UNIVERSITY OF CALIFORNIA
RIVERSIDE

Essays in Testing and Forecasting With Nested Predictive Regression Models Using Encompassing Principle

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Yan Ge

June 2015

Dissertation Committee:
Dr. Tae-Hwy Lee, Chairperson
Dr. Gloria Gonzales-Rivera
Dr. Jang-Ting Guo
Dr. Shujie Ma
Dr. Aman Ullah

The Dissertation of Yan Ge is approved:

____

____

____

____

____

Committee Chairperson

University of California, Riverside

ACKNOWLEDGEMENT

First of all, I would like to express my sincere gratitude and appreciation to my advisor Prof. Tae-Hwy Lee for his continuous support, patience and motivation of my pursuance in PhD study and research. His guidance helped me a lot not only in the aspect of research and writing of this thesis but also being an integrated person. It's my most fortunate thing to have Prof. Tae-Hwy Lee to be my advisor.

Besides my advisor, I would like to thank the rest of my committee members: Prof. Aman Ullah, Prof. Gloria Gonzáles-Rivera, Prof. Jang-ting Guo, Prof Shujie Ma for their encouragement and precious pieces of advice

My sincere thanks also goes to my department for offering me the funding to attend Econometric Society, Chinese Meeting (CMES 2014), The NSF/NBER Time Series Conference, Federal Reserve Bank of St. Louis in 2014 to show my work.

Last but not the least, I would like to thank my family supporting me spiritually throughout my life.

ABSTRACT OF THE DISSERTATION

Essays in Testing and Forecasting With Nested Predictive Regression Models
Using Encompassing Principle

by

Yan Ge

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2015
Dr. Tae-Hwy Lee, Chairperson

Out-of-sample tests for equal predictive accuracy have been widely used in economics and finance and are regarded as the "ultimate test of a forecasting model". When two non-nested models are compared, Diebold and Mariano (DM 1995) point out that the t-statistic of the mean squared-error loss-differential is asymptotically standard normal. When two models are nested, however, Clark and McCracken (CM 2001, 2005, 2009) point out that due to the parameter prediction error (PEE), the statistics will result in non-standard distribution. Further more Clark and West (CW 2006, 2007) point out that the DM statistic for testing the equal predictive accuracy of two nested mean regression models gives a favor to a smaller (nested) model, because the DM statistic tends to be negative under the null hypothesis, penalizing the bigger (nesting) model for the finite sample parameter estimation sampling error. They point out that the negative bias can be corrected by adding a non-negative adjustment term. The adjusted DM statistics (DM plus the adjustment term) is equivalent to the "encompassing test". The thesis consists of three chapters: The first chapter is comparing predictive accuracy and model combination using encompassing test for Nested Quantile Models, we consider using the quantile model and check loss function. We show that the adjusted DM statistics is asymptotically standard normal when out-of-sample to in-sample ratio goes to infinity. The second chapter is comparing nested predictive regression models with persistent predictors, in which we introduce a persistent estimator in the second model. We show that the adjusted DM statistics will still be asymptotically standard normal due to the faster convergence rate of the second model. The third chapter is encompassing test for nested predictive regression models with near unit root and drift, the big model contains a persistent estimator with drift. We show regardless whether drift term (deterministic trend) or the coefficient of autoregressive process of the predictor (stochastic trend) dominates the model, due to the higher than root-n convergence rate of the coefficient in the second model, the adjusted DM statistics is asymptotically standard normal.

Table of Contents

Chapter 1

Comparing Predictive Accuracy and Model Combination Using Encompassing Test for Nested Quantile Models

Yan Ge* and Tae-Hwy Lee‡

September 2014

Abstract

This paper extends Clark and McCracken (CM 2001, 2005, 2009) and Clark and West (CW 2006, 2007) from the conditional mean to conditional quantile. CM/CW point out that the statistic of Diebold and Mariano (DM 1995) for testing the equal predictive accuracy of two nested mean regression models gives a favor to a smaller (nested) model, by penalizing the bigger (nesting) model for parameter estimation error and making the DM statistic tend to be negative under the null hypothesis. In this paper, we compare two nested quantile regression models using the check-loss function. We point out that the same problem occurs in using the DM statistic for quantile regression. We show that the DM statistic of the

*Department of Economics, University of California, Riverside, CA 92521. E-mail: yge001@ucr.edu

†Department of Economics, University of California, Riverside, CA 92521. E-mail: taelee@ucr.edu

‡The authors would like to thank Gloria Gonzalez-Rivera, Mike McCracken, Yundong Tu, Aman Ullah, and participants at the Chinese Meeting of Econometric Society (CMES2014) at Xiamen, the 2014 NBER-NSF Time Series Conference at St. Louis, and UCR econometrics seminar, for many valuable suggestions and comments.

check-loss differential of the two nested quantile regression models tends to be negative under the null hypothesis of the equal predictive ability. We derive the adjustment term to be added to the DM statistic such that the adjusted statistic has the zero mean under the null. The finite sample behavior of DM and the adjusted statistic under the null and alternative hypotheses are examined by Monte Carlo simulation. We show that the adjusted statistic is the encompassing statistic for the quantile models and provides the optimal combination of the two nested quantile models in the sense that the combined forecast error has the minimum expected check-loss. It thus provides the optimal Stein-like shrinkage from the larger quantile model towards the smaller quanitle model, as discussed in CM (2009) for the mean regression. An application of the adjusted statistic to the predictive quantile regression of the equity premium shows strong predictive ability (Granger-causality) of dividend-yield ratio and dividend-price ratio for forecasting the quantiles of the equity premium, which could not have been observed by using DM without the adjustment.

Key Words: check-loss, predictive quanitle regression, parameter estimation error, DM statistic, encompassing, combination, shrinkage, Monte Carlo, equity premium.

JEL Classification: C53, E37, E27

1 Introduction

Out-of-sample tests for equal predictive accuracy have been widely used in economics and finance and are regarded as the "ultimate test of a forecasting model" (Hansen and Timmermann 2012, 2013). Even if a great deal of test have been widely used for in-sample models such as Wald test, t-test and F-test, Ashley, Granger, and Schmalensee (1980) advocate using out-of-sample forecast comparisons to test Granger causality, i.e., using forecast comparisons to determine whether one explanatory variable has predictive power on dependent variable has been commonly used since at least the influential work of Meese and Rogoff (1983, 1988). When two non-nested models are compared, Diebold and Mariano (DM 1995) point out that the t-statistic of the mean squared-error loss-differential is asymptotically standard normal. When two models are nested, however, Clark and McCracken (CM 2001, 2005, 2009) point out that the DM statistic behaves quite differently from non-nested case since both the numerator and denominator degenerate, which will result in non-standard distribution. Clark and West (CW 2006, 2007) point out that the DM statistic for testing the equal predictive accuracy of two nested mean regression models gives a favor to a smaller (nested) model, because the DM statistic tends to be negative under the null hypothesis, penalizing the bigger (nesting) model for the finite sample parameter estimation sampling error. They point out that the negative bias can be corrected by adding a non-negative adjustment term. The adjusted DM statistics (DM plus the adjustment term) is equivalent to the "encompassing test" (henceforth, ENC) of Nelson (1972) and Harvey, Leybourne and Newbold (1998). The adjustment term not only corrects the size but also increases the power of the test. CW (2006) show via an extensive Monte Carlo study that the ENC statistic is approximately normal and has proper size and good power.[1]

Comparing predictive accuracy of two models is challenging and has long been researched, although it has been almost entirely for the conditional mean regression. Sometimes we want to investigate the Granger Causality in Distribution by checking if the unconditional CDF of y equals the conditional CDF of y on x at every quantile ranging from 0 to 1. However due to

[1] Other tests for out-of-sample comparisons of nested models include those of Chao, Corradi and Swanson (CCS 2001), Corradi and Swanson (2002), and Giacomini and White (2006), all of which exploit the martingale difference property of the error term of the correctly specified model. These conditional moment tests are shown to be asymptotically normal under some regularity conditions in each of these papers.

the complexity, we investigate the Granger Causality in certain quantiles. In financial market, the Granger Causality in tail are usually linked with the downturn or upturn of the economy, which can not be observed through Granger Causality in Mean model. In this paper, we extend CM and CW from the mean regression to quantile regression and compare two nested quantile regression models using the check loss function. We point out that the same problem occurs when the DM statistic is used to compare two nested quantile regression models. We point out that the DM statistic based on the expected check loss-differential of the two nested quantile regression models is negative under the null hypothesis of the equal predictive ability. We derive the adjustment term to be added to the DM statistic such that the adjusted statistic becomes the encompassing statistic for the two nested quantile models. We show that the adjustment term is positive and corrects the size of DM. The encompassing test has therefore correct size and better power. To see the finite sample behavior of the DM and the adjusted encompassing statistic under the null and alternative hypotheses, Monte Carlo simulation is conducted. It shows that the bias in DM is often substantial and the encompassing statistic behaves excellently both under the null and alternative hypotheses. As the encompassing test is a consistent test, its power increases as the number of the out-of-sample observations in computing the statistic increases.

The encompassing test gives the estimates of the optimal weights of combining two nested quantile regression models in the sense that the combined forecast error has the minimum expected check loss value, it also provides the optimal Stein-like shrinkage from the large model towards the small model, as discussed in CM (2009) for the mean regression. We note that testing for equal predictive accuracy is a special case of model combination in that the two models under comparison will be given weights of either 0 or 1 (depending on the contribution of the additional predictor). However, the weights can be any real numbers as considered by Bates and Granger (1969). For nested models, CM (2009) set the coefficient of the additional variable to be "local to zero" and examine how the optimal weights depend on the variance of the error term (noise) and that of the additional variable (signal). The encompassing statistic also produces the optimal shrinkage and the optimal combined forecasts.

An application of the adjusted DM test (encompassing test) to the predictive quantile regression of the equity premium shows strong predictive ability (Granger-causality) of dividend-yield ratio for forecasting the quantiles of the equity premium under smaller in-sample observation to out-of-sample observation ratio case, which could not have been observed by using DM without the adjustment. also we observe strong predictive ability in tails, which could not have been tested using mean regression. We also find that the encompassing test based on the asymptotic distribution and the results from bootstrap method are consistent, which validates the asymptotic normality of ENC in comparing the two nested quantile regression models.[2]

The paper is organized as follows. Section 2 is a review of CM/CW regarding the Granger-causality in mean (GCM), to motive our work. Section 3 is the main part of the paper where we extend the CM and CW methods for testing GCM to testing Granger-causality in quantile (GCQ). Section 4 is the Monte Carlo simulation to examine the finite sample size and power behavior of the DM and the adjusted statistics in comparing two nested quantile models. In Section 5 we present the empirical analysis for Goyal and Welch (2008) in comparing the two nested conditional quantile models. Section 6 concludes.

2 Comparing Nested Conditional Mean Models

To test for the out-of-sample predictive ability of x_t for the conditional mean of y_{t+1}, CW (2006) compare the two nested mean models

$$\text{Model 1} \quad : \quad y_{t+1} = e_{t+1}^{(1)} \tag{1}$$

$$\text{Model 2} \quad : \quad y_{t+1} = c + bx_t + e_{t+1}^{(2)}, \tag{2}$$

where the dependent variable y_{t+1} is a martingale difference series with respect to the predictor x_t for the first model. The null hypothesis that $c = b = 0$ would be equivalent to the equal predictive ability of the two model under the squared error loss, namely,

$$\mathbb{H}_0 : \mathbb{E}\left[\left(e_{t+1}^{(1)}\right)^2 - \left(e_{t+1}^{(2)}\right)^2\right] = 0 \tag{3}$$

[2]Other papers regarding the comparison of two nested quantile regression models include Lee and Yang (2012) who use Giacomini and White (2006) for the quantile models, Giacomini and Komunjer (2005) who use a framework of the generalized method of moments (GMM), Chuang et al (2009) who study the stock return-volume relations, and Jeong et al (2012) who use a nonparametric test.

and the expectation is taken over the out-of-sample (OOS) period. Model 2 is estimated in a rolling window scheme with window size R ending at time t (starting at $t-R+1$). The out-of-sample evaluation period is $t = R, \ldots, T$ (hence the out-of-sample size is $P = T - R + 1$). For a fixed number of in-sample estimation sample R, the above population moment of the squared error loss-differential can be estimated by the following OOS moment over $t = R, \ldots, T$. The Diebold-Mariano squared error loss-differential is defined as

$$\hat{D}_P = P^{-1} \sum_{t=R}^{T} \left[\left(\hat{e}_{t+1}^{(1)} \right)^2 - \left(\hat{e}_{t+1}^{(2)} \right)^2 \right].$$

We have the following proposition:

Proposition 1: *Under* $\mathbb{H}_0$, $\mathbb{E}\hat{D}_P < 0$.

Proof:

$$\begin{aligned}
\hat{D}_P &= P^{-1} \sum_{t=R}^{T} \left[\left(\hat{e}_{t+1}^{(1)} \right)^2 - \left(\hat{e}_{t+1}^{(2)} \right)^2 \right] \\
&= P^{-1} \sum_{t=R}^{T} \left[(y_{t+1})^2 - \left(y_{t+1} - \hat{c}_t + \hat{b}_t x_t \right)^2 \right] \\
&= P^{-1} \sum_{t=R}^{T} \left[2y_{t+1} \left(\hat{c}_t + \hat{b}_t x_t \right) - \left(\hat{c}_t + \hat{b}_t x_t \right)^2 \right] \\
&= P^{-1} \sum_{t=R}^{T} 2e_{t+1}^{(1)} \left(\hat{c}_t + \hat{b}_t x_t \right) - P^{-1} \sum_{t=R}^{T} \left(\hat{c}_t + \hat{b}_t x_t \right)^2 \\
&\equiv \hat{B}_P - \hat{A}_P \\
&\overset{p}{\to} 0 - \mathbb{E} \left(\hat{c}_t + \hat{b}_t x_t \right)^2 < 0 \qquad \text{as } P \to \infty.
\end{aligned}$$

The first term vanishes because $\mathbb{E}\left(e_{t+1}^{(1)} | x_t\right) = 0$ under $\mathbb{H}_0$, which implies that $\mathbb{E}\left(\hat{B}_P\right) = \mathbb{E}\left[e_{t+1}^{(1)} \left(\hat{c}_t + \hat{b}_t x_t\right)\right] = 0$. Therefore, the squared error loss-differential tends to be negative under $\mathbb{H}_0$, resulting in downward bias in size. ■

Remark: Since, at each $t = R, R+1 \ldots, T$, $y_{t+1} = e_{t+1}^{(1)} = \hat{c}_t + \hat{b}_t x_t + \hat{e}_{t+1}^{(2)}$, we have $e_{t+1}^{(1)} \left(\hat{c}_t + \hat{b}_t x_t\right) = e_{t+1}^{(1)} \left(e_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)$. Therefore

$$P^{-1} \sum_{t=R}^{T} e_{t+1}^{(1)} \left(\hat{c}_t + \hat{b}_t x_t \right) = P^{-1} \sum_{t=R}^{T} e_{t+1}^{(1)} \left(e_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right),$$

which gives the encompassing test statistic as we see below.

However, the first model is restrictive since the martingale difference (and thus zero-mean) property of y must be known. CW (2007) relax this restriction by adding a constant term to Model 1:

$$\text{Model 1} \quad : \quad y_{t+1} = c_1 + e_{t+1}^{(1)} \tag{4}$$

$$\text{Model 2} \quad : \quad y_{t+1} = c_2 + bx_t + e_{t+1}^{(2)}, \tag{5}$$

where c_i are constant terms for Model i. Let $\hat{e}_{t+1}^{(1)} = y_{t+1} - \hat{c}_{1,t}$ and $\hat{e}_{t+1}^{(2)} = y_{t+1} - \hat{c}_{2,t} - \hat{b}_t x_t$, and the squared error loss $L\left(\hat{e}_{t+1}^{(i)}\right) \equiv \left(\hat{e}_{t+1}^{(i)}\right)^2$, $i = 1, 2$. To test for the equal predictive accuracy of the two conditional mean models, the null hypothesis is $\mathbb{H}_0$ as in (3). The test statistic of DM (1995) is based on the mean squared forecast error (MSFE) differential

$$\hat{D}_P = P^{-1} \sum_{t=R}^{T} \left[\left(\hat{e}_{t+1}^{(1)}\right)^2 - \left(\hat{e}_{t+1}^{(2)}\right)^2 \right]. \tag{6}$$

CW (2007) show that even when x has no predictive power, the MSFE loss-differential $\hat{D}_P$ is expected to be negative, i.e.: $\mathbb{E}\hat{D}_P < 0$ under $\mathbb{H}_0$ as shown below in Proposition 2. Therefore they introduce a non-negative adjustment term

$$\hat{A}_P = P^{-1} \sum_{t=R}^{T} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)^2 \tag{7}$$

and propose the adjusted MSFE loss-differential

$$\begin{aligned} \hat{B}_P &\equiv \hat{D}_P + \hat{A}_P \\ &= \frac{1}{P} \sum_{t=R}^{T} \left[\left(\hat{e}_{t+1}^{(1)}\right)^2 - \left(\hat{e}_{t+1}^{(2)}\right)^2 + \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)^2 \right] \\ &= \frac{1}{P} \sum_{t=R}^{T} \left[2\hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) \right], \end{aligned} \tag{8}$$

which has zero mean under $\mathbb{H}_0$ as shown in Proposition 2. CW (2007) use Monte Carlo simulation to demonstrate that $\mathbb{E}\hat{D}_P < 0$ under $\mathbb{H}_0$ and thus the size of the DM statistic based on $\hat{D}_P$ is biased downwards, while the adjusted DM based on $\hat{B}_P$ has zero mean and thus the correct size.

The DM statistic is $DM_P = \hat{S}_P^{-1/2} \sqrt{P} \hat{D}_P$, where $\hat{S}_P$ is a consistent estimator of $var\left(\sqrt{P}\hat{D}_P\right)$. The CW statistic is $ENC_P = \hat{Q}_P^{-1/2} \sqrt{P} \hat{B}_P$, where $\hat{Q}_P$ is a consistent estimator of $var\left(\sqrt{P}\hat{B}_P\right)$. CW (2006, 2007) show that DM_P is under-sized while ENC_P has proper size (and better power).

Proposition 2: *(a) Under* $\mathbb{H}_0$ *the limiting distribution of* $ENC_P = \hat{Q}_P^{-1/2}\sqrt{P}\hat{B}_P$ *is standard normal distribution when* $\lim_{P,R\to\infty} P/R$ *is large. Hence,* $\mathbb{E}\hat{B}_P = 0$*. (b) Under* $\mathbb{H}_0$, $\mathbb{E}\hat{D}_P < 0$.

Proof: As $\hat{B}_P \equiv \hat{D}_P + \hat{A}_P$ and $\hat{A}_P \geq 0$, we have $\mathbb{E}\hat{B}_P \geq \mathbb{E}\hat{D}_P$ and hence (a) implies (b) and it suffices to prove (a). Let $\lim_{P,R\to\infty} P/R \equiv \pi$. For $0 < \pi < \infty$, we refer to the Not-for-publication-Appendix of CM (2001), which gives the asymptotic distribution ENC_P as shown in Equation (9) below. CM (2001) also point out that for the rolling window scheme with $\pi = 0$, ENC_P is asymptotically standard normal under $\mathbb{H}_0$. If $\pi = \infty$, the $o\left(P/R\right)$ terms may not be $o\left(1\right)$ and hence may not be eliminated in the process of deriving the asymptotic distribution of ENC_P. However when π is large, we can show that the distribution of ENC_P approaches to standard normal. To see this when P/R is large, Let $\xi = R/(R+P)$. For simplicity, we consider the 1-dimensional case, i.e., we have only one additional variable x in Model 2. CM (2001) give the following equation:

$$ENC_P \Rightarrow \frac{\int_\xi^1 \xi^{-1}\left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_\xi^1 \xi^{-2}\left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}}, \tag{9}$$

where $W(s)$ is a Wiener process and $s \in [0,1]$. For large P/R, the rolling window scheme is the limiting case of CM (2001) with $\xi \longrightarrow 0$. We can show that this case will restore the asymptotic standard normality of the ENC_P statistic under the null hypothesis, i.e.:

$$\lim_{\xi\to 0} \frac{\int_\xi^1 \xi^{-1}\left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\xi^{-2}\int_\xi^1 \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \sim^d N(0,1). \tag{10}$$

We firstly consider the numerator of equation (10) by dividing $[0,1]$ to n equal segments and let $t = [sn]$, where $[sn]$ is the integer part of sn and $s \in [0,1]$. Since ξ is sufficiently small, we can write $\xi \equiv 1/n$. We discretise both the numerator and the denominator. Let $\{u_i\}_{i=1}^n$ is mixing sequence with $E\left(u\right) = 0$ and var$(u) = 1$. Let $V_t = \sum_{i=1}^t u_i$ be the partial sum. Then we have $V_t = \sum_{i=1}^t u_i \sim N(0,t)$ and therefore

$$\frac{V_t}{\sqrt{n}} = \frac{\sum_{i=1}^t u_i}{\sqrt{n}} \equiv V_n\left(s\right) \Rightarrow W(s),$$

where $V_n(s)$ is a CADLAG function and $W(s)$ is a Wiener process. Note that

$$\begin{aligned} n^{-1}\sum_{t=1}^{n} u_{t-1}u_t &= n^{-1}\sum_{t=1}^{n} V_{t-1}u_t - n^{-1}\sum_{t=1}^{n} V_{t-2}u_t \\ &\Rightarrow \int_{\xi}^{1} W(s)\mathrm{d}W(s) - \int_{\xi}^{1} W(s-\xi)\mathrm{d}W(s) \\ &= \int_{\xi}^{1} \left[W(s) - W(s-\xi)\right]\mathrm{d}W(s). \end{aligned}$$

Considering the term $\int_{\xi}^{1}\left[W(s) - W(s-\xi)\right]^2\mathrm{d}s$ of the denominator, we have

$$n^{-2}\sum_{t=1}^{n} u_{t-1}^2 = n^{-2}\sum_{t=1}^{n}\left(V_{t-1} - V_{t-2}\right)^2 \Rightarrow \int_{\xi}^{1}\left[W(s) - W(s-\xi)\right]^2\mathrm{d}s.$$

We construct the an AR(1) regression model, regressing $\{u_{t+1}\}$ on $\{u_t\}$:

$$u_{t+1} = \delta u_t + e_t$$

The estimator $\hat{\delta}$ equals $\left(\sum_{t=1}^{n} u_{t-1}u_t\right) / \left(\sum_{t=1}^{n} u_{t-1}^2\right)$ and the variance $\hat{\delta}$ equals

$$\left(\sum_{t=1}^{n} u_{t-1}^2\right)^{-1} \text{var}(u) = \left(\sum_{t=1}^{n} u_{t-1}^2\right)^{-1}.$$

Therefore,by using CLT, Equation (10) can be approximated by

$$\begin{aligned} & \frac{\int_{\xi}^{1}\left[W(s) - W(s-\xi)\right]\mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1}\left[W(s) - W(s-\xi)\right]^2\mathrm{d}s}} \\ \Rightarrow\ & \frac{\sum_{t=1}^{n} u_{t-1}u_t}{\sqrt{\sum_{t=1}^{n} u_{t-1}^2}} \\ =\ & \frac{\sum_{t=1}^{n} u_{t-1}u_t / \sum_{t=1}^{n} u_{t-1}^2}{\sqrt{\sum_{t=1}^{n} u_{t-1}^2}} \sim^d N(0,1). \end{aligned}$$

■

The DM and ENC test are compared with the CCS test, which is constructed as follows. Under $\mathbb{H}_0$, note that we have $b = 0$. If the model $y_{t+1} = e_{t+1}^{(1)}$ (CW 2006) or $y_{t+1} = c_1 + e_{t+1}^{(1)}$ (CW 2007) is correctly specified, then it should be the case that $\mathbb{E}\left(e_{t+1}^{(1)}|x_t\right) = 0$, which implies $\mathbb{E}\left(e_{t+1}^{(1)}\ x_t\right) = 0$. Therefore we construct the consistent out-of-sample conditional moment test statistic

$$\hat{M}_P = P^{-1}\sum_{t=R}^{T} \hat{e}_{t+1}^{(1)} x_t. \tag{11}$$

Under $\mathbb{H}_0$, $\hat{M}_P$ has zero mean and CCS statistic is $CCS_P = \hat{W}_P^{-1/2}\sqrt{P}\hat{M}_P$, where $\hat{W}_P$ is a consistent estimator of $var\left(\sqrt{P}\hat{M}_P\right)$. Chao, Corradi and Swanson (2001) showed that under the null hypothesis, the asymptotic distribution of CCS_P is standard normal and

$$\hat{W}_P \to^d S_{11} + \left(\pi - \pi^2/3\right) F'MS_{22}MF - \pi/2\left(F'MS_{12} + S_{12}'MF\right),$$

where

$$\begin{aligned}
S_{11} &= \sum_{j=-\infty}^{\infty} \mathbb{E}\left(\left((1,x_t)' e_{t+1} - \mu\right)\left((1,x_{t-j})' e_{t+1-j} - \mu\right)'\right) \\
\mu &= \mathbb{E}\left((1,x_t)' e_{t+1}\right) \\
S_{22} &= \sum_{j=-\infty}^{\infty} \mathbb{E}\left((Y_{t-1}e_t)\left(Y_{t-1-j}e_{t-j}\right)'\right) \\
S_{12} &= \sum_{j=-\infty}^{\infty} \mathbb{E}\left(\left((1,x_t)' e_{t+1} - \mu\right)\left(Y_{t-1-j}e_{t-j}\right)'\right) \\
F &= \mathbb{E}\left(Y_{t+1}\left(1,x_t\right)\right) \\
M &= \lim\left(Y_tY_t'\right).
\end{aligned}$$

3 Comparing Nested Conditional Quantile Models

3.1 Encompassing Test for Equal Predictive Accuracy

We extend the out-of-sample predictive ability of x_t for y_{t+1} and use the similar notations as the paper of CW (2006, 2007), and Elliot, Komunjer and Timmermann (2005). One of the main contribution of this paper is that we find the bias term as shown in Equation (18). Given $\alpha \in (0,1)$, we compare the two nested quantile models:

$$\text{Model 1} \quad : \quad y_{t+1} = c_{1,\alpha} + e_{t+1,\alpha}^{(1)} \qquad (12)$$

$$\text{Model 2} \quad : \quad y_{t+1} = c_{2,\alpha} + b_\alpha x_t + e_{t+1,\alpha}^{(2)}, \qquad (13)$$

where $c_{i,\alpha}$ is the constant for model i, x_t is the additional covariate in the second model. Below the subscript α is omitted for simplicity. At time t, both c_i and b need to be estimated. Let $f_{t+1}^{(1)} = \hat{c}_1$ the forecasts for Model 1 and $f_{t+1}^{(2)} = \hat{c}_2 + \hat{b}x_t$ the forecast for Model 2 at time t.

Also let $\hat{e}^{(1)}_{t+1} = y_{t+1} - f^{(1)}_{t+1}$ and $\hat{e}^{(2)}_{t+1} = y_{t+1} - f^{(2)}_{t+1}$ be the forecast errors of the two models, the check loss function is

$$\rho_\alpha\left(\hat{e}^{(i)}_{t+1}\right) \equiv g\left(\hat{e}^{(i)}_{t+1}\right)\hat{e}^{(i)}_{t+1}, \qquad i = 1, 2, \tag{14}$$

where $g(z) = [\alpha - 1(z < 0)]$. If x does not Granger-cause y, then $b = 0$. To test for equal predictive accuracy of the two quantile models, the null hypothesis is

$$\mathbb{H}_0 : \mathbb{E}\left[\rho_\alpha\left(\hat{e}^{(1)}_{t+1}\right) - \rho_\alpha\left(\hat{e}^{(2)}_{t+1}\right)\right] = 0. \tag{15}$$

If x_t Granger-causes y_{t+1}, i.e., $b \neq 0$, we have the alternative hypothesis

$$\mathbb{H}_1 : \mathbb{E}\left[\rho_\alpha\left(\hat{e}^{(1)}_{t+1}\right) - \rho_\alpha\left(\hat{e}^{(2)}_{t+1}\right)\right] > 0. \tag{16}$$

The DM statistic based on the check loss-differential is

$$\hat{D}_P = P^{-1}\sum_{t=R}^{T}\left[g\left(\hat{e}^{(1)}_{t+1}\right)\hat{e}^{(1)}_{t+1} - g\left(\hat{e}^{(2)}_{t+1}\right)\hat{e}^{(2)}_{t+1}\right], \tag{17}$$

where R is the number of observations for the in-sample estimation, P is the number of out-of-sample forecasts, also $R + P = T + 1$.

Below we will show that when comparing the first model that does not contain x with the second model that contains the additional covariate x, under the null hypothesis that x has no predictive ability, the check loss-differential in (17) is expected to be negative. In other words, we will show that $\hat{D}_P \xrightarrow{p} \mathbb{E}\hat{D}_P$ where $\mathbb{E}\hat{D}_P < 0$ under $\mathbb{H}_0$, which will distort the size of the DM statistic downwards. The following propositions will imply that $\mathbb{E}\hat{D}_P < 0$ under $\mathbb{H}_0$.

Proposition 3: *Let*

$$\hat{A}_P \equiv P^{-1}\sum_{t=R}^{T}\left[g\left(\hat{e}^{(2)}_{t+1}\right) - g\left(\hat{e}^{(1)}_{t+1}\right)\right]\hat{e}^{(2)}_{t+1}. \tag{18}$$

Then, $\hat{A}_P \geq 0$.

Proof: To see that $\hat{A}_P$ is non-negative we show that $\left[g\left(\hat{e}^{(2)}_{t+1}\right) - g\left(\hat{e}^{(1)}_{t+1}\right)\right]\hat{e}^{(2)}_{t+1}$ is non-negative at each t. We consider four cases: (i) When both $\hat{e}^{(1)}_{t+1}$ and $\hat{e}^{(2)}_{t+1}$ are positive, $g\left(\hat{e}^{(2)}_{t+1}\right) = g\left(\hat{e}^{(1)}_{t+1}\right) = \alpha$ and thus $\left[g\left(\hat{e}^{(2)}_{j+1}\right) - g\left(\hat{e}^{(1)}_{j+1}\right)\right]\hat{e}^{(2)}_{t+1} = 0$. (ii) When both $\hat{e}^{(1)}_{t+1}$ and $\hat{e}^{(2)}_{t+1}$ are negative, $g\left(\hat{e}^{(2)}_{t+1}\right) = g\left(\hat{e}^{(1)}_{t+1}\right) = \alpha - 1$ and thus $\left[g\left(\hat{e}^{(2)}_{j+1}\right) - g\left(\hat{e}^{(1)}_{j+1}\right)\right]\hat{e}^{(2)}_{t+1} = 0$. (iii) When $\hat{e}^{(1)}_{j+1} < 0$ and $\hat{e}^{(2)}_{j+1} > 0$, $\left[g\left(\hat{e}^{(2)}_{t+1}\right) - g\left(\hat{e}^{(1)}_{t+1}\right)\right]\hat{e}^{(2)}_{t+1} = [\alpha - (\alpha - 1)]\,\hat{e}^{(2)}_{j+1} = \hat{e}^{(2)}_{j+1} > 0$. (iv)

When $\hat{e}^{(1)}_{t+1} > 0$ and $\hat{e}^{(2)}_{j+1} < 0$, $\left[g\left(\hat{e}^{(2)}_{t+1}\right) - g\left(\hat{e}^{(1)}_{t+1}\right)\right]\hat{e}^{(2)}_{t+1} = [(\alpha - 1) - \alpha]\,\hat{e}^{(2)}_{j+1} = -\hat{e}^{(2)}_{j+1} > 0$. Therefore $\hat{A}_P \geq 0$. ■

With the adjusted check loss-differential denoted as

$$\begin{aligned}
\hat{B}_P &\equiv \hat{D}_P + \hat{A}_P \\
&= P^{-1}\sum_{t=R}^{T}\left[g\left(\hat{e}^{(1)}_{t+1}\right)\hat{e}^{(1)}_{t+1} - g\left(\hat{e}^{(2)}_{t+1}\right)\hat{e}^{(2)}_{t+1}\right] + P^{-1}\sum_{t=R}^{T}\left[g\left(\hat{e}^{(2)}_{t+1}\right) - g\left(\hat{e}^{(1)}_{t+1}\right)\right]\hat{e}^{(2)}_{t+1} \\
&= P^{-1}\sum_{t=R}^{T}\left[g\left(\hat{e}^{(1)}_{t+1}\right)\left(\hat{e}^{(1)}_{t+1} - \hat{e}^{(2)}_{t+1}\right)\right],
\end{aligned} \tag{19}$$

we consider three standardized statistics to compare the predictive ability of the two nested quantile models: First, the DM statistic is $DM_P \equiv \hat{S}_P^{-1/2}\sqrt{P}\hat{D}_P$ and the encompassing statistic $ENC_P \equiv \hat{Q}_P^{-1/2}\sqrt{P}\hat{B}_P$ is based on P, where $\hat{S}_P, \hat{Q}_P$ are the consistent estimators of $S_P = var\left(\sqrt{P}\hat{D}_P\right)$ and $Q_P = var\left(\sqrt{P}\hat{B}_P\right)$ respectively. We have the following proposition.

Proposition 4: *(a)* Under the null hypothesis, the asymptotic distribution of $ENC_P \equiv \hat{Q}_P^{-1/2}\sqrt{P}\hat{B}_P$ is

$$\frac{\int_\xi^1 \xi^{-1}\left[W(s) - W(s-\xi)\right]\mathrm{d}W(s)}{\sqrt{\int_\xi^1 \xi^{-2}\left[W(s) - W(s-\xi)\right]^2\mathrm{d}s}}.$$

For large P/R, the limiting asymptotic distribution of ENC_P is standard normal. *(b)* Under the null hypothesis, $\mathbb{E}\hat{D}_P < 0$.

Proof: As $\hat{A}_P \geq 0$ from Proposition 3, *(a)* implies *(b)*. The proof of *(a)* can be found in the first part of Section 7. ■

We will compare the DM and ENC test with the CCS test for conditional quantile and the latter one is constructed as follows. Under $\mathbb{H}_0$, we have $b = 0$. If the quantile model (12) is correctly specified, then $\mathbb{E}\left[g\left(e^{(1)}_{t+1}\right)|x_t\right] = 0$, which implies $\mathbb{E}\left[g\left(e^{(1)}_{t+1}\right)x_t\right] = 0$. Therefore we construct the out-of-sample consistent conditional *quantile* test statistic that is analogue to mean model case:

$$\hat{M}_P = \frac{1}{P}\sum_{t=R}^{T} g\left(\hat{e}^{(1)}_{t+1}\right)x_t. \tag{20}$$

The CCS statistic $CCS_P \equiv \hat{W}_P^{-1/2}\sqrt{P}\hat{M}_P$, where $\hat{W}_P$ is the consistent estimator of $W_P = var\left(\sqrt{P}\hat{M}_P\right)$.

3.2 Encompassing and Forecast Combination

The purpose of combining forecast is to find the optimal combining weight of the two models such that the combining forecast error has minimum loss. As more information is given in the second model, the weight of Model 1 will shrink and we are in favor of the second model. We consider the following combining forecast:

$$f_{t+1}^{(c)} = (1-\lambda) f_{t+1}^{(1)} + \lambda f_{t+1}^{(2)} \tag{21}$$

where the combining forecast error is

$$\varepsilon_{t+1} \equiv y_{t+1} - f_{t+1}^{(c)} = (1-\lambda) \hat{e}_{t+1}^{(1)} + \lambda \hat{e}_{t+1}^{(2)}. \tag{22}$$

We rewrite the equation above as

$$\hat{e}_{t+1}^{(1)} = \lambda \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) + \varepsilon_{t+1}, \tag{23}$$

and treat $\hat{e}_{t+1}^{(1)}$ as the dependent variable and $\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right)$ as the regressor, thus ε_{t+1} is the error term and $\hat{\lambda}$ is estimated by regressing $\left\{ \hat{e}_{t+1}^{(1)} \right\}_{t=R}^{T}$ on $\left\{ \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) \right\}_{t=R}^{T}$ using quantile regression, i.e., to minimize the expectation of the check loss of ε_{t+1}:

$$\hat{\lambda} = \arg\min_{\lambda} \mathbb{E} \rho_\alpha (\varepsilon_{t+1}). \tag{24}$$

In the second part of Section 7, we show the first order condition (FOC) of equation (24) gives the following conditional moment

$$\mathbb{E} \left[g(\varepsilon_{t+1}) \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) \right] = 0. \tag{25}$$

Under the null hypothesis where the two models have equal predictive accuracy, λ can be arbitrary number, however for out-of-sample forecast, "due to the noise (the variance) of the additional variable in the second model that pushes up the noise" (CW 2006), the out of sample check loss from the second model is higher than that of the first model. Hence when combining two models, we are in favor of the first model and endow the second model with zero or even negative weight. Therefore, $\lambda = 0$ is a critical value of checking whether two models are equal, in this case, we have $\hat{e}_{t+1}^{(1)} = \varepsilon_{t+1}$ and $\mathbb{E} \left[g\left(\hat{e}_{t+1}^{(1)} \right) \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) \right] = 0$. Hence $\hat{B}_P = P^{-1} \sum_{t=R}^{T} \left[g\left(\hat{e}_{t+1}^{(1)} \right) \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) \right]$ will converge to zero as P/R is large under $\mathbb{H}_0$, which is in accordance with Proposition 4. This is an extension of the encompassing test of Nelson (1972), Mizon and Richard (1986), CM (2001), and CW (2006, 2007), from conditional mean to conditional quantile.

4 Simulation Results on Encompassing and Forecast Combination

4.1 Simulation Design

We consider the follow data generating process in quantile Model 1 and Model 2: the additional variable x_t in the second model is an AR(1) stationary process $x_t = \phi x_{t-1} + v_t$, where $|\phi| < 1$ and $\mathbb{E}(v_t|x_{t-1}) = 0$, v_t is i.i.d, normally distributed and has zero mean and variance σ_v^2. Hence x has zero mean also and variance is $\sigma_x^2 = \sigma_v^2 / (1-\phi^2)$. The error term $e_{t+1}^{(2)}$ of the second model satisfies

$$\mathbb{E}\left(\alpha - \mathbf{1}\left(e_{t+1}^{(2)} < 0\right) \mid x_t\right) = 0, \tag{26}$$

which implies that the g function of $e_{t+1}^{(2)}$ is a martingale difference series and the conditional α-quantile of $e_{t+1}^{(2)}$ given x_t is zero.

In our DGP, we generate $e_{t+1}^{(2)}$ following normal distributions. To ensure that the α-quantile of $e_{t+1}^{(2)}$ is zero as required in (26), the error term $e_{t+1}^{(2)}$ should satisfy

$$\frac{\mathbb{E}\left(e_{t+1}^{(2)}\right)}{\sqrt{Var\left(e_{t+1}^{(2)}\right)}} = \frac{-\Phi^{-1}(\alpha)\,\sigma_e}{\sigma_e} = -\Phi^{-1}(\alpha).^3 \tag{27}$$

Therefore, if we set $Var\left(e_{t+1}^{(2)}\right) = \sigma_e^2$, then $\mathbb{E}\left(e_{t+1}^{(2)}\right) = -\Phi^{-1}(\alpha)\,\sigma_e$. Hence, $e_{t+1}^{(2)}$ will be simulated from $N\left(-\Phi^{-1}(\alpha)\,\sigma_e, \sigma_e^2\right)$. Then $\{y_{t+1}\}$ is generated from $y_{t+1} = c_2 + bx_t + e_{t+1}^{(2)}$ and has the mean and variance as follows:

$$\begin{aligned}
\mathbb{E}\left(e_{t+1}^{(1)}\right) &= \mathbb{E}(y_{t+1} - c_1) = \mathbb{E}\left(c_2 + bx_t + e_{t+1}^{(2)} - c_1\right) = c_2 + 0 - \Phi^{-1}(\alpha)\,\sigma_e - c_1 \\
Var\left(e_{t+1}^{(1)}\right) &= Var(y_{t+1} - c_1) = Var\left(c_2 + bx_t + e_{t+1}^{(2)} - c_1\right) = b^2\sigma_x^2 + \sigma_e^2.
\end{aligned}$$

Due to the same reason as in (27), under normality for the simulation design, $e_{t+1}^{(1)}$ should satisfy

$$\frac{\mathbb{E}\left(e_{t+1}^{(1)}\right)}{\sqrt{Var\left(e_{t+1}^{(1)}\right)}} = \frac{c_2 - c_1 - \Phi^{-1}(\alpha)\,\sigma_e}{\sqrt{b^2\sigma_x^2 + \sigma_e^2}} = -\Phi^{-1}(\alpha), \tag{28}$$

[3] If the error term e is normally distributed with distribution function $F(\cdot)$ and the coefficient of variation should equal $-\Phi^{-1}(\alpha)$ such that the α-quantile of e is zero. Let σ be its standard deviation, thus the mean equals $-\Phi^{-1}(\alpha)\,\sigma$. We have $F(0) = \int_{-\infty}^{0} f(e)\mathrm{d}e = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left\{\frac{e}{\sigma} - \left(-\Phi^{-1}(\alpha)\right)\right\}^2\right]\mathrm{d}e$ $= \int_{-\infty}^{\Phi^{-1}(\alpha)\sigma} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-z^2/2\sigma^2}\mathrm{d}z = \int_{-\infty}^{\Phi^{-1}(\alpha)} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\mathrm{d}w = \Phi\left(\Phi^{-1}(\alpha)\right) = \alpha.$

which implies that $c_2 - c_1 = \Phi^{-1}(\alpha)\,\sigma_e - \Phi^{-1}(\alpha)\sqrt{b^2\sigma_x^2 + \sigma_e^2}$. Note that when $b = 0$ or when $\alpha = 0.5$, we have $c_1 = c_2$. Therefore, when $\alpha \neq 0.5$ and $b \neq 0$, we have $c_1 \neq c_2$. Without loss of generality, we set $c_2 = 1$, and therefore $c_1 = c_2 - \left[\Phi^{-1}(\alpha)\,\sigma_e - \Phi^{-1}(\alpha)\sqrt{b^2\sigma_x^2 + \sigma_e^2}\right]$. We consider $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $\phi \in \{0, 0.95\}$, $\sigma_e \in \{0.1, 1.0\}$, $\sigma_v \in \{1\}$, and $b \in \{0, 0.1\}$.

We use the MATLAB package by Roger Koenker to estimate the models. The first model is estimated by regressing $\{y_s\}_{s=t-R+1}^{t}$ on constant term to obtain $\hat{c}_{1,t}$, where $t = R, \ldots, T$. The second model is estimated by regressing $\{y_s\}_{s=t-R+1}^{t}$ on $\{1, x_{s-1}\}_{s=t-R+1}^{t}$ to obtain $\left(\hat{c}_{2,t}\ \hat{b}_t\right)$. The forecast errors from the two models are $\hat{e}_{t+1}^{(1)} = y_{t+1} - \hat{c}_{1,t}$ and $\hat{e}_{t+1}^{(2)} = y_{t+1} - \hat{c}_{2,t} - \hat{b}_t x_t$ over the forecast evaluation period at $t = R, \ldots, T$. The number of observations for the rolling windows for estimation are chosen from $R \in \{60, 120, 240\}$. Let $P = T - R + 1 \in \{48, 240, 1200\}$. From these, we compute the three statistics DM_P, ENC_P, and CCS_P. The above procedure is repeated 2000 times to find out the Monte Carlo distributions of DM_P, ENC_P, and CCS_P, and to compute their size and power.

4.2 Simulation Result on Size and Power

The tables and figures below show the Monte Carlo distribution and size and powers of the ENC_P, DM_P and CCS_P statistics under different settings. We can see that for size and power of test, DM_P has a downward size that is much less than 5% under 5% nominal size, since DM_P has a negative mean. Also both CCS_P and ENC_P statistic have good size under 5% nominal size, and the distribution are both asymptotically normal especially when P/R is large. However for smaller P/R ratio, CCS_P and ENC_P may suffer a extremely high kurtosis that arise from degeneracy problem. Higher signal-to-noise ratio of the second model lead to higher ENC_P.[4]

Tables 1.1-1.8 About Here

Figures 1.1-1.3 About Here

The main findings of these tables and figures are summarized as follows.

[4]Signal-to-noise ratio is defined as $b^2\sigma_x^2/\sigma_e^2$ where b is the coefficient of the additional covariate, σ_x^2 is the variance of additional covariate and σ_e^2 is the variance of the error term of the second model.

1. Tables 1.1-1.4: DM_P tests have downward size, leading to an undersized problem under $\mathbb{H}_0$, whereas both ENC_P and CCS_P test have good size. The size of test of ENC_P and CCS_P will approach to the nominal size (10%) for large P/R.

2. Figure 1.1 represents the sampling distribution of DM_P has the negative mean under $\mathbb{H}_0$, while the sampling distributions of ENC_P and CCS_P seem to be symmetric centered at zero under $\mathbb{H}_0$, also they approach to standard normal. Note that the pick of the density is 0.4 $\left(\approx 1/\sqrt{2\pi}\right)$ at zero.

3. Tables 1.5-1.8 and Figures 1.2-1.3 present the power of test for different b, ϕ and σ_e. Figure 1.2 shows the distribution of the three statistics for low b, ϕ and high σ_e. Figure 1.3 shows the distribution of the three statistics for high b, ϕ and low σ_e We see that increasing the signal-to-noise ratio by increasing b, ϕ or lowering σ_e will lead to higher ENC_P and CCS_P, also DM_P test will have a higher power.

4.3 Simulation Result on Combining Forecasts

To see how combining forecast beats any of the models, we consider the out-of-sample *average* of the check loss for Model 1 and Model 2:

$$\begin{aligned}
\hat{\rho}_a^{(1)} &\equiv \frac{1}{P}\sum_{t=R}^{T}\left[\alpha - 1\left(\hat{e}_{t+1}^{(1)} < 0\right)\right]\hat{e}_{t+1}^{(1)}, \\
\hat{\rho}_a^{(2)} &\equiv \frac{1}{P}\sum_{t=R}^{T}\left[\alpha - 1\left(\hat{e}_{t+1}^{(2)} < 0\right)\right]\hat{e}_{t+1}^{(2)}.
\end{aligned}$$

For combining forecast, $\hat{\lambda}$ is estimated from (23) by running a quantile regression of $\left\{\hat{e}_{t+1}^{(1)}\right\}_{t=R}^{T}$ on $\left\{\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)\right\}_{t=R}^{T}$ and $\varepsilon_{t+1} = \left(1-\hat{\lambda}\right)\hat{e}_{t+1}^{(1)} + \hat{\lambda}\hat{e}_{t+1}^{(2)}$. The out-of-sample *average* of the check loss for the combined model is

$$\hat{\rho}_a^{(c)} \equiv \frac{1}{P}\sum_{t=R}^{T}\left\{\left[\alpha - 1\left(\varepsilon_{t+1} < 0\right)\right]\varepsilon_{t+1}\right\}.$$

Reported in Tables 1.9-1.16 are the out-of-sample *average* of $\hat{\rho}_a^{(1)}$, $\hat{\rho}_a^{(2)}$ and $\hat{\rho}_a^{(c)}$ as well as the asymptotic distribution of $\hat{\lambda}_P$ from the 2000 Monte Carlo simulations. We will use Tables 1.7-1.11 to show the estimator $\hat{\lambda}$ under different configurations and also to show that how combined forecast beats the two nested models when $b \neq 0$. For each R and P we have a block containing

6 numbers in 2 columns, the first column of the block represent the mean, skewness and kurtosis of $\hat{\lambda}_P$ from Monte Carlo simulation, the second column of the block represents the average check loss $\hat{\rho}_a^{(1)}$ of the first model, $\hat{\rho}_a^{(2)}$ of the second model, and $\hat{\rho}_a^{(c)}$ of the combined model.

Tables 1.9-1.16 About Here

Figures 1.4-1.6 About Here

The main results from Tables 1.9-1.16 and Figures 1.4-1.6 are summarized as follows.

1. Tables 1.9-1.12 and Figure 1.4: Under $\mathbb{H}_0$, other things equal, the optimal weight of the second model is increasing as P increases. When P is lower (e.g.: 48), the optimal weight on the second model is negative, when P is larger (e.g.: 1200), the optimal weight is approached to 0.

2. Comparing Tables 1.9-1.12 with Tables 1.1-1.4 we see that under the null hypothesis, as P/R is large, ENC_P has the correct size, meanwhile, the average of $\hat{\lambda}$ is approached to 0, Both of which imply that we are in favor of the first model under the null.

3. Tables 1.13-1.16 and Figures 1.5-1.6: Under $\mathbb{H}_1$, increasing b, ϕ or lowering σ_e leads to higher optimal weight on the second model since the additional variable x gives more information or "signal". Combined with the analysis of Tables 1.5-1.8, other things equal, we see that as b increases, the center of the distribution of $\hat{\lambda}$ shifts from 0 to 1. Also similar to (1), other things equal, the optimal weight of the second model is increasing as P increases. This is because when R is large or P is small, we can predict y_{t+1} using the previous information of y up to time t, so we endow lower weight of the second model. While for large P and small R, the previous y up to time t does not contain full information therefore can not completely predict y_{t+1}, hence we need to rely on additional variable x to predict y_{t+1}, thereby increasing the weight of the second model.

4. Tables 1.9-1.16 and Figures 1.4-1.6 conclude how the gradual change of signal-to-noise ratio affects the weight of the second model. We see that when "noise" σ_e of the second model decreases, both the power of ENC_P and the the average of $\hat{\lambda}$ increase to 1, which implies that we are in favor of the second model.

5 Predictive Quantile Regression for the Equity Premium

We apply the three statistics to the Goyal and Welch (2008) study and construct nested quantile models to test if dividend-yield ratio (DY) or dividend-price ratio (DP) Granger-causes the equity premium. The dividend-yield ratio at time t is defined as the most recent dividend at t divided by stock price at time t, the dividend-price ratio at time t is defined as the most recent dividend at $t-1$ divided by stock price at time t. The two nested quantile models are as shown in (12) and (13), where y_{t+1} is the equity premium and x_t is the dividend-yield ratio or the dividend-price ratio. We use monthly data ranging from 1926 to 2011, containing 1032 observation for both DP and DY models. In the first model, we only have a constant term, therefore at time t, we predict the future equity premium by solely using the α-quantile of previous R observations of the equity premium from time $t-R+1$ to t. In Model 2, we use the 1-lag dividend-yield ratio or dividend-price ratio to forecast the equity premium in the next month. See Goyal and Welch (2008) for more on data descriptions. We intend to check if two nested models have the same predictive accuracy under nine quantiles with $\alpha \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. We use rolling windows scheme and the total number of observations is 1032. The in-sample observations ranges from 258 ($R/P = 1/3$) to 774 ($R/P = 3/1$). The red line represents DM_P under different allocation of R and P. The thick blue line and dotted line represent ENC_P statistic and CCS_P statistic respectively.

Figures 1.7-1.8 About Here

We conduct the analysis using the asymptotic normality and also using the bootstrap distribution. The horizontal lines represent 90% and 95% percent confidence interval, corresponding to ± 1.282 and ± 1.96. We find that for all figures. the ENC_P have the highest power. Also by comparing ENC_P with DM_P, we find that (i). ENC_P lies above DM_P almost everywhere. When ENC_P falls into 90% or 95% confidence interval, DM_P is negative. By comparing ENC_P with CCS_P, ENC_P has better power. (ii). The three statistics vary depending on the in-sample to out-of-sample ratio. When R is small, for example. $R = 258$ or 516, both DY and DP Granger Causes the equity premium under 5% or 10% significant level. When R is large, for example, $R = 774$, we find that both ENC_P and CCS_P are in the 90% and 95% confidence interval for $\alpha = 0.3$, 0.5 and 0.7. This is because when R is small, we are unable to account for the y_{t+1} by solely using the previous y up to time t since the information available is scarce, therefore we need to exploit the property of additional variable x. In this way, x has predictive power and both ENC_P and DM_P lie above the upper bound of the confidence interval. As the number of in-sample observations increases, the previous y has contained rich information and can be used for forecasting and weakens the predictive power of x. Therefore large R leads the ENC_P curve fall into the confidence interval for $\alpha = 0.3, 0.5, 0.7$. (iii). For tail cases such as $\alpha = 0.1$ and $\alpha = 0.9$, we see that the ENC_P curve lies above the upper bound of 90% or 95% confidence interval. the reason is that for tail cases, there are scarce y lying above or below the historical quantile up to time t, hence we still need to rely on additional variable when making the forecast, i.e.: the additional variable x has power even for large P. This can not be observed using mean regression.

To see how the combining forecast beats any of the models and the relationship between the combination factor and ENC_P for different P and R, we do the following procedures:

1. Obtaining the empirical data that contains $T = R + P$ observations.

2. Doing the quantile regression of the two models and obtaining the forecast errors $\hat{e}_{t+1}^{(1)} = y_{t+1} - f_{t+1}^{(1)}$ and $\hat{e}_{t+1}^{(2)} = y_{t+1} - f_{t+1}^{(2)}$ from $t = R$ to T.

3. The optimal combination weight $\hat{\lambda}$ is solved using the following quantile regression $\hat{e}_{t+1}^{(1)} = \lambda \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) + \varepsilon_{t+1}$, for $t = R, \ldots, T$ and then calculate $\{\varepsilon_{t+1}\}_{t=R}^{T}$.

4. Compute the average of the check loss from forecasting of the first model $\rho_\alpha^{(1)}$, the average of the check loss from forecasting of the second model $\rho_\alpha^{(2)}$ and the average of the check loss from forecasting of the combined model $\rho_\alpha^{(c)}$ defined in the previous section.

Table 1.17 About Here

We have different R and P in Table 1.17, which reports $\hat{\lambda}$, $\rho_\alpha^{(1)}$, $\rho_\alpha^{(2)}$, $\rho_\alpha^{(c)}$ under 5 quantiles: $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, The number of in-sample observation R to the number of out-of-sample observations P are $(R, P) = (258, 774)$, $(516, 516)$, and $(774, 258)$ with $R/P = 1/3, 2/2$, and $3/1$. Table 1.17 shows how combination factor $\hat{\lambda}$ changes with different R and P. We see that for small R and large P (column 1 to column 4), the weight on the second model is greater than 0.5, also the average of the check loss from the second model is lower than that of the first model for both DY and DP. While for large R and small P, column 9 to column 12 show that the weight on the second model is around 0 or even negative when α is 0.3, 0.5 and 0.7, however for tail cases such as $\alpha = 0.1$ and 0.9, we still have higher $\hat{\lambda}$. Taking into account of column 5 to 8, we conclude that when α is not at tail, the optimal weight on the second model will decrease as R increase since more information of previous y is available to use for forecasting. However for tail cases, even for large R, we may still have higher optimal weight on the second model.

Steps 1 to 4 above give the procedure of finding the optimal weight $\hat{\lambda}$ under given R and P. We are also interested in exploiting the confidence interval of λ, testing if $\lambda = 0$. For example, in Table 1.17, we see that under $\alpha = 0.9$, $R/P = 3/1$, $\hat{\lambda} = 0.41$, we are interesting in finding the confidence interval. To do this, we follow the bootstrap method for quantile model by Feng, He and Hu (2001) to find out the distribution and bootstrap confidence interval of $\hat{\lambda}$ as well as the bootstrap t-statistic of $\hat{\lambda}$. We do the following steps:

1. Obtaining the empirical data that contains $T = R + P$ observations.
2. Doing the quantile regression of the two models and obtaining the forecast errors $\hat{e}^{(1)}_{t+1} = y_{t+1} - f^{(1)}_{t+1}$ and $\hat{e}^{(2)}_{t+1} = y_{t+1} - f^{(2)}_{t+1}$ from $t = R$ to T.
3. The optimal combination weight $\hat{\lambda}$ is solved using the following quantile regression $\hat{e}^{(1)}_{t+1} = \lambda \left(\hat{e}^{(1)}_{t+1} - \hat{e}^{(2)}_{t+1} \right) + \varepsilon_{t+1}$, for $t = R, \ldots, T$ and then calculate $\{\varepsilon_{t+1}\}^{T}_{t=R}$.
4. Fixing $\left\{ \hat{e}^{(1)}_{t+1} - \hat{e}^{(2)}_{t+1} \right\}^{T}_{t=R}$, we obtain the $B = 1000$ bootstrap samples. For each bootstrap, we draw randomly from $\{\varepsilon_{t+1}\}^{T}_{t=R}$ with replacement, and denote them by ε^*_{R+1}, $\varepsilon^*_{R+2}, \ldots \varepsilon^*_{T}$ and generate $\hat{e}^{(1)*}_{t+1} = \hat{\lambda} \left(\hat{e}^{(1)}_{t+1} - \hat{e}^{(2)}_{t+1} \right) + \tilde{\varepsilon}^*_{t+1}$ for $t = R, \ldots, T$, following Feng, He and Hu (2011), where $\tilde{\varepsilon}^*_{t+1} = w_{t+1} \left| \varepsilon^*_{t+1} \right|$ and w_{t+1} is the weight mentioned below. By replicating the regression B times we get $\hat{\lambda}^*_b$, where $b = 1, \ldots, B$. Let $\overline{\hat{\lambda}^*} = B^{-1} \sum_{b=1}^{B} \hat{\lambda}^*_b$. Let $\tau = \mathbb{E}\left(\hat{\lambda} - \lambda \right)$ is the bias of $\hat{\lambda}$. The bootstrap estimator of the bias τ is $\left(\overline{\hat{\lambda}^*} - \hat{\lambda} \right)$ and the bias-corrected estimator of λ is given by $\tilde{\lambda}^* = \hat{\lambda} - \left(\overline{\hat{\lambda}^*} - \hat{\lambda} \right) = 2\hat{\lambda} - \overline{\hat{\lambda}^*}$. The bootstrap standard error of $\hat{\lambda}$ is $\tilde{\sigma} = \sqrt{B^{-1} \sum_{b=1}^{B} \left(\hat{\lambda}^*_b - \overline{\hat{\lambda}^*} \right)^2}$.
5. Finding the θ-quantile and $(1 - \theta)$-quantile of the bootstrap value from $\left\{ \hat{\lambda}^*_1, \hat{\lambda}^*_2 \ldots \hat{\lambda}^*_B \right\}$ and denoting them by $\hat{\lambda}^*_\theta$ and $\hat{\lambda}^*_{1-\theta}$, the $(1 - 2\theta)$ of confidence interval of λ would be

$\left(2\hat{\lambda}-\hat{\lambda}^*_{1-\theta},2\hat{\lambda}-\hat{\lambda}^*_{\theta}\right)$. We will report the bias-corrected $\tilde{\lambda}$ and the estimated standard error $\tilde{\sigma}$ from step 4, the 90% and 95% confidence interval from step 5 by setting $\theta=0.025$ and 0.05.

The bootstrap procedure is different from that in GCM, see Inoue and Kilian (2005). We follow Feng, He and Hu (2011) to construct the weight w_t, the first method is the two mass distribution with probability α and $1-\alpha$ at point 2α and $2(1-\alpha)$ respectively. The second method is to follow the section 3 of Feng, He and Hu (2011), called two-part continuous weight distribution and w follows

$$f(w)=\begin{cases} -kw \ \left(-2\alpha-1/(4k)\leq w\leq -2\alpha+1/(4k)\right) \\ kw \ \left(2(1-\alpha)-1/4k\leq w\leq 2(1-\alpha)+1/4k\right) \end{cases}. \quad (29)$$

We need to be cautious for the tail cases of equation (29) since the method is applicable for $1/(8k)\leq\alpha\leq 1-1/(8k)$. In the paper of Feng, He and Hu (2011), they take $k=1$, which will lose some tail case for $\alpha\leq 1/8$ or $\alpha\geq 7/8$. Therefore in our bootstrap, we set up $k=2$ and 4 such that the bootstrap enables us to check the 10% and 90% quantile case. When $k\to+\infty$, the distribution of the weight w approaches to the two point mass weight distribution.

Tables 1.18-1.20 About Here

Comparing confident interval of $\tilde{\lambda}^*$ from the bootstrap result in Tables 1.18-1.20 with Figures 1.7 and 1.8 that depict the how different R and P as well as α impact the three statistics, we see that the bootstrap evidence and empirical results are consistent, which implies that under $\mathbb{H}_0$, by using the bootstrap method, we can not reject that $\tilde{\lambda}^*=0$. Specifically, in Tables 1.18-1.20, when $R:P=1:3$ or $1:1$ we can see that 0 is not contained in the 90% or 95% bootstrap confidence interval of $\tilde{\lambda}^*$ for all the five quantile values, meanwhile Figures 1.7 and 1.8 show that the ENC_P is above 1.645 (or 1.96); When $R:P=3:1$, we see from the Tables 18-20 that 0 lies within the 90% or 95% bootstrap confidence interval. Meanwhile, from the Figures 1.7 and 1.8, ENC_P is in the ±1.645 (or ±1.96) when $\alpha\in\{0.3,0.5,0.7\}$. However for tail cases $\alpha\in\{0.1,0.9\}$ the lower bounds of 90% or 95% bootstrap confidence interval of $\tilde{\lambda}^*$ are above 0 from the tables and correspondingly, ENC_P is above 1.645 (or 1.96).

Using Tables 1.17 and 1.18-1.20, we can calculate the t-statistic using $\hat{\lambda}/\tilde{\sigma}$ and compare it with ENC_P, In Table 1.21, $\tilde{\sigma}$ is chosen from two point mass weight distribution shown in Table 1.20. We report the t-statistic using bootstrap method under five different quantiles in Table 1.21.

Table 1.21 About Here

Comparing Table 1.21 with Figures 1.7 and 1.8, we see that the t-statistic of the optimal weight $\hat{\lambda}$ and ENC_P are highly positively correlated, e.g.: For small R and large P, such as $R/P = 1/3$, ENC_P lie between 5 and 10 for $\alpha \in \{0.1, 0.3, 0.5, 0.7\}$ and between 2 and 3 for $\alpha \in \{0.9\}$. Accordingly, the t-statistic of the optimal weight $\hat{\lambda}$ are greater than 5 for $\alpha \in \{0.1, 0.3, 0.5, 0.7\}$ but only 3.6 for $\alpha = 0.9$. For large R and small P, such as $R/P = 3/1$, we see that both ENC_P and the t-statistic of the optimal weight $\hat{\lambda}$ falls into 90% or 95% nominal confidence interval for $\alpha \in \{0.3, 0.5, 0.7\}$, however for $\alpha \in \{0.1, 0.9\}$, for both DP and DY models, the t-statistic of the optimal weight $\hat{\lambda}$ are around 2.5 and 5 respectively, and ENC_P are above 1.96, both of which imply that the DP or DY ratio has predictive power.

6 Conclusions

This paper is the extension of Clark and West (2006, 2007) from nested mean model comparison to nested quantile model comparison. The contributions contain the following: Firstly we point out that similar to Granger-causality in Mean models, due to the parameter estimation error, the Diebold Mariano test for nested quantile models also has negative mean and distort the size downward. Secondly, enlighten by Clark and McCracken (2001), Clark and West (2006, 2007), we find out a non-negative adjustment term as shown in Equation (18) that pushes up the Diebold Mariano test from negative mean to zero mean, the adjusted Diebold Mariano test is also the encompassing test. What is more, we show that after adding the adjustment term on unadjusted check loss-differential, our test has zero mean and good size. Also we point out that under alternative hypothesis, our test has good power, which increases with higher signal-to-noise ratio. We apply our test to Goyal-Welch Study data to check if the additional variable (dividend-price ratio or dividend-yield ratio) Granger Causes equity premium under different quantile. We find that the our test improves the predictive accuracy compared with

DM test, also the empirical results using our test are consistent with the findings using bootstrap method. We see that when the out of sample observation P is large, ENC_P has high power when detect the predictive ability of the predictor for the quantiles of the equity premium, we have high bootstrap t-statistic of the optimal weight of the second model, which is greater than 2, meanwhile the 90% or 95% confidence interval of the optimal weight of the second model is greater than 0. Note that when R is large, we have relatively smaller $P = T - R + 1$ and thus we have the smaller power of ENC_P as depicted in the figures as well as smaller bootstrap t-statistic. The predictive ability of the dividend-yield is generally stronger in tail quantiles with $\alpha \in \{0.1, 0.9\}$, which can not be detected using mean regression.

7 Appendix

This appendix collects two derivations. Part 1 is the proof of proposition 4(a) in Section 3. Part 2 is the derivation of the first order condition (25) for (24).

7.1 Proof of Proposition 4(a)

Under $\mathbb{H}_0$, $e^{(1)}_{t+1} = e^{(2)}_{t+1} = e_{t+1}$. We have defined $\hat{e}^{(1)}_{t+1} = y_{t+1} - \hat{c}_{1,t} \equiv y_{t+1} - x'_{1,t}\hat{\beta}_{1,t}$ and $\hat{e}^{(2)}_{t+1} = y_{t+1} - \hat{c}_{2,t} - \hat{b}_t x_t \equiv y_{t+1} - x'_{2,t}\hat{\beta}_{2,t}$, where $x'_{1,t} = 1$ and $x'_{2,t} = (1, x_t)$. We also define $q_{i,t} = x_{i,t}x'_{i,t}$ and $B_i = \mathbb{E}\left[q_{i,t}\right]$ where $i = 1, 2$. Therefore $B_1 = 1$ and

$$B_2 = \begin{pmatrix} 1 & 0 \\ 0 & \mathbb{E}\left(x_t^2\right) \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & \sigma_v^2/\left(1-\phi^2\right) \end{pmatrix}^{-1},$$

$$h_{i,t} = \left[f\left(0\right)\right]^{-1}\left[\alpha - 1\left(e^{(i)}_t < 0\right)\right] x'_{1,t-1}.$$

The estimated parameter $\hat{\beta}_{i,t}$ ($t = 1, 2, 3 \ldots R$) following the Bahadur Representation

$$\sqrt{R}\left(\hat{\beta}_{i,t} - \beta_{i,t}\right) = \sqrt{R}\left[B_i\left(t\right)H_i\left(t\right) + Res\left(t\right)\right],$$

where $B_i\left(t\right) = \left(R^{-1}\sum_{j=t-R+1}^{t} q_{i,t}\right)^{-1}$ and

$$H_i\left(t\right) = R^{-1}\sum_{j=t-R+1}^{t} h_{i,t}, Res\left(t\right) = O(R^{-3/4+\varepsilon})$$

where $\varepsilon > 0$. Hansen (Ch13, 2013) shows that

$$h_{2,t} \sim N\left(0, [f(0)]^{-2}\alpha(1-\alpha)B_2\right),$$

and by central limit theorem

$$\sqrt{R}\left[R^{-1}\sum_{j=t-R+1}^{t} h_{2,t}\right] \sim N\left(0, [f(0)]^{-2}\alpha(1-\alpha)B_2\right).$$

Now define $\sigma^2 \equiv [f(0)]^{-2}\alpha(1-\alpha)$. Let $k_1 = 1$ the number of parameters to be estimated in the first model and $k_2 = 1$ the additional number of parameters to be estimated in model 2. Let J be a selection matrix $(1,0)$ such that $Jh_{2,t} = h_{1,t}$. Let $\sup_t$ denote $\sup_{t-R+1\leq\cdot\leq t}$. And matrix A and C will defined later and $\tilde{h}_{2,t} = \sigma^{-1}A'CB_2^{1/2}h_{2,t}$, $\tilde{H}_2(t) = \sigma^{-1}A'CB_2^{1/2}H_2(t)$. For any $m\times n$ matrix A with element $a_{i,j}$ and column vector a_j. Let $vec(A)$ denote $mn\times 1$ vector $\left[a'_{1,}a'_{2\ldots}a'_n\right]'$ and let $|A|$ denote $\max_{i,j}|A_{i,j}|$. Let $W(s,\Omega)$ denote one-dimensional Brownian Motion with covariance kernel Ω and let $W(s)$ denote the same when $\Omega = 1$. The sequence U defined similar to $H_i(t)$. Let

$$U_t = [[f(0)]^{-1}g(e_t), x'_{2,t} - \mathbb{E}\left(x'_{2,t}\right)$$

and

$$vec\left(\tilde{h}_{2,t}\tilde{h}'_{2,t} - \mathbb{E}\left(\tilde{h}_{2,t}\tilde{h}'_{2,t}\right)\right)', vec\left(q_{2,t} - \mathbb{E}(q_{2,t})\right)']'$$

Then we have $\mathbb{E}(U_t) = 0, \mathbb{E}(q_{2,t}) < \infty$ is p.d. and $\mathbb{E}\left(g^2(e_t)\right) = \sigma^2$. Define $\tilde{U}_t$ the nonredundant elements of U_t, then $R^{-1}\mathbb{E}\left(\sum_{j=t-R+1}^{t}\tilde{U}_j\right)\left(\sum_{j=t-R+1}^{t}\tilde{U}_j\right)' = \Omega < \infty$ is p.d. We also define $\lim_{P,R\to\infty} P/R = \pi$ and $\xi = (1+\pi)^{-1}$. We have the following lemmas.

