Lawrence Berkeley National Laboratory

Title

EBQ CODE TRANSPORT OF SPACE-CHARGE BEAMS IN AXIALLY SYMMETRIC DEVICES

Permalink

https://escholarship.org/uc/item/6bn084d5

Author

Paul, A.C.

Publication Date

1982-11-01

Peer reviewed

DISCLAIMER

LBL--13241

DE83 011629

EBQ CODE:

TRANSPORT OF SPACE-CHARGE BEAMS IN

AXIALLY SYMMETRIC DEVICES

Arthur C. Paul

Computer Applications Department
Engineering and Technical Services Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

November 1982

-iii-

CONTENTS

EBQ CODE
TRANSPORT OF SPACE CHARGE BEAMS IN
AXIALLY SYMMETRIC DEVICFS

Arthur C. Paul

Computer Applications Department
Engineering and Technical Services Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

Such general-purpose space charge codes as EGUN,[1] BATES,[2] WOLF,[3] and TRANSPORT[4] do not gracefully accommodate the simulation of relativistic space-charged beams propagating a long distance in axially symmetric devices wheie a high degree of cancellation has ɔccurred between the self-magnetic and self-electric forces of the beam.

The EBQ code was written specifically to follow high current beam particles where space charge is important in long distance flight in axially symmetric machines possessing external electric and magnetic field. FBQ simultaneously tracks all trajectories so as to allow procedures for charge deposition based on inter-ray separations.

---

[1]Electron Trajectory Program, W. B. Herrmannsfeld, SLAC-166 (1973).

[2]A Computer Program for the Design of High Perveance Ion Sources, D. G. Bates, Culham Laboratories, CLM-R-53 (1966).

[3]WOLF, A Computer Ion Beam Simulation Package, A. C. Paul, W. S. Cooper, LBL-5373 (1976); Mathematical Models and Algorithms for the Computer Program WOLF, Klaus Halbach, LBL-4444 (1975).

[4]TRANSPORT, An Ion Optic Program, LBL Version, A. C. Paul, LBL-2697 (1975).

The orbits are treated in Cartesian geometry (position and momentum)
with z as the independent variable. Poisson's equation is solved in
cylindrical geometry on an orthogonal rectangular mesh.

EBQ can also handle problems involving multiple ion species where
the space charge from each must be included. Such problems arise in
the design of ion sources where different charge and mass states are
present.

INTRODUCTION

The EBQ (electric field $\underline{E}$, magnetic fields $\underline{B}$, and space charge Q) code simulates steady state problems envolving space charge transport of charged particles in cylindrically symmetric devices, providing a fairly flexible and forgiving data input structure.

This two dimensional program accepts data specifying the externally applied electric and magnetic fields. The electric and magnetic self fields of the particles are used to obtain a self-consistent azimuthally symmetric charge and current distributions. The code follows particles trajectories and employs a unique method of assigning values of the charge density to grid points. This method provides sufficient accuracy to model the cancellation that occurs between radial electric and magnetic self-forces in a relativistic beam.

A problem can be conceptually divided into four parts, specified by the 1) electrostatic boundary of the problem, 2) the externally applied magnetic fields, 3) the rays (charged particles) comprising the beam, and 4) special input instructing the computer code on output and plotting options, etc. The mathematical procedures used in processing the problem are described on pages 2 - 62, followed by a detailed description of the data input, pages 77 - 205, and twelve examples, pages 207 - 299.

COMPUTATIONAL PROCEDURE

The EBQ code solves Läplace's equation in cylindrical geometry by relaxation using the modified Lieberman procedure. Having thus obtained the electrostatic potential, the rays are tracked simultaneously using the local gradient of the potential to find the electric field from the electrodes. Space charge is calculated either by application of Gauss's Law or by the potential found from solution of Poisson's equation with a known charge distribution. The particles are ordered in ascending radius. The current enclosed by a ray is then used to find the radial electrical space charge field by Gauss's Law and the self-magnetic field by Ampere's Law. The inter-ray spacing is used to map the appropriate charge density onto a lattice which is used on the next cycle to fi i the space charge field in addition to the electrode field by solution of Poisson's equation. The axially symmetric external magnetic field is found from the local magnetic vector potential.

The next cycle is started by solving Poisson's equation. The rays are then reinitialized and simultaneous tracking performed with the rays depositing charge on the lattice for use on the next cycle. This set of calculation is repeated until a predetermined number of cycles have been performed or until the first moment of the particle distribution fails to change by a predetermined amount.

EQUATIONS OF MOTION

EBQ simultaneously track up to 100 trajectories paraxially propa-
gating along some device made up of cylindrically symmetric electrodes
and magnetic field lenses (solenoids). The independent variable is the
axial position z. Due to the simultaneous processing of all rays, no
retrograde motion is allowed. EBQ simulates the beam's space charge by
solving Poisson's equation and the beam's self magnetic field by appli-
cation of Ampere's Law to the current distribution within the beam.

The equations of motion are implemented in Cartesian coordinates to
avoid the singularity at the origin ($R = 0$), while Poisson's equation
and the expressions for the magnetic field are implemented in polar
coordinates to take advantage of the assumed axial symmetry of the
geometry. Seven equations of motion are simultaneously integrated for
each ray, yielding the three dimensional position and momentum of the
particle, its path length, and the time of flight.

The equations of motion are derived from the Lorentz equation:

$$\vec{\dot{P}} = \frac{eq}{c} (\vec{E} + \vec{v} \times \vec{B})$$

Eliminating time by the relation $d/dt = v_z \, d/dz$, we obtain for the
components of the Lorentz equation:

$$P_x' = \frac{eq}{v_z} E_x + eq \left(\frac{v_y}{v_z} B_z - B_y \right)$$

$$P_y' = \frac{eq}{v_z} E_y + eq \left(B_x - \frac{v_x}{v_z} B_z \right)$$

$$P_z' = \frac{eq}{\nu_z} E_z + eq \left(\frac{\nu_x}{\nu_z} B_y - \frac{\nu_y}{\nu_z} B_x \right)$$

where ' denotes differentiation with respect to the longitudinal
coordinate, z. The spatial coordinates can be found from their
differentials

$$x' = Px/Pz$$

$$y' = Py/Pz$$

The path length differential is given by

$$s' = P/Pz$$

where P is the total momentum of the particle, derivable from the energy
by the relativistic relation

$$E^2 = E_0^2 + c^2 P^2$$

The velocity of the particle can readily be found as

$$\nu = \frac{P}{E} c^2$$

c is the velocity of light, E_0 the rest energy in ev and P the
momentum in ev/C. The flight time differential is now found to be

$$t' = s'/\nu$$

rewriting the equations:

$$P_x' = \frac{eq}{c^2} \frac{E}{P_z} E_x + \frac{eq}{c}\left(y' B_z - B_y\right)$$

$$P_y' = \frac{eq}{c^2} \frac{E}{P_z} E_y + \frac{eq}{c}\left(B_x - x' B_z\right)$$

$$P_z' = \frac{eq}{c^2} \frac{E}{P_z} E_z + \frac{eq}{c}\left(x' B_y - y' B_x\right)$$

$$x' = P_x/P_z$$

$$y' = P_y/P_z$$

$$S' = P/P_z$$

$$t' = S'/\upsilon$$

subject to the specification of the initial position in phase space, x, y, z, Px, Py, Pz.

The units for E and P are ergs and ergs sec/cm respectively, B in the gauss, c in cm/sec (3×10^{10}), e in esu (4.8×10^{-10}) and E in statvolts. Practically, for computer implementation, we will work in units of energy and momentum of ev and ev/c, and electric field in Volts/cm, the momentum equations now become

$$P_x'\left(\frac{ev}{cm}\right) = q \frac{E(ev)}{P_z(ev/c)} E_x\left((V/cm)\right) + 300q\left[y' B_z(g) - B_y(g)\right]$$

$$P_y'\left(\frac{ev}{cm}\right) = q \frac{E(ev)}{P_z(ev/c)} E_y\left((V/cm)\right) + 300q\left[B_x(g) - x' B_z(g)\right]$$

$$P_z'\left(\frac{ev}{cm}\right) = q \frac{E(ev)}{P_z(ev/c)} E_z\left((V/cm)\right) + 300q\left[x' B_y(g) - y' B_x(g)\right]$$

SELF-FIELDS

The beam carries charge along the z direction which is deposited on the mesh whenever a ray crosses a mesh line. The current enclosed by each ray is used to find the self magnetic fields by application of Ampere's Law. The paraxial flow guarantees only a $B\phi$ component
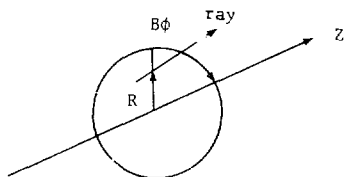
$$\oint B\phi d\ell = \mu_o i$$

where i is the beam current contained inside the orbit

$$B\phi = \frac{\mu_o i}{2\pi R}$$

where $\mu_o = 4\pi \times 10^{-7}$ mks units. i is the total beam current enclosed by the ray and c consideration at radius R.

$$B\dagger(gauss) = 0.2 \frac{i \ (amps)}{R \ (cm)}$$

From symmetry, $B\phi$ has an x and y component which enters the equations of motion depending on the ϕ coordinate of the ray, but no B_z component. This is what I mean by paraxial approximation.

The space charge by application of Gauss's Law would give a radial electric field

$$\int E_r \, ds = \frac{1}{\epsilon_0} \, \Sigma \, Q$$

$$2\pi r E_r = \frac{Q}{\epsilon_0}$$

$$E_r = \frac{i}{2\pi\epsilon_0 r \nu}$$

$$E_r(V/cm) = \frac{50}{\pi\epsilon_0} \quad \frac{i \quad (Amps)}{r(cm) \quad \nu(cm/sec)}$$

The constant has the numerical value 1.795775×10^{12} cm/farad; i is the same enclosed beam current used in calculating the self-magnetic field.

Gauss's Law is not a good approximation to the space charge in the presence of grid electrodes, since these structures short out any radial electric field. A scale factor is therefore made available for use with Gauss's Law to reduce the average field value on the first cycle when gridded guns are simulated. The subsequent cycles are based on solution of Poisson's equation and properly reflect the radial self-electric field in the vicinity of conductors.

The calculational procedure for self fields on any cycle n after the first uses the beam current i_n to find the self-magnetic field and the electric potential U_{N-1} of the preceeding cycle to find the self-radial electric field. Care must be used, since for a beam initially slightly too small, the electric field will be too large and it will

blow up, while on the next cycle, the beam will see too small an elec-
tric field to balance the self-magnetic field and will therefore pinch
on itself. This procedural instability can be damped by applying a
correction to the beam radius used in determining the self-magnetic
field based on the approximation of the radius of the beam on the
previous cycle (N-1) locally determined from the radial electric field.

$$B\phi = 0.2 \frac{i}{R_N + C_{50}(R_{N-1} - R_N)}$$

Where R_N is the ray radius on this cycle, C_{50} is an input constant,
and R_{N-1} is an approximation to the beam radius on cycle N-1
determined from the local radial electric field by Gauss's Law.

$$R_{N-1} \quad \alpha \quad \frac{i}{E_r \upsilon}$$

For very high current beams it is necessary to keep the self-magnetic
and electric field balanced, so $C_{50} = 1$ is a good guess; with not so
high a current where the problem is dominated by the self-electric field
the value of C_{50} is unimportant. Note, when iteratively calculated,
the radial value is R_N regardless of the value of C_{50}.

CHARGE DEPOSITION

Self fields for intense beams must be started in a reasonable way. EQB provides several options.

1) Gauss's Law

2) Charge 3

3) Averaging Over Many Cycles

4) Self Force Damping

5) Charge Limit.

The space charge and self magnetic forces must be in proper balance or the beam will blow up from excessive space changes or self pinch from insufficient space charge. Since the space charge is evaluated for the previous cycle some care must be used to prevent the magnetic self field from pinching the beam. In reality as the beam pinches, the space charge provides a compensating restoring force. In the simulation code, when not using Gauss's Law, this restoring force is evaluated on the previous cycle and so is non-responsive till the next cycle. In order to stabilize the runs so they do not oscillate the self magnetic force can be artificially damped.

The charge and enclosed current can be calculated by one of two different methods.

Standard Method - The standard method calculates the enclosed current as the sum of current for all rays whose radius is less than that of the ray under consideration. The self current of a given ray is taken as $(4J-3)/(8J-4)$ times its current, so:

$$I_{INSIDE_j} = \sum_{\substack{K=1 \\ K \neq J}}^{MAX} I_K(R_K < R_j) + \frac{4J-3}{8J-4} I_j$$

The space charge is deposited on the two nearest mesh intersections by a linear interpolation based on the distance of approach. The charge carried by the J^{TH} ray is

$$Q_j = \frac{I_j}{V_j}$$

where V_j is the velocity of the ray and I_j its current. If the ray is δ from the Kth mesh point, and Δ is the mesh interval, then

$$Q_{K,J} = Q_J \left(1 - \frac{\delta}{\Delta}\right)$$

$$Q_{K+1,J} = Q_J \left(\frac{\delta}{\Delta}\right)$$

The self magnetic field in equilibrium with this space charge is

$$B\phi = 0.2 \frac{I_{inside\ j}}{R}$$

Neil-charge Deposition - The Neil-charge deposition method is used when data is provided giving the charge density at the point of beam origin via the "DENSITY" input specification. This input gives the current density J in amperes/cm^2 as a function of R in cm at the place of beam origin. Conservation of charge requires that the charge between two adjacent rays be conserved,

$$Q = J(r)/V$$

the charge to be deposited on a mesh line of radius R_m separated from the adjacent mesh lines by the internal Δ is the charge between the cylinders of radius $Rm \pm \Delta/2$. This charge is calculated from a mapping of the known charge density at the location of beam origin onto the local lattice map, Figure A. Let $J_o(r_o)$ be the current density at the origin of the beam as a function of radius r_o at which a given ray is born. Let r be the radius of the beam at the current location. Rays may in fact have crossed as shown in the figure represented by a multi-valued contribution of $J_o(r_o)$ to the current value at r. Figure B shows the charge deposition mesh lattice superimposed over the radial mapping function. The charge to be deposited on any mesh point is the integral of the charge along this mapping function.

Consider the charge to be deposited on mesh line n_1, figure B

$$Q_{R1} = \frac{1}{n_1 \Delta^2} \int_A^B \frac{r_o J(r_o) dr_o}{V(r_o)}$$

while the charge to be deposited on mesh line n_2 figure B is

$$Q_{R2} = \frac{1}{n_2 \Delta^2} \left[\int_C^D \frac{r_o J(r_o) dr_o}{V(r_o)} + \int_E^{\bar{F}} \frac{r_o J(r_o) dr_o}{V(r_o)} + \int_G^{\bar{H}} \frac{r_o J(r_o) dr_o}{V(r_o)} \right]$$

The charge deposited on the axis is given by

$$Q_o = \frac{8}{\Delta^2} \int_o^{\Delta/2} \frac{r_o J(r_o) dr_o}{V(r_o)}$$
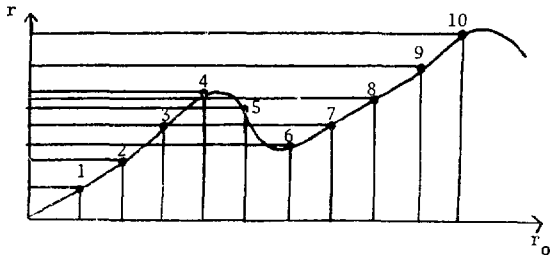
XBL 823-1900

Figure A. Radial mapping function of Neil charge deposition.
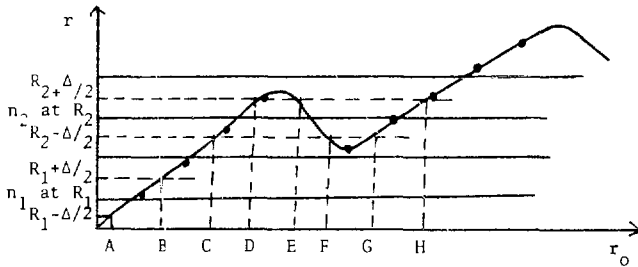
XBL 823-1901

Figure B. MESH lattice superimposed over the radial mapping function.

The self-magnetic field is determined from the current enclosed by a given ray. The enclosed current used with the Neil charge procedure is found from the ratio of areas of the ray as determined by the radial mapping function and the various ray currents which lie inside. For example, consider the current interior to ray 3 in Figure C. Here, all of the current rays 1, 2, and 6 and some fraction of the current carried by rays 3, 5, and 7 are inside ray 3.
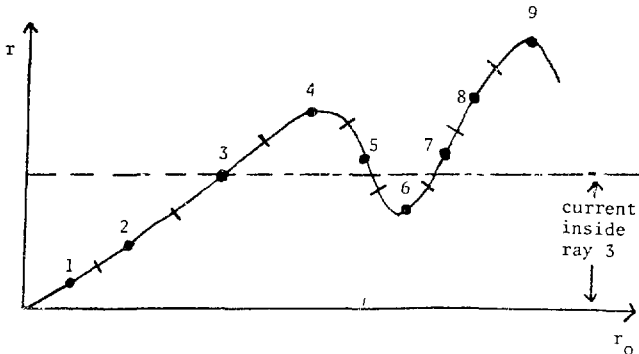
XBL 823-1902

Figure C. Finding current interior to a ray from the radial mapping function.

Each ray is assumed to be carrying its current over a radial interval extending half way to its nearest neighbors.

