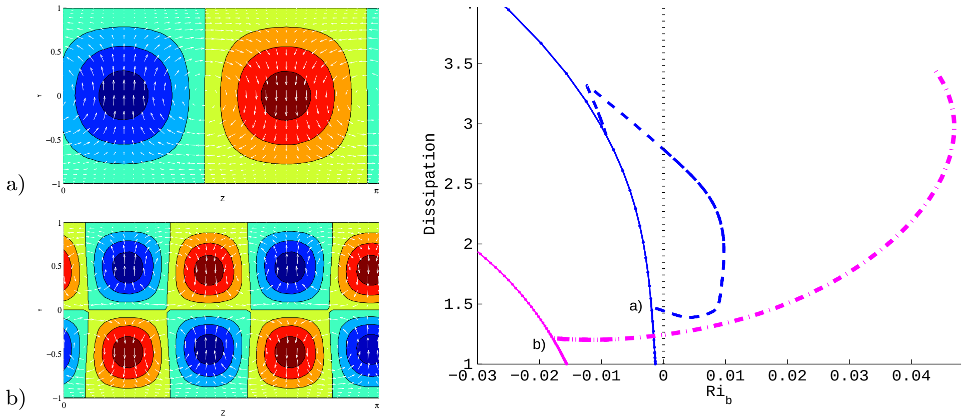

Figure 3: Continuations in Richardson number Ri_b at fixed $Re = 300$, measuring dissipation as $D = \frac{1}{V}\int_V |\nabla \times \mathbf{u}|^2 dV$, in the small domain $[L_x, L_z] = [2\pi, \pi]$. Dashed line, continuation in Ri_b of a known solution found at $Ri_b = 0$ (Schneider et al., 2008), turning point at $Ri_b = 9.7 \times 10^{-3}$. Dot-dashed line, stratified solution originally found by edge tracking at $Ri_b = 5.0 \times 10^{-3}$ (*ECS1*), turning point at $Ri_b = 4.7 \times 10^{-2}$. Continuation to negative Ri_b is also performed, both solutions connect to Rayleigh-Benard 2D solutions showed on left as contours of yz cross-sections of streamwise component of perturbation velocity $\mathbf{u}'$ (white arrows, velocity field in plane).

The structure of *ECS1* is shown as yz cross-sections in figure 4 for $Re = 10^3$ and $Re = 10^4$. As Re increases, *ECS1* becomes localised in the spanwise direction and localised at the centre of the the channel. Because *ECS1* is a steady state it has a flat critical layer exactly at the centre of the channel $y = 0$. It seems that this characteristic allows this solution to 'survive' in strongly stratified conditions.

Through continuation in Ri_b, turning points of *ECS1* for several Re are tracked and plotted in figure 4 as red circles to delimit the region in parameter space in which *ECS1* exists. The maximum at $Ri_b \approx 0.06$ for $Re = 500$, is far above where we can expect any turbulent flow. This raises the question of what happens when ECS exist but there is no turbulence. We explore that situation in the next section.

3 Optimal perturbations and bursting

The nonlinear optimal growth procedure (Kerswell et al., 2014) is designed to generate the initial state of maximum growth for an initial condition with specified energy. This technique has been implemented for finding 'minimal seeds', which are initial conditions of minimum energy that can reach turbulence. Recently, the effects of stable stratification on 'minimal seeds' have been studied by Eaves and Caulfield (2015). We have applied the same principle in strongly stratified conditions to explore the influence of ECS.

Figure 5 shows time evolutions computed at $Re = 400$ in weakly ($Ri_b = 1.0 \times 10^{-6}$) and strongly ($Ri_b = 0.04$) stratified conditions, in a domain size $[L_x, L_z] = [2\pi, \pi]$. Remarkably, even at $Ri_b = 0.04$ it is possible to generate a single 'turbulent' burst (black dot-dashed line) of similar level of energy as in the weakly stratified case (red bold solid line). By bisecting the initial energy between a relaminarisation and a bursting event, we can find the critical initial energy that evolves into a state that approaches the laminar-bursting boundary (blue solid line). This state can be converged to an ECS, in this case, *ECS1* (thin dotted line).