Lemma 1. For each $i = 1, 2$, we have $\sup_t T^{1/2}|U_t| = O(1)$.

Proof: Note that $T^{1/2}U(t) = T^{1/2}\sum_{j=t-R+1}^{t} U_j$. Recall that $|\cdot|$ denote the maximum norm and hence we have $|U(t)| = \left|\tilde{U}(t)\right|$ and hence

$$\sup_t T^{1/2}|U_t| = (T/R)\sup_t\left(T^{-1/2}\sum_{j=t-R+1}^{t}\tilde{U}_j\right).$$

By assumption that T/R is bounded even if we may have large P and small R. Given Corollary 29.19 of Davidson (1994) we know that

$$\begin{aligned} T^{-1/2}\sum\nolimits_{j=t-R+1}^{t}\tilde{U}_j &= T^{-1/2}\sum_{j=1}^{t}\tilde{U}_j - T^{-1/2}\sum_{j=1}^{t-R}\tilde{U}_j \\ &\Longrightarrow W(s;\Omega) - W(s-\xi;\Omega) \sim O(1). \end{aligned}$$

■

Lemma 2. For $i = 1, 2$, $\sum_t h'_{i,t} B_i(t) H_i(t) = \sum_t h'_{i,t} B_i H_i(t) + o(1)$.

Proof: We need to show that $\sum_t h'_{i,t}(B_i(t) - B_i) H_i(t) = o(1)$. Note that

$$
\begin{aligned}
& \sum_t h'_{i,t}(B_i(t) - B_i) H_i(t) \\
= \; & T^{-1/2} \sum_t (T/R)\, vec[T^{1/2}(B_i(t) - B_i)] \otimes \left(T^{-1/2} h_{i,t} \sum_{j=t-R+1}^{t} h_{i,j} \right),
\end{aligned}
$$

where the term $\sum_t (T/R)\, vec[T^{1/2}(B_i(t) - B_i)] \otimes \left(T^{-1/2} h_{i,t} \sum_{j=t-R+1}^{t} h_{i,j} \right)$ is $O(1)$ following 3.1 of Hansen (1992). ■

Lemma 3. Let $M = -J\prime B_1 J + B_2$. We can verify that $Q \equiv B_2^{-1/2} M B_2^{-1/2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$,

which is an idempotent matrix with rank 1. Define matrix $A = (0, 1)'$ and C=$I_{2\times 2}$ Then $Q = CAA'C$. We have the following result

$$
\begin{aligned}
&\textit{(a).} \quad T^{-1/2} \sum_{j=t-R+1}^{t} \tilde{h}_{2,j} 2, \tilde{h}'_{2,j} \to I_{1\times 1} \\
&\textit{(b).} \quad T^{-1/2} \sum_{j=t-R+1}^{t} \tilde{h}_{2,j} \to \xi^{-1}[W(s) - W(s - \xi)]
\end{aligned}
$$

Proof: (a). Note that $\mathbb{E}\left(\tilde{h}_{2,j}\right) = \mathbb{E}\left(\sigma^{-1} A' C B_2^{1/2} h_{2,t}\right) = 0$ and

$$
\begin{aligned}
& Var\left(T^{-1/2} \sum_{j=t-R+1}^{t} \tilde{h}_{2,j} \tilde{h}'_{2,j} \right) \\
= \; & T^{-1} \left(\sigma^{-1} A' C B_2^{1/2}\right) \left(\sum_{j=t-R+1}^{t} Var\left(h_{2,j} h'_{2,j}\right) \right) \left(\sigma^{-1} A' C B_2^{1/2}\right)' \\
= \; & \sigma^{-2} \left(A' C B_2^{1/2}\right) \left(\sigma^2 B_2^{-1}\right) \left(A' C B_2^{1/2}\right)' \\
= \; & A' C B_2^{1/2} B_2^{-1} B_2^{1/2} C' A \\
= \; & A' C C' A = Tr\left(C' A A' C\right) = Tr(Q) = I_{1\times 1}.
\end{aligned}
$$

Then

$$
\begin{aligned}
& T^{-1/2} \sum_{j=t-R+1}^{t} \tilde{h}_{2,j} \\
= \; & T^{-1/2} \sum_{j=1}^{t} \tilde{h}_{2,j} - T^{-1/2} \sum_{j=1}^{t-R} \tilde{h}_{2,j} \\
\Longrightarrow \; & W(s) - W(s - \xi).
\end{aligned}
$$

■

Lemma 4. Show that $\sum_t \tilde{H}_2'(t)\,\tilde{h}_{2,t+1} \longrightarrow^d \xi^{-1}\int_\xi^1 [W(s)-W(s-\xi)]\mathrm{d}W(s)$, where $\sum_t$ denotes $\sum_{j=t-R+1}^{t}$ (similarly hereinafter).

Proof: We note that

$$\sum\nolimits_t \tilde{H}_2'(t)\,\tilde{h}_{2,t+1} = \frac{T}{R}\left[\sum\nolimits_t \left(T^{-1/2}\sum\nolimits_{j=1}^{t}\tilde{h}_{2,j}' - T^{-1/2}\sum\nolimits_{j=1}^{t-R}\tilde{h}_{2,j}'\right)\left(T^{-1/2}\tilde{h}_{2,t+1}\right)\right].$$

Using $T/R=\xi^{-1}$ and the continuous mapping theorem we have

$$T^{-1/2}\sum\nolimits_{j=1}^{t}\tilde{h}_{2,j}' \Longrightarrow W(s), T^{-1/2}\sum\nolimits_{j=1}^{t-R}\tilde{h}_{2,j}' \Longrightarrow W(s-\xi)$$

and $T^{-1/2}\tilde{h}_{2,t+1} \Longrightarrow \mathrm{d}W(s)$ can we finish the proof. ■

Lemma 5. Show that $\sum_t \tilde{H}_2'(t)\,\tilde{H}_2(t) \longrightarrow^d \xi^{-2}\int_\xi^1 [W(s)-W(s-\xi)]\mathrm{d}W(s)$.

Proof: We note that

$$\begin{aligned}
& \sum\nolimits_t \tilde{H}_2'(t)\,\tilde{H}_2(t) \\
= \;& \frac{1}{T}(\frac{T}{R})^2\left[\sum\nolimits_t \left(T^{-1/2}\sum\nolimits_{j=1}^{t}\tilde{h}_{2,j}' - T^{-1/2}\sum\nolimits_{j=1}^{t-R}\tilde{h}_{2,j}'\right)'\right. \\
& \left.\times\left(T^{-1/2}\sum\nolimits_{j=1}^{t}\tilde{h}_{2,j} - T^{-1/2}\sum\nolimits_{j=1}^{t-R}\tilde{h}_{2,j}\right)\right].
\end{aligned}$$

By using Lemma 4 and the fact that $1/T \Longrightarrow \mathrm{d}s$. We have the proof. ■

Lemma 6. Show that $\sum_t g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) = \sigma^2\sum_t \tilde{H}_2'(t)\,\tilde{h}_{2,t+1} + o(1)$.

Proof: We notice that

$$\hat{e}_{t+1}^{(1)} = e_{t+1}^{(1)} - x_{1,t}'\left(\hat{\beta}_{1,t} - \beta_{1,t}\right)$$

and

$$\hat{e}_{t+1}^{(2)} = e_{t+1}^{(2)} - x_{2,t}'\left(\hat{\beta}_{2,t} - \beta_{2,t}\right).$$

Under $\mathbb{H}_0$, $e_{t+1}^{(1)} = e_{t+1}^{(2)} = e_{t+1}$. Recall that

$$\sqrt{R}\left(\hat{\beta}_{i,t} - \beta_{i,t}\right) = \sqrt{R}\left[B_i(t)\,H_i(t) + R(t)\right].$$

Then we have the following equation

$$\begin{aligned}
& \sum\nolimits_t g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)}-\hat{e}_{t+1}^{(2)}\right) \\
= & \sum\nolimits_t g\left(\hat{e}_{t+1}^{(1)}\right)\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)-x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right] \\
= & \sum\nolimits_t \left[g\left(e_{t+1}^{(1)}\right)+g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)-x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right] \\
= & \left\{\underbrace{\sum\nolimits_t[-h_{1,t+1}'B_1(t)H_1(t)+h_{2,t+1}'B_2(t)H_2(t)]}_{Term(A)}+\right. \\
& \left.\underbrace{\sum\nolimits_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)-x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right]}_{Term(B)}\right\} \\
= & \sum\nolimits_t[-h_{2,t+1}'J'B_1(t)JH_2(t)+h_{2,t+1}'B_2(t)H_2(t)]+Term(B) \\
= & \sum\nolimits_t[-h_{2,t+1}'J'B_1JH_2(t)+h_{2,t+1}'B_2(t)H_2(t)]+Term(B)+o(1) \\
= & \sum\nolimits_t[h_{2,t+1}'MH_2(t)]+Term(B).
\end{aligned}$$

We need to show that the second term (B) is $o(1)$. We consider

$$\sum\nolimits_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)\right]$$

and

$$\sum\nolimits_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right].$$

Knowing that $\hat{e}_{t+1}^{(1)}-e_{t+1}^{(1)}=x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\sim O\left(R^{-1/2}\right)$ Therefore if $g\left(\hat{e}_{t+1}^{(1)}\right)$ and $g\left(e_{t+1}^{(1)}\right)$ have different values, $e_{t+1}^{(1)}$ and $\hat{e}_{t+1}^{(1)}$ must scatter at the opposite sites of the origin. Using Chebyshev's Inequality we know that $P\left(g\left(\hat{e}_{t+1}^{(1)}\right)\neq g\left(e_{t+1}^{(1)}\right)\right)<O(R^{-1})$. Similar to the proof by Davidson (1994) and Theorem 3.1 of Hansen (1992),

$$\begin{aligned}
& \left|\sum\nolimits_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)\right]\right| \\
\leq & \sum\nolimits_t \left|g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right|\left|x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)\right|\sim o(1).
\end{aligned}$$

Similarly, $\sum_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right]=o(1)$. Therefore by Lemma 3 and the definition of $h_{2,t+1}'$ and $H_2(t)$, we have $\sum_t[h_{2,t+1}'MH_2(t)]\Longrightarrow\sigma^2\sum_t\tilde{H}_2'(t)\tilde{h}_{2,t+1}$. ■

Lemma 7. Show that $\sum_t\left[g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)}-\hat{e}_{t+1}^{(2)}\right)\right]^2=\sigma^4\sum_t\tilde{H}_2'(t)\tilde{H}_2(t)+o(1)$.

Proof: The proof is similar to Lemma 6. We have

$$\begin{aligned}
& \sum\nolimits_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)}-\hat{e}_{t+1}^{(2)}\right)\right]^2 \\
= & \sum\nolimits_t \left\{g\left(\hat{e}_{t+1}^{(1)}\right)\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)-x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right]\right\}^2 \\
= & \sum\nolimits_t \left\{\left[g\left(e_{t+1}^{(1)}\right)+g\left(\hat{e}_{t+1}^{(1)}\right)-g\left(e_{t+1}^{(1)}\right)\right]\left[x_{2,t}'\left(\hat{\beta}_{2,t}-\beta_{2,t}\right)-x_{1,t}'\left(\hat{\beta}_{1,t}-\beta_{1,t}\right)\right]\right\}^2 \\
= & \sum\nolimits_t \left\{\left[-h_{1,t+1}'B_1(t)H_1(t)+h_{2,t+1}'B_2(t)H_2(t)\right\}^2+o(1) \\
= & \sum\nolimits_t H_1'(t)B_1'(t)h_{1,t+1}h_{1,t+1}'B_1(t)H_1(t)+\sum\nolimits_t H_2'(t)B_2'(t)h_{2,t+1}h_{2,t+1}'B_2(t)H_2(t) \\
& -2\sum\nolimits_t H_1'(t)B_1'(t)h_{1,t+1}h_{2,t+1}B_2(t)H_2(t)+o(1) \\
\sim & \sum\nolimits_t H_1'(t)B_1'(t)\mathbb{E}\left(h_{1,t+1}h_{1,t+1}'\right)B_1(t)H_1(t) \\
& +\sum\nolimits_t H_2'(t)B_2'(t)\mathbb{E}\left(h_{2,t+1}h_{2,t+1}'\right)B_2(t)H_2(t) \\
& -2\sum\nolimits_t H_1'(t)B_1'(t)\mathbb{E}\left(h_{1,t+1}h_{2,t+1}'\right)B_2(t)H_2(t)+o(1). \qquad (30)
\end{aligned}$$

Note that $\mathbb{E}\left(h_{2,t+1}h_{2,t+1}'\right)=\sigma^2 B_2^{-1}$, $\mathbb{E}\left(h_{1,t+1}h_{2,t+1}'\right)=\sigma^2 JB_2^{-1}$, $\mathbb{E}\left(h_{1,t+1}h_{1,t+1}'\right)=\sigma^2 JB_2^{-1}J'$. Therefore we have

$$\begin{aligned}
& \sum\nolimits_t H_1'(t)B_1'(t)\mathbb{E}\left(h_{1,t+1}h_{1,t+1}'\right)B_1(t)H_1(t) \\
= & \sigma^2\sum\nolimits_t H_1'(t)B_1H_1(t)+o(1) \\
= & \sigma^2\sum\nolimits_t H_2'(t)J'B_1JH_2(t)+o(1),
\end{aligned}$$

$$\begin{aligned}
& \sum\nolimits_t H_2'(t)B_2'(t)\mathbb{E}\left(h_{2,t+1}h_{2,t+1}'\right)B_2(t)H_2(t) \\
= & \sigma^2\sum\nolimits_t H_2'(t)B_2H_2(t)+o(1),
\end{aligned}$$

and

$$\begin{aligned}
& \sum\nolimits_t H_1'(t)B_1'(t)\mathbb{E}\left(h_{1,t+1}h_{2,t+1}'\right)B_2(t)H_2(t) \\
= & \sigma^2\sum\nolimits_t H_2'(t)J'B_1\left(JB_2^{-1}\right)B_2H_2(t)+o(1) \\
= & \sigma^2\sum\nolimits_t H_2'(t)J'B_1JH_2(t)+o(1),
\end{aligned}$$

and Equation (30) equals

$$\begin{aligned}
& \sigma^2 \sum\nolimits_t H_2'(t)\, J' B_1 J H_2(t) + \sigma^2 \sum\nolimits_t H_2'(t)\, B_2 H_2(t) \\
& -2\sigma^2 \sum\nolimits_t H_2'(t)\, J' B_1 J H_2(t) + o(1) \\
= \; & \sigma^2 \sum\nolimits_t H_2'(t) \left[-J' B_1 J + B_2 \right] H_2(t) + o(1) \\
= \; & \sigma^2 \sum\nolimits_t H_2'(t)\, M H_2(t) + o(1) \\
= \; & \sigma^2 \sum\nolimits_t H_2'(t)\, B_2^{1/2} C A A' C B_{\mathbf{2}}^{\mathbf{1/2}} H_2(t) + o(1) \\
= \; & \sigma^4 \sum\nolimits_t \tilde{H}_2'(t)\, \tilde{H}_2(t) + o(1).
\end{aligned}$$

■

Based on Lemmas 4-7, we have

$$\begin{aligned}
\sum\nolimits_t g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) &= \sigma^2 \sum\nolimits_t \tilde{H}'(t)\, \tilde{h}_{2,t+1} + o(1) \\
\sum\nolimits_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)\right]^2 &= \sigma^4 \sum\nolimits_t \tilde{H}_2'(t)\, \tilde{H}_2(t) + o(1).
\end{aligned}$$

Also $P\bar{C}^2 \sim O\left(P^{-1}\right)$, where $\bar{C} = P^{-1}\left[\sum_t g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)\right]$. Therefore

$$\begin{aligned}
ENC_P &\equiv \hat{Q}_P^{-1/2} \sqrt{P} \hat{B}_P \\
&= \frac{\sum_t g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)}{\sqrt{\sum_t \left[g\left(\hat{e}_{t+1}^{(1)}\right)\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)\right]^2 - P\bar{C}^2}} \\
&\sim \frac{\xi^{-1} \int_\xi^1 \left[W(s) - W(s-\xi) \right] dW(s)}{\sqrt{\xi^{-2} \int_\xi^1 \left[W(s) - W(s-\xi) \right] dW(s)}}.
\end{aligned}$$

7.2 The First Order Condition of Equation (24) is Equation (25)

Let $\rho_\alpha(\varepsilon_{t+1}) = \left[\alpha - \mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} < 0\right)\right]\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1}\right)$. In this section we show that the first order condition of $\mathbb{E}\rho_\alpha(\varepsilon_{t+1})$ is equation (25). Let $\hat{\lambda} \in \Lambda$ the solution and.let $\Lambda^0 = \left\{\lambda \in \Lambda | (1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} = 0\right\}$. Therefore $\mathbb{E}\rho_\alpha(\varepsilon_{t+1})$ is differentiable at $\lambda \in \Lambda \backslash \Lambda^0$. Assume that $\mathbb{E}\rho_\alpha(\varepsilon_{t+1}) < \infty$ so that we can exchange integral and derivative symbols.

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}\mathbb{E}\rho_\alpha(\varepsilon_{t+1}) \\
=\ & \frac{\partial}{\partial\lambda}\mathbb{E}\left\{\mathbb{E}\rho_\alpha(\varepsilon_{t+1})|I_t\right\} \\
=\ & \mathbb{E}\frac{\partial}{\partial\lambda}\left\{\mathbb{E}\rho_\alpha(\varepsilon_{t+1})|I_t\right\} \\
=\ & \mathbb{E}\left(\frac{\partial}{\partial\lambda}\rho_\alpha(\varepsilon_{t+1}) \times \mathbb{E}\left[\mathbf{1}\left(\lambda \in \Lambda\backslash\Lambda^0\right)\right]|I_t\right) \\
& + \mathbb{E}\left(\frac{\partial}{\partial\lambda}\rho_\alpha(\varepsilon_{t+1}) \times \mathbb{E}\left[\mathbf{1}\left(\lambda \in \Lambda^0\right)\right]|I_t\right). \qquad (31)
\end{aligned}$$

Since $\mathbf{1}\left(\lambda \in \Lambda^0\right) = 0$ has zero measure, we have $\mathbb{E}\left[\mathbf{1}\left(\lambda \in \Lambda^0\right)\right] = 0$, the second term is of equation (31) is zero and thus $\mathbb{E}\rho_\alpha(\varepsilon_{t+1})$ is continuous and differentiable at $\lambda \in \Lambda\backslash\Lambda^0$. a.e. Following Elliott, Komunjer and Timmermann (2005), we have

$$\begin{aligned}
&\frac{\partial}{\partial\lambda}\mathbb{E}\rho_\alpha(\varepsilon_{t+1}) \\
=\ & \mathbb{E}\left[-\frac{\partial}{\partial\lambda}\mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} < 0\right)\right]\left[(1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1}\right] \\
& + \mathbb{E}\left[\alpha - \mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} < 0\right)\right] \times \frac{\partial}{\partial\lambda}\left[(1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1}\right],
\end{aligned}$$

where $\partial/\partial\lambda\left[\mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{2}_{t+1} < 0\right)\right]$ is the Dirac function, which is not equal to 0 for $\lambda \in \Lambda^0$. Therefore

$$\mathbb{E}\left\{\left[-\frac{\partial}{\partial\lambda}\mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} < 0\right)\right] \times \left[(1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1}\right]\right\} = 0$$

and

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\mathbb{E}\rho_\alpha(\varepsilon_{t+1}) &= \mathbb{E}\left\{\left[\alpha - \mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} < 0\right)\right] \times \frac{\partial}{\partial\lambda}\left[(1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1}\right]\right\} \\
&= \mathbb{E}\left\{\left[\alpha - \mathbf{1}\left((1-\lambda)\hat{e}^{(1)}_{t+1} + \lambda\hat{e}^{(2)}_{t+1} < 0\right)\right]\left(\hat{e}^{(2)}_{t+1} - \hat{e}^{(1)}_{t+1}\right)\right\}.
\end{aligned}$$

References

Ashley, R., C.W.J. Granger and R. Schmalensee (1980). “Advertising and Aggregate Consumption: An Analysis of Causality.” *Econometrica* 48 (7): 1149-67.

Chao, J. C., V. Corradi and N. Swanson (2001). “An Out of Sample Test for Granger Causality.” *Macroeconomic Dynamics* 5(4): 598-620.

Chuang, C.-C., C.-M. Kuan, and H.-Y. Lin (2009). “Causality in quantiles and dynamic stock return-volume relations.” *Journal of Banking and Finance* 33(7): 1351-1360.

Clark, T. E. and M. McCracken (2001). “Tests of equal forecast accuracy and encompassing for nested models.” *Journal of Econometrics* 105(1): 85-110.

Clark, T. E. and M. McCracken (2005). “Evaluating Direct Multistep Forecasts.” *Econometric Reviews* 24(4): 369-404.

Clark, T. E. and M. McCracken (2009). “Combining Forecasts from Nested Models.” *Oxford Bulletin of Economics and Statistics* 71(3): 303-329.

Clark, T. E. and K. D. West (2006). “Using out-of-sample mean squared prediction errors to test the martingale difference hypothesis.” *Journal of Econometrics* 135(1-2): 155-186.

Clark, T. E. and K. D. West (2007). “Approximately normal tests for equal predictive accuracy in nested models.” *Journal of Econometrics* 138(1): 291-311.

Corradi, V. and N. R. Swanson (2002). “A consistent test for nonlinear out of sample predictive accuracy.” *Journal of Econometrics* 110(2): 353-381.

Davidson, J. (1994). *Stochastic Limit Theory*. Oxford University Press, New York.

Diebold, F.X. (2015), “Comparing predictive accuracy, twenty years later: a personal perspective on the use and abuse of Diebold-Mariano tests”, forthcoming, *Journal of Business and Economic Statistics*.

Diebold, F. X. and R. S. Mariano (1995). “Comparing Predictive Accuracy.” *Journal of Business and Economic Statistics* 13(3): 253-263.

Elliott, Graham, Ivana Komunjer and Allan Timmermann (2005). “Estimation and Testing of Forecast Rationality under Flexible Loss.” *Review of Economic Studies* 72(4): 1107-1125.

Feng, X., X. He and J. Hu (2011). “Wild bootstrap for quantile regression.” *Biometrika* 98: 995-999.

Giacomini, R. and H. White (2006). “Tests of Conditional Predictive Ability.” *Econometrica* 74(6): 1545-1578.

Giacomini, R. and I. Komunjer (2005). “Evaluation and Combination of Conditional Quantile Forecasts.” *Journal of Business and Economic Statistics* 23: 416-431.

Goyal, A. and Welch, I. (2008). “A Comprehensive Look at the Empirical Performance of Equity Premium Prediction.” *Review of Financial Studies* 21: 1455-1508.

Hansen, B. E. (1992). “Convergence to stochastic integrals for dependent heterogeneous processes.” *Econometric Theory* 8(4): 489-500.

Hansen, P. R and A. Timmermann (2012). “Choice of Sample Split in Out-of-Sample Forecast Evaluation.” *Economics Working Papers ECO2012/10, European University Institute.*

Hansen, P. R and A. Timmermann (2013). “Equivalence Between Out-of-Sample Forecast Comparisons and Wald Statistics.” *Economics Working Papers ECO2012/24, European University Institute.*

Harvey, D. S., S. J. Leybourne and P. Newbold (1998). “Tests for Forecast Encompassing.” *Journal of Business and Economic Statistics* 16(2): 254-259.

Inoue. A and L. Kilian (2005). ”In-Sample or Out-of-Sample Tests of Predictability: Which One Should We Use?” *Econometric Reviews* 23(4): 371-402.

Jeong, K., W. K. Hädle, and S. Song (2012). “A Consistent Nonparametric Test for Causality in Quantile.” *Econometric Theory* 28(4): 861-887.

Koenker, R. and G. Bassett Jr. (1978). “Regression Quantiles.” *Econometrica* 46(1): 33-50.

Lee, T.-H. and W. Yang (2012). “Money-Income Granger-Causality in Quantiles.” in *Advances in Econometrics*, Millimet, D. and Terrell, D. (ed.), Emerald Publishers.

Meese, R. A. and K. Rogoff (1983). “Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?.” *Journal of International Economics* 14(2): 3-24.

Meese, R. A. and K. Rogoff (1988). “Was It Real? The Exchange Rate-Interest Differential Relation Over the Modern Floating-Rate Period.” *Journal of Finance* 43(9): 933-948.

Mizon, G. E. and J-F. Richard (1986). “The Encompassing Principle and Its Application to Non-nested Hypotheses.” *Econometrica* 54(3): 657-678.

Nelson, C. R (1972). “The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy.” *American Economic Review* 62(5): 902-17.

Newey, W. K. and J. L. Powell (1987). “Asymmetric Least Squares Estimation and Testing.” *Econometria* 55(4): 819-847.

Phillips, P. C. B. and J. L. Lee (2013). “Predictive Regression under Various Degrees of Persistence and Robust Long-horizon Regression.” *Journal of Econometrics* 177(2): 250-264.

Phillips, P. C. B. and J. L. Lee "Predictive Quantile Regression with Persistent Covariates." *Working Papers*.

West, K. D. (1996). "Asymptotic Inference about Predictive Ability." *Econometrica* 64(5): 1067-1084.

Table 1.1: Size of test under nominal level 0.05, $c_2 = 1$, $b = 0$, $\phi = 0$, $\sigma_e = 1$

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.013	0.049	0.043	0.001	0.053	0.051	0.000	0.071	0.048
$\alpha = 0.1$	$R = 120$	0.041	0.068	0.045	0.002	0.049	0.061	0.000	0.056	0.038
	$R = 240$	0.046	0.065	0.041	0.010	0.050	0.055	0.000	0.050	0.052
	$R = 60$	0.011	0.043	0.057	0.001	0.054	0.058	0.000	0.086	0.056
$\alpha = 0.3$	$R = 120$	0.024	0.045	0.057	0.001	0.045	0.052	0.000	0.063	0.053
	$R = 240$	0.024	0.047	0.056	0.008	0.036	0.046	0.000	0.048	0.049
	$R = 60$	0.006	0.039	0.048	0.001	0.054	0.058	0.000	0.086	0.056
$\alpha = 0.5$	$R = 120$	0.017	0.042	0.066	0.003	0.036	0.053	0.000	0.059	0.048
	$R = 240$	0.025	0.044	0.060	0.005	0.033	0.049	0.001	0.052	0.059
	$R = 60$	0.012	0.045	0.056	0.002	0.050	0.055	0.000	0.086	0.052
$\alpha = 0.7$	$R = 120$	0.024	0.050	0.055	0.002	0.033	0.054	0.000	0.057	0.053
	$R = 240$	0.029	0.053	0.058	0.006	0.033	0.053	0.001	0.059	0.062
	$R = 60$	0.018	0.054	0.043	0.001	0.056	0.049	0.000	0.065	0.043
$\alpha = 0.9$	$R = 120$	0.032	0.061	0.035	0.003	0.046	0.051	0.000	0.063	0.050
	$R = 240$	0.038	0.054	0.043	0.010	0.052	0.063	0.001	0.046	0.050

Notes: The tables show the size of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times. We have five quantiles: 0.1, 0.3, 0.5, 0.7 and 0.9.

Table 1.2: Size of test under nominal level 0.05, $c_2 = 1$, $b = 0$, $\phi = 0$, $\sigma_e = 0.1$

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.013	0.048	0.043	0.001	0.053	0.051	0.000	0.070	0.048
$\alpha = 0.1$	$R = 120$	0.041	0.067	0.045	0.002	0.049	0.061	0.000	0.057	0.038
	$R = 240$	0.046	0.065	0.041	0.010	0.050	0.055	0.000	0.050	0.052
	$R = 60$	0.010	0.044	0.057	0.001	0.054	0.058	0.000	0.087	0.056
$\alpha = 0.3$	$R = 120$	0.024	0.045	0.057	0.001	0.045	0.052	0.000	0.063	0.053
	$R = 240$	0.024	0.047	0.056	0.008	0.036	0.046	0.000	0.048	0.049
	$R = 60$	0.006	0.040	0.048	0.001	0.054	0.058	0.000	0.087	0.056
$\alpha = 0.5$	$R = 120$	0.016	0.042	0.066	0.003	0.036	0.053	0.000	0.059	0.048
	$R = 240$	0.024	0.044	0.060	0.005	0.033	0.049	0.001	0.051	0.059
	$R = 60$	0.012	0.046	0.056	0.002	0.050	0.055	0.000	0.086	0.052
$\alpha = 0.7$	$R = 120$	0.024	0.050	0.055	0.002	0.033	0.054	0.000	0.056	0.053
	$R = 240$	0.029	0.053	0.058	0.006	0.032	0.053	0.001	0.059	0.062
	$R = 60$	0.018	0.055	0.043	0.001	0.056	0.049	0.000	0.065	0.043
$\alpha = 0.9$	$R = 120$	0.033	0.061	0.035	0.003	0.045	0.051	0.000	0.063	0.050
	$R = 240$	0.037	0.054	0.043	0.010	0.052	0.063	0.001	0.045	0.050

Notes: The tables show the size of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times. We have five quantiles: 0.1, 0.3, 0.5, 0.7 and 0.9.

Table 1.3: Size of test under nominal level 0.05, $c_2 = 1$, $b = 0$, $\phi = 0.95$, $\sigma_e = 1$

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.014	0.079	0.077	0.000	0.059	0.047	0.000	0.069	0.035
$\alpha = 0.1$	$R = 120$	0.032	0.068	0.087	0.003	0.054	0.050	0.000	0.055	0.043
	$R = 240$	0.050	0.075	0.102	0.004	0.041	0.051	0.000	0.044	0.037
	$R = 60$	0.008	0.048	0.050	0.000	0.048	0.032	0.000	0.077	0.039
$\alpha = 0.3$	$R = 120$	0.018	0.045	0.060	0.001	0.040	0.045	0.000	0.058	0.040
	$R = 240$	0.028	0.046	0.063	0.006	0.034	0.043	0.000	0.051	0.053
	$R = 60$	0.009	0.032	0.045	0.000	0.048	0.032	0.000	0.077	0.039
$\alpha = 0.5$	$R = 120$	0.018	0.042	0.055	0.001	0.033	0.047	0.000	0.053	0.046
	$R = 240$	0.022	0.039	0.054	0.003	0.034	0.050	0.000	0.034	0.052
	$R = 60$	0.012	0.041	0.039	0.001	0.056	0.037	0.000	0.073	0.036
$\alpha = 0.7$	$R = 120$	0.021	0.042	0.049	0.002	0.049	0.046	0.000	0.066	0.048
	$R = 240$	0.019	0.040	0.054	0.005	0.037	0.058	0.000	0.055	0.045
	$R = 60$	0.021	0.074	0.075	0.000	0.060	0.038	0.000	0.072	0.037
$\alpha = 0.9$	$R = 120$	0.033	0.084	0.101	0.002	0.054	0.045	0.000	0.055	0.050
	$R = 240$	0.053	0.082	0.101	0.004	0.044	0.046	0.000	0.055	0.046

Table 1.4: Size of test under nominal level 0.05, $c_2 = 1$, $b = 0$, $\phi = 0.95$, $\sigma_e = 0.1$

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.014	0.078	0.077	0.000	0.060	0.047	0.000	0.069	0.034
$\alpha = 0.1$	$R = 120$	0.032	0.068	0.087	0.003	0.054	0.050	0.000	0.056	0.043
	$R = 240$	0.050	0.075	0.102	0.005	0.041	0.051	0.000	0.044	0.037
	$R = 60$	0.008	0.048	0.051	0.000	0.048	0.032	0.000	0.077	0.039
$\alpha = 0.3$	$R = 120$	0.019	0.045	0.060	0.001	0.040	0.045	0.000	0.058	0.040
	$R = 240$	0.028	0.046	0.063	0.006	0.034	0.043	0.000	0.051	0.053
	$R = 60$	0.008	0.032	0.045	0.000	0.048	0.032	0.000	0.077	0.039
$\alpha = 0.5$	$R = 120$	0.017	0.042	0.055	0.001	0.033	0.047	0.000	0.052	0.046
	$R = 240$	0.022	0.039	0.054	0.003	0.034	0.050	0.000	0.034	0.052
	$R = 60$	0.012	0.041	0.039	0.001	0.056	0.037	0.000	0.073	0.036
$\alpha = 0.7$	$R = 120$	0.021	0.042	0.049	0.002	0.049	0.046	0.000	0.066	0.048
	$R = 240$	0.019	0.040	0.054	0.005	0.037	0.058	0.000	0.055	0.045
	$R = 60$	0.021	0.074	0.074	0.000	0.060	0.038	0.000	0.072	0.037
$\alpha = 0.9$	$R = 120$	0.033	0.085	0.101	0.002	0.054	0.045	0.000	0.055	0.050
	$R = 240$	0.052	0.080	0.101	0.005	0.044	0.046	0.000	0.055	0.046

Table 1.5: Power of test under nominal level 0.05, $c_2 = 1$, $b = 0.1$, $\phi = 0.95$, $\sigma_e = 1$

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.067	0.244	0.167	0.046	0.523	0.484	0.047	0.972	0.995
$\alpha = 0.1$	$R = 120$	0.127	0.317	0.218	0.184	0.687	0.603	0.537	0.996	1.000
	$R = 240$	0.185	0.349	0.251	0.325	0.783	0.675	0.870	1.000	1.000
	$R = 60$	0.080	0.321	0.239	0.148	0.760	0.699	0.411	0.999	1.000
$\alpha = 0.3$	$R = 120$	0.161	0.407	0.313	0.346	0.869	0.808	0.929	1.000	1.000
	$R = 240$	0.183	0.443	0.327	0.519	0.930	0.876	0.994	1.000	1.000
	$R = 60$	0.088	0.341	0.262	0.148	0.760	0.699	0.411	0.999	1.000
$\alpha = 0.5$	$R = 120$	0.172	0.447	0.334	0.425	0.901	0.856	0.954	1.000	1.000
	$R = 240$	0.217	0.493	0.369	0.566	0.943	0.898	0.995	1.000	1.000
	$R = 60$	0.079	0.315	0.227	0.153	0.778	0.711	0.414	1.000	1.000
$\alpha = 0.7$	$R = 120$	0.138	0.397	0.288	0.379	0.880	0.849	0.934	1.000	1.000
	$R = 240$	0.200	0.457	0.339	0.526	0.922	0.870	0.989	1.000	1.000
	$R = 60$	0.063	0.247	0.180	0.053	0.530	0.482	0.039	0.973	0.991
$\alpha = 0.9$	$R = 120$	0.130	0.319	0.217	0.183	0.678	0.622	0.555	0.998	0.999
	$R = 240$	0.193	0.339	0.256	0.332	0.783	0.684	0.885	0.999	1.000

Notes: The tables show the power of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times. We have five quantiles: 0.1, 0.3, 0.5, 0.7 and 0.9.

Table 1.6: Power of test under nominal level 0.05, $c_2 = 1$, $b = 0.1$, $\phi = 0.95$, $\sigma_e = 0.1$

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.990	1.000	0.804	1.000	1.000	0.999	1.000	1.000	1.000
$\alpha = 0.1$	$R = 120$	0.993	1.000	0.790	1.000	1.000	0.992	1.000	1.000	1.000
	$R = 240$	0.995	1.000	0.771	1.000	1.000	0.980	1.000	1.000	1.000
	$R = 60$	0.999	1.000	0.900	1.000	1.000	1.000	1.000	1.000	1.000
$\alpha = 0.3$	$R = 120$	0.998	1.000	0.928	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.999	1.000	0.938	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	0.912	1.000	1.000	1.000	1.000	1.000	1.000
$\alpha = 0.5$	$R = 120$	1.000	1.000	0.945	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.999	1.000	0.975	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.999	1.000	0.893	1.000	1.000	0.999	1.000	1.000	1.000
$\alpha = 0.7$	$R = 120$	1.000	1.000	0.927	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.999	1.000	0.942	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.992	1.000	0.814	1.000	1.000	0.999	1.000	1.000	1.000
$\alpha = 0.9$	$R = 120$	0.998	1.000	0.794	1.000	1.000	0.993	1.000	1.000	1.000
	$R = 240$	0.995	1.000	0.773	1.000	1.000	0.980	1.000	1.000	1.000

Notes: The tables show the power of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times. We have five quantiles: 0.1, 0.3, 0.5, 0.7 and 0.9.

Table 1.7: Power of test under nominal level 0.05, $c_2 = 1$, $b = 0.1$, $\phi = 0.95$, $\sigma_e = 1$

		$P = 48$			$P = 240$			$P = 1200$	
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.067	0.254	0.227	0.050	0.545	0.544	0.041	0.969	0.989
$\alpha = 0.1$	$R = 120$	0.141	0.323	0.264	0.193	0.683	0.673	0.534	0.998	0.999
	$R = 240$	0.180	0.360	0.302	0.315	0.780	0.730	0.851	0.999	0.999
	$R = 60$	0.079	0.323	0.281	0.142	0.764	0.715	0.406	1.000	0.999
$\alpha = 0.3$	$R = 120$	0.151	0.406	0.365	0.370	0.867	0.847	0.933	1.000	1.000
	$R = 240$	0.198	0.460	0.400	0.508	0.924	0.894	0.994	1.000	1.000
	$R = 60$	0.081	0.339	0.289	0.166	0.799	0.755	0.488	1.000	1.000
$\alpha = 0.5$	$R = 120$	0.158	0.415	0.379	0.413	0.879	0.866	0.971	1.000	1.000
	$R = 240$	0.213	0.469	0.409	0.547	0.949	0.916	0.995	1.000	1.000
	$R = 60$	0.073	0.312	0.279	0.139	0.756	0.728	0.395	0.999	1.000
$\alpha = 0.7$	$R = 120$	0.152	0.393	0.351	0.366	0.856	0.839	0.931	1.000	1.000
	$R = 240$	0.204	0.454	0.410	0.522	0.928	0.886	0.993	1.000	1.000
	$R = 60$	0.063	0.227	0.201	0.051	0.532	0.524	0.044	0.964	0.988
$\alpha = 0.9$	$R = 120$	0.132	0.341	0.272	0.181	0.695	0.678	0.533	0.998	0.999
	$R = 240$	0.178	0.355	0.304	0.335	0.787	0.738	0.870	0.999	1.000

Table 1.8: Power of test under nominal level 0.05, $c_2 = 1$, $b = 0.1$, $\phi = 0.95$, $\sigma_e = 0.1$

		$P = 48$			$P = 240$			$P = 1200$	
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.990	1.000	0.778	1.000	1.000	0.998	1.000	1.000	1.000
$\alpha = 0.1$	$R = 120$	0.994	1.000	0.801	1.000	1.000	0.991	1.000	1.000	1.000
	$R = 240$	0.998	1.000	0.768	1.000	1.000	0.984	1.000	1.000	1.000
	$R = 60$	0.998	1.000	0.862	1.000	1.000	1.000	1.000	1.000	1.000
$\alpha = 0.3$	$R = 120$	1.000	1.000	0.914	1.000	1.000	0.999	1.000	1.000	1.000
	$R = 240$	1.000	1.000	0.938	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	0.885	1.000	1.000	1.000	1.000	1.000	1.000
$\alpha = 0.5$	$R = 120$	1.000	1.000	0.932	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	0.970	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	0.870	1.000	1.000	1.000	1.000	1.000	1.000
$\alpha = 0.7$	$R = 120$	0.998	1.000	0.901	1.000	1.000	0.999	1.000	1.000	1.000
	$R = 240$	1.000	1.000	0.940	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.989	1.000	0.781	1.000	1.000	0.998	1.000	1.000	1.000
$\alpha = 0.9$	$R = 120$	0.995	1.000	0.784	1.000	1.000	0.994	1.000	1.000	1.000
	$R = 240$	0.995	1.000	0.787	1.000	1.000	0.984	1.000	1.000	1.000

Table 1.9: Results from combining nested quantile models: $c_2 = 1$, $b = 0$, $\phi = 0$, $\sigma_e = 1$

$Repeat = 2000$		$P = 48$		$P = 240$		$P = 1200$	
		λ	Average Loss	λ	Average Loss	λ	Average Loss
$\alpha = 0.1$	$R = 60$	-0.2079 [a]	0.1790[d]	-0.0202	0.1797	0.0210	0.1793
		0.1887 [b]	0.1841[e]	-0.4236	0.1848	-0.2737	0.1844
		14.9659 [c]	0.1738[f]	3.4100	0.1786	2.9573	0.1790
$\alpha = 0.3$	$R = 60$	-0.2831	0.3528	-0.0456	0.3532	0.0396	0.3525
		-2.1525	0.3584	-0.5902	0.3587	-0.2695	0.3578
		24.1660	0.3467	3.5404	0.3520	2.9580	0.3522
$\alpha = 0.5$	$R = 60$	-0.3181	0.4033	-0.0765	0.4041	-0.0004	0.4037
		-0.6048	0.4093	-0.7516	0.4099	-0.2537	0.4095
		5.5195	0.3973	4.4117	0.4027	2.9048	0.4034
$\alpha = 0.7$	$R = 60$	-0.2269	0.3533	-0.0259	0.3532	0.0379	0.3526
		-1.1172	0.3586	-0.7243	0.3585	-0.3289	0.3578
		8.9691	0.3474	4.0574	0.3520	2.9418	0.3523
$\alpha = 0.9$	$R = 60$	-0.2146	0.1799	-0.0355	0.1797	0.0232	0.1793
		-1.0807	0.1848	-0.3508	0.1848	-0.2476	0.1844
		12.4164	0.1749	3.1660	0.1787	3.0045	0.1791
$\alpha = 0.1$	$R = 120$	-0.3951	0.1773	-0.3435	0.1763	-0.0696	0.1763
		-1.3113	0.1798	-0.9517	0.1774	-0.9398	0.1775
		20.4623	0.1723	5.1343	0.1752	4.7639	0.1761
$\alpha = 0.3$	$R = 120$	-0.5392	0.3499	-0.4446	0.3491	-0.0732	0.3486
		-3.1170	0.3523	-1.4893	0.3504	-0.9285	0.3498
		42.8294	0.3440	7.6674	0.3478	4.3749	0.3483
$\alpha = 0.5$	$R = 120$	-0.7103	0.3997	-0.4562	0.4006	-0.0976	0.4004
		-1.2326	0.4026	-1.4289	0.4019	-0.9821	0.4017
		8.2131	0.3934	7.4111	0.3993	4.5905	0.4001
$\alpha = 0.7$	$R = 120$	-0.5816	0.3504	-0.3988	0.3495	-0.0852	0.3487
		-1.7324	0.3527	-1.3523	0.3507	-0.7442	0.3500
		15.9498	0.3443	9.4568	0.3483	3.8285	0.3485
$\alpha = 0.9$	$R = 120$	-0.3452	0.1777	-0.1525	0.1777	-0.0741	0.1764
		-1.9311	0.1801	-0.6198	0.1801	-0.7440	0.1775
		40.9560	0.1725	3.5860	0.1766	3.9606	0.1762
$\alpha = 0.1$	$R = 240$	-0.7020	0.1762	-0.3435	0.1763	-0.0696	0.1763
		-2.4686	0.1773	-0.9517	0.1774	-0.9398	0.1775
		26.1175	0.1712	5.1343	0.1752	4.7639	0.1761
$\alpha = 0.3$	$R = 240$	-1.0431	0.3477	-0.4446	0.3491	-0.0732	0.3486
		-1.5079	0.3489	-1.4893	0.3504	-0.9285	0.3498
		10.6018	0.3414	7.6674	0.3478	4.3749	0.3483
$\alpha = 0.5$	$R = 240$	-1.1361	0.4006	-0.4562	0.4006	-0.0976	0.4004
		-1.9511	0.4021	-1.4289	0.4019	-0.9821	0.4017
		14.0436	0.3938	7.4111	0.3993	4.5905	0.4001
$\alpha = 0.7$	$R = 240$	-0.9199	0.3477	-0.3988	0.3495	-0.0852	0.3487
		-1.1151	0.3491	-1.3523	0.3507	-0.7442	0.3500
		13.6289	0.3413	9.4568	0.3483	3.8285	0.3485
$\alpha = 0.9$	$R = 240$	-0.5111	0.1757	-0.3080	0.1767	-0.0741	0.1764
		1.6066	0.1770	-1.0702	0.1778	-0.7440	0.1775
		40.3058	0.1704	6.4055	0.1756	3.9606	0.1762

[a] The average of the combination factor $\hat{\lambda}$.
[b] The skewness of the estimator $\hat{\lambda}$.
[c] The kurtosis of the estimator $\hat{\lambda}$.
[d] The average of the check loss from forecasting of the first model $\rho_\alpha^{(1)}$.
[e] The average of the check loss from forecasting of the second model $\rho_\alpha^{(2)}$.
[f] The average of the check loss from forecasting of the combined model $\rho_\alpha^{(c)}$.

Table 1.10: Results from combining nested quantile models: $c_2 = 1$, $b = 0$, $\phi = 0$, $\sigma_e = 0.1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		-0.2538	0.0353	-0.0446	0.0179	0.0183	0.0179
$\alpha = 0.1$	$R = 60$	-0.8435	0.0358	-0.5295	0.0184	-0.1659	0.0185
		8.2425	0.0347	3.4388	0.0178	2.8330	0.0179
		-0.3227	0.0405	-0.0377	0.0352	0.0402	0.0353
$\alpha = 0.3$	$R = 60$	-1.3736	0.0411	-0.7065	0.0357	-0.2841	0.0358
		13.0880	0.0398	4.1472	0.0351	3.1183	0.0352
		-0.3227	0.0405	-0.0900	0.0405	-0.0026	0.0403
$\alpha = 0.5$	$R = 60$	-1.3736	0.0411	-0.6578	0.0410	-0.3193	0.0409
		13.0880	0.0398	3.9475	0.0403	3.0461	0.0403
		-0.2286	0.0353	-0.0268	0.0353	0.0377	0.0353
$\alpha = 0.7$	$R = 60$	-1.1154	0.0359	-0.7237	0.0358	-0.3273	0.0358
		8.8870	0.0347	4.0485	0.0352	2.9395	0.0352
		-0.2472	0.0180	-0.0226	0.0179	0.0218	0.0179
$\alpha = 0.9$	$R = 60$	-0.7696	0.0186	-0.3812	0.0184	-0.2712	0.0185
		12.5303	0.0175	3.2307	0.0178	3.1546	0.0179
		-0.4027	0.0178	-0.1272	0.0178	-0.0158	0.0178
$\alpha = 0.1$	$R = 120$	-3.6798	0.0180	-0.8854	0.0180	-0.3892	0.0180
		57.5047	0.0172	5.3609	0.0176	2.9870	0.0177
		-0.5446	0.0350	-0.2028	0.0350	-0.0120	0.0350
$\alpha = 0.3$	$R = 120$	-1.3092	0.0353	-1.0681	0.0353	-0.4519	0.0353
		15.1541	0.0344	5.4876	0.0349	3.2440	0.0350
		-0.5606	0.0402	-0.2500	0.0402	-0.0433	0.0402
$\alpha = 0.5$	$R = 120$	-0.9918	0.0404	-0.8591	0.0405	-0.5851	0.0405
		10.6147	0.0395	4.2503	0.0400	3.5165	0.0401
		-0.5832	0.0350	-0.1661	0.0351	-0.0014	0.0350
$\alpha = 0.7$	$R = 120$	-1.7345	0.0353	-1.1048	0.0353	-0.5150	0.0353
		15.8652	0.0344	5.4091	0.0349	3.5371	0.0350
		-0.2882	0.0177	-0.1411	0.0178	-0.0024	0.0177
$\alpha = 0.9$	$R = 120$	-0.3703	0.0180	-1.0301	0.0180	-0.4660	0.0180
		10.5248	0.0172	5.6858	0.0177	3.5049	0.0177
		-0.6488	0.0176	-0.3124	0.0177	-0.0612	0.0176
$\alpha = 0.1$	$R = 240$	-1.5001	0.0177	-1.1133	0.0178	-0.7143	0.0178
		18.0745	0.0171	6.9348	0.0176	4.0447	0.0176
		-0.7171	0.0348	-0.4467	0.0350	-0.0776	0.0349
$\alpha = 0.3$	$R = 240$	0.7163	0.0349	-1.8657	0.0351	-0.8407	0.0350
		25.5898	0.0342	14.1154	0.0348	4.2084	0.0349
		-0.9954	0.0398	-0.4501	0.0401	-0.1064	0.0400
$\alpha = 0.5$	$R = 240$	-1.6778	0.0400	-1.9424	0.0402	-0.8615	0.0401
		13.5171	0.0392	13.3723	0.0399	4.1970	0.0400
		-0.9196	0.0348	-0.3993	0.0349	-0.0857	0.0349
$\alpha = 0.7$	$R = 240$	-1.1103	0.0349	-1.3538	0.0351	-0.7412	0.0350
		13.6486	0.0341	9.4409	0.0348	3.8247	0.0348
		-0.6003	0.0176	-0.3200	0.0177	-0.0763	0.0177
$\alpha = 0.9$	$R = 240$	-5.1930	0.0177	-0.7899	0.0178	-0.5904	0.0178
		108.8263	0.0171	7.0292	0.0176	3.4340	0.0176

Table 1.11: Results from combining nested quantile models: $c_2 = 1$, $b = 0$, $\phi = 0.95$, $\sigma_e = 1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		-0.1297	0.1787	-0.0343	0.1791	0.0120	0.1794
$\alpha = 0.1$	$R = 60$	-0.2751	0.1869	-0.3637	0.1868	-0.1071	0.1872
		7.9819	0.1740	3.1279	0.1780	2.7983	0.1792
		-0.2028	0.3527	-0.0280	0.3522	0.0220	0.3527
$\alpha = 0.3$	$R = 60$	-0.5578	0.3609	-0.6913	0.3602	-0.3388	0.3608
		8.8395	0.3470	4.1367	0.3509	3.2333	0.3524
		-0.2830	0.4048	-0.0693	0.4034	-0.0097	0.4037
$\alpha = 0.5$	$R = 60$	-1.9165	0.4136	-0.6089	0.4121	-0.4429	0.4124
		17.2374	0.3986	3.4409	0.4021	3.3392	0.4034
		-0.1297	0.1787	-0.0343	0.1791	0.0120	0.1794
$\alpha = 0.7$	$R = 60$	-0.2751	0.1869	-0.3637	0.1868	-0.1071	0.1872
		7.9819	0.1740	3.1279	0.1780	2.7983	0.1792
		-0.1580	0.1801	-0.0257	0.1792	0.0212	0.1794
$\alpha = 0.9$	$R = 60$	-0.6525	0.1881	-0.4265	0.1869	-0.0929	0.1870
		8.0496	0.1751	3.2685	0.1782	2.9377	0.1792
		-0.2770	0.1775	-0.0918	0.1775	-0.0179	0.1776
$\alpha = 0.1$	$R = 120$	-1.0048	0.1806	-0.6363	0.1808	-0.3756	0.1809
		17.2826	0.1725	3.9686	0.1765	3.1655	0.1774
		-0.4121	0.3499	-0.1710	0.3504	-0.0124	0.3505
$\alpha = 0.3$	$R = 120$	-0.9591	0.3534	-0.9975	0.3539	-0.4075	0.3540
		22.1054	0.3439	4.6898	0.3492	3.2718	0.3502
		-0.6381	0.4007	-0.2262	0.4016	-0.0438	0.4016
$\alpha = 0.5$	$R = 120$	-2.5034	0.4044	-0.9526	0.4055	-0.6341	0.4054
		29.8084	0.3944	4.5047	0.4003	3.8386	0.4013
		-0.4768	0.3497	-0.1544	0.3505	-0.0121	0.3503
$\alpha = 0.7$	$R = 120$	-1.7670	0.3530	-0.7785	0.3541	-0.5453	0.3538
		18.0953	0.3436	4.1320	0.3493	3.5316	0.3500
		-0.3008	0.1772	-0.1066	0.1779	-0.0145	0.1775
$\alpha = 0.9$	$R = 120$	-0.2516	0.1805	-0.9516	0.1810	-0.4441	0.1807
		15.1935	0.1719	5.3877	0.1768	3.3119	0.1773
		-0.5273	0.1763	-0.2932	0.1769	-0.0511	0.1764
$\alpha = 0.1$	$R = 240$	-0.4174	0.1776	-1.1093	0.1782	-0.6401	0.1777
		16.3982	0.1712	6.5952	0.1759	3.5943	0.1762
		-1.0051	0.3483	-0.4419	0.3495	-0.0729	0.3488
$\alpha = 0.3$	$R = 240$	-3.8433	0.3500	-1.6042	0.3511	-0.7752	0.3503
		57.7100	0.3423	8.3761	0.3483	3.9992	0.3485
		-1.0507	0.3988	-0.4899	0.4006	-0.0915	0.4001
$\alpha = 0.5$	$R = 240$	-1.2573	0.4005	-1.4400	0.4023	-0.7564	0.4017
		11.9141	0.3921	7.6429	0.3993	3.8261	0.3998
		-0.9932	0.3476	-0.4131	0.3493	-0.0782	0.3489
$\alpha = 0.7$	$R = 240$	-1.1336	0.3491	-1.6503	0.3509	-0.7517	0.3504
		10.9003	0.3414	11.8186	0.3481	3.6225	0.3486
		-0.6917	0.1759	-0.3106	0.1766	-0.0656	0.1765
$\alpha = 0.9$	$R = 240$	-2.1988	0.1772	-1.3190	0.1780	-0.6213	0.1779
		24.2136	0.1707	8.6549	0.1756	3.4799	0.1763

Table 1.12: Results from combining nested quantile models: $c_2 = 1$, $b = 0$, $\phi = 0.95$, $\sigma_e = 0.1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		-0.1304	0.0179	-0.0344	0.0179	0.0121	0.0179
$\alpha = 0.1$	$R = 60$	-0.2165	0.0187	-0.3556	0.0187	-0.1117	0.0187
		8.1932	0.0165	3.1024	0.0178	2.7987	0.0184
		-0.2048	0.0353	-0.0280	0.0352	0.0220	0.0353
$\alpha = 0.3$	$R = 60$	-0.5875	0.0361	-0.7013	0.0360	-0.3409	0.0361
		9.1553	0.0336	4.1410	0.0351	3.2311	0.0363
		-0.2821	0.0405	-0.0693	0.0403	-0.0095	0.0404
$\alpha = 0.5$	$R = 60$	-1.8917	0.0414	-0.6150	0.0412	-0.4431	0.0412
		17.0391	0.0388	3.4523	0.0394	3.3374	0.0415
		-0.2208	0.0354	-0.0291	0.0353	0.0183	0.0353
$\alpha = 0.7$	$R = 60$	-0.8027	0.0362	-0.5382	0.0361	-0.3276	0.0361
		7.7581	0.0343	3.3923	0.0341	3.0990	0.0357
		-0.1629	0.0180	-0.0259	0.0179	0.0212	0.0179
$\alpha = 0.9$	$R = 60$	-1.0979	0.0188	-0.4137	0.0187	-0.0917	0.0187
		13.4110	0.0168	3.2612	0.0174	2.9351	0.0175
		-0.2775	0.0178	-0.0916	0.0178	-0.0178	0.0178
$\alpha = 0.1$	$R = 120$	-1.0024	0.0181	-0.6458	0.0181	-0.3729	0.0181
		17.3628	0.0169	3.9922	0.0170	3.1651	0.0186
		-0.4098	0.0350	-0.1711	0.0350	-0.0122	0.0350
$\alpha = 0.3$	$R = 120$	-0.9619	0.0353	-0.9967	0.0354	-0.4095	0.0354
		22.0625	0.0347	4.6958	0.0338	3.2772	0.0362
		-0.6320	0.0401	-0.2264	0.0402	-0.0437	0.0402
$\alpha = 0.5$	$R = 120$	-2.4457	0.0404	-0.9539	0.0406	-0.6335	0.0405
		30.0329	0.0396	4.5124	0.0390	3.8468	0.0416
		-0.4734	0.0350	-0.1551	0.0351	-0.0120	0.0350
$\alpha = 0.7$	$R = 120$	-1.7607	0.0353	-0.7786	0.0354	-0.5390	0.0354
		18.0692	0.0343	4.1342	0.0340	3.5213	0.0365
		-0.2977	0.0177	-0.1071	0.0178	-0.0145	0.0177
$\alpha = 0.9$	$R = 120$	-0.2528	0.0181	-0.9705	0.0181	-0.4357	0.0181
		15.1007	0.0169	5.3821	0.0168	3.2748	0.0183
		-0.5340	0.0176	-0.2942	0.0177	-0.0509	0.0176
$\alpha = 0.1$	$R = 240$	-0.4237	0.0178	-1.1061	0.0178	-0.6422	0.0178
		16.1116	0.0175	6.6107	0.0174	3.6019	0.0175
		-0.9866	0.0348	-0.4404	0.0350	-0.0731	0.0349
$\alpha = 0.3$	$R = 240$	-3.8203	0.0350	-1.5208	0.0351	-0.7730	0.0350
		58.1925	0.0349	7.6454	0.0346	3.9970	0.0345
		-1.0584	0.0399	-0.4918	0.0401	-0.0915	0.0400
$\alpha = 0.5$	$R = 240$	-1.2665	0.0400	-1.4402	0.0402	-0.7592	0.0402
		12.1551	0.0397	7.6347	0.0401	3.8347	0.0398
		-0.9984	0.0348	-0.4121	0.0349	-0.0788	0.0349
$\alpha = 0.7$	$R = 240$	-1.0918	0.0349	-1.6552	0.0351	-0.7539	0.0350
		10.9969	0.0345	11.8592	0.0347	3.6207	0.0344
		-0.6743	0.0176	-0.3124	0.0177	-0.0655	0.0177
$\alpha = 0.9$	$R = 240$	-2.2487	0.0177	-1.3119	0.0178	-0.6223	0.0178
		24.0805	0.0166	8.6195	0.0173	3.4797	0.0176

Table 1.13: Results from combining nested quantile models: $c_2 = 1$, $b = 0.1$, $\phi = 0$, $\sigma_e = 1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		-0.0756	0.1798	0.0794	0.1800	0.1505	0.1803
$\alpha = 0.1$	$R = 60$	-0.5428	0.1842	-0.6452	0.1844	-0.1786	0.1846
		7.6280	0.1745	3.6668	0.1787	2.8697	0.1798
		-0.0852	0.3543	0.1525	0.3539	0.2448	0.3544
$\alpha = 0.3$	$R = 60$	-0.9517	0.3580	-0.8369	0.3574	-0.4095	0.3580
		7.3348	0.3474	4.2894	0.3520	3.0587	0.3535
		-0.1197	0.4066	0.1288	0.4053	0.2262	0.4056
$\alpha = 0.5$	$R = 60$	-1.6872	0.4106	-0.8007	0.4092	-0.4375	0.4096
		12.2036	0.3996	3.9977	0.4033	3.3563	0.4048
		-0.0945	0.3551	0.1672	0.3543	0.2436	0.3544
$\alpha = 0.7$	$R = 60$	-1.1104	0.3593	-0.6862	0.3578	-0.4750	0.3580
		9.3809	0.3485	3.5687	0.3524	3.5839	0.3535
		-0.1515	0.1809	0.0959	0.1802	0.1487	0.1803
$\alpha = 0.9$	$R = 60$	-0.8224	0.1856	-0.5620	0.1843	-0.1675	0.1846
		10.1451	0.1752	3.6113	0.1788	2.9119	0.1798
		-0.1305	0.1784	0.0500	0.1784	0.2355	0.1785
$\alpha = 0.1$	$R = 120$	-2.1966	0.1801	-0.9870	0.1799	-0.6954	0.1801
		33.7838	0.1727	4.9891	0.1769	4.0836	0.1780
		-0.1332	0.3515	0.1621	0.3521	0.3566	0.3522
$\alpha = 0.3$	$R = 120$	-1.3990	0.3526	-1.3178	0.3530	-0.6189	0.3531
		10.3417	0.3443	6.0604	0.3501	3.2693	0.3512
		-0.1265	0.4027	0.1817	0.4036	0.3637	0.4036
$\alpha = 0.5$	$R = 120$	-1.6797	0.4036	-1.4140	0.4045	-0.7586	0.4044
		12.9162	0.3952	6.8728	0.4015	3.7459	0.4025
		-0.0962	0.3515	0.1542	0.3522	0.3538	0.3520
$\alpha = 0.7$	$R = 120$	-2.8510	0.3524	-1.3069	0.3531	-0.6246	0.3529
		36.8570	0.3443	5.9969	0.3501	3.3702	0.3510
		-0.1540	0.1782	0.0577	0.1787	0.2445	0.1784
$\alpha = 0.9$	$R = 120$	-1.4722	0.1798	-1.0633	0.1804	-0.4986	0.1799
		18.9987	0.1724	5.9632	0.1773	3.2002	0.1778
		-0.1802	0.1771	0.0661	0.1776	0.3497	0.1773
$\alpha = 0.1$	$R = 240$	-6.5548	0.1774	-1.2669	0.1780	-0.9450	0.1775
		95.3777	0.1717	6.2634	0.1762	4.2285	0.1766
		0.1712	0.3500	0.2709	0.3511	0.5010	0.3506
$\alpha = 0.3$	$R = 240$	-2.0061	0.3494	-1.4914	0.3509	-1.3814	0.3501
		28.4228	0.3426	8.4228	0.3491	6.8904	0.3493
		0.0666	0.4008	0.2529	0.4025	0.5226	0.4021
$\alpha = 0.5$	$R = 240$	2.5542	0.4001	-4.0318	0.4021	-1.2685	0.4015
		86.4484	0.3929	49.2723	0.4002	5.9151	0.4007
		0.0978	0.3494	0.2386	0.3509	0.5029	0.3506
$\alpha = 0.7$	$R = 240$	-1.4663	0.3487	-1.9143	0.3506	-1.1675	0.3501
		11.1317	0.3421	12.8961	0.3488	5.2037	0.3494
		-0.2584	0.1767	0.0999	0.1775	0.3518	0.1774
$\alpha = 0.9$	$R = 240$	-6.9202	0.1770	-1.7151	0.1777	-1.0941	0.1777
		127.9914	0.1710	10.9887	0.1760	4.9227	0.1768

Table 1.14: Results from combining nested quantile models: $c_2 = 1$, $b = 0.1$, $\phi = 0$, $\sigma_e = 0.1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		0.9401	0.0254	0.9248	0.0254	0.9258	0.0254
$\alpha = 0.1$	$R = 60$	0.3948	0.0185	-0.1875	0.0185	-0.1848	0.0185
		4.8386	0.0179	4.2035	0.0184	3.1287	0.0184
		0.9708	0.0499	0.9607	0.0499	0.9612	0.0499
$\alpha = 0.3$	$R = 60$	0.2879	0.0358	-0.1531	0.0359	-0.2005	0.0358
		3.8091	0.0353	3.4959	0.0358	3.0662	0.0357
		0.9745	0.0571	0.9644	0.0571	0.9663	0.0571
$\alpha = 0.5$	$R = 60$	0.2800	0.0411	-0.1859	0.0410	-0.0674	0.0409
		3.9113	0.0405	3.3255	0.0410	2.9629	0.0409
		0.9720	0.0498	0.9591	0.0499	0.9610	0.0498
$\alpha = 0.7$	$R = 60$	0.2437	0.0359	-0.1533	0.0358	-0.0818	0.0358
		3.6522	0.0353	3.2813	0.0358	3.1002	0.0357
		0.9456	0.0254	0.9257	0.0254	0.9253	0.0253
$\alpha = 0.9$	$R = 60$	0.1009	0.0185	-0.3341	0.0185	-0.2434	0.0184
		4.2515	0.0179	4.5591	0.0184	3.1196	0.0184
		0.9887	0.0251	0.9611	0.0251	0.9608	0.0251
$\alpha = 0.1$	$R = 120$	0.3954	0.0180	-0.0316	0.0180	-0.2411	0.0180
		5.1576	0.0174	3.2745	0.0179	3.3112	0.0180
		0.9993	0.0495	0.9812	0.0496	0.9802	0.0496
$\alpha = 0.3$	$R = 120$	0.3230	0.0352	0.1101	0.0353	-0.1120	0.0353
		3.8168	0.0346	3.1411	0.0352	2.8493	0.0353
		0.9976	0.0567	0.9833	0.0568	0.9830	0.0568
$\alpha = 0.5$	$R = 120$	0.1624	0.0404	0.0707	0.0405	-0.0079	0.0405
		3.1891	0.0398	3.1605	0.0404	3.0643	0.0404
		0.9962	0.0495	0.9802	0.0495	0.9801	0.0496
$\alpha = 0.7$	$R = 120$	0.2441	0.0353	0.0557	0.0353	-0.1704	0.0353
		3.5696	0.0346	3.6204	0.0352	3.0072	0.0353
		0.9929	0.0251	0.9596	0.0251	0.9599	0.0251
$\alpha = 0.9$	$R = 120$	0.2023	0.0180	-0.1157	0.0180	0.0039	0.0180
		3.6305	0.0174	3.7617	0.0179	3.0051	0.0180
		1.0024	0.0248	0.9850	0.0250	0.9793	0.0249
$\alpha = 0.1$	$R = 240$	0.0031	0.0177	0.1489	0.0178	-0.2412	0.0177
		3.5427	0.0172	3.5307	0.0177	3.6428	0.0177
		0.9978	0.0492	0.9921	0.0494	0.9900	0.0493
$\alpha = 0.3$	$R = 240$	0.2418	0.0349	0.0715	0.0350	-0.0708	0.0350
		3.5586	0.0343	2.9847	0.0349	3.4034	0.0350
		0.9980	0.0565	0.9935	0.0566	0.9911	0.0566
$\alpha = 0.5$	$R = 240$	0.2217	0.0400	0.0942	0.0402	-0.0019	0.0401
		3.2844	0.0393	3.1424	0.0401	3.1089	0.0401
		0.9936	0.0492	0.9918	0.0493	0.9895	0.0493
$\alpha = 0.7$	$R = 240$	0.1637	0.0349	0.0338	0.0351	-0.0440	0.0350
		3.2809	0.0342	3.4276	0.0350	3.2063	0.0350
		1.0030	0.0249	0.9816	0.0249	0.9794	0.0249
$\alpha = 0.9$	$R = 240$	0.0713	0.0177	0.0465	0.0178	0.0456	0.0178
		3.6235	0.0171	3.4019	0.0177	3.1834	0.0177