EXTERNAL MAGNETIC FIELD

The magnetic field from external lenses (solenoids) is calculated from the magnetic vector potential. This potential $\tilde{A}$ can be found from the known radii a and the position Z_L of concentric circular loops.

$$\tilde{A} = \frac{\mu_o}{4\pi} \sum i \int \frac{d\ell}{\rho}$$
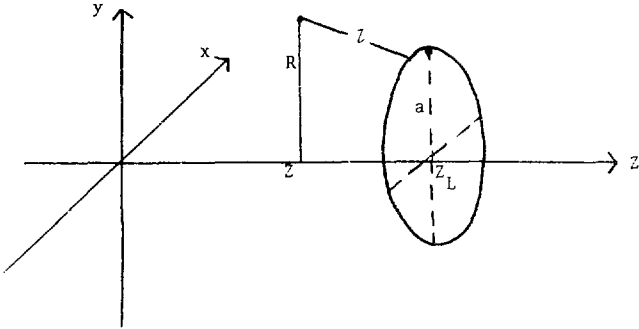
The distance from a point on this loop to some point in space at R, Z, ϕ is

$$\ell = \left[(Z_L - Z)^2 + a^2 + R^2 - 2aR \cos\phi \right]^{1/2}$$

The only component which does not cancel is the tangential component in contribution to A at (R, Z). The value of dl in this direction is $ad\phi\cos\phi$. Thus

$$A_\phi = \frac{\mu_o i}{4\pi} \; 2 \int_o^\pi \frac{a \cos\phi \; d\phi}{\left[(Z_L - Z)^2 + a^2 + R^2 - 2aR \cos\phi \right]^{1/2}}$$

Evaluation of these elliptic integrals yield the magnetic vector potential.

The magnetic field is obtained from

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \left[\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial}{\partial z}(rA\phi)\right]\hat{r} + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\hat{\phi} + \frac{1}{r}\left[\frac{\partial(rA\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right]\hat{z}$$

The azimuthal symmetry guarantees $A_r = A_z = 0$

$$B_z = \frac{\partial A\phi}{\partial r} + \frac{A\phi}{r} \qquad\qquad B_r = -\partial A\phi/\partial z$$

The vector potential is known on the lattice points of the magnetic potential mesh defined by COILGEN data. The value of the potential and the field values within a lattice cell are derived by finite difference approximation. Consider a Taylor series expansion of the vector potential about some point R,Z. The central difference equation is obtained by representing $R = r_0 + r$, $Z = z_0 + z$ so that r, z are small displacements from r_0, z_0.

$$A\phi(r,z) = A\phi(r_0,z_0) + \frac{\partial A\phi}{\partial z}\bigg| z + \frac{\partial A\phi}{\partial r}\bigg| r + \frac{1}{2}\frac{\partial^2 A\phi}{\partial r}\bigg| r^2$$

$$+ \frac{1}{2}\frac{\partial^2 A\phi}{\partial z^2}\bigg| z^2 + \frac{\partial^2 A\phi}{\partial r \partial z}\bigg| rz$$

Defining the expansion coefficients through second order by

$$A\phi(r,z) = a_0 + a_1 z + a_2 r + a_3 r^2 + a_4 z^2 + a_5 rz$$

results in the finite difference approximation to the vector potential

$$A\phi(r,z) = A_o(r_o z_o) + \left(\frac{A_4-A_2}{2\ell}\right)z + \left(\frac{A_1-A_3}{2h}\right)r + \left(\frac{A_1+A_3-2A_0}{2h^2}\right)r^2$$

$$+ \left(\frac{A_2+A_4-2A_o}{2\ell^2}\right)z^2 + \left(\frac{A_5+A_o-A_1-A_4}{h\ell}\right)rz$$
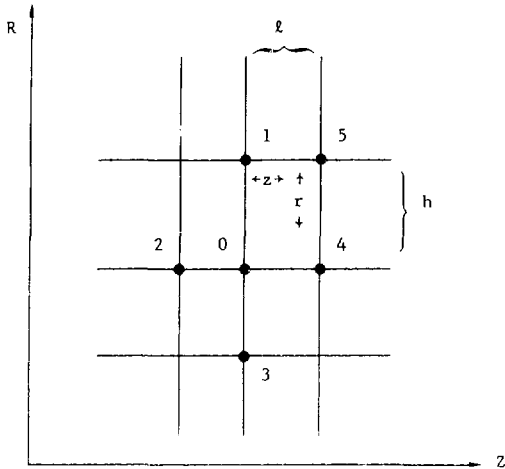
where A_o, A_1, A_2, A_3, A_4, and A_5 are the potential values at
the star mesh intersections 0, 1, 2, 3, 4, and 5 as shown above. h and
ℓ are the radial and longitudinal mesh sizes and r and z are small
displacements from the central point r_o, z_c.

The magnetic field components are

$$B_r = -a_1 - 2a_4 z - a_5 r$$

$$B_z = a_2 + 2a_3 r + a_5 z + \frac{A\phi}{R}$$

Near the axis of cylindrical symmetry $R \rightarrow o$ and a power series expansion for $A\phi$ based on the known symmetry must be used. Only even powers of R can exist in the expression for B_z, since it must be the same for $\pm r$.

Assuming the following expansion near the axis for B_z

$$B_z = 2\left[a_o + a_3 z + a_5 z^2\right] + 4\left[a_2 + a_4 z + a_6 z^2\right] r^2$$

The vector potential is found by integrating B_z

$$A\phi = (a_o + a_3 z + a_5 z^2) r + (a_2 + a_4 z + a_6 z^2) r^3$$

Dropping third order terms, we have

$$A\phi = a_o r + a_3 r z$$

yielding

$$B_r = -a_3 r$$

$$B_z = 2(a_1 + a_3 z)$$

In terms of the lattice point values, we have
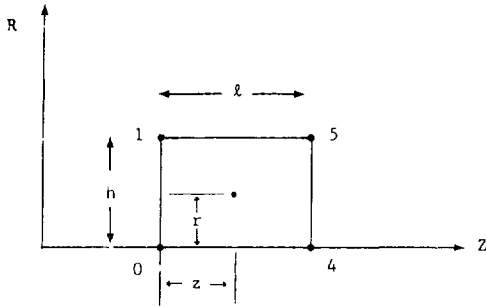
$$A_\phi = \left(\frac{A_1}{h}\right) r + \left(\frac{A_5 - A_1}{h\ell}\right) zr$$

$$B_r = -\left(\frac{A_5 - A_1}{h\ell}\right) r$$

$$B_z = \left(\frac{2A_1}{h}\right) + 2\left(\frac{A_5 - A_1}{h\ell}\right) z$$

ELECTRODYNAMIC POTENTIAI

Poisson's equation is solv˞d by the standard finite difference
approximation in cylindrical coordinates. Consider the usual
rectangular mesh shown:
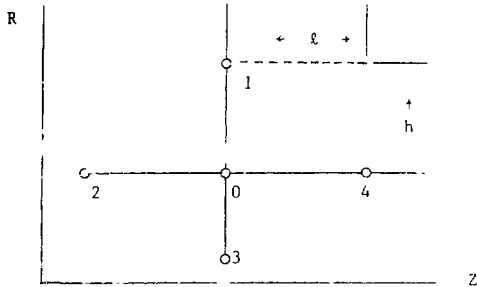
Here ℓ is the longitudinal mesh size and h the radial mesh size.
The solution for the potential at point o satisfied the finite
difference equation approximating $\nabla^2 U = \rho$

$$U_1\left[\frac{1}{h^2}\left(1 + \frac{h}{2r_o}\right)\right] + U_3\left[\frac{1}{h^2}\left(1 - \frac{h}{2r_o}\right)\right] + U_2\,\frac{1}{\ell^2} + U_4\,\frac{1}{\ell^2} - U_o\left[\frac{2}{h^2} + \frac{2}{\ell^2}\right] = \rho$$

where U_0, U_1, U_2, U_3 and U_4 are the potentials at the mesh
intersections 0, 1, 2, 3, and 4 shown in the star diagram above.
Near the axis, the singularity generated by the $h/2r_0$ terms is removed
hy taking a power series expansion for u in even powers or r

$$U = a + cr^2 + \ldots\ldots$$

The appropriate finite difference equation is

$$U_1\left(\frac{2}{h^2}\right) + U_3\left(\frac{2}{h^2}\right) + U_2\left(\frac{1}{\ell^2}\right) + U_4\left(\frac{1}{\ell^2}\right) - U_0\left[\frac{4}{h^2} + \frac{2}{\ell^2}\right] = \rho$$

Several classes of mesh stars result when a boundary intersects the mesh. The irregular stars shown below result. Define β and β_5 as the unit normalized boundary intersections (if the boundary is dz from point o, then $\beta = dz/\ell$ such that when $0 \leq dz \leq \ell$, $0 \leq \beta \leq 1$) the Taylor series expansion coefficients for the potential becomes

$$U = a_0 + a_1z + a_2r + a_3\iota^2 + a_4z^2 + a_5rz$$

where

$$a_0 = U_0$$

$$a_1 = (U_Q - U_N)/2D$$

$$a_J = (U_N + U_Q - 2U_0)/2D_1^2$$

$$a_K = (U_B - \beta^2 U_p - (1-\beta^2)U_0)/D_2^2(1+\beta)$$

$$a_L = (U_p + U_L - (1+\beta)U_0)/D_2^2\beta(1+\beta)$$

$$a_5 = \left[\left[U_0 - \left(\frac{\beta_5-\beta}{1+\beta}\right)U_p + \left[\frac{1+\beta_5}{\beta(1+\beta)} - \frac{1}{\beta_5}\right]U_B - \left(\frac{1-\beta+\beta_5}{\beta}\right)U_0\right]\right]/D_1D_2$$

$$a_5 = (U_5 + U_0 - U_1 - U_4/D_1D_2$$

The value of I, J, K, L, D_1 and D_2 are determined by the boundary intersection quadrant. U_R is the fixed boundary point potential and is not relaxed during solution of the equations. The electric fields are:

$$E_r = \frac{\partial u}{\partial r} = a_2 + 2a_3 r + a_5 z$$

$$E_z = \frac{\partial u}{\partial z} = a_1 + 2a_4 z + a_5 r$$

For internal (not boundary points or electrode points) points, higher order terms can be included in the expression for E_r and E_z. This will slow down the calculation but might be necessary for very high current beams where significant depression of the potential on axis occurred. Control of this option is selected by the value of CONST(86). The additional terms are:

$$U_{ez} = U + a_6 r^3 + (a_7 r + a_8 z) \, rz$$

$$E_r = E_r + 3a_6 r^2 + 2a_7 rz + a_8 z^2$$

$$E_z = E_z + a_7 r^2 + 2a_8 rz$$

For axial points we have, to third order in the potential

$$U = U_o + a_2 z + a_4 z^2 + (a_1 + a_3 z) \, r^2 + a_6 z^3$$

$$\varepsilon_r = 2r \, (a_1 + a_3 z)$$

$$\epsilon_z = a_2 + 2a_4 z + a_3 r^2 + 3a_6 z^2$$

Higher order terms may also be included. These additional terms are:

$$u = u + a_5 z^2 r^2 + a_7 r^4$$

$$\epsilon_r = E_r + 2a_5 z^2 r + 4a_7 r^3$$

$$\epsilon_z = E_z + 2a_5 r^2 z$$
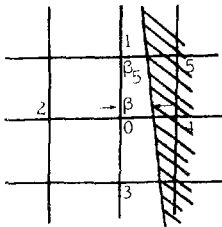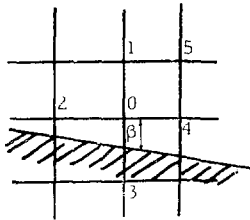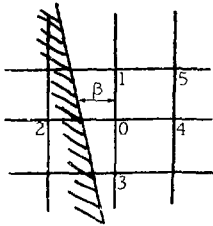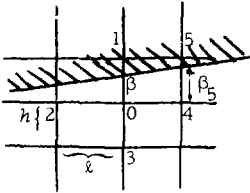
	I	J	K	L	M	N	P	Q	a_5	D_1	D_2
1	4	2	3	1	2	3	4	$-^1a_5$	ℓ	h
2	3	-1	4	2	3	4	1	$^2a_5$	h	ℓ
1	4	-2	3	3	2	1	4	$^2a_5$	ℓ	h
2	3	1	4	4	3	2	1	$-^1a_5$	h	ℓ

BOUNDARY MESH

During processing of the boundary an array is set up specifying if a point is totally interior (i.e., not on the boundary or bordering an electrode, or a boundary point). If a point is interior, then the value of this mesh point is the index ICOEF used to find the finite difference coefficient for solving Poisson's equation. If the point is a boundary point, then three added quantities are masked into this array specifying the type of boundary IB, the index for the potential of the electrode IPOT, and the unit normalized displacement of the boundary β from the central mesh point.

The structure of this MESH ( ) array in the sixty bit word of the CDC computer environment is

MESH ( ) =	β		IB	IPOT	ICOEF
60	31	30	22	13	1
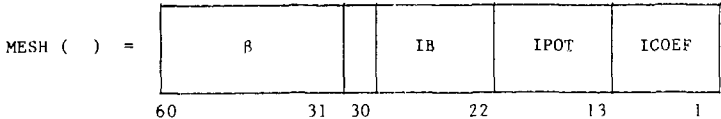
A boundary point is any point, I, whose mesh value, MESH(I), is greater than the maximum value allowed for coefficients, 4095-7777B. However, the array size of 6000 allows only 6000/6=1000 boundary points. The maximum order of mesh points is determined by the size of the U, MESH and RH arrays currently 6000 points.

A Neumann boundary is identified by a zero potential index flag
(i.e., bits 13-21). The type of boundary is given by bits 22-29. A
Neumann boundary on the axis of cylindrical symmetry is designated as
type 003, while a left Neumann boundary is type 002 and a right end
Neumann boundary is type 004.

A Drichlet boundary refers to an electrode, frozen at a potential
value given by POT(IPOT) where POT ( ) is an user input array of
potentials and IPOT is bits 13-21 of the mesh value associated with the
given point.

A table of the mesh array can be printed by use of MAP 1 data
input. The distance β of the electrode from the mesh point is stored in
the high order 30 bits of the mesh array and can be printed by use of
MAP 5 data input. This boundary intersection distance is associated
with a given leg of the mesh according to the value of the boundary
type, bits 22-29.

Bit 30 is used only during orbit tracking and is normally zero.
If, however, bit 30 has been turned on, orbit tracking will terminate
for any ray reaching this point.

Representation of the boundary intersection by 30 bits yields
results accurate to five significant figures, as shown in table 1.
Columns 2 and 3 show the 60 bit value in base 10 and octal while columns
4 and 5 show the effects of truncation of the word to the high order 30
bits.

TABLE 1

0 1.0E-8 1.0E-7 1.0E-6 1.0E-5 1.0E-4 1.0E-3 .01 .1
.25 .333333333333333   .5 .666666666666666   .75 .9375
.99 .999 .9999 .99999 .999999 .9999999 .99999999
1.0
S
1	0.00000000	0300030000000000000000	0.00030000	5000000000	0000000000
2	.00000001	1665521461670430 2132	.00000001	1665527451	0000000000
3	.00000010	1670655376246536 2557	.00000010	1670655376	0000000000
4	.00000100	1674414336750132 7544	.00000100	1674414336	0000000000
5	.00001000	1677517426542161 5503	.00001000	1677517426	0000000000
6	.00010000	1702643334272616 1025	.00010000	1702643334	0000000000
7	.00110000	1706406111564576 6514	.00099999	1706406111	0000000000
8	.01000000	1711507534121727 0243	.00999999	1711507534	0000000000
9	.10000000	1714631463146314 6314	.09999990	1714631463	0000000000
10	.25000000	1716400000000000 0000	.25000000	1716400000	0000000000
11	.33333333	1716525252525252 5251	.33333206	1716525252	0000000000
12	.50000000	1717400000000000 0000	.50000000	1717400000	0000000000
13	.66666666	1717525252525252 5251	.66666412	1717525252	0000000000
14	.75000000	1717600000000000 0000	.75000000	1717600000	0000000000
15	.93750000	1717740000000000 0000	.93750000	1717740000	0000000000
16	.99000000	1717772702436560 5075	.98999786	1717772702	0000000000
17	.99900000	1717773716662132 071	.99899673	1717773716	0000000000
18	.99990000	1717777456221642 470	.99989700	1717777456	0000000000
19	.99999000	1717777753016451 670	.99998836	1717777775	0000000000
20	.99999900	1717777757162041 37	.99999619	1717777777	0000000000
21	.99999990	1717777776245003 26	.99999619	1717777777	0000000000
22	.99999999	1717777777765206 342	.99999959	1717777777	0000000000
23	1.00000000	1720400000000000 0000	1.00000000	1720400000	0000000000

The following section now defines the individual mesh star types, their finite difference Poisson equation coefficients and electric field expansion coefficients used in the EBQ code.

The finite difference equations approximating Poisson's equation can be defined in terms of coupling coefficients sampling the potentials of points surrounding the center of the relaxation star.

$$\nabla^2 \phi \;=\; \rho$$

becomes

$$C_1 \phi_1 \,+\, C_2 \phi_2 \,+\, C_3 \phi_3 \,+\, C_4 \phi_4 \,-\, C_5 \phi_0 \,-\, C_6 \rho \;=\; 0$$

For regular stars there are no boundary or electrode intersections and the appropriate equations are: $r_0 \neq 0$

$$\nabla^2 \phi \;=\; \frac{1}{h^2}\left(1 + \frac{h}{2r_0}\right)\phi_1 + \frac{1}{h^2}\left(1 - \frac{h}{2r_0}\right)\phi_3 + \frac{1}{\ell^2}\,\phi_2 + \frac{1}{\ell^2}\,\phi_4 - \left(\frac{2}{h^2} + \frac{2}{\ell^2}\right)\phi_0 \;=\; \rho$$
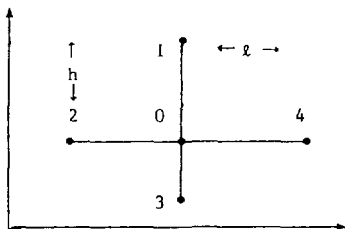
When solving Poisson's equation on the axis we take advantage of the required symmetry and the potential is developed in even powers of r. For regular stars with no electrode intersections the appropriate equation at $r_0 = 0$ is:

$$\nabla^2 \phi = \frac{4}{h^2} \phi_1 + \frac{1}{\ell^2} \phi_2 + \frac{1}{\ell^2} \phi_4 - \left[\frac{4}{h^2} + \frac{2}{\ell^2}\right]\phi_0 = \rho$$

When boundaries or electrodes occur in a mesh star we type them according to the following scheme.

