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Essays on Unconventional Monetary Policies

DISSERTATION

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by

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DEDICATION

To my parents, brother, and friends.

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ABSTRACT OF THE DISSERTATION

Essays on Unconventional Monetary Policies

By

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Doctor of Philosophy in Economics

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The three chapters in this dissertation analyze the unconventional monetary policy tools that were utilized in response to the global financial crisis of 2007-2009. Chapter 1 examines the degree of misspecification in a mainstream DSGE model with unconventional monetary policy using the DSGE-VAR approach. The findings indicate that this type of model exhibits a high level of misspecification. For instance, estimation results point to the data favoring an unrestricted vector autoregression model over a DSGE model with unconventional monetary policy. Thus, policymakers should exercise caution when using new macroeconomic models that incorporate unconventional monetary policy.

Chapter 2 examines the link between expectations formation and the effectiveness of central bank forward guidance. In a standard New Keynesian model, agents form expectations about future macroeconomic variables via either the standard rational expectations hypothesis or a more plausible theory of expectations formation called adaptive learning. The results show that the efficacy of forward guidance depends on the manner in which agents form their expectations. During an economic crisis (e.g. a recession), for example, the assumption of rational expectations overstates the effects of forward guidance relative to adaptive learning. Specifically, the output gap is higher under rational expectations than adaptive learning.

Thus, if monetary policy is based on a model with rational expectations, which is the standard assumption in the macroeconomic literature, the results of forward guidance could be potentially misleading.

Chapter 3 investigates the effectiveness of forward guidance while relaxing two standard macroeconomic assumptions: rational expectations and frictionless financial markets. A standard DSGE model is extended to include the financial accelerator mechanism. The results show that the addition of financial frictions amplifies the differences between rational expectations and adaptive learning to forward guidance. During a period of economic crisis (e.g. a recession), output under rational expectations displays more favorable responses to forward guidance than under adaptive learning. These differences are exacerbated when compared to a similar analysis without financial frictions. Thus, monetary policymakers should consider the way in which expectations and credit market frictions are modeled when examining the effects of forward guidance.

Chapter 1

Rearming Monetary Policy: A DSGE-VAR Evaluation of Recent Central Bank Action

1.1 Introduction

Since the Great Recession of 2007-2009, the conventional monetary policy tool of changing short-term interest rates has been exhausted. For instance, the federal funds rate has approached its zero percent lower bound. In response, central banks around the world initiated unfamiliar monetary policy. According to Chen, Cúrdia, and Ferrero (2012), the European Central Bank purchased about €60 billion in Euro area covered bonds in 2009. Chen, Cúrdia, and Ferrero (2012) describe the Bank of England creating an asset purchase program totaling £275 billion, or $445 billion. D'Amico and King (2010) state the Federal Reserve purchased $1.7 trillion in financial assets, such as mortgage-backed securities and debt of government-sponsored entities. The US central bank's policy served to liquify financial markets since changing the federal funds rate was no longer an option. In addition, macroeconomists incorporated these unconventional monetary actions in models to try and understand its effects on the economy. For instance, the recent and comprehensive model by Gertler and Karadi (2011), which will henceforth be known as GK (2011), created a New Keynesian model that allowed the central bank to intervene in financial markets. These types of new monetary policy models provide intuitive elements from the recent recession, but are relatively young in age. Thus, policymakers would like to know the reliability of the new models.

The goals and contributions of this paper are twofold. The first objective is to estimate the GK (2011) model using US data and Bayesian techniques. The GK (2011) model is used because of its comprehensive framework and is a popular model that incorporates unconventional asset purchases by the central bank. The second and significant contribution of this current paper evaluates the GK (2011) model using the Dynamic Stochastic General Equilibrium-Vector Autoregression (DSGE-VAR) technique presented by Del Negro and Schorfheide (2004) to examine the degree of model misspecification.

The model presented in this paper describes a New Keynesian DSGE model with a financial sector, which is taken from GK (2011). There are households who maximize expected discounted utility as in Smets and Wouters (2003, 2007). However, a part of each household are bankers who facilitate lending between households and intermediate goods producers, while the remaining household members are workers. The production side of the model includes intermediate goods producers who create products for retail firms. However, the intermediate goods producers buy capital and pay for it by borrowing funds from the financial sector. The central bank follows a monetary policy rule, which depends on the previous period's nominal interest rate, current inflation, output, and a monetary policy shock. The central bank also intervenes in the financial sector by supplying funds to non-financial firms.

The DSGE-VAR approach combines two types of frameworks in order to evaluate the DSGE model. This paper uses information from a DSGE model as *a priori* knowledge for a vector autoregression (VAR) estimation. According to Del Negro, Schorfheide, Smets, and Wouters (2007) this process produces a parameter, λ, which governs the degree of information from the DSGE model used in VAR estimation. For instance, if λ is small, the DSGE model provides little information for VAR estimation, and thus, this result suggests DSGE model misspecification. Other estimation results (e.g. impulse responses and parameter results) between the DSGE-VAR model and the DSGE model can be compared to further examine the degree of misspecification in the DSGE model.

The DSGE and DSGE-VAR estimation produce interesting results. First, DSGE estimation of the model produces results that are largely in line with values common in the literature. More importantly, the DSGE-VAR evaluation shows a high degree of misspecification in the DSGE model mentioned above. The estimate about how influential the DSGE model is to the VAR is low, which indicates minimal *a priori* information from the DSGE model. There exists larger estimates of standard deviations of the shocks under the DSGE than under DSGE-VAR model. This result indicates model misspecification manifesting itself in higher estimates of the standard deviation of the shocks. Moreover, Del Negro et al. (2007) articulate their hypothesis that DSGE-VAR posterior mean estimates should be closer to the prior mean while DSGE posterior mean estimates should be farther away from the prior mean. The majority of the posterior mean estimates shown in section 4 indicate this previously stated fact. When comparing the impulse responses between the optimal DSGE-VAR and DSGE models, misspecification is also observed. For example, the DSGE model continually leaves the DSGE-VAR($\hat{\lambda}$) implied probability intervals. Furthermore, these results suggest misspecification of the GK (2011) model, and that policymakers should exercise caution when utilizing new unconventional monetary policy models.

1.1.1 Previous Literature

The New Keynesian financial model presented in this paper has both similarities and differences to other works in the field. The structure of the model incorporates standard New Keynesian elements, which include habit formation and price stickiness, found in Smets and Wouters (2003, 2007) and Christiano, Eichenbaum, and Evans (2005); however, in the previously mentioned journal articles, a financial sector is noticeably absent. This omission can be a significant drawback since the recent recession included financial disturbances and financial interventions by government. The foundation for a New Keynesian model with financial frictions is found in the seminal work of Bernanke, Gali, and Gertler (1999). In their work, the inclusion of an entrepreneurial sector defines one of the key elements distinguishing the model from a simple New Keynesian framework. The entrepreneurs play a key role in the propagation of shocks. Adrian and Shin (2009) empirically investigate the role of financial intermediaries. They conclude that post-recession monetary policy needs to be concerned with the balance sheets of financial intermediaries. The balance sheets include repurchase agreements and commercial paper. New unconventional monetary policy has also been formulated in the work by Kiyotaki and Moore (2012). In their paper, central bank operations change the liquidity of the private sector's portfolio. When the central bank increases liquidity, its actions can motivate new investment. Christiano, Motto, and Rostagno (2009) include a financial sector into a standard DSGE model and allow the monetary authority to react to changes in the size of the interest rate spread. Their work highlights the key role that financial disturbances and shock channels have for the economy. Recently, Villa and Yang (2011) use the GK (2011) model to run Bayesian estimation on the United Kingdom's economy. Their results show that the addition of the model's financial sector helps the fit of the model and to explain the fluctuation in the United Kingdom's output during the recent recession. Furthermore, the work in this current paper differs from the previously mentioned articles. First, the model in this paper estimates the GK (2011) framework with US data. This paper also evaluates the GK (2011) model using a DSGE-VAR framework.

The results of this paper add to the economic literature on DSGE model misspecification. Del Negro et al. (2007) used a DSGE-VAR model to evaluate DSGE New Keynesian models. The misspecification result is present, but not as worrisome as in standard DSGE models with New Keynesian elements. Del Negro and Schorfheide (2006) considered a modified version of the Smets and Wouters (2003) model, and evaluated it using the DSGE-VAR approach. Brzoza-Brzezina and Kolasa (2012) used a DSGE-VAR model to evaluate three models: Bernanke, Gali, and Gertler (1999), Kiyotaki and Moore (1997), and a standard New Keynesian model similar to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). Their results showed misspecification in the financial friction models of Bernanke, Gali, and Gertler (1999) and Kiyotaki and Moore (1997), and the standard New Keynesian model. The main contribution of the paper in this current read is the evaluation of DSGE models with a new monetary policy tool. The addition of new monetary policy represents recent action the Federal Reserve took to combat the recession. Because recent models, like the one of GK (2011), have incorporated new central bank action, it is necessary to evaluate the degree of misspecification in a DSGE model with new monetary policy.

The rest of the manuscript is organized as follows. Section 2 describes the theoretical setup and the log-linearized equations of the model. Section 3 details the DSGE-VAR approach. Section 4 presents the empirical results, which include DSGE misspecification evidence in the form of posterior estimates and impulse response functions. Section 5 provides the conclusion.

1.2 Model

The model presented in this section is taken from GK (2011) and combines financial frictions under the influences of Bernanke et al. (1999), standard New Keynesian DSGE models, and bank capital as stated by Villa and Yang (2011). The main agents are households, financial intermediaries, intermediate producers, retail firms, capital producers, and a central bank. In light of central banks using unconventional means to combat the most recent recession, the model of GK (2011) also adds another tool for the central bank to intervene in the economy. The following sections describe the sectors of the model with key equations.[1]

Households: In GK (2011), a household consumes, saves, and supplies labor to intermediate goods firms. They pay lump-sum taxes to the government. Each household is composed of members who are either workers or bankers. A worker receives a wage for his or her labor. Every banker is also in charge of a financial intermediary, which will be described in a later section. Households deposit their money into riskless one-period ahead bonds issued by either a banker or govnerment.[2] The banker transfers earnings back to its household. At the beginning of each period, the household members can switch between professions. The probability that a banker stays a banker in the next period is independent of previous history and is defined as θ. Thus, (1-θ) of bankers switch to become workers each period. In addition, each new banker is endowed with a start-up transfer from the family household. This transfer is a small fraction, χ, of total assets.

The representative household maximizes expected discounted utility given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^t \left[ln(C_{t+i} - hC_{t+i-1}) - \frac{\omega}{1+\iota} L_{t+i}^{1+\iota} \right] \tag{1.1}$$

[1]The interested reader can refer to GK(2011) and Appendix A.2 for a detailed description of the model.
[2]A household deposits money with a banker who is not a part of its household.

where ι is the inverse of Frisch elasticity of labor supply, ω defines the influence of leisure on utility, h is the habit formation parameter, and $\beta > 0$. The budget constraint is given by

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \tag{1.2}$$

where Π_t is the net transfer the family household endows new bankers. T_t defines lump sum taxes, W_t real wages, C_t consumption, and L_t family labor supply. B_t represents the riskless one-period ahead bonds. Furthermore, R_t is the riskless gross real return from t-1 to t paid on real bonds. Thus, $R_t B_t$ is the total value of deposits and government debt that the household earned when it invested in those financial assets in the previous period. B_{t+1} is the amount of real bonds bought.

Financial Intermediaries: Financial intermediaries, or bankers, represent both investment and commercial banks. They take in deposits from households and in return issue riskless B_{t+1} bonds that pay R_{t+1}. They lend these funds to non-financial firms and earn R_{kt+1} over the period t to $t+1$. In order to receive the funds, the non-financial firms issue financial claims S_t, which have a value Q_t. The bankers balance sheet is defined by

$$Q_t S_t = N_t + B_{t+1} \tag{1.3}$$

where N_t is net worth of a financial intermediary. The bankers maximize expected terminal wealth. The key result is the following incentive constraint

$$V_{jt} \geq \lambda^h Q_t S_{jt} \tag{1.4}$$

where V_{jt} is expected terminal wealth of the financial intermediary. λ^h characterizes the fraction of assets that the banker diverts back to the household. Thus, equation (1.4) states that the expected terminal wealth needs to be at least as large as the amount of assets the financial intermediary gives back to his or her household. Otherwise, a financial intermediary would not have an incentive to conduct business, and instead, would divert all funds from households back to its household.

GK (2011) show that if the previous constraint binds it can be rewritten as

$$Q_t S_{jt} = \phi_t N_{jt} \tag{1.5}$$

where ϕ_t is referred to as the private leverage ratio. In other words, ϕ_t is the ratio of privately intermediated assets to equity. Equation (1.5) also states financial intermediaries are constrained by their net worth when lending funds to intermediate goods firms.

The aggregate net worth N_t is the sum of existing and new bankers. The net worth of current bankers is expressed as

$$N_{et} = \{\theta[(R_{kt} - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1}\} \tag{1.6}$$

where θ is the probability that a banker stays a banker, R_{kt} is what is earned from financial assets, and R_t is the amount paid to households for deposits. Note that the premium $(R_{kt} - R_t)$ plays an important role in net worth. If this value increases, then the amount that bankers receive from lending and taking in deposits increases and thus, overall net worth becomes larger. Recall also that new bankers receive a nominal transfer from the household. This transfer is a fraction of the value of total financial assets. Thus, the net worth for new bankers can be written as

$$N_{nt} = \chi Q_t S_{t-1} \tag{1.7}$$

where χ is the fraction of the value of total financial assets. Adding equations (1.6) and (1.7) yields the aggregate net worth equation

$$N_t = \theta[(R_{kt} - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1} + \chi Q_t S_t \tag{1.8}$$

New Central Bank Policy: In addition to following a monetary policy rule, the central bank conducts new monetary policy by intervening in the financial market.[3] The central bank raises funds from households by issuing government bonds that pay R_t, and then lends this money to non-financial firms, who pay R_t^k for the funds. There also exists an efficiency cost of τ per unit of intermediated assets. This efficiency cost can be thought of as the cost of finding appropriate private sector agents who will buy its debt. The debt issued is always honored.

The total value of intermediated financial assets in economy is composed of private and public lending

$$Q_t S_t = Q_t S_{pt} + Q_t S_{gt} \tag{1.9}$$

where S_{pt} denotes intermediated assets issued by the private sector. The central bank funds a fraction ψ_t of total intermediated assets

$$Q_t S_{gt} = \psi_t Q_t S_t \tag{1.10}$$

[3]In addition to the monetary policy rule, the central bank follows an additional rule regarding its intervention in the financial market. This rule will be described later.

By combining equations (1.5), (1.9), and (1.10), we get

$$Q_t S_t = \phi_t N_t + \psi_t Q_t S_t = \phi_{ct} N_t \tag{1.11}$$

where

$$\phi_{ct} = \frac{1}{1 - \psi_t} \phi_t \tag{1.12}$$

where GK (2011) define ϕ_t as the ratio of privately intermediated assets to equity and ϕ_{ct} as the ratio of total intermediated assets to equity.

Intermediate Goods Firms: The intermediate goods firms use capital and labor to make their product in a competitive market. They sell their product to retail firms. In order to fund their operations to create capital, they borrow funds from bankers by issuing state-contingent claims S_t. They issue as many state-contingent claims as necessary to purchase the necessary capital, K_{t+1}. The bankers receive R_t^k from lending. At the end of the period, they choose the amount of capital K_{t+1} needed for production in the next period. Intermediate goods firms also sell unused capital to capital producing firms. In addition, the production function for the representative intermediate goods firms follows the Cobb-Douglas form

$$Y_t = A_t (U_t \varepsilon_t^b K_t)^\alpha l_t^{1-\alpha} \tag{1.13}$$

where U_t is the utilization rate, A_t is total factor productivity, and ε_t^b is a shock to the value of capital.

Capital Producing Firms: The capital producing firms buy leftover capital from intermediate goods firms, refurbish it, and produce new capital I_{nt}. This new capital adds to the capital stock

$$K_{t+1} = \varepsilon_t^b K_t + I_{nt} \tag{1.14}$$

Capital producing firms maximize their expected discounted profits

$$max\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{T-t}\Lambda_{t,\tau}\left\{(Q_\tau - 1)I_{n\tau} - f\left(\frac{I_{n\tau}+I_{ss}}{I_{n_{\tau-1}}+I_{ss}}\right)(I_{n\tau}+I_{ss})\right\} \tag{1.15}$$

with

$$I_{nt} \equiv I_t - \delta(U_t)\varepsilon_t^b K_t \tag{1.16}$$

where $f(1)$=$f^{'}$=0 and $f^{''}(1)$ >0. I_t is gross capital created, I_{ss} is steady-state investment, and I_{nt} characterizes net capital created. The first order condition for net capital created results in an equation for the price of a unit of capital

$$Q_t = 1 + f(\cdot) + \frac{I_{nt}+I_{ss}}{I_{nt-1}}f^{'}(\cdot) - \mathbb{E}_t\beta\Lambda_{t,t+1}\left(\frac{I_{n\tau}+I_{ss}}{I_{n_{\tau-1}}}\right)^2 f^{'}(\cdot) \tag{1.17}$$

Retail Firms: The retail firms vary from the intermediate goods firms. Retail firms purchase intermediate goods in order to produce the final output, Y_t. There exists a continuum of retail firms indexed by f on the unit interval who each produce differentiated goods Y_{ft}. Retailers also are subject to a Calvo (1983) pricing scheme. In every period, each final good firm has a probability $(1-\sigma)$ of being able to adjust its price. If a firm cannot reoptimize, then it has a probability σ_p of indexing its price to the previous period's inflation rate. The retailer's optimization problem is then

$$max\mathbb{E}_t \sum_{i=0}^{\infty} \sigma^i\beta^i\Lambda_{t,t+i}\left[\frac{P_t^*}{P_{t+i}}\prod_{k=1}^{i}(1+\pi_{t+k-1})^{\sigma_p} - P_{mt+i}\right]Y_{ft+i} \tag{1.18}$$

With $\mu = \frac{1}{1-\frac{1}{\epsilon}}$, the first-order condition with respect to P_t^* is given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \sigma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1+\pi_{t+k-1})^{\sigma_p} - \mu P_{mt+i} \right] Y_{ft+i} = 0 \tag{1.19}$$

The familiar pricing equation is given by

$$P_t = [(1-\sigma)(P_t^*)^{1-\epsilon} + \sigma(\Pi_{t-1}^{\sigma_p} P_{t-1})^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{1.20}$$

Government Policy: The central bank follows both conventional and unconventional procedures. The monetary authority adjusts the short-term nominal interest rate according to a monetary policy rule

$$i_t = i_{t-1}^{\rho} (\pi_t^{i_\pi} y_t^{i_y})^{1-\rho} exp(\varepsilon_t^i) \tag{1.21}$$

As described above, ψ_t defines the fraction of intermediated assets that the central bank is willing intervene and is characterized by the following rule

$$\psi_t = \psi + \nu \mathbb{E}_t[(R_{t+1}^k - R_t) - (R^k - R)] \tag{1.22}$$

where R_t^k is the lending rate and R_t is the interest rate earned on deposits. Thus, R^k-R is the steady-state premium. ψ defines the steady-state fraction of publicly intermediated assets. The previous equation shows that the central bank intervenes more in the financial market and issues more government bonds when the cost of borrowing funds R_{t+1}^k increases relative to the interest paid on deposits R_t. Since the cost of borrowing has increased, the central bank will issue more government bonds to increase the supply of funds available to intermediate goods firms. In other words, equation (1.22) can be viewed as an additional tool for the central bank to respond to a recession.

1.2.1 Log-linearized Model

This section contains the log-linearized equations of the model presented in the previous section. Variables with a tilde denote percentage deviation from steady state, and those with a $'*'$ and no time subscript denote steady-state values. From the household's first-order conditions, we get an expression for consumption

$$\tilde{c}_t = c_1\tilde{r}_t + c_2\tilde{c}_{t-1} + c_3\mathbb{E}_t\tilde{c}_{t+1} - c_4\widetilde{\Xi}_{t-1} \tag{1.23}$$

where $c_1 = \frac{(1-h)(1-\beta h)}{1+\beta h^2}$, $c_2 = \frac{h}{1+\beta h^2}$, $c_3 = \frac{\beta h}{1+\beta h^2}$, and $c_4 = \frac{(1-h)(1-\beta h)}{1+\beta h^2}$. The household's marginal utility of consumption is defined as

$$\widetilde{\Xi}_t = \Xi_1\tilde{c}_t + \Xi_2\tilde{c}_{t-1} + \Xi_3\mathbb{E}_t\tilde{c}_{t+1} \tag{1.24}$$

where $\Xi_1 = \frac{-(1+\beta h^2)}{(1-\beta h)(1-h)}$, $\Xi_2 = \frac{h}{(1-\beta h)(1-h)}$, and $\Xi_3 = \frac{\beta h}{(1-\beta h)(1-h)}$. The amount of labor supplied to intermediate goods firms is given by

$$\tilde{l}_t = l_1(\widetilde{\Xi}_t + \tilde{w}_t) \tag{1.25}$$

where $l_1 = \frac{1}{\iota}$. Next, total net worth is composed of the net worth of both existing and new financial intermediaries and is given by

$$\tilde{n}_t = \theta z^*(\tilde{n}_{t-1} + \tilde{z}_t) + \frac{\chi s^*}{n^*}\Big(\tilde{q}_t + (\tilde{\phi}_{t-1} + \tilde{n}_{t-1} - \tilde{q}_{t-1})\Big) \tag{1.26}$$

GK (2011) call $\tilde{z}_t$ the gross growth rate of net worth, which is

$$\tilde{z}_t = \tilde{n}_t - \tilde{n}_{t-1} \tag{1.27}$$

As mentioned in the previous section, $\tilde{\phi}_t$ characterizes the ratio of financial claims to equity of the financial intermediaries and is given by

$$\tilde{\phi}_t = \tilde{\eta}_t + \phi_1 \tilde{v}_t \tag{1.28}$$

where $\phi_1 = \frac{(1-\theta)\beta(r^{k*}-r^*)/(1-\beta\theta)}{\lambda^h-(1-\theta)\beta(r^{k*}-r^*)/(1-\beta\theta)}$ and

$$\tilde{\eta}_t = \beta\theta\mathbb{E}_t((\tilde{\Xi}_{t+1} - \tilde{\Xi}_t) + \tilde{z}_{t+1} + \tilde{\eta}_{t+1}) \tag{1.29}$$

GK (2011) describe $\tilde{v}_t$ as the expected discounted additional benefit a banker receives from issuing another asset, while $\tilde{\eta}_t$ is the expected discounted additional benefit of the marginal increase of a banker's net worth. Gertler and Karadi (2011) define $\tilde{q}_t$ as the price of a financial claim, and thus, the price of capital. The incentive constraint faced by bankers is given by

$$\tilde{s}_t + \tilde{q}_t - s^*\tilde{\psi}_t = \tilde{\phi}_t + \tilde{n}_t \tag{1.30}$$

where $\tilde{s}_t$ is the sum of both private and government intermediated assets and $\tilde{\psi}_t$ is the fraction of central bank intermediated assets.

Turning to the supply side of the model, the production functions is defined as

$$\tilde{y}_t = \alpha(\tilde{u}_t + \tilde{k}_t) + (1-\alpha)\tilde{l}_t + \alpha\tilde{\varepsilon}_t^q + \tilde{\varepsilon}_t^a \tag{1.31}$$

where $\tilde{u}_t$ and $\tilde{k}_t$ represent capital utilization and capital, respectively. The variables $\tilde{\varepsilon}_t^q$ and $\tilde{\varepsilon}_t^a$ are shocks to capital and total factor productivity, respectively. α defines the fraction of effective capital used in production. The buying and selling of capital leads to

$$\tilde{s}_t = \mathbb{E}_t\tilde{k}_{t+1} \tag{1.32}$$

The wage rate is a function of the markup, output, and labor supply

$$\tilde{w}_t = -\tilde{\mu}_t + \tilde{y}_t - \tilde{l}_t \tag{1.33}$$

The markup $\tilde{\mu}_t$ equals

$$(\alpha - 1)\tilde{w}_t - \frac{\alpha q^*}{q^* - r^{k*}}\tilde{q}_{t-1} - \frac{\alpha r^{k*}}{q^* - r^{k*}}\tilde{r}_t^k + \tilde{\varepsilon}_t^a + \alpha\tilde{u}_t + \alpha\tilde{\varepsilon}_t^q \tag{1.34}$$

The inflation equation is

$$\tilde{\pi}_t = \pi_1\tilde{\pi}_{t-1} + \pi_2\mathbb{E}_t\tilde{\pi}_{t+1} - \pi_3\tilde{\mu}_t \tag{1.35}$$

where $\pi_1 = \frac{\sigma_p}{1+\sigma_p\beta}, \pi_2 = \frac{\beta}{1+\sigma_p\beta}$, and $\pi_3 = \frac{(1-\beta\sigma)(1-\sigma)}{(1+\sigma_p\beta)\sigma}$. As stated above, σ is the probability of a retail firm not being allowed to adjust its price each period. σ_p is the probability of a retail firm indexing its price to lagged inflation. The log-linearized equation for net capital created is given by

$$\tilde{I}_{nt} = \tilde{I}_t - \delta'(U^*)K^*\tilde{u}_t - \delta\tilde{k}_t \tag{1.36}$$

Capital utilization is

$$\tilde{u}_t = \left(\frac{1}{1+\zeta}\right)(-\tilde{\mu}_t + \tilde{y}_t - \tilde{k}_t - \tilde{\varepsilon}_t^q) \tag{1.37}$$

where ζ describes the elasticity of marginal depreciation with respect to capital utilization. The equation for the amount of capital accumulated is given by

$$\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} - \delta^{'}(U^*)\tilde{u}_{t-1} + \delta\tilde{I}_{t-1} + \tilde{\varepsilon}^q_t \tag{1.38}$$

The next equation gives the rental rate of capital

$$\tilde{r}^k_t = \left(\frac{1}{r^{k*}}\right)(\delta^{'}(U^*)(\tilde{y}_t - \tilde{k}_t - \tilde{u}_t - \tilde{\mu}_t) + \tilde{\varepsilon}^q_t + \tilde{q}_t) - \tilde{q}_{t-1} \tag{1.39}$$

The equation for the value of capital is defined as

$$\tilde{q}_t = \left(\frac{I^*_n}{(I^*_n + I^*)}\right)\upsilon((1+\beta)\tilde{I}_{nt} - \tilde{I}_{nt-1} - \beta\mathbb{E}_t\tilde{I}_{nt+1}) \tag{1.40}$$

where the parameter υ defines investment adjustment costs. Monetary policy is governed by two equations. The first is a standard Taylor Rule

$$\tilde{i}_t = \rho\tilde{i}_{t-1} + (1-\rho)(i_\pi\tilde{\pi}_t + i_y\tilde{y}_t) + \varepsilon^m_t \tag{1.41}$$

The second monetary policy equation regards the fraction of financial assets that the central bank intermediates

$$\tilde{\psi}_t = \left(\frac{\nu}{\psi^*}\right)(r^{k*}\mathbb{E}_t\tilde{r}^k_{t+1} - r^*\mathbb{E}_t\tilde{r}_{t+1}) \tag{1.42}$$

The resource constraint is defined as

$$\tilde{y}_t = c^*_y\tilde{c}_t + I^*_y\tilde{I}_t + \tau\psi^*K^*_y(\tilde{\psi}_t + \tilde{q}_t + \mathbb{E}_t\tilde{k}_{t+1}) + \tilde{\varepsilon}^g_t \tag{1.43}$$

where τ represents an efficiency cost of government intervening in the financial markets.