Table 1.15: Results from combining nested quantile models: $c_2 = 1$, $b = 0.1$, $\phi = 0.95$, $\sigma_e = 1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		0.3191	0.1863	0.4358	0.1871	0.4906	0.1870
$\alpha = 0.1$	$R = 60$	0.1013	0.1865	-0.8111	0.1873	-0.4253	0.1871
		20.2249	0.1721	4.1367	0.1854	3.3408	0.1834
		0.4475	0.3676	0.5811	0.3679	0.6270	0.3673
$\alpha = 0.3$	$R = 60$	-1.4400	0.3609	-0.9925	0.3615	-0.5592	0.3606
		14.6568	0.3460	5.7239	0.3561	3.8074	0.3540
		0.4702	0.4212	0.5937	0.4212	0.6426	0.4204
$\alpha = 0.5$	$R = 60$	-1.0812	0.4133	-1.0940	0.4132	-0.5025	0.4122
		7.1388	0.4045	5.4297	0.4077	3.6247	0.4039
		0.4553	0.3678	0.5686	0.3679	0.6276	0.3673
$\alpha = 0.7$	$R = 60$	-1.0678	0.3611	-1.3024	0.3616	-0.5887	0.3607
		8.8736	0.3506	8.5597	0.3582	3.6752	0.3546
		0.3101	0.1872	0.4289	0.1867	0.4864	0.1868
$\alpha = 0.9$	$R = 60$	-0.7424	0.1881	-0.8206	0.1869	-0.4202	0.1870
		8.3002	0.1738	4.8079	0.1762	3.2136	0.1775
		0.6429	0.1854	0.6443	0.1861	0.7021	0.1860
$\alpha = 0.1$	$R = 120$	0.3474	0.1804	-1.2641	0.1809	-0.7369	0.1809
		45.3352	0.1763	7.2125	0.1846	4.3044	0.1806
		0.7646	0.3658	0.7786	0.3668	0.8075	0.3671
$\alpha = 0.3$	$R = 120$	-0.6710	0.3535	-0.8976	0.3538	-0.4481	0.3540
		17.2464	0.3422	7.9984	0.3608	3.4997	0.3605
		0.7735	0.4197	0.7981	0.4209	0.8218	0.4208
$\alpha = 0.5$	$R = 120$	-1.9538	0.4052	-1.1301	0.4055	-0.2875	0.4055
		23.2532	0.3897	8.0592	0.4107	3.2173	0.4112
		0.7507	0.3667	0.7763	0.3674	0.8108	0.3671
$\alpha = 0.7$	$R = 120$	-1.2535	0.3539	-1.0166	0.3541	-0.3810	0.3539
		16.2892	0.3407	6.7857	0.3537	3.4049	0.3587
		0.6093	0.1857	0.6529	0.1864	0.7042	0.1859
$\alpha = 0.9$	$R = 120$	-3.2795	0.1805	-1.5595	0.1810	-0.5966	0.1807
		56.6386	0.1729	10.6889	0.1681	3.9911	0.1868
		0.9155	0.1843	0.8301	0.1851	0.8434	0.1850
$\alpha = 0.1$	$R = 240$	0.1889	0.1772	-0.9115	0.1780	-0.5772	0.1777
		26.1892	0.1690	10.3321	0.1711	4.1165	0.1750
		0.9497	0.3649	0.9131	0.3663	0.9053	0.3658
$\alpha = 0.3$	$R = 240$	0.7793	0.3492	-0.2371	0.3507	-0.4064	0.3501
		20.6424	0.3329	7.4334	0.3396	3.7972	0.3483
		0.9932	0.4185	0.9130	0.4201	0.9154	0.4195
$\alpha = 0.5$	$R = 240$	1.0518	0.4002	-0.2535	0.4024	-0.2882	0.4015
		12.5876	0.3816	6.7039	0.3900	3.6502	0.3975
		0.9484	0.3650	0.9015	0.3665	0.9084	0.3660
$\alpha = 0.7$	$R = 240$	-0.3695	0.3493	-0.2941	0.3511	-0.1141	0.3502
		25.2087	0.3293	7.1005	0.3445	3.2999	0.3452
		0.9893	0.1849	0.8365	0.1852	0.8451	0.1853
$\alpha = 0.9$	$R = 240$	2.2680	0.1772	-0.5299	0.1780	-0.4803	0.1779
		37.4155	0.1685	10.9022	0.1744	3.7784	0.1774

Table 1.16: Results from combining nested quantile models: $c_2 = 1$, $b = 0.1$, $\phi = 0.95$, $\sigma_e = 0.1$

		$P = 48$		$P = 240$		$P = 1200$	
$Repeat = 2000$		λ	Average Loss	λ	Average Loss	λ	Average Loss
		0.9857	0.0537	0.9799	0.0538	0.9795	0.0537
$\alpha = 0.1$	$R = 60$	0.0162	0.0187	0.0800	0.0187	-0.0818	0.0187
		5.8984	0.0173	3.9811	0.0183	3.0333	0.0192
		0.9922	0.1089	0.9924	0.1092	0.9919	0.1089
$\alpha = 0.3$	$R = 60$	-0.1473	0.0361	0.0134	0.0360	-0.0380	0.0361
		6.3464	0.0346	3.4933	0.0360	3.0936	0.0370
		0.9962	0.1245	0.9937	0.1253	0.9943	0.1258
$\alpha = 0.5$	$R = 60$	0.1334	0.0413	-0.1378	0.0413	-0.0613	0.0412
		4.9985	0.0410	3.3931	0.0413	3.0224	0.0407
		0.9951	0.1086	0.9921	0.1091	0.9923	0.1094
$\alpha = 0.7$	$R = 60$	0.2641	0.0361	-0.0504	0.0362	-0.0612	0.0361
		5.7399	0.0357	3.3261	0.0358	3.0781	0.0358
		0.9845	0.0537	0.9803	0.0538	0.9796	0.0538
$\alpha = 0.9$	$R = 60$	0.2314	0.0187	-0.0595	0.0187	-0.0134	0.0187
		6.5209	0.0182	3.2654	0.0184	3.1141	0.0185
		0.9967	0.0579	0.9910	0.0573	0.9907	0.0573
$\alpha = 0.1$	$R = 120$	-0.1105	0.0181	-0.0345	0.0181	-0.0883	0.0181
		6.2782	0.0173	3.6637	0.0173	3.1405	0.0187
		0.9951	0.1160	0.9962	0.1153	0.9967	0.1151
$\alpha = 0.3$	$R = 120$	0.0049	0.0353	0.0729	0.0354	0.0154	0.0354
		5.4411	0.0354	4.4169	0.0339	2.9469	0.0363
		0.9979	0.1305	0.9977	0.1318	0.9976	0.1321
$\alpha = 0.5$	$R = 120$	-0.0467	0.0405	-0.0590	0.0405	-0.0486	0.0405
		4.7017	0.0390	3.6670	0.0410	3.1718	0.0411
		0.9974	0.1134	0.9967	0.1149	0.9972	0.1149
$\alpha = 0.7$	$R = 120$	0.0362	0.0354	0.0306	0.0354	0.0316	0.0354
		5.3566	0.0340	3.7280	0.0355	3.1202	0.0358
		0.9966	0.0564	0.9903	0.0571	0.9911	0.0571
$\alpha = 0.9$	$R = 120$	0.1531	0.0181	-0.0662	0.0181	-0.1930	0.0181
		4.9594	0.0173	3.6470	0.0181	3.1298	0.0184
		1.0029	0.0587	0.9964	0.0586	0.9959	0.0587
$\alpha = 0.1$	$R = 240$	-0.1000	0.0178	-0.1474	0.0178	-0.1580	0.0178
		4.8734	0.0173	4.1090	0.0176	3.2096	0.0176
		1.0022	0.1167	0.9990	0.1165	0.9986	0.1166
$\alpha = 0.3$	$R = 240$	0.1793	0.0350	0.0467	0.0351	-0.0429	0.0350
		4.7544	0.0352	3.7011	0.0350	3.1915	0.0348
		0.9989	0.1342	0.9982	0.1335	0.9991	0.1334
$\alpha = 0.5$	$R = 240$	0.1639	0.0400	-0.0367	0.0402	-0.0048	0.0401
		4.5769	0.0380	3.6447	0.0390	2.9946	0.0398
		0.9992	0.1169	0.9983	0.1163	0.9986	0.1162
$\alpha = 0.7$	$R = 240$	0.1298	0.0349	0.0511	0.0351	0.0459	0.0350
		5.2992	0.0329	3.7471	0.0345	2.9975	0.0346
		0.9999	0.0588	0.9956	0.0584	0.9961	0.0584
$\alpha = 0.9$	$R = 240$	-0.0797	0.0177	0.0058	0.0178	-0.0948	0.0178
		7.7455	0.0165	4.3853	0.0176	3.0710	0.0179

Table 1.17: Model combination result from Goyal-Welch Study

	$R:P=25\%:75\%$				$R:P=50\%:50\%$				$R:P=75\%:25\%$			
	DP		DY		DP		DY		DP		DY	
	λ	Loss	λ	Loss	λ	Loss	λ	Loss	λ	Loss	λ	Loss
		0.0105[b]		0.0105		0.0124		0.0124		0.0094		0.0094
$\alpha=0.1$	0.6512[a]	0.0101[c]	0.7146	0.0101	0.3022	0.0130	0.3317	0.0129	1.0335	0.0092	1.5594	0.0092
		0.0098[d]		0.0098		0.0122		0.0122		0.0092		0.0093
		0.0184		0.0184		0.0223		0.0223		0.0170		0.0170
$\alpha=0.3$	0.7670	0.0176	0.7530	0.0176	0.4199	0.0226	0.4574	0.0226	-0.3839	0.0173	-0.6411	0.0174
		0.0175		0.0175		0.0222		0.0222		0.0170		0.0170
		0.0202		0.0202		0.0242		0.0242		0.0180		0.0180
$\alpha=0.5$	0.8360	0.0190	0.8161	0.0191	0.6019	0.0241	0.5969	0.0241	0.0256	0.0190	0.0272	0.0190
		0.0190		0.0190		0.0238		0.0238		0.0180		0.0180
		0.0169		0.0169		0.0199		0.0199		0.0155		0.0155
$\alpha=0.7$	0.9931	0.0160	0.9516	0.0161	0.9416	0.0192	0.8801	0.0192	0.0343	0.0171	0.0382	0.0171
		0.0160		0.0160		0.0192		0.0192		0.0155		0.0155
		0.0085		0.0085		0.0102		0.0102		0.0089		0.0089
$\alpha=0.9$	0.7291	0.0083	0.6724	0.0084	1.0933	0.0096	1.1256	0.0096	0.4101	0.0098	0.4759	0.0097
		0.0083		0.0083		0.0096		0.0096		0.0084		0.0084

[a] The optimal weight on the second model. $\hat{\lambda}$.
[b] The average of the check loss from forecasting of the first model $\rho_\alpha^{(1)}$.
[c] The average of the check loss from forecasting of the second model $\rho_\alpha^{(2)}$.
[d] The average of the check loss from forecasting of the combined model $\rho_\alpha^{(c)}$.

Table 1.18: Bootstrap result by using two-part continuous weight distribution with $k = 2$

		$R:P = 25\% : 75\%$				$R:P = 50\% : 50\%$				$R:P = 75\% : 25\%$			
	α	DP		DY		DP		DY		DP		DY	
$\tilde{\lambda}, \tilde{\sigma}$		0.641[a]	0.123[b]	0.704	0.127	0.316	0.107	0.338	0.118	1.348	0.446	1.929	0.541
90% CI	0.1	0.437[c]	0.820 [d]	0.498	0.888	0.110	0.483	0.121	0.523	0.443	1.925	0.870	2.680
95% CI		0.405[e]	0.842 [f]	0.458	0.911	0.053	0.517	0.072	0.557	0.241	1.992	0.688	2.797
$\tilde{\lambda}, \tilde{\sigma}$		0.767	0.091	0.753	0.091	0.290	0.153	0.338	0.150	0.513	0.630	0.231	0.554
90% CI	0.3	0.616	0.091	0.598	0.896	0.031	0.530	0.082	0.577	-0.525	1.508	-0.641	1.165
95% CI		0.587	0.937	0.566	0.921	-0.012	0.567	0.032	0.612	-0.706	1.668	-0.824	1.332
$\tilde{\lambda}, \tilde{\sigma}$		0.825	0.097	0.803	0.096	0.582	0.160	0.579	0.159	-0.081	0.192	-0.076	0.193
90% CI	0.5	0.666	0.985	0.639	0.963	0.324	0.857	0.322	0.850	-0.420	0.231	-0.421	0.238
95% CI		0.633	1.013	0.615	0.992	0.287	0.905	0.281	0.898	-0.475	0.281	-0.472	0.285
$\tilde{\lambda}, \tilde{\sigma}$		1.020	0.122	0.982	0.128	1.020	0.180	0.954	0.190	0.059	0.109	0.064	0.111
90% CI	0.7	0.818	1.232	0.770	1.196	0.722	1.310	0.639	1.258	-0.116	0.239	-0.112	0.249
95% CI		0.775	1.277	0.730	1.240	0.651	1.376	0.574	1.324	-0.207	0.291	-0.194	0.293
$\tilde{\lambda}, \tilde{\sigma}$		0.741	0.194	0.685	0.174	1.123	0.189	1.152	0.192	0.482	0.096	0.550	0.091
90% CI	0.9	0.445	1.050	0.421	0.972	0.804	1.420	0.815	1.440	0.306	0.623	0.406	0.684
95% CI		0.402	1.100	0.376	1.033	0.733	1.470	0.744	1.485	0.257	0.646	0.326	0.711

[a] Bias-corrected estimator of λ.
[b] Standard deviation of λ.
[c] Lower bound of 90% Confidence Interval of λ.
[d] Upper bound of 90% Confidence Interval of λ.
[e] Lower bound of 95% Confidence Interval of λ.
[f] Upper bound of 90% Confidence Interval of λ.

Table 1.19: Bootstrap result by using two-part continuous weight distribution with $k = 4$

		$R : P = 25\% : 75\%$[5]				$R : P = 50\% : 50\%$				$R : P = 75\% : 25\%$			
	α	DP		DY		DP		DY		DP		DY	
$\tilde{\lambda}, \tilde{\sigma}$		0.642	0.110	0.709	0.113	0.304	0.113	0.324	0.123	1.018	0.465	1.566	0.535
90% CI	0.1	0.457	0.808	0.512	0.878	0.084	0.457	0.095	0.496	0.117	1.629	0.587	2.275
95% CI		0.422	0.828	0.476	0.900	0.037	0.488	0.050	0.534	-0.024	1.697	0.415	2.340
$\tilde{\lambda}, \tilde{\sigma}$		0.736	0.092	0.719	0.091	0.419	0.150	0.459	0.148	-0.755	0.572	-1.013	0.521
90% CI	0.3	0.586	0.885	0.569	0.872	0.171	0.666	0.210	0.705	-1.581	0.250	-1.785	-0.069
95% CI		0.552	0.917	0.538	0.900	0.127	0.698	0.158	0.747	-1.716	0.600	-1.908	0.202
$\tilde{\lambda}, \tilde{\sigma}$		0.855	0.094	0.834	0.091	0.611	0.164	0.602	0.162	-0.064	0.190	-0.062	0.189
90% CI	0.5	0.692	1.005	0.679	0.987	0.331	0.868	0.327	0.860	-0.387	0.235	-0.372	0.236
95% CI		0.659	1.034	0.646	1.013	0.283	0.911	0.280	0.904	-0.459	0.271	-0.448	0.277
$\tilde{\lambda}, \tilde{\sigma}$		1.020	0.129	0.982	0.136	0.928	0.185	0.871	0.193	0.033	0.167	0.036	0.172
90% CI	0.7	0.802	1.234	0.745	1.196	0.602	1.215	0.542	1.184	-0.198	0.329	-0.206	0.340
95% CI		0.769	1.270	0.705	1.239	0.523	1.275	0.459	1.252	-0.243	0.416	-0.248	0.423
$\tilde{\lambda}, \tilde{\sigma}$		0.767	0.220	0.713	0.199	1.145	0.157	1.172	0.162	0.471	0.112	0.542	0.100
90% CI	0.9	0.415	1.101	0.390	1.029	0.861	1.376	0.870	1.404	0.268	0.628	0.367	0.687
95% CI		0.377	1.156	0.353	1.088	0.790	1.421	0.796	1.448	0.212	0.663	0.294	0.716

Table 1.20: Bootstrap result by using two point mass weight distribution

		$R:P=25\%:75\%$				$R:P=50\%:50\%$				$R:P=75\%:25\%$			
	α	DP		DY		DP		DY		DP		DY	
$\tilde{\lambda}, \tilde{\sigma}$		0.642	0.109	0.706	0.117	0.316	0.115	0.341	0.127	1.173	0.412	1.688	0.539
90% CI	0.1	0.452	0.800	0.506	0.872	0.092	0.500	0.113	0.543	0.323	1.660	0.679	2.376
95% CI		0.424	0.821	0.468	0.892	0.048	0.522	0.058	0.569	0.160	1.715	0.505	2.434
$\tilde{\lambda}, \tilde{\sigma}$		0.770	0.089	0.757	0.089	0.390	0.149	0.436	0.145	-0.171	0.666	-0.510	0.586
90% CI	0.3	0.616	0.911	0.601	0.900	0.130	0.617	0.171	0.651	-1.301	0.923	-1.400	0.546
95% CI		0.585	0.938	0.570	0.927	0.075	0.672	0.117	0.716	-1.473	1.088	-1.619	0.689
$\tilde{\lambda}, \tilde{\sigma}$		0.846	0.096	0.826	0.094	0.444	0.147	0.445	0.149	-0.049	0.214	-0.040	0.208
90% CI	0.5	0.682	1.004	0.668	0.981	0.210	0.709	0.207	0.695	-0.457	0.244	-0.428	0.252
95% CI		0.655	1.038	0.636	1.016	0.176	0.768	0.166	0.749	-0.540	0.288	-0.505	0.290
$\tilde{\lambda}, \tilde{\sigma}$		0.984	0.126	0.941	0.133	0.987	0.183	0.935	0.195	-0.099	0.140	-0.098	0.143
90% CI	0.7	0.777	1.193	0.726	1.158	0.677	1.289	0.599	1.243	-0.323	0.143	-0.329	0.153
95% CI		0.738	1.234	0.675	1.199	0.595	1.360	0.523	1.307	-0.365	0.171	-0.372	0.181
$\tilde{\lambda}, \tilde{\sigma}$		0.748	0.201	0.690	0.182	1.119	0.160	1.154	0.163	0.537	0.087	0.602	0.087
90% CI	0.9	0.433	1.067	0.410	0.994	0.848	1.369	0.860	1.403	0.399	0.652	0.461	0.719
95% CI		0.393	1.123	0.370	1.051	0.773	1.409	0.786	1.448	0.346	0.665	0.425	0.734

Table 1.21: t statistic of $\hat{\lambda}$ using bootstrap

	$R:P=25\%:75\%$		$R:P=50\%:50\%$		$R:P=75\%:25\%$	
	DP	DY	DP	DY	DP	DY
$\alpha=0.1$	5.974	6.108	2.628	2.612	2.508	2.893
$\alpha=0.3$	8.618	8.461	2.818	3.154	-0.576	-1.094
$\alpha=0.5$	8.708	8.682	4.095	4.006	0.120	0.131
$\alpha=0.7$	7.882	7.155	5.145	4.513	0.245	0.267
$\alpha=0.9$	3.627	3.695	6.833	6.906	4.714	5.470

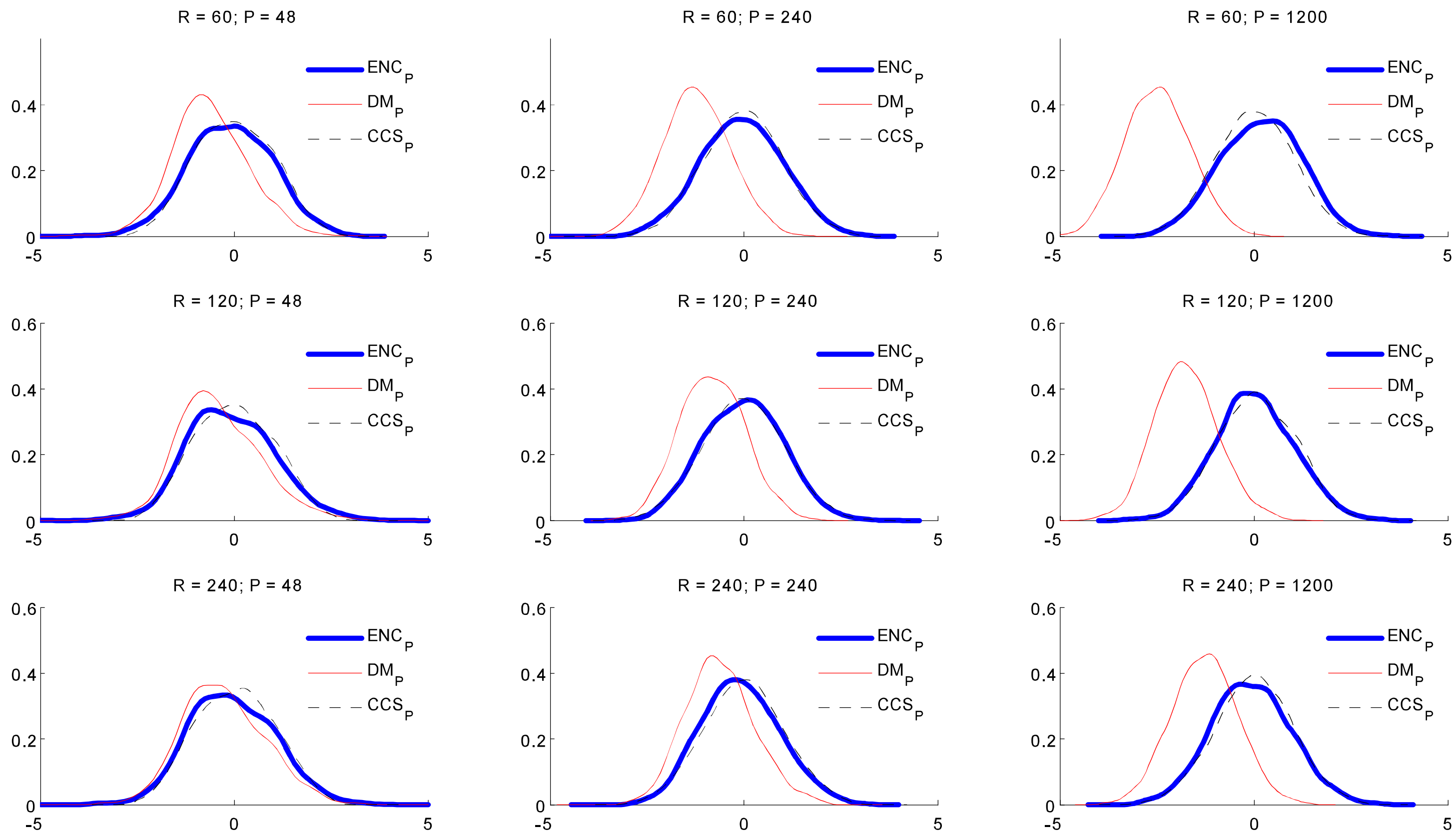

Figure 1.1: The distributions of three statistics, $\phi = 0$, $\alpha = 0.1$, $\sigma_e = 1$. 2000 Repeats. Notes: The solid lines on the left of the nine figures show the asymptotic distribution of DM_P statistics, the dotted lines of the figures show the asymptotic distribution of CCS_P statistics, the solid lines lying on the center or the right of each figures show the asymptotic distribution of ENC_P statistics.

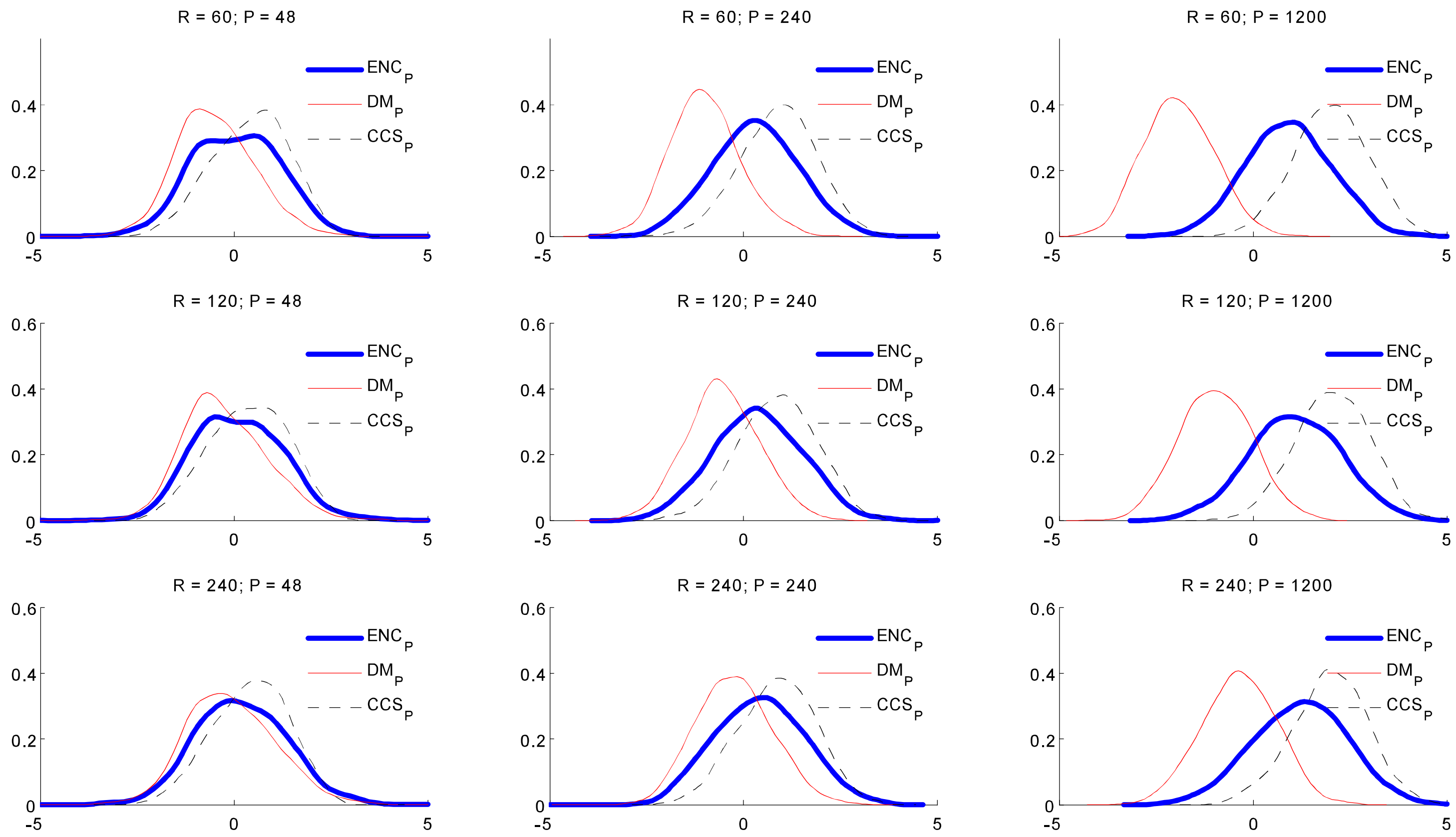

Figure 1.2: The distributions of three statistics under 10% nominal size (Low Signal-Noise Ratio), $\phi = 0, b = 0.1, \sigma_e = 1, \alpha = 0.1$. 2000 Repeats. Notes: The solid lines on the left of the nine figures show the asymptotic distribution of DM_P statistics, the dotted lines of the figures show the asymptotic distribution of CCS_P statistics, the solid lines lying on the center or the right of each figures show the asymptotic distribution of ENC_P statistics.

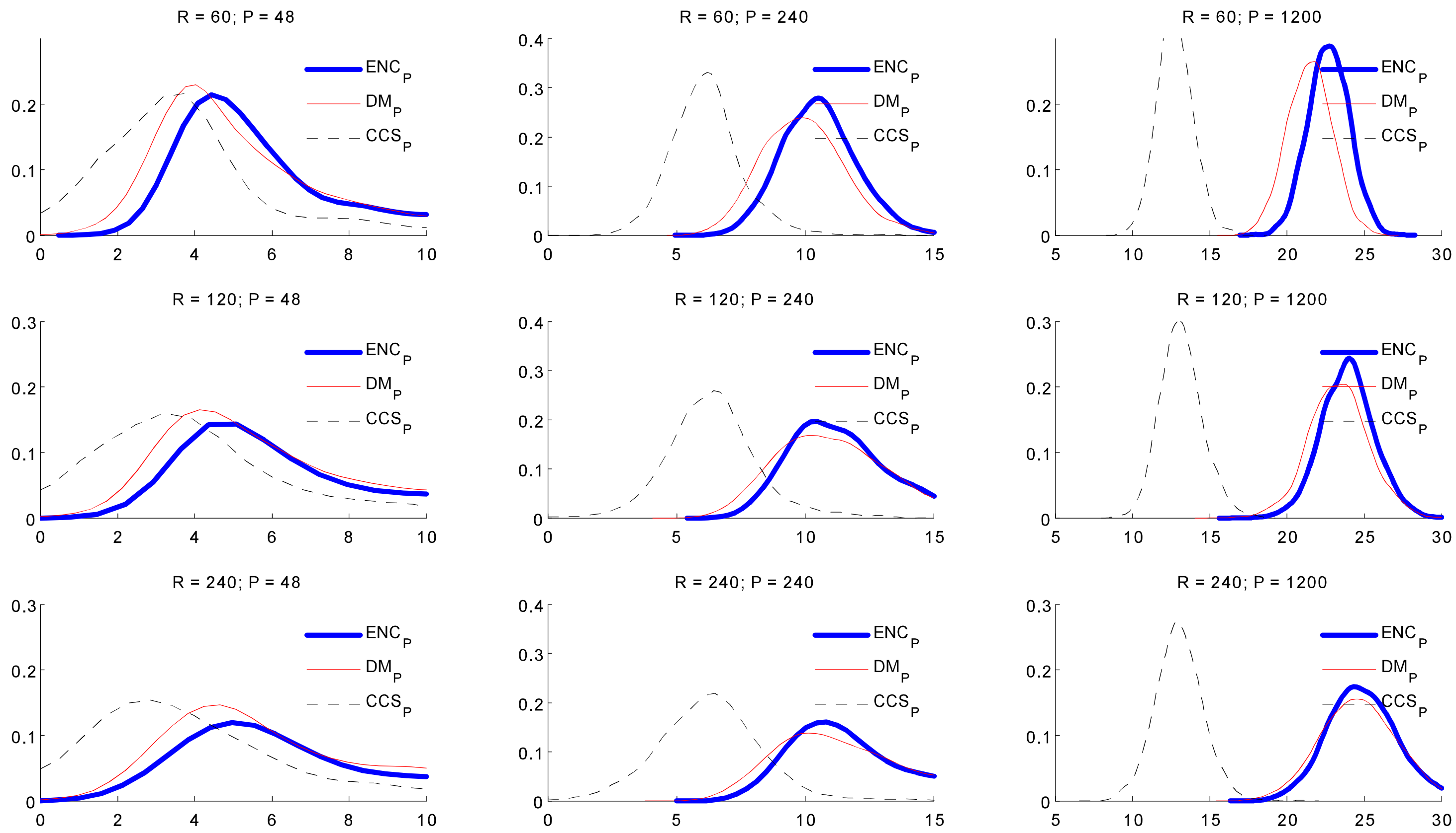

Figure 1.3: The distributions of three statistics (High Signal-Noise Ratio), $\phi = 0.95, b = 0.1, \sigma_e = 0.1, \alpha = 0.1$, 2000 Repeats. The dotted lines on the left of the nine figures show the asymptotic distribution of CCS_P statistics, the solid lines in the middle of each figures show the asymptotic distribution of DM_P statistics, the solid lines lying on the right of each figures show the asymptotic distribution of ENC_P statistics.

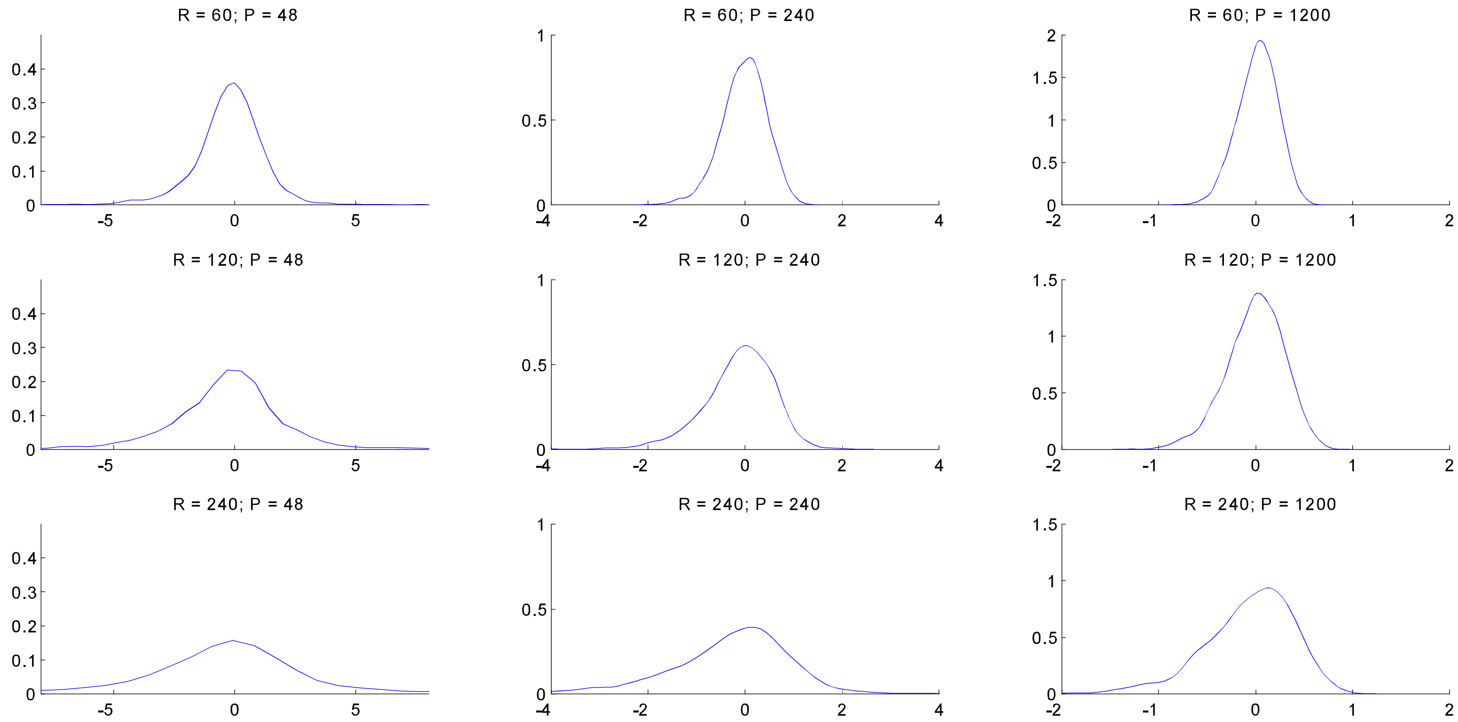

Figure 1.4: Distribution of combination weight λ, $b = 0, \phi = 0, \sigma_e = 1, \alpha = 0.1$

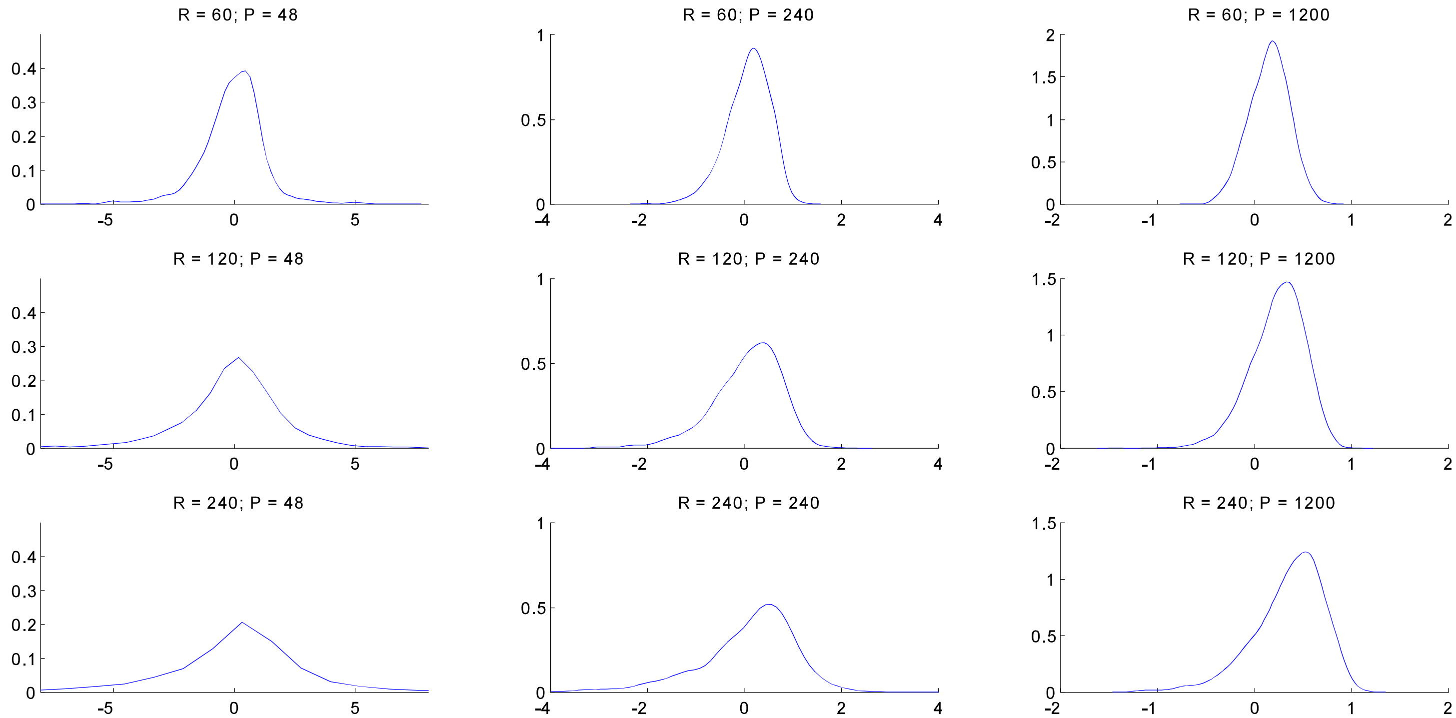

Figure 1.5: Distribution of combination weight λ, $b = 0.1, \phi = 0, \sigma_e = 1, \alpha = 0.1$ (LOW SNR)

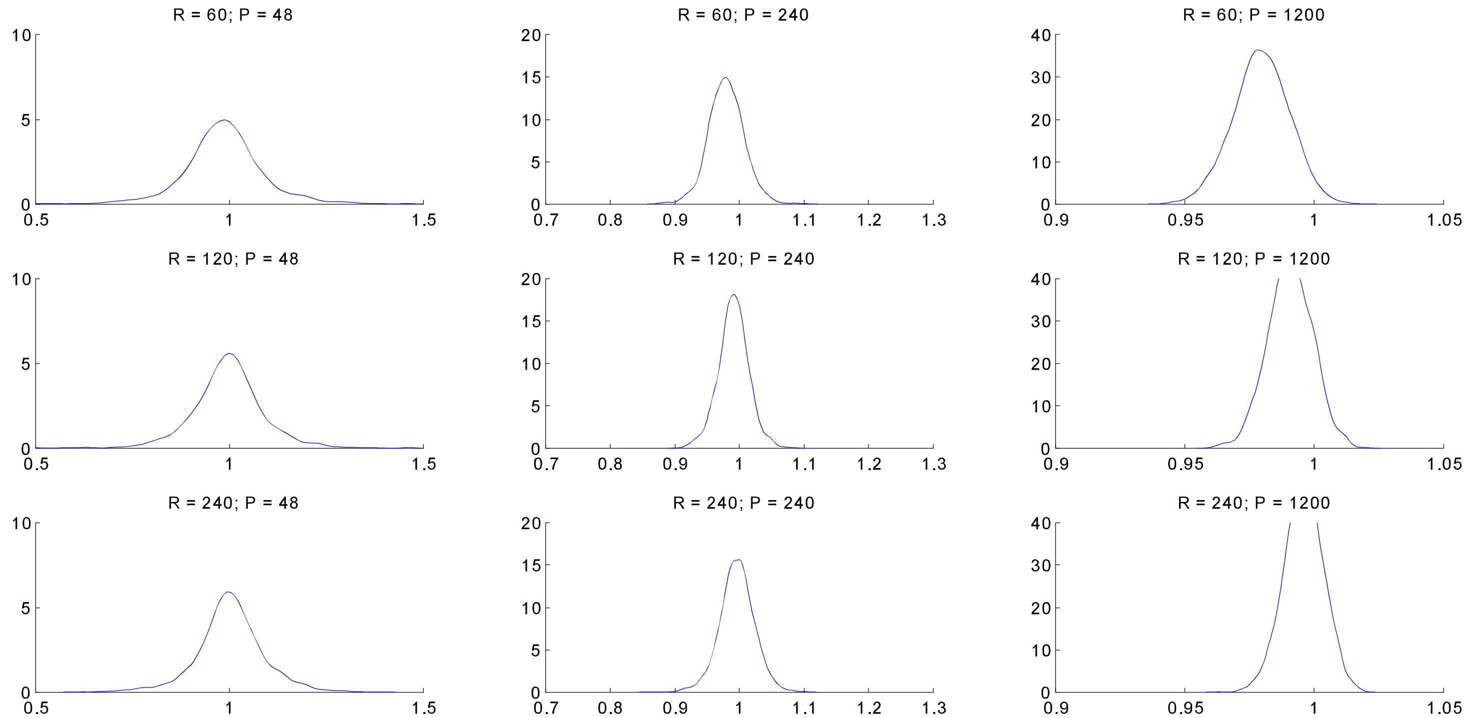

Figure 1.6: Distribution of combination weight λ, $b = 0.1, \phi = 0.95, \sigma_e = 0.1, \alpha = 0.1$ (HIGH SNR)

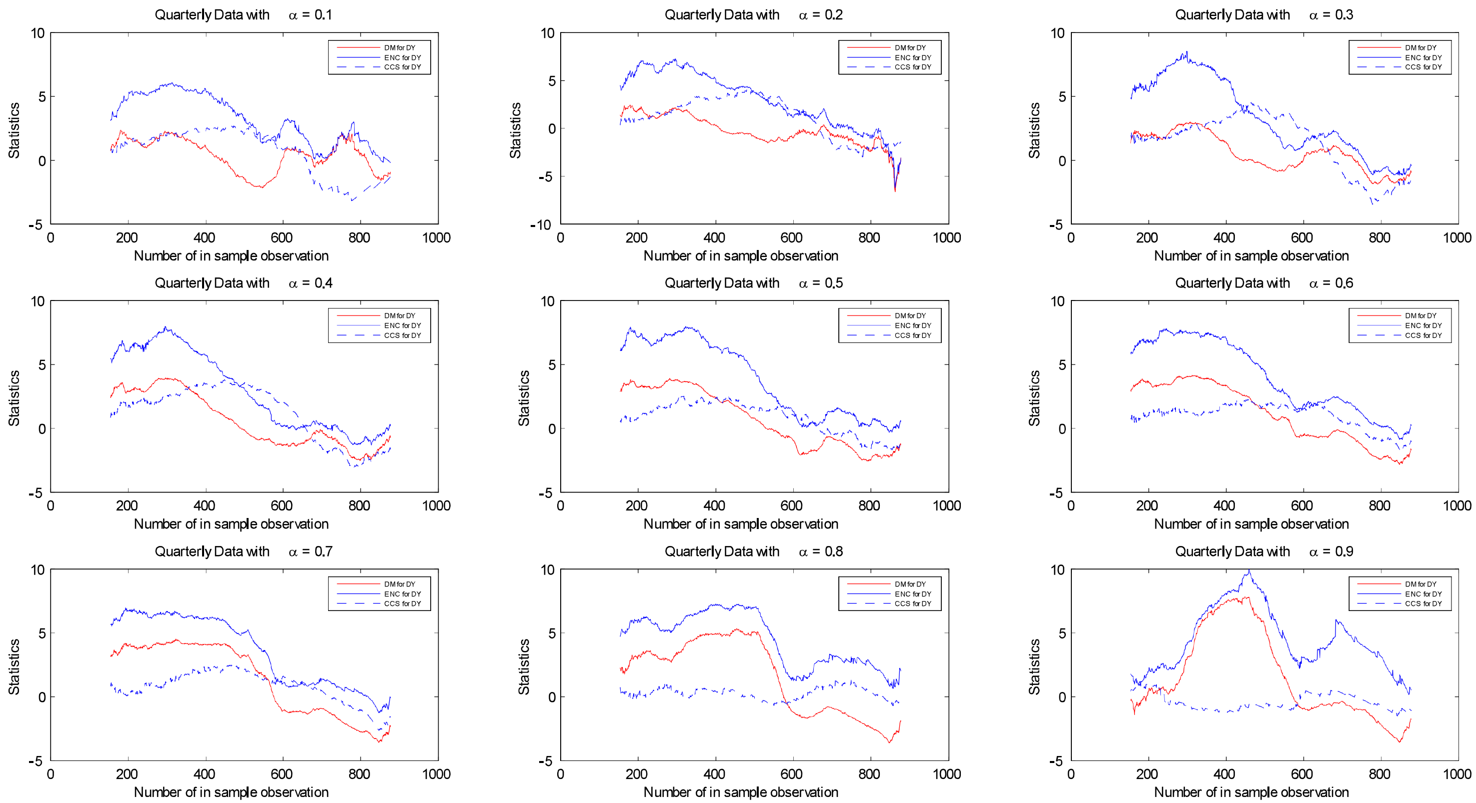

Figure 1.7: Predictive quantile regression of Equity Premium on Dividend-Yield ratio

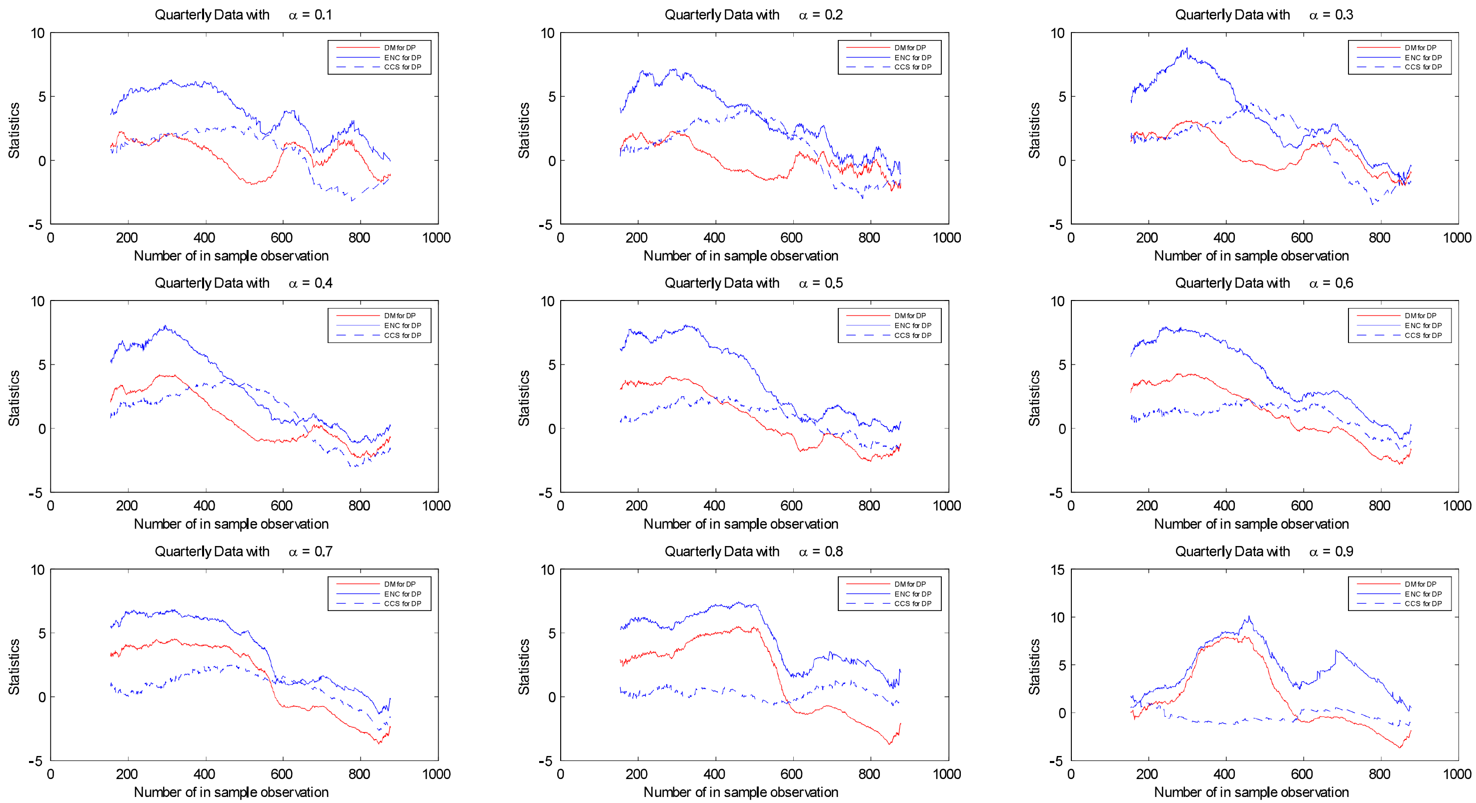

Figure 1.8: Predictive quantile regression of Equity Premium on Dividend-Price ratio

Chapter 2

Comparing Nested Predictive Regression Models with Persistent Predictors*

Yan Ge[†] and Tae-Hwy Lee[‡]

March 2015

Abstract

While Clark and McCracken (CM 2001, 2005, 2009) and Clark and West (CW 2006, 2007) assume that a predictor is weakly stationary, this paper considers a highly persistent predictor with an autoregressive (AR) root local to unity. CW find that the statistic of Diebold-Mariano (1995 DM) is under-sized because the DM statistic tends to be negative under the null hypothesis of the equal predictive ability. We find this under-sized problem in DM becomes even more severe when the predictor is persistent with the AR root closer to unity. We find, however, this problem does not exist in the encompassing (ENC) statistic of CM (2001) even when a predictor follows the Ornstein-Uhlenbeck process. This means that ENC remains correct in size and high in power. The size of ENC is robust to near-unit-root persistence in the predictor. We also examine the statistic of Chao, Corradi, and Swanson (2001 CCS) in comparison with DM and ENC. It is shown that CCS is not robust to the

*We thank Aman Ullah for suggesting to consider persistent predictors for the encompassing test.

[†]Department of Economics, University of California, Riverside, CA 92521. E-mail: yge001@ucr.edu

[‡]Department of Economics, University of California, Riverside, CA 92521. E-mail: taelee@ucr.edu

persistent predictor as it becomes severely under-sized when the predictor becomes persistent (even if it has correct size with a weakly stationary predictor) and it loses the power substantially when the predictor is persistent. An application to the predictive regression of the equity premium reveals strong predictive ability of several persistent predictors (such as inflation and interest rate) by ENC, but with little or none from DM and CCS.

Key Words: predictive regression, local to unit root process, Ornstein-Uhlenbeck process, encompassing, size and power, equity premium.

JEL Classification: C53, E37, E27

1 Introduction

When two nonnested models are compared, Diebold and Mariano (DM 1995) point out that the t-statistic of the mean squared forecast error (MSPE) loss-differential is asymptotically standard normal. However, when two nested models in which a weakly stationary predictor is added in Model 2 are compared, Clark and McCracken (CM 2001, 2005, 2009) point out that the t-statistic of DM behaves quite differently from non-nested case since both the numerator and denominator degenerate, which may result in non-standard normal distribution. They point out that due to the finite sample parameter estimation error (PEE), the DM statistic tends to be negative under the null hypothesis of the equal predictive ability. They correct the negative bias by adding a positive term and propose a test that is equivalent to encompassing (ENC thereafter) test of Nelson (1972) and Harvey, Leybourne and Newbold (1998) and the t-statistic. Under the null hypothesis in which the stationary covariate has no predictive power, CM (2001) show that the test is asymptotically standard normal when the in-sample number of observations (R) to the out-of-sample number of forecasts (P) goes to infinity $(R/P \to \infty)$. In this paper we show that under the null hypothesis, the ENC test restores the asymptotically standard normal when the ratio of the out-of-sample number of forecasts (P) to the in-sample number of observations (R) goes to infinity $(P/R \to \infty)$. Under the null hypothesis, the encompassing test also has good power.

CM (2001, 2005, 2009) and Clark and West (CW 2006, 2007) considered predictive mean regression with weak stationary predictor. This paper considers the predictive mean regression with a highly persistent predictor with an AR root local to unity. We compare two nested regression models using the squared-loss function. We show that DM statistic still tends to be negative under the null hypothesis of the equal predictive ability and is more severely undersized if the predictor is a highly persistent predictor. The t-statistic of encompassing test, in which a positive term is added to correct the negative bias of DM, is a robust test and has the correct size under the null hypothesis. We analytically show that the robustness arises from the super consistency property of the additional predictor from Model 2 that follows Ornstein–Uhlenbeck process, thus the convergent rate of the forecast error from Model 2 is faster than that from Model 1 and the asymptotic distribution of ENC statistic has the same asymptotic distribution as shown in CW (2006, 2007). We use Monte Carlo simulation to compare three different statistics and show that when the highly persistent estimator is added in Model 2, the ENC statistic is robust and has the correct size, whereas both DM test and conditional moment test (CCS test) are seriously undersized. An application to the predictive regression of the equity premium reveals strong predictive ability of several persistent predictors (such as inflation and interest rate) by ENC, but with little or none can be seen from DM or CCS.

The paper is organized as follows. Section 2 illustrates the methods of testing out-of-sample Granger-causality in mean using rolling scheme. Section 3 illustrates the asymptotic distribution of the encompassing test with a weak stationary predictor from CM (2001). Section 4 presents the asymptotic distribution of the encompassing test with a weak stationary predictor when the ratio of out-of-sample to in-sample observation is infinite. Section 5 presents the asymptotic distribution of encompassing test with a highly persistent estimator when the ratio of out-of-sample to in-sample observation is infinite. Section 6 is Monte Carlo simulation to examine the finite sample size and power behavior of the DM, ENC and CCS statistics. In Section 7 we present the empirical analysis for Goyal and Welch (2008) in comparing the two nested mean models. Section 8 concludes.

2 Comparing Nested Conditional Mean Models

To test for the out-of-sample predictive ability of x_t for y_{t+1}, we consider the following two nested models with the predictor x_t in Model 2 being local to unit root process:

$$\text{Model 1} \quad : \quad y_{t+1} = x_{1,t}'\beta_{1,t} + e_{t+1}^{(1)} = c_1 + e_{t+1}^{(1)}, \tag{1}$$

$$\text{Model 2} \quad : \quad y_{t+1} = x_{2,t}'\beta_{2,t} + e_{t+1}^{(2)} = c_2 + bx_t + e_{t+1}^{(2)}, \tag{2}$$

where c_i is the constant term for Model i, x_t is the predictor with local to unit autoregressive (AR) root process $x_{t+1} = \phi x_t + v_{t+1}$. We will consider the simple case when $x_{1,t}' = 1$ and $x_{2,t}' = (1\ x_t)'$. Under the null hypothesis, $b = 0$ and $e_{t+1}^{(1)} = e_{t+1}^{(2)}$, denoted as e_{t+1}. At each time t, both c_i and b are estimated with the rolling window of size R up to time t. Therefore

$$\begin{aligned} \hat{c}_{1,t} - c_1 &= B_1(t) H_1(t) \\ \left(\hat{c}_{2,t}, \hat{b}_t\right)' - (c_2, b)' &= B_2(t) H_2(t) \end{aligned}$$

where

$$x'_{1,t} = 1, x'_{2,t} = (1\ x_t)', q_{i,t} = x'_{i,t}x_{i,t}$$

for Model i at time t, and $B_i(t) = \left(R^{-1}\sum_{j=t-R}^{t-1} q_{i,j}\right)^{-1}$, $h_{i,t} = x'_{i,t}e_{t+1}$ and $H_i(t) = R^{-1}\sum_{j=t-R}^{t-1} h_{i,t}$. Let $f^{(1)}_{t+1} = \hat{c}_{1,t}$ be the forecasts for Model 1 and $f^{(2)}_{t+1} = \hat{c}_{2,t} + \hat{b}_t x_t$ be the forecast for Model 2 at time t and $\hat{e}^{(1)}_{t+1} = y_{t+1} - f^{(1)}_{t+1}$, $\hat{e}^{(2)}_{t+1} = y_{t+1} - f^{(2)}_{t+1}$ be the forecast errors with the squared forecast-error loss

$$L\left(\hat{e}^{(i)}_{t+1}\right) \equiv \left(\hat{e}^{(i)}_{t+1}\right)^2, \qquad i = 1, 2.$$

To test for equal predictive accuracy of the two models, the null hypothesis is

$$\mathbb{H}_0 : \mathbb{E}\left[L\left(\hat{e}^{(1)}_{t+1}\right) - L\left(\hat{e}^{(2)}_{t+1}\right)\right] = 0. \tag{3}$$

Under $\mathbb{H}_0$, x_t does not Granger-cause y_{t+1} in mean and thus $b = 0$. If x_t Granger-causes y_{t+1}, i.e., $b \neq 0$, thus the alternative hypothesis is

$$\mathbb{H}_1 : \mathbb{E}\left[L\left(\hat{e}^{(1)}_{t+1}\right) - L\left(\hat{e}^{(2)}_{t+1}\right)\right] > 0. \tag{4}$$

The Diebold-Mariano square loss differential is defined as

$$\hat{D}_P = P^{-1}\sum_{t=R}^{T} L\left(\hat{e}^{(1)}_{t+1}\right) - L\left(\hat{e}^{(2)}_{t+1}\right), \tag{5}$$

and the adjusted MSFE loss-differential is defined as

$$\hat{B}_P = P^{-1}\sum_{t=R}^{T} \hat{e}^{(1)}_{t+1}\left(\hat{e}^{(1)}_{t+1} - \hat{e}^{(2)}_{t+1}\right),$$

where R is the number of observations in the rolling windows for the in-sample estimation, P is the number of out-of-sample forecasts, and $R + P = T + 1$. We also compare DM and ENC with CCS test by Chao et al (CCS 2001), which is constructed as follows: Under $\mathbb{H}_0$, $b = 0$, which implies $\mathbb{E}\left(e^{(1)}_{t+1}x_t\right) = 0$. The out-of-sample test statistic of CCS is constructed from

$$\hat{M}_P = P^{-1}\sum_{t=R}^{T} \hat{e}^{(1)}_{t+1}x_t. \tag{6}$$

The three statistics are standardized to form the DM statistic $DM_P \equiv \hat{S}_P^{-0.5}\sqrt{P}\hat{D}_P$, the encompassing statistic $ENC_P \equiv \hat{Q}_P^{-0.5}\sqrt{P}\hat{B}_P$, and the CCS statistic $CCS_P \equiv \hat{W}_P^{-0.5}\sqrt{P}\hat{M}_P$, where $\hat{S}_P$, $\hat{Q}_P$ and $\hat{W}_P$ are the consistent estimators of $S_P = var\left(\sqrt{P}\hat{D}_P\right)$, $Q_P = var\left(\sqrt{P}\hat{B}_P\right)$, and $W_P = var\left(\sqrt{P}\hat{M}_P\right)$, respectively.

3 Asymptotic Distribution of ENC with a Stationary Predictor (CM 2001)

First, we consider a stationary predictor as in CM (2001, 2005), CW (2006, 2007), and CCS (2001).

Assumption 1a. $\{x_t\}$ is a weakly stationary process and $\mathbb{E}\left(q_{i,t}\right)$ is bounded for all t and $i = 1,2$. We define $B_i = \left(\mathbb{E}q_{i,t}\right)^{-1}$ for model $i = 1,2$.

Let $\pi = \lim_{P,R\to\infty} P/R$ and $\xi = R/T = R/\left(P+R\right)$. Note that $1/\xi - 1 \to \pi$. We consider three cases on π :

Assumption 2a. $0 < \pi < \infty$.

Assumption 2b. $\pi = 0$ (or $\xi \to 1$).

Assumption 2c. $\pi = \infty$ (or $\xi \to 0$).

Proposition 1 (CM 2001). Under Assumption 1a and Assumption 2a,

$$ENC_P \Rightarrow \frac{\int_{\xi}^{1} \xi^{-1}\left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1} \xi^{-2}\left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}},$$

under $\mathbb{H}_0$, where $W(s)$ is a Wiener process and $s \in [0,1]$. When Assumption 2a holds, the RHS of Equation (7) is *not* standard normal.

Proposition 2 (CM 2001). Under Assumption 1a and Assumption 2b,

$$ENC_P \Rightarrow \lim_{\xi\to 1} \frac{\int_{\xi}^{1} \xi^{-1}\left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1} \xi^{-2}\left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \sim N\left(0,1\right), \tag{7}$$

under $\mathbb{H}_0$, where $W(s)$ is a Wiener process and $s \in [0,1]$. When Assumption 2b holds, the RHS of Equation (7) is standard normal.

Remark 1. CM (2001) shows that when Assumption 2b holds ($\xi \to 1, \pi \to 0$) then ENC_P is asymptotically standard normal. However, CM (2001) does not consider the case when Assumption 2c holds ($\xi \to 0, \pi \to \infty$). In Section 4 below, we consider this case and show that ENC_P is still asymptotically standard normal.

Remark 2: CM (2001) assumes Assumption 1a that the predictor $\{x_t\}$ is weakly stationary and shows that ENC_P is asymptotically standard normal under Assumption 2b. In Section 5, we show that ENC_P is asymptotically standard normal under Assumption 2c when the predictor has a root local to unity.

4 Asymptotic Distribution of ENC with a Stationary Predictor when $P/R \to \infty$

In this section, we will show that the asymptotic distribution of ENC_P under $\mathbb{H}_0$, as shown in Equation (7), is asymptotically standard normal under Assumption 2c (when $\lim_{P,R\to\infty} P/R \to \infty$).

Proposition 3. Under Assumption 1a and Assumption 2c,

$$ENC_P \Rightarrow \lim_{\xi \to 0} \frac{\int_{\xi}^{1} \xi^{-1} \left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1} \xi^{-2} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \sim N(0,1), \quad (8)$$

under $\mathbb{H}_0$.

Proof: We firstly consider the numerator of equation (8) by dividing $[0,1]$ to n equal segments and let $t = [Ts]$, where $[Ts]$ is the integer part of Ts and $s \in [0,1]$. Since ξ is sufficiently small, we can write $\xi \equiv 1/n = 1 + P/R$. We discretize both the numerator and the denominator. Let $\{u_i\}_{i=1}^{n}$ be a mixing sequence drawn from the standard normal distribution $N(0,1)$ with $E(u) = 0$ and var$(u) = 1$. Let $V_t = \sum_{i=1}^{t} u_i$ be the partial sum. Then we have $U_t = \sum_{i=1}^{t} u_i \sim N(0,t)$ and therefore

$$\frac{U_t}{\sqrt{n}} = \frac{\sum_{i=1}^{t} u_i}{\sqrt{n}} \equiv U_n(s) \Rightarrow W(s),$$

where $U_n(s)$ is a '*cadlag*' function and $W(s)$ is a Wiener process. Note that

$$\begin{aligned} n^{-1}\sum_{t=1}^{n} u_{t-1}u_t &= n^{-1}\sum_{t=1}^{n} U_{t-1}u_t - n^{-1}\sum_{t=1}^{n} U_{t-2}u_t \\ &\Rightarrow \int_{\xi}^{1} W(s)\mathrm{d}W(s) - \int_{\xi}^{1} W(s-\xi)\mathrm{d}W(s) \\ &= \int_{\xi}^{1} \left[W(s) - W(s-\xi)\right] \mathrm{d}W(s) \end{aligned}$$

Considering the term $\int_{\xi}^{1} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s$ in the denominator, we have

$$n^{-2}\sum_{t=1}^{n} u_{t-1}^2 = n^{-2}\sum_{t=1}^{n} \left(U_{t-1} - U_{t-2}\right)^2 \Rightarrow \int_{\xi}^{1} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}S.$$

We construct the an AR(1) regression model, regressing $\{u_{t+1}\}$ on $\{u_t\}$:

$$u_{t+1} = \delta u_t + e_t$$

The estimator $\hat{\delta}$ equals $\left(\sum_{t=1}^{n} u_{t-1}u_t\right) / \left(\sum_{t=1}^{n} u_{t-1}^2\right)$ and the variance $\hat{\delta}$ equals

$$\left(\sum_{t=1}^{n} u_{t-1}^2\right)^{-1} var\,(u) = \left(\sum_{t=1}^{n} u_{t-1}^2\right)^{-1}.$$

Therefore Equation (8) can be approximated by

$$\frac{\int_{\xi}^{1} \xi^{-1} \left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\xi^{-2} \int_{\xi}^{1} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \Rightarrow \frac{\sum\limits_{t=1}^{n} u_{t-1}u_t}{\sqrt{\sum\limits_{t=1}^{n} u_{t-1}^2}} \sim N(0,1).$$

■

5 Asymptotic Distribution of ENC with a Persistent Predictor when $P/R \to \infty$

Suppose the predictor x_t in Model 2 follows an AR process $x_{t+1} = \phi x_t + v_{t+1}$ where $\mathbb{E}\left(v_{t+1}^2\right) = \sigma_v^2$. If $|\phi| < 1$, then

$$T^{-1} \sum_{t=1}^{T} x_t^2 \xrightarrow{p} \frac{\sigma_v^2}{1-\phi^2}, \qquad T^{-0.5} \sum_{t=1}^{T} x_t v_{t+1} \Rightarrow N\left(0, \frac{\sigma_v^2}{1-\phi^2}\right),$$

as $T \to \infty$. Many recent papers generalize the above to the case when ϕ approaches to 1 as the sample size T increases, see Bobkoski (1983), Cavanagh (1985), Chan and Wei (1987), Giraitis and Phillips (2006), Mikusheva (2007, 2014), Park (2003), Phillips (1987), Phillips and Lee (2013), and Stock (1991). Let $\phi = 1 - c/T$ for some fixed constant $c \geq 0$, $t = [Tr]$, $r \in [0\ 1]$. Let $x_{[Tr]}/\sqrt{T} \Rightarrow J_x^c(r) = \int_0^r e^{(r-s)c} dB_x(s)$ be an Onstein-Uhlenbeck process and B_x is a Brownian motion. If the AR coefficient ϕ is local to unity, then

$$T^{-2} \sum_{t=1}^{T} x_t^2 \Rightarrow \int_0^1 J_x^c(r)^2 \,\mathrm{d}r, \qquad T^{-1} \sum_{t=1}^{T} x_t v_{t+1} \Rightarrow \int_0^1 J_x^c(r)\, \mathrm{d}B_x(r),$$

as $T \to \infty$. To consider the persistent predictor we take the local to unit root process in the following Assumption 1b.

Assumption 1b. $\{x_t\}$ follows an AR process with a root local to unity, $\phi = 1 - c/T$, for some fixed constant $c \geq 0$.