IP > 0

001   002   003   004

005   006   007   010B

011B   012B   015B   014B

015B   016B   017B   020B
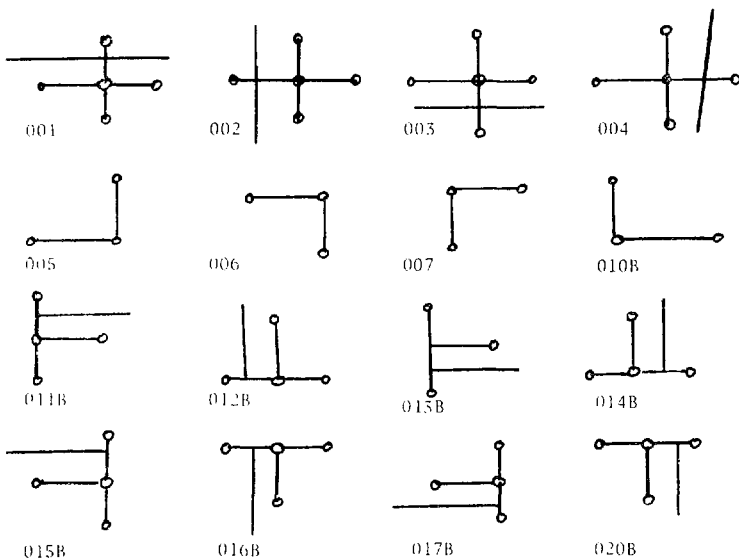
The displacement of the electrode from the central point 0 is defined as the unit normalized beta (β) while the complimented displacement from the relative star mesh point 5 is called β_5 etc.
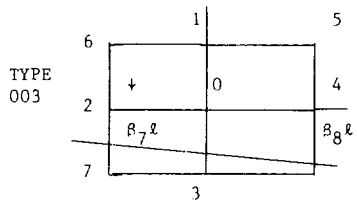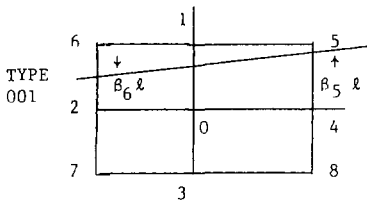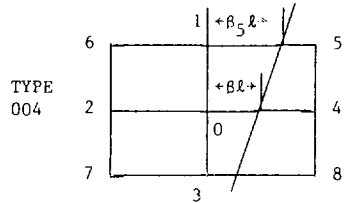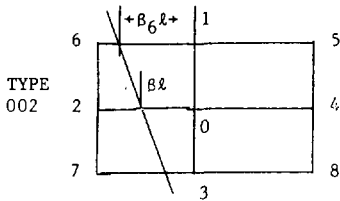
Neumann boundaries occur on the periphery of the problem and are identified by the convention of a zero potential index. Single Neumann boundaries of type 001, 002, 003 and 004 assume reflection about the mesh line upon which the boundary lies. For example, if the right end of the problem is a type 004 Neumann boundary, the potential of the star index 4 is taken as the potential of star index 2, guaranteeing left-right symmetry.
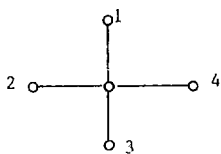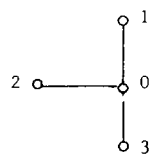
Regular Star   Neumann Star, Type 004

The single Neumann boundary types are

IP=0
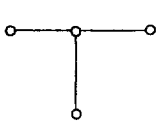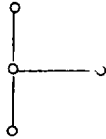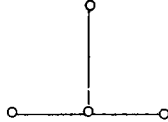
Type 001   Type 002   Type 003   Type 004
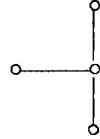
Often the corners of a problem define a double Neumann boundary
where there is both left-right and up-down reflection symmetry. In this
case, the symmetry is guaranteed by replacing the regular star potential
values outside the problem by the values inside the problem. The double
Neumann boundary types are:

IP=0
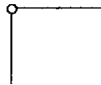
005   006   007   010B

POISSON'S EQUATION

Consider the type 001, 002, 003 and 004 irregular stars. The finite difference coupling coefficients now must reflect the electrode displacement distance β from the central point. The boundary or electrode potential will be designated ϕ_b. Then the finite difference approximations are:

$$0 < \beta < 1$$
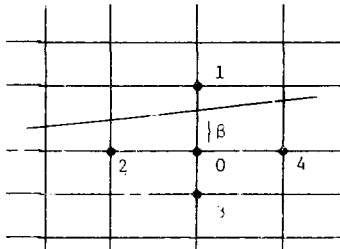
$$\nabla^2 \phi = \frac{1}{\beta(1+\beta)h^2}\left[2 + \frac{h}{r}\right]\phi_b + \frac{\phi_2}{\ell^2} + \frac{1}{(1+\beta)h^2}\left[2 - \beta\frac{h}{r}\right]\phi_3$$

$$+ \frac{\phi_4}{\ell^2} - \left[\frac{1}{\beta h^2}\left(2 + (1-\beta)\frac{h}{r}\right) + \frac{2}{\ell^2}\right]\phi_0$$

TYPE 002

$0 < \beta < 1$
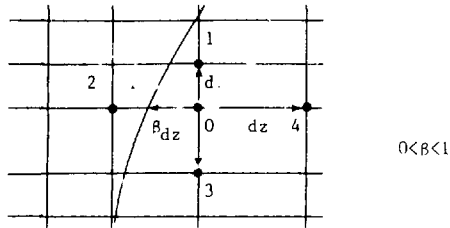
$$\nabla^2 \phi = \phi_1 \left[\frac{1}{r^2} \left(1 + \frac{dr}{2r} \right) \right] + \phi_\beta \left(\frac{2}{dz^2 \, r(1+\beta)} \right)$$

$$+ \phi_3 \left[\frac{1}{dr^2} \left(1 - \frac{dr}{2r} \right) \right] + \phi_4 \left[\frac{2}{dz^2(1+\beta)} \right] - \phi_0 \left[\frac{2}{dr^2} + \frac{2}{dz^2 \beta} \right]$$
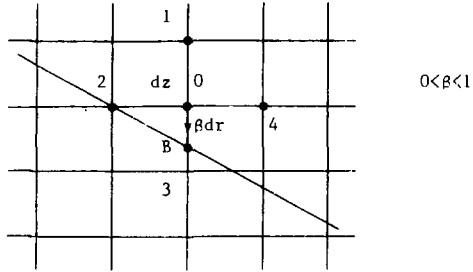
TYPE 003

$$\nabla^2 \phi = \phi_1 \left[\frac{2}{dr^2(1+\beta)} \left(1 + \frac{dr}{2r} \beta \right) \right] + \phi_2 \frac{1}{dz^2} + \phi_B \left\{ \frac{2}{dr^2(1+\beta)\beta} \left(1 - \frac{dr}{2r} \right) \right\}$$

$$+ \frac{1}{dz^2} \phi_4 - \phi_0 \left\{ \frac{2}{dz^2} + \frac{2}{dr^2\beta} \left(1 - \frac{dr}{2r} (1-\beta) \right) \right\}$$

TYPE 004

$0 < \beta < 1$
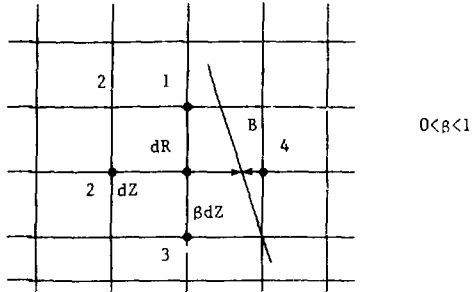
$$\nabla^2 \phi = \phi_1 \left[\frac{1}{dr^2} \left(1 + \frac{dr}{2r} \right) \right] + \phi_3 \left[\frac{1}{dr^2} \left(1 - \frac{dr}{2r} \right) \right] + \phi_2 \left[\frac{2}{(1+\beta)dz^2} \right]$$

$$+ \phi_B \left[\frac{2}{\beta(1+\beta)} \frac{1}{dz^2} \right] - \phi_0 \left[\frac{2}{dr^2} + \frac{2}{\beta dz^2} \right]$$

Poisson equation coefficients.

Type 011, 012, 013, and 014

    015, 016, 017, and 020

011
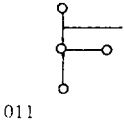
Same as type 001 with

$C_2 = 0$ and $C_4 = 2C_4$

012
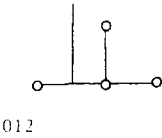
$C_1 = 4/dr^2$ $C_4 = 2/dz^2(1+\beta)$

$C_2 = C_4/\beta$ $C_5 = 4/dr^2+2/dz^2\beta$

$C_3 = 0$

013
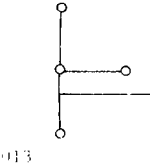
Same as type 003 with $C_2=0$, $C_4=2C_4$

014
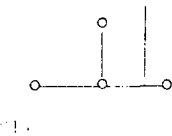
$C_1 = 4/dr^2$ $C_4 = C_2/\beta$

$C_2 = 2/dz^2(1+\beta)$ $C_5 = 4/dr^2+2/dz^2\beta$

$C_3 = 0$
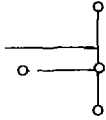
Same as type 1 with $C_2 = 2C_2$, $C_4 = 0$

015
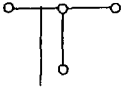
Same as type 2 with $C_1 = 0$, $C_3 = 2C_3$

016
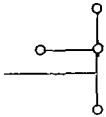
Same as type 3 with $C_4 = 0$, $C_2 = 2C_2$

017
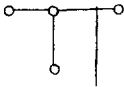
Same as type 4 with $C_1 = 0$, $C_3 = 2C_3$

020

When boundary electrodes intersect two legs of a mesh star, they are typed according to the following scheme:
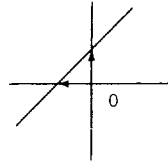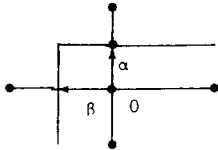
021B
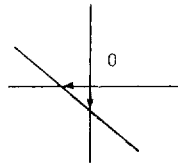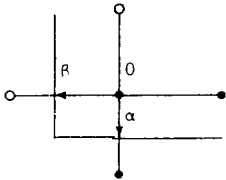
022B
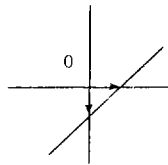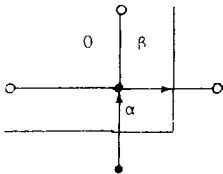
023B
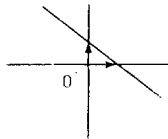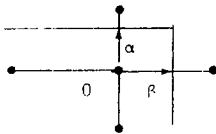
023B

Note that mathematically there is no distinction between the finite difference approximation to Poisson's equation for the inside corners and slant electrodes. These types are 021B-024B as shown above, where α is the unit normalized vertical leg electrode distance and β is the unit normalized horizontal leg electrode distance.

TYPE 021B
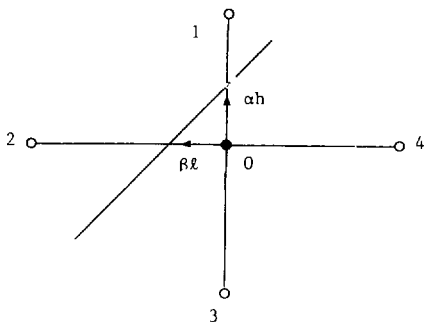
$$\nabla^2 \phi = \frac{1}{\alpha(1+\alpha)h^2}\left(2 + \frac{h}{r_0}\right)\phi_B + \frac{2}{\beta(1+\beta)\ell^2}\phi_B + \frac{1}{(1+\alpha)h^2}\left[2 - \frac{\alpha h}{r_0}\right]\phi_3$$

$$+ \frac{2}{(1+\beta)\ell^2}\phi_4 - \left[\frac{1}{\alpha h^2}\left(2 + (1-\alpha)\frac{h}{r_0}\right) + \frac{2}{\beta\ell^2}\right]\phi_0$$
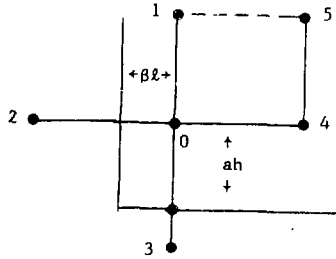
TYPE 022B

$$\nabla^2\phi = \frac{1}{h^2(1+\alpha)}\left[2 + \frac{\alpha h}{r_0}\right]\phi_1 + \frac{2}{\beta(1+\beta)\ell^2}\phi_{2_B}$$

$$+ \frac{1}{h^2(1+\alpha)\alpha}\left(2 - \frac{h}{r_0}\right)\phi_{3_B} - \left[\frac{1}{\alpha h^2}\left(2 - (1-\alpha)\frac{h}{r_0}\right) + \frac{2}{\beta\ell^2}\right]\phi_0$$

$$+ \frac{2}{(1+\beta)\ell^2}\phi_4$$
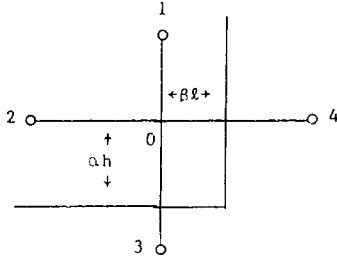
TYPE 023B

$$\nabla^2 \phi = \frac{1}{(1+\alpha)h^2} \left[2 + \frac{\alpha h}{r_0} \right] \phi_1 + \frac{2}{(1+\beta)\ell^2} \phi_2 + \frac{1}{\alpha(1+\alpha)h^2} \left[2 - \frac{h}{r_0} \right] \phi_B$$

$$+ \frac{2}{\beta(1+\beta)\ell^2} \phi_B - \left[\frac{1}{\alpha h^2} \left(2 - (1-\alpha)\frac{h}{r_0} \right) + \frac{2}{\beta \ell^2} \right] \phi_0$$

TYPF 024B
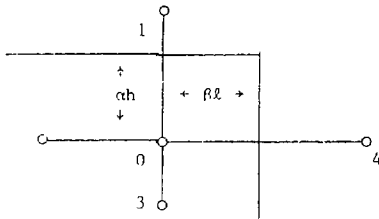
$$\nabla^2 \phi = \frac{1}{\alpha(1+\alpha)h^2} \left[2 + \frac{h}{r_0} \right] \phi_B + \frac{2}{(1+\beta)\ell^2} \phi_2 + \frac{1}{(1+\alpha)h^2} \left[2 - \frac{\alpha h}{r_0} \right] \phi_3$$

$$+ \frac{2}{\beta(1+\beta)\ell^2} \phi_B - \left[\frac{1}{\alpha h^2} \left(2 + (1-\alpha)\frac{1}{r_0} \right) + \frac{2}{\beta \ell^2} \right] \phi_0$$

Several special star mesh definitions are shown below.
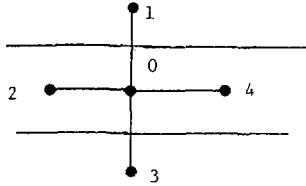
025B
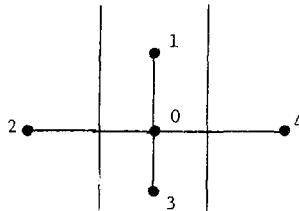
026B
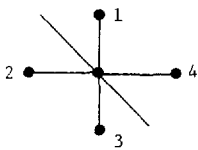
or
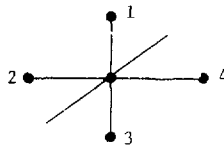
027B

When a boundary electrode intersects three legs of a mesh star, they are typed according to the following scheme:
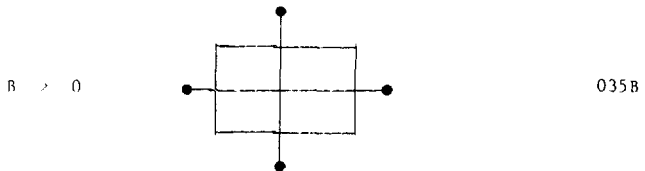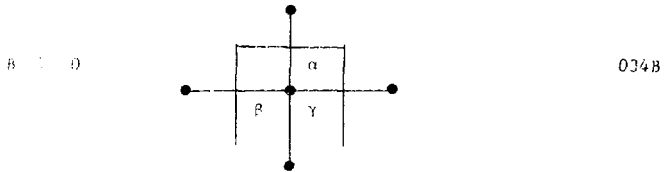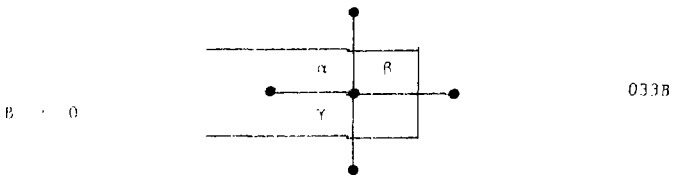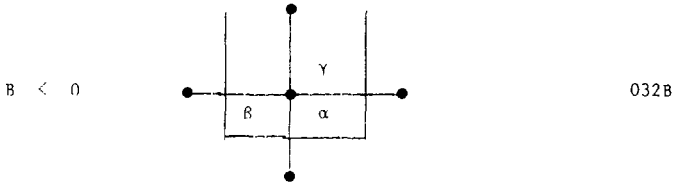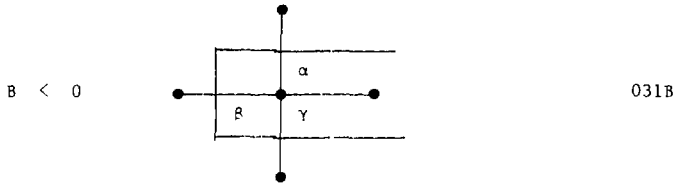
Mesh types 41B-50B define Poisson's equation coefficients and electric field approximations for slant electrodes intersecting the outer problem boundary.