There exist four shocks in this economy. The capital quality and technology shocks each follow an AR(1) process

$$\tilde{\varepsilon}_t^q = r_q \tilde{\varepsilon}_{t-1}^q + \eta_t^q, \eta_t^q \sim N(0, \sigma_q) \tag{1.44}$$

$$\tilde{\varepsilon}_t^a = r_a \tilde{\varepsilon}_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a) \tag{1.45}$$

The monetary policy and net worth shocks are both *i.i.d.*

1.3 DSGE-VAR Approach

A contribution of this paper to the existing literature is the evaluation of the New Keynesian model described above using the DSGE-VAR approach. The DSGE-VAR approach was popularized by Del Negro and Schorfheide (2004), and thus, the description below will closely follow their work. In essence, a VAR model is estimated while using DSGE information to restrict the parameter estimates. This information can be extremely helpful because of the known fact that VAR models are not parsimonious. To help overcome this issue, the VAR parameter space can be restricted using information from a theoretical DSGE model. In addition, this approach to estimation is utilized to evaluate the degree of DSGE model misspecification.

In order to motivate intuition before proceeding to the technical details, a brief description of the DSGE-VAR approach is given. For a given value of the DSGE model's parameters and realization of the shocks, artificial data can be produced. A VAR is subsequently estimated using this simulated data as well as real data. If the process is repeated for different values of the DSGE model's parameters, a mapping is created linking the VAR estimated parameters and the DSGE model's parameters. Instead of imposing a direct link between the two sets, the VAR parameters are given a prior that is centered around the DSGE model's parameters. The hyper parameter λ, which is the ratio of artificial data over actual observations, controls the tightness of the prior. If $\lambda \to 0$, the DSGE model's artificial data is not at all informative for estimation of the VAR's parameters. If $\lambda \to \infty$, the DSGE model provides superior information for VAR estimation, and thus, the DSGE model implied restrictions are useful. Therefore, the estimation of λ can serve as an indicator of how well the DSGE model agrees with the data.

1.3.1 Likelihood Function

In order to construct the posterior distribution for the model, the likelihood function needs to be defined. Consider a typical VAR model

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \varepsilon_t \tag{1.46}$$

where y_t is an n x 1 vector of endogenous variables and ε_t is an n x 1 vector of error terms. If we stack equation (1.46) over all observations, the resulting equation is

$$Y = X\Phi + \Omega \tag{1.47}$$

where the T x n matrix Y has rows $y_t^{'}$. X is a T x k matrix with rows $x_t^{'} = [1, y_{t-1}^{'}, ..., y_{t-p}^{'}]$ where $k = 1{+}np$. $\Phi = [\phi_0, \phi_1, ..., \phi_p]'$. Ω has rows $\varepsilon_t^{'}$ and is T x n. By assuming a multivariate distribution on ε_t

$$\varepsilon_t \sim N(\mathbf{0}, \mathbf{\Sigma}_\varepsilon) \tag{1.48}$$

we get the following likelihood function

$$p(Y|\Phi, \Sigma_\varepsilon) \propto |\Sigma_\varepsilon|^{\frac{-T}{2}} exp\left\{ \left(\frac{-1}{2}\right) tr[\Sigma_\varepsilon^{-1}(Y'Y - \Phi' X'Y - Y'X\Phi + \Phi' X'X\Phi)] \right\} \tag{1.49}$$

which is conditional on $y_{1-p}, ..., y_0$.

1.3.2 Prior

The prior for the DSGE-VAR model takes the form

$$p(\Phi, \Sigma_\varepsilon, \theta) = p(\Phi, \Sigma_\varepsilon|\theta)p(\theta) \tag{1.50}$$

where θ defines the vector of DSGE parameters and Φ and Σ_ε contain the VAR parameters. $p(\theta)$ defines the prior distribution on θ. The prior also involves an observed sample T and an artificial sample $T^* = \lambda T$ that are dummy observations generated from the DSGE model. Thus, the observations (Y^*, X^*) are taken from T^*. λ characterizes the influence of the artificial samples generated from the DSGE model. Therefore, by including artificial and observed samples, equation (1.49) becomes

$$p(Y^*(\theta)|\Phi, \Sigma_\epsilon) \propto |\Sigma_\epsilon|^{\frac{-\lambda T}{2}} exp\left\{ \left(\frac{-1}{2}\right) tr[\Sigma_\epsilon^{-1}(Y^{*\prime}Y^* - \Phi' X^{*\prime}Y^* - Y^{*\prime}X^*\Phi + \Phi' X^{*\prime}X^*\Phi)] \right\} \tag{1.51}$$

Del Negro and Schorfheide (2004) assume the vector y_t is covariance stationary and replace the sample moments $Y^{*'}Y^*$, $Y^{*'}X^*$, and $X^{*'}X^*$ with scaled population moments $\lambda T\Gamma^*_{yy}(\theta)$, $\lambda T\Gamma^*_{yx}(\theta)$, and $\lambda T\Gamma^*_{xx}(\theta)$, respectively. For example, $\Gamma^*_{xx}(\theta) = \mathbb{E}_\theta[x_t x_t^{'}]$. By incorporating these previous replacements and including an initial improper prior $p(\Phi, \Sigma_\varepsilon) \propto |\Sigma_\varepsilon|^{\frac{-(n+1)}{2}}$, equation (1.51) becomes

$$\begin{aligned} p(\Phi, \Sigma_\epsilon|\theta) = c^{-1}(\theta)\,|\Sigma_\epsilon|^{\frac{-(\lambda T+n+1)}{2}}\, exp\Bigg\{ \left(\frac{-1}{2}\right) tr[\lambda T\Sigma_\epsilon^{-1}(\Gamma^*_{yy}(\theta) \\ - \Phi^{'}\Gamma^*_{xy}(\theta) - \Gamma^*_{yx}(\theta)\Phi + \Phi^{'}\Gamma^*_{xx}(\theta)\Phi)]\Bigg\} \end{aligned} \tag{1.52}$$

If $\lambda T \geq k+n$ and $\Gamma_{xx}(\theta)$ are invertible, Del Negro and Schorfheide (2004) state that equation (1.52) is proper and $c(\theta)$ is defined as

$$c(\theta) = (2\pi)^{\frac{nk}{2}}\,|\lambda T\Gamma^*_{xx}(\theta)|^{\frac{-n}{2}}\,|\lambda T\Sigma^*_\epsilon(\theta)|^{\frac{-(\lambda T-k)}{2}}\,2^{\frac{n(\lambda T-k)}{2}}\,\pi^{\frac{n(n-1)}{4}}\prod_{i=1}^{n}\Gamma[(\lambda T-k+1-i)/2] \tag{1.53}$$

in order for equation (1.52) to integrate to one. In equation (1.53), $\Gamma(\bullet)$ indicates the gamma function. Furthermore, if we condition on the θ and λ, we obtain a normal-inverse-Wishart form for the prior for the VAR parameters

$$\Sigma_\epsilon|\theta, \lambda \sim IW(\lambda T\Sigma^*_\epsilon(\theta), \lambda T - k, n) \tag{1.54}$$

$$\Phi|\Sigma_\epsilon, \theta, \lambda \sim N(\Phi^*(\theta), \Sigma_\epsilon \otimes (\lambda T\Gamma^*_{xx}(\theta))^{-1}) \tag{1.55}$$

where $\Sigma^*_\epsilon(\theta) = \Gamma^*_{yy}(\theta) - \Gamma^*_{yx}(\theta)\Gamma^{*-1}_{xx}(\theta){\Gamma_{xy}}^*(\theta)$ and $\Phi^*(\theta) = {\Gamma_{xx}}^{*-1}(\theta)\Gamma^*_{xy}(\theta)$. An important element to note is the influence that λ has on the prior. As seen in equation (1.55) and described by An and Schorfheide (2007), large values of λ scale the prior towards restrictions implied by the DSGE model. As λ approaches zero, the DSGE restrictions are lessened and the prior becomes more uninformative. In addition, λ will be considered a parameter to be estimated. Thus, the DSGE-VAR model is modified to include a prior for λ. The new prior takes the form

$$p(\Phi, \Sigma_\epsilon, \theta, \lambda) = p(\Phi, \Sigma|\theta, \lambda)p(\theta)p(\lambda) \tag{1.56}$$

1.3.3 Posterior

The posterior distribution can be factored in the following way

$$p(\Phi, \Sigma_\varepsilon, \theta, \lambda|Y) = p(\Phi, \Sigma_\varepsilon|Y, \theta, \lambda)p(\theta, \lambda|Y) \tag{1.57}$$

When conditioned on θ, equations (1.54) and (1.55) characterize a conjugate prior for $p(\Phi, \Sigma_\varepsilon|Y, \theta)$ and thus, the VAR posterior distribution is from the same family of distributions. Thus, the posterior distributions for Φ and Σ_ε are defined as

$$\Sigma_\epsilon|Y, \theta, \lambda \sim IW((\lambda+1)T\tilde{\Sigma}_\epsilon(\theta, \lambda), (1+\lambda)T - k, n) \tag{1.58}$$

$$\Phi|Y, \Sigma_\epsilon, \theta, \lambda \sim N(\tilde{\Phi}(\theta, \lambda), \Sigma_\epsilon(\theta, \lambda) \otimes (\lambda T\Gamma^*_{xx}(\theta) + X'X)^{-1}) \tag{1.59}$$

where

$$\tilde{\Phi}(\theta) = (\lambda T\Gamma^*_{xx}(\theta) + X'X)^{-1}(\lambda T\Gamma^*_{xy} + X'Y) \tag{1.60}$$

$$\tilde{\Sigma}_\epsilon(\theta) = \frac{1}{(\lambda+1)T}[(\lambda T\Gamma^*_{yy}(\theta) + Y'Y) - (\lambda T\Gamma^*_{yx}(\theta) + Y'X)(\lambda T\Gamma^*_{xx}(\theta) + X'X)^{-1}(\lambda T\Gamma^*_{xy}(\theta) + X'Y)] \tag{1.61}$$

are the maximum likelihood estimates of Φ and Σ_ε, respectively. The last term in equation (1.57), $p(\theta, \lambda|Y)$ is estimated using the Markov chain Monte Carlo(MCMC) algorithm described in Del Negro and Schorfheide (2004).

1.4 Empirical Results

The model is estimated using Bayesian techniques. The Metropolis-Hastings algorithm is used to obtain posterior results, which include mean, HPD intervals, marginal likelihood, and impulse response functions. Four US quarterly time-series observable variables are utilized. As explained by Adjemian, Pariés, and Moyen (2008), the matching of the number of observable variables to the number of exogenous shocks aids in identifying the DSGE-VAR model. The four variables are real gross domestic product (GDP), investment, log difference in the GDP deflator, and interest rate spread. GDP and the log difference in the GDP deflator are standard macroeconomic variables. Investment and the interest rate spread help to match the financial sector aspect of the model. GDP and investment are in per capita terms and detrended by taking the log difference and multiplying by 100. The log difference in the GDP deflator is also multiplied by 100. The interest rate spread is defined as the difference between Baa corporate bond yield and the federal funds rate. In addition, the data are in quarters and encompasses 1954:4 to 2012:1.[4]

[4]A complete description of the variables can be found in Appendix A.1.

1.4.1 Calibration

A handful of parameters are calibrated during estimation. Most of the calibrated values derive from Villa and Yang (2011). In order to pin down steady-state values, the fraction of assets given to new bankers χ and survival rate of bankers θ are fixed at 0.002 and 0.97, respectively. The discount factor β and the depreciation rate δ are set to 0.99 and 0.025, respectively, which are common values in the literature (see GK (2011) and Smets and Wouters [2007]). The elasticity of labor supply ι is also calibrated from GK (2011) since the data contain no information about labor. The utility weight of labor and elasticity of demand are calibrated from GK (2011) and equal 3.409 and 4.167, respectively. The share of capital in production, α, is fixed at 0.33, which is a common value found in the literature. Since my observables do not contain information on consumption, I fix the habit formation parameter to 0.815 which is the value used in Villa and Yang (2011).

1.4.2 Priors

The priors on the parameters are set to distributions found in the literature and can be found in either Table 1.1 or Table 1.2 for the DSGE and DSGE-VAR models, respectively. As stated by Villa and Yang (2011), a gamma distribution defines the prior assumption for the elasticity parameter ζ. The Calvo (1983) price stickiness parameter σ and price indexation parameter σ_p both receive a beta distribution with a mean of 0.75 and 0.5, respectively, which are both from Villa and Yang (2011). The structure of the prior distribution for σ implies prices change about once every four quarters. This information roughly corresponds to the empirical work of Klenow and Malin (2010) that the frequency of price changes is at least once a year. The priors for the next set of parameters originate from Smets and Wouters (2007). The priors on the lagged terms in the AR(1) processes receive a prior beta distribution, which ensures the results lie between 0 and 1. An inverse-gamma distribution with a mean of 0.1 and standard deviation of 2 define the priors for the standard deviations of the shocks. This distribution provides a positive posterior variance as explained by Smets and Wouters (2003). The beta distribution for the lagged nominal interest rate parameter ρ ensures a result between 0 and 1. The distributions for i_π and i_y, which are from Villa and Yang (2011), imply that inflation is assumed to have a stronger effect than output on central bank policy. When estimating the DSGE-VAR model, the value for λ is assumed a uniform distribution. The lower bound is set to $\frac{21}{230} \approx 0.09$. This value stems from Paccagnini (2010) who says λ should be bounded below by $\frac{1+p*n+n}{T}$, where n defines the number of observables and p is the number of lags. The number of lags equals 4 in the present manuscript. If the lower bound is not enforced, Paccagnini (2010) states that the DSGE-VAR model will be in danger of having a degenerate prior distribution. In order not to be too informative, λ is allowed the freedom to take on values with an upper bound of 100. The parameter ν, which influences the fraction of publicly intermediated assets, characterizes a "new" parameter to the literature and is given a fairly diffuse prior. It receives a gamma prior distributions with a mean of 10, which GK (2011) state corresponds to a value similar to Federal Reserve action.

1.4.3 Posterior Estimates

1.4.3.1 DSGE Results

The DSGE posterior estimates, which are presented in Table 1.1, produce a number of results. First, most of the model's posterior estimates produce similar inferences found in the literature, such as in Smets and Wouters (2007). The financial DSGE model exhibits a high degree of price stickiness while the posterior mean on price indexation is low at 0.04.

Table 1.1: **DSGE Priors & Posterior Estimates**

		Prior Distribution			Posterior Distribution		
	Desc.	Distr.	Mean	St. Dev.	Mean	.10*	.90*
ν	Response to Credit	G	10.00	3.00	10.57	5.35	15.54
ζ	Elast. of Marg. Depr.	G	1.88	0.50	3.12	2.22	3.99
σ_p	Price Indexation	B	0.50	0.20	0.04	0.01	0.08
σ	Price Stickiness	B	0.75	0.10	0.84	0.80	0.87
ρ	Lagged Interest Rate	B	0.75	0.10	0.98	0.98	0.99
i_π	Response to Inflation	N	1.50	0.05	1.48	1.40	1.56
i_y	Response to Output	N	0.50	0.05	0.52	0.44	0.59
r_q	Lagged Capital Shock	B	0.50	0.20	0.53	0.46	0.61
r_a	Lagged Tech. Shock	B	0.50	0.20	0.99	0.99	0.99
σ_m	Monetary Shock	IG	0.10	2.00	0.18	0.12	0.24
σ_q	Capital Shock	IG	0.10	2.00	1.33	1.15	1.52
σ_a	Tech. Shock	IG	0.10	2.00	5.07	4.67	5.47

Note 1: G-Gamma Dist.; B-Beta Dist.; N-Normal Dist.; U-Uniform Dist.
IG-Inverse-Gamma Dist.
Note 2: Each estimate is cutoff at the hundreths place
Note 3: HPD Interval

The estimate for the response of the interest rate to output is given by 0.52, which is slightly higher than the value found in Villa and Yang (2011). Inflation and lagged nominal interest rates, which have posterior mean estimates of 1.48 and 0.98, respectively, play a larger role. A high value of ρ is intuitive since the central bank changes the nominal interest rate by small increments, which implies the monetary authority weighs heavily the previous period's interest rate. The estimates display more persistence in the technology shock than in the capital quality shock. This is in accordance with Villa and Yang (2011). Furthermore, the standard deviation estimates produced mixed results. σ_m produced a reasonable estimate of 0.18; however, the capital quality shock is higher at 1.33, while the technology shock is high at 5.07. This result could further point to model misspecification, which will be described in the next section. The estimate of elasticity of marginal depreciation ζ, 3.12, is higher than the value found in Villa and Yang (2011).

The estimates concerning the financial part of the model produce interesting results. The capital shock shows moderate persistence with a mean of 0.53, which is slightly higher than those found in Villa and Yang (2011). The estimate for ν, which defines the feedback parameter for central bank unconventional monetary policy, seems to match the data despite the large standard deviation assumed *a priori*. GK (2011) state that ν=10 resembles the amount of credit intervention by the Federal Reserve.

1.4.3.2 Evaluation Using DSGE-VAR

As explained above and by Del Negro and Schorfheide (2006), a DSGE-VAR model is a useful tool to evaluate DSGE models and Table 1.2 shows the posterior estimates for the DSGE-VAR($\hat{\lambda}$) model. Recall from section 3 that λ is an estimated parameter, and thus, the term DSGE-VAR($\hat{\lambda}$) signifies estimation of the DSGE-VAR model with λ as an estimated parameter. First, the estimate for λ, which will be referred to as $\hat{\lambda}$, is analyzed. Recall that λ is a measure of influence the DSGE model has on estimation of the parameters. From Del Negro and Schorfheide (2006) and equation (1.55), λ measures the tightness of the prior for the VAR parameters Φ, and the prior is influenced by the DSGE model. If λ has a large value, this implies that the prior is tight, and the DSGE model is highly informative for the VAR parameters. If λ has a small value, then the prior is very diffuse, and the DSGE model is uninformative for VAR estimation. Thus, Del Negro and Schorfheide (2006) explain that $\lambda \to 0$ implies estimating an unrestricted VAR, whereas $\lambda \to \infty$ implies estimating a VAR with restrictions imposed by the DSGE model. The DSGE-VAR posterior mean estimate for λ is 0.09. Since λ is low, the DSGE model provides little information for the VAR, and therefore, is at odds with the data. Thus, this value evidences the misspecification in the DSGE model.

Table 1.2: **DSGE-VAR Priors & Posterior Estimates**

		Prior Distribution			Posterior Distribution		
	Desc.	Distr.	Mean	St. Dev.	Mean	.10*	.90*
λ	DSGE-VAR Weight	U	0.09*	100*	0.09	0.09	0.10
ν	Response to Credit	G	10.00	3.00	12.85	7.66	18.63
ζ	Elast. of Marg. Depr.	G	1.88	0.50	2.09	1.53	2.69
σ_p	Price Indexation	B	0.50	0.20	0.13	0.02	0.25
σ	Price Stickiness	B	0.75	0.10	0.41	0.26	0.52
ρ	Lagged Interest Rate	B	0.75	0.10	0.76	0.62	0.88
i_π	Response to Inflation	N	1.50	0.05	1.52	1.45	1.60
i_y	Response to Output	N	0.50	0.05	0.49	0.43	0.55
r_q	Lagged Capital Shock	B	0.50	0.20	0.38	0.20	0.53
r_a	Lagged Tech. Shock	B	0.50	0.20	0.99	0.98	0.99
σ_m	Monetary Shock	IG	0.10	2.00	0.04	0.03	0.05
σ_q	Capital Shock	IG	0.10	2.00	0.07	0.04	0.09
σ_a	Tech. Shock	IG	0.10	2.00	0.11	0.08	0.14
σ_n	Net Worth Shock	IG	0.10	2.00	0.35	0.22	0.47

*0.09 and 100 are the upper and lower bounds for the uniform distribution
Note 1: G-Gamma Dist.; B-Beta Dist.; N-Normal Dist.; U-Uniform Dist.
IG-Inverse-Gamma Dist.
Note 2: Each estimate is cutoff at the hundreths place
Note 3: HPD interval

This paper also uses the method proposed by Paccagnini (2010) to evaluate the DSGE model. Under this approach, λ is estimated as described in the previous section. It is then used to form the ratio

$$\frac{\hat{\lambda} - \lambda_{min}}{\lambda_{min}} \tag{1.62}$$

where Paccagnini (2010) says λ should be bounded below by $\lambda_{min}=\frac{1+p*n+n}{T}$ where n is the number of observables and p is the number of lags. λ should be greater than or equal to λ_{min} in order for the prior to be non-degenerate. λ will also depend on the type of model employed (e.g. number of endogenous variables and lags).[5] If the ratio (1.62) is large, then

[5] Therefore, by using the ratio in equation (1.62), I can correct for the model inherent values that are present in the estimated value of λ.

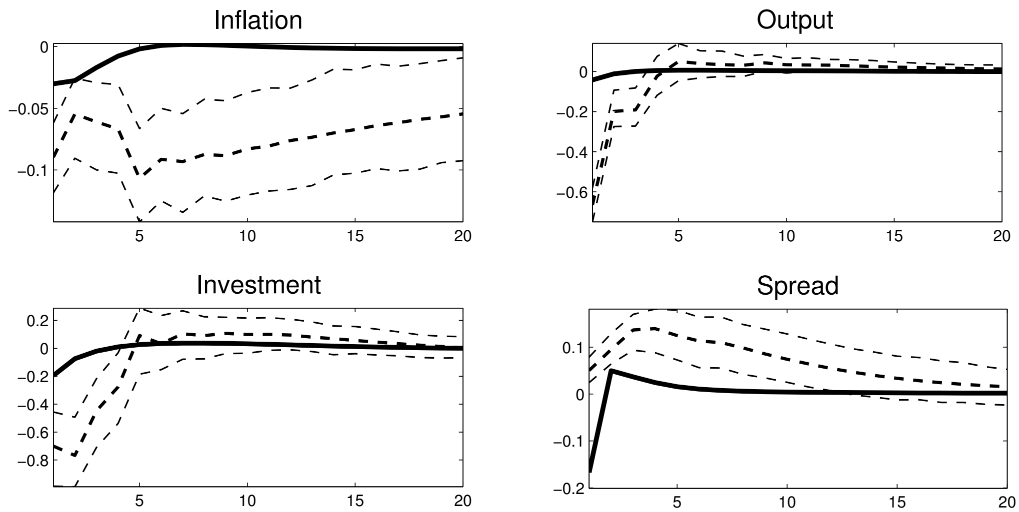
the distance between estimated λ and λ_{min} is sizable and the DSGE model fits the VAR model. By noting from section 4.2 that $\lambda_{min}=\frac{21}{230} \approx 0.09$, the ratio approximately equals 0.03. Therefore, the DSGE model does not match the data well and exhibits misspecification.

A comparison of the the estimates of the shock processes can also provide further evidence of model misspecification. In the DSGE model, the misspecification would manifest itself in the shocks; however, when the DSGE restrictions are loosened, the misspecification should lessen and estimates for the shock processes should improve. As Brzoza-Brzezina and Kolasa (2012) explain, the lower values of the standard deviation shock estimates of the DSGE-VAR($\hat{\lambda}$) model are caused by less misspecification. By examining Tables 1.1 and 1.2, the estimates for the shock processes for the shocks in the DSGE-VAR($\hat{\lambda}$) model are all lower than its counterparts in the DSGE model signaling misspecification in the DSGE model.

Another property of the DSGE-VAR versus DSGE model regards examining the distance between the posterior mean and prior mean of the two models. Del Negro et al. (2007) describe when $\lambda = 0$, the posterior of the DSGE-VAR model will be the same as the prior. In theory, for any $\hat{\lambda}$ that is less than ∞, the DSGE-VAR posterior means will be closer to their prior means than its DSGE counterparts. When comparing the posterior and prior means in Tables 1.1 and 1.2, the DSGE-VAR($\hat{\lambda}$) estimates are overall closer to their prior means. The exceptions are ν, σ, and r_q. However, with the exception of σ, the previously mentioned parameters are not improved on by the DSGE model to a large degree. For example, the posterior mean estimates of r_q are 0.53 and 0.38 for the DSGE and DSGE($\hat{\lambda}$) models, respectively, and the prior mean is 0.50.

To further investigate the degree of misspecification in the DSGE model, the impulse response function (IRF) for the different shocks are examined are presented in Figures 1.1 – 1.4. The

Figure 1.1: Estimated Mean Impulse Responses to Monetary Policy Shock

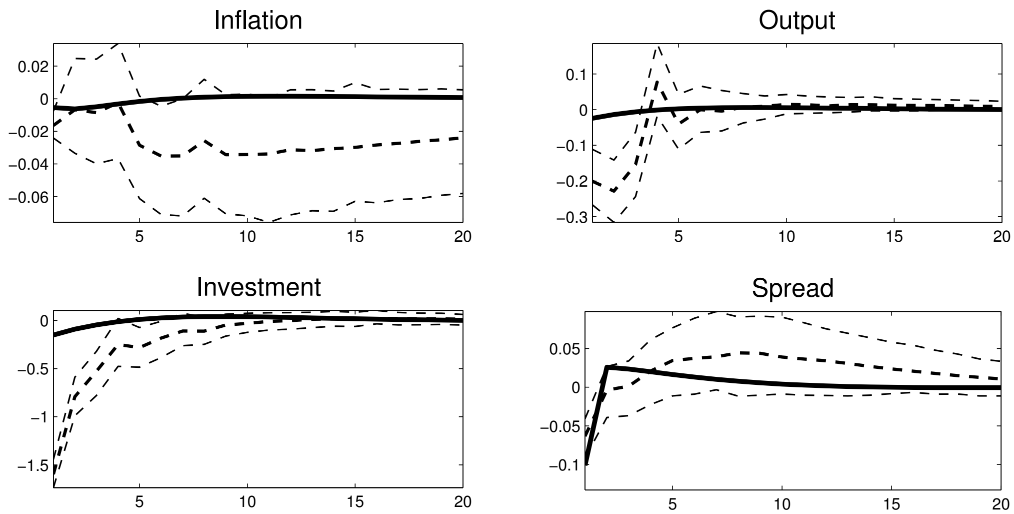


Notes: The bold solid line is the DSGE-VAR($\hat{\lambda}$) mean impulse response. The dotted lines are its 10 percent and 90 percent posterior intervals. The dashed line denotes responses obtained for the corresponding DSGE model.

DSGE-VAR($\hat{\lambda}$) and DSGE IRFs are plotted, along with 90% posterior intervals for the DSGE-VAR($\hat{\lambda}$) model.

The findings are summarized into three main points. First, Figures 1.1 – 1.4 show that the DSGE impulse responses continually leave the 90% probability bands implied by the DSGE-VAR($\hat{\lambda}$) model. The next point regards the DSGE model failing to match the persistence shown by the DSGE-VAR($\hat{\lambda}$) model. Specifically, the DSGE model's response of the inflation rate and spread to a capital quality do not match the persistence implied from the DSGE-VAR($\hat{\lambda}$) model. However, the DSGE model exhibits somewhat more persistence to a technology shock than that implied by the DSGE-VAR($\hat{\lambda}$) model. In addition, the DSGE impulse responses are not as responsive to a monetary policy shock than the DSGE-VAR($\hat{\lambda}$) impulse responses. For instance, output decreases under both models. However, the ini-

Figure 1.2: Estimated Mean Impulse Responses to (Negative) Net Worth Shock

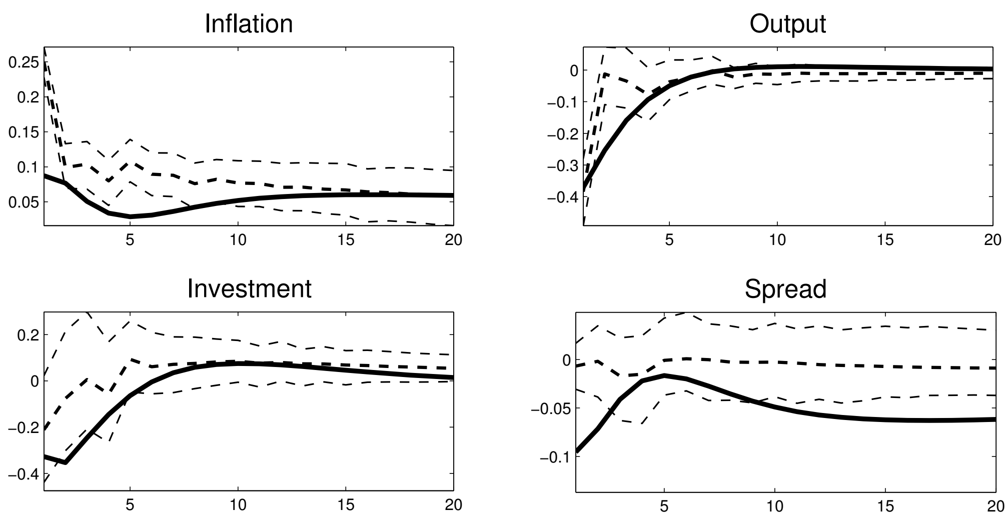


Notes: The bold solid line is the DSGE-VAR($\hat{\lambda}$) mean impulse response. The dotted lines are its 10 percent and 90 percent posterior intervals. The dashed line denotes responses obtained for the corresponding DSGE model.

tial drop in output is greater under the DSGE-VAR($\hat{\lambda}$) model than under the DSGE model. The DSGE-VAR($\hat{\lambda}$) model also seems to be display more persistent behavior than the DSGE counterparts.