Let $t \equiv [Ts]$ and $\xi \equiv R/T$. Then we have $t/T \to s$ and $(t - R + 1)/T \to (s - \xi)$. Under Assumption 1b,

$$\begin{aligned} T^{-2} \sum_{j=t-R+1}^{t} x_j^2 &\Rightarrow \int_{s-\xi}^{s} J_x^c(r)^2 \,\mathrm{d}r \\ T^{-1} \sum_{j=t-R+1}^{t} x_j v_{j+1} &\Rightarrow \int_{s-\xi}^{s} J_x^c(r) \,\mathrm{d}B_x(r), \qquad t = R, \ldots, T, \end{aligned}$$

as $T \to \infty$. Now, we state the main result, for the numerator of ENC_P, that is $\sqrt{P}\hat{B}_P$.

Proposition 4. Under Assumption 1b and Assumption 2c, we have

$$\sum_{t=R}^{T} \hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) = - \sum_{t=R}^{T} e_{t+1} \left(e_{t+1} - \hat{e}_{t+1}^{(1)} \right) + o\left(\xi^{-1}\right)$$

under $\mathbb{H}_0$.

Proof: Under the null hypothesis that $b = 0$, $e_{t+1}^{(1)} = e_{t+1}^{(2)} =: e_{t+1}$. Note that

$$\hat{e}_{t+1}^{(i)} = e_{t+1} - x_{i,t}' \left(\hat{\beta}_{i,t} - \beta_i \right)$$

for Model i. Recall $x_{1,t}' = 1$. For $\hat{B}_P$, the numerator of ENC_P, we decompose

$$\begin{aligned} & \sum_{t=R}^{T} \hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)} \right) \\ = & \sum_{t=R}^{T} \left[e_{t+1} - x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_{1,t} \right) \right] \left(e_{t+1} - x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_1 \right) - e_{t+1} + x_{2,t}' \left(\hat{\beta}_{2,t} - \beta_2 \right) \right) \\ = & \sum_{t=R}^{T} \left[e_{t+1} - x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_1 \right) \right] \left(-x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_1 \right) + x_{2,t}' \left(\hat{\beta}_{2,t} - \beta_2 \right) \right) \\ = & \sum_{t=R}^{T} e_{t+1} \left[-x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_1 \right) \right] + \sum_{t=R}^{T} e_{t+1} \left[x_{2,t}' \left(\hat{\beta}_{2,t} - \beta_2 \right) \right] \\ & + \sum_{t=R}^{T} \left(\hat{\beta}_{1,t} - \beta_1 \right) x_{1,t} x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_1 \right) - \sum_{t=R}^{T} \left(\hat{\beta}_{1,t} - \beta_2 \right) x_{1,t} x_{2,t}' \left(\hat{\beta}_{2,t} - \beta_2 \right) \\ \equiv & A_1 + A_2 + A_3 + A_4 \end{aligned} \tag{9}$$

Lemmas 1-3 show that $A_1 + A_2 + (A_3 + A_4) = O\left(\frac{T}{R}\right) + O\left(\frac{P}{T}\right) + o(1)$. Hence (9) is dominated by A_1 because $\frac{T}{R} \to \infty$ and $\frac{P}{T} \to 1$ under Assumption 2c. ∎

Lemma 1. Under Assumption 2c, $A_1 \Rightarrow -\sigma_e^2 \xi^{-1} \int_\xi^1 [W(s) - W(s-\xi)] \mathrm{d}V_e(s) = O\left(\xi^{-1}\right) = O\left(\frac{T}{R}\right)$ under $\mathbb{H}_0$.

Proof: Following Lemma A6 of CM (2001), we show

$$
\begin{aligned}
A_1 &= \sum_{t=R}^{T} e_{t+1}\left[-x_{1,t}'\left(\hat{\beta}_{1,t} - \beta_1\right)\right] \\
&= -\sum_{t=R}^{T} e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) \\
&= -\sum_{t=R}^{T} e_{t+1}\left(R^{-1}\sum_{j=t-R+1}^{t} e_j\right) \\
&= -\sum_{t=R}^{T} e_{t+1}\left(T^{-1}\sum_{j=t-R+1}^{t} e_j\right)/\xi \\
&= -\sum_{t=R}^{T}\left[\left(T^{-1/2}e_{t+1}\right)\left(T^{-1/2}\sum_{j=1}^{t} e_{,j} - T^{-1/2}\sum_{j=1}^{t-R} e_{,j}\right)\right]/\xi \\
&\Longrightarrow -\sigma_e^2\xi^{-1}\int_\xi^1 \left[W(s) - W(s-\xi)\right]\mathrm{d}V_e(s).
\end{aligned}
$$

■

Lemma 2. Under Assumption 2c, $A_2 = \sum_{t=R}^{T} e_{t+1}\left[x_{2,t}'\left(\hat{\beta}_{2,t} - \beta_{2,t}\right)\right]$ is $O(1-\xi) = O\left(\frac{P}{T}\right)$ under $\mathbb{H}_0$.

Proof: Rewrite

$$
\begin{aligned}
A_2 &= \sum_{t=R}^{T} e_{t+1}x_{2,t}'\left(\hat{\beta}_{2,t} - \beta_2\right) \\
&= \sum_{t=R}^{T} e_{t+1}x_{2,t}'\left(\sum_{j=t-R}^{t-1} x_{2,j}x_{2,j}'\right)^{-1} \\
&\quad \times \left(\sum_{j=t-R}^{t-1} x_{2,j}e_{j+1}\right) \\
&= \sum_{t=R}^{T} e_{t+1}x_{2,t}'G_T^{-1}\left[G_T^{-1}\sum_{j=t-R+1}^{t} x_{2,j}x_{2,j}'G_T^{-1}/\xi\right]^{-1} \\
&\quad \times \left[G_T^{-1}\sum_{j=t-R+1}^{t} x_{2,j}e_{j+1}/\xi\right],
\end{aligned}
$$

where $G_T = \text{diag}\left(T^{0.5}, T\right)$ as before, and for the two bracketed terms in the last line, we have

$$
\begin{aligned}
& G_T^{-1}\left(\sum_{j=t-R+1}^{t} x_{2,j}x_{2,j}'\right) G_T^{-1}/\xi \\
\Rightarrow & \begin{pmatrix} \xi & \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}r \\ \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}r & \int_{s-\xi}^{s} \left(J_x^c(r)\right)^2 \mathrm{d}r \end{pmatrix} /\xi \sim O(1), \\
& G_T^{-1} \sum_{j=t-R+1}^{t} x_{2,j}e_{j+1}/\xi \\
\Rightarrow & \begin{pmatrix} \int_{s-\xi}^{s} 1\mathrm{d}V_e(r) \\ \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}V_e(r) \end{pmatrix} /\xi \sim O(1),
\end{aligned}
$$

where $J_x^c(r)$ is an Ornstein-Uhlenbeck process, and $V_e(r)$ is a Wiener process. Hence

$$
\begin{aligned}
A_2 &= \sum_{t=R}^{T} e_{t+1}x_{2,t}'G_T^{-1}\left[G_T^{-1}\sum_{j=t-R+1}^{t} x_{2,j}x_{2,j}'G_T^{-1}/\xi\right]^{-1} \\
&\quad \times \left[G_T^{-1}\sum_{j=t-R+1}^{t} x_{2,j}e_{j+1}/\xi\right] \\
&\Rightarrow \int_{\xi}^{1}\begin{pmatrix} 1 & J_x^c(s) \end{pmatrix}\begin{pmatrix} \xi & \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}r \\ \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}r & \int_{s-\xi}^{s} J_x^c(r)^2\,\mathrm{d}r \end{pmatrix}^{-1} \\
&\quad \times \begin{pmatrix} \int_{s-\xi}^{s} 1\mathrm{d}V_e(r) \\ \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}V_e(r) \end{pmatrix} \mathrm{d}V_e(s) \\
&= \int_{\xi}^{1}\begin{pmatrix} 1 & J_x^c(s) \end{pmatrix}\begin{pmatrix} O(\xi) & O(\xi) \\ O(\xi) & O(\xi) \end{pmatrix}^{-1}\begin{pmatrix} O(\xi) \\ O(\xi) \end{pmatrix}\mathrm{d}V_e(s) \\
&= O(1-\xi)
\end{aligned}
$$

Therefore $A_2 = \sum_{t=R}^{T} e_{t+1}x_{2,t}'\left(\hat{\beta}_{2,t} - \beta_{2,t}\right) = O(1-\xi)$. ■

Lemma 3. Under Assumption 2c, $A_3 + A_4$ is $o(1)$ under $\mathbb{H}_0$.

Proof: Let $E_T = \text{diag}\left(T^0, T^{0.5}\right)$, $F_T = \text{diag}\left(T^1, T^{1.5}\right)$, $G_T = \text{diag}\left(T^{0.5}, T^1\right)$, then for any 2×2 matrix K, we have $E_T F_T = G_T G_T$ and

$$E_T \times K \times F_T = G_T \times K \times G_T,$$

because E_T, F_T, G_T are diagonal. Therefore

$$\begin{aligned} A_3 + A_4 &= \sum_{t=R}^{T} \left(\hat{\beta}_{1,t} - \beta_{1,t}\right) x_{1,t} x_{1,t}' \left(\hat{\beta}_{1,t} - \beta_{1,t}\right) \\ &\quad - \sum_{t=R}^{T} \left(\hat{\beta}_{1,t} - \beta_{1,t}\right) x_{1,t} x_{2,t}' \left(\hat{\beta}_{2,t} - \beta_{2,t}\right) \\ &= \sum_{t=R}^{T} H_1'(t) B_1(t) q_{1,t} B_1(t) H_1(t) - \sum_{t=R}^{T} H_1'(t) B_1(t) x_{1,t} x_{2,t}' B_2(t) H_2(t), \end{aligned}$$

where the second line appears to be the same as the second bracketed right-hand side term in (A7) of Lemma A10 in CM (2001), which shows that the above is $o(1)$ under Assumption 1a. However, under Assumption 1b, x_t has an AR root local to unity. We show below that the local-to-unit root in x does not affect Lemma A10 of CM (2001). This is because terms involving x can be suitably normalized as follows

$$\begin{aligned} A_3 + A_4 &= \sum_{t=R}^{T} H_1'(t) B_1(t) q_{1,t} B_1(t) H_1(t) \\ &\quad - \sum_{t=R}^{T} H_1'(t) B_1(t) x_{1,t} \left(x_{2,t}' E_T^{-1}\right) \left[E_T \times R^{-1} B_2(t) \times F_T \times \xi\right] \\ &\quad \times \left[F_T^{-1} \times R H_2(t) / \xi\right] \\ &\equiv \sum_{t=R}^{T} H_1'(t) B_1(t) q_{1,t} B_1(t) H_1(t) - \sum_{t=R}^{T} H_1'(t) B_1(t) x_{1,t} \ddot{x}_{2,t}' \ddot{B}_2(t) \ddot{H}_2(t). \end{aligned}$$

where

$$\ddot{x}_{2,t}' \equiv x_{2,t}' E_T^{-1} \Rightarrow \begin{pmatrix} 1 & J_x^c(r) \end{pmatrix} = O(1),$$

$$\begin{aligned} \ddot{B}_2(t) &\equiv E_T \times R^{-1} B_2(t) \times F_T \times \xi \\ &= G_T \left[R^{-1} B_2(t)\right] G_T \times \xi \\ &= \left[G_T^{-1} \left[R^{-1} B_2(t)\right]^{-1} G_T^{-1}\right]^{-1} \times \xi \\ &= \left[G_T^{-1} \sum_{j=t-R+1}^{t} x_{2,j} x_{2,j}' G_T^{-1}\right]^{-1} \times \xi \\ &\Rightarrow \begin{pmatrix} \xi & \int_{s-\xi}^{s} J_x^c(r)\, \mathrm{d}r \\ \int_{s-\xi}^{s} J_x^c(r)\, \mathrm{d}r & \int_{s-\xi}^{s} \left(J_x^c(r)\right)^2 \mathrm{d}r \end{pmatrix}^{-1} \times \xi \\ &= \begin{pmatrix} O(\xi) & O(\xi) \\ O(\xi) & O(\xi) \end{pmatrix}^{-1} \times O(\xi) = O(1), \end{aligned}$$

and

$$\begin{aligned}
\ddot{H}_2(t) &\equiv F_T^{-1} \times RH_2(t)/\xi \\
&= F_T^{-1} \sum_{j=t-R+1}^{t} x_{2,j} e_{j+1}/\xi \\
&= \begin{pmatrix} T^{-1}/\xi \times \sum\limits_{j=t-R+1}^{t} e_{j+1} \\ T^{-1.5}/\xi \times \sum\limits_{j=t-R+1}^{t} x_j e_{j+1} \end{pmatrix} \\
&= \begin{pmatrix} T^{-0.5}/\xi \times T^{-0.5} \sum\limits_{j=t-R+1}^{t} e_{j+1} \\ T^{-0.5}/\xi \times T^{-1} \sum\limits_{j=t-R+1}^{t} x_j e_{j+1} \end{pmatrix} \\
&\Rightarrow \begin{pmatrix} T^{-0.5}/\xi \times \int_{s-\xi}^{s} 1 \mathrm{d}V_e(r) \\ T^{-0.5}/\xi \times \int_{s-\xi}^{s} J_x^c(r) \mathrm{d}V_e(r) \end{pmatrix} \\
&= \begin{pmatrix} O\left(T^{-0.5}/\xi\right) \times O(\xi) \\ O\left(T^{-0.5}/\xi\right) \times O(\xi) \end{pmatrix} = O\left(T^{-0.5}\right).
\end{aligned}$$

Therefore, $\ddot{x}'_{2,t}, \ddot{B}_2(t), \ddot{H}_2(t)$ have the same orders of magnitude as $x'_{2,t}, B_2(t), H_2(t)$ in stationary case of Lemma A10 in CM (2001). Therefore $A_3 + A_4$ is $o(1)$ not only under Assumption 1a but also under Assumption 1b. ■

Based on Lemmas 1-3 under Assumption 1b and Assumption 2c, $\sum\limits_{t=R}^{T} \hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) = -\sum\limits_{t=R}^{T} e_{t+1} \left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) + o(1)$. Hence, this is the encompassing test for the martingale difference model $y_{t+1} = e_{t+1}$ and the constant mean model $y_{t+1} = c + e_{t+1}^{(1)}$, as studied by CW (2006). Under $\mathbb{H}_0$, ENC_P is asymptotically standard normal. Proposition 4 states this result.

Proposition 5. Under Assumption 1b and 2c, $\lim_{\xi \to 0} ENC_P \Rightarrow N(0,1)$ under $\mathbb{H}_0$.

Proof: From Proposition 2 and Lemma 1, $A_1 = O\left(\xi^{-1}\right)$ is the dominant term of in $\hat{B}_P$ and hence ENC_P is

$$
\begin{aligned}
ENC_P &= A_1/\sqrt{\text{var}\,(A_1)} + o\,(1) \\
&= \frac{-\sum_{t=R}^{T} e_{t+1}\left(e_{t+1} - \hat{e}^{(1)}_{t+1}\right)}{\sqrt{\sum_{t=R}^{T}\left[-e_{t+1}\left(e_{t+1} - \hat{e}^{(1)}_{t+1}\right) - \hat{c}_P\right]^2}} + o\,(1) \\
&\Rightarrow \lim_{\xi\to 0} \frac{-\sigma_e^2\xi^{-1}\int_\xi^1 \left[W(s) - W(s-\xi)\right]\mathrm{d}V_e(s)}{\sqrt{\sigma_e^4 \times \xi^{-2}\int_\xi^1 \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \sim N\,(0,1)\,,
\end{aligned}
$$

where $A_1 \Rightarrow -\sigma_e^2\xi^{-1}\int_\xi^1 [V_e(s) - V_e(s-\xi)]\mathrm{d}V_e(s)$, $c_{t+1} = -e_{t+1}\left(e_{t+1} - \hat{e}^{(1)}_{t+1}\right)$ and $\hat{c}_P = P^{-1}\sum_{t=R}^{T} c_{t+1} = P^{-1}A_1$. The denominator follows from Lemma 4. Therefore, ENC_P is asymptotically standard normal under $\mathbb{H}_0$ from Proposition 3. ■

Lemma 4. $\sum_{t=R}^{T}\left[-e_{t+1}\left(e_{t+1} - \hat{e}^{(1)}_{t+1}\right) - \hat{c}_P\right]^2 \Rightarrow \sigma_e^4 \times \xi^{-2}\int_\xi^1 [W(s) - W(s-\xi)]^2\mathrm{d}s.$

Proof: Following Lemma A11 of CM (2001), we have

$$
\begin{aligned}
& \sum_{t=R}^{T}\left[-e_{t+1}\left(e_{t+1} - \hat{e}^{(1)}_{t+1}\right) - \hat{c}_P\right]^2 \\
= & \sum_{t=R}^{T}\left[e_{t+1}\left(e_{t+1} - \hat{e}^{(1)}_{t+1}\right)\right]^2 - P\hat{c}_P^2 \\
= & \sum_{t=R}^{T}\left[e_{t+1}\left(R^{-1}\sum_{j=t-R+1}^{t} e_j\right)\right]^2 + O\left(P^{-1}\times\xi^{-2}\right) \\
= & \frac{T^2}{R^2}\sum_{t=R}^{T}\left[(e_{t+1})^2\left(T^{-1/2}\sum_{j=1}^{t} e_{,j} - T^{-1/2}\sum_{j=1}^{t-R} e_{,j}\right)^2\right]\frac{1}{T} + O\left(P^{-1}\times\xi^{-2}\right) \\
\Rightarrow & \ \xi^{-2}\int_\xi^1 \sigma_e^2\left[\sigma_e W(s) - \sigma_e W(s-\xi)\right]^2\mathrm{d}s,
\end{aligned}
$$

where line 3 follows from Lemma 1 for $P\hat{c}_P = A_1 = O\left(\xi^{-1}\right)$. ■

Remark 3: We have consider the nested models in which the null model contains a constant term need to be estimated. We now consider the nested model analogue to Clark and West (2006) in which the null model does not contain a constant term and the error term is martingale difference series

$$\text{Model 1} \quad : \quad y_{t+1} = 0 + e^{(1)}_{t+1}, \tag{10}$$

$$\text{Model 2} \quad : \quad y_{t+1} = x'_{2,t}\beta_{2,t} + e^{(2)}_{t+1} = c + bx_t + e^{(2)}_{t+1}, \tag{11}$$

hence in null model we impose 0 as predictors and the forecast error is the true error term. therefore $\hat{e}_{t+1}^{(1)} = y_{t+1} = e_{t+1}^{(1)}$, $\hat{e}_{t+1}^{(2)} = y_{t+1} - f_{t+1}^{(2)}$. We have the following propositions.

Proposition 6. Under Assumption 1b and 2c, CCS test, is standard normal.

Proof: Under $\mathbb{H}_0$, $\hat{e}_{t+1}^{(1)} = e_{t+1}$. Note that $\lim T/P \to 1$. Therefore

$$\begin{aligned}
\hat{M}_P &= P^{-1}\sum_{t=R}^{T} e_{t+1}^{(1)} x_t \\
&\sim N\left(0, T^2\left(P^{-2}\frac{1}{T^2}\sum_{t=R}^{T} x_t^2\right)\sigma_e^2\right) \\
&\sim N\left(0, \sigma_e^2 \int_0^1 J_x^c(r)^2 \, \mathrm{d}r\right)
\end{aligned}$$

which follows Phillips and Lee (2014), Cai and Wang (2014). Particularly, if $c = 0$, $\hat{M}_P \sim N\left(0, \sigma_e^2 \int_0^1 W(r)^2 \, \mathrm{d}r\right)$. The consistent estimator $\hat{W}_P$ is

$$\begin{aligned}
\hat{W}_P &= \frac{1}{P}\sum_{t=R}^{T}\left(e_{t+1}^{(1)} x_t - \bar{W}\right)^2 \\
&= \frac{1}{P}\left[\sum_{t=R}^{T}\left(e_{t+1}^{(1)} x_t\right)^2 - P\bar{W}^2\right]
\end{aligned}$$

where $\bar{W} = P^{-1}\sum_{t=R}^{T} e_{t+1}^{(1)} x_t \sim O(1)$. Note that

$$\begin{aligned}
&\sum_{t=R}^{T}\left(e_{t+1}^{(1)} x_t\right)^2 \\
\to\ &\sigma_e^2 \sum_{t=R}^{T} x_t^2 \\
\Longrightarrow\ &\sigma_e^2 T^2 \int_\xi^1 J_x^c(r)^2 \, \mathrm{d}r \sim O\left(T^2\right)
\end{aligned}$$

Therefore $\sum_{t=R}^{T}\left(e_{t+1}^{(1)} x_t\right)^2$ dominates $\hat{W}_P$ and

$$\hat{W}_P \to P^{-1}\sigma_e^2 T^2 \int_\xi^1 J_x^c(r)^2 \, dr \to \sigma_e^2 T \int_\xi^1 J_x^c(r)^2 \, dr,$$

we have

$$\begin{aligned}
\hat{W}_P^{-0.5}\sqrt{P}\hat{M}_P &\sim N\left(0, P\frac{\sigma_e^2 \int_0^1 J_x^c(r)^2 \, \mathrm{d}r}{\sigma_e^2 T \int_\xi^1 J_x^c(r)^2 \, dr}\right) \\
&\to {}^d N(0,1)
\end{aligned}$$

■

Remark 4. Under $\mathbb{H}_0$ and assumption 2c, ENC test is normal. Under the null hypothesis that $\{x_{1,t}\} \equiv 0$ Therefore Equation (9) only has A_2 term, whose limiting distribution is

$$\int_{\xi}^{1} \begin{pmatrix} 1 & J_x^c(s) \end{pmatrix} \begin{pmatrix} \xi & \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}r \\ \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}r & \int_{s-\xi}^{s} J_x^c(r)^2\,\mathrm{d}r \end{pmatrix}^{-1} \begin{pmatrix} \int_{s-\xi}^{s} 1\mathrm{d}V_e(r) \\ \int_{s-\xi}^{s} J_x^c(r)\,\mathrm{d}V_e(r) \end{pmatrix} \mathrm{d}V_e(s)$$

In the next section, Monte Carlo simulation shows that ENC test is standard normal.

6 Monte Carlo Simulation

We compare the three statistics by changing $\{x_t\}$ from stationary process to Ornstein-Uhlenbeck process and use the data generating process in Model 1 and Model 2 as follows: the additional variable x_t in Model 2 has the AR process: $x_t = \phi x_{t-1} + z_t$, where z_j is i.i.d, following $N(0,1)$, and $\mathbb{E}(z_t|x_{t-1}) = 0$, The error term $e_{t+1}^{(2)} \sim N(0, \sigma_e^2)$. We set $c_2 = 1$ (we also set $c_2 = 0$ for martingale difference series $\{y_t\}$ and report the result.) $b \in \{0.0, 0.1, 1.0\}$, $\phi \in \{0, 0.5, 0.9, 0.95, 0.99, 1\}$, and $\sigma_e \in \{0.1, 1.0\}$. Model 1 is estimated by regressing $\{y_j\}_{j=t-R+1}^{t}$ on constant term to obtain $\hat{c}_{1,t}$, where $t = R, \ldots, T$. Model 2 is estimated by regressing $\{y_j\}_{j=t-R+1}^{t}$ on $\{1, x_{j-1}\}_{j=t-R+1}^{t}$ to obtain $\left(\hat{c}_{2,t}\ \hat{b}_t\right)$. The forecast errors from the two models are $\hat{e}_{t+1}^{(1)} = y_{t+1} - \hat{c}_{1,t}$ and $\hat{e}_{t+1}^{(2)} = y_{t+1} - \hat{c}_{2,t} - \hat{b}_t x_t$ over the forecast evaluation period at $t = R, \ldots, T$. The number of observations for the rolling windows for estimation are chosen from $R \in \{60, 120, 240\}$. Let $P = T - R + 1 \in \{48, 240, 1200\}$. From these, we compute the three statistics DM_P, ENC_P, and CCS_P. All above tests are repeated 2000 times to find out the Monte Carlo distributions of DM_P, ENC_P, and CCS_P, and to compute their size and power. Also we consider the test statistics when $Corr(v_t, u_{t+1}) = -0.95$, $R_T = 0.8$.

The Tables 2.1 shows the size of test with additional covariate from stationary process to local to Ornstein-Uhlenbeck process. We see that DM statistics are undersized for large P/R ratio for all cases. The CCS statistic has the correct size for stationary process covariate but as ϕ increases from 0 to 0.99, the size distorts downward. ENC test is robust, having correct size for all ϕ ranging from 0 to 0.99. Table 2.2 and 2.3 show the power of test. We see that as ϕ increases, the powers approach to 1 dramatically since higher ϕ implies higher singal-to-noise ratio. Figures 2.1-2.12 show the Monte Carlo distributions of ENC_P, DM_P, and CCS_P statistics. Under $\mathbb{H}_0$, we can see that for size of test, if the predictor is a stationary process, both ENC_P and CCS_P have correct size. However, when the predictor has a near unit root, such as $\phi = 0.95$ or 0.99, both DM_P and CCS_P lower the size whereas ENC_P still has the correct size. DM_P and CCS_P have correct sizes for small P/R but suffer an undersize problem for large P/R. Under $\mathbb{H}_1$, all the tests have good powers.

Tables 2.1-2.6 About Here

Figures 2.1-2.24 About Here

7 Application

We apply the three statistics to the Goyal and Welch study (2008) and construct nested models to test if covariates such as dividend-yield ratio (DY), dividend-price ratio (DP), long term rate of yield (LTY) and inflation (INFL) Granger-causes the equity premium. The dividend-yield ratio at time t is defined as the most recent dividend at t divided by stock price at time t, the dividend-price ratio at time t is defined as the most recent dividend at $t-1$ divided by stock price at time t. The explanation of other variables are available from the homepage of A. Goyal. The two nested models are as shown in (12) and (13) below

$$\text{Model 1} \quad : \quad y_{t+1} = c_1 + e^{(1)}_{t+1}, \tag{12}$$

$$\text{Model 2} \quad : \quad y_{t+1} = c_2 + bx_t + e^{(2)}_{t+1}, \tag{13}$$

where y_{t+1} is the equity premium and x_t is the covariate. We use monthly data ranging from 1926 to 2011, containing 1032 observation for all four models. In Model 1, we only have a constant term, therefore at time t, we predict the future equity premium by solely using the historical average of previous R observations of the equity premium from time $t-R+1$ to t. In Model 2, we use the 1-lag covariate to forecast the equity premium in the next month. See Goyal and Welch (2008) for more on data descriptions. We intend to check if two nested models have the same predictive accuracy. We use the rolling window scheme of the window size R starting from the 15% of the total observations is $T = 1032$ to the 85% of T. The in-sample observation R ranges from $R = 155$ ($R/P = 155/877$) to $R = 877$ ($R/P = 877/155$). The red line represents DM_P under different allocation of R and P. The blue line and dotted line represent ENC_P statistic and CCS_P statistic respectively.

We conclude that (1) The ENC statistics always have higher statistics than DM test. (2) DP and DY figures show that ENC test is significant with small R/P ratio. Intuitively, for R/P, we are unable to account for the y_{t+1} by solely using the previous y up to time t since there is not sufficient information available, therefore we need to exploit the property of additional variable x. In this way, x has predictive power for ENC test; however when R/P is large, we can predict y_{t+1} using previous information of y up to time t, which weakens the predictive power of x. (3) LTY figure shows that long-term yield has predictive power for equity premium for all R ranging from 150 to 877 using ENC test, which can not directly been observed from DM or CCS statistic. (4) INFL figure shows that CCS test is seriously undersized if we use inflation rate as a predictor unless the number of in-sample observations exceed 600, since the statistic is lower than -1.645, whereas ENC test shows that the inflation rate has predictive power when the number of in-sample observations is below 600.

Figure 2.25 About Here

8 Conclusions

This paper extends the work of Clark and West (2006, 2007) from nested mean model with weak stationary predictor to nested mean model with a highly persistent predictor. CM (2001, 2005, 2009) and CW(2006, 2007) found that the DM statistic tends to be negative under the null hypothesis of the equal predictive ability because of parameter estimation error. We find that the DM is even more severely undersized when the predictor is highly persistent with the AR root closer to unity. We find that the ENC test is robust as it remains the correct size under the null hypothesis of the equal predictive ability, also it has high power under alternative. We show that the highly persistent predictor following the Ornstein-Uhlenbeck process implies the convergent rate of the estimator from Model 2 faster than that from a model with stationary predictor and the ENC test can shown to be asymptotically standard normal when the ratio of out-of-sample to in-sample observation is infinite. By using Monte-Carlo simulation, we see that ENC test is robust and has the correct size under null hypothesis whereas DM or CCS are severely undersized even if CCS is shown to have the correct size in CM (2001, 2005, 2009) and CW(2006, 2007) where a weak stationary predictor is used in Model 2. An application to the predictive regression of the equity premium reveals strong predictive ability of several persistent predictors (such as inflation and interest rate) by ENC, but with little or none can be seen from DM and CCS.

References

Bobkoski, M.J. (1983), "Hypothesis Testing in Nonstationary Time Series", Unpublished PhD thesis, Dept. of Statistics, University of Wisconsin, Madison.

Cavanagh, C. (1985), "Roots Local to Unity, Unpublished Manuscript", Harvard University.

Chan, N. H. and Wei, C. Z. (1987), "Asymptotic Inference for Nearly Nonstationary AR(1) Processes", *Annals of Statistics* 15(3), 1050-1063.

Chao, J. C., V. Corradi and N. Swanson (2001). "An Out of Sample Test for Granger Causality." *Macroeconomic Dynamics* 5(4): 598-620.

Clark, T. E. and M. McCracken (2001). "Tests of Equal Forecast Accuracy and Encompassing for Nested Models." *Journal of Econometrics* 105(1): 85-110.

Clark, T. E. and M. McCracken (2005). "Evaluating Direct Multistep Forecasts." *Econometric Reviews* 24(4): 369-404.

Clark, T. E. and M. McCracken (2009). "Combining Forecasts from Nested Models." *Oxford Bulletin of Economics and Statistics* 71(3): 303-329.

Clark, T. E. and K. D. West (2006). "Using Out-of-sample Mean Squared Prediction Errors to Test the Martingale Difference Hypothesis." *Journal of Econometrics* 135(1-2): 155-186.

Clark, T. E. and K. D. West (2007). "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models." *Journal of Econometrics* 138(1): 291-311.

Corradi, V. and N. R. Swanson (2002). "A Consistent Test for Nonlinear Out of Sample Predictive Accuracy." *Journal of Econometrics* 110(2): 353-381.

Diebold, F.X. (2015), "Comparing Predictive Accuracy, Twenty Years Later: A Personal Perspective on the Use and Abuse of Diebold–Mariano Tests.", *Journal of Business and Economic Statistics* 33(1): 1-8.

Diebold, F. X. and R. S. Mariano (1995). "Comparing Predictive Accuracy." *Journal of Business and Economic Statistics* 13(3): 253-263.

Giacomini, R. and H. White (2006). "Tests of Conditional Predictive Ability." *Econometrica* 74(6): 1545-1578.

Giraitis, L. and Phillips, P. C. B. (2006), "Uniform Limit Theory for Stationary Autoregression", *Journal of Time Series Analysis* 27(1), 51-60.

Goyal, A. and Welch, I. (2008). "A Comprehensive Look at the Empirical Performance of Equity Premium Prediction." *Review of Financial Studies* 21: 1455-1508.

Harvey, D. S., S. J. Leybourne and P. Newbold (1998). “Tests for Forecast Encompassing.” *Journal of Business and Economic Statistics* 16(2): 254-259.

Mikusheva, A. (2007), “Uniform inference in autoregressive models,” Econometrica 75(5), 1411-1452.

Mikusheva, A. (2014), “Second Order Expansion of the t-statistic in AR(1) Models,” *Econometric Theory*, forthcoming.

Newey, W. K. and J. L. Powell (1987). “Asymmetric Least Squares Estimation and Testing.” *Econometrica* 55(4): 819-847.

Park, J.Y. (2003): Weak Unit Roots, Unpublished Manuscript, Rice University.

Phillips, P. C. B. (1987). “Towards a Unified Asymptotic Theory for Autoregression.” *Biometrika* 74(3): 535–547.

Phillips, P. C. B. and J. H. Lee (2013). “Predictive Regression Under Various Degrees of Persistence and Robust Long-Horizon Regression.” *Journal of Econometrics* 177(2): 250-264.

Stock, J. H. (1991). “Confidence Intervals for the Largest Autoregressive Root in US Macroeconomic Time Series.” *Journal of Monetary Economics* 28: 435-459.

West, K. D. (1996). “Asymptotic Inference about Predictive Ability.” *Econometrica* 64(5): 1067-1084.

Table 2.1: Rejection frequency under 5% level, $b = 0$ (With intercept on small model)

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.010	0.039	0.060	0.000	0.037	0.054	0.000	0.043	0.056
$\rho = 0, \sigma_e = 0.1$	$R = 120$	0.009	0.030	0.048	0.001	0.029	0.057	0.000	0.042	0.053
	$R = 240$	0.025	0.040	0.059	0.006	0.034	0.055	0.000	0.032	0.045
	$R = 60$	0.005	0.027	0.044	0.000	0.026	0.050	0.000	0.030	0.045
$\rho = 0, \sigma_e = 1$	$R = 120$	0.016	0.037	0.050	0.000	0.029	0.044	0.000	0.038	0.047
	$R = 240$	0.029	0.046	0.064	0.006	0.025	0.050	0.000	0.031	0.057
	$R = 60$	0.012	0.049	0.066	0.000	0.037	0.049	0.000	0.040	0.054
$\rho = 0.1, \sigma_e = 0.1$	$R = 120$	0.018	0.040	0.055	0.000	0.029	0.044	0.000	0.031	0.045
	$R = 240$	0.018	0.033	0.048	0.006	0.030	0.056	0.000	0.029	0.050
	$R = 60$	0.008	0.036	0.059	0.000	0.037	0.053	0.000	0.040	0.049
$\rho = 0.1, \sigma_e = 1$	$R = 120$	0.018	0.038	0.052	0.002	0.034	0.052	0.000	0.034	0.047
	$R = 240$	0.025	0.037	0.057	0.003	0.029	0.055	0.000	0.031	0.049
	$R = 60$	0.005	0.033	0.060	0.000	0.028	0.049	0.000	0.038	0.054
$\rho = 0.5, \sigma_e = 0.1$	$R = 120$	0.015	0.036	0.055	0.001	0.026	0.050	0.000	0.036	0.049
	$R = 240$	0.018	0.032	0.054	0.004	0.028	0.053	0.000	0.032	0.047
	$R = 60$	0.010	0.038	0.060	0.001	0.036	0.053	0.000	0.044	0.042
$\rho = 0.5, \sigma_e = 1$	$R = 120$	0.019	0.036	0.051	0.000	0.028	0.055	0.000	0.039	0.058
	$R = 240$	0.023	0.039	0.056	0.004	0.029	0.049	0.000	0.035	0.049
	$R = 60$	0.007	0.035	0.050	0.000	0.033	0.048	0.000	0.043	0.044
$\rho = 0.9, \sigma_e = 0.1$	$R = 120$	0.011	0.034	0.064	0.001	0.026	0.040	0.000	0.044	0.052
	$R = 240$	0.023	0.044	0.052	0.004	0.024	0.043	0.000	0.033	0.051
	$R = 60$	0.007	0.035	0.050	0.000	0.034	0.049	0.000	0.048	0.055
$\rho = 0.9, \sigma_e = 1$	$R = 120$	0.016	0.035	0.057	0.000	0.027	0.046	0.000	0.037	0.056
	$R = 240$	0.023	0.040	0.051	0.003	0.032	0.065	0.000	0.036	0.058
	$R = 60$	0.002	0.024	0.041	0.000	0.028	0.040	0.000	0.047	0.034
$\rho = 0.95, \sigma_e = 0.1$	$R = 120$	0.012	0.036	0.049	0.001	0.036	0.037	0.000	0.039	0.045
	$R = 240$	0.023	0.038	0.050	0.005	0.032	0.043	0.000	0.036	0.052
	$R = 60$	0.005	0.028	0.041	0.000	0.037	0.038	0.000	0.040	0.030
$\rho = 0.95, \sigma_e = 1$	$R = 120$	0.013	0.033	0.058	0.001	0.027	0.046	0.000	0.047	0.046
	$R = 240$	0.017	0.031	0.059	0.003	0.017	0.048	0.000	0.034	0.048
	$R = 60$	0.008	0.034	0.055	0.000	0.031	0.012	0.000	0.037	0.003
$\rho = 0.99, \sigma_e = 0.1$	$R = 120$	0.012	0.036	0.058	0.001	0.034	0.030	0.000	0.043	0.011
	$R = 240$	0.026	0.049	0.058	0.001	0.019	0.036	0.000	0.040	0.032
	$R = 60$	0.003	0.033	0.043	0.000	0.032	0.013	0.000	0.045	0.004
$\rho = 0.99, \sigma_e = 1$	$R = 120$	0.010	0.032	0.052	0.000	0.031	0.031	0.000	0.040	0.018
	$R = 240$	0.021	0.036	0.050	0.003	0.030	0.043	0.000	0.035	0.031

Notes: The table shows the size of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times.

Table 2.2: Rejection frequency under 5% level, $b = 0.1$ (With intercept on small model)

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	0.954	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0, \sigma_e = 0.1$	$R = 120$	0.948	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.943	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.023	0.107	0.121	0.004	0.233	0.342	0.001	0.601	0.935
$\rho = 0, \sigma_e = 1$	$R = 120$	0.049	0.124	0.106	0.015	0.247	0.322	0.021	0.728	0.918
	$R = 240$	0.070	0.142	0.111	0.060	0.325	0.339	0.129	0.823	0.927
	$R = 60$	0.949	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.1, \sigma_e = 0.1$	$R = 120$	0.943	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.945	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.020	0.097	0.107	0.006	0.214	0.336	0.000	0.614	0.932
$\rho = 0.1, \sigma_e = 1$	$R = 120$	0.040	0.108	0.102	0.016	0.255	0.327	0.032	0.731	0.926
	$R = 240$	0.069	0.149	0.108	0.055	0.331	0.347	0.110	0.829	0.927
	$R = 60$	0.983	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.5, \sigma_e = 0.1$	$R = 120$	0.977	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.975	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.032	0.122	0.123	0.006	0.284	0.422	0.002	0.751	0.970
$\rho = 0.5, \sigma_e = 1$	$R = 120$	0.057	0.159	0.141	0.031	0.347	0.408	0.064	0.848	0.968
	$R = 240$	0.081	0.164	0.120	0.079	0.404	0.416	0.238	0.923	0.976
	$R = 60$	1.000	1.000	0.985	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.9, \sigma_e = 0.1$	$R = 120$	0.999	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.078	0.311	0.260	0.128	0.771	0.824	0.511	1.000	1.000
$\rho = 0.9, \sigma_e = 1$	$R = 120$	0.133	0.390	0.293	0.301	0.865	0.859	0.936	1.000	1.000
	$R = 240$	0.170	0.445	0.322	0.435	0.933	0.913	0.993	1.000	1.000
	$R = 60$	1.000	1.000	0.957	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.95, \sigma_e = 0.1$	$R = 120$	1.000	1.000	0.968	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	0.992	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.110	0.441	0.328	0.317	0.927	0.888	0.926	1.000	1.000
$\rho = 0.95, \sigma_e = 1$	$R = 120$	0.220	0.571	0.440	0.619	0.976	0.946	1.000	1.000	1.000
	$R = 240$	0.255	0.586	0.455	0.709	0.990	0.976	1.000	1.000	1.000
	$R = 60$	1.000	1.000	0.838	1.000	1.000	0.956	1.000	1.000	1.000
$\rho = 0.99, \sigma_e = 0.1$	$R = 120$	1.000	1.000	0.885	1.000	1.000	0.973	1.000	1.000	1.000
	$R = 240$	1.000	1.000	0.910	1.000	1.000	0.976	1.000	1.000	1.000
	$R = 60$	0.252	0.623	0.400	0.676	0.986	0.735	1.000	1.000	0.999
$\rho = 0.99, \sigma_e = 1$	$R = 120$	0.434	0.770	0.533	0.928	1.000	0.879	1.000	1.000	1.000
	$R = 240$	0.519	0.824	0.615	0.970	1.000	0.926	1.000	1.000	1.000

Notes: The table shows the power of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times.

Table 2.3: Rejection frequency under 5% level, $b = 1$ (With intercept on small model)

$Repeat = 2000$		$P=48$ DM_P	ENC_P	CCS_P	$P=240$ DM_P	ENC_P	CCS_P	$P=1200$ DM_P	ENC_P	CCS_P
$\rho=0, \sigma_e=0.1$	$R=60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0, \sigma_e=1$	$R=60$	0.964	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	0.950	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	0.953	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.1, \sigma_e=0.1$	$R=60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.1, \sigma_e=1$	$R=60$	0.953	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	0.956	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	0.939	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.5, \sigma_e=0.1$	$R=60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.5, \sigma_e=1$	$R=60$	0.979	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	0.980	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	0.970	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.9, \sigma_e=0.1$	$R=60$	1.000	1.000	0.991	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.9, \sigma_e=1$	$R=60$	1.000	1.000	0.986	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.95, \sigma_e=0.1$	$R=60$	1.000	1.000	0.945	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	0.983	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.95, \sigma_e=1$	$R=60$	1.000	1.000	0.943	1.000	1.000	1.000	1.000	1.000	1.000
	$R=120$	1.000	1.000	0.974	1.000	1.000	1.000	1.000	1.000	1.000
	$R=240$	1.000	1.000	0.986	1.000	1.000	1.000	1.000	1.000	1.000
$\rho=0.99, \sigma_e=0.1$	$R=60$	1.000	1.000	0.845	1.000	1.000	0.957	1.000	1.000	1.000
	$R=120$	1.000	1.000	0.893	1.000	1.000	0.965	1.000	1.000	1.000
	$R=240$	1.000	1.000	0.921	1.000	1.000	0.980	1.000	1.000	1.000
$\rho=0.99, \sigma_e=1$	$R=60$	1.000	1.000	0.848	1.000	1.000	0.953	1.000	1.000	1.000
	$R=120$	1.000	1.000	0.897	1.000	1.000	0.968	1.000	1.000	1.000
	$R=240$	1.000	1.000	0.915	1.000	1.000	0.981	1.000	1.000	1.000

Notes: The table shows the power of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times.

Table 2.4: Rejection frequency under 5% level, $b = 0$ (Without intercept on small model)

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\rho = 0, \sigma_e = 0.1$	$R = 60$	0.006	0.042	0.053	0.000	0.045	0.055	0.000	0.047	0.047
	$R = 120$	0.016	0.040	0.056	0.000	0.033	0.052	0.000	0.047	0.049
	$R = 240$	0.019	0.042	0.051	0.002	0.035	0.049	0.000	0.032	0.054
$\rho = 0, \sigma_e = 1$	$R = 60$	0.004	0.032	0.051	0.000	0.035	0.041	0.000	0.048	0.046
	$R = 120$	0.014	0.043	0.048	0.001	0.032	0.050	0.000	0.038	0.054
	$R = 240$	0.020	0.051	0.058	0.003	0.039	0.050	0.000	0.032	0.049
$\rho = 0.1, \sigma_e = 0.1$	$R = 60$	0.003	0.033	0.053	0.000	0.035	0.053	0.000	0.038	0.052
	$R = 120$	0.013	0.046	0.052	0.001	0.026	0.057	0.000	0.045	0.057
	$R = 240$	0.019	0.038	0.056	0.002	0.034	0.052	0.000	0.042	0.057
$\rho = 0.1, \sigma_e = 1$	$R = 60$	0.006	0.042	0.050	0.000	0.031	0.050	0.000	0.042	0.053
	$R = 120$	0.013	0.046	0.048	0.001	0.029	0.049	0.000	0.041	0.053
	$R = 240$	0.023	0.052	0.060	0.003	0.030	0.049	0.000	0.040	0.053
$\rho = 0.5, \sigma_e = 0.1$	$R = 60$	0.006	0.038	0.047	0.000	0.035	0.046	0.000	0.041	0.043
	$R = 120$	0.010	0.039	0.051	0.000	0.025	0.044	0.000	0.036	0.045
	$R = 240$	0.018	0.041	0.060	0.001	0.033	0.052	0.000	0.034	0.041
$\rho = 0.5, \sigma_e = 1$	$R = 60$	0.006	0.039	0.056	0.000	0.038	0.055	0.000	0.036	0.052
	$R = 120$	0.009	0.034	0.058	0.000	0.031	0.063	0.000	0.040	0.047
	$R = 240$	0.017	0.036	0.061	0.002	0.029	0.053	0.000	0.034	0.056
$\rho = 0.9, \sigma_e = 0.1$	$R = 60$	0.003	0.039	0.046	0.000	0.040	0.058	0.000	0.047	0.048
	$R = 120$	0.008	0.037	0.054	0.000	0.038	0.052	0.000	0.036	0.045
	$R = 240$	0.023	0.043	0.052	0.002	0.032	0.046	0.000	0.038	0.052
$\rho = 0.9, \sigma_e = 1$	$R = 60$	0.003	0.032	0.044	0.000	0.044	0.047	0.000	0.045	0.052
	$R = 120$	0.008	0.040	0.052	0.000	0.029	0.053	0.000	0.036	0.049
	$R = 240$	0.018	0.040	0.056	0.002	0.028	0.048	0.000	0.035	0.044
$\rho = 0.95, \sigma_e = 0.1$	$R = 60$	0.003	0.031	0.050	0.000	0.036	0.050	0.000	0.052	0.052
	$R = 120$	0.005	0.030	0.048	0.000	0.034	0.053	0.000	0.042	0.049
	$R = 240$	0.018	0.038	0.061	0.002	0.031	0.051	0.000	0.037	0.053
$\rho = 0.95, \sigma_e = 1$	$R = 60$	0.003	0.038	0.053	0.000	0.040	0.046	0.000	0.055	0.055
	$R = 120$	0.012	0.042	0.063	0.000	0.028	0.045	0.000	0.037	0.056
	$R = 240$	0.011	0.032	0.052	0.002	0.032	0.059	0.000	0.031	0.042
$\rho = 0.99, \sigma_e = 0.1$	$R = 60$	0.003	0.028	0.056	0.000	0.035	0.051	0.000	0.049	0.050
	$R = 120$	0.008	0.039	0.057	0.000	0.035	0.050	0.000	0.040	0.051
	$R = 240$	0.014	0.035	0.051	0.002	0.031	0.041	0.000	0.036	0.054
$\rho = 0.99, \sigma_e = 1$	$R = 60$	0.002	0.027	0.054	0.000	0.036	0.053	0.000	0.045	0.052
	$R = 120$	0.005	0.030	0.054	0.000	0.028	0.055	0.000	0.045	0.052
	$R = 240$	0.015	0.037	0.061	0.002	0.023	0.049	0.000	0.030	0.055

Notes: The table shows the size of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times.

Table 2.5: Rejection frequency under 5% level, $b = 0.1$. (Without intercept on small model)

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\rho = 0, \sigma_e = 0.1$	$R = 60$	0.937	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 120$	0.948	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.943	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0, \sigma_e = 1$	$R = 60$	0.013	0.078	0.100	0.001	0.157	0.311	0.000	0.463	0.935
	$R = 120$	0.037	0.103	0.109	0.006	0.225	0.337	0.001	0.595	0.936
	$R = 240$	0.050	0.131	0.109	0.026	0.268	0.316	0.031	0.758	0.938
$\rho = 0.1, \sigma_e = 0.1$	$R = 60$	0.932	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 120$	0.940	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.935	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.1, \sigma_e = 1$	$R = 60$	0.014	0.073	0.100	0.002	0.182	0.355	0.000	0.484	0.941
	$R = 120$	0.031	0.112	0.111	0.007	0.214	0.349	0.001	0.624	0.935
	$R = 240$	0.051	0.124	0.104	0.035	0.263	0.324	0.027	0.731	0.937
$\rho = 0.5, \sigma_e = 0.1$	$R = 60$	0.973	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 120$	0.974	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.973	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.5, \sigma_e = 1$	$R = 60$	0.010	0.102	0.119	0.001	0.230	0.428	0.000	0.616	0.978
	$R = 120$	0.038	0.129	0.124	0.012	0.296	0.434	0.007	0.753	0.977
	$R = 240$	0.062	0.150	0.132	0.051	0.366	0.431	0.075	0.873	0.982
$\rho = 0.9, \sigma_e = 0.1$	$R = 60$	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 120$	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.9, \sigma_e = 1$	$R = 60$	0.052	0.298	0.342	0.050	0.753	0.924	0.119	1.000	1.000
	$R = 120$	0.099	0.356	0.321	0.191	0.838	0.917	0.726	1.000	1.000
	$R = 240$	0.144	0.401	0.335	0.369	0.902	0.918	0.967	1.000	1.000
$\rho = 0.95, \sigma_e = 0.1$	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.95, \sigma_e = 1$	$R = 60$	0.114	0.468	0.518	0.264	0.941	0.989	0.818	1.000	1.000
	$R = 120$	0.194	0.552	0.519	0.521	0.972	0.986	0.995	1.000	1.000
	$R = 240$	0.228	0.577	0.495	0.678	0.989	0.991	1.000	1.000	1.000
$\rho = 0.99, \sigma_e = 0.1$	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.99, \sigma_e = 1$	$R = 60$	0.473	0.795	0.811	0.875	0.998	1.000	1.000	1.000	1.000
	$R = 120$	0.544	0.831	0.812	0.953	0.999	1.000	1.000	1.000	1.000
	$R = 240$	0.547	0.827	0.791	0.968	1.000	1.000	1.000	1.000	1.000

Notes: The table shows the power of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times.

Table 2.6: Rejection frequency under 5% level, $b = 1$. (Without intercept on small model)

		$P = 48$			$P = 240$			$P = 1200$		
$Repeat = 2000$		DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0, \sigma_e = 0.1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.939	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0, \sigma_e = 1$	$R = 120$	0.940	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.939	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.1, \sigma_e = 0.1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.934	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.1, \sigma_e = 1$	$R = 120$	0.939	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.940	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.5, \sigma_e = 0.1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	0.975	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.5, \sigma_e = 1$	$R = 120$	0.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.970	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.9, \sigma_e = 0.1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.9, \sigma_e = 1$	$R = 120$	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.95, \sigma_e = 0.1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.95, \sigma_e = 1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.99, \sigma_e = 0.1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 60$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\rho = 0.99, \sigma_e = 1$	$R = 120$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$R = 240$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: The table shows the power of DM_P, ENC_P and CCS_P test under 5% nominal size from Monte-Carlo Simulation of 2000 times.

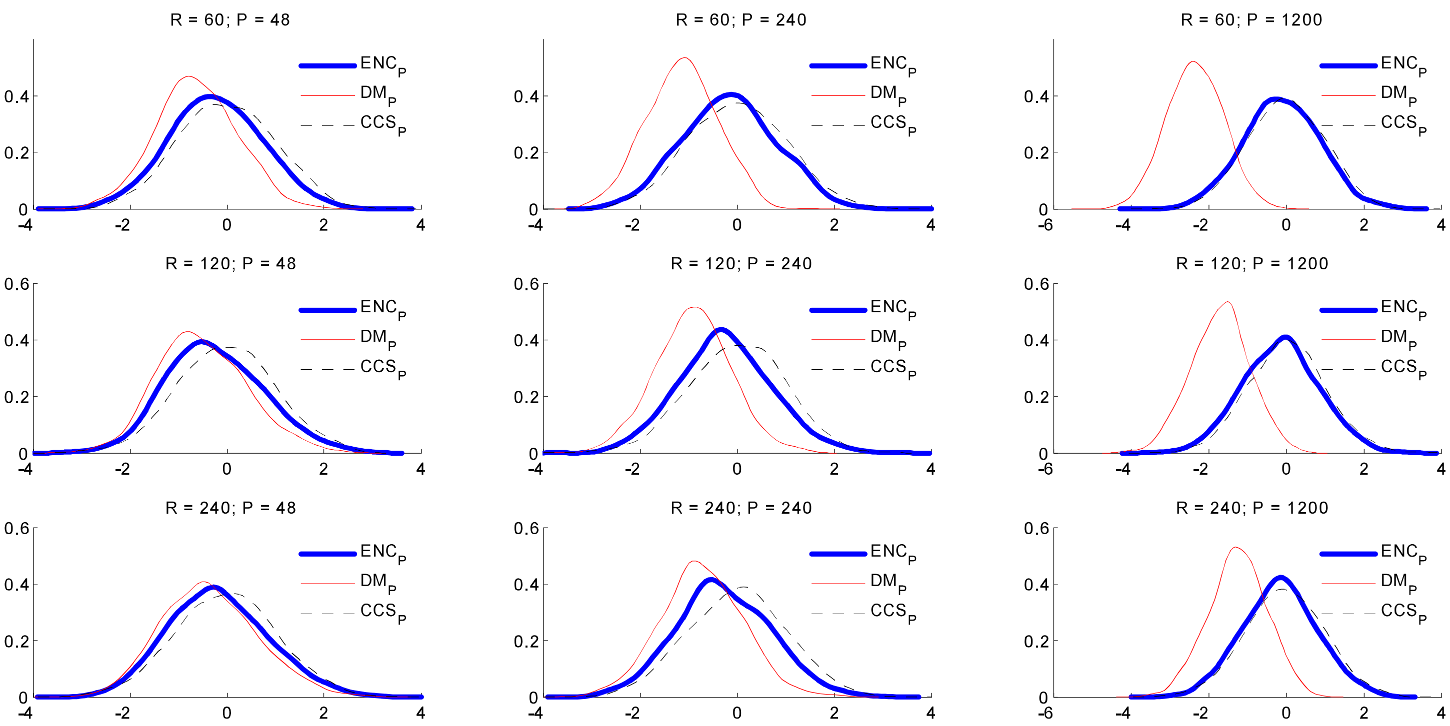

Figure 2.1: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0$, $b = 0$, $\sigma_e = 1$, with intercept on small model, 2000 Repeats.

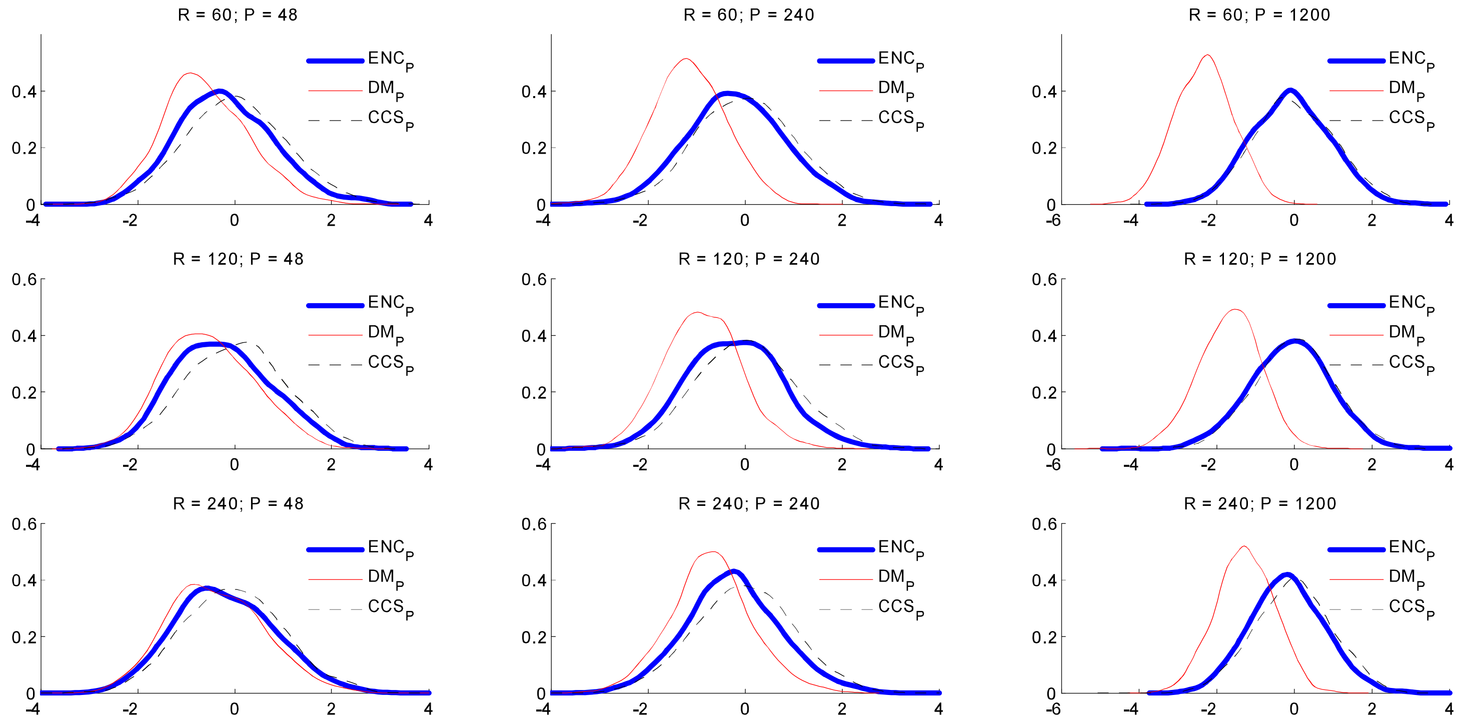

Figure 2.2: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0$, $b = 0$, $\sigma_e = 0.1$, with intercept on small model, 2000 Repeats.

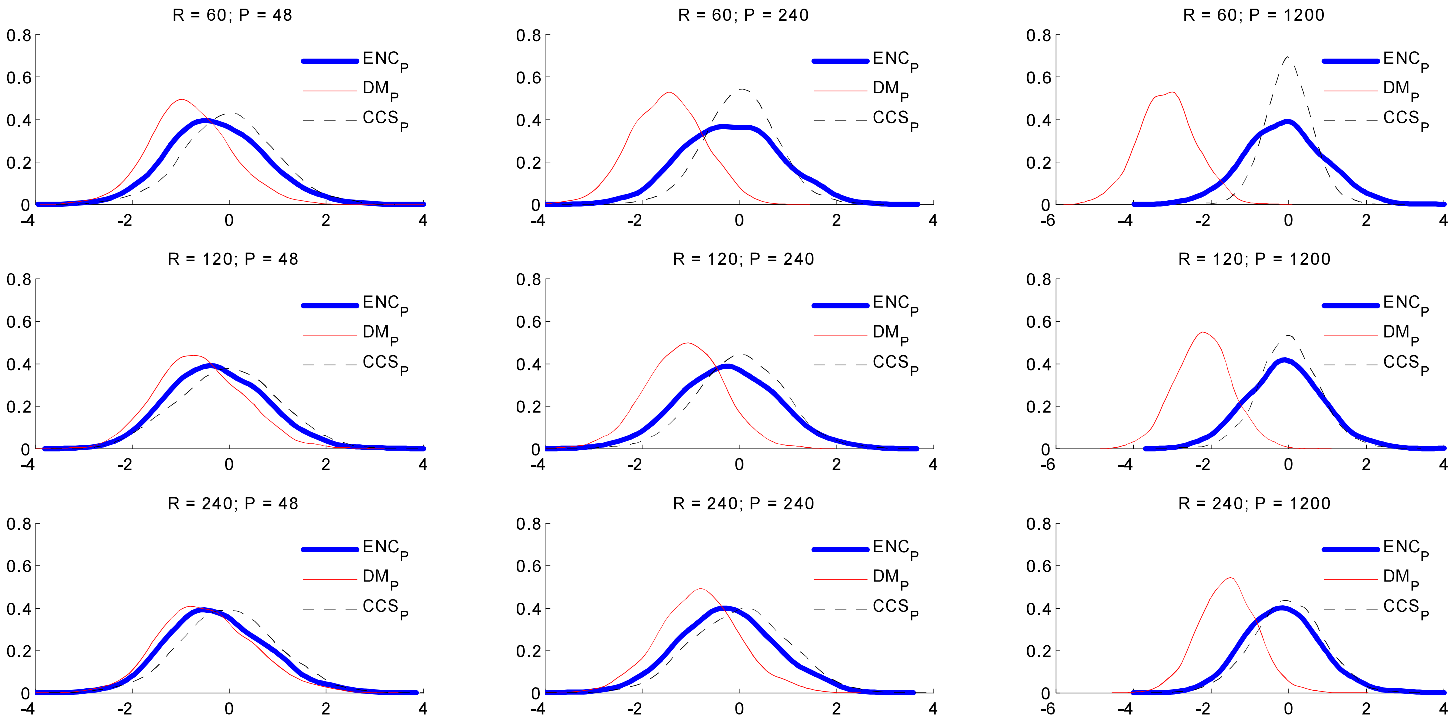

Figure 2.3: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0.99$, $b = 0$, $\sigma_e = 1$, with intercept on small model, 2000 Repeats.

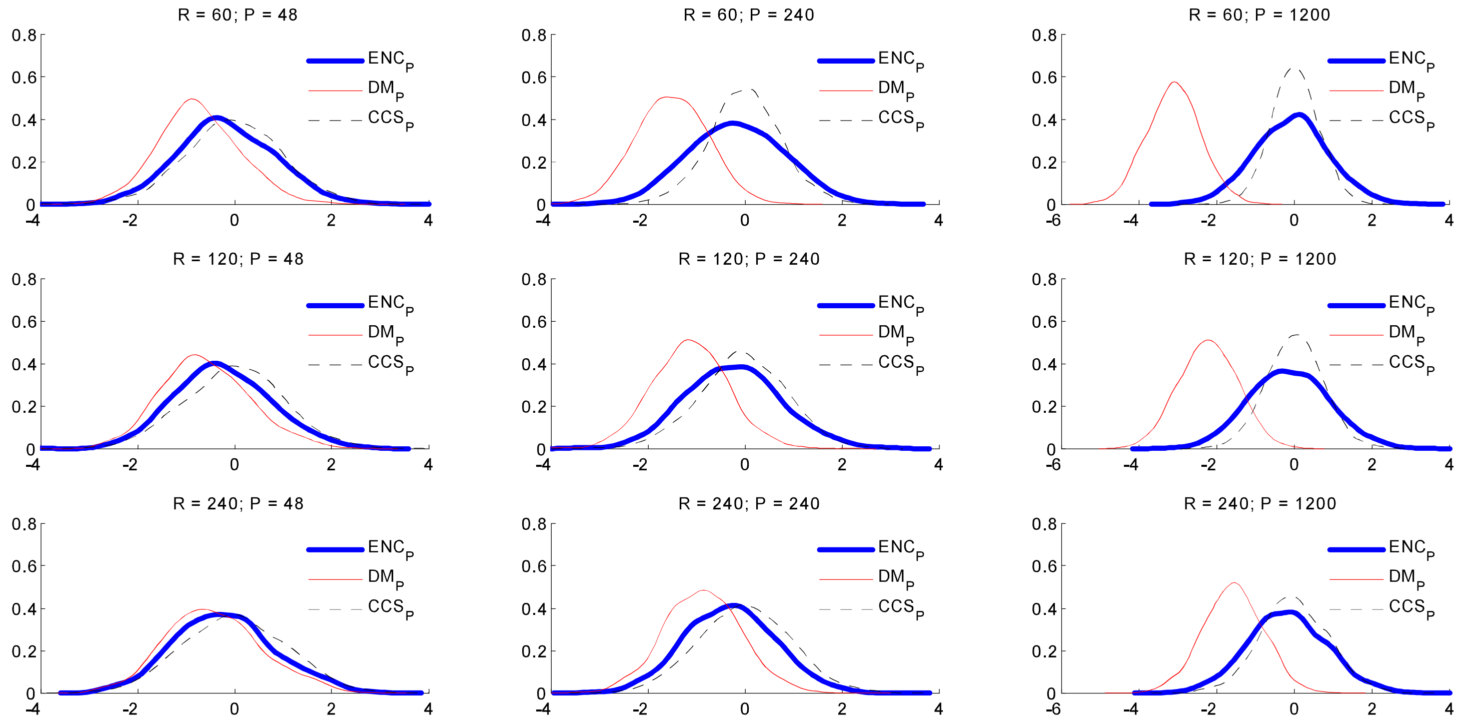

Figure 2.4: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0.99$, $b = 0$, $\sigma_e = 0.1$, with intercept on small model, 2000 Repeats.

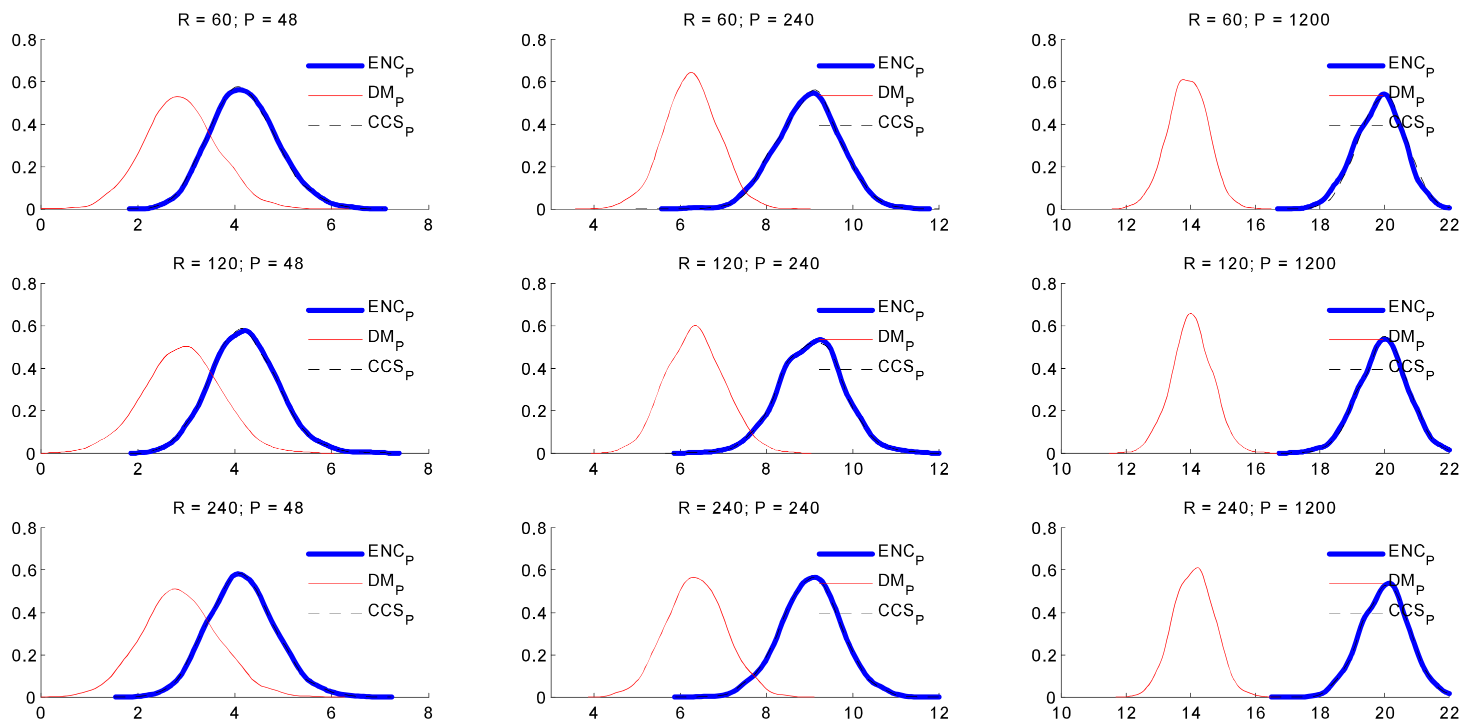

Figure 2.5: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 0.1$, $\sigma_e = 0.1$, with intercept on small model, 2000 Repeats.