41B   R=0
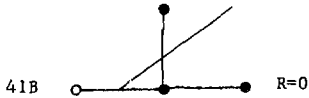
Treat as type 21B with $C_3=0$, $C_1=2C_1$

42B   R=0
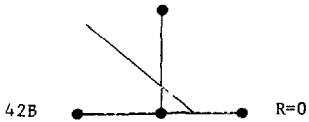
Treat as type 24B with $C_3=0$, $C_1=2C_1$

43B   Z=0
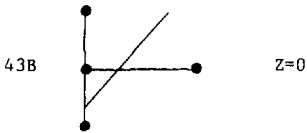
Treat as type 23B with $C_2=0$, $C_4=2C_4$

44B   Z=0
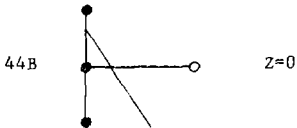
Treat as type 24B with $C_2=0$, $C_4=2C_4$

45B   Z=ZMAX
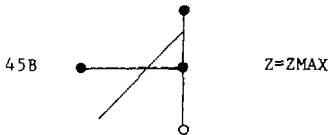
Treat as type 21B with $C_4=0$, $C_2=2C_2$

46B   R=RMAX
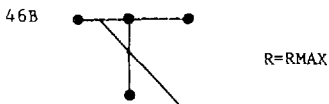
Treat as type 22B with $C_1=0$, $C_3=2C_3$

47B   R=RMAX   Treat as type 23B with

$C_1=0$, $C_3=2C_3$

50B   Z=ZMAX   Treat as type 22B with

$C_4=0$, $C_2=2C_2$
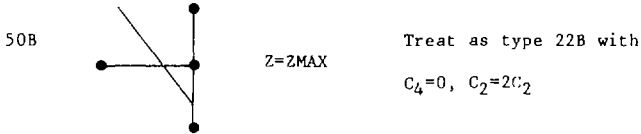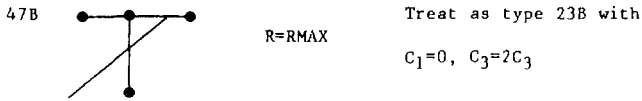
Mesh types 51B-54B define slanted electrodes running along the outer boundary. The Poisson equation coefficients for these types are modified 2, 3, and 4 types.

51B   R=0

52B   Z=XMAX   Treat as type 2,

$C_4=0$, $C_2=2C_2$

53B   R=RMAX   Treat as type 3,

$C_1=0$, $C_3=2C_3$

54B   Z=0   Treat as type 4

$C_2=0$, $C_4=2C_4$
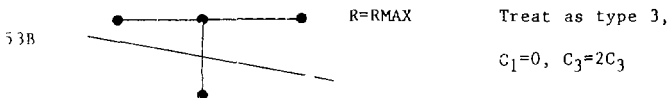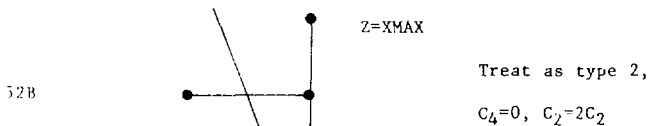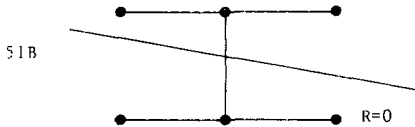
Mesh types 55B-60B define single point electrodes. They are treated as type 001-004 for Poisson's equation and totally interior (non-electrode intersecting) points for the electric field calculation. If a single point is defined with a displacement off the mesh in both r and z, it will be treated by convention as a vertical mesh pair, generating a type 55B and 57B mesh point.

55 B

56 B

57 B

60 B
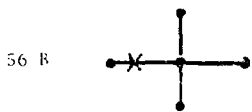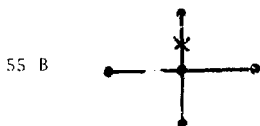
Single point on mesh intersection, set as type 55B.

If $DR>10^{-5}$, treat as vertical pair. type 55B and 57B.

57B

55B
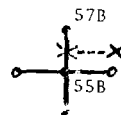
ELECTRIC FIELD COEFFICIENTS

Electric fields are calculated from a power series expansion of U around the central mesh star located at R_0, Z_0 such that

$$-\frac{1}{2} \text{DRMESH} + R_o \leq R \leq R_c + \frac{1}{2} \text{DRMESH}$$

$$-\frac{1}{2} \text{DZMESH} + Z_o \leq Z \leq Z_o + \frac{1}{2} \text{DZMESH}$$

Defining $R = R_0$ r and $Z = Z_0 + z$, then r and z are small displacements from R_0, Z_0.

The expansion used depends on the mesh star type. For regular stars, type 0 an interior electrode intersection stars, type 001, 002, 003, 004, 021, 0 2, 023, and 024, the expansion is

$$u = u_o + a_1 z + a_2 r + a_3 r^2 + a_4 z^2 + a_5 rz$$

$$E_r = a_2 + 2a_3 r + a_5 z$$

$$E_z \quad a_1 + 2a_4 z + a_5 r$$

The coefficients a_1 a_2, a_3, a_4 and a_5 are given in Table 2. The equation for a_5 depends in which quadrant of the star the off diagonal term is located.
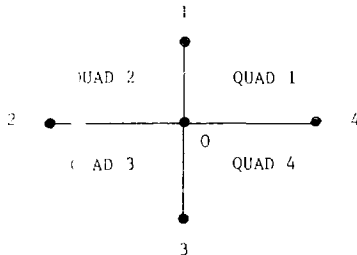
The expansion used for type 011B and 013B stars must be even in powers of z to reflect the left-right symmetry at the left edge and of the problem. Here

$$u = u_o + a_2 r + a_3 r^2 + (a_4 + a_5 r) z^2$$

$$E_r = a_2 + 2a_3 r + a_5 z^2$$

$$E_z = 2a_4 z + 2a_5 rz$$

The expansion used for type 012B and 014B stars must be even in powers of r, since we are on the axis of rotational symetry.

$$u = u_o + a_1 z + a_4 z^2 + (a_3 + a_5 z) r^2$$

$$E_r = 2a_3 r + 2a_5 rz$$

$$E_z = a_1 + 2a_4 z + a_5 r^2$$

These coefficients a_1, a_2, a_3, a_4 and a_5 are given in Table 3. Electric field expansions for stars in which an electrode passes through and parallel to mesh points can take advantage of the required symmetry. For types 001B and 003B we have $\beta = 0$ and $U_o = U_2 = U_4 = POT$.

$$u = U_o + a_1 z + a_2 r + a_3 r^2 + a_4 z^2 + a_5 rz$$
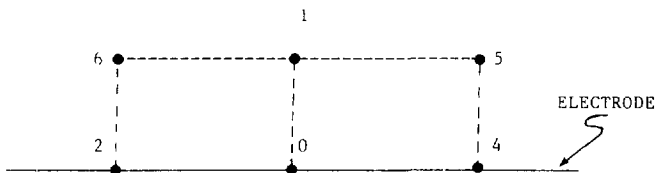
Assuming only a linear r dependence, then $A_3 = 0$. Symmetry requires $A_1 = A_4 = 0$ so;

$$a_2 = \frac{u_1 - u_0}{h}$$

$$a_5 = \frac{u_5 - u_6}{2h\ell}$$

$$u = a_0 + a_2 r + a_5 rz$$

$$E_r = a_2 + a_5 z$$

$$E_z = a_5 r$$

For types 002B and 004B we have $\beta = 0$ and $u_1 = u_0 = u_3 = POT$. Assuming only a linear z dependence requires $A_4 = 0$, while symmetry demands $A_2 = A_3 = 0$, so;
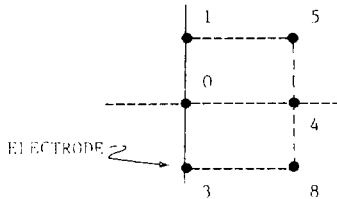
$$a_1 = \frac{u_4 - u_0}{h}$$

$$a_5 = \frac{u_5 - u_8}{2h\ell}$$

$$u = u_0 + a_1 z + a_5 zr$$

$$E_r = a_5 z$$

$$E_z = a_1 + a_5 r$$

Electric field expansions are not necessary for types 015B, 017B, 016B and 020B stars since $R = R_o + r$, $Z = Z_o + z$ would lie outside the problem boundary.
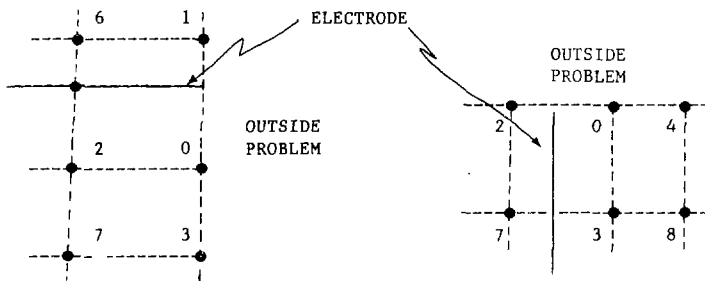
TYPE 015B, 017B   TYPE 016B, 020B

Electric field calculations for types 41B-54B

The electric field calculations for these types of mesh points are approximated by moving to an adjacent interior mesh point, ID, and treating as an expansion about that point in the appropriate displaced quadrant. This somewhat sloppy procedure is justified in that details finer than a mesh cell cannot be accurately modeled and really require a finer mesh.

41 B   Increment R   QUAD 4

47 B   Back step R   QUAD 1

42 B   Increment R   QUAD 4

50B   Back step Z   QUAD 1

43 B   Increment Z   QUAD 2

51 B   Increment R   QUAD 4

44 B   Increment Z   QUAD 2

52 B   Back step Z   QUAD 1

45 B   Back step Z   QUAD 1

53 B   Back step R   QUAD 1

46 B   Back step Z   Back step R

54 B   Increment Z   QUAD 2
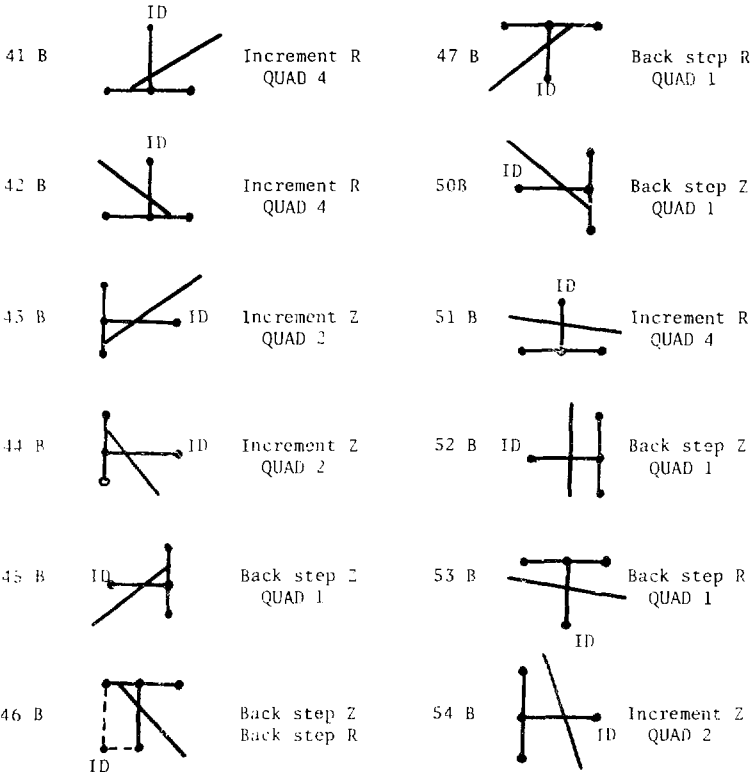
Electric field calculations for mesh types 55B-60B

Electric field expansions for boundary mesh types 55B-60B are approximated by those for mesh types 001-004. The expansions are totally interior, as if the single electrode intersection generating the 55B-60B type were not present. The electrode effects the fields only insofar as it determined the value of the potentials on the adjacent mesh points and therefore effects the calculations through its Poisson equation expansion coefficients.

Equipotential Calculation

Equipotential lines are calculated with the condition that the total derivative of the potential be zero. The initial value of the potential line is specified as input or taken as a tenth increment of the maximum potential of the problem. The axis of symmetry ($r=0$) is searched for a potential U_o at Z_o greater th the desired value U_D. The exact axial position z of the potential line U_D is then found as:

$$Z = Z_o - \left(\frac{U_o - U_D}{E_z} \right) \quad ; \quad R = 0$$

where E_z is the longitudinal electric field dU/dz. This linear correction to the starting position of the line is applied iteratively until sufficient accuracy is obtained.

The line is then tracked by advancing R or Z by a predetermined step ds according to the relative size of E_R or E_z.

$$dU = \frac{\partial U}{\partial r} \, dr + \frac{\partial U}{\partial Z} \, dz = 0$$

$$E_r dr + E_z dz = 0$$

Table 2. Electric field coefficients, types 1-4.

	001	002	003	004
A1	$\dfrac{U_4 - U_2}{2\ell}$	$\dfrac{-U_B + \beta^2 U_4 + U_o(1-\beta^2)}{\beta(1+\beta)\ell}$	$\dfrac{U_4 - U_2}{2\ell}$	$\dfrac{U_B - \beta^2 U_2 - (1-\beta^2)U_o}{\beta(1+\beta)\ell}$
A2	$\dfrac{U_B - U_3\beta^2 - (1-\beta^2)U_o}{\beta(1+\beta)h}$	$\dfrac{U_1 - U_3}{2h}$	$\dfrac{-U_B + U_1\beta^2 + (1-\beta^2)U_o}{\beta(1+\beta)h}$	$\dfrac{U_1 - U_3}{2h}$
A3	$\dfrac{U_B + \beta U_3 - (1+\beta)U_o}{\beta(1+\beta)h^2}$	$\dfrac{U_1 + U_3 - 2U_o}{2h^2}$	$\dfrac{\beta U_1 + U_B - (1+\beta)U_o}{\beta(1+\beta)h^2}$	$\dfrac{U_1 + U_B - 2U_o}{2h}$
A4	$\dfrac{U_2 + U_4 - 2U_o}{2\ell^2}$	$\dfrac{U_B + \beta U_4 - U_o(1+\beta)}{\beta(1+\beta)\ell^2}$	$\dfrac{U_2 + U_4 - 2U_o}{2\ell^2}$	$\dfrac{\beta U_2 + U_B - (1+\beta)U_o}{\beta(1+\beta)\ell^2}$

Table 2. (continued)

	001	002	003	004
QUAD 1 AS	$\dfrac{1}{h\ell}\left\{-\left(\dfrac{1+\beta_5}{\beta(1+\beta)}-\dfrac{1}{\beta_5}\right)u_B -\dfrac{u_4}{\beta_5}+\dfrac{1+\beta_5-\beta}{\beta}u_o\right\}+\dfrac{\beta-\beta_5}{1+\beta}u_3$	$\dfrac{1}{h\ell}\left\{-u_1-u_4+u_5+u_o\right\}$	$\dfrac{1}{h\ell}\left\{-u_1+u_5-u_4-u_o\right\}$	$\dfrac{1}{h\ell}\left\{-\dfrac{u_1}{\beta_5}-\left(\dfrac{1+\beta_5}{\beta(1+\beta)}-\dfrac{1}{\beta_5}\right)u_B\right\}+\dfrac{\beta-\beta_5}{1+\beta}u_2+\dfrac{1+\beta_5-\beta}{\beta}u_o$
QUAD 2 AS	$\dfrac{1}{h\ell}\left\{\dfrac{u_2}{\beta_6}+\left(\dfrac{1+\beta_6}{\beta(1+\beta)}-\dfrac{1}{\beta_6}\right)u_B\right\}+\dfrac{\beta-\beta_6}{1+\beta}u_3-\dfrac{1+\beta-\beta}{\beta}u_o$	$\dfrac{1}{h\ell}\left\{\dfrac{u_1}{\beta_6}-\dfrac{1+\beta_6}{1+\beta}u_4+\left(\dfrac{1+\beta_6}{\beta(1+\beta)}-\dfrac{1}{\beta_6}\right)u_B-\dfrac{1+\beta_6-\beta}{\beta}u_o\right\}$	$\dfrac{1}{h\ell}\left\{u_2-u_5+u_1-u_o\right\}$	$\dfrac{1}{h\ell}\left\{u_1-u_6+u_2-u_o\right\}$
QUAD 3 AS	$\dfrac{1}{h\ell}\left\{u_3+u_7-u_2-u_o\right\}$	$\dfrac{1}{h\ell}\left\{-\dfrac{u_3}{\beta_7}+\dfrac{\beta-\beta_7}{1+\beta}u_4-\left(\dfrac{1+\beta_7}{\beta(1+\beta)}-\dfrac{1}{\beta_7}\right)u_B+\dfrac{1+\beta_7-\beta}{\beta}u_o\right\}$	$\dfrac{1}{h\ell}\left\{\dfrac{\beta-\beta_7}{1+\beta}u_1-\left(\dfrac{1+\beta_7}{\beta(1+\beta)}-\dfrac{1}{\beta_7}\right)u_B-\dfrac{u_2}{\beta_7}+\dfrac{1+\beta_7}{\beta}u_o\right\}$	$\dfrac{1}{h\ell}\left\{u_7-u_3+u_2-u_o\right\}$
QUAD 4 AS	$\dfrac{1}{h\ell}\left\{u_3-u_8+u_4-u_o\right\}$	$\dfrac{1}{h\ell}\left\{u_3-u_8+u_4-u_o\right\}$	$\dfrac{1}{h\ell}\left\{-\dfrac{u_4}{\beta_8}-\left(\dfrac{1+\beta}{\beta(1+\beta)}-\dfrac{1}{\beta_8}\right)u_B+\dfrac{1+\beta-\beta}{\beta}u_o\right\}$	$\dfrac{1}{h\ell}\left\{\dfrac{u_3}{\beta_8}+\left(\dfrac{1+\beta_8}{(\beta+1)\beta}-\dfrac{1}{\beta_8}\right)u_B\right\}-\dfrac{\beta-\beta_8}{1+\beta}u_2-\dfrac{1+\beta_8-\beta}{\beta}u_o$

Table 3. Electric field coefficients.