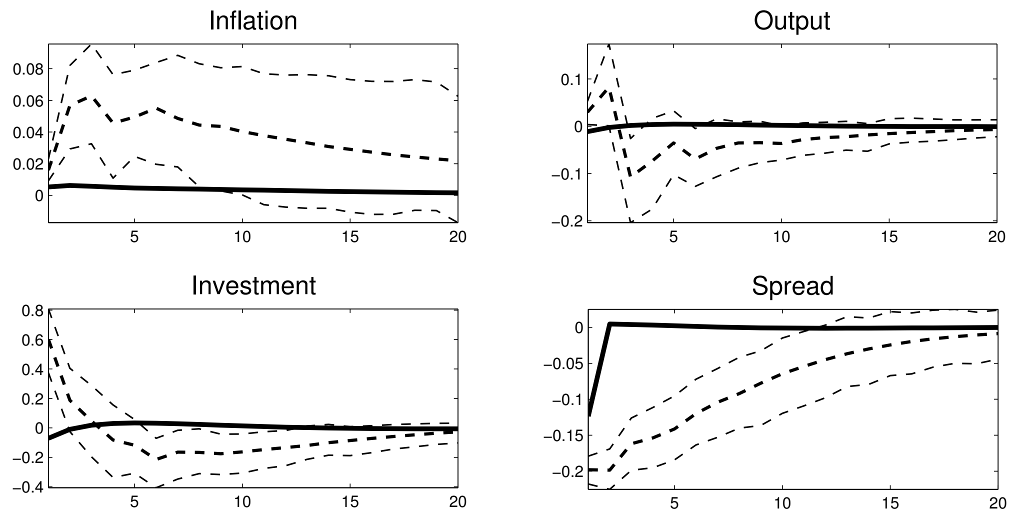
In summary, the results of this DSGE-VAR evaluation of the DSGE model with unconventional monetary policy show that the latter seems to be misspecified. Posterior estimates and impulse responses help to support the conclusion that this new DSGE model fails to capture key elements of the data. Thus, even though the model contains attractive features, the results show that policymakers should be cautious when utilizing this new DSGE model with unconventional monetary policy.

Figure 1.3: Estimated Mean Impulse Responses to Technology Shock



Notes: The bold solid line is the DSGE-VAR($\hat{\lambda}$) mean impulse response. The dotted lines are its 10 percent and 90 percent posterior intervals. The dashed line denotes responses obtained for the corresponding DSGE model.

Figure 1.4: Estimated Mean Impulse Responses to Capital Quality Shock



Notes: The bold solid line is the DSGE-VAR($\hat{\lambda}$) mean impulse response. The dotted lines are its 10 percent and 90 percent posterior intervals. The dashed line denotes responses obtained for the corresponding DSGE model.

1.5 Concluding Remarks

In conclusion, the prominent and recent New Keynesian DSGE financial friction model of GK (2011), which incorporates intuitive elements from the most recent crisis, is estimated using US data and evaluated using the DSGE-VAR approach. The main finding is that this DSGE model with financial frictions and a new monetary policy tool shows a high degree of misspecification. The estimate of λ, which governs the extent that the DSGE model influences the VAR, is extremely low suggesting that the unrestricted VAR model is preferred to the financial friction DSGE model. There exists higher standard deviation estimates for the shock process for the DSGE model than the DSGE-VAR($\hat{\lambda}$). A higher estimate for the standard deviations of the shocks indicates misspecification manifests itself in the shocks. DSGE-VAR($\hat{\lambda}$) posterior mean estimates also are closer to the prior mean than the DSGE counterparts, which agrees with Del Negro et al. (2007). In addition, the impulse responses of the DSGE modell in general do not agree with the DSGE-VAR($\hat{\lambda}$) model. This is evidenced mainly by the DSGE impulse responses often leaving the probability intervals implied by the DSGE-VAR($\hat{\lambda}$) model. These results suggest that policymakers should exercise caution when utilizing new DSGE models that model LSAPs.

This paper has interesting elements for future macroeconomic modeling. The work presented in this paper has added to the current literature by extending the model of GK (2011) by estimating the model with US data as well as evaluating the model using the DSGE-VAR approach. Even with the degree of misspecification present in this DSGE model, the model described in this manuscript and GK (2011) displays key elements, which include financial frictions and a new monetary policy tool, from the recent recession. Future related research should look into building from this type of model while improving the fit with the data.

Chapter 2

Learning and the Effectiveness of Central Bank Forward Guidance

2.1 Introduction

Once U.S. short-term interest rates effectively reached the zero lower bound (ZLB) during the 2007-2009 global financial crisis, monetary policymakers exhausted the conventional policy tool as overnight interest rates could not be lowered. In response, central banks pursued "unconventional" policies. One of these alternatives pursued by the Federal Reserve was large-scale asset purchases (LSAPs) where the central bank purchases longer-term securities in hopes of lowering long-term yields. Another unconventional policy was forward guidance, where the central bank communicates to the public information about the future course of the policy rate. Forward guidance has been pursued by central banks such as the Federal Reserve, Bank of Canada, Bank of England, and the European Central Bank. An example of forward guidance was given in the September 2012 Federal Open Market Committee (FOMC) statement: "the Committee also . . . anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." In addition, Eggertsson and Woodford (2003) and Woodford (2012) argue that committing to an interest rate path that is lower than what one would commit to under normal circumstances (i.e. when overnight interest rates are away from the ZLB) can have additional stimulative economic effects. Standard New Keynesian models (e.g. Woodford [2003]) predict consumption, investment, and pricing decisions are sensitive to the expected path of short term interest rates. If agents expect low interest rates in the future, current consumption and prices all increase. This stimulative effect can be limited by a conventional monetary policy rule that adjusts interest rates in response to target variables, such as the output gap and inflation. Households and firms may rationally expect higher interest rates in response to future expansions. If a forward guidance statement, instead, keeps a low policy rate through part of the expansion, consumption today will not be as limited.

The effectiveness of forward guidance hinges on how private sector expectations about economic state variables (e.g. output and inflation) and interest rates respond to forward guidance. Therefore, it is important to study whether the economic effects of forward guidance are sensitive to the rational expectations assumption that is the standard benchmark in macroeconomic models.[1] While a reasonable benchmark that is popular among macroeconomic models, rational expectations makes strong assumptions about the amount of knowledge agents possess when forming beliefs. It is natural then to examine how effective forward guidance policies can be under a more plausible theory of expectations formation.

[1]A related issue is the credibility of policymakers to commit to a future path of interest rates (see, for instance, see Woodford [2012]). In part, because of credibility concerns, Woodford (2012) prefers forward guidance policies that explicitly state the criteria that will underlie future policy rules. This current paper abstracts from this subject.

This paper studies the effectiveness of forward guidance in an environment where rational expectations has been replaced by an adaptive learning rule similar to one proposed by Marcet and Sargent (1989) and Evans and Honkapohja (2001). In particular, the economic environment is based on Preston (2005) who derives a New Keynesian model with (potentially) non-rational expectations. Households and firms formulate spending and pricing decisions, respectively, that depend on their subjective expectations about future economic conditions and interest rates. The novelty of this paper is to incorporate policy communication about future interest rates into agents' subjective expectations. The central bank sets interest rates according to a monetary policy rule that responds positively to the output gap and inflation. The rule is augmented with anticipated shocks as in Del Negro, Giannoni, and Patterson (2012) and Laseen and Svensson (2011).[2] The anticipated shocks define central bank communication about future deviations from a normal interest rate rule that agents know today. The shocks also represent time-contingent forward guidance in which the central bank communicates a definitive forward guidance end date. In this case, communication about the future path of interest rates is for a fixed amount of periods into the future and is independent of economic conditions.[3]

Agents are assumed to form expectations via either the rational expectations hypothesis or an adaptive learning rule. The former is a strong assumption and assumes agents construct expectations with respect to the true probability distribution of the model. Rational expectations agents must know the model's deep parameters, structure of the model, beliefs of other agents, and distribution of the error terms. A popular alternative to rational expectations is adaptive learning. This approach builds from the cognitive consistency principle that agents behave as real-life economists (see, for instance, Evans and Honkapohja [2013]).

[2]The anticipated shocks are similar to the news shocks of Schmitt-Grohé and Uribe (2012).

[3]This type of forward guidance is in contrast to state-contingent forward guidance where the duration of a constant interest rate path is linked to economic conditions.

An econometrician, for example, would produce forecasts of future economic variables by forming an econometric model. He or she would estimate the parameters using standard econometric techniques. As new data arrives, these forecasts would be revised. Thus, a real-life economist is engaging in a process of learning about the economy. Analogously, adaptive learning agents are assumed to behave as econometricians and formulate forecasts of future endogenous variables using standard econometric techniques. The variables in their econometric model are based on the solution found under rational expectations, but adaptive learning agents estimate the parameters using ordinary least squares. Their beliefs about future endogenous variables are appropriately revised as new data arrive.[4]

The results of this paper show that the desired effect of forward guidance depends on the manner in which agents form their expectations. This outcome is first shown during normal economic times.[5] The impulse responses of the endogenous variables under adaptive learning fail to capture the precise effects a forward guidance shock has on the economy. There exists more persistence in the paths of the output gap and inflation under adaptive learning than rational expectations. Differences also occur when the central bank communicates to both rational expectations and adaptive learning agents the same forward guidance information such that the interest rate will equal zero for an extended period of time. The output gap and inflation return to long-run equilibrium quicker under rational expectations than adaptive learning. Under adaptive learning, the paths of the output gap and inflation overshoot and undershoot the rational expectations paths. Consequently, there exists larger variation of the paths of the output gap and inflation under adaptive learning than rational expectations. These effects occur because rational expectations agents fully understand the precise and

[4]Adaptive learning agents do not take into account they will update their beliefs in future periods. They believe that the beliefs they form every period are optimal. This methodology follows from the anticipated utility discussion from Kreps (1998).

[5]As will be discussed in Section 2.3, forward guidance is assumed to start after a large number of periods have passed, that is, after a period of economic stability.

positive effects of forward guidance on the economy. However, adaptive learning agents fail to understand the positive effects and must continually make adjustments to their beliefs causing them to overshoot and undershoot the rational expectations paths of the output gap and inflation.

The effectiveness of forward guidance is also examined under a period of economic crisis (e.g. a recession). The policy experiment includes a scenario where forward guidance is implemented to combat the effects of a downturn in the economy. The results show the effects of forward guidance under rational expectations are overstated relative to adaptive learning. Specifically, the value of the output gap is higher under the assumption of rational expectations than adaptive learning. The reason is that rational expectations agents base their expectations of future values of the endogenous variables on the true model of the economy. They understand the economic downturn and how forward guidance will precisely alleviate the economy. However, adaptive learning agents observe the economic downturn, but fail to fully understand how forward guidance will improve the economy. They are *estimating* the effects of forward guidance on the economy as their forecasts are based on an econometric model.

Overall, the results of the paper suggest a main finding: policymakers should exercise caution when recommending forward guidance policy. If monetary policy is based on a model with the standard rational expectations hypothesis, which assumes agents know the true structure of the model, the results may be misleading relative to a more plausible theory of expectations formation (e.g. adaptive learning). Specifically, during an economic crisis, the predicted effects of forward guidance under the rational expectations assumption are overstated in comparison to adaptive learning.

2.1.1 Previous Literature

This paper contributes to the growing literature on unconventional monetary policy. Eggertsson and Woodford (2003) explain that the expectations channel plays a key role on the economy when interest rates are at the ZLB and at any level. Specifically, they describe that the future path of short-term interest rates affects long-term interest rates and asset prices, and thus, the management of expectations about future interests rates affects agents' optimal decisions. De Graeve, Ilbas, and Wouters (2014) find that the effectiveness of forward guidance does not necessarily work through decreasing the long-run interest rate, contrary to previous studies. The type of forward guidance and lack of information about the underlying reasons for implementing forward guidance (e.g. monetary stimulus or sign of future economic crisis) can dampen the effects of this monetary policy tool. In addition, recent literature has found large effects from forward guidance. Carlstrom, Fuerst, and Paustian (2012) show that standard New Keynesian models with the interest rate fixed for a finite period of time result in extreme responses of output and inflation. Del Negro et al. (2012) construct a Dynamic Stochastic General Equilibrium (DSGE) model with forward guidance, which produces large responses of macroeconomic variables to forward guidance. Del Negro et al. (2012) state that the long-term bond yield drives these unusually high responses. As will be discussed in Section 2.3.3, this current paper suggests that the exceedingly large responses to forward guidance found in the previously mentioned articles could be due to the manner in which expectations are modeled.

The model in this paper utilizes time-contingent forward guidance since there has been recent evidence of its effectiveness. Gürkaynak, Sack, and Swanson (2005) find empirical evidence that FOMC statements about the future path of the policy rate greatly contribute to the changes in the long-term interest rates. Swanson and Williams (2014) show that Federal Reserve forward guidance announcements affect market expectations about future policy. Woodford (2012) also explains that forward guidance has had an impact on market participants. Using overnight interest rate swaps (OIS) to measure market expectations about the policy rate in Canada, Woodford (2012) displays that OIS rates immediately changed upon release of the Bank of Canada's forward guidance statement. The work of Chang and Feunou (2013) show that the Bank of Canada's forward guidance statement in 2009 had positive effects on the economy by reducing uncertainty about future monetary policy rates. A reduction in interest rate uncertainty can affect levels of investment, output, and unemployment in the economy as described by Baker, Bloom, and Davis (2013). Femia, Friedman, and Sack (2013) show evidence that financial variables, such as Treasury yields and equity prices, reacted favorably to the Federal Reserve's time-contingent forward guidance announcements.

By analyzing the role of expectations formation on forward guidance, this paper builds on the adaptive learning and policy literature. Mitra, Evans, and Honkapohja (2012) examine the effects of the fiscal authority giving guidance on the future course of government purchases and taxes. The results show that a temporary change in fiscal policy leads to different effects on adaptive learning and rational expectations agents. The adaptive learning output multipliers seem to match empirical data more than its rational expectations counterparts. Eusepi and Preston (2010) investigate the link between adaptive learning and central bank communication strategies. Increased central bank communication, such as communicating the monetary policy rule and the variables within the rule, can lead to increased macroeconomic stability. Preston (2006) studies forecast-based monetary policy rules and adaptive learning. He finds that a central bank that understands the basis of private sector forecasts can aid in increasing macroeconomic stability.

The remaining sections of the paper are organized as follows. Section two presents the New Keynesian model with forward guidance. Section three discusses expectations formation under both rational expectations and adaptive learning. Section four presents the outcomes of forward guidance under both rational expectations and adaptive learning. Section five examines the results under different parameter schemes. Section six concludes.

2.2 Model

The aggregate dynamics of the economy are described by a New Keynesian model derived under (potentially) non-rational expectations (see Preston [2005]). There exists a continuum of households indexed by $i \in [0,1]$. Households maximize expected future discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i;\xi_T) - \int_0^1 v(h_T^i(j);\xi_T)dj \right] \tag{2.1}$$

where β is the discount factor and is bounded between zero and one. Utility depends on C_T^i, which is consumption by household i of goods in the economy. Households also receive a disutility when supplying labor, $h_T^i(j)$, for the production of each good j. ξ_T denotes an aggregate preference shock. $\hat{E}_t^i$ denotes (potentially) non-rational expectations that satisfy standard probability laws, such as $\hat{E}_t^i \hat{E}_{t+1}^i = \hat{E}_t^i$. Beliefs are assumed to be homogeneous across agents, but agents do not know this fact.

A household is subject to a budget constraint that takes the following form

$$M_t^i + B_t^i \leq (1+i_{t-1}^m)M_{t-1}^i + (1+i_{t-1})B_{t-1}^i + P_t Y_t^i - T_t - P_t C_t^i \tag{2.2}$$

where T_t denotes lump-sum taxes and transfers, M_t^i is money holdings, and i_t^m denotes interest paid on money balances. Asset markets are assumed to be incomplete such that household's can transfer wealth between periods through a one-period riskless bond B_t^i. Accordingly, i_t is the interest paid on bonds. Y_t^i is household i's real income. P_t is the aggregate price index, and $P_tY_t^i$ denotes household i's nominal income which is given by

$$P_tY_t^i = \int_0^1 [w_t(j)h_t^i(j) + \Pi_t(j)]dj \tag{2.3}$$

A household receives wages $w_t(j)$ for hours worked towards the production of good j, $h_t^i(j)$. Since each household owns an equal part of each firm, it receives profits from the sale of good j, $\Pi_t(j)$. Furthermore, even though it is present in the budget constraint, money does not show up in the utility function. It is assumed that money balances do not relieve any transactional frictions. However, a household may choose to hold money balances because it provides a financial return.

The aggregate variables C_t^i and P_t are assumed to be defined by the Dixit-Stiglitz constant-elasticity-of-substitution aggregator

$$C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{2.4}$$

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \tag{2.5}$$

where $\theta > 1$ is the elasticity of substitutions across differentiated goods, $c_t^i(j)$ describes household i's consumption of good j, and $p_t(j)$ is the price of good j.

By log-linearizing the intertemporal budget constraint and Euler equation, the following results are obtained

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \bar{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i \tag{2.6}$$

$$\hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1}) + g_t - \hat{E}_t^i g_{t+1} \tag{2.7}$$

where $\hat{\pi}_t$ is current inflation, $\sigma \equiv \frac{-U_c}{U_{cc}\bar{C}}$ defines the intertemporal elasticity of substitution, $g_t \equiv \sigma \frac{U_{c\xi}\xi_t}{U_c}$ denotes a preference shock, and $\bar{w}_t^i \equiv \frac{W_t^i}{P_t\bar{Y}}$ is share of real wealth ($W_t^i \equiv (1 + i_{t-1})B_{t-1}^i$) as a fraction of steady-state income. The " ˆ " symbol over variables denotes log deviations from steady state. By solving (2.7) backwards from date T to t, taking expectations at time t, plugging the result into (2.6), and integrating over i, the following equation for aggregate consumption emerges

$$\hat{C}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right] \tag{2.8}$$

Note that $\int_i \bar{w}_t^i di = 0$ since bonds are in zero net supply from market clearing. $\hat{E}_t = \int_i \hat{E}_t^i di$ denotes the average expectations operator. By imposing the market equilibrium condition $\hat{Y}_t = \hat{C}_t$ and defining the resulting equation in terms of the output gap $\hat{x}_t \equiv \hat{Y}_t - \hat{Y}_t^n$, the following equation emerges

$$\hat{x}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)\hat{x}_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \hat{r}_T^n] \tag{2.9}$$

where

$$\hat{r}_t^n \;=\; \rho_n \hat{r}_{t-1}^n + \varepsilon_t^n \tag{2.10}$$

and $\varepsilon_t^n \overset{iid}{\sim} N(0, \sigma_n^2)$. $\hat{Y}_t^n$ is the natural rate of output, that is, output prevailing under flexible prices, and $\hat{r}_t^n \equiv (\hat{Y}_{t+1}^n - g_{t+1}) - (\hat{Y}_t^n - g_t)$. Equation (2.9) relates the current output gap $\hat{x}_t$ to current and future expected values of the output gap, interest rate $\hat{i}_t$, inflation rate $\hat{\pi}_t$, and natural real interest rate shock $\hat{r}_t^n$. Households take into account the future values of the endogenous variables infinitely far into the future when choosing optimal consumption today. Intuitively, the expected course of a household's consumption pattern matters to its optimal consumption today. A household also knows future consumption patterns are affected by future values of income, interest rates, and inflation. Thus, expectations of these variables are important for decisions today.

The production side of the economy is populated by firms that operate in a monopolistically competitive environment. Each good is produced using labor from households. A firm is subject to a Calvo (1983) pricing scheme. Each period a fraction $0 < 1-\alpha < 1$ of producers can optimally reset their prices. The remaining α producers retain the same prices from the previous period. Furthermore, a good is produced following the production function $y_t(i) = A_t f(h_t(i))$ where A_t is a technology shock. The demand curve for good i is given by $y_t(i) = Y_t(p_t(i)/P_t)^{-\theta}$. The following Dixit-Stiglitz aggregate price index is assumed

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha) p_t^{*1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.11}$$

A firm maximizes its expected present discounted value of profits

$$\hat{E}_t^i \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T}[\Pi_T^i(p_t(i))] \tag{2.12}$$

where $Q_{t,T}$ describes the stochastic discount factor showing how firms value its future stream of income. The stochastic discount factor is given by

$$Q_{t,T} = \beta^{T-t} \frac{P_t}{P_T} \frac{U_c(Y_T, \xi_T)}{U_c(Y_t, \xi_t)} \tag{2.13}$$

The profit function is defined by

$$\Pi_T^i(p_t(i)) = Y_t P_t^{\theta} p_t(i)^{1-\theta} - w_t(i) f^{-1}(Y_t P_t^{\theta} p_t(i)^{-\theta}/A_t) \tag{2.14}$$

Maximizing (2.12) with respect to $p_t(i)$ yields the following first order condition

$$\hat{E}_t^i \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} Y_T P_T^{\theta} [\hat{p}_t^*(i) - \bar{\mu} P_T s_{t,T}(i)] = 0 \tag{2.15}$$

where $\bar{\mu} = \frac{\theta}{\theta-1}$, and $s_{t,T}$ is the firm's real marginal cost function. Furthermore, by substituting in the stochastic discount factor and real marginal costs into the firm's first order condition and then log linearizing around a zero inflation steady state, the following result is produced

$$\hat{p}_t^*(i) = \hat{E}_t^i \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\frac{1-\alpha\beta}{1+\omega\theta}(\omega + \sigma^{-1})\hat{x}_T + \alpha\beta\hat{\pi}_{T+1} \right] \tag{2.16}$$

ω defines the elasticity of a firm's real marginal cost function with respect to its output and θ measures the elasticity of substitution between differentiated goods. Note also that log linearizing (2.11) yields

$$\hat{\pi}_t = \hat{p}_t^*(1-\alpha)/\alpha \tag{2.17}$$

where $\hat{\pi}_t$ is current inflation. Integrating over i and plugging (2.17) into (2.16) yields the following equation for inflation

$$\hat{\pi}_t \;=\; \kappa\hat{x}_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\kappa\alpha\beta\hat{x}_{T+1} + (1-\alpha)\beta\hat{\pi}_{T+1} + \hat{\mu}_T] \tag{2.18}$$

where

$$\hat{\mu}_t \;=\; \rho_\mu \hat{\mu}_{t-1} + \varepsilon_t^\mu \tag{2.19}$$

and $\varepsilon_t^\mu \overset{iid}{\sim} N(0, \sigma_\mu^2)$.[6] Equation (2.18) defines the inflation rate as a function of current and future values of the output gap, inflation rate, and cost-push shock $\hat{\mu}_t$. ω describes

[6] As in Preston (2006), a supply shock μ_t is added.

the elasticity of a firm's real marginal cost function with respect to its own output, and $\kappa \equiv \frac{(1-\alpha)}{\alpha}\frac{(1-\alpha\beta)}{(1+\omega\theta)}(\omega+\sigma^{-1}) > 0$. The optimal decisions by firms are shown to depend on the long-run expected path of macroeconomic variables because of the assumption of sticky prices. A firm must be concerned that it will not be able to adjust its price in future periods regardless of future economic conditions. Thus, optimal pricing decisions today require firms to forecast future states and values of economic variables.[7]

The model is closed by describing the central bank of the economy. The central bank follows a monetary policy rule that takes the following form

$$\hat{i}_t = \chi_\pi \hat{\pi}_t + \chi_x \hat{x}_t + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^{R} \tag{2.20}$$

The short-term nominal interest rate changes based on the output gap, inflation rate, monetary policy shock, and forward guidance shocks. ε_t^{MP} defines an unanticipated monetary policy shock and is *i.i.d.* In order to incorporate forward guidance into the model, the monetary policy rule is augmented with anticipated shocks following Del Negro et al. (2012) and Laseen and Svensson (2011). Each anticipated or forward guidance shock ($\varepsilon_{l,t-l}$) is contained in the last term in equation (2.20) and is *i.i.d.* Intuitively, the forward guidance shock can be thought of as an announcement by the central bank in period $t-l$ that the interest rate will change l periods later, i.e. in period t. If the central bank has been communicating guidance on the interest rate for L periods ahead, there would be $1, 2, 3, \ldots, L$ forward guidance shocks that affect the monetary policy rule in period t. Thus, L corresponds to the length of the

[7] Another approach to modeling learning and (potentially) non-rational expectations in macroeconomic models regards the "Euler-equation" method presented in Evans and Honkapohja (2001), where only one period ahead forecasts of the endogenous variables show up in the model's equations under both rational expectations and adaptive learning. For a comparison between the "infinite-horizon" and Euler-equation approach to learning, see Evans, Honkapohja, and Mitra (2013)

forward guidance horizon announced by the central bank. The last term in equation (2.20) can also be thought of as the sum of all forward guidance commitments stated by the central bank $1, 2, ...,$ and L periods ago that affect the nominal interest rate in period t. Following Del Negro et al. (2012) and Laseen and Svensson (2011), the system is also augmented with L state variables $v_{1,t}, v_{2,t}, ..., v_{L,t}$. The law of motion for each of these state variables is given by

$$v_{1,t} = v_{2,t-1} + \varepsilon^R_{1,t} \tag{2.21}$$

$$v_{2,t} = v_{3,t-1} + \varepsilon^R_{2,t} \tag{2.22}$$

$$v_{3,t} = v_{4,t-1} + \varepsilon^R_{3,t} \tag{2.23}$$

$$\vdots$$

$$v_{L,t} = \varepsilon^R_{L,t} \tag{2.24}$$

In other words, each component of $v_t = [v_{1,t}, v_{2,t}, ..., v_{L,t}]'$ is the sum of all central bank forward guidance commitments known in period t that affect the interest rate $1, 2, ...,$ and L periods into the future, respectively.[8] It should be noted that equations (2.21) – (2.24) can be simplified to find that $v_{1,t-1} = \sum_{l=1}^{L} \varepsilon^R_{l,t-l}$. In addition, equations (2.20) – (2.24) provide a computationally tractable method to model forward guidance. Since the forward guidance shocks in equation (2.20) equal $v_{1,t-1}$, the forward guidance shocks can be put into a vector of predetermined variables in standard state-space form. As described by Laseen and Svensson (2011), standard solution techniques then can be used to solve the final system of equations. Another reason to model forward guidance in this way is that it relieves the concern of the existence of multiple solutions. As described in Honkapohja and Mitra (2005)

[8]In the terminology of Laseen and Svensson (2011), $v_{1,t}, v_{2,t}, ..., v_{L,t}$ are described as central bank "projections" (p. 10) of what $\sum_{l=1}^{L} \varepsilon^R_{l,t-l}$ will be $1, 2, ...,$ and L periods into the future, respectively.

and Woodford (2005), indeterminacy can arise if forward guidance is instead modeled as pegging the interest rate to a certain value.[9] For instance, without a monetary policy that responds to economic fluctuations, real disturbances to the economy can produce a multitude of equilibrium responses of the endogenous variables.