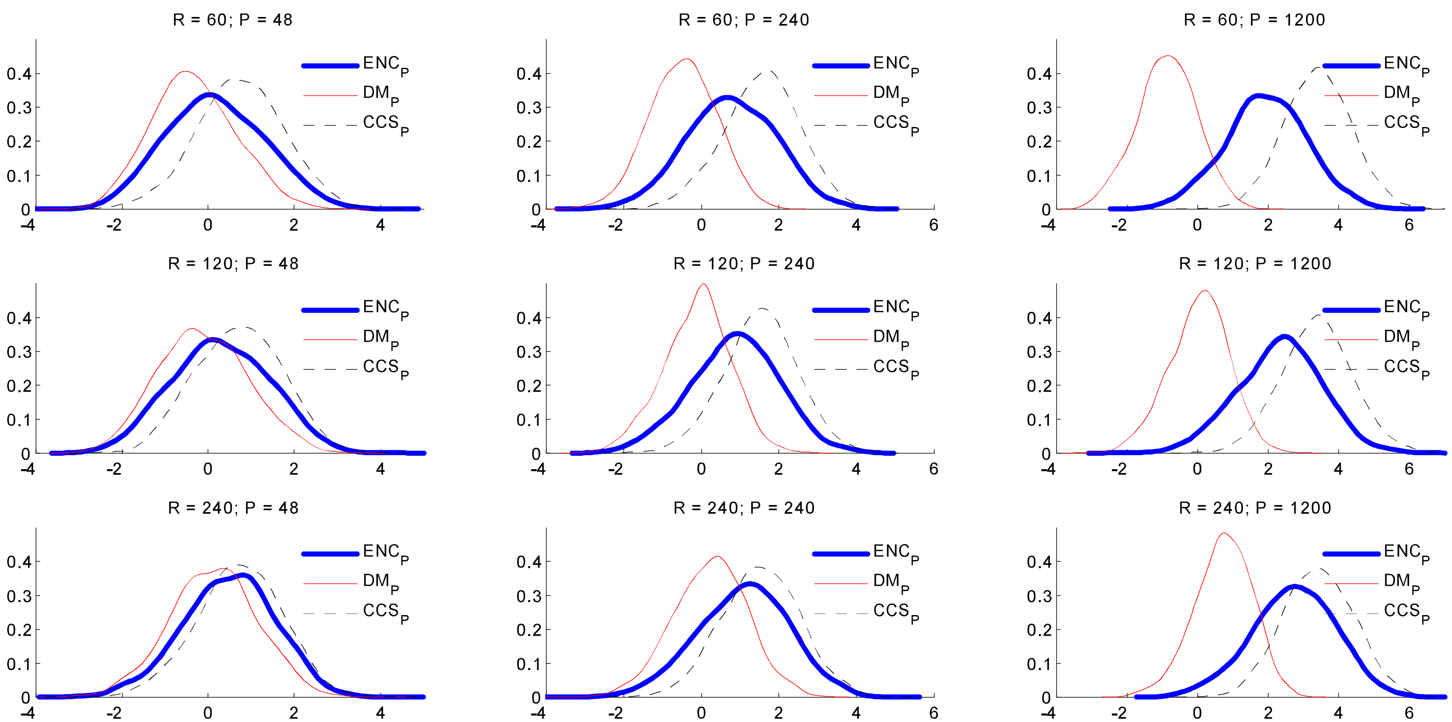

Figure 2.6: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 0.1$, $\sigma_e = 1$, with intercept on small model, 2000 Repeats.

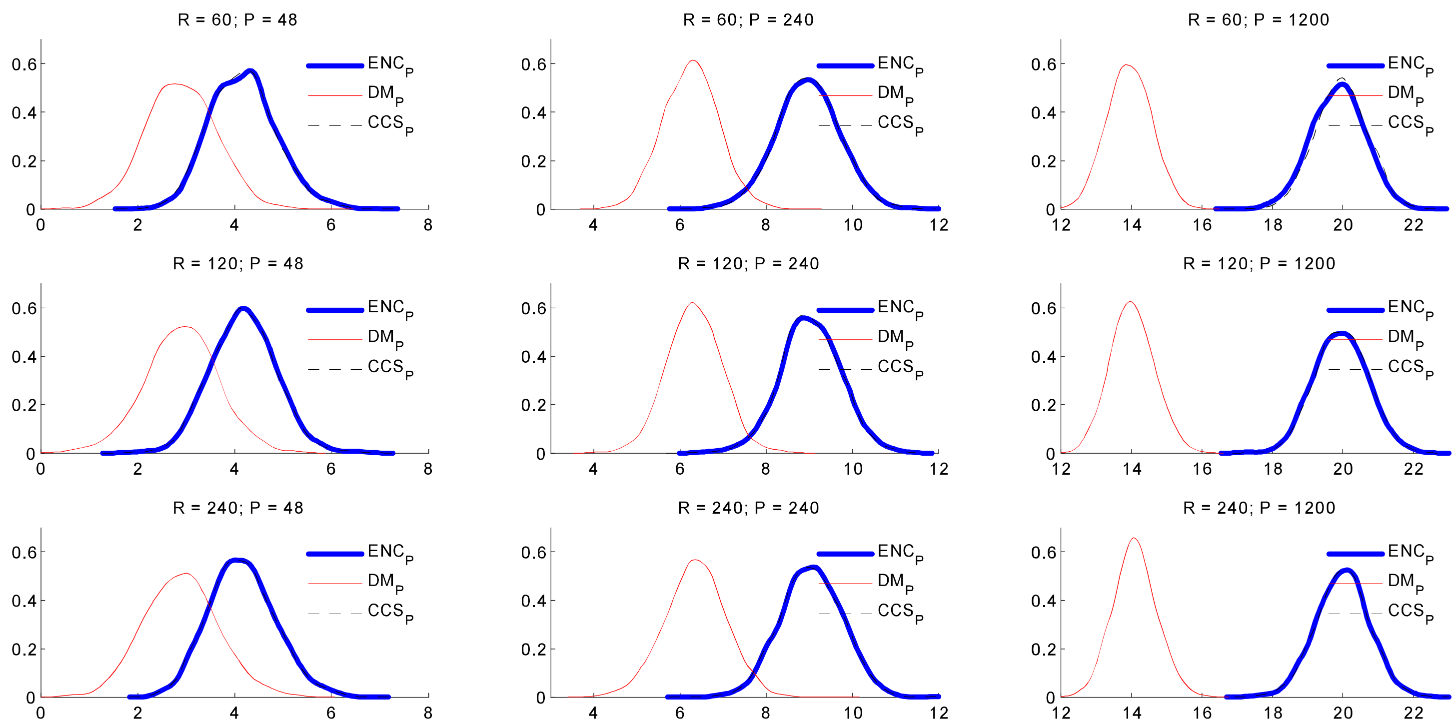

Figure 2.7: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 1$, $\sigma_e = 1$, with intercept on small model, 2000 Repeats.

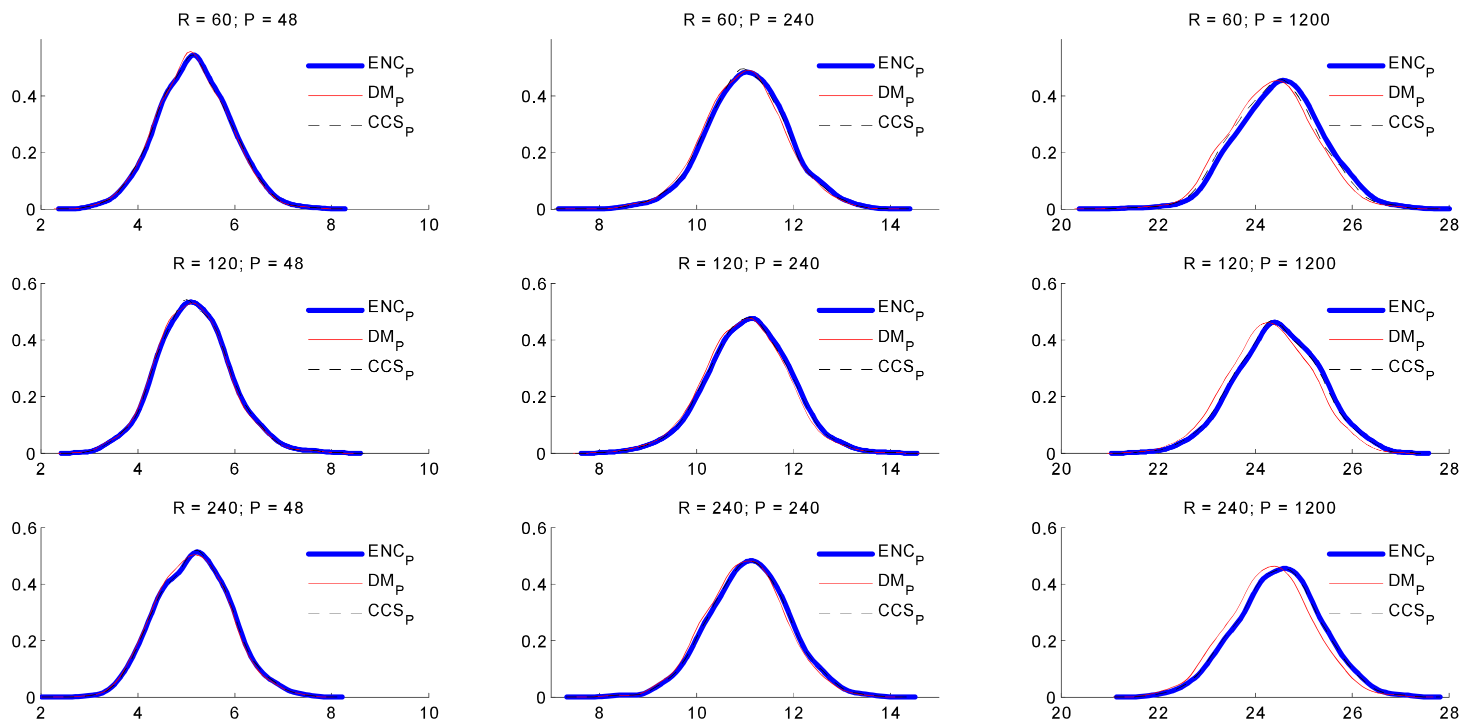

Figure 2.8: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 1$, $\sigma_e = 0.1$, with intercept on small model, 2000 Repeats.

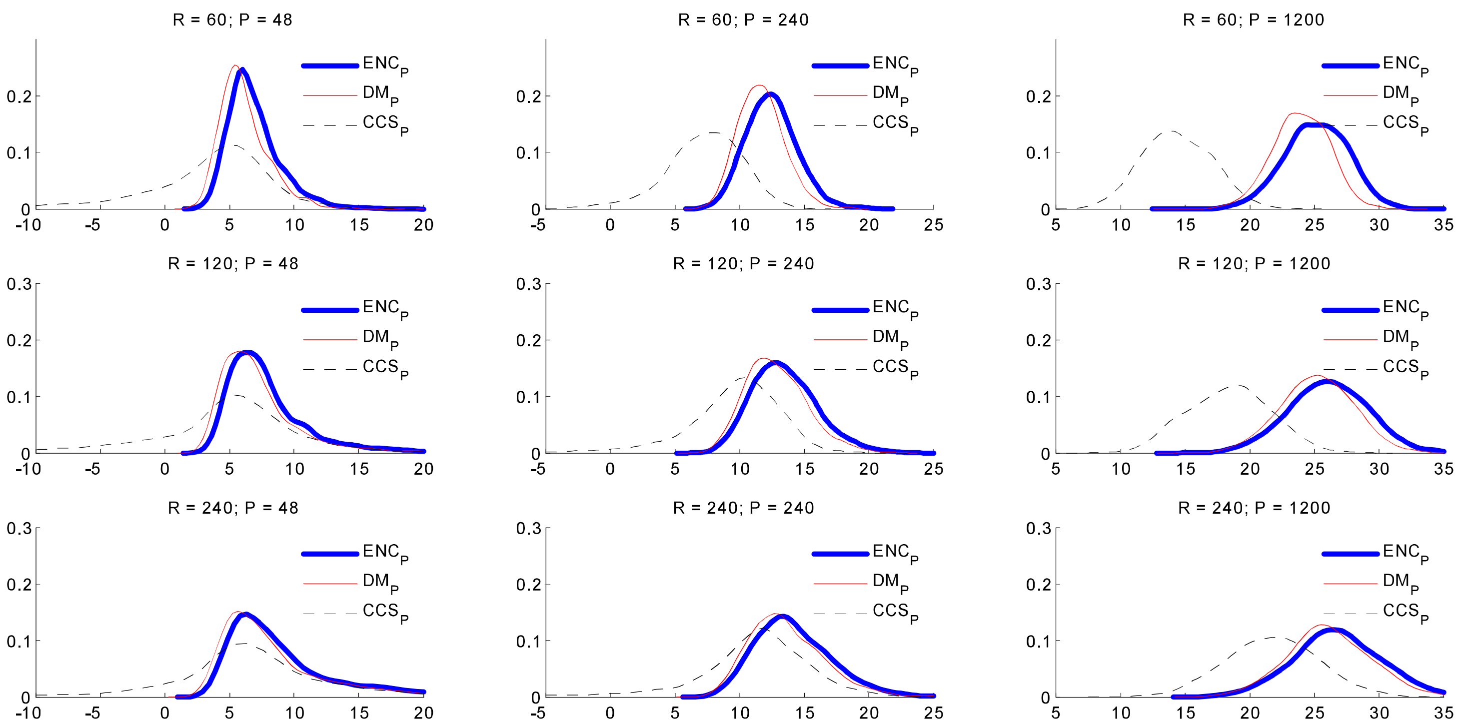

Figure 2.9: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 0.1$, $\sigma_e = 0.1$, with intercept on small model, 2000 Repeats.

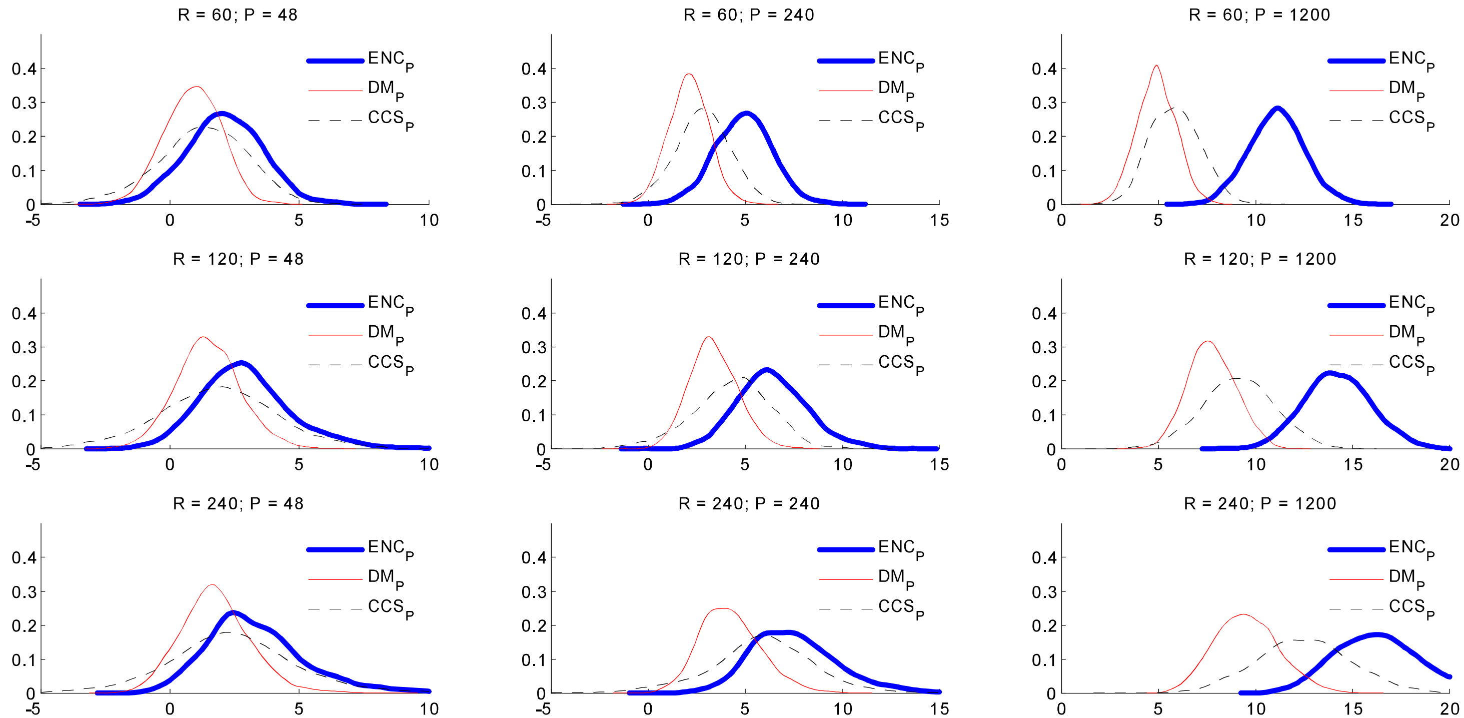

Figure 2.10: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 0.1$, $\sigma_e = 1$, with intercept on small model, 2000 Repeats.

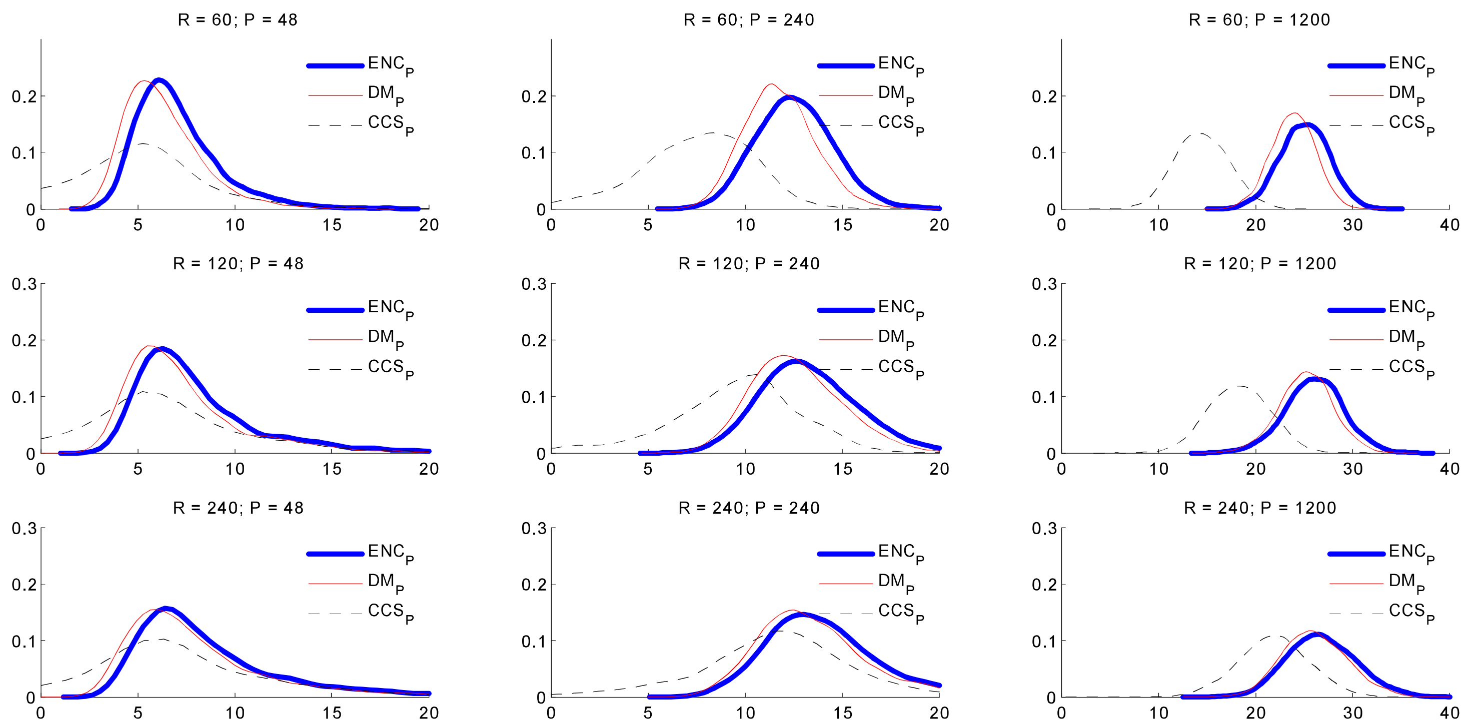

Figure 2.11: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 1$, $\sigma_e = 1$, with intercept on small model, 2000 Repeats.

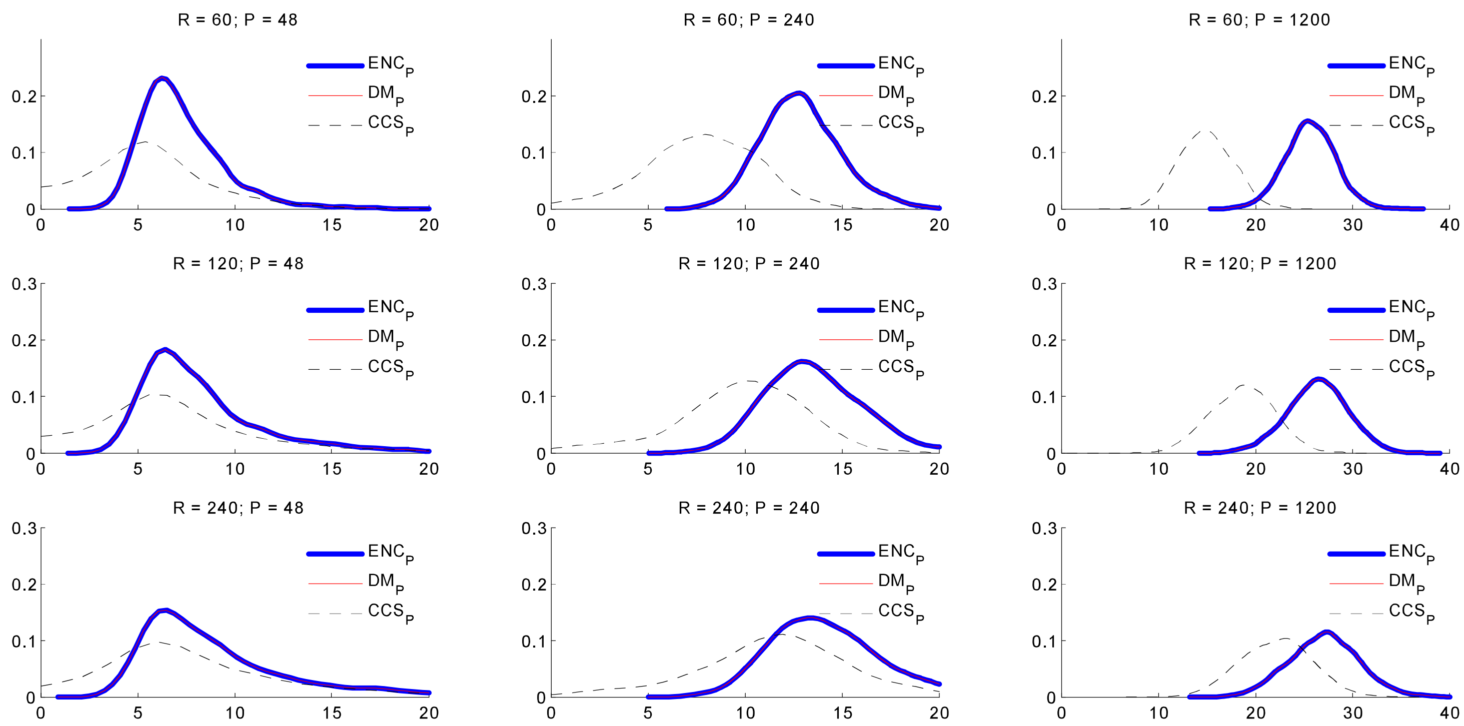

Figure 2.12: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 1$, $\sigma_e = 0.1$, with intercept on small model, 2000 Repeats.

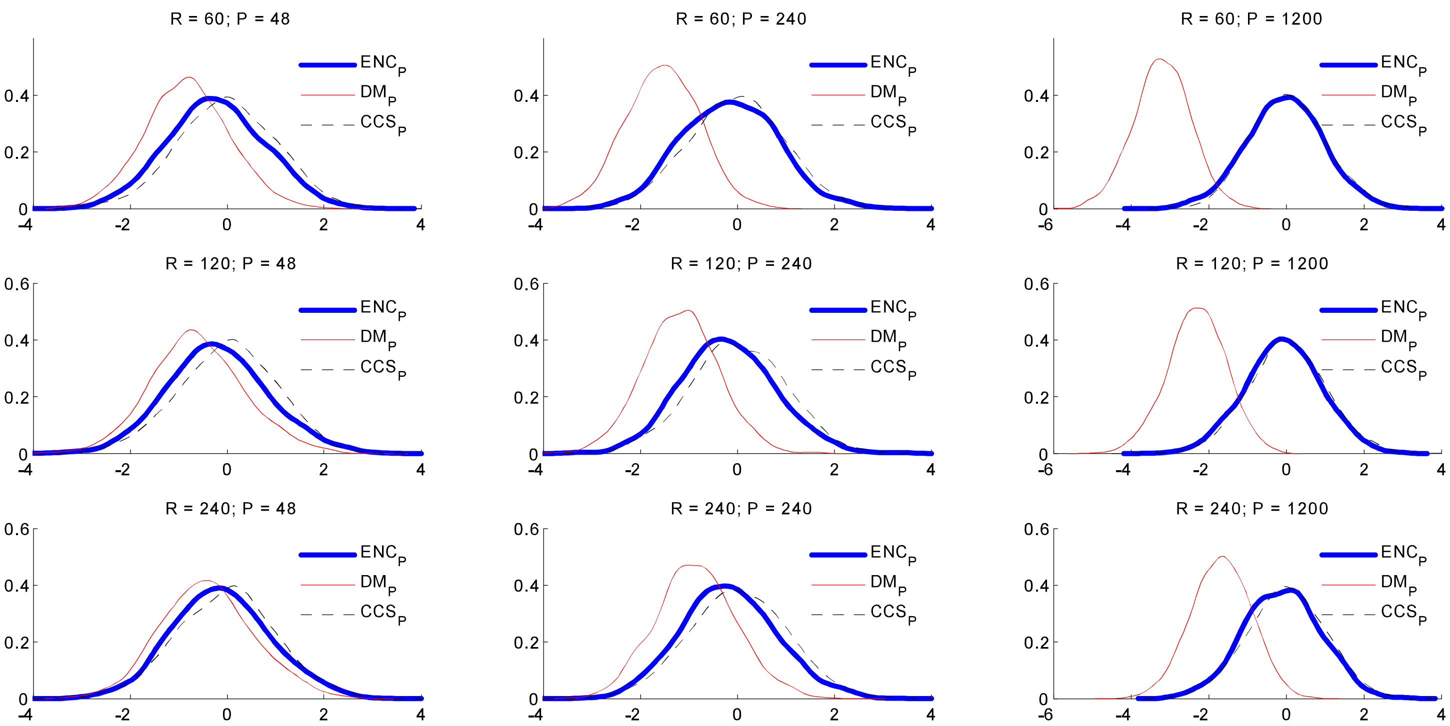

Figure 2.13: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0$, $b = 0$, $\sigma_e = 1$, without intercept on small model, 2000 Repeats.

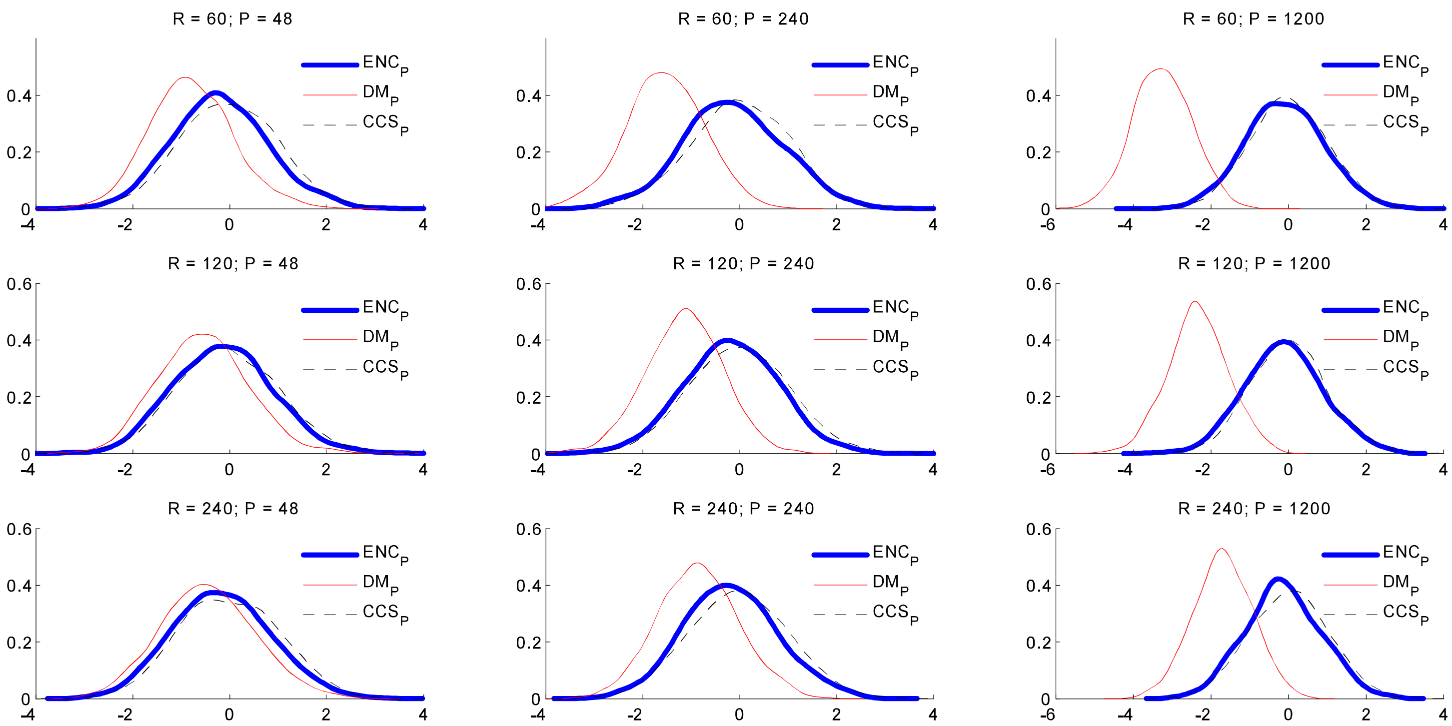

Figure 2.14: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0$, $b = 0$, $\sigma_e = 0.1$, without intercept on small model, 2000 Repeats.

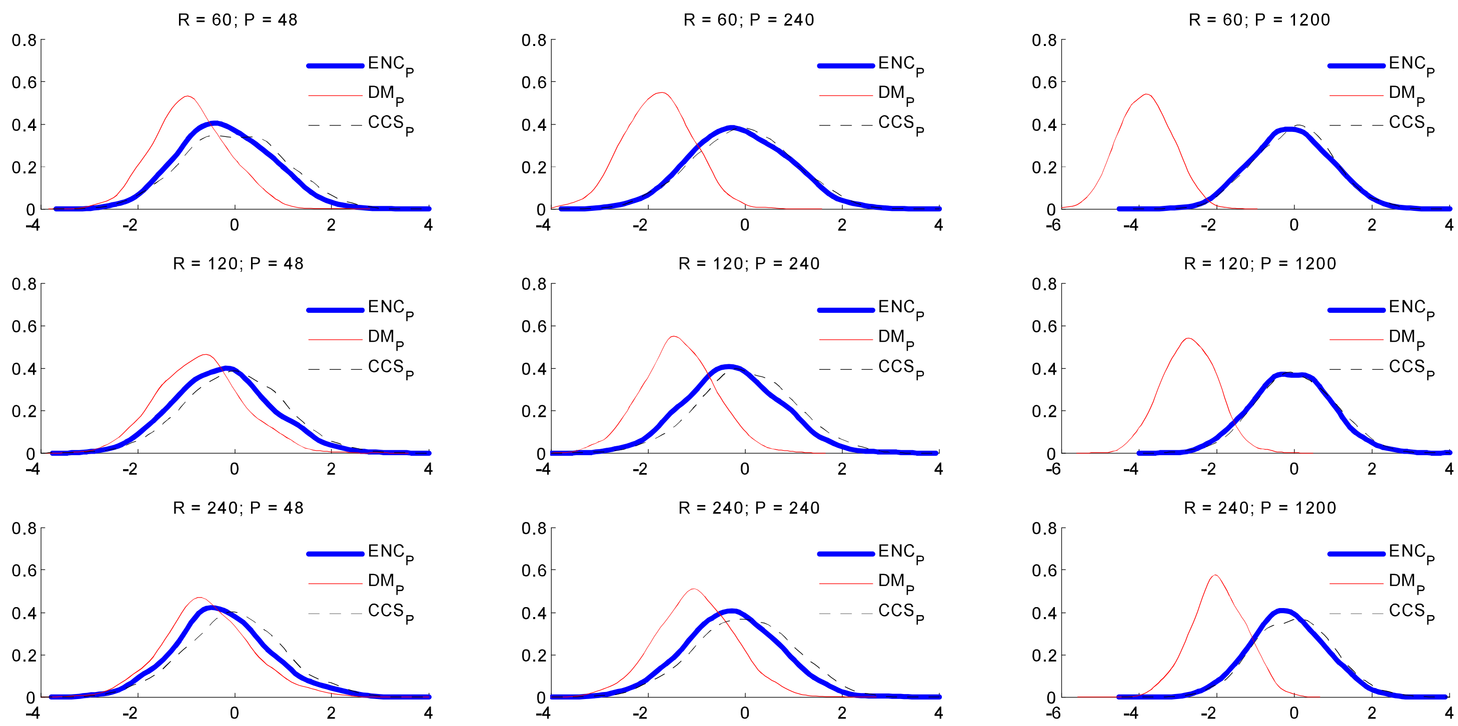

Figure 2.15: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0.99$, $b = 0$, $\sigma_e = 1$, without intercept on small model, 2000 Repeats.

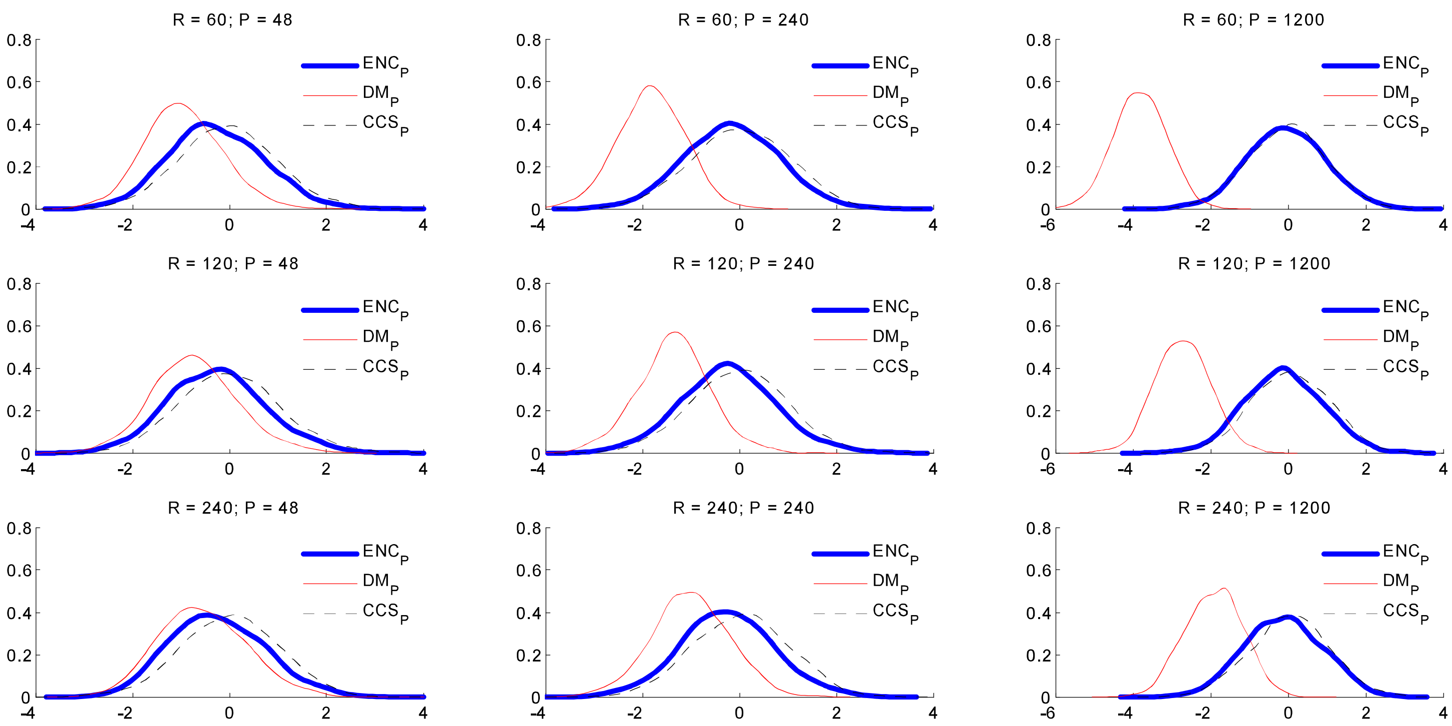

Figure 2.16: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_0$, $\phi = 0.99$, $b = 0$, $\sigma_e = 0.1$, without intercept on small model, 2000 Repeats.

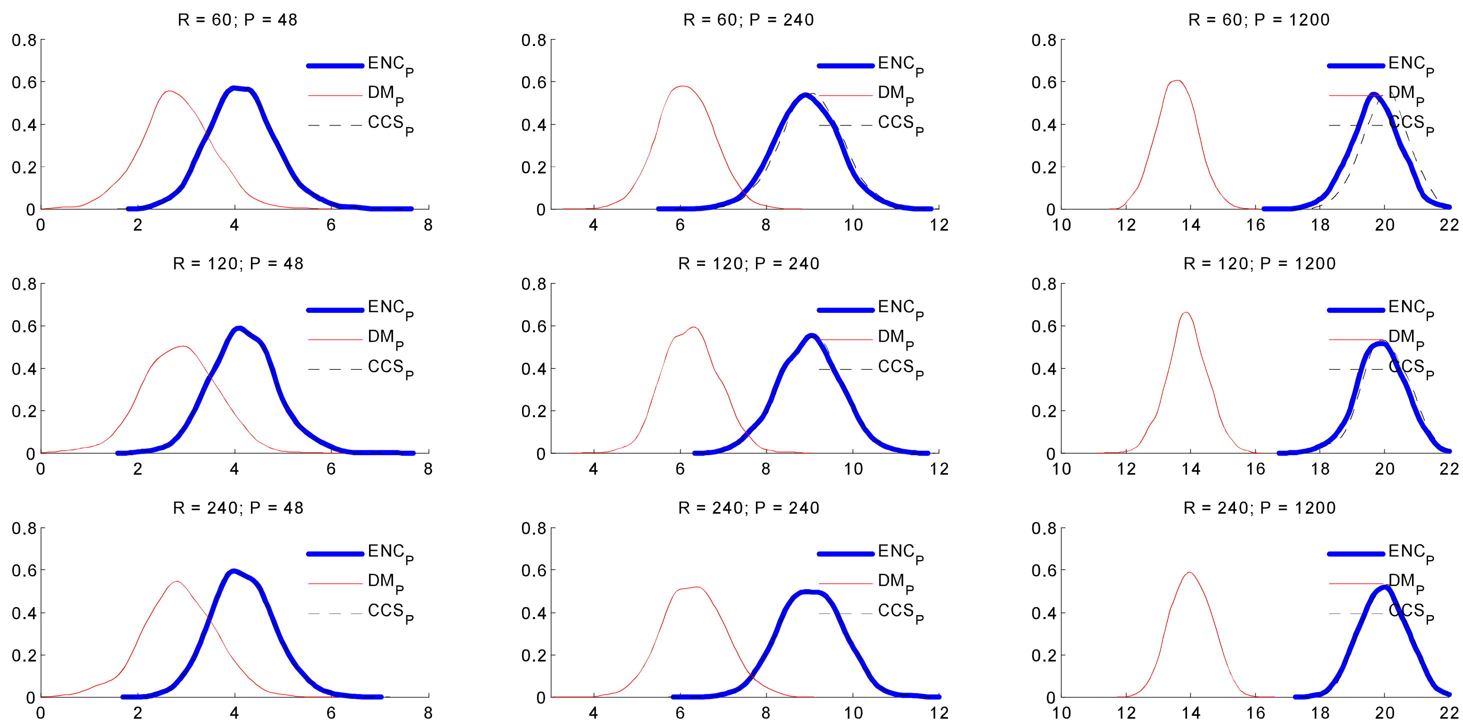

Figure 2.17: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 0.1$, $\sigma_e = 0.1$, without intercept on small model, 2000 Repeats.

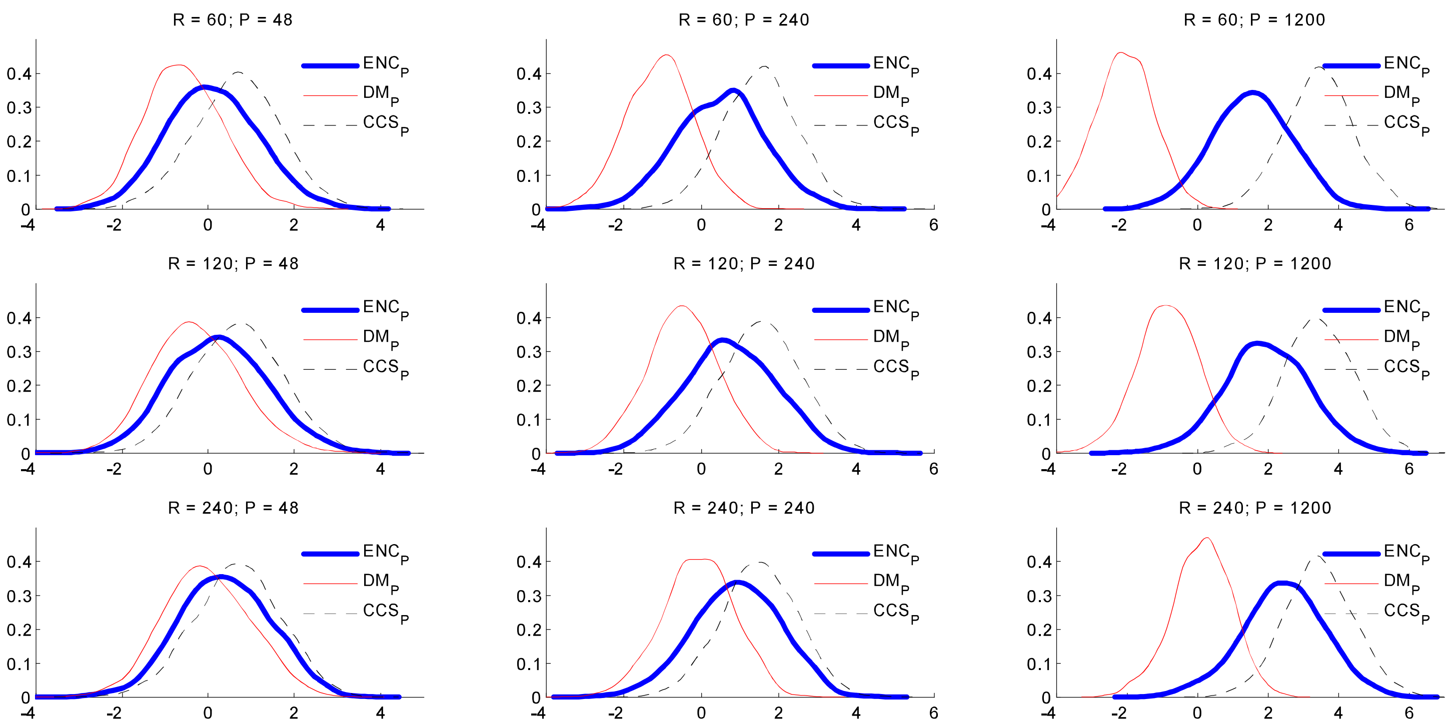

Figure 2.18: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 0.1$, $\sigma_e = 1$, without intercept on small model, 2000 Repeats.

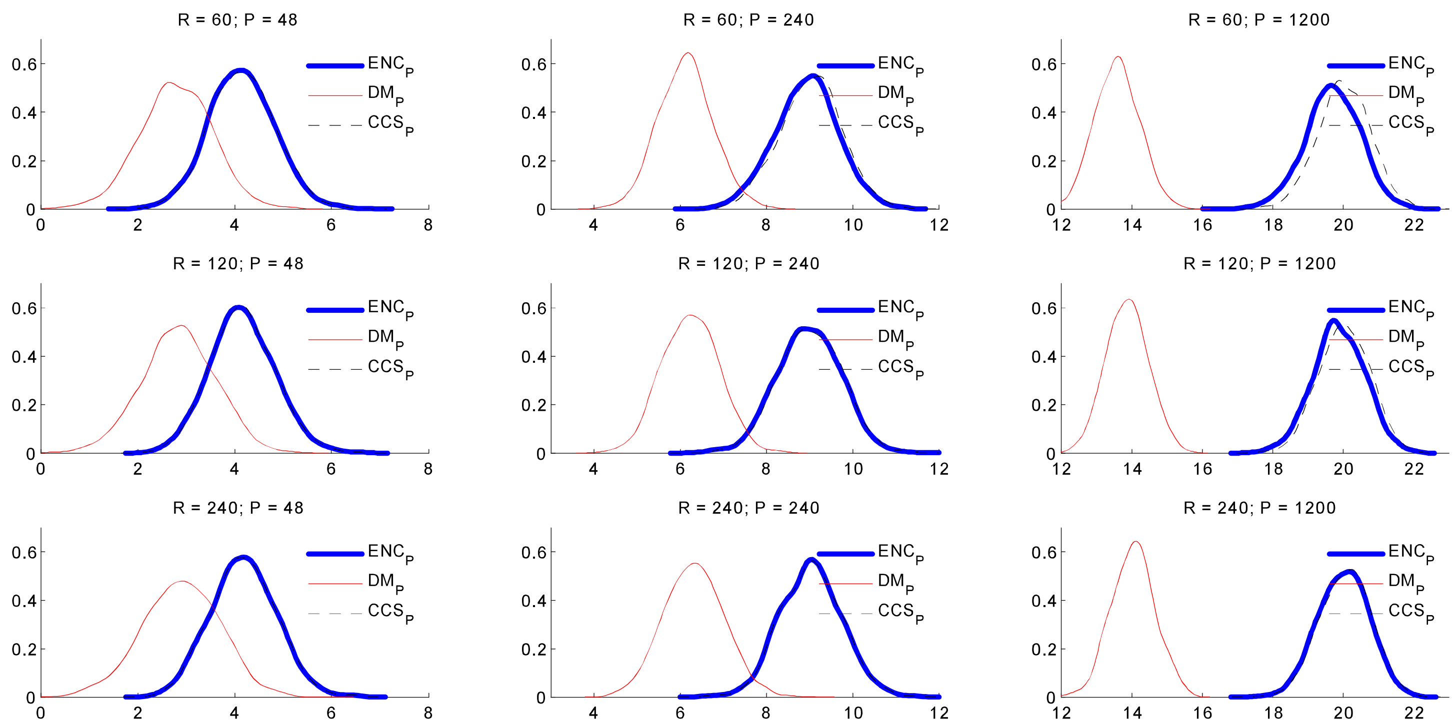

Figure 2.19: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 1$, $\sigma_e = 1$, without intercept on small model, 2000 Repeats.

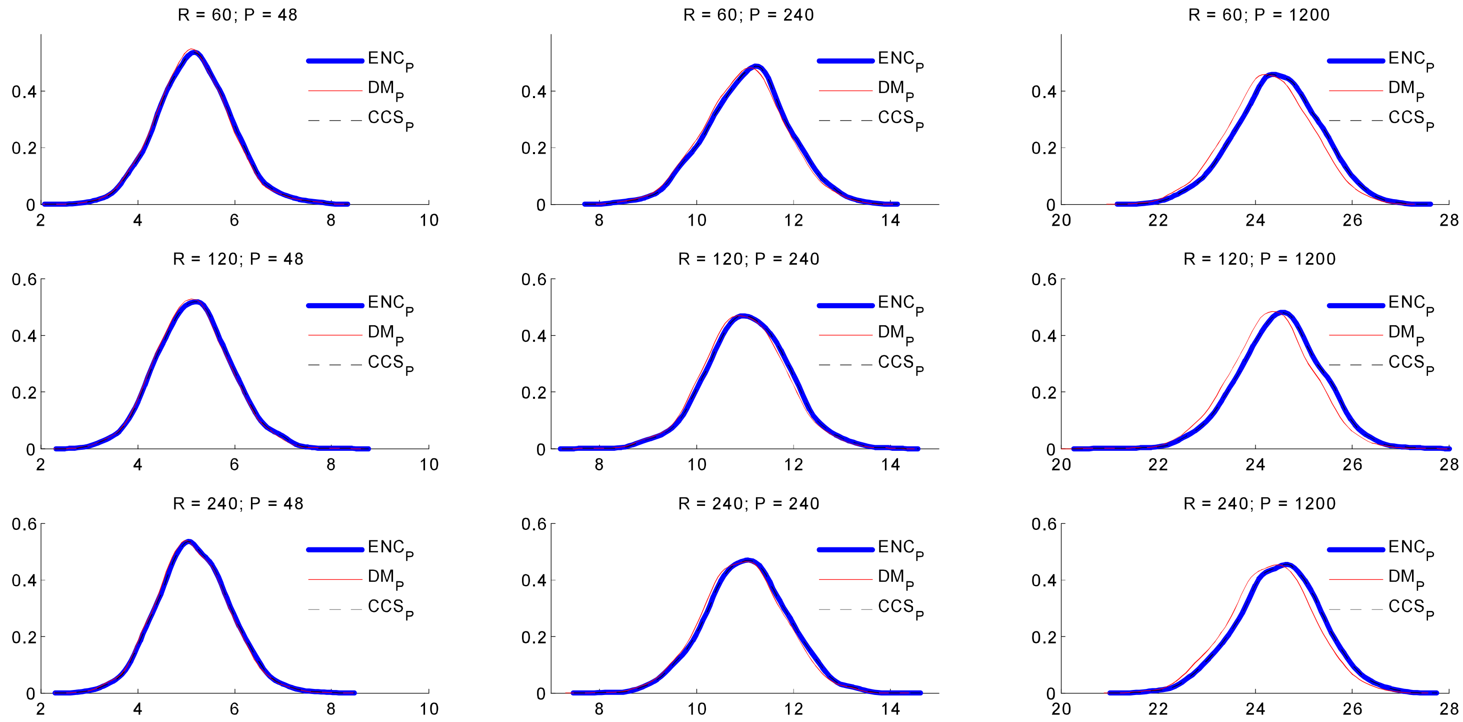

Figure 2.20: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0$, $b = 1$, $\sigma_e = 0.1$, without intercept on small model, 2000 Repeats.

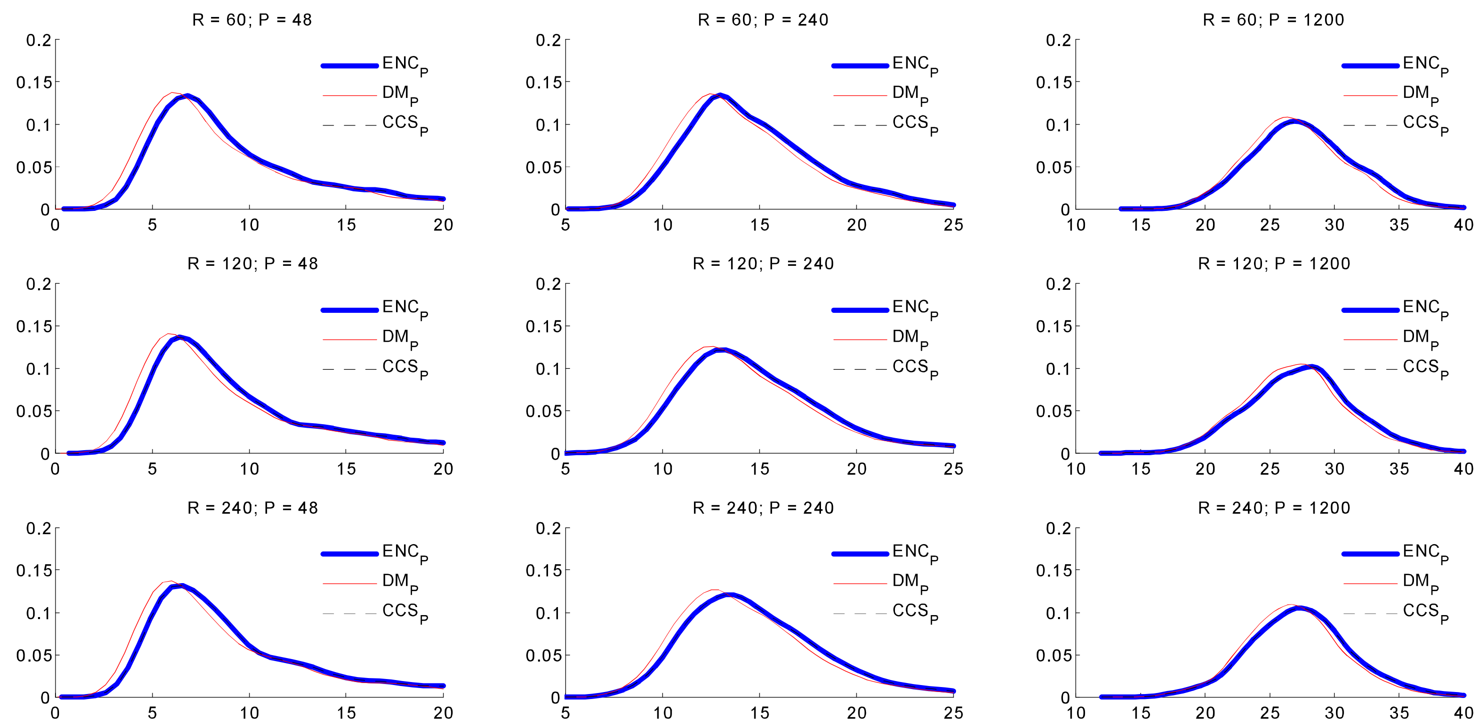

Figure 2.21: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 0.1$, $\sigma_e = 0.1$, without intercept on small model, 2000 Repeats.

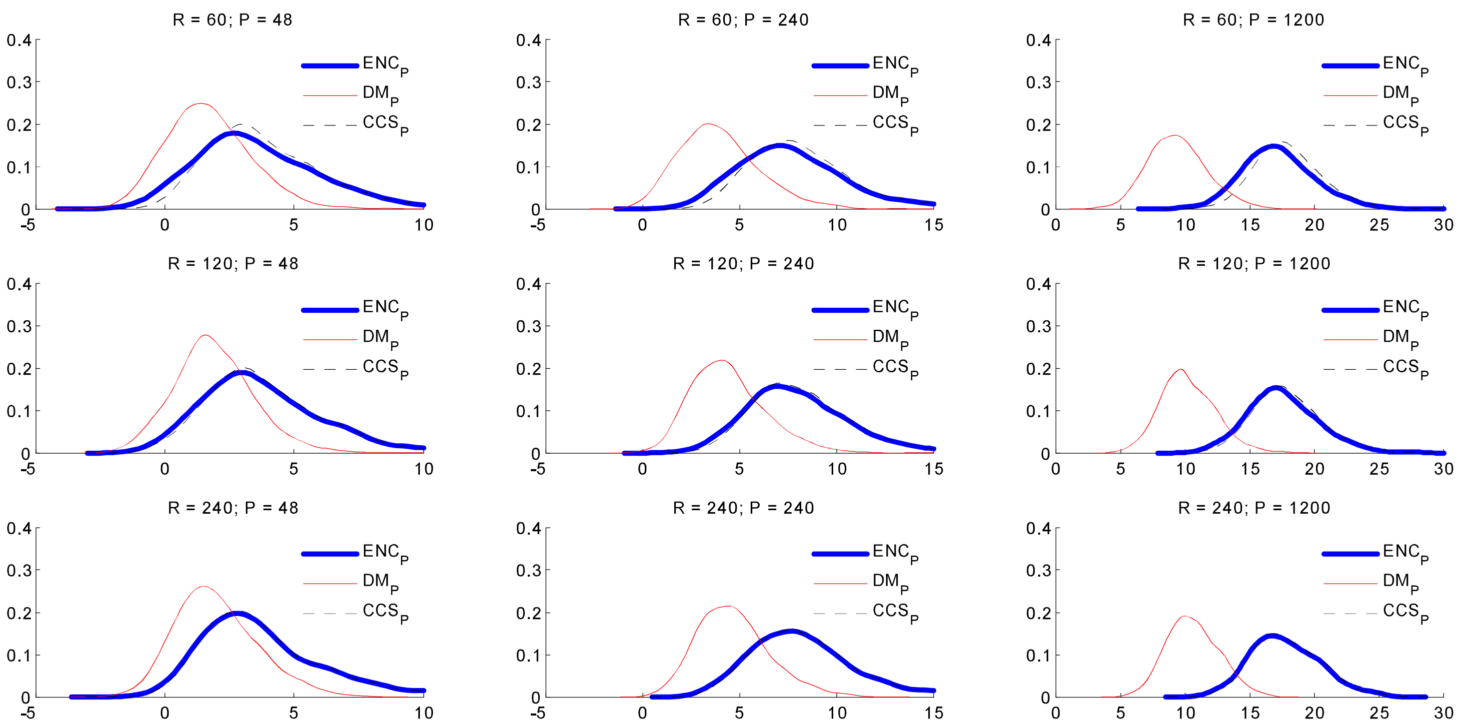

Figure 2.22: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 0.1$, $\sigma_e = 1$, without intercept on small model, 2000 Repeats.

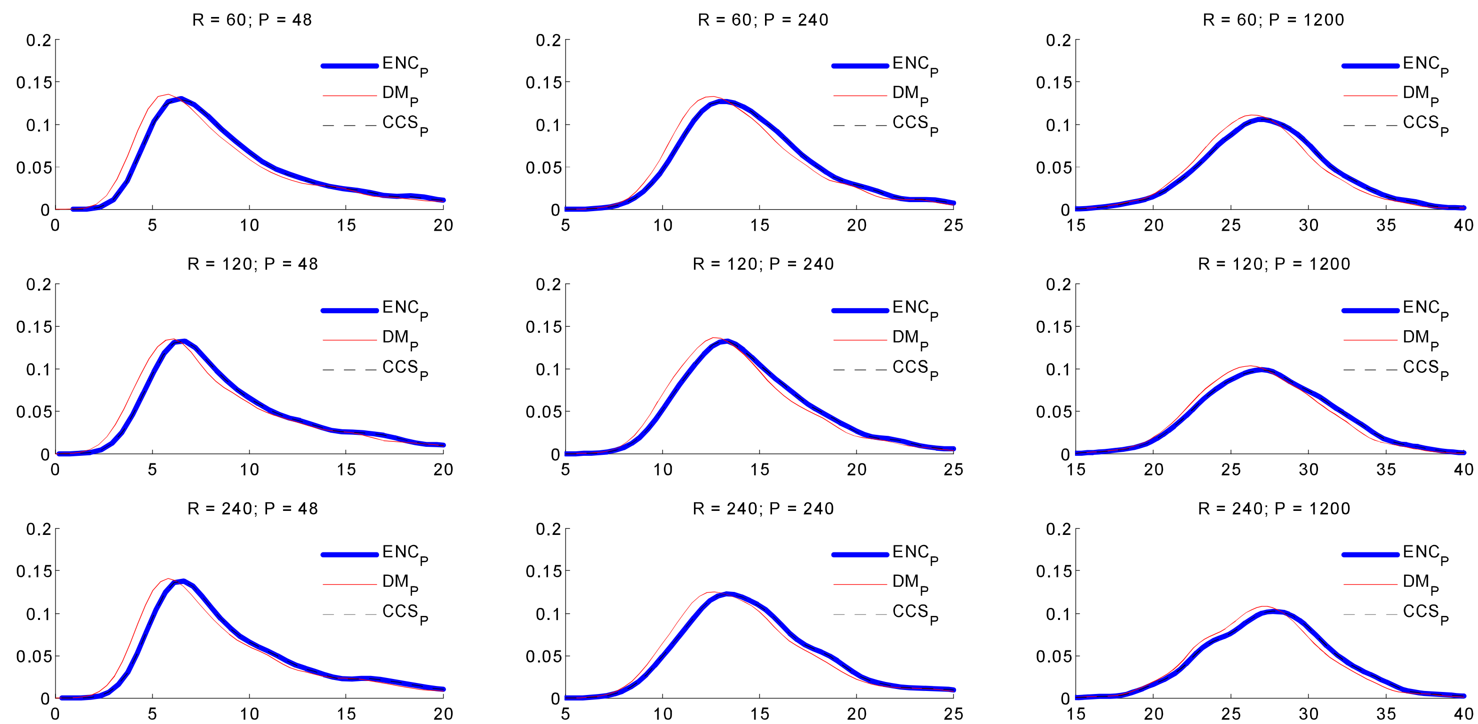

Figure 2.23: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 1$, $\sigma_e = 1$, without intercept on small model, 2000 Repeats.

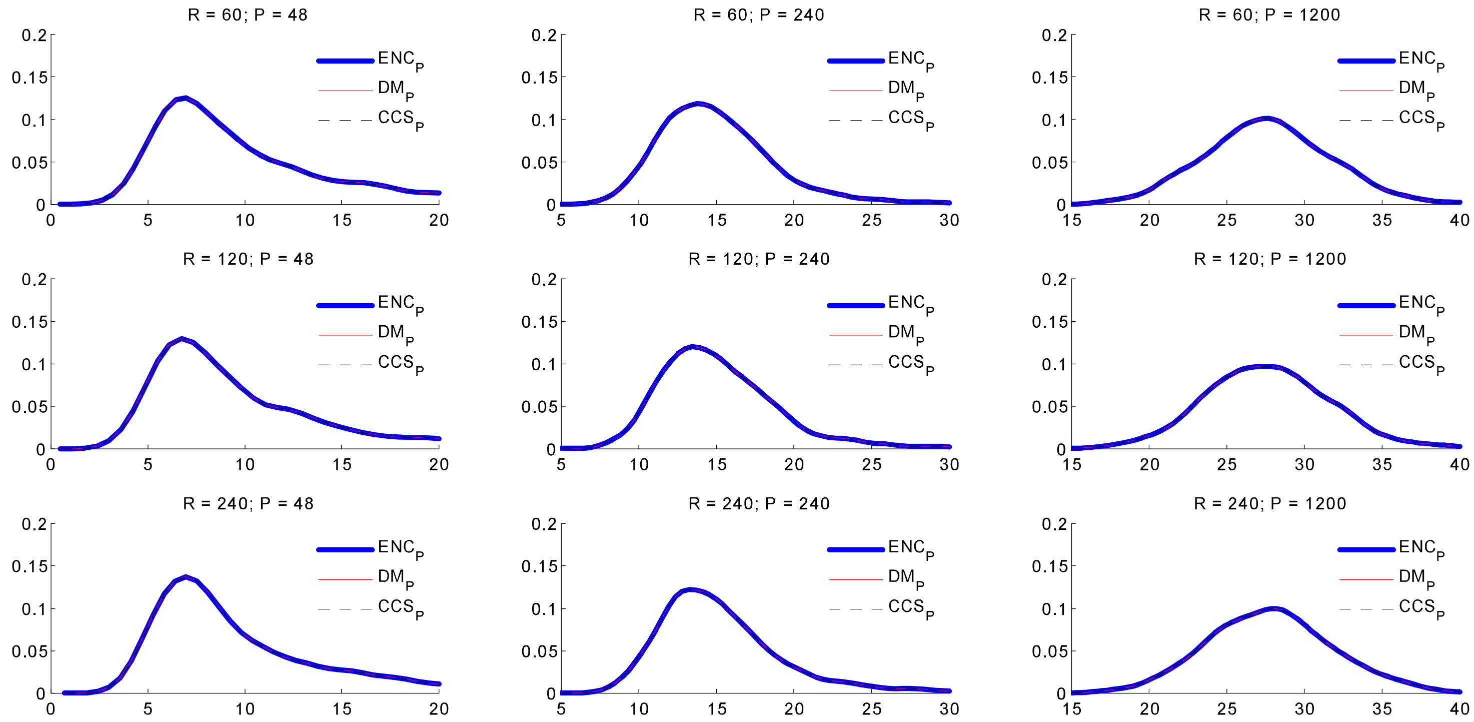

Figure 2.24: Monte Carlo distribution of ENC (blue line), DM (red line), and CCS (dashed line) under $\mathbb{H}_1$, $\phi = 0.99$, $b = 1$, $\sigma_e = 0.1$, without intercept on small model, 2000 Repeats.

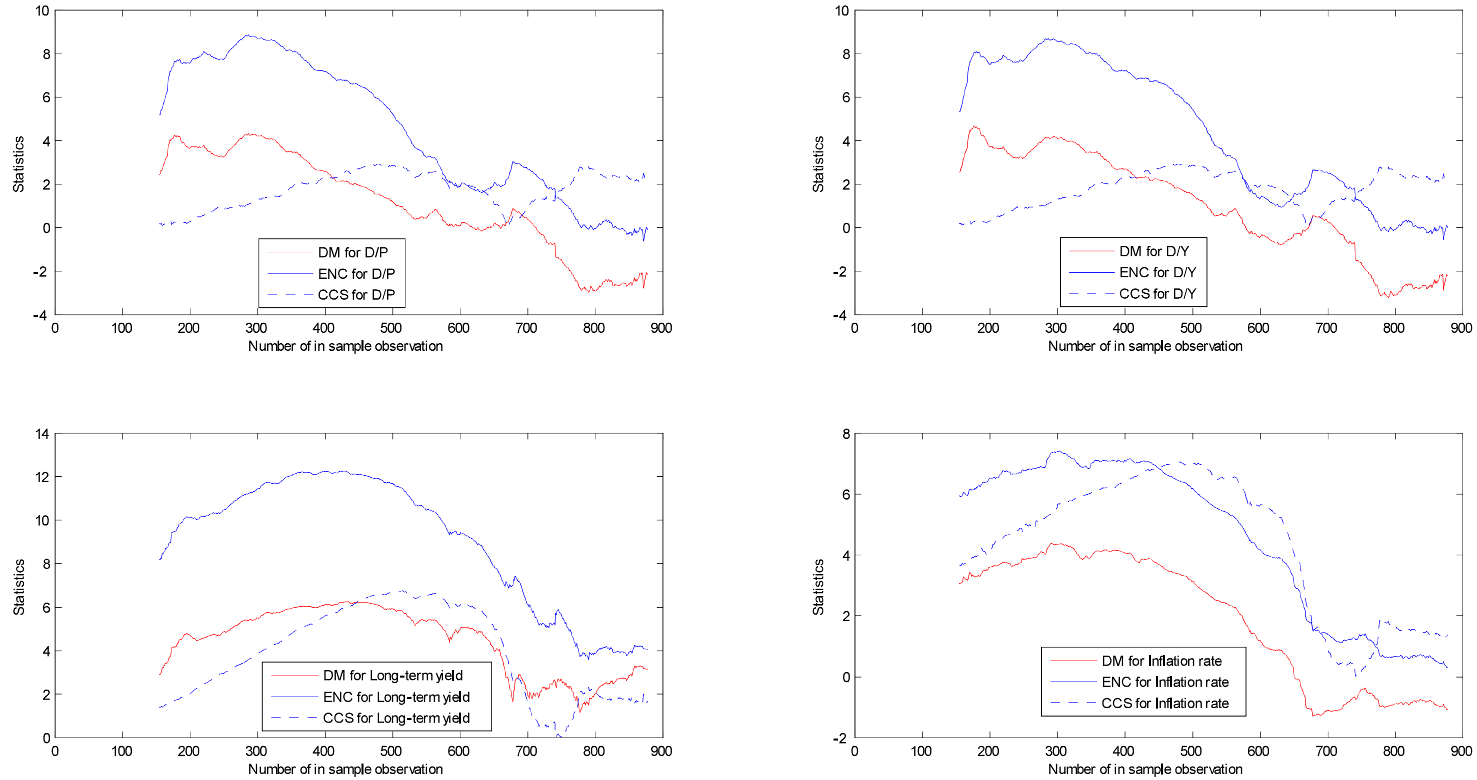

Figure 2.25: Testing for predictive ability of persistent predictors for equity premium

Encompassing Test for Nested Predictive Regression Models with Near Unit Root and Drift

Yan Ge* and Tae-Hwy Lee†

June 2015

Abstract

Phillips and Magdalinos (2007, 2009), Phillips (2014), and Phillips and Lee (2013, 2014) consider limit theory for predictive regressions when the predictor is persistent with an AR root in the vicinity of unity. Also, Phillips and Chen (2014) consider the limit theory when the persistent predictor contains drift. These authors show that the in-sample inference of the predictive regression is based on its asymptotic distribution which is mixed normal. In this paper, we adopt these predictive regression models for the out-of-sample comparison. We examine the asymptotic and finite sample properties of the encompassing statistic to test for out-of-sample Granger-causality with this type of predictors (with an AR root in the vicinity of unity and with drift). In contrast to the asymptotic mixed normal distribution, we show that the out-of-sample inference of the predictive regression using the encompassing test has the asymptotic standard normal distribution due to the faster convergence rate of the coefficient in big model. A Monte Carlo simulation shows that the asymptotic results also hold in finite sample.