	011	012	013	014
A1	0	$\dfrac{\beta U_4 - U_B + (1-\beta^2)U_o}{\beta(1+\beta)\ell}$	0	$\dfrac{U_B - \beta^2 U_2 - (1-\beta^2)U_o}{\beta(1+\beta)\ell}$
A2	$\dfrac{U_B - \beta^2 U_3 - (1+\beta^2)U_o}{\beta(1+\beta)h}$	0	$\dfrac{\beta^2 U_1 - U_B + (1-\beta^2)U_o}{\beta(1+\beta)h}$	0
A3	$\dfrac{U_B + \beta U_3 - (1+\beta)U_o}{\beta(1+\beta)h}$	$\dfrac{U_1 - U_o}{h^2}$	$\dfrac{U_B + \beta U_1 - (1+\beta)U_o}{\beta(1+\beta)h^2}$	$\dfrac{U_1 - U_o}{h^2}$
A4	$\dfrac{U_4 - U_o}{\ell^2}$	$\dfrac{U_3 + \beta U_4 - (1+\beta)U_o}{\beta(1+\beta)\ell^2}$	$\dfrac{U_4 - U_o}{\ell^2}$	$\dfrac{U_B + \beta U_2 - (1+\beta)U_o}{\beta(1+\beta)\ell^2}$

Table 3. (continued)

	011	012	013	014
	$-\dfrac{1}{h_\ell^2}\left\{-\dfrac{U_4}{\beta_5}+\dfrac{\beta-\alpha_5}{1+\beta}U_3 \right.$ $\left. -\left(\dfrac{1+\beta_5}{\beta(\beta+1)}-\dfrac{1}{\beta_5}\right)U_B \right.$ $\left. +\dfrac{1+\beta_5-\beta}{\beta}U_o\right\}$	$\dfrac{1}{h_\ell^2}\left\{U_5-U_1-U_4+U_o\right\}$	$-\dfrac{1}{h_\ell^2}\left\{U_5-U_4-U_1+U_o\right\}$	$\dfrac{1}{h_\ell^2}\left\{-\dfrac{U_1}{\beta_5}+\dfrac{\beta-\beta_5}{1+\beta}U_2 \right.$ $\left. -\left(\dfrac{1+\beta_5}{\beta(1+\beta)}-\dfrac{1}{\beta_5}\right)U_B \right.$ $\left. +\dfrac{1+\beta_5-\beta}{\beta}U_o\right\}$
A5	$-\dfrac{1}{h_\ell^2}\left(U_4-U_8+U_3-U_o\right)$	$\dfrac{1}{h_\ell^2}\left\{\dfrac{U_1}{\beta_6}-\dfrac{\beta-\beta_6}{1+\beta}U_4 \right.$ $\left. +\left(\dfrac{1+\beta_6}{\beta(1+\beta)}-\dfrac{1}{\beta_5}\right)U_B-\dfrac{1+\beta_6-\beta}{\beta}U_o\right\}$	$\dfrac{1}{h_\ell^2}\left\{\dfrac{U_4}{\beta_8}-\dfrac{\beta-\beta_8}{1+\beta}U_1 \right.$ $\left. +\left(\dfrac{1+\beta_8}{\beta(1+\beta)}-\dfrac{1}{\beta_8}\right)U_B-\dfrac{1+\beta_8-\beta}{\beta}U_o\right\}$	$\dfrac{1}{h_\ell^2}\left[U_2-U_6+U_1-U_o\right]$

Table 4. Electric field coefficients
α vertical displacements
β Horizontal displacements

	021	022	023	024
A1	$\dfrac{1}{\beta(1+\beta)\ell^2}\left\{\begin{array}{l}-U_{B2} + \beta^2 U_4\\ + (1-\beta^2)U_o\end{array}\right\}$	$\dfrac{1}{\beta(1+\beta)\ell}\left\{\begin{array}{l}-U_{B2} + \beta^2 U_4\\ + (1-\beta^2)U_o\end{array}\right\}$	$\dfrac{1}{\beta(1+\beta)\ell}\left\{\begin{array}{l}U_B - \beta^2 U_2\\ - (1-\beta^2)U_o\end{array}\right\}$	$\dfrac{1}{\beta(1+\beta)\ell}\left\{\begin{array}{l}U_{B4} - \beta^2 U_2\\ - (1-\beta^2)U_o\end{array}\right\}$
A2	$\dfrac{1}{\alpha(1+\alpha)h}\left\{\begin{array}{l}U_{B1} - \alpha^2 U_3\\ - (1-\alpha^2)U_o^2\end{array}\right\}$	$\dfrac{1}{\alpha(1+\alpha)h}\left\{\begin{array}{l}-U_{B3} + \alpha^2 U_1\\ + (1-\alpha^2)U_o\end{array}\right\}$	$\dfrac{1}{\alpha(1+\alpha)h}\left\{\begin{array}{l}-U_{B3} + \alpha^2 U_1\\ + (1-\alpha^2)U_o\end{array}\right\}$	$\dfrac{1}{\alpha(1+\alpha)h}\left\{\begin{array}{l}U_{B1} - \alpha^2 U_3\\ - (1-\alpha^2)U_o\end{array}\right\}$
A3	$\dfrac{1}{\alpha(1+\alpha)h^2}\left\{\begin{array}{l}U_{B1} + \alpha U_3\\ - (1+\alpha)U_o\end{array}\right\}$	$\dfrac{1}{\alpha(1+\alpha)h^2}\left\{\begin{array}{l}U_{B3} + \alpha U_1\\ - (1+\alpha)U_o\end{array}\right\}$	$\dfrac{1}{\alpha(1+\alpha)h^2}\left\{\begin{array}{l}U_{B3} + \alpha U_1\\ - (1+\alpha)U_o\end{array}\right\}$	$\dfrac{1}{\alpha(1+\alpha)h^2}\left\{\begin{array}{l}U_{B1} + \alpha U_3\\ - (1+\alpha)U_o\end{array}\right\}$
A4	$\dfrac{1}{\beta(1+\beta)\ell^2}\left\{\begin{array}{l}U_{B2} + \beta U_4\\ - (1+\beta)U_o\end{array}\right\}$	$\dfrac{1}{\beta(1+\beta)\ell^2}\left\{\begin{array}{l}U_{B2} + \beta U_4\\ - (1+\beta)U_o\end{array}\right\}$	$-\dfrac{1}{\beta(1+\beta)\ell^2}\left\{\begin{array}{l}U_{B4} + \beta U_2\\ - (1+\beta)U_o\end{array}\right\}$	$\dfrac{1}{\beta(1+\beta)\ell^2}\left\{\begin{array}{l}U_{B4} + \beta U_2\\ - (1+\beta)U_o\end{array}\right\}$

Table 4. (continued)

	021	022	023	024
A5	$\frac{1}{h\ell}\left\{U_o + a_1\ell + a_2h + a_3h^2 + a_4\ell^2 - U_8\right\}$ with $h < 0$	$\frac{1}{h\ell}\left\{U_o + a_1\ell + a_2h + a_3h^2 + a_4\ell^2 - U_5\right\}$ with $h,\ell > 0$	$\frac{1}{h\ell}\left\{U_o + a_1\ell + a_2h + a_3h^2 + a_4\ell^2 - U_6\right\}$ with $\ell < 0$	$\frac{1}{h\ell}\left\{U_o + a_1\ell + a_2h + a_3h^2 + a_4\ell^2 - U_7\right\}$ with $h,\ell < 0$

Table 5. Poisson equation coefficients.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_0
001	$\frac{1}{\beta(1+\beta^2)h^2}\left(2+\frac{h}{r}\right)$	$\frac{1}{\ell^2}$	$\frac{1}{(1+\beta)h^2}\left(2-\beta\frac{h}{r}\right)$	$\frac{1}{\ell^2}$	$-\frac{1}{\beta h^2}\left[2+(1-\beta)\frac{h}{r}\right]-\frac{2}{\ell^2}$
002	$\frac{1}{2h^2}\left(2+\frac{h}{r}\right)$	$\frac{2}{\beta(1+\beta)\ell^2}$	$\frac{1}{2h^2}\left(2-\frac{h}{r}\right)$	$\frac{2}{(1+\beta)\ell^2}$	$-\frac{2}{h^2}-\frac{2}{\beta\ell^2}$
003	$\frac{1}{(1+\beta)h^2}\left(2+\frac{\beta h}{r}\right)$	$\frac{1}{\ell^2}$	$\frac{1}{\beta(1+\beta)h^2}\left(2-\frac{h}{r}\right)$	$\frac{1}{\ell^2}$	$-\frac{1}{\beta h^2}\left[2-(1-\beta)\frac{h}{r}\right]-\frac{2}{\ell^2}$
004	$\frac{1}{2h^2}\left(2+\frac{h}{r}\right)$	$\frac{2}{(1+\beta)\ell^2}$	$\frac{1}{2h^2}\left(2-\frac{h}{r}\right)$	$\frac{2}{\beta(1+\beta)\ell^2}$	$-\frac{2}{h^2}-\frac{2}{\beta\ell^2}$
021	$\frac{1}{\alpha(1+\alpha)h^2}\left(2+\frac{h}{r_o}\right)$	$\frac{2}{\beta(1+\beta)\ell^2}$	$\frac{1}{\alpha(1+\alpha)h^2}\left(2-\frac{\alpha h}{r_o}\right)\alpha$	$\frac{2}{\beta(1+\beta)\ell^2}\beta$	$-\frac{1}{\alpha h^2}\left[2+(1-\alpha)\frac{h}{r_o}\right]-\frac{2}{\beta\ell^2}$
022	$\frac{1}{\alpha(1+\alpha)h^2}\left(2+\frac{\alpha h}{r_o}\right)\alpha$	$\frac{2}{\beta(1+\beta)\ell^2}$	$\frac{1}{\alpha(1+\alpha)h^2}\left(2-\frac{\alpha h}{r_o}\right)$	$\frac{2}{\beta(1+\beta)\ell^2}\beta$	$-\frac{1}{\alpha h^2}\left[2-(1-\alpha)\frac{h}{r_o}\right]-\frac{2}{\beta\ell^2}$
023	$\frac{1}{\alpha(1+\alpha)h^2}\left(2+\frac{\alpha h}{r_o}\right)\alpha$	$\frac{2}{\beta(1+\beta)\ell^2}\beta$	$\frac{1}{\alpha(1+\alpha)h^2}\left(2-\frac{h}{r_o}\right)$	$\frac{2}{\beta(1+\beta)\ell^2}$	$-\frac{1}{\alpha h^2}\left[2-(1-\alpha)\frac{h}{r_o}\right]-\frac{2}{\beta\ell^2}$
024	$\frac{1}{\alpha(1+\alpha)h^2}\left(2+\frac{\alpha h}{r_o}\right)$	$\frac{2}{\beta(1+\beta)\ell^2}\beta$	$\frac{1}{\alpha(1+\alpha)h^2}\left(2-\frac{\alpha h}{r_o}\right)\alpha$	$\frac{2}{\beta(1+\beta)\ell^2}$	$-\frac{1}{\alpha h^2}\left[2+(1-\alpha)\frac{h}{r_o}\right]-\frac{2}{\beta\ell^2}$

When the radial electric field exceeds the longitudinal field,

$$z = z + ds \; ; \qquad r = r - \frac{Ez}{Er} ds \; ; \qquad Ez < Er$$

and when the radial electric field is less than the longitudinal field

$$r = r + ds \; ; \qquad z = z - \frac{Er}{Ez} ds \; ; \qquad Er < Ez$$

The equipotential line is tracked until it closes on itself, or until it leaves the problem boundary, or until more than 200 points have been found on a given line, or until both Er and Ez vanish.

These calculations are controlled by the following data elements.

$$\begin{aligned}
&\text{CONST (47)} = \text{NEQUIP} \\
&\text{CONST (48)} = \text{DSEQUIP} \\
&\text{CONST (49)} = \text{PRINT EQUIP LINE} \\
&\text{CONST (51)} = \text{EQUIP ERROR E} \\
&\text{CONST (52)} = \text{ITER MAX} \\
&\text{PLOT EQUIP S} \\
&\text{CONST (76)} = \text{R or Z OF LINE SEARCH}
\end{aligned}$$

NEQUIP specifies how many equipotential lines are to be followed. If zero, no lines will be generated. DSEQUIP is the interval of advancement for R or Z in following the line. The value of R, Z and U(R,Z) will be printed as output if CONST (49) = I; otherwise, no printed output is generated. The maximum error in the line is given by CONST (51) such that an iterative correction will be applied a maximum of CONST (52) times such that

$$\frac{U - U_{\perp}}{U_D} \; < \; E$$

A CalComp plot of the lines will appear on any trajectory plots specified by CONST (34) = 11. If no trajectories are calculated, a plot can still be generated by setting CONST (34) = 11 and using the PLOT EQUIP S data card.

The equipotential plots are made by a search along the Z direction at R=CONST (76) if the step, CONST (48), is positive or along the R direction at Z=CONST (76) if the step, CONST (48), is negative.

NORMAL OUTPUT FROM EBQCODE

Each data card encountered is printed into the output stream. After generation of MESH by boundary input, a map of the mesh array giving the index for the coupling coefficients is printed.

After Ray input, a table of the standard values of the RAYS, i.e., N, X, Y, Z, Px, Py, Pz,.and current is printed followed by a summary table giving the average value of beam energy (ev), current (amps), momentum (ev/c), magnetic ridigity (g-cm), γ and β.

After TRACK input, a table of the current value of the CONST array is printed, followed by the standard orbit output. At the conclusion of ray tracing for each cycle, the longitudinal velocity V_z(cm/sec) of each ray will be printed. If a ray was lost on some aperture, its velocity will be printed as zero. If a ray encountered a stopping flag, bit 30 turned on, its velocity will be printed as -2. If a ray exceeded the allowed energy interval given by CONST (64-65), its velocity will be printed as -3. A map of the potential and charge array is then given.

After END a summary of the number of calls and accumulated CP time in seconds is printed.

Orbit output at a specified z value consists of a table of

RAY E(ev) X(cm) Y(cm) S(cm) P_x(ev/c) P_y(ev/c) P_z(ev/c) T(sec) R(cm)

For each ray traced, orbit output is printed for the initial particle distribution and the final particle distribution. If more than one Poisson-ray trace cycle is performed, then orbit output is generated for the particle distribution at the end of each cycle. Special output can be requested on the last cycle by the following data input:

CONST	19	DZOUT	output particle map every DZOUT cm.
CONST	20	1	output table of fields along with orbit
OUTPUT	Z( )	S	output particle map at each given z value
PUNCH	Z( )	S	the particle distribution is printed and written to tape 4 at each given z value
FIELD	Z( )	S	a map of the electric and magnetic field experienced by each ray is printed for each Z value
PLOT PHASE	Z( )	S	a plot of r vs dr/dz is made for each z value
PLOT DENSITY	Z( )	S	a plot of ρ vs r is made for each Z value

If detailed output is desired on any intermediate cycle, the continue option on the track card should be used. Consider output at 10, 20, 22.5 cm on the 1st, 7th, and 8th cycles.

TRACK X 1	first iteration with output
TRACK CONTINUE 6	track next 6 cycles, output on last, 7th cycle
TRACK CONTINUE 1	track, 8th cycle

The amount of output generated during orbit integration is determined by the above specified output specifications. If a specified Z value does not fall on a usual RK step interval, the integrator will make a special stop at the required value. When tracking many rays, it is often useful to know the value of certain moments of the particle distribution

CONST   *42   dZMOMENT

specifies a calculation of R, R_{rms}, Pr, P_{rms} every dZMOMENT cm along
the beam path.

CalComp plots are available for the trajectory profiles of the rays
selected by the PLOT TRAJ cards according to the value of CONST (34).
CalComp phase plots are available for Z values selected by the PLOT
PHASE card according to the value of CONST(37). CalComp plot of the
charge density versus r is available for the Z values selected by the
PLOT DENSITY card according to the value of CONST (44). If a density
plot is desired on a regular interval, then alternatively the interval
can be specified by

CONST   *45   dZDENSITY

where dZDENSITY is in cm.

CONTROLLING THE VOLUME AND TYPE OF OUTPUT FROM EBQ

Print of static fields can be specified in the following ways.

PRINT FIELDS   –   Prints fields generated at specified locations B_r, B_z, A_ϕ, E_r, E_z, and U

MAP 1   –   Output map of MESH ARRAY

MAP 2   –   Output electrostatic potential array

MAP 3   –   Output charge density array

MAP 4   –   Output POISSON equation coefficients

MAP 5   –   Output map of boundary-mesh displacements R

MAP 11   –   Output map of BR,BZ fields on a specified grid

MAP 13   –   Output map of vector potential on a specified grid

MAP 14   –   Output map of ER,EZ fields on a specified grid

MAP 15   –   Output map of electrostatic potential on a

                    specified grid

OUTPUT POISSON –   Output potentials at four R,Z locations during convergence of Poisson's equation

CONST (18)   –   Poisson output print interval for residual and cycle number

CONST (28)   –   COILGEN output giving the value of A_ϕ, B_r, B_z generated during evaluated of the elliptic integral while calculating the value of A_ϕ from the coil geometry.

Orbit output is controlled by the following data types:

CONST (19)   –   Orbit print interval

CONST (20)   –   Field output with orbit output

CONST (27)   –   Diagnostic output ray number

CONST (42)   –   Calculate moments for distribution CONST(42) cm.

FIELD Z( ) S - Output field values for values given in the
                  parameter list during orbit calculations

OUTPUT Z ( ) S - Output rays for Z values given in the parameter
                   list during orbit tracking

PRINT RAYS   - Output table of the initial values of the rays,
                   X, Y, Z, Px, Py, Pz.

PRINT CONST   - Prints out the values of the CONST ( ) array

PRINT POT   - Prints out the electrode potential array

PRINT FIELDS   - Prints out the fields on a specified grid

Plotted output is available on the CalComp device and on the printer. This output is controlled by the data summarized in the following list. The data controlling the plotting occurs in the input stream preceding the TRACK command - e.g.,

> PLOT TRAJ S
>
> PLOT PHASE 2.4 6.7 23.5 S
>
> TRACK

The one exception is the PLOT PHASE S data with no z values producing a plot of the final ray data at whatever values are left after termination of the RK integration.