The following example presents the case where the central bank's forward guidance horizon is 2 periods ahead, i.e. $L = 2$. The model's system of equations consists of $v_{1,t}$ and $v_{2,t}$ whose laws of motion are defined as

$$v_{1,t} = v_{2,t-1} + \varepsilon^R_{1,t} = \varepsilon^R_{2,t-1} + \varepsilon^R_{1,t} \tag{2.25}$$

$$v_{2,t} = \varepsilon^R_{2,t} \tag{2.26}$$

Thus, $v^R_{1,t}$ defines the sum of all forward guidance commitments by the central bank known in period t that affect the interest rate one period later. $v^R_{1,t}$ consists of current period forward guidance affecting the interest rate one period later, $\varepsilon^R_{1,t}$, and previous period's forward guidance affecting the interest rate two periods later, $v_{2,t-1} = \varepsilon^R_{2,t-1}$. $v_{2,t}$ is the sum of all forward guidance commitments by the central bank known in period t that affect the interest rate two periods later. Since the forward guidance horizon is two periods, $v_{2,t}$ consists of current period forward guidance affecting the interest rate two periods later, $\varepsilon^R_{2,t}$.[10]

The ZLB on interest rates is also enforced. Forward guidance has gained attention due to interest rates effectively reaching the ZLB because of the 2007-2009 global financial recession.

[9] Carlstrom, Fuerst, and Paustian (2012) show that determinacy can arise from an interest rate peg if terminal conditions are known and a standard monetary policy rule is followed after the interest rate peg. However, unusually large responses of the output and inflation are found through this process.

[10] A constant interest rate path can still be achieved by modeling forward guidance with equations (2.20)-(2.24). As will be described in Section 2.3.2.2, the forward guidance shocks can be chosen such that the interest rate equals a certain value for a fixed amount of periods into the future.

Thus, it seems natural to model the ZLB on nominal interest rates when simulating forward guidance. Specifically, equations (2.9) and (2.20) become

$$\hat{x}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}[(1-\beta)\hat{x}_{T+1} - \sigma(i_T - i^* - \hat{\pi}_{T+1}) + \hat{r}^n_T] \tag{2.27}$$

$$i_t = max\{i^* + \chi_\pi \hat{\pi}_t + \chi_x \hat{x}_t + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon^R_{l,t-l}, 0\} \tag{2.28}$$

where $i^* = r^* + \pi^*$ is the steady-state nominal interest rate.[11]

To summarize, the aggregate dynamics of the economy with forward guidance are defined by the output gap, inflation rate, AR(1) shock processes, monetary policy rule with forward guidance, and the laws of motion of the sum of central bank commitments, that is, equations $(2.9), (2.10), (2.18), (2.19),$ and $(2.20) - (2.24)$. With enforcement of the ZLB, equations (2.27) and (2.28) are used instead of (2.9) and (2.20). To simplify notation, the " ˆ " symbol over the variables is removed for the remainder of the paper.

2.3 Expectation Formation

This paper assumes agents form expectations following either the rational expectations hypothesis or adaptive learning. The difference between the two types of expectations formation regards the amount of knowledge agents hold about the economy (See, for example, Marcet and Sargent (1989), Evans and Honkapohja (2001), and Evans, Honkapohja, and Mitra (2009).). Under rational expectations, agents know the structure of the model, parameters

[11] In a zero steady-state inflation rate, $\pi^* = 0$. The model implied steady-state real interest rate $r^* = \beta^{-1} - 1$.

of the model (e.g. σ, κ, etc.), distribution of the error terms, and beliefs of other agents. They compute expectations based off the true model of the economy. Under adaptive learning, agents do not know the true model of the economy, and thus, cannot compute precise expectations as under rational expectations. Instead, they operate as econometricians by forming an econometric model to forecast values of the endogenous variables. Their model includes the variables in the rational expectations solution. Adaptive learning agents estimate the values of the model's parameters using standard econometric methods. As new information becomes available every period, they appropriately adjust their forecasts.

Rational Expectations–The model defined by equations $(2.9), (2.10), (2.18), (2.19)$, and $(2.20) - (2.24)$ can be simplified under the assumption of rational expectations. Agents with rational expectations understand the beliefs of other agents and are able to compute the aggregate probabilities of the model. As shown in Preston (2005), this additional information simplifies the infinite horizon model to the "benchmark" one step ahead New Keynesian model. Specifically, equations (2.9) and (2.18) become

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + r_t^n \tag{2.29}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t \tag{2.30}$$

The model with rational expectations can be solved using standard techniques, such as one suggested by Sims (2002). The model can be written in general state-space form as suggested by Sims (2002). This form is defined as

$$\widetilde{\Gamma}_0 \widetilde{Y}_t = C + \widetilde{\Gamma}_1 \widetilde{Y}_{t-1} + \widetilde{\Gamma}_2 \widetilde{\epsilon}_t + \widetilde{\Gamma}_3 \zeta_t \tag{2.31}$$

where

$$\widetilde{Y}_t = [x_t, \pi_t, i_t, r_t^n, \mu_t, v_{1,t}, v_{2,t}, \ldots, v_{L,t}, E_t x_{t+1}, E_t \pi_{t+1}]' \quad (2.32)$$

$$\widetilde{\epsilon}_t = [\varepsilon_t^n, \varepsilon_t^\mu, \varepsilon_t^{MP}, \varepsilon_{1,t}^R, \varepsilon_{2,t}^R, \ldots, \varepsilon_{L,t}^R]' \quad (2.33)$$

C defines a vector of constants of required dimensions. ζ_t defines the vector of expectational errors (e.g. $\zeta_t^\pi = \pi_t - E_{t-1}\pi_t$) of required dimensions. Using standard techniques to solve the model with rational expectations (e.g. Sims [2002]) and the parameter values in Table 2.1, the solution to the system under rational expectations is

$$\widetilde{Y}_t = \widetilde{C} + \xi_1 \widetilde{Y}_{t-1} + \xi_2 \widetilde{\epsilon}_t \quad (2.34)$$

where the matrices $\widetilde{C}$, ξ_1, and ξ_2 are defined in Appendix B.1.[12]

Adaptive Learning–In order to evaluate the expectations in equations (2.9) and (2.18) under adaptive learning, agents act as econometricians by forming a model based on variables that appear in the rational expectations solution and estimate the coefficients. This model is labeled the "Perceived Law of Motion" (PLM) and is constructed from the minimum state variable (MSV) solution that exists under rational expectations.[13] The PLM is defined as

$$Y_t = a + bv_t + cw_t + dv_{1,t-1} + \varepsilon_t \quad (2.35)$$

[12]Discussion of the parameter values can be found in Table 2.1 in Section 2.3.1.

[13]This paper focuses on a version of the model that is determinate so that the PLM is based on the unique non-explosive rational expectations equilibrium. The parameter values in Table 2.1 verify that the rational expectations solution is determinate.

where

$$Y_t = [x_t, \pi_t, i_t]' \tag{2.36}$$

$$v_t = [v_{1,t}, v_{2,t}, ..., v_{L,t}]' \tag{2.37}$$

The vector $w_t = [r_t^n, \mu_t]'$ is defined by

$$w_t = \widetilde{\phi} w_{t-1} + \bar{\varepsilon}_t \tag{2.38}$$

where

$$\widetilde{\phi} = \begin{bmatrix} \rho_n & 0 \\ 0 & \rho_\mu \end{bmatrix} \tag{2.39}$$

$$\bar{\varepsilon}_t = [\varepsilon_t^n, \varepsilon_t^\mu]' \tag{2.40}$$

By rewriting equations (2.21) − (2.24), the vector v_t becomes

$$v_t = \Phi v_{t-1} + \eta_t \tag{2.41}$$

where

$$\eta_t = [\varepsilon^R_{1,t}, \ldots, \varepsilon^R_{L,t}]' \tag{2.42}$$

and Φ is an L x L matrix given by

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & & & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \tag{2.43}$$

(2.44)

a, b, c, and d are unknown coefficient matrices of appropriate dimensions that agents estimate and learn about over time.[14] Furthermore, the addition of $v_{1,t-1}$ is a necessary component of the PLM since it is present in the rational expectations solution shown in Appendix B.1 and not contained in the vector v_t.[15]

An important component of adaptive learning models regards the information available to agents when they form expectations. In this paper, adaptive learning agents are assumed to

[14] In the PLM, the time subscript is left off the coefficients to emphasize that adaptive learning agents believe current period forecasts are optimal and do not take into account they will be updating their beliefs every period. However, as will be described later, the PLM coefficients will evolve over time.

[15] Since this paper restricts attention to fundamentals solutions and Y_{t-1} does not appear in equations (2.9), (2.18), and (2.20), the PLM does not contain Y_{t-1}.

know the values of the regressors in the PLM and previous period's coefficient estimates when forming beliefs about the future. They update their parameter estimates at the end of the period. This assumption avoids the simultaneous determination of current period coefficient estimates and endogenous variables when forming expectations and making optimal decisions.[16] The *i.i.d.* monetary policy shock is also assumed to be unobserved.[17] Furthermore, the following is the timeline of events:

1. At the beginning of period t, v_t, and w_t are observed by the agents and added to their information set.

2. Agents use v_t, w_t, and $v_{1,t-1}$ as well as previous period's estimates (i.e. a_{t-1}, b_{t-1}, c_{t-1}, and d_{t-1}) to form expectations about the future.

3. Y_t is realized.

4. In order to update their parameter estimates, agents compute a least squares regression of Y_t on 1, v_t, w_t, and $v_{1,t-1}$.

Agents update their parameter estimates of the PLM by following the recursive least squares (RLS) formula

$$\phi_t = \phi_{t-1} + \tau_t R_t^{-1} z_t (Y_t - \phi_{t-1}^{'} z_t)' \quad (2.45)$$

$$R_t = R_{t-1} + \tau_t (z_t z_t^{'} - R_{t-1}) \quad (2.46)$$

[16] An alternative is to assume that agents use the coefficient estimates from the current period when forming expectations. This results in expectations and current period parameter estimates determined simultaneously when making optimal decisions.

[17] This is similar to Milani (2007).

where $\phi = (a, b, c, d)'$ contains the PLM coefficients to be estimated. R_t defines the precision matrix of the regressors in the PLM $z_t \equiv [1, v_t, w_t, v_{1,t-1}]'$. τ_t is known as the "gain" parameter and controls the response of ϕ_t to new information. The last expression in equation (2.45) defines the recent prediction error of the endogenous variables.

The gain parameter in equations (2.45) and (2.46) can either decrease over time or be fixed at certain values. In the decreasing gain or RLS case, $\tau_t = t^{-1}$ and past observations are equally weighted. Evans and Honkapohja (2001) explain that as $t \to \infty$ the coefficients in the PLM converge to the rational expectations coefficients with probability one. As is assumed in this current paper, the gain parameter can also be fixed at a certain value. Under this method called discounted or constant gain learning (CGL), $\tau_t = \bar{\tau}$ and the most recent observations play a larger role when updating agents' coefficients and expectations. Evans and Honkapohja (2001) describe that the coefficients in the PLM converge in distribution to their rational expectations values with a variance that is proportional to the constant gain parameter. CGL may be a more realistic way to model learning since it allows agents to update their beliefs every period to new information as a real-life econometrician revising his or her forecasts every period.

Agents solve for $\hat{E}_t Y_{T+1}$ by using equation (2.35). For any $T \geq t$, their expectations infinite periods ahead are given by

$$\begin{aligned}\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} Y_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} a_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} b_{t-1} v_{T+1} \\ + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} c_{t-1} w_{T+1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} d_{t-1} v_{1,T}\end{aligned} \tag{2.47}$$

$$\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} Y_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} a_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} b_{t-1} v_{T+1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} c_{t-1} w_{T+1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} d_{t-1} v_{1,T} \tag{2.48}$$

By noting the geometric sums and expectations of v_t twelve periods ahead or greater equal the zero vector, equations (2.47) and (2.48) simplify to equal

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} Y_{T+1} = (1-\beta)^{-1} a_{t-1} + b_{t-1}\Phi(I_L - \beta\Phi)^{-1}(I_L - (\beta\Phi)^{11})v_t + c_{t-1}(I_2 - \beta\widetilde{\phi})^{-1}\widetilde{\phi} w_t + d_{t-1}[1, \beta, \beta^2, \ldots, \beta^{11}]v_t \tag{2.49}$$

$$\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} Y_{T+1} = (1-\alpha\beta)^{-1} a_{t-1} + b_{t-1}\Phi(I_L - \alpha\beta\Phi)^{-1}(I_L - (\alpha\beta\Phi)^{11})v_t + c_{t-1}(I_2 - \alpha\beta\widetilde{\phi})^{-1}\widetilde{\phi} w_t + d_{t-1}[1, \alpha\beta, (\alpha\beta)^2, \ldots, (\alpha\beta)^{11}]v_t \tag{2.50}$$

Equations (2.49) and (2.50) are substituted into equations (2.9) and (2.18) to give

$$Y_t = \Gamma_0(\phi_{t-1}) + \Gamma_1(\phi_{t-1})Y_{t-1} + \Gamma_2(\phi_{t-1})v_t + \Gamma_3(\phi_{t-1})\widetilde{w}_t \tag{2.51}$$

where

$$\widetilde{w}_t = [w_t, \varepsilon_t^{MP}]' \tag{2.52}$$

Equation (2.51) is called the "Actual Law of Motion" (ALM) and describes the actual evolution of the endogenous variables implied by the PLM (2.35).

2.4 Results

2.4.1 Parameterization

This section details the calibration values for the model's parameters, which are shown in Table 2.1. The discount rate, β, is set to equal 0.99 which is a common value found in the literature. The parameter representing the intertemporal elasticity of substitution is fixed at one. This value has been assumed *a priori* in Smets and Wouters (2003). κ is set to equal 0.1. This number roughly corresponds to a high degree of price stickiness, α, found in empirical work by Klenow and Malin (2010), a value of ω found in Giannoni and Woodford (2004), and a value of θ found in the literature (e.g. Gertler and Karadi [2011]). Monetary policy positively responds to the output gap, and positively adjusts at more than a one-to-one rate to the inflation rate. $\chi_x = 0.125$ follows from Branch and Evans (2013). The value of χ_π closely follows empirical adaptive learning work by Milani (2007). The structural disturbances are not assumed to exhibit high persistence. The distribution of the white noise shocks is not assumed to be highly dispersed. There also is no covariance between the structural shocks.

The current paper examines results for the CGL case. In regards to choosing the CGL parameter $\bar{\tau}$, this paper uses 0.02. This choice is close to the results used in the literature, such as Orphanides and Williams (2005), Milani (2007), and Branch and Evans (2006). For robustness, the current methodology also examines the results under different values of $\bar{\tau}$.

Table 2.1: Parameter Values

	Description	Value
σ	IES	1
β	Discount Factor	0.99
κ	Function of Price Stickiness	0.1
α	Price Stickiness	0.75
χ_π	Feedback Inflation	1.4
χ_x	Feedback Output Gap	0.125
ρ_n	Autoregressive Demand	0.5
ρ_μ	Autoregressive Cost-Push	0.5
σ_n	Demand Shock	0.001
σ_μ	Cost-Push Shock	0.001
σ_i	M.P Shock	0.001
$\sigma_{1,i}$	1 Period Ahead FG Shock	0.001
$\sigma_{2,i}$	2 Period Ahead FG Shock	0.001
$\sigma_{3,i}$	3 Period Ahead FG Shock	0.001
$\sigma_{4,i}$	4 Period Ahead FG Shock	0.001
$\sigma_{5,i}$	5 Period Ahead FG Shock	0.001
$\sigma_{6,i}$	6 Period Ahead FG Shock	0.001
$\sigma_{7,i}$	7 Period Ahead FG Shock	0.001
$\sigma_{8,i}$	8 Period Ahead FG Shock	0.001
$\sigma_{9,i}$	9 Period Ahead FG Shock	0.001
$\sigma_{10,i}$	10 Period Ahead FG Shock	0.001
$\sigma_{11,i}$	11 Period Ahead FG Shock	0.001
$\sigma_{12,i}$	12 Period Ahead FG Shock	0.001
L	FG Horizon	12
$\bar{\tau}$	CGL	0.02

Note: FG stands for forward guidance.

The value for the length of the forward guidance horizon L is chosen to match time-contingent forward guidance by the Federal Reserve. This is based off the FOMC September 2012 statement:"the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." This announcement was one of the last FOMC statements to exclusively use time-contingent forward guidance language.

By taking "mid-2015" to be at most the end of the third quarter of 2015, the number of quarters from September 2012 to "mid-2015" is twelve. Thus, $L = 12$.

2.4.2 Normal Economic Times

2.4.2.1 Impulse Responses

In this section, impulse responses of the output gap and inflation rate to negative one unit monetary policy and forward guidance shocks under different expectation assumptions are examined in Figures 2.1 and 2.2.[18] The forward guidance shocks are the anticipated shocks found in equations (2.21) - (2.24). Since equation (2.51) exhibits a nonlinear structure, standard linear techniques to compute impulse responses under adaptive learning do not apply. To remedy this situation, this paper follows Eusepi and Preston (2011) by proceeding in the following manner. The model is simulated twice for $T + K$ periods, where K is the impulse response function horizon. The impulse responses are calculated starting in period $T + 1$.[19] In the first simulation, time period $T + 1$ includes a negative one unit shock. The K-period impulse response function is given by the difference between the first and second simulations over the final K periods. The process is then repeated for $5,000$ simulations and the mean impulse response across the $5,000$ simulations is calculated to arrive at the final impulse response trajectory. The impulse response function horizon is chosen to be twenty periods, that is, $K = 20$.

Impact–As seen in Figures 2.1 and 2.2, the initial response of the macroeconomic variables is approximately the same under both adaptive learning and rational expectations. This result

[18] A projection facility is utilized to ensure beliefs are not explosive.

[19] T is chosen to be a large number so that the adaptive learning coefficients converge to its stationary distribution.

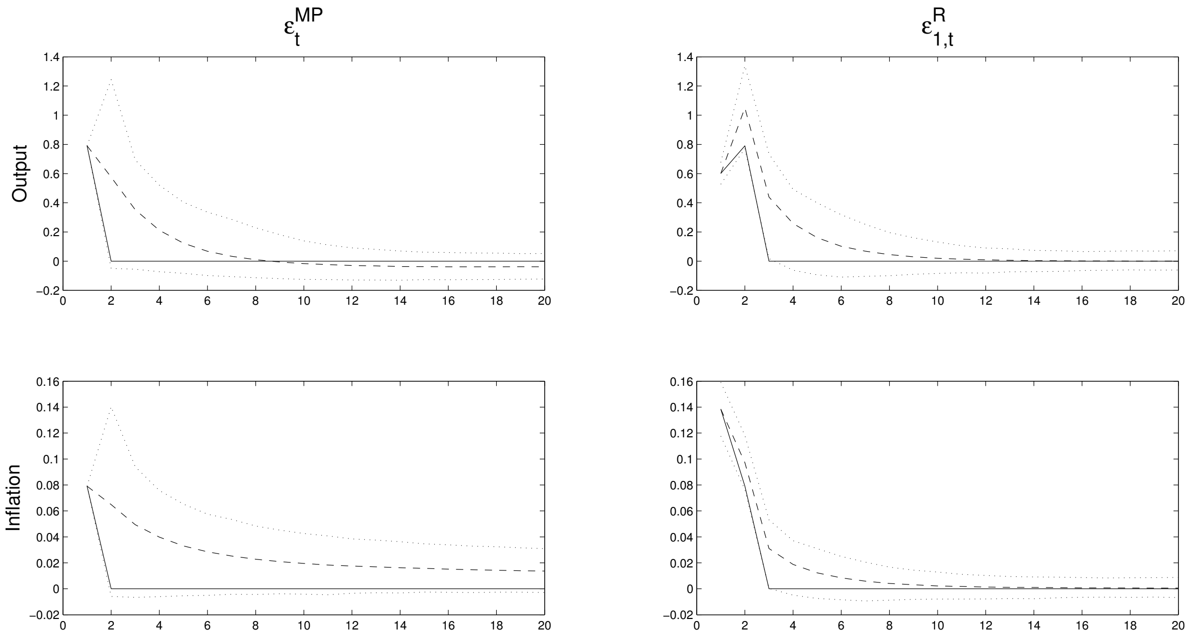


Figure 2.1: Impulse Responses of Endogenous Variables to Unanticipated and Forward Guidance Shocks. Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Lines: 95% Confidence Bands.

is not surprising since Evans and Honkapohja (2001) state that CGL coefficients converge to a Normal distribution centered around its rational expectations counterparts. Thus, the initial impact under adaptive learning could be greater or less than the initial impact under rational expectations.

After Impact–Figures 2.1 and 2.2 also display the impulse responses after the forward guidance announcement is known to agents. From the household's perspective, they must optimally allocate consumption across time based on their expectations of future variables. Since they know that the interest rate will decrease in the future, a household changes its optimal consumption across time and increases current consumption. In addition, firms know they may not be able to change their price in the future regardless of the state of the economy. Thus, they take into account expectations of future variables as seen in equation

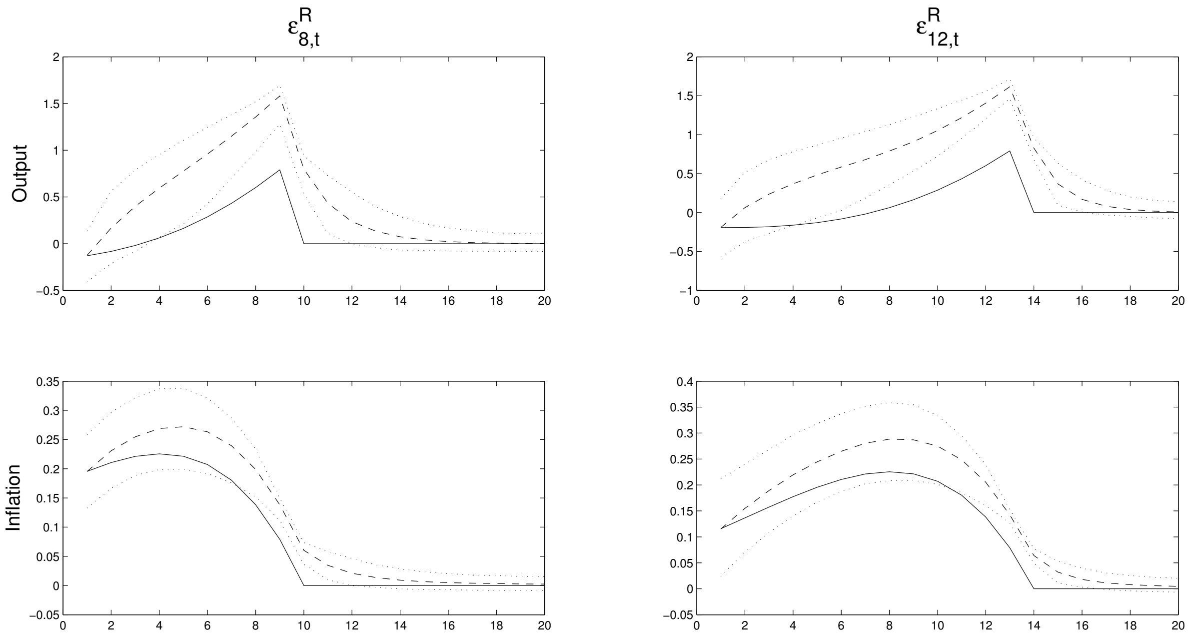


Figure 2.2: Impulse Responses of Endogenous Variables to Forward Guidance Shocks. Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Lines: 95% Confidence Bands.

(2.18). When the central bank announces that the interest rate will increase in the future, a firm knows that the future output gap and inflation will be affected, and thus, this action affects current pricing decisions. Furthermore, there exists a larger and more delayed effect on the economy under a forward guidance shock than under an unanticipated monetary policy shock. This result is similar to Milani and Treadwell (2012).

The impulse responses show that adaptive learning agents fail to understand the precise effect an announcement to lower the future interest rate will have on the economy. Adaptive learning agents know the forward guidance announcement announced by the central bank. However, since they do not understand the precise effect this shock will have on the economy, adaptive learning agents are continually readjusting their forecasts each period causing the impulse responses to exhibit more persistence than under rational expectations. In addition, when the forward guidance shock has been realized upon the economy, there exists a greater substitution effect under adaptive learning than rational expectations. Adaptive learning agents substitute into more consumption than rational expectations agents. The former agents overshoot their rational expectations counterparts. This conclusion occurs because rational expectations agents precisely know how the anticipated changes in monetary policy will affect the endogenous variables at later dates. However, adaptive learning agents imprecisely understand how a commitment to lower the future interest rate will have on the economy since they do not know the true model of the economy.

After Shock Realized–The impulse response graphs of rational expectations and adaptive learning do not follow the same path after the shock is realized upon the economy. The impulse responses with rational expectations agents converge quicker to zero percentage deviation from the unshocked series. Rational expectations agents understand that the shock will not occur in the future and they quickly adjust their expectations. However, the impulse responses under adaptive learning exhibit more persistence than the impulse responses under rational expectations. This outcome is present because the dynamics of the impulse responses under adaptive learning are driven by adjustments in the beliefs of the agents. Adaptive learning agents revise their estimates of the parameters of the economy each period, while rational expectations agents fully understand the model's parameters. The impulse responses of a conventional monetary policy shock shown in the first column of Figure 2.1 also display the same difference in persistence.

The results coincide with the literature on adaptive learning. The outcomes match Eggertsson (2008) who found that temporary policy shifts do not have as large of an effect on the economy as permanent policy shifts under the assumption of rational expectations. The persistence results also coincide with Milani (2007) who found that a DSGE model with constant-gain learning generates persistence in the macroeconomic variables.

To summarize, the message from this section is that adaptive learning agents fail to understand the precise effect a forward guidance announcement has on the economy. When the forward guidance shock is known to agents, the output gap and inflation rate under adaptive learning proceed in a different path than under rational expectations. After the shock has been realized, rational expectations agents quickly adjust their expectations to the knowledge that the shock is gone, while adaptive learning agents' beliefs are more persistent. These results are attributed to rational expectations agents precisely understanding the effects forward guidance has on the economy, while the beliefs of adaptive learning agents slowly adjust.

2.4.2.2 Policy Exercise

The results displayed by the impulse response functions showed that adaptive learning agents failed to understand the precise effects forward guidance has on the economy. This current section shows this conclusion through a different scenario. Specifically, the central bank would like to keep the interest rate fixed at a certain level $\bar{i}$ for $L+1$ periods. The experiment is described next and is motivated by the policy exercise described in Del Negro et al. (2012).

Suppose at the beginning of period T, the central bank implements forward guidance such that the interest rate will be fixed at $\bar{i} = 0$ in period T and L periods into the future. This announcement corresponds to an unanticipated shock in period T and news about the future interest rate $1, 2, \ldots, L$ periods into the future. In this scenario, the monetary policymaker's job is to choose ε_T^{MP} and $\eta_T = [\varepsilon_{1,T}^R, \ldots, \varepsilon_{L,T}^R]'$ such that the interest rate in periods T to $T+L$ equals $\bar{i}$. The central bank also believes that agents hold rational expectations, which is a common assumption in macroeconomic literature. To show that adaptive learning agents respond differently to the same forward guidance information, the adaptive learning agents are given the same guidance on the interest rate as under rational expectations. Furthermore, the exercise is assumed to start in period T.[20] The model is then simulated from T to the end of the forward guidance horizon $T + L$. The process is then repeated $5,000$ times and the mean across the $5,000$ simulations is calculated.