*Department of Economics, University of California, Riverside, CA 92521. E-mail: yge001@ucr.edu

†Department of Economics, University of California, Riverside, CA 92521. E-mail: taelee@ucr.edu

Key Words: predictive regression, persistent predictor, drift, encompassing test, asymptotic normality.

JEL Classification: C53, E37, E27

1 Introduction

A regression with a highly persistent estimator is being studied by a lot of econometricians, Phillips and Lee (2013, PL thereafter) considered the predictive regression model with local to unit root (LUR) process and derived the asymptotic distribution of the estimator. Phillips and Magdalinos (2009) use the instrumental variable method when the predictor is endogenous. Chen and Deo (2009, 2010, 2011) studied the restricted maximum likelihood when the estimator is a unit root process, Phillips and Chen (2014, PC thereafter) considered the predictive mode in which the autoregressive predictor contains both stochastic trend and deterministic trend. They derived the convergence rate of the predictive regression equation and pointed out that a highly persistent estimator $\{x_t\}$ leads to faster than or equal to $\sqrt{n}$ convergence rate of the estimator, they also showed how the scale of the drift term interact the convergence rate of the estimator. Our paper is based on the model and conclusions from PC (2014), extending from in-sample model to out-of-sample model, we compare the equal predictive accuracy between two nested models. We follow PL (2013), PC (2014), using the rolling scheme. We use encompassing test and show that if the predictor is persistent, due to faster than or equal to the standard $\sqrt{n}$ rate, the encompassing test is robust as it has the correct size under the null hypothesis.

The organization of the paper is as follows. Section 1 is the introduction. Section 2 presents the nested model and three test statistics. Section 3 presents the asymptotic distribution of the encompassing test if the predictor is weak stationary and the ratio of out-of-sample to in-sample observation is finite. Section 4 presents the asymptotic distribution of the encompassing test if the predictor is weak stationary and the ratio of out-of-sample to in-sample observation is infinite. Section 5 considers predictive regression with persistent predictor containing drift and we consider six cases. Section 6 presents the theorems. Section 7 is Monte-Carlo Simulation, Section 8 concludes.

2 Comparing Nested Conditional Mean Models

We create the following nested model to test if the regressor x_t has predictive power on y_{t+1}

$$\text{Model 1} \quad : \quad y_{t+1} = x'_{1,t}\Gamma_1 + e^{(1)}_{t+1} = \pi_1 + e^{(1)}_{t+1}, \tag{1}$$

$$\text{Model 2} \quad : \quad y_{t+1} = x'_{2,t}\Gamma_2 + e^{(2)}_{t+1} = \pi_2 + \beta x_t + e^{(2)}_{t+1}, \tag{2}$$

where π_i is the constant term for Model i, x_t is the predictor defined in Equation (10). Under the null hypothesis, $\beta = 0$ and $e^{(1)}_{t+1} = e^{(2)}_{t+1}$ (denoted as e_{t+1}). At each time t, both π_i and β are estimated with the rolling window of size R up to time t. Therefore

$$\begin{aligned} \hat{\pi}_{1,t} - \pi_1 &= B_1(t) H_1(t) \\ \left(\hat{\pi}_{2,t}, \hat{\beta}_t\right)' - (\pi_2, \beta)' &= B_2(t) H_2(t) \end{aligned}$$

where $x'_{1,t} = 1$, $x'_{2,t} = (1\ x_t)$, $q_{i,t} = x'_{i,t} x_{i,t}$ for Model i at time t, and

$$B_i(t) = \left(R^{-1} \sum_{j=t-R}^{t-1} q_{i,j} \right)^{-1},$$

$h_{i,t} = x'_{i,t} e_{t+1}$ and $H_i(t) = R^{-1} \sum_{j=t-R}^{t-1} h_{i,t}$. Let $f^{(1)}_{t+1} = \hat{\pi}_{1,t}$ be the forecasts for Model 1 and $f^{(2)}_{t+1} = \hat{\pi}_{2,t} + \hat{\beta}_t x_t$ be the forecast for Model 2 at time t and $\hat{e}^{(1)}_{t+1} = y_{t+1} - f^{(1)}_{t+1}$, $\hat{e}^{(2)}_{t+1} = y_{t+1} - f^{(2)}_{t+1}$ be the forecast errors with the squared forecast-error loss

$$L\left(\hat{e}^{(i)}_{t+1}\right) \equiv \left(\hat{e}^{(i)}_{t+1}\right)^2, \qquad i = 1, 2.$$

To test for equal predictive accuracy of the two models, the null hypothesis is

$$\mathbb{H}_0 : \mathbb{E}\left[L\left(\hat{e}^{(1)}_{t+1}\right) - L\left(\hat{e}^{(2)}_{t+1}\right) \right] = 0. \tag{3}$$

Under $\mathbb{H}_0$, x_t does not Granger-cause y_{t+1} in mean. Under the alternative hypothesis that x_t Granger-causes y_{t+1} in mean,

$$\mathbb{H}_1 : \mathbb{E}\left[L\left(\hat{e}^{(1)}_{t+1}\right) - L\left(\hat{e}^{(2)}_{t+1}\right) \right] > 0. \tag{4}$$

The Diebold-Mariano squared-error loss differential is defined as

$$\hat{D}_P = P^{-1} \sum_{t=R}^{T} L\left(\hat{e}^{(1)}_{t+1}\right) - L\left(\hat{e}^{(2)}_{t+1}\right), \tag{5}$$

and the adjusted MSFE loss-differential (ENC) is defined as

$$\begin{aligned}\hat{B}_P &= \hat{D}_P + P^{-1}\sum_{t=R}^{T}\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right)^2 \\ &= 2P^{-1}\sum_{t=R}^{T}\hat{e}_{t+1}^{(1)}\left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right),\end{aligned}$$

where R is the number of observations in the rolling windows for the in-sample estimation, P is the number of out-of-sample forecasts, and $R + P = T + 1$. We also compare DM and ENC with CCS test by Chao et al (CCS 2001), which is constructed as follows: Under $\mathbb{H}_0$, $\beta = 0$, which implies $\mathbb{E}\left(e_{t+1}^{(1)}x_t\right) = 0$. The out-of-sample test statistic of CCS is constructed from

$$\hat{M}_P = P^{-1}\sum_{t=R}^{T}\hat{e}_{t+1}^{(1)}x_t. \tag{6}$$

The three statistics are standardized to form the DM statistic $DM_P \equiv \hat{S}_P^{-0.5}\sqrt{P}\hat{D}_P$, the encompassing statistic $ENC_P \equiv \hat{Q}_P^{-0.5}\sqrt{P}\hat{B}_P$, and the CCS statistic $CCS_P \equiv \hat{W}_P^{-0.5}\sqrt{P}\hat{M}_P$, where $\hat{S}_P, \hat{Q}_P$ and $\hat{W}_P$ are the consistent estimators of $S_P = var\left(\sqrt{P}\hat{D}_P\right), Q_P = var\left(\sqrt{P}\hat{B}_P\right)$, and $W_P = var\left(\sqrt{P}\hat{M}_P\right)$, respectively.

3 Asymptotic Distribution of ENC with a Stationary Predictor when P/R is finite

We consider a stationary predictor as in CM (2001, 2005), CW (2006, 2007), and CCS (2001).

Assumption 1a. $\{x_t\}$ is a weakly stationary process and $\mathbb{E}\left(q_{i,t}\right)$ is bounded for all t and $i = 1, 2$. We define $B_i = \left(\mathbb{E}q_{i,t}\right)^{-1}$ for model $i = 1, 2$.

Let $\pi = \lim_{P,R\to\infty} P/R$ and $\xi = R/T = R/\left(P + R\right)$. Note that $1/\xi - 1 \to \pi$. We consider three cases on π :

Assumption 2a. $0 < \pi < \infty$.

Assumption 2b. $\pi = 0$ (or $\xi \to 1$).

Assumption 2c. $\pi = \infty$ (or $\xi \to 0$).

Proposition 1 (CM 2001). Under Assumption 1a and Assumption 2a,

$$ENC_P \Rightarrow \frac{\int_{\xi}^{1} \xi^{-1} \left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1} \xi^{-2} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}},$$

under $\mathbb{H}_0$, where $W(s)$ is a Wiener process and $s \in [0,1]$. When Assumption 2a holds, the RHS of Equation (7) is *not* standard normal.

Proposition 2 (CM 2001). Under Assumption 1a and Assumption 2b,

$$ENC_P \Rightarrow \lim_{\xi \to 1} \frac{\int_{\xi}^{1} \xi^{-1} \left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1} \xi^{-2} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \sim N(0,1), \tag{7}$$

under $\mathbb{H}_0$, where $W(s)$ is a Wiener process and $s \in [0,1]$. When Assumption 2b holds, the RHS of Equation (7) is standard normal.

Remark 1. CM (2001) shows that when Assumption 2b holds ($\xi \to 1, \pi \to 0$) then ENC_P is asymptotically standard normal. However, CM (2001) does not consider the case when Assumption 2c holds ($\xi \to 0, \pi \to \infty$). In Section 4 below, we consider this case and show that ENC_P is still asymptotically standard normal.

Remark 2: CM (2001) assumes Assumption 1 that the predictor $\{x_t\}$ is weakly stationary and shows that ENC_P is asymptotically standard normal under Assumption 2b. In Section 6, we show that ENC_P is asymptotically standard normal under Assumption 2c when the predictor has a root local to unity.

4 Asymptotic Distribution of ENC with a Stationary Predictor when $P/R \to \infty$

In this section, we will show that the asymptotic distribution of ENC_P under $\mathbb{H}_0$, as shown in Equation (7), is asymptotically standard normal under Assumption 2c (when $\lim_{P,R\to\infty} P/R \to \infty$).

Proposition 3. Under Assumption 1a and Assumption 2c,

$$ENC_P \Rightarrow \lim_{\xi \to 0} \frac{\int_{\xi}^{1} \xi^{-1} \left[W(s) - W(s-\xi)\right] \mathrm{d}W(s)}{\sqrt{\int_{\xi}^{1} \xi^{-2} \left[W(s) - W(s-\xi)\right]^2 \mathrm{d}s}} \sim N(0,1), \tag{8}$$

under $\mathbb{H}_0$.

Proof: We firstly consider the numerator of equation (8) by dividing $[0, 1]$ to n equal segments and let $t = [Ts]$, where $[Ts]$ is the integer part of Ts and $s \in [0, 1]$. Since ξ is sufficiently small, we can write $\xi \equiv 1/n = 1 + P/R$. We discretize both the numerator and the denominator. Let $\{u_i\}_{i=1}^{n}$ be a mixing sequence drawn from the standard normal distribution $N(0, 1)$ with $E(u) = 0$ and $\text{var}(u) = 1$. Let $V_t = \sum_{i=1}^{t} u_i$ be the partial sum. Then we have $U_t = \sum_{i=1}^{t} u_i \sim N(0, t)$ and therefore

$$\frac{U_t}{\sqrt{n}} = \frac{\sum_{i=1}^{t} u_i}{\sqrt{n}} \equiv U_n(s) \Rightarrow W(s),$$

where $U_n(s)$ is a '*cadlag*' function and $W(s)$ is a Wiener process. Note that

$$\begin{aligned} n^{-1} \sum_{t=1}^{n} u_{t-1} u_t &= n^{-1} \sum_{t=1}^{n} U_{t-1} u_t - n^{-1} \sum_{t=1}^{n} U_{t-2} u_t \\ &\Rightarrow \int_{\xi}^{1} W(s) \mathrm{d}W(s) - \int_{\xi}^{1} W(s-\xi) \mathrm{d}W(s) \\ &= \int_{\xi}^{1} [W(s) - W(s-\xi)] \, \mathrm{d}W(s) \end{aligned}$$

Considering the term $\int_{\xi}^{1} [W(s) - W(s-\xi)]^2 \mathrm{d}s$ in the denominator, we have

$$n^{-2} \sum_{t=1}^{n} u_{t-1}^2 = n^{-2} \sum_{t=1}^{n} (U_{t-1} - U_{t-2})^2 \Rightarrow \int_{\xi}^{1} [W(s) - W(s-\xi)]^2 \, \mathrm{d}s.$$

We construct the an AR(1) regression model, regressing $\{u_{t+1}\}$ on $\{u_t\}$:

$$u_{t+1} = \delta u_t + v_{t+1}$$

The estimator $\hat{\delta}$ equals $\left(\sum_{t=1}^{n} u_{t-1} u_t\right) / \left(\sum_{t=1}^{n} u_{t-1}^2\right)$ and the variance $\hat{\delta}$ equals

$$\left(\sum_{t=1}^{n} u_{t-1}^2 \right)^{-1} var(u) = \left(\sum_{t=1}^{n} u_{t-1}^2 \right)^{-1}.$$

Therefore Equation (8) can be approximated by

$$\frac{\int_{\xi}^{1} \xi^{-1} [W(s) - W(s-\xi)] \, \mathrm{d}W(s)}{\sqrt{\xi^{-2} \int_{\xi}^{1} [W(s) - W(s-\xi)]^2 \, \mathrm{d}s}} \Rightarrow \frac{\sum_{t=1}^{n} u_{t-1} u_t}{\sqrt{\sum_{t=1}^{n} u_{t-1}^2}} \sim N(0, 1).$$

∎

5 Predictive Regression with Persistent Predictor and Drift

We consider the out-of-sample predictive ability using persistent estimator with local to zero drift μ in the following Phillips and Chen (2014) in the following sense.

$$y_{t+1} = \pi + \beta x_t + e_{t+1} \quad (9)$$

$$x_t = \rho^t x_0 + \mu \sum_{i=1}^{t-1} \rho^i + \tilde{x}_t \quad (10)$$

$$\tilde{x}_t = \rho\, \tilde{x}_{t-1} + e_{xt} \quad (11)$$

where

$$\rho = 1 + \frac{c}{T^\alpha}$$

$$\mu = \frac{\tilde{\mu}}{T^\gamma}$$

where $c < 0$, r is the coefficient of correlation of e_{xt} and e_{t+1}, α determines the magnitude of the stochastic trend. Equation (10) indicated that the autoregressive process $\{x_t\}$ is impacted by stochastic and deterministic trend. We see that when $\alpha = 0$ and $0 < -c < 2$, we have $|\rho| < 1$ and $\tilde{x}_t$ is a weak stationary process. When $\alpha = 1$, $\tilde{x}_t$ is a local to unit root process.. When $0 < \alpha < 1$, $\tilde{x}_t$ is moderately integrated to unit root process. The parameter γ determines the scale of drift, hence controls the deterministic trend of the predictor x_t. We category the interaction of drift and coefficient of AR process $\{x_t\}$ into seven cases. Phillips (2014) considered the local to unit root process $\{x_t\}$ with coefficient $\rho = 1 + c/T$, where T is the number of the predictors. he showed that the confidence interval can not be estimated using ADF test if $\{x_t\}$ is a weak stationary process. Phillips and Lee provided the asymptotic distribution of the coefficient of the regression of y on x where x is an AR(1) process with persistent estimator but without drift ($\mu = 0$). They show that the coefficient follows Onstein-Uhlenbeck process and is T-convergence. Phillips and Chen (PC 2014 thereafter) consider the local to unit root process $\{x_t\}$ with a local to zero drift and pointed out how convergence rate of the coefficient of predictive regression of y on x is higher than or equal to $\sqrt{n}$.

Case 1 ($\rho < 1$ and $\mu = \tilde{\mu} T^{-\gamma}$)

$$\rho = 1 + \frac{c}{T^\alpha}, \text{ with } -2 < c < 0,\ \alpha = 0$$

$$\mu = \frac{\tilde{\mu}}{T^\gamma}, \text{ with } \tilde{\mu} \neq 0,\ \gamma \in \left[0, \frac{1}{2}\right) \text{ or } \gamma \in \left[\frac{1}{2}, \infty\right)$$

In Case 1 $\{x_t\}$ is weakly stationary with $\rho = 1 + cT^{-\alpha} = 1 + c < 1$. We treat this as a separate case. For simulation, we consider $\rho \in \{0, 0.1, 0.3, 0.5, 0.7, 0.9, 0.95\}$ and $\gamma \in \{0, 0.3, 0.5, 0.6, 0.9, 1\}$ we notice that when $\{x_t\}$ is a weak stationary process, we have

$$\left(\hat{\pi}_t, \hat{\beta}_t\right)' - (\pi, \beta)' = O\left(R^{=0.5}\right)$$

Case 2 (PL, $\rho = 1$ and $\mu = 0$): This is a special case of Case 5. We separately report this case as Case 2 because this is when there is no drift term. This case has the AR unit root without drift.

$$\begin{aligned}
\rho &= 1, \text{ with } c = 0 \text{ (No } \alpha) \\
\mu &= \frac{\tilde{\mu}}{T^\gamma} = 0 \qquad (\tilde{\mu} = 0 \text{ or } \gamma = \infty) \\
\tilde{x}_t &= \tilde{x}_{t-1} + e_{xt} \\
x_t &= x_0 + \tilde{x}_t
\end{aligned}$$

PC (2014) show that when $c = 0, \gamma \geq 0.5$, $\hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(R^{-1}\right)$

Case 3 (PL, $\rho = 1 + cT^{-1}$ and $\mu = 0$): This is a special case of Case 6. We separately report this case as Case 3 because this is when there is no drift term. This case has the AR root local to unity but no drift.

$$\begin{aligned}
\rho &= 1 + \frac{c}{T}, \text{ with } c < 0 \text{ and } \alpha = 1 \\
\mu &= \frac{\tilde{\mu}}{T^\gamma} = 0 \qquad (\tilde{\mu} = 0 \text{ or } \gamma = \infty)
\end{aligned}$$

Case 4 (PL, $\rho = 1 + cT^{-\alpha}$ and $\alpha \in (0, 1)$): This is a special case of Case 7. We separately report this case as Case 4 because this is when there is no drift term. This case has the AR root moderately integrated to unity but no drift.

$$\begin{aligned}
\rho &= 1 + \frac{c}{T^\alpha}, \text{ with } c < 0 \text{ and } \alpha \in (0, 1) \\
\mu &= \frac{\tilde{\mu}}{T^\gamma} = 0 \qquad (\tilde{\mu} = 0 \text{ or } \gamma = \infty)
\end{aligned}$$

Case 5 (PC Lemma A.1, $\rho = 1$ and $\mu = \tilde{\mu}T^{-\gamma}$):

$$\begin{aligned}
\rho &= 1, \text{ with } c = 0 \\
\mu &= \frac{\tilde{\mu}}{T^\gamma}, \text{ with } \tilde{\mu} \neq 0, \ \gamma \in \left[0, \frac{1}{2}\right) \text{ or } \gamma \in \left[\frac{1}{2}, \infty\right)
\end{aligned}$$

PC (2014) show that when $\gamma \in \left[0, \frac{1}{2}\right)$, $\hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-(1.5-\gamma)}\right)$. When $\gamma \in \left[\frac{1}{2}, \infty\right)$, $\hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-1}\right)$. For simulation, we consider the following combination:

$$\gamma \in \{0, 0.3, 0.5, 0.6, 0.9, 1\} \text{ and } \tilde{\mu} \in \{3, 5, 10\}.$$

We consider

$$\begin{aligned} 1.\quad c &= 0, 0 < \gamma < 0.5 : \hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-(1.5-\gamma)}\right) \\ 2.\quad c &= 0, \gamma \geq 0.5 : \hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-1}\right) \end{aligned}$$

Case 6 (PC Lemma A.2, $\rho = 1 + cT^{-1}$ and $\mu = \tilde{\mu}T^{-\gamma}$)

$$\begin{aligned} \rho &= 1 + \frac{c}{T}, \text{ with } c < 0 \text{ and } \alpha = 1 \\ \mu &= \frac{\tilde{\mu}}{T^{\gamma}}, \text{ with } \tilde{\mu} \neq 0,\ \gamma \in \left[0, \frac{1}{2}\right) \text{ or } \gamma \in \left[\frac{1}{2}, \infty\right) \end{aligned}$$

PC (2014) show that when $\gamma \in \left[0, \frac{1}{2}\right)$, $\hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-(\alpha-\gamma+0.5)}\right) = O\left(T^{-(1.5-\gamma)}\right)$. When $\gamma \in \left[\frac{1}{2}, \infty\right)$, $\hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-(\alpha/2+0.5)}\right) = O\left(T^{-1}\right)$. The convergent rate is identical to Case 3, depending on γ. For simulation, we consider the following combination:

$$(\alpha, \gamma) \in \{(1, 0), (1, 0.3), (1, 0.5), (1, 0.6), (1, 0.9), (1, 1)\}$$

and

$$\tilde{\mu} \in \{3, 5, 10\}.$$

Case 7 (PC Lemma A.4, $\rho = 1 + cT^{-\alpha}$ and $\mu = \tilde{\mu}T^{-\gamma}$)

$$\begin{aligned} \rho &= 1 + \frac{c}{T^{\alpha}}, \text{ with } c < 0,\ \alpha \in (0, 1) \\ \mu &= \frac{\tilde{\mu}}{T^{\gamma}}, \text{ with } \tilde{\mu} \neq 0,\ \gamma \in \left[0, \frac{1}{2}\right) \text{ or } \gamma \in \left[\frac{1}{2}, \infty\right) \end{aligned}$$

PC (2014) show that when $\gamma \geq 0.5$ or $0 < \alpha/2 < \gamma < 0.5$, $\hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-(\alpha/2+0.5)}\right)$. When $0 < \gamma \leq \alpha/2 < 0.5 : \hat{\beta}_{1,t} - \beta_{1,t} \sim O\left(T^{-(\alpha-\gamma+0.5)}\right)$. For simulation, we consider the following combination:

$$(\alpha, \gamma) \in \{(0.9, 0), (0.9, 0.3)\}$$

$$(\alpha, \gamma) \in \{(0.9, 0.5), (0.9, 0.6), (0.9, 0.9), (0.9, 1)\}$$

and

$$\tilde{\mu} \in \{3, 5, 10\}.$$

To summarize, when using the whole data set, the table below show the convergent rate of estimator $\hat{\beta}$

	Cases	c	α	$\tilde{\mu}$	γ	Convergence rate
1	Stationary	$c > 0$	$\alpha = 0$	any	NA	$T^{0.5}$
2	UR	$c = 0$	NA	$\tilde{\mu} = 0$	NA	T
3	LUR	$c > 0$	$\alpha = 1$	$\tilde{\mu} = 0$	NA	T
4	MIUR	$c > 0$	$\alpha \in (0, 1)$	$\tilde{\mu} = 0$	NA	$T^{\alpha/2+0.5}$
5	UR+Drift	$c = 0$	NA	$\tilde{\mu} \neq 0$	$\gamma > 0$	$\max\left\{T^{1.5-\gamma}, T\right\}$
6	LUR+Drift	$c > 0$	$\alpha = 1$	$\tilde{\mu} \neq 0$	$\gamma > 0$	$\max\left\{T^{1.5-\gamma}, T\right\}$
7	MIUR+Drift	$c > 0$	$\alpha \in (0, 1)$	$\tilde{\mu} \neq 0$	$\gamma > 0$	$\max\left\{T^{\alpha-\gamma+0.5}, T^{\alpha/2+0.5}\right\}$

6 Asymptotic Distribution of ENC with a Persistent Predictor when $P/R \to \infty$

Suppose the predictor x_t in Model 2 follows an AR process $x_{t+1} = \phi x_t + v_{t+1}$ where $\mathbb{E}\left(v_{t+1}^2\right) = \sigma_v^2$. If $|\phi| < 1$, then

$$T^{-1}\sum_{t=1}^{T} x_t^2 \xrightarrow{p} \frac{\sigma_v^2}{1-\phi^2}, \qquad T^{-0.5}\sum_{t=1}^{T} x_t v_{t+1} \Rightarrow N\left(0, \frac{\sigma_v^2}{1-\phi^2}\right),$$

as $T \to \infty$. Many recent papers generalize the above to the case when ϕ approaches to 1 as the sample size T increases, see Bobkoski (1983), Cavanagh (1985), Chan and Wei (1987), Giraitis and Phillips (2006), Mikusheva (2007, 2014), Park (2003), Phillips (1987), Phillips and Lee (2013), and Stock (1991). Specifically for $\alpha = 1$, let $\phi = 1 - c/T$ for some fixed constant $c \geq 0$, $t = [Tr]$, $r \in [0\ 1]$. Let $x_{[Tr]}/\sqrt{T} \Rightarrow J_x^c(r) = \int_0^r e^{(r-s)c} dB_x(s)$ be an Onstein-Uhlenbeck process and B_x is a Brownian motion. and

$$T^{-2}\sum_{t=1}^{T} x_t^2 \Rightarrow \int_0^1 J_x^c(r)^2\,\mathrm{d}r, \qquad T^{-1}\sum_{t=1}^{T} x_t v_{t+1} \Rightarrow \int_0^1 J_x^c(r)\,\mathrm{d}B_x(r), \tag{12}$$

as $T \to \infty$. To consider the persistent predictor we take the *moderately* local to unit root process in the following Assumption 1b.

Assumption 1b. $\{x_t\}$ follows an AR process

$$x_{t+1} = \phi x_{t-1} + v_t$$

with a root *moderately* local to unity, $\phi = 1 - c/T^{\alpha}$, for some fixed constant $c \geq 0$ and $\alpha \in (0, 1]$.

Let $t \equiv [Ts]$ and $\xi \equiv R/T$. Then we have $t/T \to s$ and $(t - R + 1)/T \to (s - \xi)$. Under Assumption 1b when $\alpha = 1$, if $\{x_t\}$ does not contain drift term, from Equation (12), for $t = R, \ldots, T$,we have

$$T^{-2} \sum_{j=t-R+1}^{t} x_j^2 \Rightarrow \int_{s-\xi}^{s} J_x^c(r)^2 \, \mathrm{d}r, \qquad T^{-1} \sum_{j=t-R+1}^{t} x_j v_{j+1} \Rightarrow \int_{s-\xi}^{s} J_x^c(r) \, \mathrm{d}B_x(r),$$

as $T \to \infty$. Now, we state the main result, for the numerator of ENC_P, that is $\sqrt{P}\hat{B}_P$.

In the previous section, Case 1 can be referred to Clark and McCraken (2001). Case 2 and Case 3 are based on Assumption, Yan and Lee (2014) show that due to the faster convergence rate of $\hat{\beta}$, the forecast error of Model 2 will be more approached to the true error term compared with that of Model 1, leading to the encompassing have the same asymptotic distribution as Clark and West (2006). Case 2 is a special scenario of Case 3 as $c = 0$.

Under Assumption 1b, PC (2014) show that

$$T^{-(\alpha+1)} \sum_{t=1}^{T} x_t^2 = O_p(1), \qquad T^{-\left(\frac{\alpha+1}{2}\right)} \sum_{t=1}^{T} x_t v_{t+1} = O_p(1)$$

This corresponds to Case 4.

Now, we extend Assumption 1b to allow the drift term in the predictor.

Assumption 1c. $\{x_t\}$ follows an AR process

$$x_{t+1} = \mu + \phi x_{t-1} + v_t$$

with $\phi = 1 - c/T^{\alpha}$, for some fixed constant $c \geq 0, \alpha \in (0, 1]$ and the drift term $\mu = \tilde{\mu} T^{-\gamma}$ with $\tilde{\mu} \neq 0, \gamma > 0$.

Assumption 1c includes Assumption 1b as a special case when $\tilde{\mu} = 0$. Cases 2-4 are based on Assumption 1b. For Case 2 and Case 3, let $z = 1$. For Case 4, let $z = (\alpha + 1)/2$. Cases

5-7 are based on Assumption 1c. For Case 5 and Case 6, let $z = \frac{3}{2} - \gamma$ when $\gamma \in \left[0, \frac{1}{2}\right)$, and let $z = 1$ when $\gamma \in \left[\frac{1}{2}, \infty\right)$. For Case 7, let $z = \alpha - \gamma + \frac{1}{2}$ when $0 < \gamma \leq \frac{\alpha}{2} < \frac{1}{2}$, and let $z = \frac{\alpha}{2} + \frac{1}{2}$ when $0 < \frac{\alpha}{2} < \gamma < \frac{1}{2}$. Note that for all six cases, $z > \frac{1}{2}$. Following PC (2014), we can show that

$$\begin{pmatrix} \sum_{i=1}^{T} 1 & \sum_{i=1}^{T} x_i \\ \sum_{i=1}^{T} x_i & \sum_{i=1}^{T} x_i^2 \end{pmatrix} = \begin{pmatrix} O_p(T) & O_p\left(T^{z+0.5}\right) \\ O_p\left(T^{z+0.5}\right) & O_p\left(T^{2z}\right) \end{pmatrix},$$

and therefore

$$\begin{pmatrix} \sum_{i=t-R+1}^{t} 1 & \sum_{i=t-R+1}^{t} x_i \\ \sum_{i=t-R+1}^{t} x_i & \sum_{i=t-R+1}^{t} x_i^2 \end{pmatrix} = \begin{pmatrix} O_p(T)\,\xi & O_p\left(T^{z+0.5}\right)\xi \\ O_p\left(T^{z+0.5}\right)\xi & O_p\left(T^{2z}\right)\xi \end{pmatrix}.$$

We have the following two propositions under Assumption 1c. These propositions also hold under Assumption 1b as Assumption 1c includes Assumption 1b as a special case.

Proposition 4 . Under Assumption 1c and Assumption 2c, we have

$$\sum_{t=R}^{T} \hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) = -\sum_{t=R}^{T} e_{t+1} \left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) + o_p\left(\xi^{-1}\right)$$

under $\mathbb{H}_0$.

Proof: Under the null hypothesis that $\beta = 0$, $e_{t+1}^{(1)} = e_{t+1}^{(2)} =: e_{t+1}$. Note that

$$\hat{e}_{t+1}^{(i)} = e_{t+1} - x_{i,t}' \left(\hat{\Gamma}_{i,t} - \Gamma_i\right)$$

for Model i. Recall $x_{1,t}' = 1$. For $\hat{B}_P$, the numerator of ENC_P, we decompose

$$\begin{aligned} & \sum_{t=R}^{T} \hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) \\ = \; & \sum_{t=R}^{T} \left[e_{t+1} - x_{1,t}'\left(\hat{\Gamma}_{1,t} - \Gamma_1\right)\right] \left(e_{t+1} - x_{1,t}'\left(\hat{\Gamma}_{1,t} - \Gamma_1\right) - e_{t+1} + x_{2,t}'\left(\hat{\Gamma}_{2,t} - \Gamma_2\right)\right) \\ = \; & \sum_{t=R}^{T} \left[e_{t+1} - x_{1,t}'\left(\hat{\Gamma}_{1,t} - \Gamma_1\right)\right] \left(-x_{1,t}'\left(\hat{\Gamma}_{1,t} - \Gamma_1\right) + x_{2,t}'\left(\hat{\Gamma}_{2,t} - \Gamma_2\right)\right) \\ = \; & \sum_{t=R}^{T} e_{t+1} \left[-x_{1,t}'\left(\hat{\Gamma}_{1,t} - \Gamma_1\right)\right] + \sum_{t=R}^{T} e_{t+1} \left[x_{2,t}'\left(\hat{\Gamma}_{2,t} - \Gamma_2\right)\right] \\ & + \sum_{t=R}^{T} \left(\hat{\Gamma}_{1,t} - \Gamma_1\right) x_{1,t} x_{1,t}' \left(\hat{\Gamma}_{1,t} - \Gamma_1\right) - \sum_{t=R}^{T} \left(\hat{\Gamma}_{1,t} - \Gamma_1\right) x_{1,t} x_{2,t}' \left(\hat{\Gamma}_{2,t} - \Gamma_2\right) \\ \equiv \; & A_1 + A_2 + A_3 + A_4 \end{aligned} \tag{13}$$

Lemmas 1-3 show that $A_1 + A_2 + (A_3 + A_4) = O\left(\frac{T}{R}\right) + O\left(\frac{P}{T}\right) + o(1)$. Hence (13) is dominated by A_1 because $\frac{T}{R} \to \infty$ and $\frac{P}{T} \to 1$ under Assumption 2c. ∎

Lemma 1. Under Assumption 2c, $A_1 \Rightarrow -\sigma_e^2 \xi^{-1} \int_\xi^1 [W(s) - W(s-\xi)] \mathrm{d}V_e(s) = O_p\left(\xi^{-1}\right) = O_p\left(\frac{T}{R}\right)$ under $\mathbb{H}_0$.

Proof: Following Lemma A6 of CM (2001), we show

$$
\begin{aligned}
A_1 &= \sum\nolimits_{t=R}^{T} e_{t+1}\left[-x_{1,t}'\left(\hat{\Gamma}_{1,t} - \Gamma_1\right)\right] \\
&= -\sum\nolimits_{t=R}^{T} e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) \\
&= -\sum\nolimits_{t=R}^{T} e_{t+1}\left(R^{-1}\sum\nolimits_{j=t-R+1}^{t} e_j\right) \\
&= -\sum\nolimits_{t=R}^{T} e_{t+1}\left(T^{-1}\sum\nolimits_{j=t-R+1}^{t} e_j\right)/\xi \\
&= -\sum\nolimits_{t=R}^{T}\left[\left(T^{-1/2}e_{t+1}\right)\left(T^{-1/2}\sum\nolimits_{j=1}^{t} e_{,j} - T^{-1/2}\sum\nolimits_{j=1}^{t-R} e_{,j}\right)\right]/\xi \\
&\Longrightarrow -\sigma_e^2\xi^{-1}\int_\xi^1 \left[W(s) - W(s-\xi)\right]\mathrm{d}V_e(s).
\end{aligned}
$$

Lemma 2. Under Assumption 1c and Assumption 2c,

$$
A_2 = \sum\nolimits_{t=R}^{T} e_{t+1}\left[x_{2,t}'\left(\hat{\Gamma}_{2,t} - \Gamma_2\right)\right] = O_p\left(1-\xi\right) = O_p\left(\frac{P}{T}\right)
$$

under $\mathbb{H}_0$.

Proof: Rewrite

$$
\begin{aligned}
A_2 &= \sum\nolimits_{t=R}^{T} e_{t+1}x_{2,t}'\left(\hat{\Gamma}_{2,t} - \Gamma_2\right) \\
&= \sum\nolimits_{t=R}^{T} e_{t+1}x_{2,t}'\left(\sum_{j=t-R}^{t-1} x_{2,j}x_{2,j}'\right)^{-1}\left(\sum_{j=t-R}^{t-1} x_{2,j}e_{j+1}\right) \\
&= \sum\nolimits_{t=R}^{T} e_{t+1}x_{2,t}'G_T^{-1}\left[G_T^{-1}\sum\nolimits_{j=t-R+1}^{t} x_{2,j}x_{2,j}'G_T^{-1}/\xi\right]^{-1} \\
&\quad \times\left[G_T^{-1}\sum\nolimits_{j=t-R+1}^{t} x_{2,j}e_{j+1}/\xi\right],
\end{aligned}
$$

where $G_T = \text{diag}\left(T^{0.5}, T^z\right)$. For the two bracketed terms in the last line, we have

$$
\begin{aligned}
& G_T^{-1}\left(\sum\nolimits_{j=t-R+1}^{t} x_{2,j}x_{2,j}'\right)G_T^{-1}/\xi \\
\Rightarrow\ & G_T^{-1}\begin{pmatrix} O\left(T\right)\times\xi & O\left(T^{z+0.5}\right)\times\xi \\ O\left(T^{z+0.5}\right)\times\xi & O\left(T^{2z}\right)\times\xi \end{pmatrix}G_T^{-1}/\xi = O_p\left(1\right), \\
& G_T^{-1}\sum\nolimits_{j=t-R+1}^{t} x_{2,j}e_{j+1}/\xi \\
\Rightarrow\ & G_T^{-1}\begin{pmatrix} \int_{s-\xi}^{s} 1\mathrm{d}V_e\left(r\right) \\ O\left(T^z\right)\times\xi \end{pmatrix}/\xi = O_p\left(1\right),
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_2 &= \sum\nolimits_{t=R}^{T} e_{t+1}x_{2,t}'G_T^{-1}\left[G_T^{-1}\sum\nolimits_{j=t-R+1}^{t} x_{2,j}x_{2,j}'G_T^{-1}/\xi\right]^{-1} \\
&\quad \times\left[G_T^{-1}\sum\nolimits_{j=t-R+1}^{t} x_{2,j}e_{j+1}/\xi\right] \\
&= \sum\nolimits_{t=R}^{T} e_{t+1}x_{2,t}'G_T^{-1}\times O\left(1\right) \\
&= \sum\nolimits_{t=R}^{T}\begin{pmatrix} e_{t+1} \\ e_{t+1}x_t \end{pmatrix}G_T^{-1}\times O\left(1\right) = O\left(1-\xi\right)
\end{aligned}
$$

Therefore $A_2 = \sum_{t=R}^{T} e_{t+1}x_{2,t}'\left(\hat{\Gamma}_{2,t}-\Gamma_2\right) = O\left(1-\xi\right)$. ■

Lemma 3. Under Assumption 1c and Assumption 2c, $A_3 + A_4$ is $o_p\left(1\right)$ under $\mathbb{H}_0$.

Proof: Let $E_T = \text{diag}\left(T^0, T^{z-0.5}\right)$, $F_T = \text{diag}\left(T^1, T^{z+0.5}\right)$, $G_T = \text{diag}\left(T^{0.5}, T^z\right)$, then for any 2×2 matrix K, we have $E_TF_T = G_TG_T$ and

$$E_T \times K \times F_T = G_T \times K \times G_T,$$

because E_T, F_T, G_T are diagonal. Therefore

$$
\begin{aligned}
A_3 + A_4 &= \sum\nolimits_{t=R}^{T}\left(\hat{\Gamma}_{1,t}-\Gamma_1\right)x_{1,t}x_{1,t}'\left(\hat{\Gamma}_{1,t}-\Gamma_1\right) \\
&\quad -\sum\nolimits_{t=R}^{T}\left(\hat{\Gamma}_{1,t}-\Gamma_1\right)x_{1,t}x_{2,t}'\left(\hat{\Gamma}_{2,t}-\Gamma_2\right) \\
&= \sum\nolimits_{t=R}^{T} H_1'\left(t\right)B_1(t)q_{1,t}B_1(t)H_1\left(t\right) - \sum\nolimits_{t=R}^{T} H_1'\left(t\right)B_1(t)x_{1,t}x_{2,t}'B_2(t)H_2\left(t\right),
\end{aligned}
$$

where the second line appears to be the same as the second bracketed right-hand side term in (A7) of Lemma A10 in CM (2001), which shows that the above is $o\left(1\right)$ under Assumption 1a.

However, under Assumption 1b, x_t has an AR root local to unity. We show below that the local-to-unit root in x does not affect Lemma A10 of CM (2001). This is because terms involving x can be suitably normalized as follows

$$\begin{aligned} A_3 + A_4 &= \sum\nolimits_{t=R}^{T} H_1'(t) B_1(t) q_{1,t} B_1(t) H_1(t) \\ &\quad - \sum\nolimits_{t=R}^{T} H_1'(t) B_1(t) x_{1,t} \left(x_{2,t}' E_T^{-1}\right) \left[E_T \times R^{-1} B_2(t) \times F_T \times \xi\right] \\ &\quad \times \left[F_T^{-1} \times R H_2(t) / \xi\right] \\ &\equiv \sum\nolimits_{t=R}^{T} H_1'(t) B_1(t) q_{1,t} B_1(t) H_1(t) - \sum\nolimits_{t=R}^{T} H_1'(t) B_1(t) x_{1,t} \ddot{x}_{2,t}' \ddot{B}_2(t) \ddot{H}_2(t). \end{aligned}$$

where

$$\ddot{x}_{2,t}' \equiv x_{2,t}' E_T^{-1} \Rightarrow O(1),$$

$$\begin{aligned} \ddot{B}_2(t) &\equiv E_T \times R^{-1} B_2(t) \times F_T \times \xi \\ &= G_T \left[R^{-1} B_2(t)\right] G_T \times \xi \\ &= \left[G_T^{-1} \left[R^{-1} B_2(t)\right]^{-1} G_T^{-1}\right]^{-1} \times \xi \\ &= \left[G_T^{-1} \sum\nolimits_{j=t-R+1}^{t} x_{2,j} x_{2,j}' G_T^{-1}\right]^{-1} \times \xi \\ &= O(1), \end{aligned}$$

and

$$\begin{aligned} \ddot{H}_2(t) &\equiv F_T^{-1} \times R H_2(t) / \xi \\ &= F_T^{-1} \sum\nolimits_{j=t-R+1}^{t} x_{2,j} e_{j+1} / \xi \\ &\Rightarrow \begin{pmatrix} T^{-0.5}/\xi \times \int_{s-\xi}^{s} 1 \mathrm{d} V_e(r) \\ T^{-z-0.5}/\xi \times \sum_{j=t-R+1}^{t} x_{2,j} e_{j+1} \end{pmatrix} \\ &\Rightarrow T^{-0.5} \begin{pmatrix} \int_{s-\xi}^{s} 1 \mathrm{d} V_e(r)/\xi \\ T^{-z}/\xi \times \sum_{j=t-R+1}^{t} x_{2,j} e_{j+1} \end{pmatrix} = O\left(T^{-0.5}\right). \end{aligned}$$

Therefore, $\ddot{x}_{2,t}', \ddot{B}_2(t), \ddot{H}_2(t)$ have the same orders of magnitude as $x_{2,t}', B_2(t), H_2(t)$ in stationary case of Lemma A10 in CM (2001). Therefore $A_3 + A_4$ is $o_p(1)$ under Assumption 1b or Assumption 1c. ■

Based on Lemmas 1-3 and Proposition 4, we have

$$\sum\nolimits_{t=R}^{T} \hat{e}_{t+1}^{(1)} \left(\hat{e}_{t+1}^{(1)} - \hat{e}_{t+1}^{(2)}\right) = -\sum\nolimits_{t=R}^{T} e_{t+1} \left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) + o_p(1).$$

Hence, this is the encompassing test for the martingale difference model $y_{t+1} = e_{t+1}$ and the constant mean model $y_{t+1} = c + e_{t+1}^{(1)}$, as studied by CW (2006). Under $\mathbb{H}_0$, ENC_P is asymptotically standard normal. Proposition 5 states this result.

Proposition 5. Under Assumption 1c and Assumption 2c, $\lim_{\xi\to 0} ENC_P \Rightarrow N(0,1)$ under $\mathbb{H}_0$.

Proof: From Proposition 2 and Lemma 1, $A_1 = O\left(\xi^{-1}\right)$ is the dominant term of in $\hat{B}_P$ and hence ENC_P is

$$\begin{aligned} ENC_P &= A_1/\sqrt{\text{var}\left(A_1\right)} + o\left(1\right) \\ &= \frac{-\sum_{t=R}^{T} e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right)}{\sqrt{\sum_{t=R}^{T}\left[-e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) - \hat{c}_P\right]^2}} + o\left(1\right) \\ &\Rightarrow \lim_{\xi\to 0} \frac{-\sigma_e^2\xi^{-1}\int_\xi^1\left[W(s) - W(s-\xi)\right]\mathrm{d}V_e(s)}{\sqrt{\sigma_e^4 \times \xi^{-2}\int_\xi^1\left[W(s) - W(s-\xi)\right]^2\mathrm{d}s}} \sim N\left(0,1\right), \end{aligned}$$

where $A_1 \Rightarrow -\sigma_e^2\xi^{-1}\int_\xi^1\left[V_e(s) - V_e(s-\xi)\right]\mathrm{d}V_e(s)$, $c_{t+1} = -e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right)$ and $\hat{c}_P = P^{-1}\sum_{t=R}^{T} c_{t+1} = P^{-1}A_1$. The denominator follows from Lemma 4. Therefore, ENC_P is asymptotically standard normal under $\mathbb{H}_0$ from Proposition 3. ■

Lemma 4. Under Assumptions 1c and 2c,

$$\sum_{t=R}^{T}\left[-e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) - \hat{c}_P\right]^2 \Rightarrow \sigma_e^4 \times \xi^{-2}\int_\xi^1\left[W(s) - W(s-\xi)\right]^2 ds.$$

Proof: Following Lemma A11 of CM (2001), we have

$$\begin{aligned} & \sum_{t=R}^{T}\left[-e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right) - \hat{c}_P\right]^2 \\ = & \sum_{t=R}^{T}\left[e_{t+1}\left(e_{t+1} - \hat{e}_{t+1}^{(1)}\right)\right]^2 - P\hat{c}_P^2 \\ = & \sum_{t=R}^{T}\left[e_{t+1}\left(R^{-1}\sum_{j=t-R+1}^{t} e_j\right)\right]^2 + O_p\left(P^{-1}\xi^{-2}\right) \\ = & \frac{T^2}{R^2}\sum_{t=R}^{T}\left[\left(e_{t+1}\right)^2\left(T^{-1/2}\sum_{j=1}^{t} e_{,j} - T^{-1/2}\sum_{j=1}^{t-R} e_{,j}\right)^2\right]\frac{1}{T} + O_p\left(P^{-1}\xi^{-2}\right) \\ \Rightarrow & \ \xi^{-2}\int_\xi^1 \sigma_e^2\left[\sigma_e W(s) - \sigma_e W(s-\xi)\right]^2\mathrm{d}s, \end{aligned}$$

where line 3 follows from Lemma 1 for $P\hat{c}_P = A_1 = O_p\left(\xi^{-1}\right)$. ■

7 Monte Carlo

We will examine the finite sample properties of the above asymptotic results by comparing three statistics: DM, ENC and CCS test for size and power of test, the three tests have been mentioned in CW (2006, 2007), Lee and Ge (2013, 2014). For simulation, We set $Var\,(e_{xt}) = \sigma_x^2 = 1$, $Var\,(e_{t+1}) = \sigma_e^2 \in \{0.1, 1\}$, $R \in \{50, 100, 200, 400\}$, $P \in \{200, 400, 800, 1600\}$, $\gamma \in \{0.0, 0, 2, 0.3, 0.5, 0.6, 1.0, \infty\}$, $\alpha \in \{0.9, 1.0\}$, $c \in \{0, -5\}$, $\tilde{x}_0 = 0$, $x_0 \sim N(0, \sigma_x^2)$. $\tilde{\mu} \in \{0, 10\}$, $\pi = 1$, $\beta \in \{0, 0.1, 1\}$. For size of test, the error terms $\{e_{xt}\}$ and $\{e_{t+1}\}$ are uncorrelated, we set $r = 0$. For power of test, $r = -0.95$, simulating the high negative correlation between stock returns and many commonly used predictors (PC 2014). The tables below show the results under different settings

Tables 3.1-3.27 About Here

Tables 3.1-3.23 correspond to Cases 1-6 and Tables 3.24-3.27 show the test power.

1. Case 1 is researched by Clark and McCraken (2001), Clark and West (2006).
2. Case 2 and Case 3 are simulated by Lee and Ge, see Ge and Lee (2014b)
3. We report the simulation results for Cases 4-7.

Tables 3.1-3.23 show that both DM and CCS statistics distort downward for large P/R ratio for all cases. The ENC test is robust, having the correct size regardless of the interaction of the scale of stochastic trend and deterministic trend.Tables 3.24-3.27 show the power of test. We see that as ϕ increases, the powers approach to 1 dramatically since higher ϕ implies higher singal-to-noise ratio.

8 Conclusions

In this paper we propose the encompassing statistic for the out-of-sample inference on comparing two nested predictive regression models where the predictor has both an AR root local to unity and the drift local to zero. The convergence rate of the predictive regression parameter estimator is impacted by the scales of the stochastic trend or deterministic trend. With the presence of the drift in addition to the near unit root, the convergence rate of the parameter estimation can become even faster, and it essentially treat the unknown parameter values of the predictive regression as if they were known. The nonstationary behavior of the predictor with a unit root and drift would make the parameter estimator follow asymptotically a non-normal distribution. However, the faster convergence rate of the parameter estimation make the out-of-sample encompassing test statistic be asymptotically standard normal under the null hypothesis. We conduct a Monte Carlo simulation and show that the asymptotic results also hold in finite sample size.

In the present paper, we consider the predictive model when the predictor is not endogenous. For our immediate future work, we are working on the predictive regression model with the endogenous predictor by extending the IVX method by Phillips and Magdalinos (2007, 2009) and Phillips and Lee (2014) to the out-of-sample encompassing inference.

References

Amihud, Y., and C. M. Hurvich, (2004), "Predictive Regressions: A Reduced-Bias Estimation Method", *Journal of Financial and Quantitative Analysis* 39(04): 813-841.

Amihud, Y., Hurvich, C. M., and Y. Wang, (2010), "Predictive Regression with Order-P Autoregressive Predictor", *Journal of Empirical Finance* 17(3): 513-525.

Campbell, J. Y., and R. J. Shiller, (1988), "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies* 1(3): 195-228.

Campbell, J. and M. Yogo, (2006), "Efficient Tests of Stock Return Predictability", *Journal of Financial Economics* 81(1): 27-60.

Cavanagh, C. L., Elliott, G., and J. H. Stock, (1995), "Inference in Models with Nearly Integrated Regressors", *Econometric theory* 11(5): 1131-1147.

Chao, J. C., V. Corradi and N. Swanson, (2001), "An Out of Sample Test for Granger Causality" *Macroeconomic Dynamics* 5(4): 598-620.

Cheang, W. K. and G. Reinsel, (2000), "Bias Reduction of Autoregressive Estimates in Time Series Regression Model through Restricted Maximum Likelihood", *Journal of American Statistical Association* 95: 1173-1184.

Chen, W. and R. Deo, (2009a), "Bias Reduction and Likelihood Based Almost-Exactly Sized Hypothesis Testing in Predictive Regressions using the Restricted Likelihood", *Econometric Theory* 25(5): 1143-1179.

Chen, W. and R. Deo, (2009b), "The Restricted Likelihood Ratio Test at the Boundary in Autoregressive Series", *Journal of Time Series* Analysis 30(6): 618-630.

Chen, W. and R. Deo, (2010), "Weighted Least Squares Approximate Restricted Likelihood Estimation for Vector Autoregressive Processes", *Biometrika* 97(1): 231-237.

Chen, W. and R. Deo, (2011), "The Restricted Likelihood Ratio Test at the Boundary in Autoregressive Series", *Journal of Time Series Analysis* 33(2): 325-339.

Chen, W. and R. R. Shamendu, & Y. Yi, (2013), "Uniform Inference in Predictive Regression Models", *Journal of Business and Economic Statistics* 31(4): 525-533.

Clark, T. E. and M. McCracken, (2001). "Tests of Equal Forecast Accuracy and Encompassing for Nested Models"" *Journal of Econometrics* 105(1): 85-110.

Clark, T. E. and M. McCracken, (2005). "Evaluating Direct Multistep Forecasts." *Econometric Reviews* 24(4): 369-404.

Clark, T. E. and M. McCracken, (2009). "Combining Forecasts from Nested Models." *Oxford Bulletin of Economics and Statistics* 71(3): 303-329.

Clark, T. E. and K. D. West, (2006). "Using Out-of-Sample Mean Squared Prediction Errors to Test the Martingale Difference Hypothesis." *Journal of Econometrics* 135(1-2): 155-186.

Clark, T. E. and K. D. West, (2007). "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models." *Journal of Econometrics* 138(1): 291-311.

Cochrane, J. H. (2008), "The Dog That Did Not Bark: A Defense of Return Predictability", *Review of Financial Studies* 21(4): 1533-1575.

Cooper, D. M., and R. Thompson, (1977), "A Note on the Estimation of the Parameters of the Autoregressive Moving Average Process" *Biometrika*, 64(3): 625-628.

Corradi, V. and N. R. Swanson, (2002). "A Consistent Test for Nonlinear Out of Sample Predictive Accuracy." *Journal of Econometrics* 110(2): 353-381.

Diebold, F.X. (2015), "Comparing Predictive Accuracy, Twenty Years Later: A Personal Perspective on the Use and Abuse of Diebold–Mariano Tests", *Journal of Business and Economic Statistics* 33(1): 1-8.

Diebold, F. X. and R. S. Mariano, (1995). "Comparing Predictive Accuracy." *Journal of Business and Economic Statistics* 13(3): 253-263.

Jansson, M. and M. Moreira, (2006), "Optimal Inference in Regression Models with Nearly Integrated Regressors", *Econometrica* 74(3): 681-714.

Phillips, P. C. B., (1987a), "Time Series Regression with a Unit Root", *Econometrica* 55(2): 277-302.

Phillips, P. C. B., (1987b), "Towards a Unified Asymptotic Theory for Autoregression", *Biometrika* 74(3): 535-547.

Phillips, P. C. B., (2014), "On Confidence Intervals for Autoregressive Roots and Predictive Regression", *Econometrica* 82(3): 1177-1195.

Phillips, P. C. B. and J. H. Lee, (2013), "Predictive Regression Under Various Degrees of Persistence and Robust Long-Horizon Regression", *Journal of Econometrics* 177(2): 250-264.

Phillips, P. C. B. and J. H. Lee, (2014), "Robust Econometric Inference with Mixed Integrated and Mildly Explosive Regressors", Working Paper.

Phillips, P. C. B. and T. Magdalinos, (2007), "Limit Theory for Moderate Deviations from a Unit Root", *Journal of Econometrics* 137(1): 115-130.

Phillips, P. C. B. and T. Magdalinos, (2009), "Econometric Inference in the Vicinity of Unity" Working Paper.

Phillips, P. C. B. and Y. Chen, (2014), "Restricted Likelihood Ratio Tests in Predictive Regression: Online Supplement" Working Paper, Yale University and Singapore Management University.

Stambaugh, R. F., (1999), "Predictive Regressions", *Journal of Financial Economics* 54(3): 375-421.

Zhu, M., (2012), "Jackknife for Bias Reduction in Predictive Regressions", *Journal of Financial Econometrics* 11(1): 193-220.

Table 3.1: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (UR with $\gamma < 0.5$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	1.000	3.000	50	0.000	0.032	0.001	0.000	0.034	0.000	0.000	0.043	0.000	0.000	0.042	0.000
$c = 0$	1.000	3.000	100	0.000	0.033	0.003	0.000	0.037	0.000	0.000	0.048	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	3.000	200	0.001	0.023	0.027	0.000	0.028	0.002	0.000	0.039	0.001	0.000	0.044	0.000
$\gamma = 0$	1.000	3.000	400	0.001	0.023	0.046	0.000	0.021	0.026	0.000	0.028	0.004	0.000	0.031	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	1.000	5.000	50	0.000	0.031	0.001	0.000	0.034	0.000	0.000	0.042	0.000	0.000	0.041	0.000
$c = 0$	1.000	5.000	100	0.000	0.033	0.003	0.000	0.037	0.000	0.000	0.048	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	5.000	200	0.001	0.023	0.027	0.000	0.028	0.002	0.000	0.040	0.001	0.000	0.044	0.000
$\gamma = 0$	1.000	5.000	400	0.001	0.023	0.047	0.000	0.021	0.025	0.000	0.029	0.004	0.000	0.031	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	1.000	10.000	50	0.000	0.030	0.001	0.000	0.035	0.000	0.000	0.042	0.000	0.000	0.042	0.000
$c = 0$	1.000	10.000	100	0.000	0.033	0.003	0.000	0.037	0.000	0.000	0.049	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	10.000	200	0.001	0.023	0.027	0.000	0.028	0.002	0.000	0.040	0.001	0.000	0.043	0.000
$\gamma = 0$	1.000	10.000	400	0.001	0.023	0.047	0.000	0.022	0.025	0.000	0.028	0.004	0.000	0.031	0.000
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	1.000	0.928	50	0.000	0.031	0.001	0.000	0.033	0.000	0.000	0.042	0.000	0.000	0.043	0.000
$c = 0$	1.000	0.754	100	0.000	0.033	0.004	0.000	0.036	0.000	0.000	0.046	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.612	200	0.002	0.023	0.027	0.000	0.028	0.002	0.000	0.037	0.001	0.000	0.042	0.000
$\gamma = 0.3$	1.000	0.497	400	0.001	0.021	0.047	0.000	0.020	0.027	0.000	0.027	0.004	0.000	0.032	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	1.000	1.546	50	0.000	0.034	0.001	0.000	0.034	0.000	0.000	0.044	0.000	0.000	0.042	0.000
$c = 0$	1.000	1.256	100	0.000	0.033	0.004	0.000	0.036	0.000	0.000	0.049	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	1.020	200	0.002	0.023	0.027	0.000	0.028	0.002	0.000	0.040	0.001	0.000	0.044	0.000
$\gamma = 0.3$	1.000	0.829	400	0.001	0.021	0.047	0.000	0.021	0.027	0.000	0.027	0.004	0.000	0.033	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	1.000	3.092	50	0.000	0.032	0.001	0.000	0.034	0.000	0.000	0.043	0.000	0.000	0.042	0.000
$c = 0$	1.000	2.512	100	0.000	0.033	0.003	0.000	0.037	0.000	0.000	0.048	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	2.040	200	0.001	0.023	0.027	0.000	0.028	0.002	0.000	0.040	0.001	0.000	0.043	0.000
$\gamma = 0.3$	1.000	1.657	400	0.001	0.023	0.047	0.000	0.021	0.026	0.000	0.028	0.004	0.000	0.031	0.000

Table 3.2: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (UR with $\gamma \geq 0.5$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	1.000	0.424	50	0.000	0.030	0.001	0.000	0.033	0.000	0.000	0.039	0.000	0.000	0.043	0.000
$c = 0$	1.000	0.300	100	0.000	0.034	0.004	0.000	0.032	0.000	0.000	0.042	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.212	200	0.001	0.022	0.028	0.000	0.027	0.004	0.000	0.036	0.001	0.000	0.037	0.000
$\gamma = 0.5$	1.000	0.150	400	0.001	0.022	0.046	0.000	0.020	0.026	0.000	0.022	0.004	0.000	0.031	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	1.000	0.707	50	0.000	0.031	0.001	0.000	0.034	0.000	0.000	0.042	0.000	0.000	0.042	0.000
$c = 0$	1.000	0.500	100	0.000	0.032	0.004	0.000	0.033	0.000	0.000	0.043	0.000	0.000	0.054	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.354	200	0.002	0.025	0.028	0.000	0.028	0.003	0.000	0.037	0.001	0.000	0.040	0.000
$\gamma = 0.5$	1.000	0.250	400	0.001	0.021	0.046	0.000	0.020	0.028	0.000	0.024	0.004	0.000	0.033	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	1.000	1.414	50	0.000	0.034	0.001	0.000	0.033	0.000	0.000	0.045	0.000	0.000	0.041	0.000
$c = 0$	1.000	1.000	100	0.000	0.033	0.004	0.000	0.035	0.000	0.000	0.048	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.707	200	0.002	0.024	0.027	0.000	0.028	0.002	0.000	0.039	0.001	0.000	0.042	0.000
$\gamma = 0.5$	1.000	0.500	400	0.001	0.021	0.047	0.000	0.020	0.027	0.000	0.027	0.004	0.000	0.032	0.000
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	1.000	0.287	50	0.000	0.029	0.000	0.000	0.035	0.000	0.000	0.031	0.000	0.000	0.042	0.000
$c = 0$	1.000	0.189	100	0.000	0.031	0.005	0.000	0.032	0.000	0.000	0.038	0.000	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.125	200	0.002	0.022	0.028	0.000	0.025	0.006	0.000	0.040	0.001	0.000	0.034	0.000
$\gamma = 0.6$	1.000	0.082	400	0.002	0.025	0.045	0.000	0.019	0.030	0.000	0.022	0.007	0.000	0.028	0.001
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	1.000	0.478	50	0.000	0.030	0.001	0.000	0.032	0.000	0.000	0.040	0.000	0.000	0.043	0.000
$c = 0$	1.000	0.315	100	0.000	0.034	0.004	0.000	0.031	0.000	0.000	0.042	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.208	200	0.001	0.023	0.028	0.000	0.027	0.004	0.000	0.036	0.001	0.000	0.038	0.000
$\gamma = 0.6$	1.000	0.137	400	0.001	0.022	0.046	0.001	0.020	0.026	0.000	0.022	0.004	0.000	0.031	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	1.000	0.956	50	0.000	0.031	0.001	0.000	0.033	0.000	0.000	0.043	0.000	0.000	0.043	0.000
$c = 0$	1.000	0.631	100	0.000	0.033	0.004	0.000	0.036	0.000	0.000	0.044	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.416	200	0.002	0.024	0.028	0.000	0.028	0.003	0.000	0.038	0.001	0.000	0.042	0.000
$\gamma = 0.6$	1.000	0.275	400	0.001	0.021	0.046	0.000	0.020	0.028	0.000	0.024	0.004	0.000	0.032	0.000

Table 3.3: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (UR with $\gamma \geq 0.5$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	1.000	0.089	50	0.000	0.028	0.006	0.000	0.039	0.001	0.000	0.037	0.000	0.000	0.035	0.000
$c = 0$	1.000	0.048	100	0.000	0.028	0.017	0.000	0.036	0.003	0.000	0.042	0.002	0.000	0.045	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.025	200	0.003	0.024	0.029	0.001	0.021	0.018	0.000	0.040	0.008	0.000	0.037	0.004
$\gamma = 0.9$	1.000	0.014	400	0.009	0.032	0.047	0.004	0.031	0.039	0.001	0.026	0.014	0.001	0.029	0.006
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	1.000	0.148	50	0.000	0.026	0.002	0.000	0.039	0.000	0.000	0.037	0.000	0.000	0.036	0.000
$c = 0$	1.000	0.079	100	0.000	0.024	0.013	0.000	0.036	0.002	0.000	0.039	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.042	200	0.003	0.024	0.031	0.001	0.025	0.011	0.000	0.038	0.008	0.000	0.036	0.001
$\gamma = 0.9$	1.000	0.023	400	0.009	0.031	0.047	0.002	0.028	0.037	0.001	0.027	0.014	0.001	0.027	0.003
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	1.000	0.296	50	0.000	0.031	0.000	0.000	0.035	0.000	0.000	0.030	0.000	0.000	0.043	0.000
$c = 0$	1.000	0.158	100	0.000	0.028	0.006	0.000	0.033	0.001	0.000	0.037	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.085	200	0.002	0.019	0.033	0.000	0.026	0.011	0.000	0.038	0.002	0.000	0.032	0.000
$\gamma = 0.9$	1.000	0.046	400	0.008	0.030	0.049	0.001	0.029	0.036	0.000	0.022	0.008	0.000	0.025	0.003
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	1.000	0.060	50	0.000	0.030	0.007	0.000	0.038	0.001	0.000	0.041	0.001	0.000	0.034	0.000
$c = 0$	1.000	0.030	100	0.000	0.028	0.021	0.000	0.034	0.004	0.000	0.043	0.001	0.000	0.044	0.001
$\sigma_u = 1, \sigma_x = 1$	1.000	0.015	200	0.003	0.026	0.034	0.001	0.021	0.019	0.000	0.039	0.012	0.000	0.035	0.005
$\gamma = 1$	1.000	0.008	400	0.007	0.030	0.047	0.003	0.030	0.040	0.001	0.024	0.014	0.001	0.030	0.006
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	1.000	0.100	50	0.000	0.028	0.005	0.000	0.040	0.001	0.000	0.036	0.000	0.000	0.036	0.000
$c = 0$	1.000	0.050	100	0.000	0.028	0.016	0.000	0.035	0.003	0.000	0.042	0.002	0.000	0.045	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.025	200	0.003	0.024	0.028	0.001	0.021	0.018	0.000	0.040	0.009	0.000	0.037	0.004
$\gamma = 1$	1.000	0.013	400	0.008	0.031	0.046	0.003	0.031	0.038	0.001	0.027	0.013	0.001	0.029	0.006
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	1.000	0.200	50	0.000	0.027	0.001	0.000	0.037	0.000	0.000	0.036	0.000	0.000	0.040	0.000
$c = 0$	1.000	0.100	100	0.000	0.024	0.011	0.000	0.032	0.002	0.000	0.036	0.001	0.000	0.037	0.000
$\sigma_u = 1, \sigma_x = 1$	1.000	0.050	200	0.003	0.023	0.032	0.001	0.025	0.014	0.000	0.037	0.007	0.000	0.032	0.001
$\gamma = 1$	1.000	0.025	400	0.009	0.032	0.048	0.001	0.029	0.035	0.001	0.026	0.015	0.001	0.028	0.004