TRAJECTORIES   - PLOT TRAJ N( ) S

PLOT RAY N( ) S

CONST(34) - CalComp TRAJ if value is 10 or 11

CONST(35) - CalComp R axis length, inches

CONST(36) - CalComp Z axis length, inches

CONST(43) - Plot BZ on frame if value is 1.

PHASE PLOTS   - PLOT PHASE Z( ) S - specifies plot of phase
                                           space for Z values given
                                           in parameters list

CONST(37) - CalComp phase plot if value is 10
                     or 11

CONST(38) - CalComp dR/dZ axis length, inches

CONST(39) - CalComp R axis length, inches

DENSITY PLOTS   - CONST(44) - CalComp density profile if value
                               is 10 or 11
                               Printer plot if value is 1 or 11

CONST(45) - DZ interval of plot of density
                     profile

PLOT DENSITY Z( ) S – specifies a plot of the charge density
profile for Z values given in parameter list.

PLOT BOUNDARY   – CalComp plot of the electrode boundaries

PLOT EQUIP S   – CalComp plot of electrode boundaries and all
equipotential lines specified by CONST(47),
EQUIP, and PLOT EQUIP data

PLOT ENERGY S   – CalComp plot of the ion energy versus beam
radius

PLOT AMPS S   – CalComp plot of the beam current versus beam
radius

PLOT PSI S   – CalComp plot of the beam rotational angle ψ
versus beam radius

PLOT JDENSITY Z( ) S – specifies a plot of the current density
profile for the Z values specified.

PHASE PLOTTING

EBQ can generate any one or combination of X-X', Y-Y', and r-r'
phase plots according to the value of CONST(57). These phase plots can
be produced for the rays which have not hit an electrode or exceed the
maximum radius, all at a given Z, or all at a given energy, or all at
arriving on a special surface. If more than one ray group has been
generated, these plots can be produced separately for each ray group or
for all groups together, according to the value of CONST(66).

Rays are eliminated from plotting at a given time when they have
previously been lost by exceeding a maximum radial value, CONST(31), or
by hitting a boundary region defined as "OUTSIDE". Rays which have not
been so eliminated will be plotted at the longitudinal Z values speci-
fied on the PLOT PHASE Z( ) data entry. These phase plots might appear
distorted by the space charge depression of the beam energy. CONST(64)
and (65) may be set in order to produce a plot of all the rays at a given
energy. CONST(64) and (65) specify the minimum and maximum beam energy,
such that any ray whose energy falls outside this range is simply no
longer integrated. After conclusion of the tracking, a phase plot made
will be for values, which are left from the time the rays fell outside
the allowed energy range. Consider producing a phase plot at 250 kV.

CONST *65 250E3 S

TRACK BEAM

PLOT PHASE S

Rays can also be plotted at the end of a TRACK by the PLOT PHASE S
command when they have terminated by passing a "STOP" flag set in the
boundary.

FIELD-FREE DATA INPUT

All data input are read in field-free format. As each card is read it is printed into the output stream. The field-free input may begin in any column of the card. All entries after a $ are ignored by the program and may be used to place comments on the data cards. An unlimited number of comment cards, beginning with a $ may be placed anywhere in the data deck. Alpha-numerical blocks are separated by blanks, equal signs, or commas. Numeric blocks consist of numbers or group of numbers which constitute the data input. These numbers may be integers (which will be considered as having unspecified decimal points and as such really be floating point numbers) floating point numbers, or exponential numbers in any mixed order with provision for repetition by use of the repeat specification R. Individual elements of an array may be entered by specifying the array element index preceded by an asterisk and the element value. A second value will be considered to be the next element of the array.

Blank cards in the data stream are ignored. Certain lists of data will be read on as many cards as necessary to satisfy the list or until an "S" is encountered designating the end of the data list. Such data lists are explicitly shown in the detailed description of each data type by specification of the S in the data stream.

Examples of legitimate numbers are:

INTEGERS   -   1,2,25 (interpreted as 1., 2., 25.)

FLOATING POINT   -   3.1456-5.2   .667 -0.0234

EXPONENTIALS   -   2.01E-6   1E8   2.3E+4

REPEATS   -   0R6,   6.75R3,   1.42E-3R4

ARRAY ELEMENTS   -   *7   3.14159   *72   1.0   2.0

DATA INPUT

Data for the EBQ code consists of one or more data cases. Each data case starts with a title cards and ends with an END card. A repeated END card (really a title card for the next case) signals the end of all data and terminates the execution of EBQ.

Between the title card and the END card any number of data entries can be made. These entries begin with a mnemonic name (called the data name or data type), followed by a parameter list, an array of values, or one or more special cards peculiar to the type of da being entered. The data types are:

BOUNDARY	IGNOR	PUNCHB
BSCALE	INITIALIZE	RAYGEN
CALL	LAPLACE	RAYS
CHARGE	LEDGE	REWIND
COILGEN	MAP	RKSTEP
CONST	MESH	SEGMENT
CURRENT	MOMENT	SKIP
DEFINE	OUTPUT	SLOPE
DENSITY	PAGE	STOP
DUMP	PLOT	TIMEE
END	POISSON	TITLE
ENDTRACK	POLAR	TRACK
EQUIP	POT	UNIT
FIELD	PRINT	WALL
IDRAY	PUNCH	XEQ
		XYZ

Before discussing the input parameter or arrays associated with each of these data types in detail, I shoul make a few general remarks on the data input conventions that will b used.

A)   If a data type requires a parameter list, this list must appear on
     the same card as the mnemonic name.

B)   If a data type requires an array input, this array data can appear
     on the mnemonic name card and/or any number of subsequent cards
     until an S is encountered in the data stream – e.g.,

                    CONST   *10   1.25

                    *1   .511E6   *35   11   S

or

                    CONST

                    *1   .511E6   *10   1.25   *35   11

                    S

C)   In the detailed discussions to follow on the individual data types,
     the input of an array requiring a terminator S is written

                    <u>CONST   STUF(   )   S</u>

     where the ( ) signifies an array of name STUF and S is the explicit
     terminator – e.g., CONST 1.1 3.7 3.0 67 S gives the array the
     following values STUF(1)=1.1, STUF(2)=3.7, STUF(3)=3.0, and
     STUF(4)=67.

D)   The input of a parameter list is written in this report explicitly
     giving the list entries. Some data types do not require a para-
     meter list, or some lists are optional. These choices are indi-
     cated by repeated data name entries as headings in the text – e.g.,

                    LAPLACE

                    LAPLACE SWITCH

                    LAPLACE SWITCH INITIALIZE

For the most part, the data input is order independent. There are some obvious exceptions which will be listed below. The general rule is input data types before they are to be used in execution called by some subsequent data entry. The recommended order is

```
            TITLE
            BOUNDARY
               {
            CONST
               {
            RAYS
               {
            TRACK
            END
```

Obviously, if one were to solve Laplace's equation and print a potential map, he should first input his boundary and potentials

```
            TITLE
            BOUNDARY
               {
            POT
               {
            LAPLACE
            MAP 2
            END
```

COMPUTER FIELD ARRAY LIMITS

Several types of limits are imposed on the size of problems
capable of simulation, size of array storage of rays and boundary
impose the most severe limits. Additionally, the bit masking procedure
used with the MESH array restricts the size of the potential array to
777B (511 decimal) and the size of the coefficient boundary array to
7777B (4095 decimal).

QUANTITY	ARRAY SIZE	COMMON BLOCK
MESH,U,RH	6000	BLANK COMMON
COEF	6000	BLANK COMMON
POT	300	BLANK COMMON
BOUNDARY POINTS	1000	LARGE CORE
XORB,YORB,ZORB PXORB,PYORB,PZORB, ZORB,TORB,CINSIDE	100	RAYS
ZPHASE EQUIPLOT ZDENPLT PUNCH PUNCHB	20	TRAJCC
BC ZONES	10	FGRIDCM
MAGNET COILS IN		
, GENERATION	500	JUNK
APHI	6000	BAMAPX
RTRAJ	350	RAYPLTC
ZTRAJ	350	
EQUIP	100	
QLIMIT	100	QLIMIT

EBQ DATA INPUT

BOUNDARY

A boundary is made up of one or more segments defining a closed region inside of which Poisson's equation shall be solved. EBQ automatically generates a Neumann boundary around the outer edge of the problem. Neumann segments (if explicitly specified by the user) should be given first, followed by any Dirichlet segments. A boundary segment is a sequence of points ended by a "S". A point is specified by three numbers, (IPOT, R,Z) giving the potential index for the point and its R,Z coordinate in cm. If IPOT=0 the point is a Neumann point - e.g.,

IPOT R Z---------------------------------IPOT R Z S

These points will be connected by straight line segments. IPOT can take several special values:

IPOT=0	--------------	Neumann point.
IPOT>0	--------------	Dirichlet point - POT index.
IPOT=FIT	--------------	Following two points will be fit by cubic.
IPOT=OUTSIDE	--------------	R, Z point at an exterior point.
IPOT=INCREMENT	--------------	IPOTMIN-IPOTMAX, any subsequent IPOT I will be incremented.
IPOT=I	--------------	Use incremental IPOT.
IPOT=S	--------------	End of line segment.
IPOT=STOP	--------------	Followed by normal boundary data IPOT,R,Z...Defines a surface upon which rays will terminate.
IPOT=REMOVE	--------------	Followed by normal boundary data-replaces a boundary by internal points.

If a single point is defined, then a test is made for

$$\left| dr \right| \; < \; 10^{-5}$$ ●——— X — ——●

otherwise   X

End of all boundary input is specified by an "3."

          IPOT R Z --------------- S
          IPOT R Z --------------- S
          S

The potential array, $\psi(\;)$, is initialized to the value of CONST(30), normally zero. IPOT specifies the index of the POT array in which the potential of the boundary is given. The IPOT index is given for the line segment beginning with the specified point--i.e., consider a connected segment specified as:

          3 0 0 0 10 0 5 10 20 S

Potential 3 is associated with the line $(0,0) + (10,0)$ Neumann boundary from $(10,0) + (10,20)$ and potential 5 on the last point $(10,20)$.

     If a graded Dirichlet boundary is desired, this can be conveniently implemented by the INCREMENT procedure. Say the points along some line are to be associated with the sequence of potentials 15, 16 --------- 30 defining a fringing field at the end of some problem. The data could be given as:

```
-
-
-
INCREMENT 15 30
-
-
-
I 0 0  30  15.  0  S
-
-
-
-
```

The IPOT=I will cause IPOT=15 and be incremented by one up to the value

of 30 for each subsequent point generated on the line segment.

Currently there is a limit of 6000 points in a problem and 000

Boundary Points (6000/6). The maximum number of potentials is 300.

The Boundary data consists of at least four cards - the BOUNDARY

card, the mesh size card, the line segment card, and ending S card.

There can be an unlimited number of line segment cards.

```
    BOUNDARY

    NR NZ RMIN RMAX ZMIN ZMAX UNIT
    }
    {       Line segment cards, each segment ending with an S
    S
```

RMIN is the minimum radius and must be 0 (zero). RMAX is the maximum

radius in cm of the boundary. ZMIN and ZMAX are the longitudinal extent

of the problem in cm. A mesh will be generated with NR X NZ points.

The mesh size is:

$$dR = \frac{RMAX}{NR-1}$$

$$dZ = \frac{ZMAX-ZMIN}{NZ-1}$$

The boundary points are processed in the order they are en. untered in the data stream. A point will be redefined by a subsequent int   t with a line segment. For this reason it is necessary that N boundaries be processed first, followed by Dirichlet seg m   he segments can be defined in any direction, any order, ar i do ot be continuous or sequentia' Electrodes can be   ed. However continuous outer Neumann boundary will automaticall   gen processing the user specified boundary segments.

If a seventh parameter is included the sec   boundary data, it will multiply all distances durin iput to onver from the data units to the EBQ distance unit of cent   rs.

BOUNDARY X NO.NIT

When the BOUNDARY data card has a third parameter n   cio, charge and potential arrays U( ) and RH( ) wil. not b   . Only the MESH( ) array will be generated by the bounda p s   i is useful when the charge and potential arrays are ase to a s at left by some previous case, but several mesh modifications a: made.
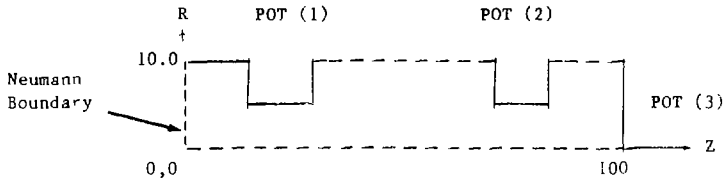
```
11   101  0  10.  0.  100  $  1CMX 1CM grid

1   R Z  1  R  Z-------S  $  Electrode, POT(1)

2   R Z  2  R  Z-------S  $  Electrode, POT(2)

3   0  100.  3  10.  100.  S  $  Electrode, POT(3)

5   $  END  ail Boundary data
```

Note that in this example, the Neumann boundary is a rectangle around the problem defined by the code before processing the specified Dirichlet (electrode) points on the outer periphery.

When IPOT is the literal FIT, three points will be taken to find the radius and center of a circle. The section of the circle connecting the points will be used to generate the boundary.

```
BOUNDARY
 -
 -
 -
FIT  i_1  r_1  z_1  i_2  r_2  z_2  i_3  r_3  z_3  . . .  S
 -
 -
```

The points 1, 2, and 3 will be fitted. EBQ will print the radius
and center of the circle it used, R, r_0, z_0

BOUNDARY

—

—

...... i_1 r_1 z_1 FIT i_2 r_2 z_2 i_3 r_3 z_3 ... S

—

—

Here, FIT appears inside the boundary card, not the first entry, but
will still fit points 1, 2 and 3 where point 1 is the point preceeding
the FIT.
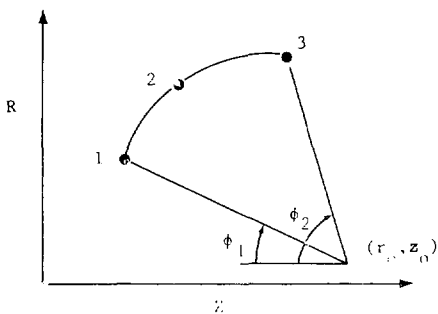
If one is inputting a circle, use diametrically opposite points, with
the first and last point the same.

For example, --- 5 5 8 FIT 5 9 8 5 5 8 --- would produce the result
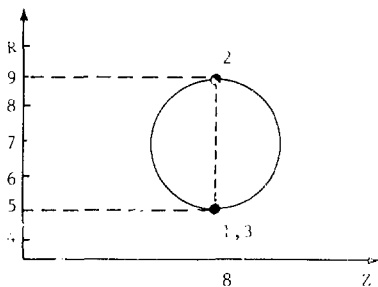
Alternatively, the fitting can be done by specification of the center and radius of the circle to be used. This is done by use of the FIT CURVE data.

--- i_1 r_1 z_1 FIT CURVE I R_o Z_o A PHI_1 PHI_2 ---
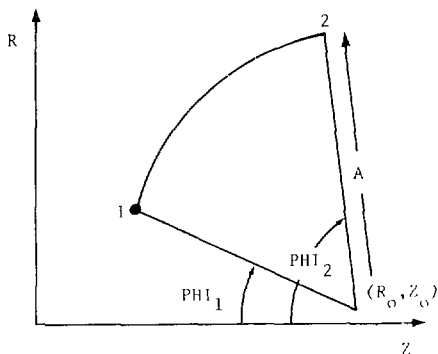
OUTSIDE RC ZC I_1 R_1 Z_1 ---- I_N R_N Z_N S

OUTSIDE data defines a simply connected closed Dirichlet region, inside of which all MESH( ) types are zero. The effect is to prevent solution of Poisson's equation and ray tracing in these areas. RC, ZC defines a point inside the closed region used for determining which side of the boundary is outside. This test can be ambiguous if the region is too complicated. For each line segment making up the region, one of the following two tests are performed depending on if the line segment is being followed by incrementing R or Z (more vertical or more horizontal).

HORIZONTAL:

$$R = R_N + \left(\frac{R_{N+1}-R_N}{Z_{N+1}-Z_N}\right)\left(Z_C - Z_N\right)$$

"outside" is above line segment if $R < RC$

VERTICAL:

$$Z = Z_N + \left(\frac{Z_{N+1}-Z_N}{R_{N+1}-R_N}\right)\left(RC - R_N\right)$$

"outside" is to right of line segment if $Z < ZC$.

Consider the slit shown below, designated electrode 3, which is to restrict beam flow from the cathode - extractor electrodes 1 and 2.
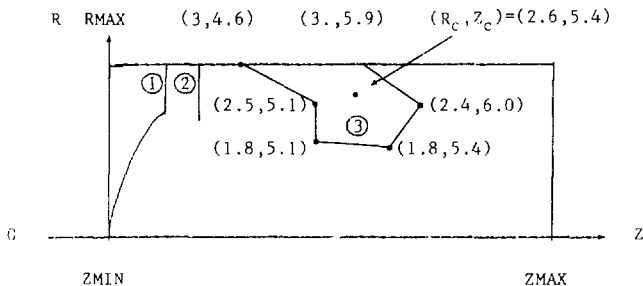
The data specification for this case would be:

```
BOUNDARY
NR  NZ  0   RMAX  ZMIN  ZMAX
1  0  0 ----------- S         $ CATHODE
2       ----------- S         $ EXTRACTOR
OUTSIDE  2.6  5.4  3  3.0  4.6  3  2.5  5.1  3  1.8  5.1
         3  1.8  5.4  3  2.4  6.0  3  3.0  5.9  3  3.0  4.6  S
S
```

The potential of the region outside the problem boundary can be optionally specified by giving either the potential value or the potential index from which the potential will be taken. In this case, the data following the "OUTSIDE" is either "I" or "U" and the value, followed by the value of R,Z pointing outside.

```
OUTSIDE R Z                  $ NOT SETTING POTENTIAL AT R,Z
OUTSIDE I IPOT R Z           $ U(OUTSIDE) = POT(IPOT)
OUTSIDE U POT R Z            $ U(OUTSIDE) = POT
```

See example 4.