This policy exercise also assumes that the central bank is committed to its goal of $\bar{i}$ every period during the forward guidance horizon. Rational expectations agents precisely understand the central bank's guidance, and thus, the interest rate each period implied by rational expectations equals $\bar{i}$. Since the adaptive learning process is different than rational expectations, the same forward guidance will not give a model implied $\bar{i}$ during the forward guidance horizon. To model a commitment to $\bar{i} = 0$, the central bank chooses ε_t^{MP} each period over the forward guidance horizon to ensure the interest rate equals $\bar{i}$.[21]

Figure 2.3 compares the dynamics under rational expectations and adaptive learning for the output gap and inflation. The values of the output gap and inflation during the forward guidance horizon are averaged across simulations. The solid line represents rational expectations while the dashed line line displays the adaptive learning path. Under both expectations assumptions, forward guidance has an obvious stimulative effect on impact. Since interest rates are lowered, the output gap and inflation increase. As time elapses and the forward guidance horizon draws to an end, the stimulative effects of this central bank policy fade away. In addition, since adaptive learning agents' beliefs are distributed around its rational expectations counterparts, the initial effect is approximately the same under both series. However, the effect under adaptive learning could vary from rational expectations depending on adaptive learning agents' beliefs used at time T to forecast future variables.

[20] T is chosen to be a large number so that the adaptive learning coefficients converge to its stationary distribution.

[21] This adjustment seems fair since agents' expectations in real life about the future interest rate might not respond as exactly as the central bank would want, and thus, the interest rate might not equal a model implied $\bar{i}$.

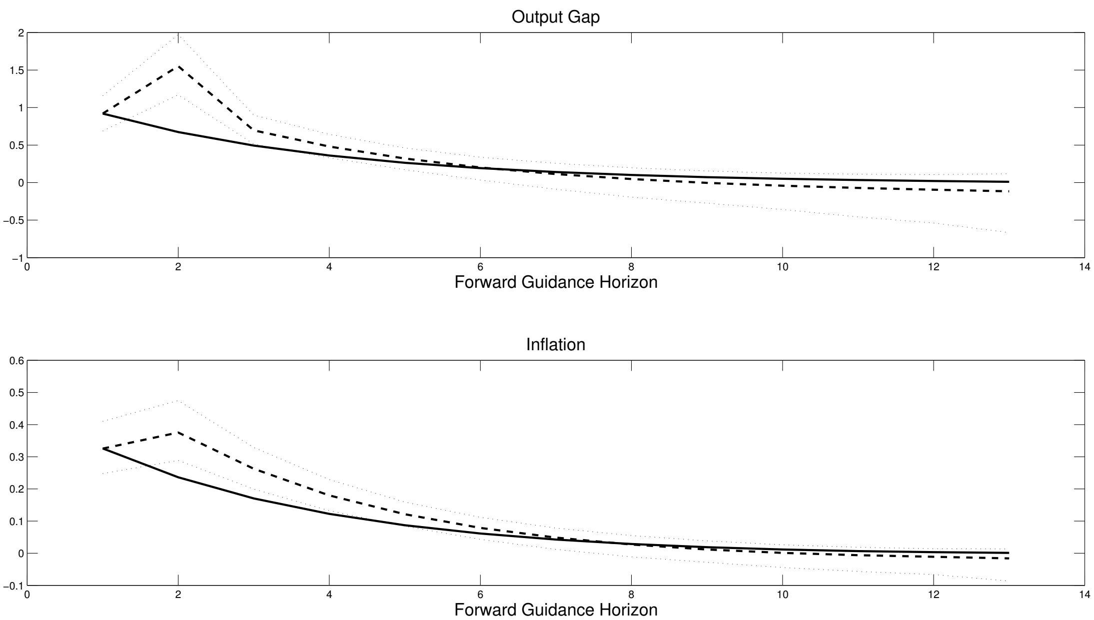


Figure 2.3: Dynamics of the Output Gap and Inflation in Response to Forward Guidance. Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Lines: 95% Confidence Bands.

Figure 2.3 also shows that adaptive learning agents fail to understand how the same forward guidance commitments made under rational expectations will impact the economy under learning. This results in larger variation in both the output gap and inflation and a slower speed back to long-run equilibrium under adaptive learning than under rational expectations. The adaptive learning agents observe the unanticipated lowering of the interest rate in period T. In the next period, they adjust their expectations of the output gap and inflation upwards due to this previous information. The adaptive learning path then continues at a downward path quicker than under rational expectations. Furthermore, the effect from central bank forward guidance results in more pessimism under adaptive learning than under rational expectations at longer horizons. By having only partial information about the true model of the economy, adaptive learning agents fail to foresee the precise positive impact the forward guidance information has on the dynamics of the output gap and inflation. Thus, this aspect leads to a period of undershooting on the part of adaptive learning agents. The output gap and inflation under adaptive learning fall short of the paths of rational expectations at longer horizons. Rational expectations agents, however, precisely understand the effects of forward guidance on the output gap and inflation. They understand the stimulative effect forward guidance has on the economy, and thus, the output gap and inflation is higher than under adaptive learning over this latter period.

The source of this difference between the two paths regards the assumptions made under rational expectations and adaptive learning. The rational expectations agents know the true model and aggregate probabilities. They can infer the precise effect forward guidance has on the economy. However, the expectations of adaptive learning agents do not respond in the same way. Adaptive learning agents do not know the true model of the economy, and thus, cannot infer the proper aggregate probabilities and expectations. Even though they know the changes in the future path of interest rates implemented by the central bank, adaptive learning agents imprecisely understand how that guidance impacts the economy. Since they readjust their forecasts each period, adaptive learning agents overshoot and undershoot the paths implied by rational expectations. Moreover, this result shows a consequence of the decision of monetary policymakers. If the central bank assumes agents have rational expectations, which is standard in the macroeconomic literature, the predicted outcomes seem to be misleading. The more realistic assumption of adaptive learning displays a different path than what would occur under rational expectations.

2.4.3 Economic Crisis

In response to the 2007-2009 Great Recession, forward guidance was implemented by central banks around the world. With that event in mind, this section builds upon the previous subsection's exercise by considering forward guidance during an economic recession. The economy is assumed to start in period T, that is, after a period of economic stability (corresponding to say the period before the recent Great Recession).[22] The model is then simulated from T to the end of the forward guidance horizon $T+L$. As in the previous subsection, the central bank implements forward guidance by choosing the unanticipated monetary policy and anticipated forward guidance shocks such that the nominal interest rate equals zero from periods T to $T+L$. To capture features from the recent Great Recession, a large negative demand shock impacts the economy in period T, and causes a recession. A sequence of five more negative demand shocks follows so that the recession lasts six periods.[23] In the following periods, the shocks are drawn from a normal distribution. Thus, the forward guidance horizon spans a recession and normal times. The process is then repeated $5,000$ times and the mean across the $5,000$ simulations is calculated.

Figure 2.4 displays the macroeconomic effects of forward guidance during an economic recession. The graph shows the value of the output gap under adaptive learning minus the value of the output gap under rational expectations.[24] Under both expectations formation assumptions, the negative demand shocks cause the output gap to drop below its steady state value. However, the positive effects of forward guidance are overstated under the assumption of rational expectations relative to adaptive learning. Throughout the forward guidance horizon, the value of the output gap under rational expectations is higher than under adaptive learning. The former agents know the economy is in a recession and precisely understand how forward guidance will alleviate the economy as their expectations are based on the true model of the economy. However, adaptive learning agents observe the economic downturn, but fail to completely understand the positive effects of forward guidance. They

[22] This strategy also ensures the adaptive learning coefficients converge to its stationary distribution.

[23] This length is based on the duration of the Great Recession as defined by the National Bureau of Economic Research.

[24] A positive value on Figure 2.4 indicates the output gap under adaptive learning is higher than under rational expectations. A negative value implies the output gap under rational expectations is higher than under adaptive learning.

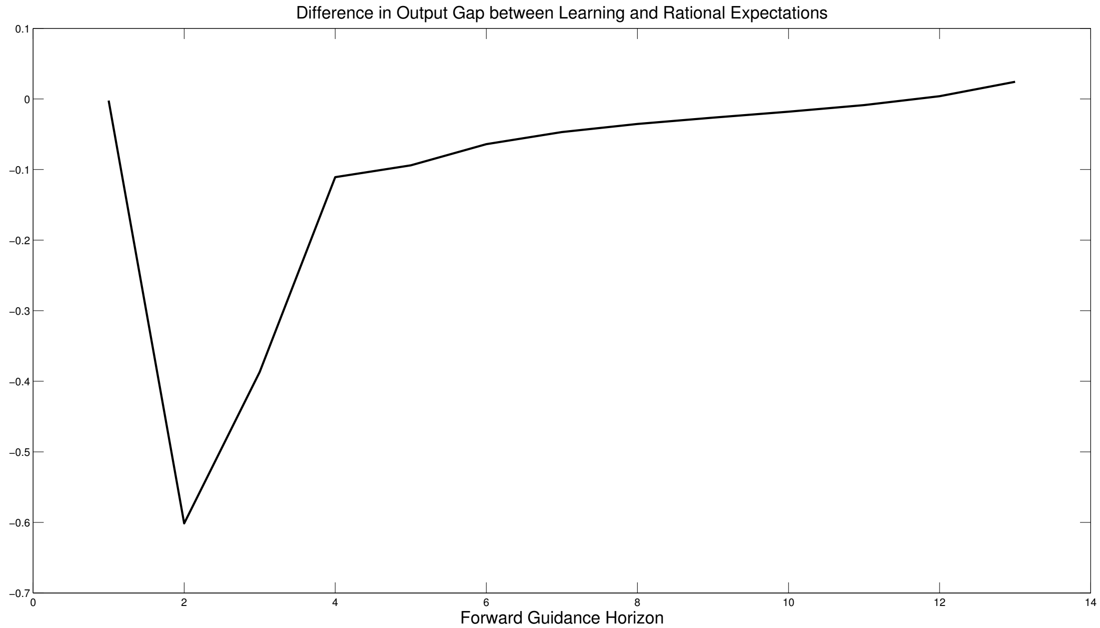


Figure 2.4: Macroeconomic Effects of Forward Guidance during an Economic Crisis

must *estimate* the effects of forward guidance on the economy as their forecasts are based on an econometric model. Thus, adaptive learning agents are slower to understand how forward guidance will alleviate the downturn in the economy.

Additional intuition for the results of this section is displayed in Figure 2.5, which shows the values of adaptive learning minus rational expectations of the discounted long-run expectations of the output gap, inflation and the interest rate across the forward guidance horizon. The adaptive learning agents are more pessimistic about the future output gap and inflation as their long-run expectations are lower than under rational expectations. The former are overestimating the ramifications of the downturn in the economy and their estimates of the effects of forward guidance are not strong enough to overcome this negative reaction. However, rational expectations agents understand the effects of the economic downturn and how forward guidance will precisely alleviate the economy.

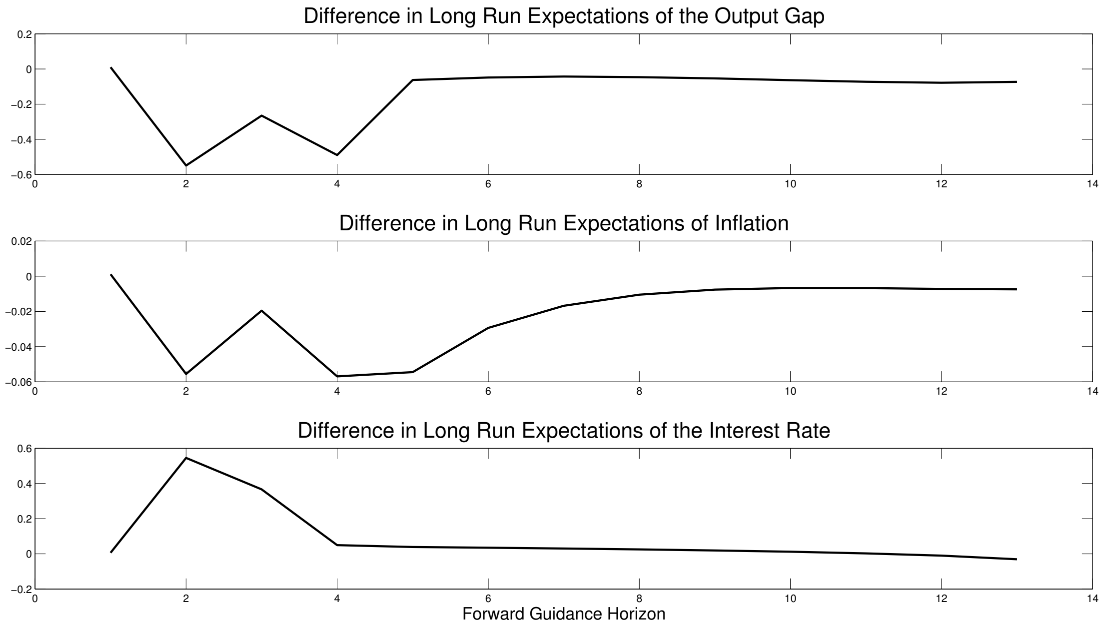


Figure 2.5: Difference between Adaptive Learning and Rational Expectations Infinite Horizon Expectations of the Macroeconomic Variables. A positive value indicates the value under adaptive learning is higher than under rational expectations. A negative value indicates the variable's value under adaptive learning is lower than under rational expectations.

The results in Figure 2.5 also relate to the empirical findings of Del Negro et al. (2012). Their model, which was solved under the assumption of rational expectations, produced an exceedingly large reaction of the macroeconomic variables to forward guidance statements. Del Negro et al. (2012) argued that the source of the excessive responses was an unusually large drop of the long-run interest rate to forward guidance statements relative to the data. In this current paper, the bottom panel of Figure 2.5 shows a comparable result: the long-run expectation of the interest rate is lower under rational expectations than adaptive learning. This produces larger responses of long-run expectations of the output gap and inflation, and consequently, the current output gap and inflation under rational expectations than adaptive learning. Thus, this result suggests two additional takeaways. A forward guidance model better matches the data under the assumption of adaptive learning than rational expectations. In addition, unusually large responses of the macroeconomic variables to forward guidance found in Del Negro et al. (2012) could be due to the way in which expectations are modeled.

Overall, the results suggest a main finding for policymakers. If monetary policy is based on a model with rational expectations, which is the standard assumption in the macroeconomic literature, the results may be misleading. This section shows that the assumption of rational expectations overstates the effects of forward guidance relative to adaptive learning during an economic recession. The adaptive learning results also match the data better than rational expectations.

2.5 Extensions

2.5.1 Alternative Parameterization

The results of this paper are investigated under different values of σ, the intertemporal elasticity of substitution parameter. σ measures the effect current and future real interest rates have on current consumption and output. This parameter is important since forward guidance involves statements about future nominal interest rates, and consequently, the real interest rate. Furthermore, this paper investigates the outcomes of the model when $\sigma = 0.15$, $\sigma = 1$, and $\sigma = 1.5$.[25] These results are displayed via adaptive learning impulse responses of the output gap and inflation to negative one unit forward guidance shocks similar to Section 2.3.2.1.

[25] $\sigma = 1$ is the baseline case used in this paper. For illustrative purposes, this paper chooses the two other values of σ to be 0.15 and 1.5.

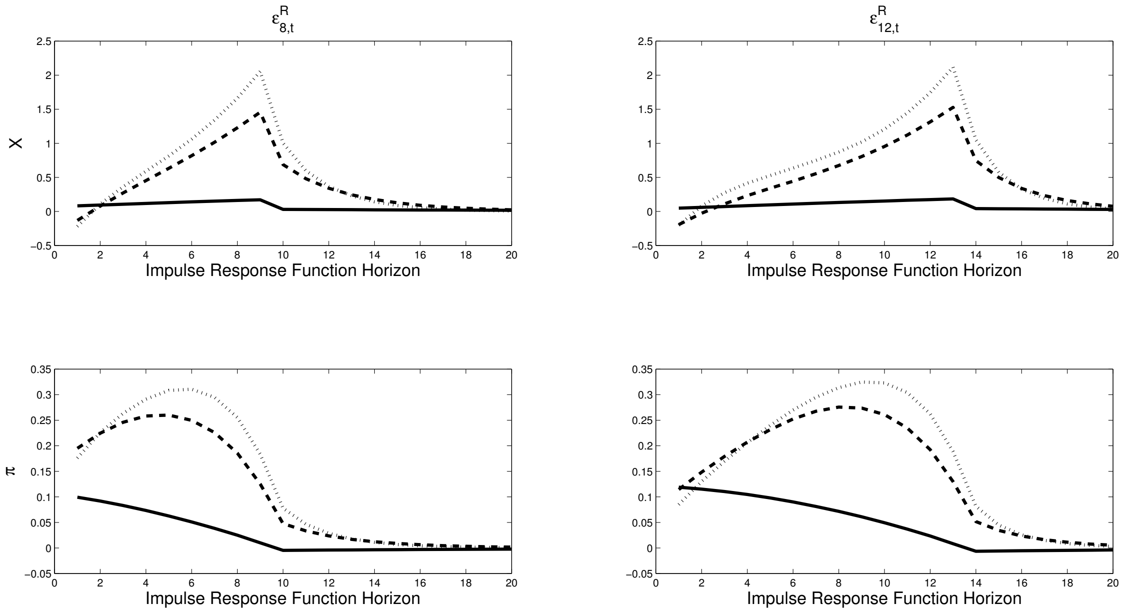


Figure 2.6: Impulse Response Functions to Forward Guidance Shocks Under Different Values of σ. Solid Line: $\sigma = 0.15$; Dashed Line: $\sigma = 1$; Dotted Line: $\sigma = 1.5$

The results displayed in Figure 2.6 show that higher values of σ produce greater forward guidance effects than lower values of σ. As σ increases, the output gap responds more to current and future real interest rates. Thus, since forward guidance involves information about future nominal interest rates, demand responds more to news that the interest rate will decrease in the future. As σ decreases, the output gap does not respond as much to changes in current and expected future interest rates. Therefore, the impact of policy shocks on the economy is less pronounced. Overall, the impulse responses of the output gap and inflation are not as responsive to forward guidance news in comparison to results under a higher value of σ.

2.5.2 Alternative Constant Gains

In this section, a robustness exercise is simulated to examine the effects of forward guidance policy when adaptive learning agents vary the degree in which they discount previous observations. Specifically, higher and lower values of the gain parameter, $\bar{\tau}$, are used. In addition to $\bar{\tau} = 0.02$, the other constant gains assumed are $\bar{\tau} = 0.01$ and $\bar{\tau} = 0.05$.

The results in Figure 2.7 show that the responses of the macroeconomic variables to forward guidance under adaptive learning depend on the value of $\bar{\tau}$. From the time of the forward guidance announcement to when the shock is realized, agents with higher constant gains seem to misvalue more the effects of forward guidance than agents with lower constant gains. Under higher values of $\bar{\tau}$, agents place more weight on new information, and thus, exhibit a stronger reaction to forward guidance news. Each period's estimates and beliefs should vary more from the previous period's estimates. However, under lower values of $\bar{\tau}$, agents do not misvalue the effects of forward guidance as much as under higher values of $\bar{\tau}$. They do not exhibit as strong of a reaction to forward guidance news as agents with a higher value of $\bar{\tau}$. Moreover, after the shock is realized on the economy, agents with a higher $\bar{\tau}$ are quicker to realize the shock is not present as they weight previous observations more than agents with a lower $\bar{\tau}$. Thus, the impulse responses under higher values of $\bar{\tau}$ are quicker to return to zero percentage deviation from the unshocked series than under lower values of $\bar{\tau}$.

2.6 Conclusion

In order to combat the effects of the 2007-2009 global financial crisis, central banks around the world have instituted forward guidance. Because the effectiveness of forward guidance hinges on how expectations respond to forward guidance, it is of interest to investigate

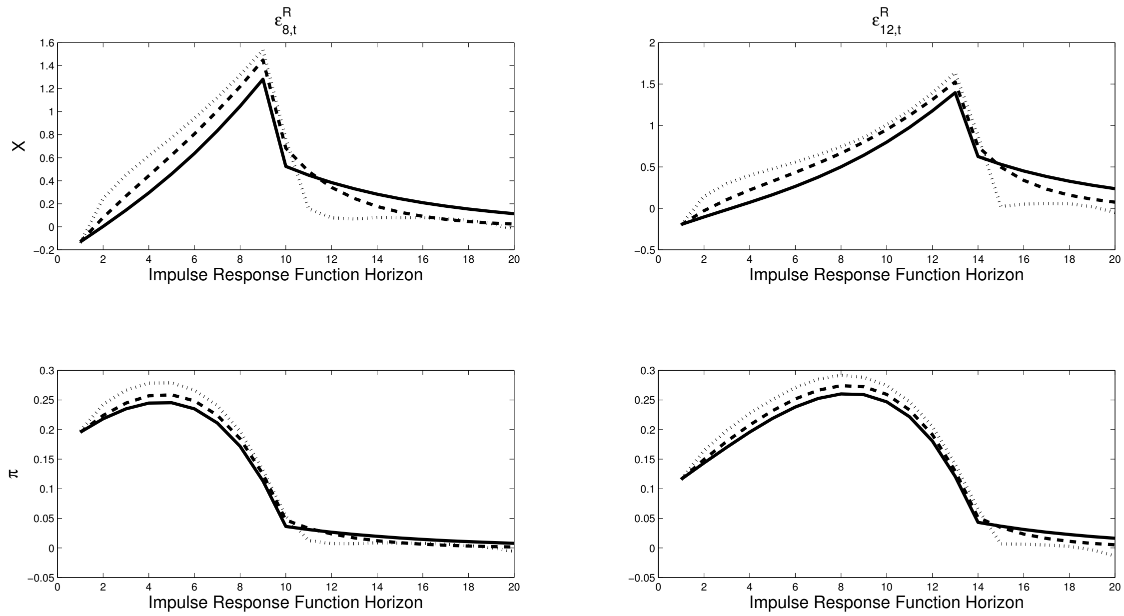


Figure 2.7: Impulse Response Functions to Forward Guidance Shocks under Different Values of $\bar{\tau}$. Solid Line: CGL with $\bar{\tau} = 0.01$; Dashed Line: CGL with $\bar{\tau} = 0.02$; Dotted Line: CGL with $\bar{\tau} = 0.05$.

the link between expectation assumptions and forward guidance. The standard way to model expectations in the macroeconomic literature is the rational expectations hypothesis. However, if agents form expectations using a more plausible theory of expectations formation (e.g. adaptive learning), the forward guidance results are different.

This paper presents an infinite horizon New Keynesian model with forward guidance and compares the results under two types of expectation assumptions. Under the assumption of rational expectations, Evans and Honkapohja (2001) state agents form expectations based on the true model of the economy. However, adaptive learning agents do not know this information, and instead, act as real-life economists and construct their expectations using standard econometric techniques. The results of this paper show that the desired effect of forward guidance depends on the manner in which agents form their expectations. When the central bank gives the same forward guidance information such that the interest rate equals zero for an extended period of time to both types of agents, the adaptive learning paths of the output gap and inflation overshoot and undershoot the ones of rational expectations. In addition, the impulse responses of the macroeconomic variables show that adaptive learning agents miss the precise response to forward guidance shocks. The impulse responses under adaptive learning display a more persistent effect than its rational expectations counterparts. During a period of economic crisis (e.g. a recession), the effects of forward guidance under rational expectations are overstated relative to adaptive learning. Specifically, the output gap is larger under the assumption of rational expectations than adaptive learning. These results occur because rational expectations agents precisely understand the effects forward guidance has on the economy as they form their expectations from the true model of the economy. However, adaptive learning agents must *estimate* the effects of forward guidance on the economy as they do not know the true model of the economy. Furthermore, these latter results have implications for policymakers. If the effects of forward guidance are based on a model with rational expectations, which is the standard assumption in the macroeconomic literature, the results may be misleading relative to a more plausible theory of expectations formation (e.g. adaptive learning).

There are other modifications to the model presented in this paper that are worth noting. For instance, this paper allows agents to know the end date of forward guidance. Another type of forward guidance policy allows the central bank to link the expiration date of forward guidance to economic conditions. For instance, the unemployment rate is a criterion that the Federal Reserve has used to link to its forward guidance policy. The RLS formula also could be altered to include a gain parameter that changes based on recent forecast errors as discussed in Milani (2014) and Marcet and Nicolini (2003). This formation of the gain parameter allows agents to better track structural breaks in the economy. In addition, agents can be assumed to have heterogeneous expectations as in Branch and McGough (2009).

Branch (2004) uses survey data and shows evidence that respondents have heterogeneous expectations. Overall, the role of expectations formation is especially crucial to understand the effects of forward guidance.

Chapter 3

The Effectiveness of Central Bank Forward Guidance without the Rational Expectations Hypothesis and Frictionless Financial Markets

3.1 Introduction

Once U.S. short-term interest rates effectively reached the zero lower bound (ZLB) during the 2007-2009 global financial crisis, monetary policymakers exhausted the conventional policy tool as overnight interest rates could not be lowered. In response, central banks pursued "unconventional" policies. One of these alternatives pursued by the Federal Reserve was large-scale asset purchases (LSAPs) where the central bank purchases longer-term securities in hopes of lowering long-term yields. Another unconventional policy was forward guidance, where the central bank communicates to the public information about the future course of the policy rate. Forward guidance has been pursued by central banks such as the Federal Reserve, Bank of Canada, Bank of England, and the European Central Bank. An example of forward guidance was given in the September 2012 Federal Open Market Committee (FOMC) statement: "the Committee also ... anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." In addition, Eggertsson and Woodford (2003) and Woodford (2012) argue that committing to an interest rate path that is lower than what one would commit to under normal circumstances (i.e. when overnight interest rates are away from the ZLB) can have additional stimulative economic effects. Since standard New Keynesian models (e.g. Woodford [2003]) predict agents being forward looking, households and firms may rationally expect higher interest rates in response to future expansions. If a forward guidance statement, instead, keeps a low policy rate through part of the expansion, consumption today will not be as limited.

The effectiveness of forward guidance hinges on two key channels–financial markets and expectations–that are largely overlooked in standard macroeconomic models. The addition of credit frictions in macroeconomic models is not a standard assumption. Frictionless financial markets are largely assumed for simplicity and not to model realistic features of an economy. However, this absence removes the prominent role that financial frictions play in the macroeconomy and a key medium in which forward guidance can influence the economy. In addition, the way in which private sector expectations about economic state variables (e.g. output and inflation) respond to forward guidance defines a key avenue through which forward guidance operates. Therefore, it is important to study whether the economic effects of forward guidance are sensitive to the rational expectations assumption that is the standard benchmark in macroeconomic models.[1] While a reasonable benchmark that is popular

[1]A related issue is the credibility of policymakers to commit to a future path of interest rates (see, for instance, see Woodford [2012]). In part, because of credibility concerns, Woodford (2012) prefers forward

among macroeconomic models, rational expectations makes strong assumptions about the amount of knowledge agents possess when forming beliefs. It is natural then to examine how effective forward guidance policies can be under a more plausible theory of expectations formation.

This paper studies the effectiveness of forward guidance in an environment where credit market frictions persist and rational expectations has been replaced by an adaptive learning rule similar to one proposed by Marcet and Sargent (1989) and Evans and Honkapohja (2001). In particular, the economic environment is based on the Federal Reserve Bank of New York-Dynamic Stochastic General Equilibrium (FRBNY-DSGE) model presented in Del Negro, Giannoni, and Patterson (2012) and Del Negro et al. (2013). The novelty of the model is to incorporate policy communication about future interest rates into agents' expectations. The central bank sets interest rates according to a monetary policy rule that responds to the output, inflation, and lagged nominal interest rates. The rule is augmented with anticipated shocks as in Laseen and Svensson (2011).[2] The anticipated shocks define central bank communication about future deviations from a normal interest rate rule that agents know today. The shocks also represent time-contingent forward guidance in which the central bank communicates a definitive forward guidance end date. In this case, communication about the future path of interest rates is for a fixed amount of periods into the future and is independent of economic conditions.[3]

Agents are assumed to form expectations via either the rational expectations hypothesis or an adaptive learning rule. The former is a strong assumption as agents construct expectations with respect to the true probability distribution of the model. Rational expectations

guidance policies that explicitly state the criteria that will underlie future policy rules. This current paper abstracts from this subject.