Table 3.4: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (LUR with $\alpha = 1$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.900	3.000	50	0.000	0.036	0.000	0.000	0.042	0.000	0.000	0.042	0.000	0.000	0.047	0.000
$c = 5$	0.950	3.000	100	0.001	0.033	0.003	0.000	0.035	0.000	0.000	0.041	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	3.000	200	0.005	0.030	0.025	0.001	0.028	0.002	0.000	0.034	0.000	0.000	0.039	0.000
$\alpha = 1, \gamma = 0$	0.988	3.000	400	0.016	0.033	0.044	0.006	0.028	0.021	0.001	0.029	0.002	0.000	0.033	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.900	5.000	50	0.000	0.035	0.000	0.000	0.042	0.000	0.000	0.042	0.000	0.000	0.046	0.000
$c = 5$	0.950	5.000	100	0.001	0.033	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	5.000	200	0.006	0.031	0.025	0.001	0.028	0.002	0.000	0.034	0.000	0.000	0.038	0.000
$\alpha = 1, \gamma = 0$	0.988	5.000	400	0.017	0.034	0.044	0.007	0.030	0.021	0.001	0.029	0.002	0.000	0.032	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.900	10.000	50	0.000	0.036	0.000	0.000	0.042	0.000	0.000	0.042	0.000	0.000	0.046	0.000
$c = 5$	0.950	10.000	100	0.001	0.032	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	10.000	200	0.007	0.030	0.025	0.001	0.028	0.002	0.000	0.035	0.000	0.000	0.038	0.000
$\alpha = 1, \gamma = 0$	0.988	10.000	400	0.017	0.035	0.044	0.007	0.030	0.021	0.001	0.028	0.002	0.000	0.032	0.000
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.900	0.928	50	0.000	0.037	0.000	0.000	0.042	0.000	0.000	0.043	0.000	0.000	0.046	0.000
$c = 5$	0.950	0.754	100	0.001	0.031	0.004	0.000	0.035	0.000	0.000	0.041	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	0.612	200	0.004	0.028	0.026	0.001	0.029	0.003	0.000	0.034	0.000	0.000	0.039	0.000
$\alpha = 1, \gamma = 0.3$	0.988	0.497	400	0.011	0.032	0.044	0.004	0.028	0.022	0.001	0.030	0.003	0.000	0.033	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.900	1.546	50	0.000	0.036	0.000	0.000	0.042	0.000	0.000	0.042	0.000	0.000	0.047	0.000
$c = 5$	0.950	1.256	100	0.001	0.032	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	1.020	200	0.004	0.029	0.025	0.001	0.029	0.003	0.000	0.034	0.000	0.000	0.039	0.000
$\alpha = 1, \gamma = 0.3$	0.988	0.829	400	0.012	0.033	0.044	0.003	0.028	0.021	0.001	0.030	0.003	0.000	0.033	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.900	3.092	50	0.000	0.036	0.000	0.000	0.042	0.000	0.000	0.042	0.000	0.000	0.047	0.000
$c = 5$	0.950	2.512	100	0.001	0.033	0.003	0.000	0.035	0.000	0.000	0.041	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	2.040	200	0.005	0.029	0.025	0.001	0.028	0.002	0.000	0.034	0.000	0.000	0.039	0.000
$\alpha = 1, \gamma = 0.3$	0.988	1.657	400	0.015	0.034	0.044	0.004	0.028	0.021	0.001	0.030	0.002	0.000	0.033	0.000
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.900	0.424	50	0.000	0.038	0.004	0.000	0.043	0.002	0.000	0.042	0.001	0.000	0.047	0.000
$c = 5$	0.950	0.300	100	0.001	0.031	0.010	0.000	0.035	0.003	0.000	0.041	0.001	0.000	0.044	0.001
$\sigma_u = 1, \sigma_x = 1$	0.975	0.212	200	0.003	0.028	0.031	0.001	0.030	0.010	0.000	0.034	0.003	0.000	0.040	0.002
$\alpha = 1, \gamma = 0.5$	0.988	0.150	400	0.009	0.030	0.045	0.003	0.029	0.028	0.000	0.030	0.009	0.000	0.033	0.004
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.900	0.707	50	0.000	0.037	0.001	0.000	0.042	0.000	0.000	0.043	0.000	0.000	0.047	0.000
$c = 5$	0.950	0.500	100	0.001	0.031	0.006	0.000	0.034	0.000	0.000	0.041	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	0.354	200	0.003	0.028	0.027	0.001	0.029	0.005	0.000	0.034	0.000	0.000	0.040	0.000
$\alpha = 1, \gamma = 0.5$	0.988	0.250	400	0.009	0.031	0.044	0.003	0.029	0.024	0.000	0.030	0.004	0.000	0.033	0.001
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.900	1.414	50	0.000	0.036	0.000	0.000	0.042	0.000	0.000	0.043	0.000	0.000	0.047	0.000
$c = 5$	0.950	1.000	100	0.001	0.031	0.003	0.000	0.035	0.000	0.000	0.041	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	0.707	200	0.004	0.029	0.025	0.001	0.029	0.003	0.000	0.034	0.000	0.000	0.039	0.000
$\alpha = 1, \gamma = 0.5$	0.988	0.500	400	0.011	0.033	0.044	0.004	0.028	0.022	0.001	0.031	0.003	0.000	0.033	0.000

Table 3.5: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (LUR with $\alpha = 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.900	0.287	50	0.000	0.037	0.012	0.000	0.042	0.007	0.000	0.042	0.005	0.000	0.047	0.005
$c = 5$	0.950	0.189	100	0.001	0.031	0.020	0.000	0.035	0.013	0.000	0.041	0.008	0.000	0.044	0.006
$\sigma_u = 1, \sigma_x = 1$	0.975	0.125	200	0.003	0.027	0.037	0.001	0.030	0.022	0.000	0.035	0.014	0.000	0.040	0.010
$\alpha = 1, \gamma = 0.6$	0.988	0.082	400	0.008	0.030	0.050	0.003	0.028	0.037	0.000	0.031	0.023	0.000	0.033	0.015
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.900	0.478	50	0.000	0.038	0.002	0.000	0.042	0.001	0.000	0.043	0.000	0.000	0.047	0.000
$c = 5$	0.950	0.315	100	0.001	0.031	0.010	0.000	0.035	0.003	0.000	0.041	0.001	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	0.208	200	0.003	0.028	0.032	0.001	0.030	0.010	0.000	0.034	0.003	0.000	0.040	0.002
$\alpha = 1, \gamma = 0.6$	0.988	0.137	400	0.009	0.030	0.046	0.003	0.029	0.030	0.000	0.031	0.011	0.000	0.033	0.005
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.900	0.956	50	0.000	0.037	0.000	0.000	0.042	0.000	0.000	0.043	0.000	0.000	0.047	0.000
$c = 5$	0.950	0.631	100	0.001	0.031	0.005	0.000	0.034	0.000	0.000	0.041	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	0.416	200	0.003	0.028	0.027	0.001	0.029	0.004	0.000	0.034	0.000	0.000	0.039	0.000
$\alpha = 1, \gamma = 0.6$	0.988	0.275	400	0.010	0.031	0.044	0.003	0.029	0.023	0.000	0.030	0.004	0.000	0.033	0.001
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.900	0.089	50	0.000	0.038	0.040	0.000	0.043	0.037	0.000	0.042	0.035	0.000	0.047	0.036
$c = 5$	0.950	0.048	100	0.001	0.031	0.044	0.000	0.035	0.041	0.000	0.041	0.040	0.000	0.044	0.038
$\sigma_u = 1, \sigma_x = 1$	0.975	0.025	200	0.003	0.028	0.048	0.001	0.030	0.047	0.000	0.035	0.044	0.000	0.040	0.043
$\alpha = 1, \gamma = 0.9$	0.988	0.014	400	0.008	0.030	0.052	0.003	0.028	0.048	0.000	0.031	0.048	0.000	0.033	0.042
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.900	0.148	50	0.000	0.038	0.031	0.000	0.042	0.026	0.000	0.042	0.023	0.000	0.047	0.023
$c = 5$	0.950	0.079	100	0.001	0.031	0.040	0.000	0.035	0.034	0.000	0.041	0.032	0.000	0.044	0.030
$\sigma_u = 1, \sigma_x = 1$	0.975	0.042	200	0.003	0.028	0.047	0.001	0.030	0.045	0.000	0.035	0.040	0.000	0.040	0.038
$\alpha = 1, \gamma = 0.9$	0.988	0.023	400	0.008	0.030	0.052	0.003	0.028	0.048	0.000	0.031	0.045	0.000	0.033	0.039
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.900	0.296	50	0.000	0.037	0.011	0.000	0.042	0.007	0.000	0.042	0.004	0.000	0.047	0.005
$c = 5$	0.950	0.158	100	0.001	0.031	0.025	0.000	0.035	0.018	0.000	0.041	0.013	0.000	0.044	0.011
$\sigma_u = 1, \sigma_x = 1$	0.975	0.085	200	0.003	0.027	0.042	0.001	0.030	0.031	0.000	0.035	0.026	0.000	0.040	0.023
$\alpha = 1, \gamma = 0.9$	0.988	0.046	400	0.008	0.030	0.052	0.003	0.028	0.045	0.000	0.031	0.037	0.000	0.033	0.033
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.900	0.060	50	0.000	0.038	0.044	0.000	0.042	0.041	0.000	0.042	0.040	0.000	0.047	0.040
$c = 5$	0.950	0.030	100	0.001	0.031	0.046	0.000	0.035	0.043	0.000	0.041	0.044	0.000	0.044	0.042
$\sigma_u = 1, \sigma_x = 1$	0.975	0.015	200	0.003	0.028	0.048	0.001	0.030	0.049	0.000	0.035	0.045	0.000	0.040	0.045
$\alpha = 1, \gamma = 1$	0.988	0.008	400	0.008	0.029	0.052	0.003	0.028	0.049	0.000	0.031	0.048	0.000	0.033	0.043
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.900	0.100	50	0.000	0.038	0.038	0.000	0.043	0.035	0.000	0.042	0.033	0.000	0.047	0.033
$c = 5$	0.950	0.050	100	0.001	0.031	0.044	0.000	0.035	0.040	0.000	0.041	0.040	0.000	0.044	0.038
$\sigma_u = 1, \sigma_x = 1$	0.975	0.025	200	0.003	0.028	0.048	0.001	0.030	0.047	0.000	0.035	0.044	0.000	0.040	0.043
$\alpha = 1, \gamma = 1$	0.988	0.013	400	0.008	0.030	0.052	0.003	0.028	0.049	0.000	0.031	0.048	0.000	0.033	0.042
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.900	0.200	50	0.000	0.037	0.021	0.000	0.042	0.018	0.000	0.042	0.014	0.000	0.047	0.014
$c = 5$	0.950	0.100	100	0.001	0.031	0.036	0.000	0.035	0.030	0.000	0.041	0.027	0.000	0.044	0.025
$\sigma_u = 1, \sigma_x = 1$	0.975	0.050	200	0.003	0.028	0.047	0.001	0.030	0.043	0.000	0.035	0.037	0.000	0.040	0.037
$\alpha = 1, \gamma = 1$	0.988	0.025	400	0.008	0.030	0.052	0.003	0.028	0.048	0.000	0.031	0.044	0.000	0.033	0.039

Table 3.6: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (MIUR with $\alpha = 0.9$, $\gamma < 0.5$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.852	3.000	50	0.000	0.037	0.000	0.000	0.041	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	3.000	100	0.001	0.031	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	3.000	200	0.005	0.030	0.025	0.001	0.029	0.002	0.000	0.033	0.000	0.000	0.037	0.000
$\alpha = 0.9, \gamma = 0$	0.977	3.000	400	0.014	0.035	0.044	0.004	0.029	0.021	0.001	0.029	0.002	0.000	0.032	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.852	5.000	50	0.000	0.037	0.000	0.000	0.041	0.000	0.000	0.042	0.000	0.000	0.047	0.000
$c = 5$	0.921	5.000	100	0.001	0.031	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	5.000	200	0.005	0.030	0.025	0.001	0.029	0.002	0.000	0.033	0.000	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0$	0.977	5.000	400	0.016	0.036	0.044	0.005	0.029	0.021	0.001	0.030	0.002	0.000	0.031	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.852	10.000	50	0.000	0.036	0.000	0.000	0.041	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	10.000	100	0.001	0.032	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	10.000	200	0.006	0.031	0.025	0.001	0.028	0.002	0.000	0.034	0.000	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0$	0.977	10.000	400	0.019	0.039	0.044	0.006	0.030	0.021	0.001	0.030	0.003	0.000	0.032	0.000
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.852	0.928	50	0.000	0.037	0.001	0.000	0.042	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	0.754	100	0.001	0.031	0.005	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.612	200	0.004	0.028	0.026	0.001	0.030	0.004	0.000	0.034	0.000	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0.3$	0.977	0.497	400	0.009	0.031	0.045	0.003	0.028	0.022	0.000	0.030	0.003	0.000	0.032	0.000
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.852	1.546	50	0.000	0.037	0.000	0.000	0.041	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	1.256	100	0.001	0.031	0.004	0.000	0.036	0.000	0.000	0.039	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	1.020	200	0.004	0.028	0.026	0.001	0.029	0.003	0.000	0.034	0.000	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0.3$	0.977	0.829	400	0.011	0.032	0.044	0.004	0.028	0.022	0.000	0.030	0.003	0.000	0.032	0.000
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.852	3.092	50	0.000	0.037	0.000	0.000	0.041	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	2.512	100	0.001	0.031	0.003	0.000	0.035	0.000	0.000	0.040	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	2.040	200	0.005	0.029	0.025	0.001	0.029	0.003	0.000	0.034	0.000	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0.3$	0.977	1.657	400	0.012	0.034	0.044	0.004	0.028	0.021	0.001	0.029	0.002	0.000	0.032	0.000

Table 3.7: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (MIUR with $\alpha = 0.9$, $\gamma \geq 0.5$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.852	0.424	50	0.000	0.038	0.007	0.000	0.042	0.005	0.000	0.041	0.003	0.000	0.047	0.003
$c = 5$	0.921	0.300	100	0.001	0.031	0.015	0.000	0.035	0.007	0.000	0.040	0.004	0.000	0.044	0.003
$\sigma_u = 1, \sigma_x = 1$	0.958	0.212	200	0.004	0.028	0.035	0.001	0.030	0.015	0.000	0.034	0.008	0.000	0.038	0.005
$\alpha = 0.9, \gamma = 0.5$	0.977	0.150	400	0.009	0.031	0.047	0.004	0.028	0.032	0.000	0.031	0.016	0.000	0.032	0.008
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.852	0.707	50	0.000	0.037	0.002	0.000	0.042	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	0.500	100	0.001	0.031	0.008	0.000	0.035	0.001	0.000	0.039	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.354	200	0.004	0.028	0.028	0.001	0.030	0.007	0.000	0.034	0.001	0.000	0.038	0.001
$\alpha = 0.9, \gamma = 0.5$	0.977	0.250	400	0.009	0.031	0.045	0.003	0.028	0.026	0.000	0.031	0.007	0.000	0.032	0.002
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.852	1.414	50	0.000	0.037	0.000	0.000	0.041	0.000	0.000	0.041	0.000	0.000	0.046	0.000
$c = 5$	0.921	1.000	100	0.001	0.031	0.004	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.707	200	0.004	0.028	0.026	0.001	0.030	0.004	0.000	0.034	0.000	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0.5$	0.977	0.500	400	0.009	0.031	0.045	0.003	0.028	0.022	0.000	0.030	0.003	0.000	0.032	0.000
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.852	0.287	50	0.000	0.037	0.017	0.000	0.041	0.014	0.000	0.041	0.011	0.000	0.047	0.011
$c = 5$	0.921	0.189	100	0.001	0.031	0.026	0.000	0.036	0.020	0.000	0.040	0.016	0.000	0.044	0.014
$\sigma_u = 1, \sigma_x = 1$	0.958	0.125	200	0.004	0.027	0.042	0.001	0.030	0.030	0.000	0.034	0.023	0.000	0.038	0.020
$\alpha = 0.9, \gamma = 0.6$	0.977	0.082	400	0.008	0.031	0.051	0.004	0.028	0.042	0.000	0.031	0.032	0.000	0.032	0.025
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.852	0.478	50	0.000	0.038	0.005	0.000	0.042	0.003	0.000	0.041	0.001	0.000	0.047	0.001
$c = 5$	0.921	0.315	100	0.001	0.031	0.014	0.000	0.035	0.006	0.000	0.040	0.003	0.000	0.044	0.002
$\sigma_u = 1, \sigma_x = 1$	0.958	0.208	200	0.004	0.028	0.035	0.001	0.030	0.015	0.000	0.034	0.008	0.000	0.038	0.006
$\alpha = 0.9, \gamma = 0.6$	0.977	0.137	400	0.009	0.031	0.048	0.004	0.028	0.034	0.000	0.031	0.018	0.000	0.032	0.010
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.852	0.956	50	0.000	0.037	0.001	0.000	0.042	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$c = 5$	0.921	0.631	100	0.001	0.031	0.006	0.000	0.035	0.001	0.000	0.039	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.416	200	0.004	0.028	0.027	0.001	0.030	0.006	0.000	0.034	0.001	0.000	0.038	0.000
$\alpha = 0.9, \gamma = 0.6$	0.977	0.275	400	0.009	0.031	0.045	0.003	0.028	0.025	0.000	0.030	0.006	0.000	0.032	0.001

Table 3.8: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (MIUR with $\alpha = 0.9, \gamma \geq 0.5$. (Cont))

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.852	0.089	50	0.000	0.037	0.046	0.000	0.042	0.042	0.000	0.041	0.041	0.000	0.047	0.041
$c = 5$	0.921	0.048	100	0.001	0.031	0.046	0.000	0.036	0.046	0.000	0.040	0.046	0.000	0.044	0.043
$\sigma_u = 1, \sigma_x = 1$	0.958	0.025	200	0.004	0.028	0.048	0.001	0.030	0.049	0.000	0.034	0.047	0.000	0.038	0.048
$\alpha = 0.9, \gamma = 0.9$	0.977	0.014	400	0.008	0.031	0.052	0.004	0.028	0.048	0.000	0.031	0.050	0.000	0.032	0.045
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.852	0.148	50	0.000	0.037	0.037	0.000	0.041	0.033	0.000	0.041	0.030	0.000	0.047	0.031
$c = 5$	0.921	0.079	100	0.001	0.031	0.042	0.000	0.036	0.041	0.000	0.040	0.040	0.000	0.044	0.037
$\sigma_u = 1, \sigma_x = 1$	0.958	0.042	200	0.004	0.027	0.048	0.001	0.030	0.047	0.000	0.034	0.045	0.000	0.038	0.044
$\alpha = 0.9, \gamma = 0.9$	0.977	0.023	400	0.008	0.031	0.053	0.004	0.028	0.048	0.000	0.031	0.048	0.000	0.032	0.045
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.852	0.296	50	0.000	0.037	0.016	0.000	0.041	0.013	0.000	0.041	0.010	0.000	0.047	0.010
$c = 5$	0.921	0.158	100	0.001	0.031	0.030	0.000	0.036	0.026	0.000	0.040	0.022	0.000	0.044	0.020
$\sigma_u = 1, \sigma_x = 1$	0.958	0.085	200	0.004	0.027	0.044	0.001	0.030	0.038	0.000	0.034	0.034	0.000	0.038	0.031
$\alpha = 0.9, \gamma = 0.9$	0.977	0.046	400	0.008	0.031	0.053	0.004	0.028	0.047	0.000	0.031	0.043	0.000	0.032	0.038
$\tilde{\mu} = 3, \beta = 0, \pi = 1$	0.852	0.060	50	0.000	0.037	0.048	0.000	0.042	0.045	0.000	0.041	0.044	0.000	0.047	0.045
$c = 5$	0.921	0.030	100	0.001	0.031	0.048	0.000	0.036	0.048	0.000	0.040	0.048	0.000	0.044	0.046
$\sigma_u = 1, \sigma_x = 1$	0.958	0.015	200	0.004	0.028	0.049	0.001	0.030	0.050	0.000	0.034	0.048	0.000	0.038	0.049
$\alpha = 0.9, \gamma = 1$	0.977	0.008	400	0.008	0.031	0.052	0.004	0.028	0.049	0.000	0.031	0.050	0.000	0.032	0.046
$\tilde{\mu} = 5, \beta = 0, \pi = 1$	0.852	0.100	50	0.000	0.037	0.044	0.000	0.042	0.041	0.000	0.041	0.039	0.000	0.047	0.039
$c = 5$	0.921	0.050	100	0.001	0.031	0.046	0.000	0.036	0.046	0.000	0.040	0.045	0.000	0.044	0.043
$\sigma_u = 1, \sigma_x = 1$	0.958	0.025	200	0.004	0.028	0.048	0.001	0.030	0.049	0.000	0.034	0.047	0.000	0.038	0.048
$\alpha = 0.9, \gamma = 1$	0.977	0.013	400	0.008	0.031	0.052	0.004	0.028	0.048	0.000	0.031	0.050	0.000	0.032	0.046
$\tilde{\mu} = 10, \beta = 0, \pi = 1$	0.852	0.200	50	0.000	0.037	0.029	0.000	0.041	0.025	0.000	0.041	0.022	0.000	0.047	0.021
$c = 5$	0.921	0.100	100	0.001	0.031	0.039	0.000	0.036	0.038	0.000	0.040	0.035	0.000	0.044	0.033
$\sigma_u = 1, \sigma_x = 1$	0.958	0.050	200	0.004	0.027	0.047	0.001	0.030	0.046	0.000	0.034	0.043	0.000	0.038	0.042
$\alpha = 0.9, \gamma = 1$	0.977	0.025	400	0.008	0.031	0.052	0.004	0.028	0.049	0.000	0.031	0.047	0.000	0.032	0.044

Table 3.9: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.000	3.000	50	0.000	0.039	0.001	0.001	0.035	0.000	0.000	0.040	0.000	0.000	0.050	0.000
$\pi = 1$	0.000	3.000	100	0.002	0.029	0.006	0.000	0.032	0.001	0.000	0.034	0.001	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	0.000	3.000	200	0.003	0.030	0.027	0.000	0.020	0.009	0.000	0.029	0.000	0.000	0.038	0.000
$\gamma = 0$	0.000	3.000	400	0.011	0.031	0.045	0.004	0.026	0.025	0.001	0.025	0.004	0.001	0.026	0.000
$\tilde{\mu} = 5, \beta = 0$	0.000	5.000	50	0.000	0.039	0.000	0.001	0.035	0.000	0.000	0.040	0.000	0.000	0.050	0.000
$\pi = 1$	0.000	5.000	100	0.002	0.029	0.006	0.000	0.032	0.000	0.000	0.034	0.000	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	0.000	5.000	200	0.123	0.159	0.053	0.000	0.020	0.004	0.000	0.029	0.000	0.000	0.038	0.000
$\gamma = 0$	0.000	5.000	400	0.011	0.031	0.049	0.004	0.026	0.026	0.001	0.025	0.003	0.001	0.026	0.000
$\tilde{\mu} = 10, \beta = 0$	0.000	10.000	50	0.000	0.039	0.000	0.001	0.035	0.000	0.000	0.040	0.000	0.000	0.050	0.000
$\pi = 1$	0.000	10.000	100	0.002	0.029	0.004	0.000	0.032	0.000	0.000	0.034	0.000	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	0.000	10.000	200	0.003	0.030	0.025	0.000	0.020	0.003	0.000	0.029	0.000	0.000	0.038	0.000
$\gamma = 0$	0.000	10.000	400	0.011	0.031	0.045	0.004	0.026	0.024	0.001	0.025	0.003	0.001	0.026	0.000
$\tilde{\mu} = 3, \beta = 0$	0.000	0.928	50	0.000	0.039	0.017	0.001	0.035	0.019	0.000	0.040	0.015	0.000	0.050	0.012
$\pi = 1$	0.000	0.754	100	0.002	0.029	0.027	0.000	0.032	0.036	0.000	0.034	0.022	0.000	0.043	0.025
$\sigma_u = 1, \sigma_x = 1$	0.000	0.612	200	0.003	0.030	0.040	0.000	0.020	0.038	0.000	0.029	0.044	0.000	0.038	0.030
$\gamma = 0.3$	0.000	0.497	400	0.011	0.031	0.051	0.004	0.026	0.047	0.001	0.025	0.038	0.001	0.026	0.032
$\tilde{\mu} = 5, \beta = 0$	0.000	1.546	50	0.000	0.039	0.007	0.001	0.035	0.004	0.000	0.040	0.002	0.000	0.050	0.002
$\pi = 1$	0.000	1.256	100	0.002	0.029	0.017	0.000	0.032	0.013	0.000	0.034	0.007	0.000	0.043	0.007
$\sigma_u = 1, \sigma_x = 1$	0.000	1.020	200	0.003	0.030	0.034	0.000	0.020	0.026	0.000	0.029	0.020	0.000	0.038	0.010
$\gamma = 0.3$	0.000	0.829	400	0.011	0.031	0.051	0.004	0.026	0.036	0.001	0.025	0.029	0.001	0.026	0.019
$\tilde{\mu} = 10, \beta = 0$	0.000	3.092	50	0.000	0.039	0.001	0.001	0.035	0.000	0.000	0.040	0.000	0.000	0.050	0.000
$\pi = 1$	0.000	2.512	100	0.002	0.029	0.008	0.000	0.032	0.001	0.000	0.034	0.001	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	0.000	2.040	200	0.003	0.030	0.028	0.000	0.020	0.012	0.000	0.029	0.002	0.000	0.038	0.001
$\gamma = 0.3$	0.000	1.657	400	0.011	0.031	0.044	0.004	0.026	0.028	0.001	0.025	0.011	0.001	0.026	0.003

Table 3.10: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.000	0.424	50	0.000	0.039	0.035	0.001	0.035	0.043	0.000	0.040	0.034	0.000	0.050	0.037
$\pi = 1$	0.000	0.300	100	0.002	0.029	0.042	0.000	0.032	0.056	0.000	0.034	0.048	0.000	0.043	0.053
$\sigma_u = 1, \sigma_x = 1$	0.000	0.212	200	0.003	0.030	0.041	0.000	0.020	0.052	0.000	0.029	0.055	0.000	0.038	0.048
$\gamma = 0.5$	0.000	0.150	400	0.011	0.031	0.054	0.004	0.026	0.054	0.001	0.025	0.048	0.001	0.026	0.047
$\tilde{\mu} = 5, \beta = 0$	0.000	0.707	50	0.000	0.039	0.024	0.001	0.035	0.029	0.000	0.040	0.023	0.000	0.050	0.020
$\pi = 1$	0.000	0.500	100	0.002	0.029	0.036	0.000	0.032	0.044	0.000	0.034	0.036	0.000	0.043	0.041
$\sigma_u = 1, \sigma_x = 1$	0.000	0.354	200	0.003	0.030	0.040	0.000	0.020	0.052	0.000	0.029	0.055	0.000	0.038	0.044
$\gamma = 0.5$	0.000	0.250	400	0.011	0.031	0.055	0.004	0.026	0.054	0.001	0.025	0.047	0.001	0.026	0.044
$\tilde{\mu} = 10, \beta = 0$	0.000	1.414	50	0.000	0.039	0.007	0.001	0.035	0.004	0.000	0.040	0.002	0.000	0.050	0.003
$\pi = 1$	0.000	1.000	100	0.002	0.029	0.021	0.000	0.032	0.021	0.000	0.034	0.012	0.000	0.043	0.014
$\sigma_u = 1, \sigma_x = 1$	0.000	0.707	200	0.003	0.030	0.039	0.000	0.020	0.032	0.000	0.029	0.037	0.000	0.038	0.025
$\gamma = 0.5$	0.000	0.500	400	0.011	0.031	0.051	0.004	0.026	0.047	0.001	0.025	0.038	0.001	0.026	0.032
$\tilde{\mu} = 3, \beta = 0$	0.000	0.287	50	0.000	0.039	0.043	0.001	0.035	0.050	0.000	0.040	0.040	0.000	0.050	0.045
$\pi = 1$	0.000	0.189	100	0.002	0.029	0.043	0.000	0.032	0.060	0.000	0.034	0.052	0.000	0.043	0.056
$\sigma_u = 1, \sigma_x = 1$	0.000	0.125	200	0.003	0.030	0.046	0.000	0.020	0.054	0.000	0.029	0.055	0.000	0.038	0.050
$\gamma = 0.6$	0.000	0.082	400	0.011	0.031	0.053	0.004	0.026	0.053	0.001	0.025	0.049	0.001	0.026	0.046
$\tilde{\mu} = 5, \beta = 0$	0.000	0.478	50	0.000	0.039	0.032	0.001	0.035	0.040	0.000	0.040	0.032	0.000	0.050	0.034
$\pi = 1$	0.000	0.315	100	0.002	0.029	0.042	0.000	0.032	0.055	0.000	0.034	0.048	0.000	0.043	0.051
$\sigma_u = 1, \sigma_x = 1$	0.000	0.208	200	0.003	0.030	0.041	0.000	0.020	0.052	0.000	0.029	0.055	0.000	0.038	0.048
$\gamma = 0.6$	0.000	0.137	400	0.011	0.031	0.054	0.004	0.026	0.054	0.001	0.025	0.049	0.001	0.026	0.047
$\tilde{\mu} = 10, \beta = 0$	0.000	0.956	50	0.000	0.039	0.016	0.001	0.035	0.018	0.000	0.040	0.014	0.000	0.050	0.011
$\pi = 1$	0.000	0.631	100	0.002	0.029	0.032	0.000	0.032	0.039	0.000	0.034	0.031	0.000	0.043	0.034
$\sigma_u = 1, \sigma_x = 1$	0.000	0.416	200	0.003	0.030	0.042	0.000	0.020	0.048	0.000	0.029	0.052	0.000	0.038	0.044
$\gamma = 0.6$	0.000	0.275	400	0.011	0.031	0.055	0.004	0.026	0.054	0.001	0.025	0.046	0.001	0.026	0.043

Table 3.11: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.000	0.287	50	0.000	0.039	0.043	0.001	0.035	0.050	0.000	0.040	0.040	0.000	0.050	0.045
$\pi = 1$	0.000	0.189	100	0.002	0.029	0.043	0.000	0.032	0.060	0.000	0.034	0.052	0.000	0.043	0.056
$\sigma_u = 1, \sigma_x = 1$	0.000	0.125	200	0.003	0.030	0.046	0.000	0.020	0.054	0.000	0.029	0.055	0.000	0.038	0.050
$\gamma = 0.6$	0.000	0.082	400	0.011	0.031	0.053	0.004	0.026	0.053	0.001	0.025	0.049	0.001	0.026	0.046
$\tilde{\mu} = 5, \beta = 0$	0.000	0.478	50	0.000	0.039	0.032	0.001	0.035	0.040	0.000	0.040	0.032	0.000	0.050	0.034
$\pi = 1$	0.000	0.315	100	0.002	0.029	0.042	0.000	0.032	0.055	0.000	0.034	0.048	0.000	0.043	0.051
$\sigma_u = 1, \sigma_x = 1$	0.000	0.208	200	0.003	0.030	0.041	0.000	0.020	0.052	0.000	0.029	0.055	0.000	0.038	0.048
$\gamma = 0.6$	0.000	0.137	400	0.011	0.031	0.054	0.004	0.026	0.054	0.001	0.025	0.049	0.001	0.026	0.047
$\tilde{\mu} = 10, \beta = 0$	0.000	0.956	50	0.000	0.039	0.016	0.001	0.035	0.018	0.000	0.040	0.014	0.000	0.050	0.011
$\pi = 1$	0.000	0.631	100	0.002	0.029	0.032	0.000	0.032	0.039	0.000	0.034	0.031	0.000	0.043	0.034
$\sigma_u = 1, \sigma_x = 1$	0.000	0.416	200	0.003	0.030	0.042	0.000	0.020	0.048	0.000	0.029	0.052	0.000	0.038	0.044
$\gamma = 0.6$	0.000	0.275	400	0.011	0.031	0.055	0.004	0.026	0.054	0.001	0.025	0.046	0.001	0.026	0.043
$\tilde{\mu} = 3, \beta = 0$	0.000	0.089	50	0.000	0.039	0.050	0.001	0.035	0.052	0.000	0.040	0.048	0.000	0.050	0.049
$\pi = 1$	0.000	0.048	100	0.002	0.029	0.048	0.000	0.032	0.060	0.000	0.034	0.054	0.000	0.043	0.058
$\sigma_u = 1, \sigma_x = 1$	0.000	0.025	200	0.003	0.030	0.046	0.000	0.020	0.056	0.000	0.029	0.057	0.000	0.038	0.051
$\gamma = 0.9$	0.000	0.014	400	0.011	0.031	0.056	0.004	0.026	0.052	0.001	0.025	0.050	0.001	0.026	0.046
$\tilde{\mu} = 5, \beta = 0$	0.000	0.148	50	0.000	0.039	0.048	0.001	0.035	0.051	0.000	0.040	0.047	0.000	0.050	0.048
$\pi = 1$	0.000	0.079	100	0.002	0.029	0.048	0.000	0.032	0.060	0.000	0.034	0.054	0.000	0.043	0.058
$\sigma_u = 1, \sigma_x = 1$	0.000	0.042	200	0.003	0.030	0.048	0.000	0.020	0.057	0.000	0.029	0.057	0.000	0.038	0.051
$\gamma = 0.9$	0.000	0.023	400	0.011	0.031	0.056	0.004	0.026	0.052	0.001	0.025	0.050	0.001	0.026	0.046
$\tilde{\mu} = 10, \beta = 0$	0.000	0.296	50	0.000	0.039	0.043	0.001	0.035	0.050	0.000	0.040	0.040	0.000	0.050	0.045
$\pi = 1$	0.000	0.158	100	0.002	0.029	0.044	0.000	0.032	0.061	0.000	0.034	0.052	0.000	0.043	0.056
$\sigma_u = 1, \sigma_x = 1$	0.000	0.085	200	0.003	0.030	0.048	0.000	0.020	0.055	0.000	0.029	0.055	0.000	0.038	0.051
$\gamma = 0.9$	0.000	0.046	400	0.011	0.031	0.054	0.004	0.026	0.051	0.001	0.025	0.051	0.001	0.026	0.046
$\tilde{\mu} = 3, \beta = 0$	0.000	0.060	50	0.000	0.039	0.051	0.001	0.035	0.053	0.000	0.040	0.048	0.000	0.050	0.049
$\pi = 1$	0.000	0.030	100	0.002	0.029	0.048	0.000	0.032	0.060	0.000	0.034	0.054	0.000	0.043	0.058
$\sigma_u = 1, \sigma_x = 1$	0.000	0.015	200	0.003	0.030	0.046	0.000	0.020	0.056	0.000	0.029	0.057	0.000	0.038	0.050
$\gamma = 1$	0.000	0.008	400	0.011	0.031	0.057	0.004	0.026	0.052	0.001	0.025	0.050	0.001	0.026	0.047
$\tilde{\mu} = 5, \beta = 0$	0.000	0.100	50	0.000	0.039	0.050	0.001	0.035	0.052	0.000	0.040	0.048	0.000	0.050	0.048
$\pi = 1$	0.000	0.050	100	0.002	0.029	0.048	0.000	0.032	0.060	0.000	0.034	0.054	0.000	0.043	0.058
$\sigma_u = 1, \sigma_x = 1$	0.000	0.025	200	0.003	0.030	0.046	0.000	0.020	0.056	0.000	0.029	0.057	0.000	0.038	0.051
$\gamma = 1$	0.000	0.013	400	0.011	0.031	0.056	0.004	0.026	0.052	0.001	0.025	0.050	0.001	0.026	0.047
$\tilde{\mu} = 10, \beta = 0$	0.000	0.200	50	0.000	0.039	0.047	0.001	0.035	0.051	0.000	0.040	0.045	0.000	0.050	0.046
$\pi = 1$	0.000	0.100	100	0.002	0.029	0.047	0.000	0.032	0.061	0.000	0.034	0.054	0.000	0.043	0.057
$\sigma_u = 1, \sigma_x = 1$	0.000	0.050	200	0.003	0.030	0.048	0.000	0.020	0.056	0.000	0.029	0.056	0.000	0.038	0.051
$\gamma = 1$	0.000	0.025	400	0.011	0.031	0.056	0.004	0.026	0.052	0.001	0.025	0.050	0.001	0.026	0.046

Table 3.12: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.100	3.000	50	0.000	0.038	0.000	0.000	0.034	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.100	3.000	100	0.001	0.027	0.007	0.000	0.033	0.000	0.000	0.035	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.100	3.000	200	0.003	0.029	0.025	0.000	0.022	0.007	0.000	0.029	0.000	0.000	0.036	0.000
$\gamma = 0$	0.100	3.000	400	0.010	0.028	0.045	0.005	0.025	0.026	0.001	0.026	0.004	0.001	0.027	0.000
$\tilde{\mu} = 5, \beta = 0$	0.100	5.000	50	0.000	0.038	0.000	0.000	0.034	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.100	5.000	100	0.001	0.027	0.005	0.000	0.033	0.000	0.000	0.035	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.100	5.000	200	0.003	0.029	0.025	0.000	0.022	0.003	0.000	0.029	0.000	0.000	0.036	0.000
$\gamma = 0$	0.100	5.000	400	0.010	0.028	0.048	0.005	0.025	0.026	0.001	0.026	0.003	0.001	0.027	0.000
$\tilde{\mu} = 10, \beta = 0$	0.100	10.000	50	0.000	0.038	0.000	0.000	0.034	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.100	10.000	100	0.001	0.027	0.004	0.000	0.033	0.000	0.000	0.035	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.100	10.000	200	0.003	0.029	0.024	0.000	0.022	0.003	0.000	0.029	0.000	0.000	0.036	0.000
$\gamma = 0$	0.100	10.000	400	0.010	0.028	0.045	0.005	0.025	0.024	0.001	0.026	0.003	0.001	0.027	0.000
$\tilde{\mu} = 3, \beta = 0$	0.100	0.928	50	0.000	0.037	0.012	0.000	0.034	0.016	0.000	0.039	0.012	0.000	0.049	0.008
$\pi = 1$	0.100	0.754	100	0.001	0.027	0.027	0.000	0.033	0.029	0.000	0.035	0.016	0.000	0.044	0.022
$\sigma_u = 1, \sigma_x = 1$	0.100	0.612	200	0.003	0.029	0.039	0.000	0.022	0.034	0.000	0.029	0.038	0.000	0.036	0.028
$\gamma = 0.3$	0.100	0.497	400	0.010	0.028	0.054	0.005	0.025	0.044	0.001	0.026	0.041	0.001	0.027	0.027
$\tilde{\mu} = 5, \beta = 0$	0.100	1.546	50	0.000	0.037	0.005	0.000	0.034	0.002	0.000	0.039	0.001	0.000	0.049	0.002
$\pi = 1$	0.100	1.256	100	0.001	0.027	0.016	0.000	0.033	0.011	0.000	0.035	0.006	0.000	0.044	0.003
$\sigma_u = 1, \sigma_x = 1$	0.100	1.020	200	0.003	0.029	0.032	0.000	0.022	0.022	0.000	0.029	0.017	0.000	0.036	0.008
$\gamma = 0.3$	0.100	0.829	400	0.010	0.028	0.052	0.005	0.025	0.034	0.001	0.026	0.027	0.001	0.027	0.014
$\tilde{\mu} = 10, \beta = 0$	0.100	3.092	50	0.000	0.038	0.000	0.000	0.034	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.100	2.512	100	0.001	0.027	0.007	0.000	0.033	0.001	0.000	0.035	0.001	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.100	2.040	200	0.003	0.029	0.027	0.000	0.022	0.011	0.000	0.029	0.001	0.000	0.036	0.001
$\gamma = 0.3$	0.100	1.657	400	0.010	0.028	0.042	0.005	0.025	0.029	0.001	0.026	0.010	0.001	0.027	0.002
$\tilde{\mu} = 3, \beta = 0$	0.100	0.424	50	0.000	0.037	0.035	0.000	0.034	0.042	0.000	0.039	0.034	0.000	0.049	0.033
$\pi = 1$	0.100	0.300	100	0.001	0.027	0.043	0.000	0.033	0.051	0.000	0.035	0.046	0.000	0.044	0.054
$\sigma_u = 1, \sigma_x = 1$	0.100	0.212	200	0.003	0.029	0.042	0.000	0.022	0.051	0.000	0.029	0.057	0.000	0.036	0.048
$\gamma = 0.5$	0.100	0.150	400	0.010	0.028	0.053	0.005	0.025	0.052	0.001	0.026	0.048	0.001	0.027	0.045
$\tilde{\mu} = 5, \beta = 0$	0.100	0.707	50	0.000	0.037	0.020	0.000	0.034	0.025	0.000	0.039	0.021	0.000	0.049	0.019
$\pi = 1$	0.100	0.500	100	0.001	0.027	0.035	0.000	0.033	0.038	0.000	0.035	0.034	0.000	0.044	0.036
$\sigma_u = 1, \sigma_x = 1$	0.100	0.354	200	0.003	0.029	0.043	0.000	0.022	0.049	0.000	0.029	0.051	0.000	0.036	0.044
$\gamma = 0.5$	0.100	0.250	400	0.010	0.028	0.051	0.005	0.025	0.052	0.001	0.026	0.045	0.001	0.027	0.041
$\tilde{\mu} = 10, \beta = 0$	0.100	1.414	50	0.000	0.037	0.006	0.000	0.034	0.004	0.000	0.039	0.002	0.000	0.049	0.002
$\pi = 1$	0.100	1.000	100	0.001	0.027	0.020	0.000	0.033	0.015	0.000	0.035	0.010	0.000	0.044	0.011
$\sigma_u = 1, \sigma_x = 1$	0.100	0.707	200	0.003	0.029	0.038	0.000	0.022	0.031	0.000	0.029	0.034	0.000	0.036	0.021
$\gamma = 0.5$	0.100	0.500	400	0.010	0.028	0.054	0.005	0.025	0.044	0.001	0.026	0.041	0.001	0.027	0.027

Table 3.13: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.100	0.287	50	0.000	0.037	0.041	0.000	0.034	0.048	0.000	0.039	0.041	0.000	0.049	0.043
$\pi = 1$	0.100	0.189	100	0.001	0.027	0.047	0.000	0.033	0.057	0.000	0.035	0.052	0.000	0.044	0.057
$\sigma_u = 1, \sigma_x = 1$	0.100	0.125	200	0.003	0.029	0.049	0.000	0.022	0.053	0.000	0.029	0.058	0.000	0.036	0.051
$\gamma = 0.6$	0.100	0.082	400	0.010	0.028	0.054	0.005	0.025	0.051	0.001	0.026	0.052	0.001	0.027	0.048
$\tilde{\mu} = 5, \beta = 0$	0.100	0.478	50	0.000	0.037	0.032	0.000	0.034	0.039	0.000	0.039	0.031	0.000	0.049	0.031
$\pi = 1$	0.100	0.315	100	0.001	0.027	0.043	0.000	0.033	0.051	0.000	0.035	0.045	0.000	0.044	0.053
$\sigma_u = 1, \sigma_x = 1$	0.100	0.208	200	0.003	0.029	0.043	0.000	0.022	0.051	0.000	0.029	0.057	0.000	0.036	0.048
$\gamma = 0.6$	0.100	0.137	400	0.010	0.028	0.053	0.005	0.025	0.051	0.001	0.026	0.048	0.001	0.027	0.047
$\tilde{\mu} = 10, \beta = 0$	0.100	0.956	50	0.000	0.037	0.011	0.000	0.034	0.016	0.000	0.039	0.010	0.000	0.049	0.005
$\pi = 1$	0.100	0.631	100	0.001	0.027	0.031	0.000	0.033	0.037	0.000	0.035	0.023	0.000	0.044	0.030
$\sigma_u = 1, \sigma_x = 1$	0.100	0.416	200	0.003	0.029	0.042	0.000	0.022	0.045	0.000	0.029	0.050	0.000	0.036	0.041
$\gamma = 0.6$	0.100	0.275	400	0.010	0.028	0.050	0.005	0.025	0.051	0.001	0.026	0.045	0.001	0.027	0.040
$\tilde{\mu} = 3, \beta = 0$	0.100	0.089	50	0.000	0.037	0.048	0.000	0.034	0.052	0.000	0.039	0.050	0.000	0.049	0.049
$\pi = 1$	0.100	0.048	100	0.001	0.027	0.049	0.000	0.033	0.058	0.000	0.035	0.057	0.000	0.044	0.061
$\sigma_u = 1, \sigma_x = 1$	0.100	0.025	200	0.003	0.029	0.051	0.000	0.022	0.053	0.000	0.029	0.058	0.000	0.036	0.052
$\gamma = 0.9$	0.100	0.014	400	0.010	0.028	0.054	0.005	0.025	0.052	0.001	0.026	0.052	0.001	0.027	0.048
$\tilde{\mu} = 5, \beta = 0$	0.100	0.148	50	0.000	0.037	0.048	0.000	0.034	0.050	0.000	0.039	0.047	0.000	0.049	0.047
$\pi = 1$	0.100	0.079	100	0.001	0.027	0.048	0.000	0.033	0.059	0.000	0.035	0.054	0.000	0.044	0.061
$\sigma_u = 1, \sigma_x = 1$	0.100	0.042	200	0.003	0.029	0.052	0.000	0.022	0.053	0.000	0.029	0.058	0.000	0.036	0.051
$\gamma = 0.9$	0.100	0.023	400	0.010	0.028	0.055	0.005	0.025	0.052	0.001	0.026	0.052	0.001	0.027	0.047
$\tilde{\mu} = 10, \beta = 0$	0.100	0.296	50	0.000	0.037	0.040	0.000	0.034	0.048	0.000	0.039	0.041	0.000	0.049	0.043
$\pi = 1$	0.100	0.158	100	0.001	0.027	0.047	0.000	0.033	0.058	0.000	0.035	0.054	0.000	0.044	0.059
$\sigma_u = 1, \sigma_x = 1$	0.100	0.085	200	0.003	0.029	0.050	0.000	0.022	0.051	0.000	0.029	0.058	0.000	0.036	0.051
$\gamma = 0.9$	0.100	0.046	400	0.010	0.028	0.056	0.005	0.025	0.052	0.001	0.026	0.052	0.001	0.027	0.048
$\tilde{\mu} = 3, \beta = 0$	0.100	0.060	50	0.000	0.037	0.050	0.000	0.034	0.052	0.000	0.039	0.050	0.000	0.049	0.049
$\pi = 1$	0.100	0.030	100	0.001	0.027	0.050	0.000	0.033	0.058	0.000	0.035	0.057	0.000	0.044	0.061
$\sigma_u = 1, \sigma_x = 1$	0.100	0.015	200	0.003	0.029	0.052	0.000	0.022	0.054	0.000	0.029	0.058	0.000	0.036	0.051
$\gamma = 1$	0.100	0.008	400	0.010	0.028	0.053	0.005	0.025	0.052	0.001	0.026	0.052	0.001	0.027	0.048
$\tilde{\mu} = 5, \beta = 0$	0.100	0.100	50	0.000	0.037	0.048	0.000	0.034	0.051	0.000	0.039	0.049	0.000	0.049	0.049
$\pi = 1$	0.100	0.050	100	0.001	0.027	0.049	0.000	0.033	0.058	0.000	0.035	0.057	0.000	0.044	0.061
$\sigma_u = 1, \sigma_x = 1$	0.100	0.025	200	0.003	0.029	0.051	0.000	0.022	0.053	0.000	0.029	0.058	0.000	0.036	0.052
$\gamma = 1$	0.100	0.013	400	0.010	0.028	0.054	0.005	0.025	0.052	0.001	0.026	0.052	0.001	0.027	0.048
$\tilde{\mu} = 10, \beta = 0$	0.100	0.200	50	0.000	0.037	0.045	0.000	0.034	0.049	0.000	0.039	0.046	0.000	0.049	0.046
$\pi = 1$	0.100	0.100	100	0.001	0.027	0.048	0.000	0.033	0.059	0.000	0.035	0.055	0.000	0.044	0.061
$\sigma_u = 1, \sigma_x = 1$	0.100	0.050	200	0.003	0.029	0.052	0.000	0.022	0.053	0.000	0.029	0.059	0.000	0.036	0.051
$\gamma = 1$	0.100	0.025	400	0.010	0.028	0.055	0.005	0.025	0.052	0.001	0.026	0.052	0.001	0.027	0.047

Table 3.14: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.300	3.000	50	0.000	0.036	0.000	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.300	3.000	100	0.001	0.027	0.005	0.000	0.031	0.000	0.000	0.031	0.000	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	0.300	3.000	200	0.003	0.028	0.025	0.000	0.023	0.005	0.000	0.030	0.000	0.000	0.037	0.000
$\gamma = 0$	0.300	3.000	400	0.010	0.028	0.048	0.005	0.022	0.026	0.001	0.027	0.003	0.000	0.025	0.000
$\tilde{\mu} = 5, \beta = 0$	0.300	5.000	50	0.000	0.036	0.000	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.300	5.000	100	0.001	0.027	0.004	0.000	0.031	0.000	0.000	0.031	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.300	5.000	200	0.003	0.028	0.024	0.000	0.023	0.003	0.000	0.030	0.000	0.000	0.037	0.000
$\gamma = 0$	0.300	5.000	400	0.010	0.028	0.047	0.005	0.022	0.026	0.001	0.027	0.003	0.000	0.025	0.000
$\tilde{\mu} = 10, \beta = 0$	0.300	10.000	50	0.000	0.037	0.000	0.000	0.036	0.000	0.000	0.039	0.000	0.000	0.050	0.000
$\pi = 1$	0.300	10.000	100	0.001	0.028	0.004	0.000	0.032	0.000	0.000	0.030	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.300	10.000	200	0.003	0.028	0.025	0.000	0.023	0.003	0.000	0.030	0.000	0.000	0.037	0.000
$\gamma = 0$	0.300	10.000	400	0.010	0.028	0.046	0.005	0.022	0.024	0.001	0.027	0.003	0.000	0.025	0.000
$\tilde{\mu} = 3, \beta = 0$	0.300	0.928	50	0.000	0.036	0.008	0.000	0.035	0.009	0.000	0.038	0.004	0.000	0.049	0.003
$\pi = 1$	0.300	0.754	100	0.001	0.027	0.025	0.000	0.031	0.019	0.000	0.031	0.012	0.000	0.044	0.012
$\sigma_u = 1, \sigma_x = 1$	0.300	0.612	200	0.003	0.028	0.036	0.000	0.023	0.031	0.000	0.030	0.030	0.000	0.037	0.017
$\gamma = 0.3$	0.300	0.497	400	0.010	0.028	0.053	0.005	0.022	0.038	0.001	0.027	0.037	0.000	0.025	0.020
$\tilde{\mu} = 5, \beta = 0$	0.300	1.546	50	0.000	0.036	0.002	0.000	0.035	0.000	0.000	0.038	0.001	0.000	0.049	0.001
$\pi = 1$	0.300	1.256	100	0.001	0.027	0.011	0.000	0.031	0.005	0.000	0.031	0.003	0.000	0.043	0.001
$\sigma_u = 1, \sigma_x = 1$	0.300	1.020	200	0.003	0.028	0.031	0.000	0.023	0.018	0.000	0.030	0.009	0.000	0.037	0.004
$\gamma = 0.3$	0.300	0.829	400	0.010	0.028	0.049	0.005	0.022	0.033	0.001	0.027	0.019	0.000	0.025	0.007
$\tilde{\mu} = 10, \beta = 0$	0.300	3.092	50	0.000	0.036	0.000	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\pi = 1$	0.300	2.512	100	0.001	0.027	0.006	0.000	0.031	0.000	0.000	0.031	0.000	0.000	0.043	0.000
$\sigma_u = 1, \sigma_x = 1$	0.300	2.040	200	0.003	0.028	0.026	0.000	0.023	0.009	0.000	0.030	0.000	0.000	0.037	0.001
$\gamma = 0.3$	0.300	1.657	400	0.010	0.028	0.041	0.005	0.022	0.028	0.001	0.027	0.005	0.000	0.025	0.002
$\tilde{\mu} = 3, \beta = 0$	0.300	0.424	50	0.000	0.036	0.031	0.000	0.035	0.038	0.000	0.038	0.033	0.000	0.049	0.028
$\pi = 1$	0.300	0.300	100	0.001	0.027	0.042	0.000	0.031	0.039	0.000	0.031	0.039	0.000	0.044	0.044
$\sigma_u = 1, \sigma_x = 1$	0.300	0.212	200	0.003	0.028	0.045	0.000	0.023	0.049	0.000	0.030	0.054	0.000	0.037	0.048
$\gamma = 0.5$	0.300	0.150	400	0.010	0.028	0.049	0.005	0.022	0.050	0.001	0.027	0.053	0.000	0.025	0.040
$\tilde{\mu} = 5, \beta = 0$	0.300	0.707	50	0.000	0.036	0.014	0.000	0.035	0.017	0.000	0.038	0.013	0.000	0.049	0.010
$\pi = 1$	0.300	0.500	100	0.001	0.027	0.032	0.000	0.031	0.032	0.000	0.031	0.023	0.000	0.044	0.029
$\sigma_u = 1, \sigma_x = 1$	0.300	0.354	200	0.003	0.028	0.042	0.000	0.023	0.042	0.000	0.030	0.046	0.000	0.037	0.038
$\gamma = 0.5$	0.300	0.250	400	0.010	0.028	0.050	0.005	0.022	0.046	0.001	0.027	0.049	0.000	0.025	0.038
$\tilde{\mu} = 10, \beta = 0$	0.300	1.414	50	0.000	0.036	0.002	0.000	0.035	0.000	0.000	0.038	0.001	0.000	0.049	0.001
$\pi = 1$	0.300	1.000	100	0.001	0.027	0.016	0.000	0.031	0.010	0.000	0.031	0.005	0.000	0.043	0.004
$\sigma_u = 1, \sigma_x = 1$	0.300	0.707	200	0.003	0.028	0.034	0.000	0.023	0.027	0.000	0.030	0.023	0.000	0.037	0.012
$\gamma = 0.5$	0.300	0.500	400	0.010	0.028	0.053	0.005	0.022	0.037	0.001	0.027	0.037	0.000	0.025	0.019

Table 3.15: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.300	0.287	50	0.000	0.036	0.041	0.000	0.035	0.045	0.000	0.038	0.041	0.000	0.049	0.041
$\pi = 1$	0.300	0.189	100	0.001	0.027	0.044	0.000	0.031	0.048	0.000	0.031	0.046	0.000	0.044	0.053
$\sigma_u = 1, \sigma_x = 1$	0.300	0.125	200	0.003	0.028	0.049	0.000	0.023	0.054	0.000	0.030	0.057	0.000	0.037	0.051
$\gamma = 0.6$	0.300	0.082	400	0.010	0.028	0.051	0.005	0.022	0.050	0.001	0.027	0.052	0.000	0.025	0.043
$\tilde{\mu} = 5, \beta = 0$	0.300	0.478	50	0.000	0.036	0.024	0.000	0.035	0.032	0.000	0.038	0.029	0.000	0.049	0.024
$\pi = 1$	0.300	0.315	100	0.001	0.027	0.042	0.000	0.031	0.038	0.000	0.031	0.038	0.000	0.044	0.043
$\sigma_u = 1, \sigma_x = 1$	0.300	0.208	200	0.003	0.028	0.045	0.000	0.023	0.049	0.000	0.030	0.054	0.000	0.037	0.048
$\gamma = 0.6$	0.300	0.137	400	0.010	0.028	0.048	0.005	0.022	0.048	0.001	0.027	0.052	0.000	0.025	0.040
$\tilde{\mu} = 10, \beta = 0$	0.300	0.956	50	0.000	0.036	0.007	0.000	0.035	0.009	0.000	0.038	0.003	0.000	0.049	0.003
$\pi = 1$	0.300	0.631	100	0.001	0.027	0.028	0.000	0.031	0.025	0.000	0.031	0.018	0.000	0.044	0.019
$\sigma_u = 1, \sigma_x = 1$	0.300	0.416	200	0.003	0.028	0.040	0.000	0.023	0.038	0.000	0.030	0.043	0.000	0.037	0.031
$\gamma = 0.6$	0.300	0.275	400	0.010	0.028	0.050	0.005	0.022	0.043	0.001	0.027	0.050	0.000	0.025	0.035
$\tilde{\mu} = 3, \beta = 0$	0.300	0.089	50	0.000	0.036	0.047	0.000	0.035	0.054	0.000	0.038	0.048	0.000	0.049	0.051
$\pi = 1$	0.300	0.048	100	0.001	0.027	0.052	0.000	0.031	0.054	0.000	0.031	0.052	0.000	0.044	0.056
$\sigma_u = 1, \sigma_x = 1$	0.300	0.025	200	0.003	0.028	0.053	0.000	0.023	0.055	0.000	0.030	0.058	0.000	0.037	0.053
$\gamma = 0.9$	0.300	0.014	400	0.010	0.028	0.052	0.005	0.022	0.055	0.001	0.027	0.053	0.000	0.025	0.046
$\tilde{\mu} = 5, \beta = 0$	0.300	0.148	50	0.000	0.036	0.046	0.000	0.035	0.054	0.000	0.038	0.047	0.000	0.049	0.050
$\pi = 1$	0.300	0.079	100	0.001	0.027	0.050	0.000	0.031	0.053	0.000	0.031	0.051	0.000	0.044	0.055
$\sigma_u = 1, \sigma_x = 1$	0.300	0.042	200	0.003	0.028	0.052	0.000	0.023	0.055	0.000	0.030	0.059	0.000	0.037	0.053
$\gamma = 0.9$	0.300	0.023	400	0.010	0.028	0.051	0.005	0.022	0.053	0.001	0.027	0.054	0.000	0.025	0.046
$\tilde{\mu} = 10, \beta = 0$	0.300	0.296	50	0.000	0.036	0.041	0.000	0.035	0.045	0.000	0.038	0.040	0.000	0.049	0.039
$\pi = 1$	0.300	0.158	100	0.001	0.027	0.046	0.000	0.031	0.050	0.000	0.031	0.047	0.000	0.044	0.053
$\sigma_u = 1, \sigma_x = 1$	0.300	0.085	200	0.003	0.028	0.051	0.000	0.023	0.056	0.000	0.030	0.059	0.000	0.037	0.051
$\gamma = 0.9$	0.300	0.046	400	0.010	0.028	0.050	0.005	0.022	0.053	0.001	0.027	0.054	0.000	0.025	0.046
$\tilde{\mu} = 3, \beta = 0$	0.300	0.060	50	0.000	0.036	0.047	0.000	0.035	0.055	0.000	0.038	0.048	0.000	0.049	0.051
$\pi = 1$	0.300	0.030	100	0.001	0.027	0.052	0.000	0.031	0.054	0.000	0.031	0.052	0.000	0.044	0.057
$\sigma_u = 1, \sigma_x = 1$	0.300	0.015	200	0.003	0.028	0.054	0.000	0.023	0.055	0.000	0.030	0.058	0.000	0.037	0.053
$\gamma = 1$	0.300	0.008	400	0.010	0.028	0.051	0.005	0.022	0.056	0.001	0.027	0.054	0.000	0.025	0.045
$\tilde{\mu} = 5, \beta = 0$	0.300	0.100	50	0.000	0.036	0.047	0.000	0.035	0.054	0.000	0.038	0.048	0.000	0.049	0.051
$\pi = 1$	0.300	0.050	100	0.001	0.027	0.052	0.000	0.031	0.054	0.000	0.031	0.052	0.000	0.044	0.056
$\sigma_u = 1, \sigma_x = 1$	0.300	0.025	200	0.003	0.028	0.053	0.000	0.023	0.055	0.000	0.030	0.058	0.000	0.037	0.053
$\gamma = 1$	0.300	0.013	400	0.010	0.028	0.052	0.005	0.022	0.056	0.001	0.027	0.053	0.000	0.025	0.046
$\tilde{\mu} = 10, \beta = 0$	0.300	0.200	50	0.000	0.036	0.046	0.000	0.035	0.052	0.000	0.038	0.044	0.000	0.049	0.046
$\pi = 1$	0.300	0.100	100	0.001	0.027	0.049	0.000	0.031	0.053	0.000	0.031	0.051	0.000	0.044	0.055
$\sigma_u = 1, \sigma_x = 1$	0.300	0.050	200	0.003	0.028	0.052	0.000	0.023	0.055	0.000	0.030	0.059	0.000	0.037	0.053
$\gamma = 1$	0.300	0.025	400	0.010	0.028	0.051	0.005	0.022	0.053	0.001	0.027	0.054	0.000	0.025	0.046

Table 3.16: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.500	3.000	50	0.000	0.035	0.000	0.000	0.037	0.000	0.000	0.038	0.000	0.000	0.051	0.000
$\pi = 1$	0.500	3.000	100	0.001	0.027	0.005	0.000	0.032	0.000	0.000	0.036	0.000	0.000	0.045	0.000
$\sigma_u = 1, \sigma_x = 1$	0.500	3.000	200	0.004	0.028	0.024	0.001	0.029	0.004	0.000	0.031	0.000	0.000	0.038	0.000
$\gamma = 0$	0.500	3.000	400	0.011	0.030	0.049	0.003	0.023	0.025	0.001	0.028	0.003	0.000	0.026	0.000
$\tilde{\mu} = 5, \beta = 0$	0.500	5.000	50	0.000	0.036	0.000	0.000	0.037	0.000	0.000	0.038	0.000	0.000	0.050	0.000
$\pi = 1$	0.500	5.000	100	0.001	0.027	0.004	0.000	0.032	0.000	0.000	0.036	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.500	5.000	200	0.004	0.028	0.025	0.001	0.030	0.003	0.000	0.031	0.000	0.000	0.038	0.000
$\gamma = 0$	0.500	5.000	400	0.011	0.030	0.047	0.003	0.023	0.025	0.001	0.028	0.003	0.000	0.026	0.000
$\tilde{\mu} = 10, \beta = 0$	0.500	10.000	50	0.000	0.035	0.000	0.000	0.037	0.000	0.000	0.038	0.000	0.000	0.050	0.000
$\pi = 1$	0.500	10.000	100	0.001	0.027	0.004	0.000	0.032	0.000	0.000	0.035	0.000	0.000	0.044	0.000
$\sigma_u = 1, \sigma_x = 1$	0.500	10.000	200	0.004	0.028	0.025	0.001	0.029	0.003	0.000	0.031	0.000	0.000	0.039	0.000
$\gamma = 0$	0.500	10.000	400	0.011	0.030	0.045	0.003	0.022	0.025	0.001	0.028	0.003	0.000	0.026	0.000
$\tilde{\mu} = 3, \beta = 0$	0.500	0.928	50	0.000	0.036	0.003	0.000	0.037	0.003	0.000	0.038	0.003	0.000	0.051	0.001
$\pi = 1$	0.500	0.754	100	0.001	0.027	0.018	0.000	0.032	0.009	0.000	0.036	0.007	0.000	0.045	0.006
$\sigma_u = 1, \sigma_x = 1$	0.500	0.612	200	0.004	0.028	0.035	0.001	0.028	0.025	0.000	0.031	0.018	0.000	0.038	0.010
$\gamma = 0.3$	0.500	0.497	400	0.011	0.030	0.049	0.002	0.023	0.038	0.001	0.028	0.031	0.000	0.026	0.016
$\tilde{\mu} = 5, \beta = 0$	0.500	1.546	50	0.000	0.035	0.001	0.000	0.037	0.000	0.000	0.038	0.000	0.000	0.051	0.000
$\pi = 1$	0.500	1.256	100	0.001	0.027	0.008	0.000	0.032	0.002	0.000	0.036	0.001	0.000	0.045	0.000
$\sigma_u = 1, \sigma_x = 1$	0.500	1.020	200	0.004	0.028	0.029	0.001	0.028	0.013	0.000	0.031	0.005	0.000	0.038	0.002
$\gamma = 0.3$	0.500	0.829	400	0.011	0.030	0.044	0.002	0.023	0.032	0.001	0.028	0.012	0.000	0.026	0.004
$\tilde{\mu} = 10, \beta = 0$	0.500	3.092	50	0.000	0.035	0.000	0.000	0.037	0.000	0.000	0.038	0.000	0.000	0.051	0.000
$\pi = 1$	0.500	2.512	100	0.001	0.027	0.006	0.000	0.032	0.000	0.000	0.036	0.000	0.000	0.045	0.000
$\sigma_u = 1, \sigma_x = 1$	0.500	2.040	200	0.004	0.028	0.022	0.001	0.029	0.005	0.000	0.031	0.000	0.000	0.038	0.000
$\gamma = 0.3$	0.500	1.657	400	0.011	0.030	0.043	0.003	0.023	0.028	0.001	0.028	0.004	0.000	0.026	0.001
$\tilde{\mu} = 3, \beta = 0$	0.500	0.424	50	0.000	0.036	0.022	0.000	0.037	0.028	0.000	0.038	0.023	0.000	0.051	0.019
$\pi = 1$	0.500	0.300	100	0.001	0.026	0.038	0.000	0.032	0.038	0.000	0.036	0.034	0.000	0.045	0.038
$\sigma_u = 1, \sigma_x = 1$	0.500	0.212	200	0.004	0.027	0.045	0.001	0.028	0.049	0.000	0.031	0.053	0.000	0.038	0.042
$\gamma = 0.5$	0.500	0.150	400	0.011	0.030	0.046	0.002	0.023	0.049	0.001	0.028	0.050	0.000	0.026	0.040
$\tilde{\mu} = 5, \beta = 0$	0.500	0.707	50	0.000	0.036	0.009	0.000	0.037	0.009	0.000	0.038	0.006	0.000	0.051	0.007
$\pi = 1$	0.500	0.500	100	0.001	0.027	0.028	0.000	0.032	0.022	0.000	0.036	0.018	0.000	0.045	0.018
$\sigma_u = 1, \sigma_x = 1$	0.500	0.354	200	0.004	0.027	0.041	0.001	0.028	0.043	0.000	0.031	0.044	0.000	0.038	0.029
$\gamma = 0.5$	0.500	0.250	400	0.011	0.030	0.045	0.002	0.023	0.042	0.001	0.028	0.046	0.000	0.026	0.030
$\tilde{\mu} = 10, \beta = 0$	0.500	1.414	50	0.000	0.035	0.001	0.000	0.037	0.000	0.000	0.038	0.001	0.000	0.051	0.000
$\pi = 1$	0.500	1.000	100	0.001	0.027	0.010	0.000	0.032	0.004	0.000	0.036	0.003	0.000	0.045	0.001
$\sigma_u = 1, \sigma_x = 1$	0.500	0.707	200	0.004	0.028	0.033	0.001	0.028	0.022	0.000	0.031	0.014	0.000	0.038	0.006
$\gamma = 0.5$	0.500	0.500	400	0.011	0.030	0.049	0.002	0.023	0.038	0.001	0.028	0.031	0.000	0.026	0.016