   STOP I_1 R_1 Z_1 --------------- I_N R_N Z_N S

A boundary segment starting with STOP defines a segment which will stop individual rays, whereas the same segment data not preceded by STOP is transparent to the rays. When I_1 is non-zero, an internal Dirichlet boundary will be defined in the usual way with bit 30 on. It is the unit value of bit 30 which the tracking routine tests to determine the transparency of the mesh. If I_1 is zero, the boundary segment will be followed turning bit 30 on without defining an actual boundary - i.e., leaving the Poisson coefficients set as interior points.

A STOP boundary will not be normally plotted. If it is desired for the boundary t be plotted along with the other normal boundaries, STOP must be followed by PLOT, i.e.,

$$\text{STOP PLOT } I_1 \quad R_1 \quad Z_1 \quad \text{------} \quad S$$

$$\text{REMOVE } I_1 \quad R_1 \quad Z_1 \quad \text{-----------------} \quad I_N \quad R_N \quad Z_N \quad S$$

A boundary segment can be removed by preceding the identical data which 'efined it in the first place by REMOVE. This procedure restores these boundary points to interior points.

$$\text{REMOVE STOP } \quad I_1 \quad R_1 \quad Z_1 \quad \text{---- -----} \quad I_N \quad R_N \quad Z_N \quad S$$

Preceding a boundary segment data string by REMOVE STOP llows the segment turning off bit 30 to restore transparency to t : mesh for this segment.

BOUNDARY MODIFY

An existing problem boundary can be modified. This option prevents initializing the boundary and allows addition of electrodes or definition of "OUTSIDE" regions. This is useful when desiring to stop particles on an equipotential surface for phase plotting, when a solution to the whole problem must be obtained so space charge depression is properly evaluated. Then an "OUTSIDE" region can be inserted or a STOP segment defined to stop rays on the equipotential. The boundary modify data consists only of the boundary segments which constitute changes to be made -- e.g.,

                    BOUNDARY MODIFY

                    STOP 0 R_1 Z_1 0 R_2 Z_2 ----------- S

                    OUTSIDE R_X Z_X

                              I_1 R_1 Z_1 I_2 R_2 Z_2 ---------- S

                    S

BOUNDARY MODIFY ALL

This option requires the input of the full boundary data set just as if a new problem was being defined, however, it doesn't initialize or redefine anything but the boundary data. This is useful when studying small perturbations in electrode structure.

BSCALE SCALE

Multiples ve·tor potential array used for generation of external magnetic fields by Scale -- i.e., if IRBFLED *IZBFLDI=N and NCOILS≠0, then

$$A(I) = A\phi \; (I) \; * \; SCALE$$

for I = 1 to N.

The Scale value is referenced to the original value of A4 when generated. If a run is made at .9 of the original value, then a subsequent run at 0.8 of the original value would use BSCALE 0.8. The actual scaling is done by a call to FLDINIT generated by a TRACK, MAP, or XEQ card. B_r and B_z are calculated by differentiation of the vector potential.

Their values can be printed by

PRINT FIELDS . . . . . .

or

MAP 13, 14

R1   R2   DR   Z1   Z2   DZ

A profile can be given to the vector potential by providing input of the form of a power series expansion for the scale factor.

BSCALE   A   B   C   D   E

where the scale factor use by EBQ is calculated as

$$SCALE(Z) = A + B\left(\frac{Z-ZMIN}{L}\right) + C\left(\frac{Z-ZMIN}{L}\right)^2 + D\left(\frac{Z-ZMIN}{L}\right)^3 + E\left(\frac{Z-ZMIN}{L}\right)^4$$

Z is the longitudinal position and L=ZMAX-ZMIN.

CALL NAME

See Define.

<u>CHARGE TYPE MAXRQ SCALE ADD DZSTEP ZQ1 ZQ2</u>

RH( ) S

<u>CHARGE LIMIT LQSKIP QSCALE</u>

QLIMIT( ) S

The charge array RH( ) is initiated to zero, at the ng of processing Boundary data. RH can be set initially by a rea' from a Tape 8 Dump or by the CHARGE data entry.

TYPE = 1 Input directly RH( ) ascending R, then Z, maximum of 3000 points.

RMAX
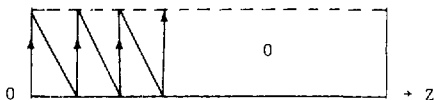
TYPE = 2 Input RH ( ) ascending Z, then R, maximum 3000 points R cut off is MAXRQ rows
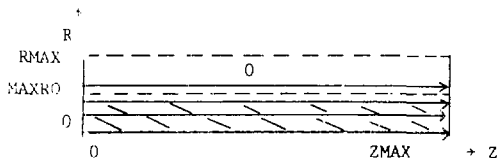
where RMAX, ZMAX are the values defined by the boundary grid mesh data. The potential produced by this charge array may be found by executing the POISSON data card. The RH ( ) array is literally the right hand side of Poisson's equation and has units of volts/cm^2. This is related to the charge density in Coulombs/cm^3 by:

$$RH = \frac{Q}{\varepsilon_o} \, 100$$

where $\varepsilon_o \approx 8.854 \times 10^{-12}$ farads/m.

The longitudinal (Z) extent for the charge input is ZMIN - ZMAX. If the seventh and eighth parameters, ZQ1 and ZQ2, are given, then the lontitudinal extent of the charge input is over the restricted range ZQ1 to ZQ2.

The values of the input array are multiplied by scale. If the fifth parameter is the literal "ADD," then the charge is added to the charge array without initialization to zero.

   TYPE = 3   Input initial charge density load based on uniform
              current density, beam profile and longitudinal beam
              energy

   CHARGE 3 J SCALE ADD DZSTEP ZQ1 ZQ2
        R( ) S $ radius in cm
        E( ) S $ energy in ev

This input is used to start problems in the extreme space charge limit where the self-magnetic and electric fields cannot be neglected on the first cycle or fudged by the CONST(3), CONST(4) scale factors. CHARGE 3 input will deposit charge according to a specified uniform current density J and a beam envelope radial profile R( ) for a beam whose energy E( ) varies along the Z direction. J is in amperes/cm^2, R( ) in cm, and E( ) in electron volts. The right hand side of Poisson's equation is then initialized as:

$$RH(\ ) \ = \ \frac{J_o R_o^{\ 2}}{\varepsilon_o c} \ \frac{E_o + E}{R^2 \sqrt{E^2 + 2E_o E}}$$

where c is the velocity of light, $\varepsilon_o = 8.845 \times 10^{-14}$ f/cm; R_o the beam radius, E_o the beam particle rest mass and RH( ) the charge density in volts/cm^2. Charge load assumes the beam energy has only a longitudinal variation, neglecting the space charge depression, but does take into account the charge density variation produced by a radial modulation of the beam profile.

If the DZSTEP is zero, then the initial charge array is specified on an irregular Z interval and a third array list must be included.

    CHARGE 3 J Scale ADD 0

    R( ) S $ radius in cm

    E( ) S $ energy in ev

    Z( ) S $ longitudinal position in cm

If SCALE is non-zero for CHARGE 3 data, it will multiply the energy array making the units arbitrary; e.g., if the beam energy is in MeV, then SCALE = 1.0E6 and the E array is the energy in MeV.

CHARGE INITIALIZE

This data command causes the initialization of the charge array RH( ) setting it to zero over the entire problem size.

CHARGE LIMIT LQSKIP QSCALE

QLIMIT(` S

This data inputs an array QLIMIT( ) specifying the maximum charge density that will be allowed in the RH( ) array. The values of QLIMIT give the maximum charge density allowable along the z axis in volts/cm^2. The input can be scaled by the value of QSCALE if non-zero. Before solving Poisson equation each radial position of the RH(R,Z) array will be checked against the value of QLIMIT(Z) and any value of the RH array exceeding this limiting value will be constrained. The charge density at this point will be set to half the the limit value and half the excess charge density will be spread uniformly between the axis and this location, the other half of the excess charge density will be uniformly deposited on the next N = CONST(77) radial mesh points. If the radius corresponds to a radial mesh point less than M = CONST (78), then the excess charge density will only be deposited outward from the axis on the next N mesh points.

LQSKIP is normally 1 so there is a one to one correspondence between the longitudinal axis z and the values in the QLIMIT array. If LQSKIP has the value 2, then each QLIMIT point will be used for two z points in the RH array, and if LQSKIP has the value 3, then the points in the QLIMIT array will be used for three RH array points, etc. A maximum of 100 points can be specified in the QLIMIT array.

This procedure is turned on by setting CONST(77) = N, the number of radial positions over which the charge should be outwardly spread, and turned off by setting CONST(77) = 0, or by the data:

CHARGE LIMIT   0   0

with no input of the QLIMIT( ) array. If no charge limit data has been included in the data deck, but CONST(77) is greater than zero, then this value will be used everywhere as the limiting charge density in volts/ cm^2. Any excess charge will simply be ignored, i e., will be discarded.

COILGEN TYPE BLEFT BRIGHT RBMAX NZONES NREPEAT UNIT BZCUTOFF

The magnetic field from external coils can be given by directly reading a map of the A_ϕ component of the vector potential in guass-cm or by reading the positions and currents of ideal, centered circular loops from which the vector potential will be calculated. During ray tracking, the vector potential is differentiated to obtain the values of B_r and B_z. A maximum of 6000 mesh points may be used in the vector potential array which can cover a region of space different from that covered by the electrostatic boundary region. The value of $A_\phi(R,Z)$ is initialized to zero before processing the COILGEN data if the value of CONST(75) is zero. The elements of the CONST array which affect the COILGEN data are elements 28, 43, 70 and 75.

The COILGEN data parameters are interpreted as follows:

TYPE   -1   Turns magnetic field back on after it has been disabled
               by TYPE 0.

           0   Turns off external magnetic field.

           1   input individual coils specifying z, r, I per card for
               each coil

           2   Coil generator, specify grid of R,Z values for
               conductors at points of grid R1, R2, DR, Z1, Z2, DZ.
               Maximum of 500 coils per COILGEN data set.

           3   input Z array values, R array values and current array
               values for the coils.

           4   Constant BZ values of BLEFT, BRIGHT with $B_r=0$.

           5   Read directly the A_ϕ array. This input must be
               terminated by an "S".

6 Calls the coil generator for conducting loops with
   excitation in ampere-turns and data input of arrays of
   RMINS( ), RMAXS( ), ZS( ), NRS( ), and NZS( ).

9 Read directly the A_ϕ array from tape 9 in field free
   formated form. This data must be terminated by an "S".

BLEFT B_z value for Z < ZBMIN, B_r taken as zero.

BRIGHT B_z value for Z > ZBMAX, B_r taken as zero.

RBMAX   maximum radius of calculation of the external magnetic
            fields. If not specified, RMAX from the boundary is used.

NZONES   number of field zones.

NREPEAT   number of repeats of field zones.

UNIT   If non-zero, multiplies the distances of the COILGEN data
            input to convert them to cm.

BZCUTOFF   Calculation of $A_\phi(R,Z)$ will be terminated for a coil loop
            for which B_z at R=0 is less than this cutoff value in
            Gauss.

MAGNETIC FIELD ZONES

The magnetic field map must be divided into several zones if strong field changes occur. Consider a long solenoid lens whose B_z field along R=0 is as shown:
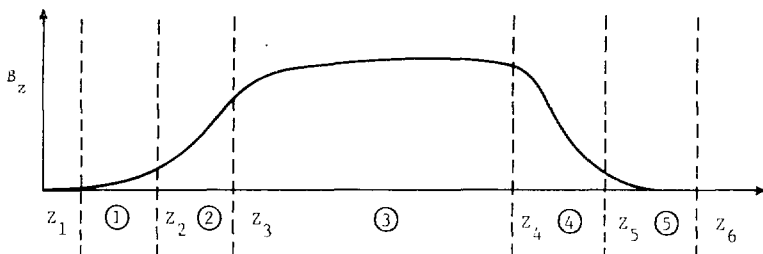
A zone of finer mesh should be used in the fringing field regions between Z2 and Z3, and between Z4 and Z5. This is accomplished by a division of the map into five zones as shown. The field grid card is now expanded to give an array of Z values, an array of DR values, and an array of DZ values.

```
COLLGEN TYPE B1   B2   RBMAX   5   0
Z1   Z2   Z3   Z4   Z5   Z6
     DR1   DR2   ---   DR5
     DZ1   DZ2   ---   DZ5
Scales S
  { coil data
```

ZBMIN is taken as Z1 and ZBMAX is taken as Z6. The Z, DR, and DZ values may appear on one or as many cards as desired. The total number of mesh points generated is:

$$N = \sum_{j=1}^{NZONES} \left[\frac{Z_{j+1} - Z_j}{dZ_j} + 3 \right] \left[\frac{RBMAX}{dR_j} + 2 \right]$$

CARDS FOLLOWING COILGEN CARD:

When $3 \geq$ TYPE ≥ 1 several cards follow the COILGEN card. These specify the magnetic field grid and scale factors, followed by the coil or vector potential data. Consider the simple coil data with NZONES=0.

```
COILGEN TYPE B1 B2 RBMAX 0 0
ZBMIN   ZBMAX   DR   DZ
SCALES      S
    Coil Data
```

A vector potential map will be generated by the coil data extending from ZBMIN-dZ $\leq$ Z $\leq$ ZBMAX+dZ and $0 \leq$ R $\leq$ RBMAX+dr. The cell size of this map is specified by DR, DZ in cm. A fictitious boundary is taken around the map — i.e., the vector potential is calculated and stored for one dz mesh line to the left and right of ZBMIN, ZBMAX and one dR mesh line above RBMAX. Magnetic field values are only calculated, however, within the real specified map values (0,RBMAX) (ZBMIN--ZBMAX)
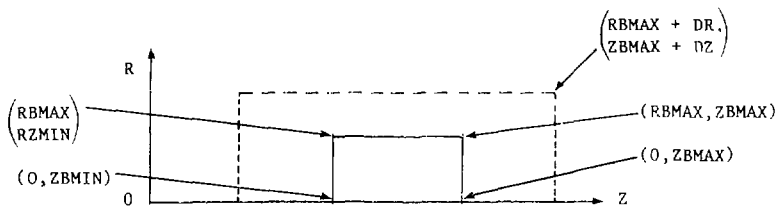
outside this map, the field values are taken as $B_r=0$, and $B_z=B1$,

$Z<ZBMIN$ or $B_z=B2$, $Z>ZBMAX$.

REPETITION OF ZONES

Magnetic field zones can be considered to slide along the Z axis in the sense that when Z is larger than ZBMAX, the zones are moved such that ZBMIN is now at the previous ZBMAX value. Consider three identical short solenoids as shown:
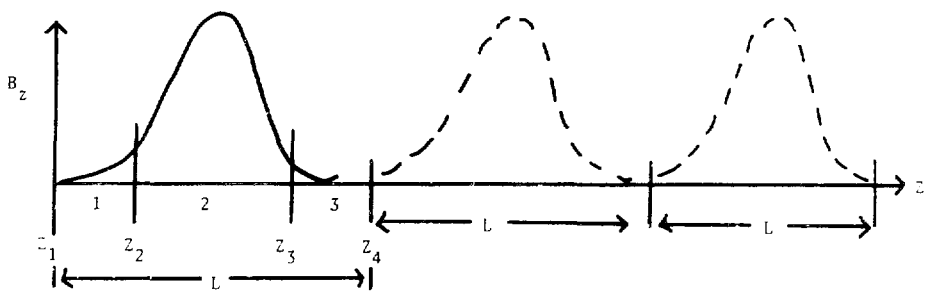
COILGEN TYPE B1 B2 RBMAX   3   3

Z1 Z2 Z3 Z4 dR$_1$ dr$_2$ dR$_3$ dZ$_1$ dZ$_2$ d7$_3$

SCALES   S

   coil data

COIL DATA

When coils are specified, their R, Z values are in cm and their current is in amperes.
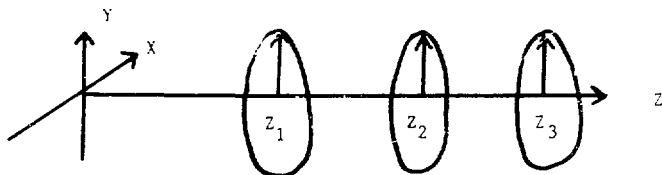
The number of COILS is given by the number of scale values read for type 1 and 3 input, and the number of COIL GROUPS is given by the number of scale values read for type 2 input with the number of coils in each group determined by the coil generator.

Consider 3 coils of 100, 250 and 100 amps then the scale and coil cards for type 1 data would be:

```
100.    250.    100.    S
Z1    R1    1.
Z2    R2    1.
Z3    R3    1.
```

or alternatively

```
1&3    S
Z1    R1    100.
Z2    R2    25.
Z3    R3    100.
```

Note, the current is the product of the scale value for each coil and the coil current.

TYPE=2 coil data generates a loop for each point of the given
grid. The scale values are really the coil currents given for the grid:

```
CURRENTS ---------- S
RMIN RMAX dR ZMIN ZMAX dZ
   ●
   ●
   ●
   ●
   ●
```

TYPE=3 coil data specifies the Z, R, and I arrays and is useful when all
coils have, say, the same R, and I values--e.g., consider seven 10 cm
radius, 563.2 amp coils at Z1---Z7. Input

```
1.0R7    S
Z1   Z2 ----- Z7
10R7
563.2R7
```

TYPE=4 input specifies only a constant BZ value with BR=0 everywhere.
Only three entries are required and no other cards of the COILGEN set
can be used.

```
COILGEN 4  BZ  0  0  0  0
```

TYPE=5 input specifies input of the vector potential directly at mesh
points compatible with the specifications on the COILGEN and subsequent
data cards. The input would be:

```
COILGEN 5 BLEFT BRIGHT RBMAX

ZBMIN ZBMAX DR DZ

Scale S

Aφ( )

S
```

Note, the Aϕ( ) array data must be terminated by an "S" and specifies
the vector potential component in ascending R, then ascending Z.
This array of numbers extend from ZBMIN-DZ to ZBMAX+DZ in the radial
range from 0 to RBMAX+DR to cover the phantom edge shown in the field
map, page 100. The scale data is also terminated by an "S" and consists
of a single number, which will multiply the Aϕ( ) data when non-zero or
non-unity.