[2]The anticipated shocks are similar to the news shocks of Schmitt-Grohé and Uribe (2012).

[3]This type of forward guidance is in contrast to state-contingent forward guidance where the duration of a constant interest rate path is linked to economic conditions.

agents must know the model's deep parameters, structure of the model, beliefs of other agents, and distribution of the error terms. A popular alternative to rational expectations is adaptive learning. This approach builds from the cognitive consistency principle that agents behave as real-life economists (see, for instance, Evans and Honkapohja [2013]). An econometrician, for example, would produce forecasts of future economic variables by forming an econometric model and estimate the parameters using standard econometric techniques. As new data arrives, these forecasts would be revised. Thus, a real-life economist is engaging in a process of learning about the economy. Analogously, adaptive learning agents are assumed to behave as econometricians and formulate forecasts of future endogenous variables using standard econometric techniques. The variables in their econometric model are based on the solution found under rational expectations, but adaptive learning agents estimate the parameters using ordinary least squares. Their beliefs about future endogenous variables are appropriately revised as new data arrive.[4]

The inclusion of financial frictions follows Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2009) and adds a realistic feature. The new components model the borrowing and lending of funds seen in the real economy by adding two types of agents to a standard medium scale DSGE model: banks and entrepreneurs. Banks take in deposits from households and lend to entrepreneurs. The latter type of agents use these funds to purchase capital and rent it to intermediate goods producers. Banks charge entrepreneurs a premium over the riskless interest rate as there is a possibility they default. This "spread" fluctuates based on entrepreneurs' leverage and an idiosyncratic shock that affects the perceived riskiness of entrepreneurs by banks. If riskiness increases, entrepreneurs have a harder time receiving funds, and thus, are constrained in the amount of capital they can funnel to

[4] Adaptive learning agents do not take into account they will update their beliefs in future periods. They believe that the beliefs they form every period are optimal. This methodology follows from the anticipated utility discussion from Kreps (1998).

the production side of the economy. The spread or riskiness shock captures how the financial sector contributed towards the Great Recession. Del Negro et al. (2013) show spread shocks caused about half the decrease in U.S. output during the Great Recession.

The results show that the addition of financial frictions amplifies the differences between rational expectations and adaptive learning to forward guidance statements. These outcomes are first shown under impulse responses of the model to one unit forward guidance shocks. Adaptive learning agents fail to understand the precise effects of forward guidance. The results are also presented via a constant interest rate scenario in which the central bank communicates that the nominal interest rate will equal zero for an extended period of time. This exercise shows that the effects of forward guidance are overstated under rational expectations relative to adaptive learning. Specifically, the value of output is higher under rational expectations than adaptive learning throughout the forward guidance horizon. During a period of economic crisis (e.g. a recession), rational expectations display a more favorable response to forward guidance than under adaptive learning. These differences between rational expectations and adaptive learning are also exacerbated relative to a model without a financial sector. For instance, in response to forward guidance announcements, the difference in output is larger between rational expectations and adaptive learning in this current paper than in a similar analysis without financial frictions in Cole (2015).

The reasons for the differences arise from the amount of knowledge that agents are assumed to hold and the additional financial variables to forecast. Under rational expectations, agents base their expectations of future macroeconomic variables off the true model of the economy. Thus, rational expectations agents compute precise expectations about how forward guidance statements affect future macroeconomic variables. However, adaptive learning agents are not endowed with this level of knowledge. Instead, they estimate the effects of forward guidance using their econometric model of the economy. In addition, the inclusion of a financial sector magnifies the differences between rational expectations and adaptive learning agents. There is more inertia in the adaptive learning forecasting model and more variables to forecast than in a model without financial frictions. The forecasts of future variables concerning not only households and firms, but also the financial sector show a more favorable response to forward guidance under rational expectations than adaptive learning. These previous reasons and the fact that adaptive learning agents are estimating the effects of forward guidance create bigger differences between the two types of expectations assumptions.

Overall, the results of the paper suggest a main finding: the effects of forward guidance depends on the manner in which expectations and financial frictions are modeled. Under the assumption of rational expectations, forward guidance produces more favorable values of output than under adaptive learning. When including credit frictions, these differences are magnified.

3.1.1 Previous Literature

This paper contributes to the growing literature on forward guidance. The seminal work by Eggertsson and Woodford (2003) explains the importance of the expectations channel on the economy. Future interest rates affect long-term interest rates and asset prices, and thus, management of interest rate expectations is pertinent for a central bank. The results from Del Negro et al. (2012) and Carlstrom, Fuerst, and Paustian (2012) display unusually large responses of the macroeconomic variables to forward guidance relative to the data. Del Negro et al. (2012) label this outcome "the forward guidance puzzle." This present paper suggests that the unusually large responses may be due to the rational expectations assumption employed in Del Negro et al. (2012), and a more realistic expectation formation assumption (e.g. adaptive learning) produces results that better match the data.

The current paper follows recent literature examining the effectiveness of forward guidance. De Graeve, Ilbas, and Wouters (2014) study the effects of forward guidance through a different lens than the expectations channel. They find that the type of forward guidance and underlying reasons for implementing forward guidance (e.g. monetary stimulus or sign of future economic crisis) can dampen the effects of this monetary policy tool. Levin, López-Salido, Nelson, and Yun (2010) explain that the effectiveness of forward guidance can vary with the type of structural shock affecting the economy. Swanson and Williams (2014) show that Federal Reserve forward guidance announcements affect market expectations about future policy. Kool and Thornton (2012) test the effectiveness of forward guidance across four countries: New Zealand, Norway, Sweden, and the United States. They find forward guidance helped market participants forecast future short-term yields. In addition, Cole (2015) examines the effects of forward guidance across rational expectations and adaptive learning assumptions, but utilizes a DSGE model without financial frictions. The present paper shows that the differences between rational expectations and adaptive learning to forward guidance are amplified when a DSGE model is expanded to include a financial sector.

This paper fits into the literature on expectation formation and policy. Caputo, Medina, and Soto (2010) use a DSGE model with adaptive learning and a financial accelerator as in Bernanke et al. (1999). They find that the financial accelerator model with adaptive learning leads to large business cycle fluctuations, but a central bank that aggressively responds to inflation can limit the volatility in output and inflation. Mitra, Evans, and Honkapohja (2012) examine the effects of the fiscal authority giving guidance on the future course of government purchases and taxes. The results show that a temporary change in fiscal policy leads to different effects depending on whether agents form forecasts via rational expectations or adaptive learning. The adaptive learning output multipliers match the empirical evidence more than its rational expectations counterparts. Eusepi and Preston (2010) investigate the link between adaptive learning and central bank communication strategies. Increased macroeconomic stability can result from increased central bank communication, such as reporting the monetary policy rule and the variables within the rule to the public. Woodford (2010) studies optimal monetary policy in which the central bank understands agents may not form forecasts via the rational expectations hypothesis. He stresses the importance of policy commitment (e.g. guaranteeing stable inflation) regardless of agents' expectations not being model consistent.

The remaining sections are organized as follows. Section two presents the medium scale DSGE model with financial frictions. Section three discusses expectations formation under both rational expectations and adaptive learning. Section four contains the results of forward guidance under both types of expectations formation. Within this section, impulse response functions and a policy simulation in which the central bank communicates to the public that the interest rate will equal zero for an extended period of time are presented. Section five concludes.

3.2 Model

The aggregate dynamics of the economy are described by a medium scale DSGE model with financial frictions following Del Negro et al. (2012) and Del Negro et al. (2013). It contains a large number of frictions found in standard DSGE models (e.g. Smets and Wouters [2007]). These include price and wage stickiness, price and wage indexation, habit formation in consumption, capital utilization, and investment adjustment costs. The model also includes credit frictions following Bernanke et al. (1999) and Christiano et al. (2009). The remainder of this section presents a description of the model followed by the log-linearized equations.

3.2.1 Description

Households: Households maximize the sum of their expected discounted utility. They receive utility from consumption and disutility from providing work to firms. The total amount of labor each household supplies is bundled by labor packers (e.g. employment agencies) to sell to intermediate goods producers. In addition, households can transfer wealth between periods by investing in government issued bonds and deposits held at banks. The frictions in the household sector take the form of habit formation in consumption and wage stickiness. Households have market power in the labor market and choose their nominal wage á la Calvo (1983). In every period, a household has probability $(1-\zeta_w)$ of choosing its wage, and a probability ζ_w of not being able to choose its wage. Under the latter scenario, wages are indexed to either previous period's inflation times last period's productivity with probability ι_w, or steady state inflation times the economy's growth rate with probability $(1-\iota_w)$.

Firms: There exist two types of firms: intermediate and final goods producers. Intermediate goods producers operate in a monopolistically competitive market and use labor and capital to create differentiated products to sell to final goods producers. The source of their labor and capital comes from households (via employment agencies) and entrepreneurs, respectively. The intermediate goods producers are subject to nominal price rigidities in the form of a Calvo (1983) pricing scheme. In each period, firms have a probability $(1-\zeta_p)$ of freely changing their price. The remaining fraction ζ_p of firms index their price to either previous period's inflation with probability ι_p or the steady state rate of inflation with probability $(1-\iota_p)$. The final goods producers conduct business in a competitive market and bundle the intermediate goods into one composite good.

Financial Sector and Capital Producers: The modeling of credit frictions starts with two agents: banks and entrepreneurs. Banks receive deposits from households and use the proceeds to issue loans to entrepreneurs. The entrepreneurs use the funds to purchase capital from capital producers and rent it to intermediate goods firms. Banks also charge entrepreneurs a premium over the risk-free interest rate as there is a risk of default. This "spread" varies with entrepreneurs' leverage, that is, the ratio of the value of capital to net worth. In every period, an idiosyncratic shock also affects the amount of capital that entrepreneurs manage. Consequently, an adverse shock affects the ability of entrepreneurs to repay their loans to the bank. In addition, there exist spread shocks which affect the volatility of the idiosyncratic shock. This latter event can reflect entrepreneurs' perceived riskiness by banks to pay back loans. As will be described later, spread shocks play a key role in disrupting financial markets as it affects entrepreneurs' ability to borrow funds and channel capital to firms.

Capital producers operate in a perfectly competitive market and are responsible for the creation of the stock of capital. They purchase a part of output from final goods producers and transform it into capital subject to adjustment costs. They also purchase a fraction of capital from entrepreneurs. These two sources of capital comprise the amount of capital for use next period. Capital producers sell capital back to entrepreneurs who then rent it to intermediate goods producers.

Government Policy: The policy component of the model includes both monetary and fiscal. The monetary authorities follow a Taylor-type rule and adjust the short-term nominal interest rate to changes in output, inflation, lagged nominal interest rate, monetary policy shock, and anticipated or forward guidance shocks. The fiscal authorities collect lump-sum taxes and satisfy a government budget constraint. There also exists a government spending shock which captures exogenous fluctuations in aggregate demand.

3.2.2 Log-linearized Equations

The following are the set of log-linearized equations that describe the DSGE model with financial frictions. The " ˆ " and " * " symbols represent log deviations from steady state and steady state values, respectively. The $\hat{E}_t$ indicates (potentially) non-rational expectations, while E_t denotes the model-consistent rational expectations operator. From the household's first-order conditions, we get the consumption Euler equation:

$$\hat{\xi}_t = \hat{R}_t + \hat{E}_t\hat{\xi}_{t+1} - \hat{E}_t\hat{\pi}_{t+1} \tag{3.1}$$

where $\hat{\xi}_t$ is the marginal utility of consumption, $\hat{R}_t$ is the nominal interest rate paid on government issued bonds and controlled by the central bank, and $\hat{\pi}_t$ is the inflation rate. Consumption is defined according to the following equation:

$$\begin{aligned}(e^\gamma - h\beta)(e^\gamma - h)\hat{\xi}_t = e^\gamma(e^\gamma - h)\hat{b}_t - (e^{2\gamma} + \beta h^2)\hat{c}_t + he^\gamma\hat{c}_{t-1} \\ - \beta h(e^\gamma - h)\hat{E}_t\hat{b}_{t+1} + \beta he^\gamma\hat{E}_t\hat{c}_{t+1}\end{aligned} \tag{3.2}$$

where $\hat{c}_t$ is consumption, $\hat{b}_t$ is a stochastic shock to household utility, β is the discount factor, h represents habit formation in consumption, and e^γ is the steady-state (gross) growth rate of the economy. The demand for money by households is given by

$$v_m\hat{m}_t = -\frac{1}{R^* - 1}\hat{R}_t - \hat{\xi}_t \tag{3.3}$$

where $\hat{m}_t$ is money.

Households have market power in the labor market. Wages are chosen by households according to a Calvo (1983) scheme. In each period, a fraction $1-\zeta_w$ of households can choose their wage. The remaining ζ_w of households index wages to either previous period's inflation times last period's productivity with probability ι_w, or steady state inflation times the economy's growth rate with probability $(1-\iota_w)$. The optimal reset wage equation is given by

$$\begin{aligned}(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\tilde{w}}_t + (1+\zeta_w\beta\nu_l(\frac{1+\lambda_w}{\lambda_w}))\hat{w}_t &= \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{E}_t(\hat{\tilde{w}}_{t+1}+\hat{w}_{t+1}) \\ &+ (1-\zeta_w\beta)(e^{2\gamma}+h^2\beta)\frac{e^{-\gamma}}{e^{\gamma}-h}\hat{b}_t + \hat{\varphi}_t + (1-\zeta_w\beta)(\nu_l\hat{L}_t - \hat{\xi}_t) \\ &- \zeta_w\beta\iota_w(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\pi}_t + \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{E}_t\hat{\pi}_{t+1}\end{aligned} \tag{3.4}$$

$\hat{\tilde{w}}_t$ represents the freely chosen wage by households, $\hat{w}_t$ is the aggregate wage, $\hat{L}_t$ is aggregate labor, and $\hat{\varphi}_t$ is a stochastic shock that affects the marginal utility of labor. λ_w defines the elasticity of substitution between differentiated labor services, and ν_l represents the inverse Frisch elasticity of labor supply. In addition, the aggregate wage equation is given by

$$\hat{w}_t = \hat{w}_{t-1} + \iota_w\hat{\pi}_{t-1} - \hat{\pi}_t + \frac{1-\zeta_w}{\zeta_w}\hat{\tilde{w}}_t \tag{3.5}$$

The production side of the economy is populated by intermediate and final goods producing firms. The intermediate goods firms operate in a monopolistically competitive market while final goods producers conduct business in a competitive market. Prices do not freely adjust in the former market. Specifically, a fraction $(1-\zeta_p)$ of firms can freely adjust its price every period. The remaining ζ_p of firms either index prices to previous period's inflation with probability ι_p or steady state rate of inflation with probability $(1 - \iota_p)$. Consequently, the Philips Curve is given by

$$\hat{\pi}_t = \frac{\iota_p \zeta_p}{1 + \iota_p \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \iota_p \beta} \hat{E}_t \hat{\pi}_{t+1} + \frac{(1 - \zeta_p \beta)(1 - \zeta_p)}{(1 + \iota_p \beta)\zeta_p} \hat{mc}_t + \frac{1}{(1 + \iota_p \beta)\zeta_p} \hat{\lambda}_{f,t} \tag{3.6}$$

where $\hat{\lambda}_{f,t}$ represents a cost-push shock. $\hat{mc}_t$ is marginal cost and is defined by

$$\hat{mc}_t = (1 - \alpha)\hat{w}_t + \alpha \hat{r}_t^k \tag{3.7}$$

where $\hat{r}_t^k$ is the rental rate of capital and α captures capital's share of output. Intermediate goods firms utilize both labor and capital in a Cobb-Douglas production function given by

$$\hat{y}_t = \frac{\alpha(y^* + \Phi)}{y^*} \hat{k}_t + \frac{(1 - \alpha)(y^* + \Phi)}{y^*} \hat{L}_t \tag{3.8}$$

where $\hat{k}_t$ represents effective capital in the economy and Φ is fixed costs in production.

The model's resource constraint satisfies

$$\hat{y}_t = \hat{g}_t + \frac{c*}{c* + i*} \hat{c}_t + \frac{i*}{c* + i*} \hat{i}_t + \frac{r^{k*} k*}{c* + i*} \hat{u}_t \tag{3.9}$$

where $\hat{i}_t$ is investment, $\hat{u}_t$ is capital utilization, and $\hat{g}_t$ is a government spending shock capturing exogenous aggregate demand fluctuations in the economy.

The capital-to-labor ratio is given by

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t \tag{3.10}$$

The financial side of the economy is populated by banks and entrepreneurs. Entrepreneurs borrow funds from banks to purchase capital from capital producers and rent it to intermediate goods producers. The amount of funds entrepreneurs can borrow is a function of their net worth, which evolves according to the following equation:

$$\begin{aligned}\hat{n}_t = \zeta_{n,\tilde{R}^k}(\hat{\tilde{R}}_t^k - \hat{\pi}_t) - \zeta_{n,R}(\hat{R}_{t-1} - \hat{\pi}_t) + \zeta_{n,qK}(\hat{q}_{t-1}^k + \hat{\bar{k}}_{t-1}) \\ + \zeta_{n,n}\hat{n}_{t-1} + \hat{\gamma}_t - \frac{\zeta_{n,\mu^e}}{\zeta_{sp,\mu^e}}\hat{\mu}_{t-1}^e - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\hat{\sigma}_{\omega,t-1}\end{aligned} \tag{3.11}$$

where $\hat{n}_t$ is net worth, $\hat{q}_t^k$ is the price of capital, $\hat{\bar{k}}_t$ measures the amount of installed capital, $\hat{\gamma}_t$ defines the time-varying exogenous fraction of entrepreneurs that survive each period shock, $\hat{\mu}_t^e$ is a bankruptcy cost shock, and $\hat{\sigma}_{\omega,t}$ is a spread shock. $\zeta_{n,\tilde{R}^k}, \zeta_{n,R}, \zeta_{n,qK}, \zeta_{n,n}, \zeta_{n,\mu^e}$, and ζ_{n,σ_ω} are the elasticities of net worth with respect to the return on capital, nominal interest rate, price of capital, net worth itself, bankruptcy cost shock, and the spread shock, respectively. ζ_{sp,σ_ω} represents the elasticity of the spread with respect to the volatility of the spread shock. ζ_{sp,μ_e} is the elasticity of the spread with respect to the bankruptcy cost shock. $\hat{\tilde{R}}_t^k$ is the gross return on capital entrepreneurs receive from renting capital to intermediate goods producers and is defined by

$$\hat{\tilde{R}}_t^k - \hat{\pi}_t = \frac{r_*^k}{r_*^k + (1-\delta)}\hat{r}_t^k + \frac{1-\delta}{r_*^k + (1-\delta)}\hat{q}_t^k - \hat{q}_{t-1}^k \tag{3.12}$$

where δ is the depreciation rate. The expected excess return on capital or spread is defined by the following equation

$$\hat{E}_t(\hat{\tilde{R}}_{t+1}^k - \hat{R}_t) = \zeta_{sp,b}(\hat{q}_t^k + \hat{\bar{k}}_t - \hat{n}_t) + \hat{\mu}_t^e + \hat{\sigma}_{\omega,t} \tag{3.13}$$

where $\hat{\sigma}_{\omega,t}$ is defined as a spread shock. It characterizes banks' perception of the riskiness of entrepreneurs. For example, if this shock increases, banks perceive entrepreneurs to be risky and thus, bank loans are harder to receive. This decrease in funds hampers the ability of entrepreneurs to funnel capital to the intermediate goods sector. $\zeta_{sp,b}$ characterizes the elasticity of the spread to entrepreneurs' leverage, which is defined as the ratio of the value of capital to nominal net worth. The amount of installed capital in the model is given by

$$\hat{\bar{k}}_t = (1 - \frac{i*}{k*})\hat{\bar{k}}_{t-1} + \frac{i*}{k*}\hat{\mu}_t + \frac{i*}{k*}\hat{i}_t \tag{3.14}$$

where $\hat{\mu}_t$ is a shock to the amount of capital. The amount of investment $\hat{i}_t$ is defined by

$$\hat{i}_t = \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{\beta}{1+\beta}\hat{E}_t\hat{i}_{t+1} + \frac{1}{(1+\beta)S''e^{2\gamma}}\hat{q}_t^k + \hat{\mu}_t \tag{3.15}$$

where $S(\bullet)$ captures the cost of adjusting capital and $S^{'} > 0$ and $S^{''} > 0$.

The amount of capital is described by the following equation:

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} \tag{3.16}$$

$\hat{u}_t$ defines the capital utilization rate and the corresponding equation is given by

$$r_*^k \hat{r}_t^k = a^{''} \hat{u}_t \tag{3.17}$$

where $a^{''}$ captures capital utilization costs.

Monetary Policy: The model is closed by describing the central bank of the economy. The central bank follows a monetary policy rule that takes the following form

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t) + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^{R} \tag{3.18}$$

The short-term nominal interest rate changes based on itself, output, inflation rate, monetary policy shock, and forward guidance shocks. ε_t^{MP} defines an unanticipated monetary policy shock and is *i.i.d.* In order to incorporate forward guidance into the model, the monetary policy rule is augmented with anticipated shocks following Del Negro et al. (2012) and Laseen and Svensson (2011). Each anticipated or forward guidance shock ($\varepsilon_{l,t-l}$) is contained in the last term in equation (3.18) and is *i.i.d.* Intuitively, the forward guidance shock can be thought of as an announcement by the central bank in period $t-l$ that the interest rate will change l periods later, i.e. in period t. If the central bank has been communicating guidance on the interest rate for L periods ahead, there would be $1, 2, 3, \ldots, L$ forward guidance shocks that affect the monetary policy rule in period t. Thus, L corresponds to the length of the forward guidance horizon announced by the central bank. The last term in equation (3.18) can also be thought of as the sum of all forward guidance commitments stated by the central bank $1, 2, ...,$ and L periods ago that affect the nominal interest rate in period t. Following Del Negro et al. (2012) and Laseen and Svensson (2011), the system is also augmented with L state variables $v_{1,t}, v_{2,t}, ..., v_{L,t}$. The law of motion for each of these state variables is given by

$$v_{1,t} = v_{2,t-1} + \varepsilon^R_{1,t} \tag{3.19}$$

$$v_{2,t} = v_{3,t-1} + \varepsilon^R_{2,t} \tag{3.20}$$

$$v_{3,t} = v_{4,t-1} + \varepsilon^R_{3,t} \tag{3.21}$$

$$\vdots$$

$$v_{L,t} = \varepsilon^R_{L,t} \tag{3.22}$$

In other words, each component of $v_t = [v_{1,t}, v_{2,t}, ..., v_{L,t}]'$ is the sum of all central bank forward guidance commitments known in period t that affect the interest rate $1, 2, ...,$ and L periods into the future, respectively.[5] It should be noted that equations (3.19) – (3.22) can be simplified to find that $v_{1,t-1} = \sum_{l=1}^{L} \varepsilon^R_{l,t-l}$. In addition, equations (3.18) – (3.22) provide a computationally tractable method to model forward guidance. Since the forward guidance shocks in equation (3.18) equal $v_{1,t-1}$, the forward guidance shocks can be put into a vector of predetermined variables in standard state-space form. As described by Laseen

[5] In the terminology of Laseen and Svensson (2011), $v_{1,t}, v_{2,t}, ..., v_{L,t}$ are described as central bank "projections" (p. 10) of what $\sum_{l=1}^{L} \varepsilon^R_{l,t-l}$ will be $1, 2, ...,$ and L periods into the future, respectively.

and Svensson (2011), standard solution techniques then can be used to solve the final system of equations. Another reason to model forward guidance in this way is that it relieves the concern of the existence of multiple solutions. As described in Honkapohja and Mitra (2005) and Woodford (2005), indeterminacy can arise if forward guidance is instead modeled as pegging the interest rate to a certain value.[6] For instance, without a monetary policy that responds to economic fluctuations, real disturbances to the economy can produce a multitude of equilibrium responses of the endogenous variables.

The following example presents the case where the central bank's forward guidance horizon is 2 periods ahead, i.e. $L = 2$. The model's system of equations consists of $v_{1,t}$ and $v_{2,t}$ whose laws of motion are defined as

$$v_{1,t} = v_{2,t-1} + \varepsilon^R_{1,t} = \varepsilon^R_{2,t-1} + \varepsilon^R_{1,t} \tag{3.23}$$

$$v_{2,t} = \varepsilon^R_{2,t} \tag{3.24}$$

Thus, $v^R_{1,t}$ defines the sum of all forward guidance commitments by the central bank known in period t that affect the interest rate one period later. $v^R_{1,t}$ consists of current period forward guidance affecting the interest rate one period later, $\varepsilon^R_{1,t}$, and previous period's forward guidance affecting the interest rate two periods later, $v_{2,t-1} = \varepsilon^R_{2,t-1}$. $v_{2,t}$ is the sum of all forward guidance commitments by the central bank known in period t that affect the interest rate two periods later. Since the forward guidance horizon is two periods, $v_{2,t}$ consists of current period forward guidance affecting the interest rate two periods later, $\varepsilon^R_{2,t}$.[7]

[6] Carlstrom, Fuerst, and Paustian (2012) show that determinacy can arise from an interest rate peg if terminal conditions are known and a standard monetary policy rule is followed after the interest rate peg. However, unusually large responses of the output and inflation are found through this process.

[7] A constant interest rate path can still be achieved by modeling forward guidance with equations (3.18)-(3.22). As will be described in Section 3.3.2.2, the forward guidance shocks can be chosen such that the interest rate equals a certain value for a fixed amount of periods into the future.

The ZLB on interest rates is also enforced. Forward guidance has gained attention due to interest rates effectively reaching the ZLB because of the 2007-2009 global financial recession. Thus, it seems natural to model the ZLB on nominal interest rates when simulating forward guidance. Specifically, the monetary policy rule becomes

$$R_t = max\{R^* + \rho_R(R_{t-1} - R^*) + (1-\rho_R)(\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t) + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^R, 0\} \quad (3.25)$$

where $R^* = \bar{R}^* + \pi^*$ is the steady-state nominal interest rate and $\bar{R}^*$ is the steady-state real interest rate. The rest of the model's equations are appropriately redefined. Furthermore, the " ˆ " symbol over the variables is removed for the remainder of the paper to simplify notation.

3.2.3 Exogenous Shocks

The model's exogenous shocks consist of a spread shock ($\sigma_{w,t}$), price mark-up shock ($\lambda_{f,t}$), labor shock (φ_t), stochastic preference shock (b_t), government spending shock (g_t), marginal efficiency of investment shock (μ_t), bankruptcy cost shock (μ_t^e), time-varying exogenous survival rate of entrepreneurs shock (γ_t), monetary policy shock (ε_t^{MP}), and forward guidance shocks ($\varepsilon_{1,t}^R, \varepsilon_{2,t}^R, \ldots, \varepsilon_{L,t}^R$). The first eight are assumed to follow an AR(1) processes with autocorrelation parameters ($\rho_{\sigma_w}, \rho_{\lambda_f}, \rho_\varphi, \rho_b, \rho_g, \rho_\mu, \rho_{\mu^e}$, and ρ_γ). The monetary policy and forward guidance shocks are assumed to be *i.i.d.*

3.3 Expectations Formation

This paper assumes agents form expectations following either the rational expectations hypothesis or adaptive learning. The difference between the two types of expectations formation regards the amount of knowledge agents hold about the economy (See, for example, Marcet and Sargent (1989), Evans and Honkapohja (2001), and Evans, Honkapohja, and Mitra (2009).). Under rational expectations, agents know the structure of the model, parameters of the model (e.g. ζ_p, h, etc.), distribution of the error terms, and beliefs of other agents. They compute expectations based on the true model of the economy. Under adaptive learning, agents do not know the true model of the economy, and thus, cannot compute precise expectations as under rational expectations. Instead, they operate as real-life economists. An econometrician, for example, produces forecasts of future economic variables by forming an econometric model and estimating the parameters using standard econometric techniques. As new data arrive, these forecasts would be revised. Thus, a real-life economist is engaging in a process of learning about the economy. Analogously, adaptive learning agents formulate forecasts of future endogenous variables using standard econometric techniques. The variables in their econometric model are based on the solution found under rational expectations, and adaptive learning agents estimate the parameters using ordinary least squares. Their beliefs about future endogenous variables are appropriately revised as new data arrive.