Table 3.17: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.500	0.287	50	0.000	0.036	0.037	0.000	0.037	0.039	0.000	0.038	0.039	0.000	0.051	0.035
$\pi = 1$	0.500	0.189	100	0.001	0.026	0.046	0.000	0.032	0.046	0.000	0.036	0.045	0.000	0.045	0.048
$\sigma_u = 1, \sigma_x = 1$	0.500	0.125	200	0.004	0.027	0.049	0.001	0.028	0.054	0.000	0.031	0.056	0.000	0.038	0.047
$\gamma = 0.6$	0.500	0.082	400	0.011	0.030	0.046	0.002	0.023	0.052	0.001	0.028	0.053	0.000	0.026	0.044
$\tilde{\mu} = 5, \beta = 0$	0.500	0.478	50	0.000	0.036	0.019	0.000	0.037	0.024	0.000	0.038	0.020	0.000	0.051	0.018
$\pi = 1$	0.500	0.315	100	0.001	0.026	0.037	0.000	0.032	0.037	0.000	0.036	0.033	0.000	0.045	0.037
$\sigma_u = 1, \sigma_x = 1$	0.500	0.208	200	0.004	0.027	0.045	0.001	0.028	0.049	0.000	0.031	0.053	0.000	0.038	0.043
$\gamma = 0.6$	0.500	0.137	400	0.011	0.030	0.046	0.002	0.023	0.049	0.001	0.028	0.052	0.000	0.026	0.041
$\tilde{\mu} = 10, \beta = 0$	0.500	0.956	50	0.000	0.036	0.003	0.000	0.037	0.002	0.000	0.038	0.003	0.000	0.051	0.001
$\pi = 1$	0.500	0.631	100	0.001	0.027	0.027	0.000	0.032	0.016	0.000	0.036	0.010	0.000	0.045	0.009
$\sigma_u = 1, \sigma_x = 1$	0.500	0.416	200	0.004	0.027	0.039	0.001	0.028	0.036	0.000	0.031	0.036	0.000	0.038	0.024
$\gamma = 0.6$	0.500	0.275	400	0.011	0.030	0.046	0.002	0.023	0.042	0.001	0.028	0.044	0.000	0.026	0.028
$\tilde{\mu} = 3, \beta = 0$	0.500	0.089	50	0.000	0.036	0.049	0.000	0.037	0.054	0.000	0.038	0.052	0.000	0.051	0.052
$\pi = 1$	0.500	0.048	100	0.001	0.026	0.049	0.000	0.032	0.055	0.000	0.036	0.052	0.000	0.045	0.055
$\sigma_u = 1, \sigma_x = 1$	0.500	0.025	200	0.004	0.027	0.054	0.001	0.028	0.056	0.000	0.031	0.058	0.000	0.038	0.051
$\gamma = 0.9$	0.500	0.014	400	0.011	0.030	0.049	0.002	0.023	0.054	0.001	0.028	0.054	0.000	0.026	0.046
$\tilde{\mu} = 5, \beta = 0$	0.500	0.148	50	0.000	0.036	0.045	0.000	0.037	0.051	0.000	0.038	0.049	0.000	0.051	0.047
$\pi = 1$	0.500	0.079	100	0.001	0.026	0.049	0.000	0.032	0.053	0.000	0.036	0.054	0.000	0.045	0.054
$\sigma_u = 1, \sigma_x = 1$	0.500	0.042	200	0.004	0.027	0.054	0.001	0.028	0.057	0.000	0.031	0.057	0.000	0.038	0.050
$\gamma = 0.9$	0.500	0.023	400	0.011	0.030	0.048	0.002	0.023	0.054	0.001	0.028	0.054	0.000	0.026	0.045
$\tilde{\mu} = 10, \beta = 0$	0.500	0.296	50	0.000	0.036	0.035	0.000	0.037	0.039	0.000	0.038	0.038	0.000	0.051	0.034
$\pi = 1$	0.500	0.158	100	0.001	0.026	0.047	0.000	0.032	0.046	0.000	0.036	0.047	0.000	0.045	0.051
$\sigma_u = 1, \sigma_x = 1$	0.500	0.085	200	0.004	0.027	0.051	0.001	0.028	0.055	0.000	0.031	0.058	0.000	0.038	0.050
$\gamma = 0.9$	0.500	0.046	400	0.011	0.030	0.045	0.002	0.023	0.054	0.001	0.028	0.054	0.000	0.026	0.045
$\tilde{\mu} = 3, \beta = 0$	0.500	0.060	50	0.000	0.036	0.049	0.000	0.037	0.056	0.000	0.038	0.052	0.000	0.051	0.052
$\pi = 1$	0.500	0.030	100	0.001	0.026	0.049	0.000	0.032	0.056	0.000	0.036	0.054	0.000	0.045	0.055
$\sigma_u = 1, \sigma_x = 1$	0.500	0.015	200	0.004	0.027	0.054	0.001	0.028	0.055	0.000	0.031	0.059	0.000	0.038	0.051
$\gamma = 1$	0.500	0.008	400	0.011	0.030	0.050	0.002	0.023	0.054	0.001	0.028	0.053	0.000	0.026	0.046
$\tilde{\mu} = 5, \beta = 0$	0.500	0.100	50	0.000	0.036	0.048	0.000	0.037	0.054	0.000	0.038	0.052	0.000	0.051	0.051
$\pi = 1$	0.500	0.050	100	0.001	0.026	0.050	0.000	0.032	0.055	0.000	0.036	0.052	0.000	0.045	0.055
$\sigma_u = 1, \sigma_x = 1$	0.500	0.025	200	0.004	0.027	0.054	0.001	0.028	0.055	0.000	0.031	0.058	0.000	0.038	0.051
$\gamma = 1$	0.500	0.013	400	0.011	0.030	0.049	0.002	0.023	0.054	0.001	0.028	0.054	0.000	0.026	0.046
$\tilde{\mu} = 10, \beta = 0$	0.500	0.200	50	0.000	0.036	0.042	0.000	0.037	0.046	0.000	0.038	0.046	0.000	0.051	0.043
$\pi = 1$	0.500	0.100	100	0.001	0.026	0.050	0.000	0.032	0.051	0.000	0.036	0.052	0.000	0.045	0.053
$\sigma_u = 1, \sigma_x = 1$	0.500	0.050	200	0.004	0.027	0.054	0.001	0.028	0.056	0.000	0.031	0.057	0.000	0.038	0.050
$\gamma = 1$	0.500	0.025	400	0.011	0.030	0.047	0.002	0.023	0.055	0.001	0.028	0.054	0.000	0.026	0.045

Table 3.18: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.700	3.000	50	0.000	0.033	0.000	0.000	0.037	0.000	0.000	0.033	0.000	0.000	0.049	0.000
$\pi = 1$	0.700	3.000	100	0.001	0.029	0.004	0.000	0.034	0.000	0.000	0.035	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.700	3.000	200	0.005	0.025	0.024	0.001	0.032	0.004	0.000	0.034	0.000	0.000	0.040	0.000
$\gamma = 0$	0.700	3.000	400	0.014	0.035	0.048	0.002	0.022	0.025	0.000	0.029	0.003	0.000	0.028	0.000
$\tilde{\mu} = 5, \beta = 0$	0.700	5.000	50	0.000	0.033	0.000	0.000	0.038	0.000	0.000	0.034	0.000	0.000	0.048	0.000
$\pi = 1$	0.700	5.000	100	0.001	0.029	0.003	0.000	0.033	0.000	0.000	0.034	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	0.700	5.000	200	0.005	0.025	0.024	0.001	0.032	0.002	0.000	0.034	0.000	0.000	0.041	0.000
$\gamma = 0$	0.700	5.000	400	0.014	0.035	0.046	0.002	0.022	0.024	0.000	0.029	0.003	0.000	0.027	0.000
$\tilde{\mu} = 10, \beta = 0$	0.700	10.000	50	0.000	0.035	0.000	0.000	0.035	0.000	0.000	0.033	0.000	0.000	0.047	0.000
$\pi = 1$	0.700	10.000	100	0.001	0.029	0.004	0.000	0.034	0.000	0.000	0.034	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	0.700	10.000	200	0.005	0.024	0.025	0.001	0.032	0.003	0.000	0.034	0.000	0.000	0.041	0.000
$\gamma = 0$	0.700	10.000	400	0.014	0.033	0.046	0.002	0.021	0.023	0.000	0.030	0.003	0.000	0.028	0.000
$\tilde{\mu} = 3, \beta = 0$	0.700	0.928	50	0.000	0.033	0.002	0.000	0.035	0.000	0.000	0.033	0.000	0.000	0.049	0.000
$\pi = 1$	0.700	0.754	100	0.001	0.029	0.011	0.000	0.033	0.004	0.000	0.034	0.003	0.000	0.052	0.001
$\sigma_u = 1, \sigma_x = 1$	0.700	0.612	200	0.005	0.026	0.030	0.001	0.032	0.018	0.000	0.034	0.009	0.000	0.040	0.004
$\gamma = 0.3$	0.700	0.497	400	0.013	0.036	0.048	0.002	0.022	0.038	0.000	0.029	0.018	0.000	0.028	0.008
$\tilde{\mu} = 5, \beta = 0$	0.700	1.546	50	0.000	0.033	0.000	0.000	0.035	0.000	0.000	0.033	0.000	0.000	0.049	0.000
$\pi = 1$	0.700	1.256	100	0.001	0.029	0.008	0.000	0.034	0.001	0.000	0.034	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.700	1.020	200	0.005	0.026	0.027	0.001	0.032	0.009	0.000	0.034	0.001	0.000	0.040	0.001
$\gamma = 0.3$	0.700	0.829	400	0.013	0.036	0.047	0.002	0.022	0.028	0.000	0.029	0.008	0.000	0.028	0.002
$\tilde{\mu} = 10, \beta = 0$	0.700	3.092	50	0.000	0.033	0.000	0.000	0.037	0.000	0.000	0.033	0.000	0.000	0.049	0.000
$\pi = 1$	0.700	2.512	100	0.001	0.029	0.005	0.000	0.034	0.000	0.000	0.035	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.700	2.040	200	0.005	0.026	0.024	0.001	0.032	0.003	0.000	0.034	0.000	0.000	0.040	0.000
$\gamma = 0.3$	0.700	1.657	400	0.013	0.036	0.046	0.002	0.022	0.025	0.000	0.029	0.003	0.000	0.028	0.001
$\tilde{\mu} = 3, \beta = 0$	0.700	0.424	50	0.000	0.033	0.016	0.000	0.035	0.013	0.000	0.033	0.012	0.000	0.049	0.011
$\pi = 1$	0.700	0.300	100	0.001	0.029	0.035	0.000	0.033	0.027	0.000	0.034	0.025	0.000	0.052	0.024
$\sigma_u = 1, \sigma_x = 1$	0.700	0.212	200	0.005	0.026	0.043	0.001	0.032	0.047	0.000	0.034	0.044	0.000	0.040	0.033
$\gamma = 0.5$	0.700	0.150	400	0.013	0.036	0.047	0.002	0.022	0.025	0.000	0.029	0.052	0.000	0.028	0.036
$\tilde{\mu} = 5, \beta = 0$	0.700	0.707	50	0.000	0.033	0.003	0.000	0.035	0.002	0.000	0.033	0.002	0.000	0.049	0.000
$\pi = 1$	0.700	0.500	100	0.001	0.029	0.022	0.000	0.033	0.012	0.000	0.034	0.009	0.000	0.052	0.007
$\sigma_u = 1, \sigma_x = 1$	0.700	0.354	200	0.005	0.026	0.034	0.001	0.032	0.033	0.000	0.034	0.030	0.000	0.040	0.019
$\gamma = 0.5$	0.700	0.250	400	0.013	0.036	0.050	0.002	0.023	0.050	0.000	0.029	0.043	0.000	0.028	0.026
$\tilde{\mu} = 10, \beta = 0$	0.700	1.414	50	0.000	0.033	0.000	0.000	0.035	0.000	0.000	0.033	0.000	0.000	0.049	0.000
$\pi = 1$	0.700	1.000	100	0.001	0.029	0.008	0.000	0.033	0.002	0.000	0.034	0.001	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.700	0.707	200	0.005	0.026	0.028	0.001	0.032	0.013	0.000	0.034	0.004	0.000	0.040	0.003
$\gamma = 0.5$	0.700	0.500	400	0.013	0.036	0.047	0.002	0.022	0.038	0.000	0.029	0.018	0.000	0.028	0.008

Table 3.19: Rejection frequency for DM, ENC and CCS test test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.700	0.287	50	0.000	0.033	0.030	0.000	0.035	0.028	0.000	0.033	0.028	0.000	0.049	0.026
$\pi = 1$	0.700	0.189	100	0.001	0.029	0.043	0.000	0.033	0.039	0.000	0.034	0.038	0.000	0.052	0.041
$\sigma_u = 1, \sigma_x = 1$	0.700	0.125	200	0.005	0.026	0.045	0.001	0.032	0.054	0.000	0.034	0.056	0.000	0.040	0.042
$\gamma = 0.6$	0.700	0.082	400	0.013	0.036	0.045	0.002	0.023	0.053	0.000	0.029	0.056	0.000	0.028	0.041
$\tilde{\mu} = 5, \beta = 0$	0.700	0.478	50	0.000	0.033	0.011	0.000	0.035	0.008	0.000	0.033	0.008	0.000	0.049	0.009
$\pi = 1$	0.700	0.315	100	0.001	0.029	0.035	0.000	0.033	0.026	0.000	0.034	0.024	0.000	0.052	0.023
$\sigma_u = 1, \sigma_x = 1$	0.700	0.208	200	0.005	0.026	0.044	0.001	0.032	0.047	0.000	0.034	0.045	0.000	0.040	0.034
$\gamma = 0.6$	0.700	0.137	400	0.013	0.036	0.047	0.002	0.023	0.050	0.000	0.029	0.053	0.000	0.028	0.038
$\tilde{\mu} = 10, \beta = 0$	0.700	0.956	50	0.000	0.033	0.001	0.000	0.035	0.000	0.000	0.033	0.000	0.000	0.049	0.000
$\pi = 1$	0.700	0.631	100	0.001	0.029	0.015	0.000	0.033	0.004	0.000	0.034	0.005	0.000	0.052	0.002
$\sigma_u = 1, \sigma_x = 1$	0.700	0.416	200	0.005	0.026	0.033	0.001	0.032	0.029	0.000	0.034	0.026	0.000	0.040	0.014
$\gamma = 0.6$	0.700	0.275	400	0.013	0.036	0.051	0.002	0.022	0.048	0.000	0.029	0.039	0.000	0.028	0.023
$\tilde{\mu} = 3, \beta = 0$	0.700	0.089	50	0.000	0.033	0.049	0.000	0.036	0.048	0.000	0.033	0.049	0.000	0.049	0.050
$\pi = 1$	0.700	0.048	100	0.001	0.029	0.052	0.000	0.033	0.051	0.000	0.034	0.057	0.000	0.052	0.055
$\sigma_u = 1, \sigma_x = 1$	0.700	0.025	200	0.005	0.026	0.049	0.001	0.032	0.056	0.000	0.034	0.061	0.000	0.040	0.048
$\gamma = 0.9$	0.700	0.014	400	0.013	0.036	0.046	0.002	0.023	0.054	0.000	0.029	0.055	0.000	0.028	0.042
$\tilde{\mu} = 5, \beta = 0$	0.700	0.148	50	0.000	0.033	0.042	0.000	0.036	0.039	0.000	0.033	0.044	0.000	0.049	0.045
$\pi = 1$	0.700	0.079	100	0.001	0.029	0.050	0.000	0.033	0.049	0.000	0.034	0.055	0.000	0.052	0.054
$\sigma_u = 1, \sigma_x = 1$	0.700	0.042	200	0.005	0.026	0.047	0.001	0.032	0.056	0.000	0.034	0.059	0.000	0.040	0.048
$\gamma = 0.9$	0.700	0.023	400	0.013	0.036	0.047	0.002	0.023	0.055	0.000	0.029	0.055	0.000	0.028	0.042
$\tilde{\mu} = 10, \beta = 0$	0.700	0.296	50	0.000	0.033	0.029	0.000	0.035	0.027	0.000	0.033	0.028	0.000	0.049	0.025
$\pi = 1$	0.700	0.158	100	0.001	0.029	0.045	0.000	0.033	0.045	0.000	0.034	0.041	0.000	0.052	0.046
$\sigma_u = 1, \sigma_x = 1$	0.700	0.085	200	0.005	0.026	0.046	0.001	0.032	0.055	0.000	0.034	0.057	0.000	0.040	0.046
$\gamma = 0.9$	0.700	0.046	400	0.013	0.036	0.046	0.002	0.023	0.055	0.000	0.029	0.057	0.000	0.028	0.042
$\tilde{\mu} = 3, \beta = 0$	0.700	0.060	50	0.000	0.033	0.051	0.000	0.036	0.049	0.000	0.033	0.052	0.000	0.049	0.050
$\pi = 1$	0.700	0.030	100	0.001	0.029	0.052	0.000	0.033	0.051	0.000	0.034	0.057	0.000	0.052	0.055
$\sigma_u = 1, \sigma_x = 1$	0.700	0.015	200	0.005	0.026	0.049	0.001	0.032	0.056	0.000	0.034	0.061	0.000	0.040	0.049
$\gamma = 1$	0.700	0.008	400	0.013	0.036	0.047	0.002	0.023	0.055	0.000	0.029	0.056	0.000	0.028	0.042
$\tilde{\mu} = 5, \beta = 0$	0.700	0.100	50	0.000	0.033	0.049	0.000	0.036	0.046	0.000	0.033	0.049	0.000	0.049	0.049
$\pi = 1$	0.700	0.050	100	0.001	0.029	0.052	0.000	0.033	0.051	0.000	0.034	0.056	0.000	0.052	0.055
$\sigma_u = 1, \sigma_x = 1$	0.700	0.025	200	0.005	0.026	0.049	0.001	0.032	0.056	0.000	0.034	0.061	0.000	0.040	0.048
$\gamma = 1$	0.700	0.013	400	0.013	0.036	0.046	0.002	0.023	0.054	0.000	0.029	0.056	0.000	0.028	0.042
$\tilde{\mu} = 10, \beta = 0$	0.700	0.200	50	0.000	0.033	0.036	0.000	0.036	0.034	0.000	0.033	0.037	0.000	0.049	0.040
$\pi = 1$	0.700	0.100	100	0.001	0.029	0.050	0.000	0.033	0.048	0.000	0.034	0.051	0.000	0.052	0.053
$\sigma_u = 1, \sigma_x = 1$	0.700	0.050	200	0.005	0.026	0.047	0.001	0.032	0.055	0.000	0.034	0.059	0.000	0.040	0.048
$\gamma = 1$	0.700	0.025	400	0.013	0.036	0.047	0.002	0.023	0.054	0.000	0.029	0.055	0.000	0.028	0.042

Table 3.20: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.900	3.000	50	0.000	0.036	0.000	0.000	0.037	0.000	0.000	0.046	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	3.000	100	0.001	0.034	0.004	0.001	0.035	0.000	0.000	0.043	0.000	0.000	0.054	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	3.000	200	0.003	0.028	0.025	0.001	0.027	0.003	0.001	0.035	0.000	0.000	0.039	0.000
$\gamma = 0$	0.900	3.000	400	0.014	0.035	0.047	0.002	0.026	0.022	0.001	0.030	0.003	0.001	0.029	0.000
$\tilde{\mu} = 5, \beta = 0$	0.900	5.000	50	0.000	0.032	0.000	0.000	0.038	0.000	0.000	0.045	0.000	0.000	0.043	0.000
$\pi = 1$	0.900	5.000	100	0.001	0.031	0.004	0.001	0.034	0.000	0.000	0.041	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	5.000	200	0.003	0.029	0.025	0.001	0.028	0.003	0.001	0.034	0.000	0.000	0.039	0.000
$\gamma = 0$	0.900	5.000	400	0.014	0.035	0.047	0.002	0.025	0.022	0.001	0.030	0.003	0.001	0.027	0.000
$\tilde{\mu} = 10, \beta = 0$	0.900	10.000	50	0.000	0.034	0.000	0.000	0.038	0.000	0.000	0.044	0.000	0.000	0.044	0.000
$\pi = 1$	0.900	10.000	100	0.001	0.033	0.004	0.001	0.034	0.000	0.000	0.043	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	10.000	200	0.004	0.032	0.025	0.001	0.027	0.003	0.001	0.035	0.000	0.000	0.039	0.000
$\gamma = 0$	0.900	10.000	400	0.014	0.036	0.046	0.002	0.026	0.023	0.001	0.028	0.003	0.001	0.026	0.000
$\tilde{\mu} = 3, \beta = 0$	0.900	0.928	50	0.000	0.036	0.000	0.000	0.034	0.000	0.000	0.043	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	0.754	100	0.001	0.033	0.006	0.001	0.035	0.001	0.000	0.046	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	0.612	200	0.003	0.029	0.027	0.001	0.027	0.007	0.001	0.034	0.001	0.000	0.039	0.000
$\gamma = 0.3$	0.900	0.497	400	0.011	0.033	0.046	0.002	0.026	0.032	0.001	0.030	0.010	0.001	0.028	0.003
$\tilde{\mu} = 5, \beta = 0$	0.900	1.546	50	0.000	0.034	0.000	0.000	0.035	0.000	0.000	0.044	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	1.256	100	0.001	0.034	0.005	0.001	0.036	0.000	0.000	0.043	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	1.020	200	0.003	0.029	0.025	0.001	0.028	0.005	0.001	0.035	0.001	0.000	0.039	0.000
$\gamma = 0.3$	0.900	0.829	400	0.012	0.033	0.047	0.002	0.026	0.024	0.001	0.030	0.005	0.001	0.028	0.001
$\tilde{\mu} = 10, \beta = 0$	0.900	3.092	50	0.000	0.036	0.000	0.000	0.037	0.000	0.000	0.046	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	2.512	100	0.001	0.034	0.004	0.001	0.035	0.000	0.000	0.044	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	2.040	200	0.004	0.028	0.025	0.001	0.028	0.004	0.001	0.035	0.000	0.000	0.039	0.000
$\gamma = 0.3$	0.900	1.657	400	0.012	0.035	0.047	0.002	0.027	0.024	0.001	0.030	0.004	0.001	0.028	0.000
$\tilde{\mu} = 3, \beta = 0$	0.900	0.424	50	0.000	0.036	0.004	0.000	0.035	0.000	0.000	0.044	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	0.300	100	0.002	0.034	0.015	0.001	0.036	0.009	0.000	0.046	0.006	0.000	0.053	0.005
$\sigma_u = 1, \sigma_x = 1$	0.900	0.212	200	0.003	0.028	0.029	0.001	0.028	0.027	0.001	0.035	0.022	0.000	0.039	0.016
$\gamma = 0.5$	0.900	0.150	400	0.013	0.033	0.057	0.002	0.026	0.046	0.001	0.029	0.042	0.001	0.027	0.020
$\tilde{\mu} = 5, \beta = 0$	0.900	0.707	50	0.000	0.035	0.000	0.000	0.035	0.000	0.000	0.043	0.000	0.000	0.043	0.000
$\pi = 1$	0.900	0.500	100	0.002	0.034	0.006	0.001	0.035	0.001	0.000	0.046	0.001	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	0.354	200	0.003	0.028	0.027	0.001	0.027	0.017	0.001	0.035	0.006	0.000	0.039	0.004
$\gamma = 0.5$	0.900	0.250	400	0.012	0.033	0.052	0.002	0.026	0.041	0.001	0.029	0.026	0.001	0.027	0.010
$\tilde{\mu} = 10, \beta = 0$	0.900	1.414	50	0.000	0.035	0.000	0.000	0.035	0.000	0.000	0.043	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	1.000	100	0.001	0.034	0.006	0.001	0.035	0.000	0.000	0.044	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	0.707	200	0.004	0.029	0.026	0.001	0.027	0.005	0.001	0.034	0.001	0.000	0.039	0.000
$\gamma = 0.5$	0.900	0.500	400	0.011	0.033	0.047	0.002	0.026	0.032	0.001	0.030	0.010	0.001	0.028	0.003

Table 3.21: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.900	0.287	50	0.000	0.035	0.010	0.000	0.037	0.007	0.000	0.044	0.005	0.000	0.042	0.004
$\pi = 1$	0.900	0.189	100	0.002	0.035	0.028	0.001	0.036	0.024	0.000	0.046	0.021	0.000	0.053	0.015
$\sigma_u = 1, \sigma_x = 1$	0.900	0.125	200	0.003	0.028	0.036	0.001	0.028	0.040	0.001	0.035	0.040	0.000	0.039	0.034
$\gamma = 0.6$	0.900	0.082	400	0.013	0.033	0.056	0.002	0.026	0.047	0.001	0.029	0.056	0.001	0.027	0.032
$\tilde{\mu} = 5, \beta = 0$	0.900	0.478	50	0.000	0.035	0.001	0.000	0.035	0.000	0.000	0.044	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	0.315	100	0.002	0.034	0.014	0.001	0.036	0.007	0.000	0.046	0.005	0.000	0.053	0.005
$\sigma_u = 1, \sigma_x = 1$	0.900	0.208	200	0.003	0.028	0.028	0.001	0.028	0.028	0.001	0.035	0.023	0.000	0.039	0.017
$\gamma = 0.6$	0.900	0.137	400	0.013	0.033	0.056	0.002	0.026	0.045	0.001	0.029	0.046	0.001	0.027	0.022
$\tilde{\mu} = 10, \beta = 0$	0.900	0.956	50	0.000	0.036	0.000	0.000	0.034	0.000	0.000	0.043	0.000	0.000	0.042	0.000
$\pi = 1$	0.900	0.631	100	0.001	0.034	0.006	0.001	0.035	0.001	0.000	0.047	0.000	0.000	0.053	0.000
$\sigma_u = 1, \sigma_x = 1$	0.900	0.416	200	0.003	0.028	0.028	0.001	0.027	0.012	0.001	0.035	0.005	0.000	0.039	0.002
$\gamma = 0.6$	0.900	0.275	400	0.012	0.033	0.051	0.002	0.026	0.040	0.001	0.029	0.024	0.001	0.027	0.009
$\tilde{\mu} = 3, \beta = 0$	0.900	0.089	50	0.000	0.037	0.033	0.000	0.038	0.039	0.000	0.043	0.040	0.000	0.043	0.035
$\pi = 1$	0.900	0.048	100	0.002	0.035	0.051	0.001	0.037	0.046	0.000	0.046	0.046	0.000	0.053	0.043
$\sigma_u = 1, \sigma_x = 1$	0.900	0.025	200	0.002	0.028	0.048	0.001	0.028	0.055	0.001	0.035	0.055	0.000	0.040	0.051
$\gamma = 0.9$	0.900	0.014	400	0.013	0.033	0.059	0.002	0.026	0.055	0.001	0.029	0.062	0.001	0.027	0.038
$\tilde{\mu} = 5, \beta = 0$	0.900	0.148	50	0.000	0.037	0.023	0.000	0.038	0.030	0.000	0.043	0.026	0.000	0.042	0.024
$\pi = 1$	0.900	0.079	100	0.002	0.036	0.048	0.001	0.036	0.045	0.000	0.046	0.040	0.000	0.053	0.037
$\sigma_u = 1, \sigma_x = 1$	0.900	0.042	200	0.002	0.028	0.047	0.001	0.028	0.053	0.001	0.035	0.053	0.000	0.040	0.050
$\gamma = 0.9$	0.900	0.023	400	0.013	0.033	0.058	0.002	0.026	0.055	0.001	0.029	0.062	0.001	0.027	0.038
$\tilde{\mu} = 10, \beta = 0$	0.900	0.296	50	0.000	0.035	0.010	0.000	0.036	0.006	0.000	0.044	0.004	0.000	0.042	0.003
$\pi = 1$	0.900	0.158	100	0.002	0.035	0.034	0.001	0.036	0.029	0.000	0.046	0.027	0.000	0.053	0.023
$\sigma_u = 1, \sigma_x = 1$	0.900	0.085	200	0.003	0.028	0.041	0.001	0.028	0.048	0.001	0.035	0.049	0.000	0.039	0.040
$\gamma = 0.9$	0.900	0.046	400	0.013	0.033	0.056	0.002	0.026	0.051	0.001	0.029	0.061	0.001	0.027	0.036
$\tilde{\mu} = 3, \beta = 0$	0.900	0.060	50	0.000	0.036	0.035	0.000	0.038	0.046	0.000	0.043	0.044	0.000	0.043	0.038
$\pi = 1$	0.900	0.030	100	0.002	0.036	0.052	0.001	0.037	0.048	0.000	0.046	0.047	0.000	0.053	0.044
$\sigma_u = 1, \sigma_x = 1$	0.900	0.015	200	0.002	0.028	0.050	0.001	0.028	0.055	0.001	0.035	0.056	0.000	0.040	0.051
$\gamma = 1$	0.900	0.008	400	0.013	0.033	0.059	0.002	0.026	0.057	0.001	0.029	0.062	0.001	0.027	0.039
$\tilde{\mu} = 5, \beta = 0$	0.900	0.100	50	0.000	0.037	0.029	0.000	0.038	0.037	0.000	0.043	0.038	0.000	0.043	0.033
$\pi = 1$	0.900	0.050	100	0.002	0.036	0.050	0.001	0.037	0.046	0.000	0.046	0.045	0.000	0.053	0.043
$\sigma_u = 1, \sigma_x = 1$	0.900	0.025	200	0.002	0.028	0.048	0.001	0.028	0.055	0.001	0.035	0.055	0.000	0.040	0.051
$\gamma = 1$	0.900	0.013	400	0.013	0.033	0.060	0.002	0.026	0.056	0.001	0.029	0.062	0.001	0.027	0.039
$\tilde{\mu} = 10, \beta = 0$	0.900	0.200	50	0.000	0.036	0.018	0.000	0.037	0.020	0.000	0.044	0.016	0.000	0.042	0.014
$\pi = 1$	0.900	0.100	100	0.002	0.036	0.045	0.001	0.036	0.042	0.000	0.046	0.039	0.000	0.053	0.033
$\sigma_u = 1, \sigma_x = 1$	0.900	0.050	200	0.002	0.028	0.046	0.001	0.028	0.052	0.001	0.035	0.052	0.000	0.040	0.050
$\gamma = 1$	0.900	0.025	400	0.013	0.033	0.058	0.002	0.026	0.054	0.001	0.029	0.062	0.001	0.027	0.038

Table 3.22: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.950	3.000	50	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.046	0.000	0.000	0.038	0.000
$\pi = 1$	0.950	3.000	100	0.001	0.037	0.004	0.001	0.035	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	3.000	200	0.004	0.030	0.025	0.001	0.033	0.003	0.001	0.037	0.000	0.000	0.038	0.000
$\gamma = 0$	0.950	3.000	400	0.015	0.032	0.048	0.004	0.027	0.022	0.001	0.029	0.003	0.001	0.026	0.000
$\tilde{\mu} = 5, \beta = 0$	0.950	5.000	50	0.001	0.036	0.000	0.000	0.039	0.000	0.000	0.046	0.000	0.000	0.039	0.000
$\pi = 1$	0.950	5.000	100	0.001	0.036	0.004	0.001	0.034	0.000	0.000	0.041	0.000	0.000	0.047	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	5.000	200	0.005	0.035	0.025	0.001	0.033	0.003	0.001	0.034	0.000	0.000	0.040	0.000
$\gamma = 0$	0.950	5.000	400	0.016	0.033	0.046	0.005	0.026	0.022	0.001	0.030	0.003	0.001	0.026	0.000
$\tilde{\mu} = 10, \beta = 0$	0.950	10.000	50	0.001	0.035	0.000	0.000	0.038	0.000	0.000	0.047	0.000	0.000	0.040	0.000
$\pi = 1$	0.950	10.000	100	0.000	0.037	0.004	0.001	0.033	0.000	0.000	0.042	0.000	0.000	0.050	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	10.000	200	0.006	0.038	0.026	0.001	0.033	0.003	0.001	0.033	0.000	0.000	0.039	0.000
$\gamma = 0$	0.950	10.000	400	0.017	0.035	0.046	0.006	0.026	0.023	0.001	0.030	0.003	0.001	0.026	0.000
$\tilde{\mu} = 3, \beta = 0$	0.950	0.928	50	0.000	0.033	0.000	0.000	0.037	0.000	0.000	0.047	0.000	0.000	0.039	0.000
$\pi = 1$	0.950	0.754	100	0.001	0.040	0.005	0.000	0.031	0.000	0.000	0.038	0.000	0.000	0.050	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	0.612	200	0.002	0.031	0.025	0.000	0.030	0.005	0.001	0.033	0.001	0.000	0.040	0.000
$\gamma = 0.3$	0.950	0.497	400	0.011	0.035	0.050	0.002	0.025	0.026	0.001	0.029	0.005	0.001	0.028	0.001
$\tilde{\mu} = 5, \beta = 0$	0.950	1.546	50	0.000	0.033	0.000	0.000	0.038	0.000	0.000	0.045	0.000	0.000	0.039	0.000
$\pi = 1$	0.950	1.256	100	0.001	0.038	0.004	0.001	0.036	0.000	0.000	0.039	0.000	0.000	0.049	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	1.020	200	0.004	0.032	0.025	0.000	0.030	0.005	0.001	0.033	0.000	0.000	0.041	0.000
$\gamma = 0.3$	0.950	0.829	400	0.011	0.033	0.046	0.002	0.025	0.023	0.001	0.028	0.004	0.001	0.028	0.000
$\tilde{\mu} = 10, \beta = 0$	0.950	3.092	50	0.000	0.035	0.000	0.000	0.039	0.000	0.000	0.046	0.000	0.000	0.038	0.000
$\pi = 1$	0.950	2.512	100	0.001	0.038	0.004	0.001	0.035	0.000	0.000	0.042	0.000	0.000	0.048	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	2.040	200	0.004	0.030	0.024	0.001	0.031	0.003	0.001	0.035	0.000	0.000	0.039	0.000
$\gamma = 0.3$	0.950	1.657	400	0.013	0.034	0.046	0.003	0.028	0.022	0.001	0.028	0.004	0.001	0.026	0.000
$\tilde{\mu} = 3, \beta = 0$	0.950	0.424	50	0.000	0.034	0.000	0.000	0.040	0.000	0.000	0.047	0.000	0.000	0.041	0.000
$\pi = 1$	0.950	0.300	100	0.000	0.036	0.007	0.000	0.032	0.002	0.000	0.040	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	0.212	200	0.002	0.029	0.029	0.000	0.032	0.019	0.001	0.032	0.011	0.000	0.041	0.010
$\gamma = 0.5$	0.950	0.150	400	0.011	0.034	0.052	0.002	0.025	0.037	0.001	0.029	0.032	0.001	0.027	0.015
$\tilde{\mu} = 5, \beta = 0$	0.950	0.707	50	0.000	0.031	0.000	0.000	0.039	0.000	0.000	0.048	0.000	0.000	0.040	0.000
$\pi = 1$	0.950	0.500	100	0.001	0.039	0.006	0.000	0.031	0.000	0.000	0.037	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	0.354	200	0.002	0.028	0.025	0.000	0.032	0.009	0.001	0.032	0.004	0.000	0.041	0.002
$\gamma = 0.5$	0.950	0.250	400	0.011	0.034	0.049	0.002	0.026	0.032	0.001	0.030	0.014	0.001	0.027	0.006
$\tilde{\mu} = 10, \beta = 0$	0.950	1.414	50	0.000	0.032	0.000	0.000	0.038	0.000	0.000	0.045	0.000	0.000	0.039	0.000
$\pi = 1$	0.950	1.000	100	0.001	0.038	0.005	0.000	0.034	0.000	0.000	0.038	0.000	0.000	0.050	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	0.707	200	0.002	0.032	0.025	0.000	0.030	0.004	0.001	0.033	0.001	0.000	0.040	0.000
$\gamma = 0.5$	0.950	0.500	400	0.011	0.035	0.050	0.002	0.025	0.026	0.001	0.029	0.005	0.001	0.028	0.001

Table 3.23: Rejection frequency for DM, ENC and CCS test under null hypothesis, $\beta = 0$ (Weak Stationary Process with $\rho < 1$, Cont)

	$Repeat = 2000$			$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \beta = 0$	0.950	0.287	50	0.000	0.035	0.003	0.000	0.040	0.001	0.000	0.049	0.001	0.000	0.042	0.000
$\pi = 1$	0.950	0.189	100	0.000	0.036	0.020	0.000	0.032	0.010	0.000	0.041	0.010	0.000	0.051	0.005
$\sigma_u = 1, \sigma_x = 1$	0.950	0.125	200	0.002	0.030	0.034	0.000	0.032	0.031	0.001	0.032	0.031	0.000	0.041	0.024
$\gamma = 0.6$	0.950	0.082	400	0.011	0.034	0.061	0.002	0.025	0.048	0.001	0.029	0.050	0.001	0.027	0.027
$\tilde{\mu} = 5, \beta = 0$	0.950	0.478	50	0.000	0.033	0.000	0.000	0.040	0.000	0.000	0.047	0.000	0.000	0.041	0.000
$\pi = 1$	0.950	0.315	100	0.000	0.037	0.007	0.000	0.031	0.002	0.000	0.040	0.000	0.000	0.052	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	0.208	200	0.002	0.029	0.029	0.000	0.032	0.019	0.001	0.032	0.011	0.000	0.041	0.010
$\gamma = 0.6$	0.950	0.137	400	0.011	0.034	0.052	0.002	0.025	0.036	0.001	0.029	0.035	0.001	0.027	0.018
$\tilde{\mu} = 10, \beta = 0$	0.950	0.956	50	0.000	0.033	0.000	0.000	0.037	0.000	0.000	0.046	0.000	0.000	0.040	0.000
$\pi = 1$	0.950	0.631	100	0.001	0.039	0.006	0.000	0.030	0.000	0.000	0.038	0.000	0.000	0.051	0.000
$\sigma_u = 1, \sigma_x = 1$	0.950	0.416	200	0.002	0.030	0.025	0.000	0.032	0.007	0.001	0.032	0.002	0.000	0.041	0.000
$\gamma = 0.6$	0.950	0.275	400	0.011	0.034	0.049	0.002	0.026	0.030	0.001	0.030	0.013	0.001	0.027	0.005
$\tilde{\mu} = 3, \beta = 0$	0.950	0.089	50	0.000	0.035	0.024	0.000	0.039	0.025	0.000	0.048	0.024	0.000	0.043	0.020
$\pi = 1$	0.950	0.048	100	0.000	0.035	0.046	0.000	0.033	0.039	0.000	0.041	0.039	0.000	0.050	0.032
$\sigma_u = 1, \sigma_x = 1$	0.950	0.025	200	0.002	0.029	0.044	0.000	0.031	0.048	0.001	0.033	0.051	0.000	0.041	0.052
$\gamma = 0.9$	0.950	0.014	400	0.011	0.034	0.055	0.002	0.024	0.052	0.001	0.029	0.062	0.001	0.027	0.035
$\tilde{\mu} = 5, \beta = 0$	0.950	0.148	50	0.000	0.035	0.016	0.000	0.040	0.011	0.000	0.049	0.008	0.000	0.043	0.007
$\pi = 1$	0.950	0.079	100	0.000	0.037	0.041	0.000	0.033	0.033	0.000	0.041	0.028	0.000	0.050	0.026
$\sigma_u = 1, \sigma_x = 1$	0.950	0.042	200	0.002	0.030	0.043	0.000	0.031	0.047	0.001	0.033	0.047	0.000	0.041	0.050
$\gamma = 0.9$	0.950	0.023	400	0.011	0.034	0.056	0.002	0.024	0.051	0.001	0.029	0.060	0.001	0.027	0.036
$\tilde{\mu} = 10, \beta = 0$	0.950	0.296	50	0.000	0.035	0.002	0.000	0.041	0.001	0.000	0.049	0.001	0.000	0.042	0.000
$\pi = 1$	0.950	0.158	100	0.000	0.036	0.024	0.000	0.033	0.018	0.000	0.041	0.015	0.000	0.050	0.010
$\sigma_u = 1, \sigma_x = 1$	0.950	0.085	200	0.002	0.030	0.038	0.000	0.031	0.036	0.001	0.033	0.040	0.000	0.041	0.038
$\gamma = 0.9$	0.950	0.046	400	0.011	0.034	0.061	0.002	0.024	0.050	0.001	0.029	0.061	0.001	0.027	0.033
$\tilde{\mu} = 3, \beta = 0$	0.950	0.060	50	0.000	0.035	0.027	0.000	0.039	0.033	0.000	0.049	0.032	0.000	0.043	0.027
$\pi = 1$	0.950	0.030	100	0.000	0.036	0.049	0.000	0.033	0.041	0.000	0.040	0.043	0.000	0.050	0.034
$\sigma_u = 1, \sigma_x = 1$	0.950	0.015	200	0.002	0.029	0.044	0.000	0.031	0.048	0.001	0.033	0.051	0.000	0.041	0.055
$\gamma = 1$	0.950	0.008	400	0.011	0.034	0.056	0.002	0.024	0.050	0.001	0.029	0.062	0.001	0.027	0.035
$\tilde{\mu} = 5, \beta = 0$	0.950	0.100	50	0.000	0.035	0.021	0.000	0.040	0.023	0.000	0.048	0.018	0.000	0.043	0.018
$\pi = 1$	0.950	0.050	100	0.000	0.035	0.046	0.000	0.033	0.037	0.000	0.041	0.038	0.000	0.050	0.031
$\sigma_u = 1, \sigma_x = 1$	0.950	0.025	200	0.002	0.029	0.044	0.000	0.031	0.048	0.001	0.033	0.051	0.000	0.041	0.052
$\gamma = 1$	0.950	0.013	400	0.011	0.034	0.055	0.002	0.024	0.052	0.001	0.029	0.062	0.001	0.027	0.035
$\tilde{\mu} = 10, \beta = 0$	0.950	0.200	50	0.000	0.035	0.010	0.000	0.040	0.005	0.000	0.048	0.003	0.000	0.043	0.001
$\pi = 1$	0.950	0.100	100	0.000	0.037	0.035	0.000	0.033	0.030	0.000	0.041	0.023	0.000	0.050	0.022
$\sigma_u = 1, \sigma_x = 1$	0.950	0.050	200	0.002	0.030	0.044	0.000	0.031	0.046	0.001	0.033	0.046	0.000	0.041	0.048
$\gamma = 1$	0.950	0.025	400	0.011	0.034	0.057	0.002	0.024	0.050	0.001	0.029	0.061	0.001	0.027	0.036

Table 3.24: Rejection frequency for DM, ENC and CCS test under alternative hypothesis, $\beta = 1$ (LUR with $\alpha = 1$)

	$Repeat = 2000$				$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	β	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \pi = 1$	0.900	3.000	50	0.156	0.009	0.663	0.051	0.002	0.799	0.003	0.001	0.949	0.000	0.000	0.998	0.000
$c = 5, \phi = -0.95$	0.950	3.000	100	0.078	0.176	0.943	0.597	0.063	0.951	0.182	0.006	0.972	0.008	0.001	0.996	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	3.000	200	0.039	0.735	1.000	0.993	0.636	1.000	0.954	0.374	1.000	0.671	0.114	1.000	0.124
$\alpha = 1, \gamma = 0$	0.988	3.000	400	0.020	0.993	1.000	1.000	0.992	1.000	1.000	0.983	1.000	1.000	0.930	1.000	0.996
$\tilde{\mu} = 5, \pi = 1$	0.900	5.000	50	0.156	0.058	0.934	0.204	0.008	0.968	0.015	0.001	0.993	0.000	0.000	1.000	0.000
$c = 5, \phi = -0.95$	0.950	5.000	100	0.078	0.687	1.000	0.968	0.441	0.999	0.740	0.154	1.000	0.155	0.014	1.000	0.002
$\sigma_u = 1, \sigma_x = 1$	0.975	5.000	200	0.039	0.993	1.000	1.000	0.988	1.000	1.000	0.965	1.000	0.997	0.830	1.000	0.857
$\alpha = 1, \gamma = 0$	0.988	5.000	400	0.020	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\tilde{\mu} = 10, \pi = 1$	0.900	10.000	50	0.156	0.785	1.000	0.946	0.474	1.000	0.500	0.095	1.000	0.014	0.004	1.000	0.000
$c = 5, \phi = -0.95$	0.950	10.000	100	0.078	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	0.999	0.930	1.000	0.692
$\sigma_u = 1, \sigma_x = 1$	0.975	10.000	200	0.039	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\alpha = 1, \gamma = 0$	0.988	10.000	400	0.020	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\tilde{\mu} = 3, \pi = 1$	0.900	0.928	50	0.156	0.001	0.296	0.032	0.000	0.556	0.022	0.000	0.869	0.077	0.000	0.991	0.908
$c = 5, \phi = -0.95$	0.950	0.754	100	0.078	0.011	0.294	0.090	0.001	0.442	0.022	0.000	0.698	0.005	0.000	0.930	0.004
$\sigma_u = 1, \sigma_x = 1$	0.975	0.612	200	0.039	0.068	0.424	0.403	0.022	0.476	0.154	0.002	0.598	0.031	0.000	0.804	0.004
$\alpha = 1, \gamma = 0.3$	0.988	0.497	400	0.020	0.188	0.732	0.803	0.138	0.720	0.646	0.053	0.719	0.310	0.012	0.814	0.058
$\tilde{\mu} = 5, \pi = 1$	0.900	1.546	50	0.156	0.002	0.374	0.025	0.000	0.622	0.006	0.001	0.888	0.005	0.000	0.994	0.103
$c = 5, \phi = -0.95$	0.950	1.256	100	0.078	0.022	0.450	0.143	0.003	0.564	0.028	0.001	0.782	0.003	0.000	0.951	0.001
$\sigma_u = 1, \sigma_x = 1$	0.975	1.020	200	0.039	0.115	0.648	0.582	0.051	0.684	0.288	0.008	0.748	0.054	0.001	0.883	0.004
$\alpha = 1, \gamma = 0.3$	0.988	0.829	400	0.020	0.338	0.914	0.932	0.266	0.891	0.826	0.135	0.904	0.522	0.038	0.931	0.144
$\tilde{\mu} = 10, \pi = 1$	0.900	3.092	50	0.156	0.011	0.682	0.055	0.002	0.812	0.003	0.001	0.953	0.000	0.000	0.998	0.000
$c = 5, \phi = -0.95$	0.950	2.512	100	0.078	0.107	0.853	0.439	0.030	0.888	0.103	0.003	0.935	0.005	0.001	0.991	0.000
$\sigma_u = 1, \sigma_x = 1$	0.975	2.040	200	0.039	0.387	0.976	0.926	0.263	0.968	0.711	0.088	0.977	0.262	0.010	0.983	0.019
$\alpha = 1, \gamma = 0.3$	0.988	1.657	400	0.020	0.764	1.000	1.000	0.676	0.998	0.994	0.518	0.998	0.923	0.311	0.999	0.610
$\tilde{\mu} = 3, \pi = 1$	0.900	0.424	50	0.156	0.001	0.263	0.056	0.000	0.540	0.108	0.000	0.860	0.688	0.000	0.990	1.000
$c = 5, \phi = -0.95$	0.950	0.300	100	0.078	0.003	0.197	0.060	0.001	0.361	0.027	0.000	0.644	0.021	0.000	0.916	0.136
$\sigma_u = 1, \sigma_x = 1$	0.975	0.212	200	0.039	0.036	0.227	0.250	0.009	0.306	0.084	0.000	0.456	0.024	0.000	0.716	0.006
$\alpha = 1, \gamma = 0.5$	0.988	0.150	400	0.020	0.097	0.439	0.606	0.064	0.453	0.416	0.018	0.467	0.160	0.002	0.626	0.026
$\tilde{\mu} = 5, \pi = 1$	0.900	0.707	50	0.156	0.001	0.276	0.039	0.000	0.546	0.042	0.000	0.864	0.242	0.000	0.990	0.999
$c = 5, \phi = -0.95$	0.950	0.500	100	0.078	0.005	0.230	0.069	0.001	0.389	0.022	0.000	0.666	0.010	0.000	0.921	0.029
$\sigma_u = 1, \sigma_x = 1$	0.975	0.354	200	0.039	0.047	0.287	0.293	0.013	0.361	0.101	0.000	0.504	0.026	0.000	0.748	0.004
$\alpha = 1, \gamma = 0.5$	0.988	0.250	400	0.020	0.121	0.529	0.665	0.078	0.534	0.480	0.027	0.543	0.193	0.005	0.686	0.032
$\tilde{\mu} = 10, \pi = 1$	0.900	1.414	50	0.156	0.002	0.353	0.025	0.000	0.606	0.008	0.001	0.879	0.007	0.000	0.994	0.218
$c = 5, \phi = -0.95$	0.950	1.000	100	0.078	0.016	0.360	0.110	0.002	0.502	0.023	0.001	0.736	0.004	0.000	0.940	0.002
$\sigma_u = 1, \sigma_x = 1$	0.975	0.707	200	0.039	0.076	0.485	0.443	0.026	0.520	0.177	0.003	0.633	0.034	0.000	0.824	0.004
$\alpha = 1, \gamma = 0.5$	0.988	0.500	400	0.020	0.189	0.732	0.805	0.140	0.721	0.647	0.053	0.720	0.310	0.013	0.814	0.060

Table 3.25: Rejection frequency for DM, ENC and CCS test under alternative hypothesis, $\beta = 0.1$ (LUR with $\alpha = 1$. (Cont))

	$Repeat = 2000$				$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	β	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \pi = 1$	0.900	0.287	50	0.156	0.001	0.262	0.076	0.000	0.535	0.205	0.000	0.859	0.892	0.000	0.989	1.000
$c = 5, \phi = -0.95$	0.950	0.189	100	0.078	0.002	0.181	0.059	0.001	0.347	0.033	0.000	0.634	0.042	0.000	0.912	0.290
$\sigma_u = 1, \sigma_x = 1$	0.975	0.125	200	0.039	0.033	0.197	0.222	0.008	0.274	0.073	0.000	0.431	0.024	0.000	0.701	0.008
$\alpha = 1, \gamma = 0.6$	0.988	0.082	400	0.020	0.082	0.385	0.555	0.055	0.388	0.379	0.013	0.420	0.138	0.001	0.581	0.023
$\tilde{\mu} = 5, \pi = 1$	0.900	0.478	50	0.156	0.001	0.264	0.050	0.000	0.541	0.088	0.000	0.860	0.599	0.000	0.989	1.000
$c = 5, \phi = -0.95$	0.950	0.315	100	0.078	0.003	0.198	0.060	0.001	0.363	0.026	0.000	0.646	0.019	0.000	0.916	0.122
$\sigma_u = 1, \sigma_x = 1$	0.975	0.208	200	0.039	0.035	0.225	0.250	0.009	0.306	0.084	0.000	0.455	0.024	0.000	0.715	0.006
$\alpha = 1, \gamma = 0.6$	0.988	0.137	400	0.020	0.094	0.430	0.597	0.063	0.438	0.409	0.018	0.455	0.157	0.002	0.618	0.026
$\tilde{\mu} = 10, \pi = 1$	0.900	0.956	50	0.156	0.001	0.301	0.031	0.000	0.558	0.020	0.000	0.870	0.068	0.000	0.991	0.882
$c = 5, \phi = -0.95$	0.950	0.631	100	0.078	0.009	0.260	0.079	0.001	0.414	0.022	0.000	0.679	0.007	0.000	0.926	0.009
$\sigma_u = 1, \sigma_x = 1$	0.975	0.416	200	0.039	0.051	0.322	0.324	0.015	0.388	0.113	0.001	0.527	0.026	0.000	0.760	0.004
$\alpha = 1, \gamma = 0.6$	0.988	0.275	400	0.020	0.128	0.554	0.677	0.082	0.553	0.498	0.031	0.560	0.206	0.005	0.697	0.032
$\tilde{\mu} = 3, \pi = 1$	0.900	0.089	50	0.156	0.001	0.260	0.123	0.000	0.534	0.450	0.000	0.859	0.996	0.000	0.990	1.000
$c = 5, \phi = -0.95$	0.950	0.048	100	0.078	0.002	0.168	0.061	0.001	0.337	0.043	0.000	0.622	0.095	0.000	0.909	0.602
$\sigma_u = 1, \sigma_x = 1$	0.975	0.025	200	0.039	0.027	0.160	0.199	0.006	0.243	0.066	0.000	0.405	0.025	0.000	0.690	0.013
$\alpha = 1, \gamma = 0.9$	0.988	0.014	400	0.020	0.071	0.335	0.512	0.049	0.335	0.340	0.011	0.375	0.119	0.001	0.542	0.021
$\tilde{\mu} = 5, \pi = 1$	0.900	0.148	50	0.156	0.001	0.262	0.111	0.000	0.531	0.355	0.000	0.859	0.991	0.000	0.990	1.000
$c = 5, \phi = -0.95$	0.950	0.079	100	0.078	0.002	0.168	0.060	0.001	0.340	0.041	0.000	0.621	0.077	0.000	0.910	0.525
$\sigma_u = 1, \sigma_x = 1$	0.975	0.042	200	0.039	0.027	0.164	0.204	0.006	0.247	0.068	0.000	0.410	0.025	0.000	0.691	0.011
$\alpha = 1, \gamma = 0.9$	0.988	0.023	400	0.020	0.072	0.342	0.520	0.049	0.343	0.345	0.011	0.378	0.121	0.001	0.549	0.021
$\tilde{\mu} = 10, \pi = 1$	0.900	0.296	50	0.156	0.001	0.262	0.075	0.000	0.536	0.196	0.000	0.858	0.883	0.000	0.989	1.000
$c = 5, \phi = -0.95$	0.950	0.158	100	0.078	0.002	0.176	0.060	0.001	0.344	0.034	0.000	0.629	0.048	0.000	0.911	0.353
$\sigma_u = 1, \sigma_x = 1$	0.975	0.085	200	0.039	0.030	0.180	0.213	0.007	0.260	0.070	0.000	0.421	0.024	0.000	0.697	0.010
$\alpha = 1, \gamma = 0.9$	0.988	0.046	400	0.020	0.076	0.361	0.536	0.052	0.359	0.358	0.011	0.395	0.131	0.001	0.560	0.021
$\tilde{\mu} = 3, \pi = 1$	0.900	0.060	50	0.156	0.001	0.261	0.138	0.000	0.534	0.501	0.000	0.860	0.998	0.000	0.990	1.000
$c = 5, \phi = -0.95$	0.950	0.030	100	0.078	0.002	0.165	0.061	0.001	0.335	0.046	0.000	0.622	0.102	0.000	0.909	0.642
$\sigma_u = 1, \sigma_x = 1$	0.975	0.015	200	0.039	0.027	0.157	0.197	0.006	0.239	0.066	0.000	0.403	0.025	0.000	0.689	0.014
$\alpha = 1, \gamma = 1$	0.988	0.008	400	0.020	0.069	0.331	0.510	0.048	0.333	0.338	0.011	0.370	0.118	0.001	0.541	0.020
$\tilde{\mu} = 5, \pi = 1$	0.900	0.100	50	0.156	0.001	0.261	0.120	0.000	0.533	0.431	0.000	0.860	0.995	0.000	0.990	1.000
$c = 5, \phi = -0.95$	0.950	0.050	100	0.078	0.002	0.168	0.061	0.001	0.337	0.042	0.000	0.622	0.092	0.000	0.909	0.595
$\sigma_u = 1, \sigma_x = 1$	0.975	0.025	200	0.039	0.027	0.160	0.199	0.006	0.243	0.066	0.000	0.405	0.025	0.000	0.690	0.013
$\alpha = 1, \gamma = 1$	0.988	0.013	400	0.020	0.070	0.335	0.512	0.049	0.334	0.339	0.011	0.374	0.119	0.001	0.542	0.021
$\tilde{\mu} = 10, \pi = 1$	0.900	0.200	50	0.156	0.001	0.263	0.094	0.000	0.534	0.294	0.000	0.859	0.967	0.000	0.990	1.000
$c = 5, \phi = -0.95$	0.950	0.100	100	0.078	0.002	0.170	0.060	0.001	0.342	0.037	0.000	0.621	0.065	0.000	0.910	0.478
$\sigma_u = 1, \sigma_x = 1$	0.975	0.050	200	0.039	0.028	0.167	0.205	0.006	0.249	0.069	0.000	0.414	0.025	0.000	0.692	0.011
$\alpha = 1, \gamma = 1$	0.988	0.025	400	0.020	0.072	0.344	0.522	0.050	0.344	0.346	0.011	0.379	0.122	0.001	0.549	0.021

Table 3.26: Rejection frequency for DM, ENC and CCS test under alternative hypothesis, $\beta = 1$ (MIUR with $\alpha = 0.9$)

	$Repeat = 2000$				$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	β	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \pi = 1$	0.852	3.000	50	0.156	1.000	1.000	0.243	0.043	1.000	0.000	0.092	1.000	0.000	0.261	1.000	0.000
$c = 5, \phi = -0.95$	0.921	3.000	100	0.078	0.037	0.870	0.001	0.042	0.997	0.000	0.091	1.000	0.000	0.263	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	3.000	200	0.039	0.068	0.662	0.023	0.052	0.955	0.001	0.073	1.000	0.000	0.208	1.000	0.000
$\alpha = 0.9, \gamma = 0$	0.977	3.000	400	0.020	0.083	0.484	0.208	0.085	0.806	0.034	0.089	0.988	0.001	0.159	1.000	0.000
$\tilde{\mu} = 5, \pi = 1$	0.852	5.000	50	0.156	0.031	0.982	0.000	0.050	1.000	0.000	0.106	1.000	0.000	0.277	1.000	0.000
$c = 5, \phi = -0.95$	0.921	5.000	100	0.078	0.058	0.921	0.003	0.060	0.998	0.000	0.107	1.000	0.000	0.288	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	5.000	200	0.039	0.118	0.833	0.100	0.102	0.981	0.002	0.125	1.000	0.000	0.264	1.000	0.000
$\alpha = 0.9, \gamma = 0$	0.977	5.000	400	0.020	0.195	0.824	0.547	0.197	0.952	0.189	0.188	0.999	0.008	0.273	1.000	0.000
$\tilde{\mu} = 10, \pi = 1$	0.852	10.000	50	0.156	0.090	0.995	0.000	0.116	1.000	0.000	0.181	1.000	0.000	0.361	1.000	0.000
$c = 5, \phi = -0.95$	0.921	10.000	100	0.078	0.238	0.996	0.066	0.236	1.000	0.001	0.272	1.000	0.000	0.472	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	10.000	200	0.039	0.449	0.999	0.728	0.450	1.000	0.205	0.464	1.000	0.001	0.581	1.000	0.000
$\alpha = 0.9, \gamma = 0$	0.977	10.000	400	0.020	0.717	1.000	0.995	0.684	1.000	0.913	0.694	1.000	0.435	0.749	1.000	0.011
$\tilde{\mu} = 3, \pi = 1$	0.852	0.928	50	0.156	0.020	0.976	0.000	0.041	1.000	0.000	0.095	1.000	0.007	0.269	1.000	0.972
$c = 5, \phi = -0.95$	0.921	0.754	100	0.078	0.034	0.859	0.000	0.037	0.998	0.000	0.089	1.000	0.000	0.264	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.612	200	0.039	0.054	0.597	0.004	0.045	0.953	0.000	0.057	1.000	0.000	0.193	1.000	0.000
$\alpha = 0.9, \gamma = 0.3$	0.977	0.497	400	0.020	0.051	0.269	0.024	0.053	0.703	0.002	0.059	0.984	0.000	0.118	1.000	0.000
$\tilde{\mu} = 5, \pi = 1$	0.852	1.546	50	0.156	0.022	0.973	0.000	0.041	1.000	0.000	0.091	1.000	0.000	0.265	1.000	0.016
$c = 5, \phi = -0.95$	0.921	1.256	100	0.078	0.032	0.853	0.000	0.036	0.998	0.000	0.087	1.000	0.000	0.260	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	1.020	200	0.039	0.052	0.582	0.006	0.045	0.944	0.000	0.057	1.000	0.000	0.192	1.000	0.000
$\alpha = 0.9, \gamma = 0.3$	0.977	0.829	400	0.020	0.047	0.256	0.033	0.051	0.689	0.003	0.059	0.980	0.000	0.116	1.000	0.000
$\tilde{\mu} = 10, \pi = 1$	0.852	3.092	50	0.156	0.024	0.976	0.000	0.043	1.000	0.000	0.093	1.000	0.000	0.263	1.000	0.000
$c = 5, \phi = -0.95$	0.921	2.512	100	0.078	0.034	0.853	0.000	0.039	0.997	0.000	0.086	1.000	0.000	0.260	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	2.040	200	0.039	0.053	0.597	0.012	0.046	0.942	0.001	0.062	1.000	0.000	0.194	1.000	0.000
$\alpha = 0.9, \gamma = 0.3$	0.977	1.657	400	0.020	0.050	0.289	0.075	0.060	0.709	0.006	0.062	0.979	0.000	0.121	1.000	0.000
$\tilde{\mu} = 3, \pi = 1$	0.852	0.424	50	0.156	0.022	0.978	0.001	0.042	1.000	0.024	0.099	1.000	0.771	0.272	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.300	100	0.078	0.036	0.873	0.000	0.040	0.998	0.000	0.092	1.000	0.000	0.269	1.000	0.023
$\sigma_u = 1, \sigma_x = 1$	0.958	0.212	200	0.039	0.057	0.634	0.004	0.045	0.961	0.000	0.058	1.000	0.000	0.195	1.000	0.000
$\alpha = 0.9, \gamma = 0.5$	0.977	0.150	400	0.020	0.059	0.299	0.017	0.059	0.738	0.002	0.059	0.989	0.000	0.120	1.000	0.000
$\tilde{\mu} = 5, \pi = 1$	0.852	0.707	50	0.156	0.021	0.976	0.000	0.041	1.000	0.001	0.098	1.000	0.116	0.270	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.500	100	0.078	0.036	0.865	0.000	0.040	0.998	0.000	0.088	1.000	0.000	0.267	1.000	0.001
$\sigma_u = 1, \sigma_x = 1$	0.958	0.354	200	0.039	0.057	0.619	0.004	0.046	0.959	0.000	0.057	1.000	0.000	0.193	1.000	0.000
$\alpha = 0.9, \gamma = 0.5$	0.977	0.250	400	0.020	0.057	0.287	0.018	0.060	0.727	0.002	0.060	0.987	0.000	0.119	1.000	0.000
$\tilde{\mu} = 10, \pi = 1$	0.852	1.414	50	0.156	0.022	0.974	0.000	0.042	1.000	0.000	0.092	1.000	0.000	0.265	1.000	0.065
$c = 5, \phi = -0.95$	0.921	1.000	100	0.078	0.034	0.854	0.000	0.035	0.998	0.000	0.087	1.000	0.000	0.262	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.707	200	0.039	0.053	0.595	0.004	0.045	0.950	0.000	0.057	1.000	0.000	0.191	1.000	0.000
$\alpha = 0.9, \gamma = 0.5$	0.977	0.500	400	0.020	0.051	0.270	0.024	0.053	0.703	0.002	0.059	0.984	0.000	0.118	1.000	0.000

Table 3.27: Rejection frequency for DM, ENC and CCS test under alternative hypothesis, $\beta = 0.1$ (MIUR with $\alpha = 0.9$. (Cont))

	$Repeat = 2000$				$P = 200$			$P = 400$			$P = 800$			$P = 1600$		
	ρ	μ	R	β	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P	DM_P	ENC_P	CCS_P
$\tilde{\mu} = 3, \pi = 1$	0.852	0.287	50	0.156	0.023	0.978	0.004	0.041	1.000	0.083	0.101	1.000	0.973	0.272	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.189	100	0.078	0.036	0.877	0.001	0.041	0.998	0.000	0.092	1.000	0.001	0.271	1.000	0.136
$\sigma_u = 1, \sigma_x = 1$	0.958	0.125	200	0.039	0.057	0.637	0.005	0.045	0.963	0.000	0.059	1.000	0.000	0.197	1.000	0.000
$\alpha = 0.9, \gamma = 0.6$	0.977	0.082	400	0.020	0.059	0.308	0.017	0.060	0.743	0.002	0.060	0.989	0.000	0.120	1.000	0.000
$\tilde{\mu} = 5, \pi = 1$	0.852	0.478	50	0.156	0.022	0.978	0.001	0.042	1.000	0.013	0.099	1.000	0.634	0.272	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.315	100	0.078	0.036	0.872	0.000	0.040	0.998	0.000	0.092	1.000	0.000	0.269	1.000	0.016
$\sigma_u = 1, \sigma_x = 1$	0.958	0.208	200	0.039	0.057	0.634	0.004	0.045	0.961	0.000	0.058	1.000	0.000	0.195	1.000	0.000
$\alpha = 0.9, \gamma = 0.6$	0.977	0.137	400	0.020	0.059	0.300	0.017	0.059	0.738	0.002	0.059	0.989	0.000	0.121	1.000	0.000
$\tilde{\mu} = 10, \pi = 1$	0.852	0.956	50	0.156	0.020	0.976	0.000	0.041	1.000	0.000	0.095	1.000	0.006	0.269	1.000	0.958
$c = 5, \phi = -0.95$	0.921	0.631	100	0.078	0.035	0.862	0.000	0.038	0.998	0.000	0.090	1.000	0.000	0.266	1.000	0.000
$\sigma_u = 1, \sigma_x = 1$	0.958	0.416	200	0.039	0.055	0.610	0.004	0.047	0.958	0.000	0.057	1.000	0.000	0.193	1.000	0.000
$\alpha = 0.9, \gamma = 0.6$	0.977	0.275	400	0.020	0.058	0.283	0.019	0.058	0.723	0.002	0.058	0.987	0.000	0.120	1.000	0.000
$\tilde{\mu} = 3, \pi = 1$	0.852	0.089	50	0.156	0.024	0.978	0.025	0.041	1.000	0.416	0.101	1.000	1.000	0.271	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.048	100	0.078	0.036	0.883	0.002	0.044	0.998	0.001	0.090	1.000	0.010	0.273	1.000	0.633
$\sigma_u = 1, \sigma_x = 1$	0.958	0.025	200	0.039	0.058	0.652	0.005	0.044	0.965	0.001	0.061	1.000	0.000	0.200	1.000	0.000
$\alpha = 0.9, \gamma = 0.9$	0.977	0.014	400	0.020	0.062	0.320	0.016	0.060	0.750	0.002	0.060	0.990	0.000	0.121	1.000	0.000
$\tilde{\mu} = 5, \pi = 1$	0.852	0.148	50	0.156	0.024	0.978	0.016	0.041	1.000	0.263	0.101	1.000	0.999	0.270	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.079	100	0.078	0.035	0.882	0.002	0.043	0.998	0.001	0.091	1.000	0.009	0.273	1.000	0.489
$\sigma_u = 1, \sigma_x = 1$	0.958	0.042	200	0.039	0.058	0.650	0.005	0.044	0.965	0.000	0.060	1.000	0.000	0.197	1.000	0.000
$\alpha = 0.9, \gamma = 0.9$	0.977	0.023	400	0.020	0.061	0.316	0.016	0.060	0.748	0.002	0.060	0.990	0.000	0.121	1.000	0.000
$\tilde{\mu} = 10, \pi = 1$	0.852	0.296	50	0.156	0.023	0.978	0.004	0.041	1.000	0.079	0.101	1.000	0.970	0.272	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.158	100	0.078	0.035	0.880	0.001	0.042	0.998	0.001	0.091	1.000	0.001	0.271	1.000	0.210
$\sigma_u = 1, \sigma_x = 1$	0.958	0.085	200	0.039	0.057	0.644	0.005	0.044	0.964	0.000	0.059	1.000	0.000	0.197	1.000	0.000
$\alpha = 0.9, \gamma = 0.9$	0.977	0.046	400	0.020	0.061	0.312	0.016	0.060	0.748	0.002	0.061	0.989	0.000	0.120	1.000	0.000
$\tilde{\mu} = 3, \pi = 1$	0.852	0.060	50	0.156	0.024	0.979	0.034	0.042	1.000	0.494	0.101	1.000	1.000	0.270	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.030	100	0.078	0.036	0.884	0.002	0.044	0.998	0.001	0.091	1.000	0.013	0.274	1.000	0.696
$\sigma_u = 1, \sigma_x = 1$	0.958	0.015	200	0.039	0.057	0.653	0.005	0.044	0.965	0.001	0.061	1.000	0.000	0.200	1.000	0.000
$\alpha = 0.9, \gamma = 1$	0.977	0.008	400	0.020	0.063	0.322	0.016	0.060	0.750	0.002	0.060	0.990	0.000	0.122	1.000	0.000
$\tilde{\mu} = 5, \pi = 1$	0.852	0.100	50	0.156	0.024	0.978	0.024	0.041	1.000	0.383	0.101	1.000	1.000	0.271	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.050	100	0.078	0.036	0.883	0.002	0.044	0.998	0.001	0.090	1.000	0.010	0.273	1.000	0.623
$\sigma_u = 1, \sigma_x = 1$	0.958	0.025	200	0.039	0.058	0.652	0.005	0.044	0.965	0.001	0.061	1.000	0.000	0.200	1.000	0.000
$\alpha = 0.9, \gamma = 1$	0.977	0.013	400	0.020	0.063	0.321	0.016	0.060	0.750	0.002	0.060	0.990	0.000	0.121	1.000	0.000
$\tilde{\mu} = 10, \pi = 1$	0.852	0.200	50	0.156	0.022	0.978	0.008	0.041	1.000	0.173	0.100	1.000	0.996	0.272	1.000	1.000
$c = 5, \phi = -0.95$	0.921	0.100	100	0.078	0.035	0.882	0.001	0.042	0.998	0.001	0.091	1.000	0.005	0.272	1.000	0.396
$\sigma_u = 1, \sigma_x = 1$	0.958	0.050	200	0.039	0.058	0.650	0.005	0.044	0.964	0.000	0.060	1.000	0.000	0.197	1.000	0.000
$\alpha = 0.9, \gamma = 1$	0.977	0.025	400	0.020	0.061	0.316	0.016	0.060	0.749	0.002	0.060	0.990	0.000	0.121	1.000	0.000