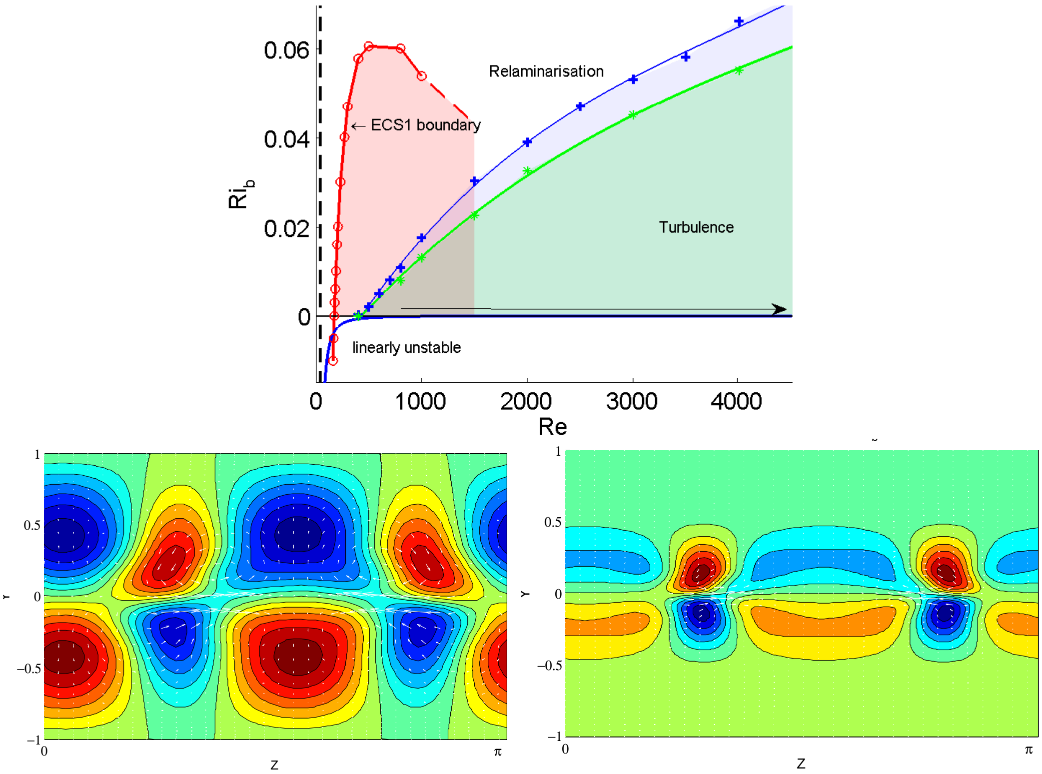

Figure 4: Top, (Re, Ri_b) plane for stratified pCf and turbulent-laminar boundary for long turbulent events (1000 h/U) in a domain size $[L_x, L_z] = [2\pi, 2\pi]$ (blue crosses) and $[L_x, L_z] = [2\pi, \pi]$ (green stars). Red circles are the turning points for several Re, which mark the boundary of the region in which *ECS1* exists. Bottom, yz cross-sections of streamwise component of perturbation velocity $\mathbf{u}'$ of *ECS1* (white arrows, velocity field in plane). Bottom left, $Re = 10^3$ at $Ri_b = 3.0 \times 10^{-4}$ bottom right, $Re = 10^4$ at $Ri_b = 3.0 \times 10^{-4}$.

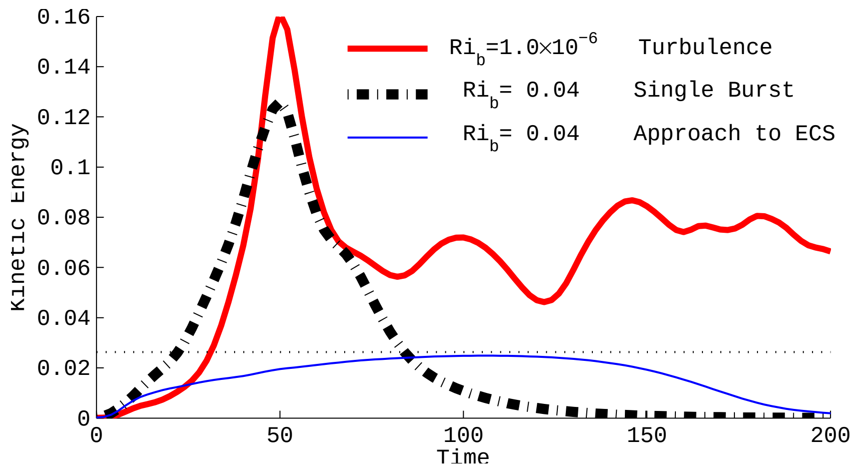

Figure 5: Time evolution of initial conditions computed in weakly ($Ri_b = 1.0 \times 10^{-6}$) and strongly ($Ri_b = 0.04$) stratified conditions, for $Re = 400$ in domain size $[L_x, L_z] = [2\pi, \pi]$. The nonlinear optimal method is able to find the initial conditions of minimum energy that trigger turbulence. When turbulence is suppressed by strong stratification, this method can also find the initial conditions of maximum growth. The black dot-dashed line shows a single 'turbulent' burst that has a similar level of kinetic energy as the turbulent event. By bisecting the initial energy of this burst, it is possible to find an state (blue solid line) that approaches the laminar-bursting boundary. This can be converged to an ECS, in this case, *ECS1* (marked as the thin dotted line).

The parameter space in which critical initial energies for triggering a 'turbulent' bursts are possible to find, correspond to the area in which *ECS1* exists (area delimited by circles in figure 4, above that region perturbations decay monotonically), at least for relatively low Re. This observation suggests that ECS still have influence on this strongly stratified region by producing bursting events.

4 Conclusions

We explored the turbulent region in the parameter space of Re and Ri_b of stratified plane Couette flow. By edge tracking and branch continuation in Ri_b, we found a stratified ECS in a small computational domain $[L_x, L_z] = [2\pi, \pi]$ which is able to exist in strongly stratified conditions (*ECS1*). We investigated the behaviour and symmetric properties of such solution.

We have successfully applied the nonlinear optimal technique for generating states that can be converged to stratified ECS. By doing so, we found that the region of strong stratification in which *ECS1* exists, also corresponds to the region in which 'turbulent' bursting can be found. This indicates that ECS still have influence on this area of parameter space where sustained turbulence doesn't exist and supports the idea that ECS are precursors of turbulence.

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