Rational Expectations–The model with rational expectations can be solved using standard techniques, such as one suggested by Sims (2002). The model can be written in general state-space form is defined as

$$\widetilde{\Gamma}_0\widetilde{Y}_t = C + \widetilde{\Gamma}_1\widetilde{Y}_{t-1} + \widetilde{\Gamma}_2\widetilde{\varepsilon}_t + \widetilde{\Gamma}_3\zeta_t \tag{3.26}$$

where

$$\widetilde{Y}_t = [Y_t, \epsilon_t, v_t, \Xi_t]' \quad (3.27)$$

$$Y_t = [\xi_t, R_t, c_t, \bar{k}_t, i_t, k_t, u_t, r_t^k, q_t, \tilde{R}_t^K, \pi_t, n_t, w_t, \tilde{w}_t, L_t, mc_t, y_t, m_t]' \quad (3.28)$$

$$\epsilon_t = [\lambda_{f,t}, \mu_t, \varphi_t, g_t, \sigma_{w,t}, b_t, \mu_t^e, \gamma_t]' \quad (3.29)$$

$$v_t = [v_{1,t}, v_{2,t}, \ldots, v_{L,t}]' \quad (3.30)$$

$$\Xi_t = [E_t\xi_{t+1}, E_t\pi_{t+1}, E_tc_{t+1}, E_ti_{t+1}, E_t\tilde{R}_{t+1}^k, E_t\tilde{w}_{t+1}, E_tw_{t+1}]' \quad (3.31)$$

$$\widetilde{\varepsilon}_t = [\varepsilon_t^\lambda, \varepsilon_t^\mu, \varepsilon_t^\varphi, \varepsilon_t^g, \varepsilon_t^{\sigma_w}, \varepsilon_t^b, \varepsilon_t^{\mu^e}, \varepsilon_t^\gamma, \varepsilon_t^{MP}, \varepsilon_{1,t}^R, \varepsilon_{2,t}^R, \ldots, \varepsilon_{L,t}^R]' \quad (3.32)$$

C defines a vector of constants of required dimensions. The vector Y_t contains the model's endogenous variables. ϵ_t is a vector of the model's exogenous processes, and Ξ_t denotes the vector of expectations. The structural disturbances are defined by the vector $\widetilde{\varepsilon}_t$. ζ_t denotes the vector of expectational errors (e.g. $\zeta_t^\pi = \pi_t - E_{t-1}\pi_t$) of required dimensions. Using standard techniques to solve the model with rational expectations (e.g. Sims [2002]) and the parameter values in Table C.1.1 in Appendix C.1, the solution to the system under rational expectations is

$$\widetilde{Y}_t = \widetilde{C} + \xi_1\widetilde{Y}_{t-1} + \xi_2\widetilde{\varepsilon}_t \quad (3.33)$$

where the matrices $\widetilde{C}$, ξ_1, and ξ_2 are nonlinear functions of the model's parameters.[8]

Adaptive Learning–In order to evaluate the expectations in equations $(3.1) - (3.17)$ under adaptive learning, agents act as econometricians by forming a model based on variables that

[8]Discussion of the parameter values can be found in Section 3.3.1.

appear in the rational expectations solution and estimate the coefficients. This model is labeled the "Perceived Law of Motion" (PLM) and is constructed from the minimum state variable (MSV) solution that exists under rational expectations.[9] The PLM is defined as

$$Y_t = a + bY_{t-1} + cv_t + d\epsilon_t + ev_{1,t-1} + \varepsilon_t \tag{3.34}$$

where Y_t, v_t, and ϵ_t are defined as in the rational expectations model. In addition, the reader should note that v_t and ϵ_t can be expressed as

$$v_t = \Phi v_{t-1} + \eta_t \tag{3.35}$$

$$\epsilon_t = \widetilde{\phi}\epsilon_{t-1} + \bar{\epsilon}_t \tag{3.36}$$

[9]This paper focuses on a version of the model that is determinate so that the PLM is based on the unique non-explosive rational expectations equilibrium. The parameter values in Table C.1.1 in Appendix C.1 verify that the rational expectations solution is determinate.

where Φ is an L x L matrix given by

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & & & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{3.37}$$

$$\tag{3.38}$$

and

$$\eta_t = [\varepsilon^R_{1,t}, \varepsilon^R_{1,t}, \dots, \varepsilon^R_{L,t}]' \tag{3.39}$$

$$\widetilde{\phi} = \begin{bmatrix} \rho_{\lambda_f} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\varphi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{\sigma_w} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\mu^e} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\gamma \end{bmatrix} \tag{3.40}$$

$$\bar{\epsilon_t} = [\varepsilon^\lambda_t, \varepsilon^\mu_t, \varepsilon^\varphi_t, \varepsilon^g_t, \varepsilon^w_t, \varepsilon^b_t, \varepsilon^{\mu^e}_t, \varepsilon^\gamma_t]' \tag{3.41}$$

a, b, c, d, and e are unknown coefficient matrices of appropriate dimensions that agents estimate and learn about over time.[10] Furthermore, the addition of $v_{1,t-1}$ is a necessary component of the PLM since it is present in the rational expectations solution and not contained in the vector v_t.

An important component of adaptive learning models regards the information available to agents when they form expectations. In this paper, adaptive learning agents are assumed to know the values of the regressors in the PLM and previous period's coefficient estimates when forming beliefs about the future. They update their parameter estimates at the end of the period. This assumption avoids the simultaneous determination of current period coefficient estimates and endogenous variables when forming expectations and making optimal decisions.[11] The *i.i.d.* monetary policy shock is also assumed to be unobserved.[12] Furthermore, the following is the timeline of events:

1. At the beginning of period t, v_t, and ϵ_t are observed by the agents and added to their information set.

2. Agents use Y_{t-1}, v_t, ϵ_t, and $v_{1,t-1}$ as well as previous period's estimates (i.e. a_{t-1}, b_{t-1}, c_{t-1}, d_{t-1}, and e_{t-1}) to form expectations about the future.

3. Y_t is realized.

4. In order to update their parameter estimates, agents compute a least squares regression of Y_t on 1, Y_{t-1}, v_t, ϵ_t, and $v_{1,t-1}$.

[10]In the PLM, the time subscript is left off the coefficients to emphasize that adaptive learning agents believe current period forecasts are optimal and do not take into account they will be updating their beliefs every period. However, as will be described later, the PLM coefficients will evolve over time.

[11]An alternative is to assume that agents use the coefficient estimates from the current period when forming expectations. This results in expectations and current period parameter estimates determined simultaneously when making optimal decisions.

[12]This is similar to Milani (2007).

Agents update their parameter estimates of the PLM by following the recursive least squares (RLS) formula

$$\phi_t = \phi_{t-1} + \tau_t R_t^{-1} z_t (Y_t - \phi_{t-1}^{'} z_t)' \quad (3.42)$$

$$R_t = R_{t-1} + \tau_t (z_t z_t^{'} - R_{t-1}) \quad (3.43)$$

where $\phi = (a, b, c, d, e)^{'}$ contains the PLM coefficients to be estimated. R_t defines the precision matrix of the regressors in the PLM $z_t \equiv [1, Y_{t-1}, v_t, \epsilon_t, v_{1,t-1}]^{'}$. τ_t is known as the "gain" parameter and controls the response of ϕ_t to new information. The last expression in equation (3.42) defines the recent prediction error of the endogenous variables.

The gain parameter in equations (3.42) and (3.43) can either decrease over time or be fixed at certain values. In the decreasing gain or RLS case, $\tau_t = t^{-1}$ and past observations are equally weighted. Evans and Honkapohja (2001) explain that as $t \to \infty$ the coefficients in the PLM converge to the rational expectations coefficients with probability one. As is assumed in this current paper, the gain parameter can also be fixed at a certain value. Under this method called discounted or constant gain learning (CGL), $\tau_t = \bar{\tau}$ and the most recent observations play a larger role when updating agents' coefficients and expectations. Evans and Honkapohja (2001) describe that the coefficients in the PLM converge in distribution to their rational expectations values with a variance that is proportional to the constant gain parameter. CGL may be a more realistic way to model learning since it allows agents to update their beliefs every period to new information as a real-life econometrician revising his or her forecasts every period.

Agents solve for $\hat{E}_t Y_{t+1}$ by using equation (3.34). Specifically, expectations are given by

$$\begin{aligned}\hat{E}_t Y_{t+1} = (I_{18} + b_{t-1})a_{t-1} + b_{t-1}^2 Y_{t-1} + (b_{t-1}c_{t-1} + c_{t-1}\Phi)v_t \\ + (b_{t-1}d_{t-1} + d_{t-1}\widetilde{\phi})\epsilon_t + b_{t-1}e_{t-1}v_{1,t-1} + e_{t-1}v_{1,t}\end{aligned} \tag{3.44}$$

Equation (3.44) is substituted into equations (3.1) – (3.17) to give

$$Y_t = \Gamma_0(\phi_{t-1}) + \Gamma_1(\phi_{t-1})Y_{t-1} + \Gamma_2(\phi_{t-1})v_t + \Gamma_3(\phi_{t-1})\widetilde{\epsilon}_t \tag{3.45}$$

where

$$\widetilde{\epsilon}_t = [\epsilon_t, \varepsilon_t^{MP}]' \tag{3.46}$$

Equation (3.45) is called the "Actual Law of Motion" (ALM) and describes the actual evolution of the endogenous variables implied by the PLM (3.34).

3.4 Results

3.4.1 Parameterization

Table C.1.1 in Appendix C.1 displays the values of the parameters used in simulation. The values largely follow from empirical work by Del Negro et al. (2012) and Del Negro et al. (2013). There exists a high degree of habit formation in consumption with $h = 0.7$. The value of the price stickiness parameter implies that prices change once a year, which corresponds to empirical work by Klenow and Malin (2010). The high degree of wage stickiness also matches empirical evidence found in Del Negro et al. (2013). $a^{''} = 0.02$ indicates a smaller reaction of the rental rate of capital to changes in the capital utilization rate. The inclusion of a financial sector also adds additional credit market parameters. The survival rate of entrepreneurs is set to 0.99. $\zeta_{sp,b}$ defines the elasticity of the spread $(E_t(\tilde{R}^K_{t+1} - R_t))$ with respect to leverage $(q^k_t + \bar{k}_t - n_t)$ and equals to 0.05. The structural disturbances also are not assumed to exhibit high persistence. The distribution of the white noise shocks is not assumed to be highly dispersed. There also is no covariance between the structural shocks.

The current paper examines results for the CGL case. In regards to choosing the CGL parameter $\bar{\tau}$, the present paper uses 0.02. This choice is close to the results used in the literature, such as Orphanides and Williams (2005), Milani (2007), and Branch and Evans (2006).

The values of the monetary policy parameters in Table C.1.1 closely match the existing literature. Monetary policy positively responds to output, and positively adjusts at more than a one-to-one rate to the inflation rate. The value of χ_x comes from Branch and Evans (2013). The value of χ_π closely follows empirical adaptive learning work by Milani (2007). The policy inertia parameter matches empirical work by Del Negro et al. (2013). The value for the length of the forward guidance horizon L equals time-contingent forward guidance by the Federal Reserve. This number is based off the FOMC September 2012 statement: "the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." This announcement was one of the last FOMC statements to exclusively use time-contingent forward guidance language. By taking "mid-2015" to be at most the end of the third quarter of 2015, the number of quarters from September 2012 to "mid-2015" is twelve. Thus, $L = 12$.

3.4.2 Normal Economics Times

3.4.2.1 Impulse Responses

In this section, impulse responses of output to negative one unit forward guidance shocks under different expectation assumptions are examined in Figure 3.1.[13] The forward guidance shocks are the anticipated shocks found in equations (3.19) - (3.22). Since equation (3.45) exhibits a nonlinear structure, standard linear techniques to compute impulse responses under adaptive learning do not apply. To remedy this situation, this paper follows Eusepi and Preston (2011) by proceeding in the following manner. The model is simulated twice for $T + K$ periods, where K is the impulse response function horizon. The impulse responses are calculated starting in period $T + 1$.[14] In the first simulation, time period $T + 1$ includes a negative one unit shock. The K-period impulse response function is given by the difference between the first and second simulations over the final K periods. The mean impulse response

[13]A projection facility is utilized to ensure beliefs are not explosive.

[14]T is chosen to be a large number so that the adaptive learning coefficients converge to its stationary distribution.

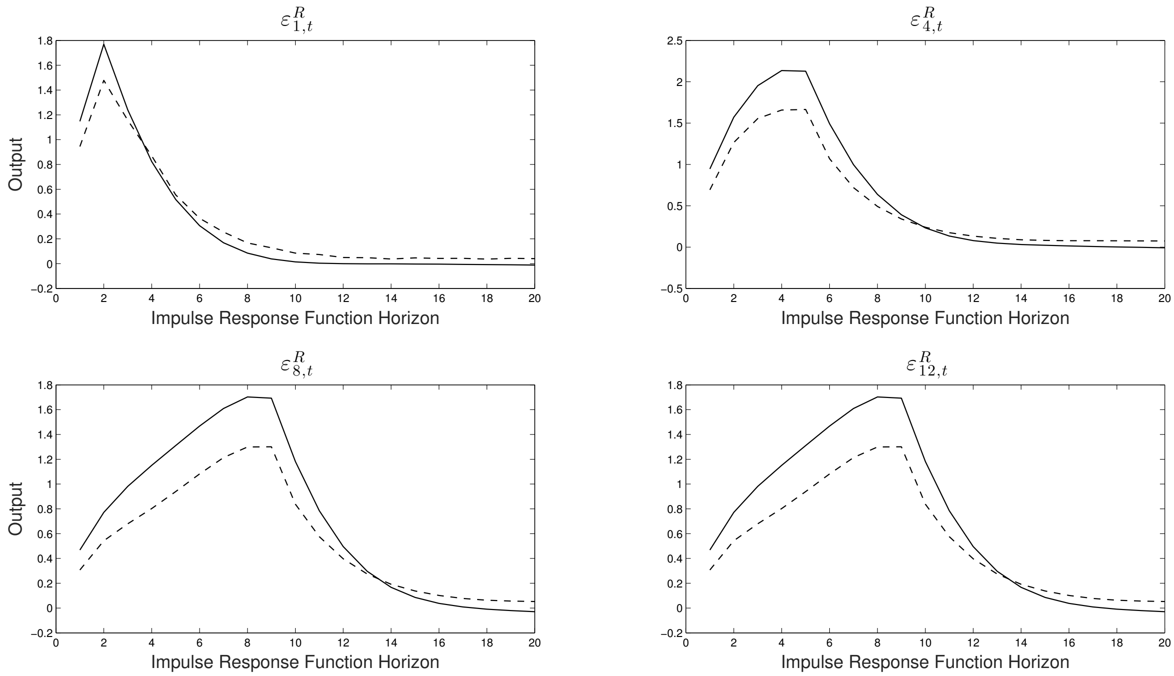


Figure 3.1: Impulse Response of Output to Forward Guidance Shocks. Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Lines: 95% Confidence Bands.

across simulations is calculated to arrive at the final impulse response trajectory. The impulse response function horizon is chosen to be twenty periods, that is, $K = 20$.

The impulse responses show that adaptive learning agents fail to understand the precise effects that forward guidance has on the economy. The period from impact to realization of the forward guidance shock displays that the adaptive learning output path undershoots the rational expectations counterpart. The reason for the difference regards the amount of knowledge the two types of agents are assumed to possess. Since rational expectations agents know precisely how the endogenous variables evolve, their expectations are based off the true model of the economy. Consequently, rational expectations agents understand the positive benefits future interest rate statements have on future macroeconomic variables. However, adaptive learning agents are unable to base their expectations off the true model of the economy as they are not endowed with that information. Instead, they estimate the effects of forward guidance utilizing an econometric model of the economy. In addition, the inclusion of a financial sector contributes to the differences between adaptive learning and rational expectations. The financial sector produces additional variables to forecast and more inertial behavior (lagged variables) in the PLM relative to a model without financial frictions (e.g. Cole[2015]). Therefore, adaptive learning agents are slower to understand the effects of forward guidance on the economy.

Overall, the message from this section is that adaptive learning agents fail to completely understand the positive benefits forward guidance has on the economy and a financial sector compounds the differences between the two types of agents. When the forward guidance shock is known to agents, output under adaptive learning proceeds on a different path than under rational expectations. These results are attributed to rational expectations agents precisely understanding the effects forward guidance has on the economy, while the beliefs of adaptive learning agents slowly adjust. The presence of financial frictions creates a slower response of adaptive learning to forward guidance than rational expectations.

3.4.2.2 Policy Exercise

This current section builds upon the previous result that one-unit forward guidance shocks produce different effects on the economy depending on the expectation assumption. While the previous section examined forward guidance via impulse responses, this section studies forward guidance through a different scenario. Specifically, the central bank would like to keep the interest rate fixed at $\bar{R}$ for $L+1$ periods. The experiment is explained next and is motivated by the policy exercise described in Del Negro et al. (2012).

Suppose at the beginning of period T, the central bank implements forward guidance such that the interest rate will be fixed at $\bar{R} = 0$ in period T and L periods into the future. This announcement corresponds to an unanticipated shock in period T and news about the future interest rate $1, 2, \ldots, L$ periods into the future. In this scenario, the monetary policymaker's job is to choose ε_T^{MP} and $\eta_T = [\varepsilon_{1,T}^R, \varepsilon_{2,T}^R, \ldots, \varepsilon_{L,T}^R]'$ such that the interest rate in periods T to $T + L$ equals $\bar{R}$. The central bank also believes that agents hold rational expectations, which is a common assumption in macroeconomic literature. To show that adaptive learning agents respond differently to the same forward guidance information, the adaptive learning agents are given the same guidance on the interest rate as under rational expectations. Furthermore, the exercise is assumed to start in period T.[15] The model is then simulated from T to the end of the forward guidance horizon $T + L$.

This policy exercise also assumes that the central bank is committed to its goal of $\bar{R}$ every period during the forward guidance horizon. Rational expectations agents precisely understand the central bank's guidance, and thus, the interest rate each period implied by rational expectations equals $\bar{R}$. Since the adaptive learning process is different than rational expectations, the same forward guidance will not give a model implied $\bar{R}$ during the forward guidance horizon. To model a promise to $\bar{R} = 0$, the central bank chooses ε_t^{MP} each period over the forward guidance horizon to ensure the interest rate equals $\bar{R}$.[16]

Figure 3.2 compares the dynamics under rational expectations and adaptive learning for output. The solid line represents the value of output under adaptive learning minus the value of output under rational expectations. A positive value in Figure 3.2 indicates output under adaptive learning is higher than under rational expectations. A negative value implies

[15] T is chosen to be a large number so that the adaptive learning coefficients converge to its stationary distribution.

[16] This adjustment seems fair since agents' expectations in real life about the future interest rate might not respond as exactly as the central bank would want, and thus, the interest rate might not equal a model implied $\bar{R}$.

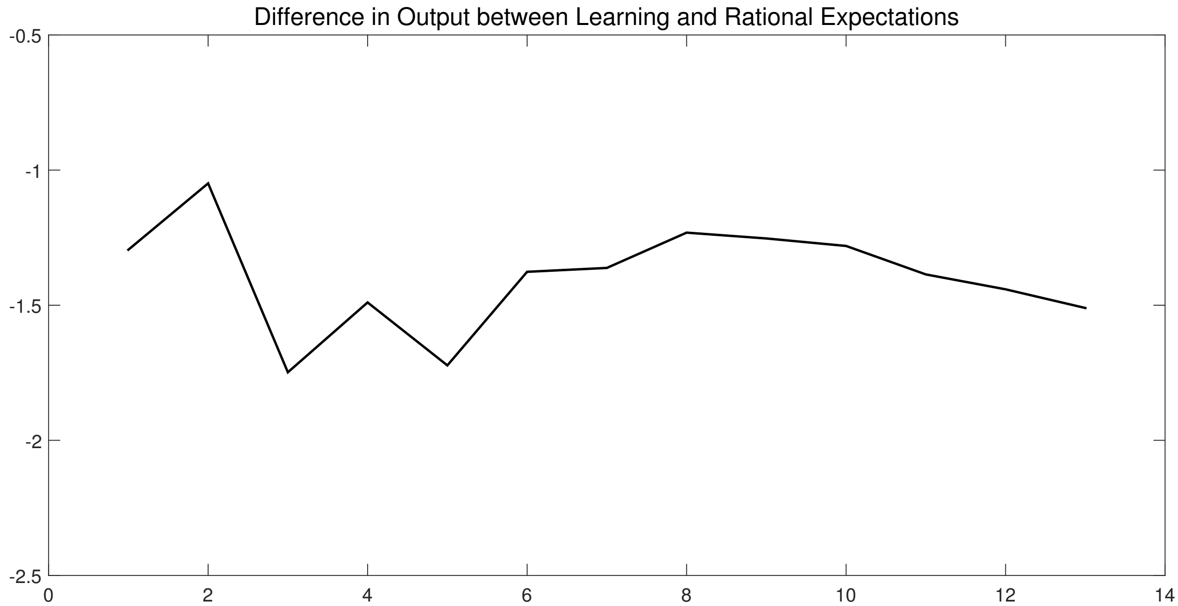


Figure 3.2: Dynamics of Output in Response to Forward Guidance

Note: The graph shows the difference in output between adaptive learning and rational expectations agents. A positive value indicates the value under adaptive learning is higher than under rational expectations. A negative value indicates the variable's value under adaptive learning is lower than under rational expectations.

output under rational expectations is higher than under adaptive learning. These differences in the value of output during the forward guidance horizon are averaged across simulations.

Figure 3.2 also shows that adaptive learning agents fail to understand how the same forward guidance promises made under rational expectations will impact the economy under learning. The effect from central bank forward guidance results in more optimism under rational expectations than under adaptive learning. Across the forward guidance horizon, the value of output is higher under rational expectations than adaptive learning. By having only partial information about the true model of the economy, adaptive learning agents fail to foresee the precise positive impact the forward guidance information has on the economy. Even though they know the changes in the future path of interest rates implemented by the central bank, adaptive learning agents imprecisely understand how that guidance impacts the economy. Rational expectations agents, however, precisely understand the effects of forward guidance on output. They understand the stimulative effect forward guidance has on the economy, and thus, output is higher under rational expectations than adaptive learning.

The results also display that the differences between rational expectations and adaptive learning to forward guidance are exacerbated when including a financial sector in the model. Cole (2015) performed a similar exercised utilizing a smaller scale DSGE model *without* a financial sector. The results showed output under adaptive learning overshooting and undershooting output under rational expectations. In the present paper, Figure 3.2 displays that the value of output is lower under adaptive learning than rational expectations across the forward guidance horizon. The existence of a financial sector causes more inertial behavior (lagged variables) in the PLM and more variables to forecast, and thus, creates a less favorable reaction of adaptive learning to forward guidance and larger differences between the two types of expectation assumptions.

3.4.3 Economic Crisis

In response to the 2007-2009 Great Recession, forward guidance was implemented by central banks around the world. With that event in mind, this section builds upon the previous subsection's exercise by considering forward guidance during an economic recession. The economy is assumed to start in period T, that is, after a period of economic stability (corresponding to say the period before the recent Great Recession).[17] The model is then simulated from T to the end of the forward guidance horizon $T+L$. As in the previous subsection, the central bank implements forward guidance by choosing the unanticipated monetary policy

[17]This strategy also ensures the adaptive learning coefficients converge to its stationary distribution.

and anticipated forward guidance shocks such that the nominal interest rate equals zero from periods T to $T + L$. To capture features from the Great Recession, a large spread shock impacts the economy in period T, and causes a recession. A sequence of five more spread shocks follows so that the recession lasts six periods.[18] In the following periods, the shocks are drawn from a normal distribution. Thus, the forward guidance horizon spans a recession and normal times.

The spread shock operates through the financial sector to cause a downturn in the economy. A higher spread implies banks perceive entrepreneurs to be riskier, and thus, borrowing costs and cost of capital for firms increases. This result hinders firms from receiving capital from entrepreneurs. Lower economic activity results from less capital being channeled to the production side of the economy. Furthermore, the modeling of a recession via a spread shock closely matches the data. Del Negro et al. (2013) show that spread shocks accounted for about half the decline in output growth during the Great Recession in the U.S.

Figure 3.3 displays the macroeconomic effects of forward guidance during an economic recession. The graph shows the value of output under adaptive learning minus the value of output under rational expectations. The positive effects of forward guidance are overstated under the assumption of rational expectations relative to adaptive learning. Throughout the forward guidance horizon, the value of output under rational expectations is higher than under adaptive learning. The former agents know the economy is in a recession and precisely understand how forward guidance will alleviate the economy as their expectations are based on the true model of the economy. However, adaptive learning agents observe the economic downturn, but fail to completely understand the positive effects of forward guidance. They must *estimate* the effects of forward guidance on the economy as their forecasts are based on

[18] This length is based on the duration of the Great Recession as defined by the National Bureau of Economic Research.

an econometric model. Thus, adaptive learning agents are slower to understand how forward guidance will alleviate the downturn in the economy.

The addition of the financial sector in the model amplifies the differences between the adaptive learning and rational expectations in comparison to a DSGE model without financial frictions (e.g. Cole [2015]). In Cole (2015), the model of the economy was based on a smaller scale DSGE model *without* financial frictions. Output was lower under adaptive learning than rational expectations. With the inclusion of the financial sector in the present paper, the differences between the two types of expectations are amplified. The addition of the financial sector includes more inertia in the PLM and more variables to forecast (e.g. $E_t \tilde{R}^k_{t+1}$). The adaptive learning agents must estimate how forward guidance will alleviate the recession by forecasting not only future variables concerning households and firms, but also the financial sector. This creates more errors by the adaptive learning agents relative to their rational expectations counterparts causing the effects of forward guidance to be greatly overstated under rational expectations relative to adaptive learning.

The results in Figure 3.4 show the difference between the expectations of adaptive learning and rational expectations of consumption, gross return on capital for entrepreneurs, and investment across the forward guidance horizon.[19] The positive response of rational expectations agents to forward guidance is overstated relative to adaptive learning showing that the latter type of agents are more pessimistic about the future. Households expect lower consumption in the future if they are forming expectations via adaptive learning than rational expectations. The agents in the financial side of the economy also exhibit similar type of behavior. Expectations about the future gross return on capital for entrepreneurs are higher under adaptive learning than rational expectations indicating a worse economic environment

[19] As a reminder, $\hat{E}_t$ denotes (potentially) non-rational expectations, while E_t represents the model-consistent rational expectations operator.

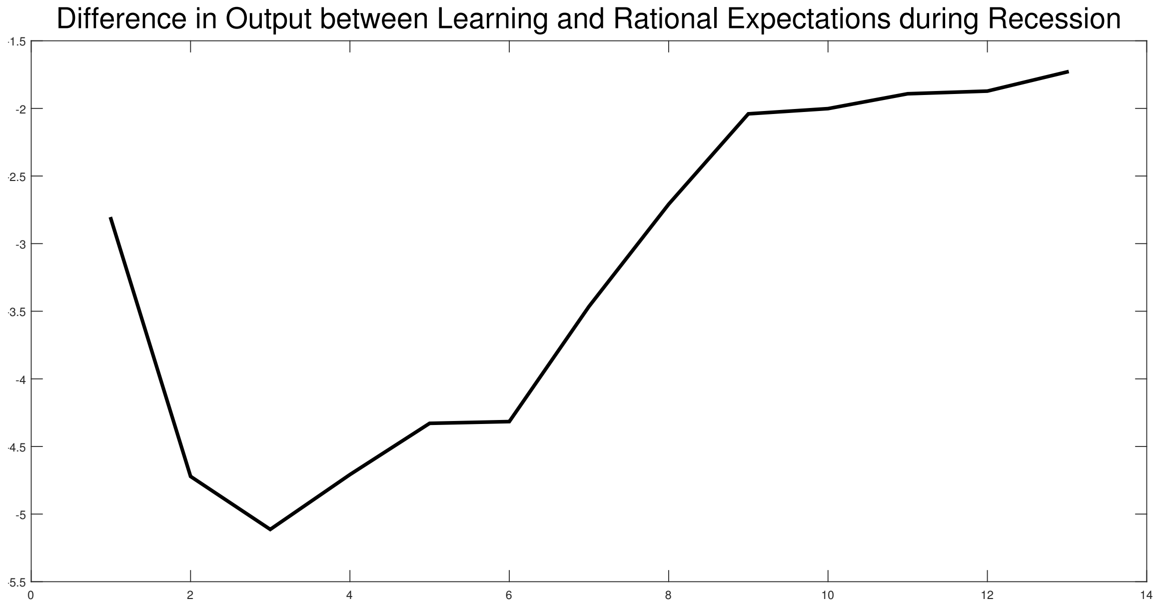


Figure 3.3: Macroeconomic Effects of Forward Guidance during an Economic Crisis

Note: The graph shows the difference in output between adaptive learning and rational expectations agents. A positive value indicates the value under adaptive learning is higher than under rational expectations. A negative value indicates the variable's value under adaptive learning is lower than under rational expectations.

under the former than latter agents. Since $R_t = 0$ over the forward guidance horizon, the spread $(E_t\tilde{R}^k_{t+1} - R_t)$ is higher under adaptive learning than rational expectations. As explained above, higher levels of the spread hurt the economy because borrowing costs hinder entrepreneurs' ability to funnel capital to intermediate goods firms. In response to forward guidance, the rational expectations forecasts about future investment also respond more favorably than under adaptive learning. The bottom subplot in Figure 3.4 shows expectations of investment are higher under rational expectations than under adaptive learning across the forward guidance horizon.

The results also relate to the findings of Del Negro et al. (2012). Their paper showed "the forward guidance puzzle" in which forward guidance statements produced an exceedingly

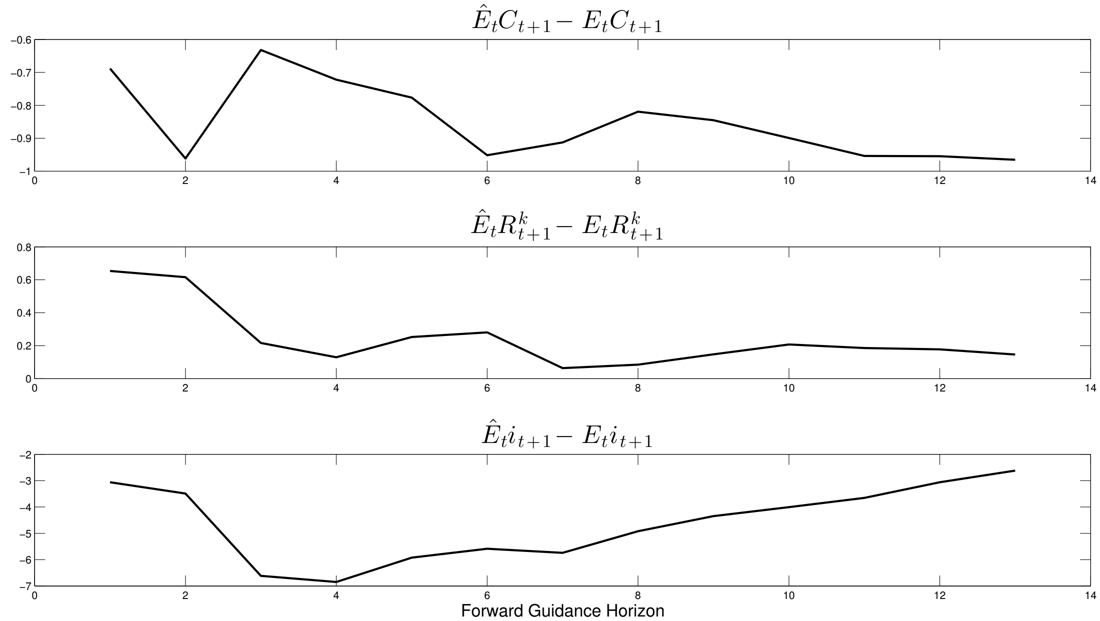


Figure 3.4: Difference between the Expectations of Adaptive Learning ($\hat{E}_t$) and Rational Expectations (E_t) Agents

Note: A positive value indicates the value under adaptive learning is higher than under rational expectations. A negative value indicates the variable's value under adaptive learning is lower than under rational expectations.

large reaction of the macroeconomic variables to forward guidance statements in relation to the data. Their model is the same one as presented above, but solved only under the assumption of rational expectations. As shown in the results of this paper, adaptive learning does not have as large of a response to forward guidance as rational expectations. Specifically, the expectations in Figure 3.4 exhibit a more favorable reaction to forward guidance under rational expectations than adaptive learning. Thus, this paper suggests that the unusually large responses of the macroeconomic variables to forward guidance found in Del Negro et al. (2012) could be due to the way in which expectations are modeled.

Overall, the response of rational expectations agents to forward guidance is overstated relative to adaptive learning. Rational expectations agents precisely know how forward guidance will affect the economy as their forecasts are based on the correct model of the economy. However, adaptive learning agents have partial knowledge about the model of the economy, and must *estimate* the effects of forward guidance on the economy. In addition, the role of financial frictions exacerbates the differences. In relation to a model without a financial sector, there is more inertia in the PLM and additional variables to forecast, and thus, more overall differences between rational expectations and adaptive learning. The expectations of the former respond more favorably to forward guidance relative to the latter agents. The results of adaptive learning to forward guidance also seem to match the data better than rational expectations. Thus, it is imperative to include credit frictions when understanding the effects of forward guidance on the economy.

3.5 Conclusion

The 2007-2009 global financial crisis caused central banks around the world to implement the unconventional monetary policy of forward guidance to stimulate their economies. The effectiveness of forward guidance hinges on two key channels–expectations and financial markets–that are largely overlooked in standard macroeconomic models. The standard expectations formation assumption is the rational expectations hypothesis, while frictionless financial markets are largely assumed for convenience. Thus, it is of interest to investigate the effectiveness of forward guidance when the rational expectations assumption has been relaxed and credit frictions are included.

This paper utilizes a medium scale DSGE model with financial frictions to compare the effects of forward guidance under both rational expectations and adaptive learning. The results show that the addition of financial markets into a DSGE model amplifies the differences between rational expectations and adaptive learning to forward guidance statements. When a one unit forward guidance shock impacts the economy, adaptive learning agents fail to understand the precise effects of forward guidance. When the central bank gives the same forward guidance information such that the interest rate equals zero for an extended period of time to both types of agents, the results display that the value of output under rational expectations is higher than under adaptive learning throughout the forward guidance horizon. During a period of economic crisis (e.g. a recession), output under rational expectations also displays more favorable responses to forward guidance than under adaptive learning. Rational expectations agents form their forecasts based off the true model of the economy, and thus, can understand how forward guidance will precisely help the economy. However, adaptive learning agents must estimate the effects of forward guidance on the economy as their forecasts are based off an econometric model of the economy. In addition, these differences are magnified when compared to an analysis without financial frictions (e.g. Cole [2015]). The additional inertia in the PLM, more financial sector variables to forecast, and the fact that adaptive learning agents estimate the effects of forward guidance create bigger differences between the two types of expectation assumptions.

There are other modifications to the model presented in this paper that are worth noting. For example, the credibility of central bank forward guidance announcements could be examined as in Dong (2014). In the model presented above, agents believe the forward guidance statements, and the central bank implemented its forward guidance promises. However, the results could be examined when agents do not completely believe the central bank will complete its forward guidance promises. The type of forward guidance could also be changed. This current paper examines time-contingent forward guidance in which the central bank informs agents of the end date of forward guidance. Another type of forward guidance is called state contingent in which the central bank links the expiration date of forward guidance to economic conditions (e.g. unemployment rate and output). The RLS formula could also be modified to allow agents to better track structural changes in the economy as described in Marcet and Nicolini (2003) and Milani (2014). Specifically, the gain parameter would be a constant if the recent prediction errors were large and decreasing if the recent prediction errors were small. Overall, the role of expectations and credit market frictions is especially crucial to understand the effects of forward guidance on the economy.

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Appendix A

Appendix to Chapter 1

A.1 Data Description

$$
\begin{aligned}
output &= LN\left(\frac{GDPC96/CNP160V}{GDPC96(-1)/CNP160V(-1)}\right) * 100 && \text{(A.1.1)} \\
investment &= LN\left(\frac{FPI/CNP160V}{FPI(-1)/CNP160V(-1)}\right) && \text{(A.1.2)} \\
inflation &= LN\left(\frac{GDPDEF}{GDPDEF(-1)}\right) * 100 && \text{(A.1.3)} \\
spread &= (BAA - FEDFUNDS)/4 && \text{(A.1.4)}
\end{aligned}
$$

Table A.1.1: **Data Series Description**

Series ID:	**Title:**	**Source:**	**Description:**
GDPC96	Real GDP	US Dept. of Commerce: Bureau of Economic Analysis	Billions of Chained 2005 Dollars
FPI	Fixed Private Investment	US Dept. of Commerce: Bureau of Economic Analysis	Billions of Dollars
CNP160V	Civilian Noninstitutional Population	US Dept. of Labor: Bureau of Labor Statistics	At least 16 years of age, noninstitutional, not on active duty
GDPDEF	GDP Implicit Price Deflator	US Dept. of Commerce: Bureau of Economic Analysis	Index 2005=100
FEDFUNDS	Effective Federal Funds Rate	Board of Governors of the Federal Reserve System	Averages of Daily Figures
BAA	Moody's Seasoned Baa Corporate Bond Yield	Board of Governors of the Federal Reserve System	Averages of daily data

A.2 Model

A.2.1 Households

In GK (2011), a household consumes, saves, and supplies labor to intermediate goods firms. They pay lump-sum taxes to the government. Each household is composed of members who are either workers or bankers. A worker receives a wage for his or her labor. Every banker is also in charge of a financial intermediary, which will be described in a later section. Households deposit their money into riskless one-period ahead bonds issued by either a banker or government.[1] The banker transfers earnings back to its household. At the beginning of each period, the household members can switch between professions. The probability that a banker stays a banker in the next period is independent of previous history and is defined as θ. Thus, (1-θ) of bankers switch to become workers each period. In addition, each new banker is endowed with a start-up transfer from the family household. This transfer is a small fraction, χ, of total assets.

The representative household maximizes expected discounted utility given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^t \left[ln(C_{t+i} - hC_{t+i-1}) - \frac{\omega}{1+\iota} L_{t+i}^{1+\iota} \right] \tag{A.2.1}$$

where ι is the inverse of Frisch elasticity of labor supply, ω defines the influence of leisure on utility, h is the habit formation parameter, and $\beta > 0$. The budget constraint is given by

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \tag{A.2.2}$$

where Π_t is the net transfer the family household endows new bankers. T_t defines lump sum taxes, W_t real wages, C_t consumption, and L_t family labor supply. B_t represents the diskless one-period ahead bonds. Furthermore, R_t is the riskless gross real return from t-1 to t paid on real bonds. Thus, $R_t B_t$ is the total value of deposits and government debt that the household earned when it invested in those financial assets in the previous period. B_{t+1} is the amount of real bonds bought.

The first order conditions with respect to consumption, labor supply, and one period ahead bonds are

$$\Xi_t = (C_t - hC_{t-1})^{-1} - \beta h \mathbb{E}_t (C_{t+1} - hC_t)^{-1} \tag{A.2.3}$$

[1] A household deposits money with a banker who is not a part of its household.

$$\Xi_t W_t = \omega L_t^{\iota} \tag{A.2.4}$$

$$\mathbb{E}_t \beta \Lambda_{t+1} R_{t+1} = 1 \tag{A.2.5}$$

where

$$\Lambda_{t,t+1} = \frac{\Xi_{t+1}}{\Xi_t} \tag{A.2.6}$$

and Ξ_t represents the marginal utility of consumption.

A.2.2 Financial Intermediaries

Financial intermediaries, or bankers, represent both investment and commercial banks. They take in deposits from households and lend these funds to non-financial firms. In order to receive the funds, the non-financial firms issue financial claims S_t, which have a value Q_t. The bankers maximize expected terminal wealth. The key result is the following incentive constraint

$$V_{jt} \geq \lambda^h Q_t S_{jt} \tag{A.2.7}$$

where V_{jt} is expected terminal wealth of the financial intermediary. λ^h characterizes the fraction of assets that the banker diverts back to the household. Thus, equation (A.2.7) states that the expected terminal wealth needs to be at least as large as the amount of assets the financial intermediary gives back to his or her household. Otherwise, the banker would have no incentive to be a banker.

GK (2011) show that if the previous constraint binds it can be rewritten as

$$Q_t S_{jt} = \phi_t N_{jt} \tag{A.2.8}$$

where ϕ_t is referred to as the private leverage ratio. In other words, ϕ_t is the ratio of privately intermediated assets to equity.

The aggregate net worth N_t is the sum of existing and new bankers. The net worth of current bankers is expressed as

$$N_{et} = \{\theta[(R_{kt} - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1}\} \tag{A.2.9}$$

where θ is the probability that a banker stays a banker, R_{kt} is what is earned from financial assets, and R_t is the amount paid to households for deposits. Note that the premium $(R_{kt} - R_t)$ plays an important role in net worth. If this value increases, then the amount that bankers receive from lending and taking in deposits increases and thus, overall net worth becomes larger. Recall also that new bankers receive a nominal transfer from the household. This transfer is a fraction of the value of total financial assets. Thus, the net worth for new bankers can be written as

$$N_{nt} = \chi Q_t S_{t-1} \tag{A.2.10}$$

where χ is the fraction of the value of total financial assets. Adding equations (A.2.9) and (A.2.10) yields the aggregate net worth equation

$$N_t = \theta[(R_{kt} - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1} + \chi Q_t S_t \quad \text{(A.2.11)}$$

A.2.3 New Central Bank Policy

In addition to following a standard Taylor rule, the central bank conducts new monetary policy by intervening in the financial market. The central bank raises funds from households by issuing debt that pays R_t, and then lends this money to non-financial firms, who pay R_t^k for the funds. There also exists an efficiency cost of τ per unit of intermediated assets.

The total value of financial assets in economy is composed of private and public lending

$$Q_t S_t = Q_t S_{pt} + Q_t S_{gt} \quad \text{(A.2.12)}$$

where the central bank funds a fraction ψ_t of intermediated assets

$$Q_t S_{gt} = \psi_t Q_t S_t \quad \text{(A.2.13)}$$

By combining equations (A.2.12), (A.2.13), (A.2.8), we get

$$Q_t S_t = \phi_t N_t + \psi_t Q_t S_t = \phi_{ct} N_t \quad \text{(A.2.14)}$$

where

$$\phi_{ct} = \frac{1}{1 - \psi_t}\phi_t \quad \text{(A.2.15)}$$

where GK (2011) define ϕ_t as the ratio of privately intermediated assets to equity and ϕ_{ct} as the ratio of total intermediated assets to equity.

A.2.4 Intermediate Goods Firms

The intermediate goods firms use capital and labor to make their product in a competitive market. At the end of the period, they choose the amount of capital K_{t+1} needed for production in the next period. Intermediate goods firms also sell unused capital to capital producing firms. The amount of unused capital is given by

$$(Q_{t+1} - 1 * \delta(U_{t+1}))\varepsilon_t^b K_{t+1} \tag{A.2.16}$$

where the price of depreciated capital is 1 and Q_{t+1} is the value of new capital. ε_t^b captures a shock to the value of capital.

In order to buy capital, they borrow funds from bankers by issuing state-contingent claims S_t. They issue as many state-contingent claims as necessary to purchase the necessary capital, K_{t+1}. The bankers receive R_t^k from lending. Intermediate goods firms are subject to two equations. First, the production function for the representative intermediate goods firms follows the Cobb-Douglas form

$$Y_t = A_t(U_t \varepsilon_t^b K_t)^\alpha l_t^{1-\alpha} \tag{A.2.17}$$

where U_t is the utilization rate, A_t is total factor productivity, and ε_t^b is a shock to the value of capital. The second constraint descthe value of capital bought, $Q_t K_{t+1}$, and the value of funds borrowed from bankers, Q_tS_t. It is given by

$$Q_tK_{t+1} = Q_tS_t \tag{A.2.18}$$

where Q_t defines the price of a unit of capital.

The first-order conditions for capital utilization, demand for labor, and K_{t+1} yield

$$P_t^m \alpha \frac{Y_t}{U_t} = \delta^{'}(U_t)\varepsilon_t^b K_t \tag{A.2.19}$$

$$P_t^m (1-\alpha)\frac{Y_t}{L_t} = W_t \tag{A.2.20}$$

$$R_{t+1}^k = \frac{\left[P_t^m \alpha \frac{Y_{t+1}}{\varepsilon_{t+1}^b K_{t+1}} + Q_{t+1} - \delta(U_{t+1})\right]\varepsilon_{t+1}^b}{Q_t} \tag{A.2.21}$$

A.2.5 Capital Producing Firms

The capital producing firms buy leftover capital from intermediate goods firms, refurbish it, and produce new capital I_{nt}. This new capital adds to the capital stock

$$K_{t+1} = \varepsilon_t^b K_t + I_{nt} \tag{A.2.22}$$

Capital producing firms maximize their expected discounted profits

$$max\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{T-t}\Lambda_{t,\tau}\left\{(Q_\tau - 1)I_{n\tau} - f\left(\frac{I_{n\tau}+I_{ss}}{I_{n_{\tau-1}}+I_{ss}}\right)(I_{n\tau}+I_{ss})\right\} \tag{A.2.23}$$

with

$$I_{nt} \equiv I_t - \delta(U_t)\varepsilon_t^b K_t \tag{A.2.24}$$

where $f(1)$=f'=0 and $f''(1)$ >0. I_t is gross capital created, I_{ss} is steady-state investment, and I_{nt} characterizes net capital created. The first order condition for net capital created results in an equation for the price of a unit of capital

$$Q_t = 1 + f(\cdot) + \frac{I_{nt}+I_{ss}}{I_{nt-1}} f'(\cdot) - \mathbb{E}_t\beta\Lambda_{t,t+1}\left(\frac{I_{n\tau}+I_{ss}}{I_{n_{\tau-1}}}\right)^2 f'(\cdot) \tag{A.2.25}$$

A.2.6 Retail Firms

The retail firms vary from the intermediate goods firms. Retail firms purchase intermediate goods in order to produce the final output, Y_t. There exists a continuum of retail firms indexed by f on the unit interval who each produce differentiated goods Y_{ft}. Retailers also are subject to a Calvo(1983) pricing scheme. In every period, each final good firm has a probability $(1-\sigma)$ of being able to adjust its price. If a firm cannot reoptimize, then it has a probability σ_p of indexing its price to the previous period's inflation rate. The retailer's optimization problem is then

$$max\mathbb{E}_t \sum_{i=0}^{\infty} \sigma^i\beta^i\Lambda_{t,t+i}\left[\frac{P_t^*}{P_{t+i}}\prod_{k=1}^{i}(1+\pi_{t+k-1})^{\sigma_p} - P_{mt+i}\right]Y_{ft+i} \tag{A.2.26}$$

With $\mu = \frac{1}{1-\frac{1}{\epsilon}}$, the first-order condition with respect to P_t^* is given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \sigma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1+\pi_{t+k-1})^{\sigma_p} - \mu P_{mt+i} \right] Y_{ft+i} = 0 \tag{A.2.27}$$

The familiar pricing equation is given by

$$P_t = [(1-\sigma)(P_t^*)^{1-\epsilon} + \sigma(\Pi_{t-1}^{\sigma_p} P_{t-1})^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{A.2.28}$$

A.2.7 Government Policy

The central bank follows both conventional and unconventional procedures. The monetary authority adjusts the short-term nominal interest rate according to a Taylor rule

$$i_t = i_{t-1}^{\rho} (\pi_t^{i_\pi} y_t^{i_y})^{1-\rho} exp(\varepsilon_t^i) \tag{A.2.29}$$

As described above, ψ_t defines the fraction of intermediated assets that the central bank is willing intervene and is characterized by the following rule

$$\psi_t = \psi + \nu \mathbb{E}_t[(R_{t+1}^k - R_t) - (R^k - R)] \tag{A.2.30}$$

where R_t^k is the lending rate and R_t is the interest rate earned on deposits. Thus, R^k-R is the steady-state premium. ψ defines the steady-state fraction of publicly intermediated assets. Shows that the central bank intervenes more in the financial market and issues more government bonds when the cost of borrowing funds R_{t+1}^k increases relative to the interest paid on deposits R_t. In other words, equation (A.2.30) can be viewed as an additional tool for the central bank to respond to a recession.

A.2.8 Resource Constraint and Capital Stock

The resource constraint includes standard and model specific terms

$$Y_t = C_t + I_t + f\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right)(I_{nt} + I_{ss}) + G + \tau\psi_t Q_t K_{t+1} \tag{A.2.31}$$

Government spending is taken as exogenous. The last term in equation (A.2.31) comes from the fact that $\tau\psi_t Q_t S_t$ is the amount the central bank spends on intervention and $S_t = K_{t+1}$ from equation (A.2.18).

The amount of capital stock is given by the following equation

$$K_{t+1} = \varepsilon_t^b K_t + I_{nt} \tag{A.2.32}$$

Combining the previous equation with equation (A.2.24), we get the final equation for flow of capital stock

$$K_{t+1} = I_t + (1 - \delta(U_t))\varepsilon_t^b K_t \tag{A.2.33}$$

Appendix B

Appendix to Chapter 2

B.1 Rational Expectations Solution

By following Sims (2002), the model consisting of equations (2.10), (2.18), (2.19), (2.21) – (2.24), (2.27), and (2.28) can be solved to yield the solution

$$\widetilde{Y}_t = \widetilde{C} + \xi_1 \widetilde{Y}_{t-1} + \xi_2 \epsilon_t \tag{B.1.1}$$

where

$$\widetilde{C} = [0, 0, 0.01, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]' \tag{B.1.2}$$

$$\xi_1 = [\xi_{1a}, \xi_{1b}] \tag{B.1.3}$$

$\xi_{1a} =$

$$\begin{bmatrix}
0 & 0 & 0 & 0.62 & -1.11 & -0.79 & -0.60 & -0.43 & -0.29 & -0.16 \\
0 & 0 & 0 & 0.12 & 0.77 & -0.08 & -0.14 & -0.18 & -0.21 & -0.22 \\
0 & 0 & 0 & 0.25 & 0.94 & 0.79 & -0.27 & -0.31 & -0.33 & -0.33 \\
0 & 0 & 0 & 0.50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.50 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.31 & -0.55 & 0 & -0.79 & -0.60 & -0.43 & -0.29 \\
0 & 0 & 0 & 0.06 & 0.39 & 0 & -0.08 & -0.14 & -0.18 & -0.21
\end{bmatrix} \tag{B.1.4}$$

$$\xi_{1b} =$$

$$\begin{bmatrix} -0.06 & 0.02 & 0.08 & 0.13 & 0.16 & 0.18 & 0.19 & 0 & 0 \\ -0.23 & -0.22 & -0.21 & -0.20 & -0.18 & -0.16 & -0.14 & 0 & 0 \\ -0.32 & -0.31 & -0.28 & -0.26 & -0.23 & -0.20 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.16 & -0.06 & 0.02 & 0.08 & 0.13 & 0.16 & 0.18 & 0 & 0 \\ -0.22 & -0.23 & -0.22 & -0.21 & -0.20 & -0.18 & -0.16 & 0 & 0 \end{bmatrix} \tag{B.1.5}$$

$$\xi_2 = [\xi_{2a}, \xi_{2b}] \tag{B.1.6}$$

$\xi_{2a} =$

$$\begin{bmatrix} 1.25 & -2.22 & -0.79 & -0.60 & -0.43 & -0.29 & -0.16 \\ 0.25 & 1.54 & -0.08 & -0.14 & -0.18 & -0.21 & -0.22 \\ 0.50 & 1.88 & 0.79 & -0.27 & -0.31 & -0.33 & -0.33 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.62 & -1.11 & 0 & -0.79 & -0.60 & -0.43 & -0.29 \\ 0.12 & 0.77 & 0 & -0.08 & -0.14 & -0.18 & -0.21 \end{bmatrix} \tag{B.1.7}$$

$$\xi_{2b} =$$

$$\begin{bmatrix}
-0.06 & 0.02 & 0.08 & 0.13 & 0.16 & 0.18 & 0.19 & 0.19 \\
-0.23 & -0.22 & -0.21 & -0.20 & -0.18 & -0.16 & -0.14 & -0.12 \\
-0.32 & -0.31 & -0.28 & -0.26 & -0.23 & -0.20 & -0.17 & -0.14 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-0.16 & -0.06 & 0.02 & 0.08 & 0.13 & 0.16 & 0.18 & 0.19 \\
-0.22 & -0.23 & -0.22 & -0.21 & -0.20 & -0.18 & -0.16 & -0.14
\end{bmatrix} \tag{B.1.8}$$

Appendix C

Appendix to Chapter 3

C.1 Parameter Values

Table C.1.1: Parameter Values

	Description	Value
α	Capital's Share of Output	0.33
ζ_p	Price Stickiness	0.75
ι_p	Price Indexation	0.5
δ	Depreciation	0.025
Φ	Share of Fixed Costs	0.8
S''	Investment Adjustment Cost	4
h	Habit Formation	0.7
a''	Capital Utilization Cost	0.2
ν_l	Elasticity of Labor Supply	2
ν_m	Money Demand	2
β	Discount Factor	0.99
ζ_w	Wage Stickiness	0.75
ι_w	Wage Indexation	0.5
λ_w	Elast. of Sub. Diff. Labor Services	0.3
ψ_π	Feedback Inflation	1.55
ψ_y	Feedback Output	0.125
ρ_r	Lagged Interest Rate	0.8
ζ_{spb}	Elast. of Spread w.r.t. Leverage	0.05
γ	Steady-State Growth Rate of Economy	2.75
χ	Money Demand	0.1
λ_f	Steady-State Price Mark-Up	0.15
g_*	Steady-State Government	0.3
$F(\bar{\omega})$	Steady-State Default Rate	0.03
ρ_μ	Autoregressive MEI	0.2
ρ_ϕ	Autoregressive Labor	0.2
ρ_g	Autoregressive Gov't	0.2
ρ_{λ_f}	Autoregressive Price Mark-up	0.2
ρ_{σ_w}	Autoregressive Spread	0.2
ρ_b	Autoregressive Preference Shifter	0.2
ρ_{μ^e}	Autoregressive Bankruptcy	0.2
ρ_γ	Autoregressive Survival Entrepreneurs	0.2
L	FG Horizon	12
$\bar{\tau}$	CGL	0.02

Note: The standard deviations of the structural shocks are set to 0.0001. FG stands for forward guidance.