COILGEN 9

Same as COILGEN 5 except data is read from tape 9. The data consists of
the COILGEN 9 card, grid card, scale card, and then as many cards as
necessary to input the field.

Type 6 input specifies input for the coil generator in the format
shown below. This data generates COILGEN 2 data which is used to
calculate the vector potential magnetic field array.

```
        COILGEN 6 ....
        ZMIN  ZMAX  DR  DZ
        AMPTURNS( ) S            $ N ENTRIES, MAX OF 100
        RMINS( )                 $ N
        RMAXS( )                 $ N
        ZS( )                    $ 2N ENTRIES
        NRS( )                   $ N
        NZS( )                   $ N
```

The first data array gives the ampere-turns of the coil set and must end
with the "S" terminator. The next three arrays give the coil cross-
section dimensions in cm. The ZS( ) array give the longitudinal posi-
tion of the start and end of each coil cross section in turn. The
NRS( ) and NZS( ) arrays specify the number of R and Z layers into which
each coil will be divided. Consider the diagram below, the data might
be:

```
COILGEN  6  0  0  7
0   150   1   2.5
1000   1500   4250   S
6.5R2   5
8R2   6.0
30   40   44   60   70   120
2   3   1
3   4   5
```
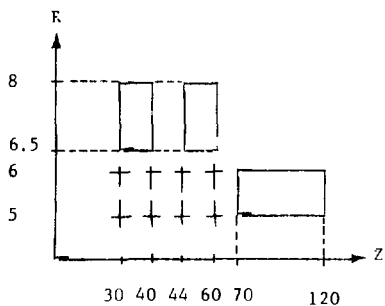
The coil cross sections
are replaced by circular
loops as shown in the
second part of the diagram.
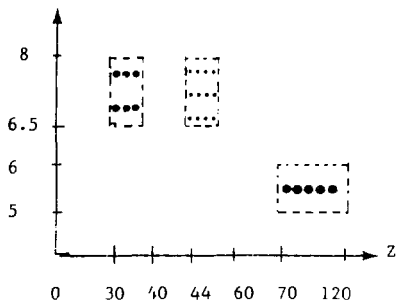
In cartesion geometry, only a constant B_y field can be used

COILGEN 4

CONST LIST ( )   S

An array of parameters whose elements have the meaning shown in the table can be set via the CONST data entry. The input is field-free and only those elements whose value are to be altered from the default value need be given. To change the 18th and 19th value:

            CONST   *18   20   25   S

or

            CONST   *18   20   25

                      S

etc.

The current values of the CONST ( ) array are printed to the output file on execution of a TRACK or PRINT CONST data entry.

Summary table of EBQ CONST( ) array with default values.

```
CONST( 1)=REST MASS IN ELECTRON VOLTS                           0.511E6
CONST( 2)=CHARGE                                                -1.0
CONST( 3)=GAUSS LAW SPACE CHARGE FACTOR, 1ST CYCLE              1.
CONST( 4)=SELF BPHI FIELD FACTOR 1-ST CYCLE ONLY               1.
CONST( 5)=C  VELOCITY OF LIGHT IN CM/SEC                        2.998E10
CONST( 6)=EQMB                                                  -299.7921
CONST( 7)=EQMEE                                                 -1.
CONST( 8)=1/EPSILON =1/8.854E-12 FARADS/METER                  1.1294E11
CONST( 9)=0 RAY INPUT IS Z WITH S=0,OTHERWISE S WITH Z=CONST(9)
CONST(10)=RUNGA KUTTA INTEGRATION STEP CM                       DZMESH
CONST(11)=RAD   .0174529                                        .74529
CONST(12)=PI                                                    3.14159
CONST(13)=DATA ANGLE INPUT RAD(1) .01745(DEG)                   1.
CONST(14)=BEAM LINE GRAPH INTERVAL IZSKIP (0 NO GRAPH)          1
CONST(15)=OMEGA POISSON OVER-RELACATION FACTOR                  1.8
CONST(16)=EPS MAXIMUM POISSON CHANGE IN VOLTS                   1E-5(PMAX-MIN)
CONST(17)=MAXIMUM NUMBER OF POISSON ITERATIONS                  200
CONST(18)=POISSON PRINT INTERVAL                                10
CONST(19)=ORBIT OUTPUT STEP CM                                  ZMAX
CONST(20)=FIELD OUTPUT ON TRAJ 1,  NO OUTPUT IF 0               0
CONST(21)=MAX NUMBER OF RAY TRACE CYCLES                        1
CONST(22)=RAY TRACE CONVERGENCE FACTOR                          .01
CONST(23)=RAY Z ORIGIN SHIFT IN CM                              0
CONST(24)=R,Z RAY INPUT FACTOR TO CONVERT TO CM                 1.
CONST(25)=Z(CM) RAY CONVERGENCE MOMENT CALL                     ZMAX
CONST(26)=MAXIMUM TRAJ LENGTH ZMAX ORBITS                       1.0E100
CONST(27)=DIAGNOSTIC ORBIT OUTPUT RAY NUMBER                    0
CONST(28)=COILGEN OUTPUT                                        0
CONST(29)=NUMBER OF QDEP INTERVALES BETWEEN RAY TRAJ PLOT PONINTS
CONST(30)=POTENTIAL INITAL MAP VALUE                            0
CONST(31)=MAX RADIUS OF ORBIT INTEGRATION                       RMAX
CONST(32)=DUMP NUMBER,TAPE8 FOR MESH(),U(),RH() ETC             1
CONST(33)=DUMP TAPE8    RAYS                                    0
CONST(34)=TRAJ CALCOMP (10), PRINT(01), BOTH (11)               10
CONST(35)=CALCOMP R AXIS LENGTH, INCHES                         6.0
CONST(36)= CALCOMP Z AXIS LENGTH, INCHES                        10.0
CONST(37)= PHASE PLOT CALCOMP (10), PRINT(01),  BOTH(11)        01
CONST(38)= CALCOMP DR/DZ AXIS LENGTH, INCEHS                    5.0
CONST(39)= CALCOMP R AXIS LENGTH, INCHES                        5.0
CONST(40)= CHARGE AVERAGING WEIGHT                              0.
CONST(41)= NUMBER OF CYCLES FOR CHARGE AVERAGING                0
CONST(42)= DZ FOR CALLS TO MOMENT (CM)                          0
CONST(43)= PLOT BZ ON FRAME 1, DO NOT PLOT 0                    1
CONST(44)= DENSITY CALCOMP (10) PRINT (01) BOTH( 11)            0
CONST(45)= DZ FOR PLOT OF DENSITY PROFILE (CM)                  0
CONST(46)=N   CALC CURRENT ENCLOSE EVERY N RK STEPS             0
CONST(47)=NUMBER OF EQUIPOTENTIAL LINES ON CALCOMP              0
CONST(48)=DS EQUIPOTS                                           .5*DRMESH
CONST(49)=EQUIP R,Z,U PRINT FLAG. PRINT IF 1                    -1
CONST(50)=SELF B FIELD FACTOR FROM ER FIELD                     0
```

Summary Table of EBQ CONST( ) Array with Default Values (Continued)

CONST(51)=EQUIPOTENTIAL ERROR	1.0E-4
CONST(52)=MAX NUMBER OF ITERATIONS TO FIND EQUIPOT	5
CONST(53)=NRSKIP U() PUNCH TAPE 4 WITH RAYS PUNCH Z S	1
CONST(54)=1 STOP ON RETROGRADE MOTION (0,DONT STOP)	0
CONST(55)=SELF B FIELD RADIAL CUTOFF	.01
CONST(56)=Z50. USE CONST(50) FOR Z.GT.Z50	0
CONST(57)=PHASE PLOT TYPE 100(X,XP) 10(Y,YP) 1(R,RP)	1
CONST(58)=N, PRINT EVERY N-TH RAY TRACKING OUTPUT	1
CONST(59)=1, OUTPUT FORCE TERMS WITH FIELD TRACKING	0
CONST(60)=Z(CM) TO THE LEFT SEGMENT RAY TRANSFER PNT.	0
CONST(61)=1, OUTPUT PREVIOUS ENVELOPED ON PLOTS DASHED	1
CONST(62)=S, SCALE RH() ARRAY FOR Z.GT.ZRETRO	.5
CONST(63)=1 CALC. EMITTANCE WITH MOMENT CALLS	1
CONST(64)=MINIMUM ENERGY (EV) FOR ORBITS	0
CONST(65)=MAXIMUM ENERGY (EV) KINETIC FOR ORBITS	1E100
CONST(66)=0 PLOT ALL RAY GROUPS TOGETHER, OTHERWISE SEPARATLY	1
CONST(67)=	
CONST(68)=Z SEGMENT BOUNDARY ADVANCE PER ITERATION	0
CONST(70)=PLOT COILGEN COILS ON CALCOMP FRAME	1
CONST(71)=PLOT RAY ENVELOP ON CALCOMP FRAME	1
CONST(72)=OUTPUT CHILDS LAW SURFACE	-1
CONST(73)=1 OUTPUT Z RELATIVE TO ZMIN, =0 REL ZSTART	0
CONST(74)=CHILDS LAW POTENTIAL UNDER-RELAXATION FACTOR	0.5
CONST(75)=0 INITLZ APHI=0, =1 APHI IS TOBE ACCUMULATED	0
CONST(76)=REQJIP, R OF LINE SEARCHED FOR EQUIPOTENTIALS	0
CONST(77)=MAX RH LIMIT V/CM**2 OR CHARGE LIMIT N	0
CONST(78)=CHARGE LIMIT MINIMUM RADIAL MESH	4
CONST(79)=CHILDS LAW RELAXATION ERROR LIMIT	0
CONST(80)= IXSW1 , POISSON ITERATION OMEGA=CONST(82)	40
CONST(81)= IXSW2 , POISSON ITERATION OMEGA=CONST(83)	100
CONST(82)= OMEGA FOR POISSON ITER=IXSW1	1.800
CONST(83)= OMEGA FOR POISSON ITER=IXSW2	1.900

ELEMENT

1 - Rest mass of beam particles in electron volts. The rest mass is related to the mass of the particle by $E_o = MC^2/a$ where M is the mass in grams, C the velocity of light in cm/sec and a is the number of ergs per electron volt $\approx 1.6 \times 10^{-12}$. Default value is the rest mass of the electron, 0.511×10^6 eV.

2 - Charge of the particle in units of an electrical engineer, the charge of a standard electron. The default value is for electrons, -1.

3 - On the first cycle of a problem, the current enclosed by a ray is used to find a radial electric field by Gauss's Law. This field is scaled by CONST(3) and added to the radial electric field found from the gradient of the electric potential found from a solution of Laplace's equation.

4 - On the first cycle of a problem, the current enclosed by a ray is used to find the self-magnetic field by applications of Ampere's Law. This field is scaled by CONST(4).

5 - The velocity of light in cm/sec, 2.998×10^{10}.

6 - The magnetic field force is multiplied by CONST(6) before contribution to the equations of motion.

7 - The electric field force is multiplied by CONST(7) before contribution to the equations of motion.

8   -   The reciprocal of the pervitivity of free space.

$1/E_\varepsilon = 36\pi \times 10^9$ meters/farad.

9   -   Ray input from cards or dump files is Z with the path

length S zero if CONST(9)=0. Otherwise, ray input is path

length S with Z=CONST(9). RAYS 8 data always inputs the

pathlength from the dump file with Z taken as the Z value

from the dump plus the value of CONST(23). The path length

for the RAYS 8 data can be set to zero by setting

CONST(9)=0.

10   -   The Runga-Kutta integration step in cm. This step must be

equal to or smaller than the longitudinal (Z) mesh size.

11   -   Radians per degree, 0.0174529.

12   -   Pi = 3.14159

13   -   Ray angle input conversion factor. This constant multi-

plies the ϕ, α, and RAYGEN angles to convert them to

radians.

14   -   A beam line paper plot is generated after each TRACK

command showing the radial extent of the beam for every

CONST(14) longitudinal mesh intervals, producing a plot of

LMAX/CONST(14) lines in length. If CONST(14)=0, no paper

plot will be generated.

15   -   Poisson equation over-relaxation factor. Must be between

1.0 and 2.0 with 1.8 a good value for most problems.

16   -   Poisson's equation will be relaxed until the maximum number

of iterations (CONST(17)) has occurred or until the largest

change in potential mesh U( ) is smaller than CONST(16).
The default value is 10^{-5} times the maximum potential
minus the minimum potential in the potential array POT( ).

17 — Maximum number of iterations in relaxing Poisson's
equations per cycle. Default value is 200.

18 — A line of output is generated every CONST(18) Poisson
iterations giving the iteration number, the sum of
residuals, and the location and value of the largest change
made in the scalar potential array U( ).

19 — An orbit output table is generated every CONST(19) cm along
the beam path. The output gives the energy E, cartesian
position X, Y, path length S, momentum components P_X,
P_Y, P_Z, flight time t, radius R, and polar azimuth ϕ
for each ray tracked.

20 — An output table of field values encountered by each ray is
printed after the table generated by CONST(19) if
CONST(20)=1, no field table is printed if CONST(20)=0.
The values printed give the electric and magnetic field at
the current position of each ray.

21 — Number of Poisson equation – ray trace cycles to be
processed.

22 — Poisson equation – ray trace cycles are processed until the
maximum number, CONST(21) has occurred or until either the
fractional change of the RMS radius of the beam at

Z=CONST(25) is smaller than CONST(22), or, if using Child's
Law Start, the fractional change of the total beam current
is smaller than CONST(22).

23 - The longitudinal origin of each ray is shifted by CONST(23)
       cm such that Z=Z(input)*CONST(24) + CONST(23).

24 - The ray input position R,Z is multiplied by CONST(24) to
       convert R,Z to centimeters.

25 - Longitudinal location at which RMS beam radius is to be
       calculated to determine convergence of the Poisson-ray
       trace cycles. Default value is ZMAX.

26 - Maximum length of beam tracking. Default is ZMAX.

27 - Diagnostic ray number

28 - Output of external magnetic fields and vector potential
       during processing of COILGEN data if CONST(28)=1. No
       output if CONST(28)=0.

29 - A maximum of 2500 points is available for ray plot
       storage. CONST(29) specifies the interval in mesh units at
       which rays are stored for plotting. CONST(29)=1 stores ray
       radii at every Z mesh crossing, while CONST(29)=2 stores
       the ray radii every other Z mesh crossing, etc.

30 - The potential array U is initialized to CONST(30) before
       solving Poisson's equation if CONST(30) is non-zero.
       Otherwise, the array is initialized to 1/2(MAX POT-MIN
       POT),

31 - Ray tracking for an individual ray stops when the radius
       exceeds CONST(31).

32 - **Dump number** identifying a TAPE8 binary problem dump used in
       restarting the problem. CONST(32) can have any integer
       value greater than 0.

33 - **Dump number** identifying a TAPE8 binary ray dump used in
       continuing a beam.

34 - Specifies desired combination of ray plot output (R,Z)
       and/or CalComp plot for the rays designated on the PLOT
       TRAJ data cards. A CalComp plot is generated if the tens
       position is unity and print output is generated if the
       units position is unity. The allowed values are 0, 1, 10,
       and 11. Default = 10.

35 - CalComp R axis (transverse dimension) length in inches.
       Maximum value is 10.5 inches. If CONST(35) is negative,
       then it gives the R axis length in centimeters. Default
       6.0 inches.

36 - CalComp Z axis (longitudinal dimension) length in inches.
       If CONST(36) is negative, then it gives the Z axis length
       in centimeters. Default 10.0 inches.

37 - Specifies desired combination of phase space plots on paper
       and/or CalComp. A CalComp plot is generated if the tens
       position is unity and a printer plot is generated if the
       units position is unity. The allowed values are 0, 1, 10,
       and 11. Default = 01.

38 - CalComp DR/DZ axis length for phase plots in inches maximum
       value is 10.5 inches. Default value is 5.0 inches.

39   -   CalComp R axis length for phase plots in inches. Default
          value is 5.0 inches.

40   -   Charge averaging weight factor - W. The ray charge density
          deposition array RH( ) is multiplied by W before ray
          tracing and divided by (W+1) after ray tracing to conserve
          charge. The effect is to reduce large fluctuations in the
          beam at the beginning of a problem. Default is 0.

41   -   Number of Poisson-ray trace cycles over which charge
          averaging shall be done. Default is 0.

42   -   Longitudinal interval in centimeters for which the ray
          moments shall be calculated. Default is 0.

43   -   Plot of the axial external magnetic field will be
          superimposed on the ray CalComp plots if CONST(43)=1.

44   -   Specifies desired combination of charge density paper and
          CalComp plots. These plots are generated for the Z values
          specified on the PLOT DENSITY data card or on the
          longitudinal intervals given by CONST(45). A CalComp plot
          is generated if the tens position is unity and a printer
          plot is generated if the units position is unity. The
          allowed combinations are 0, 1, 10, and 11. Default is 0.

45   -   Longitudinal interval in centimeters for which a charge
          density profile shall be plotted. Default value of 0 is
          interpreted to mean no plots are generated via this data.

46 - A calculation of the current enclosed by each ray is
       normally done on crossing a longitudinal mesh line.
       CONST(46) different from 0 specifies calculation of the
       enclosed current every CONST(46) Runga-Kutta integration
       steps.

47 - Specifies the number of equipotential lines to be plotted
       along with the beam trace CalComp plots. Default is 0.

48 - Equipotential line differential increment step in
       centimeters. Default value is half the radial mesh size.

49 - A print of the equipotential line, R,Z,U,E_r and E_z
       found every N^{th} step is made if CONST(49) $\geq$ 1, where
       N=CONST(49). Default is 0. CONST(49)=0 produces a summary
       list of the equipotential lines plotted. CONST(49)=-1
       turns off the summary list.

50 - Self magnetic field radial correction factor. Default is
       0. See self fields theory section for detailed discussion.

51 - Allowed error in finding equipotential line. Default is
       10^{-4}.

52 - Maximum number of iterative corrections allowed to find
       equipotential line. Default is 3.

53 - When the potentials from U( ) array are to be punched at
       some Z location every N^{th} radial mesh point, N=CONST(53).
       The punching is initiated by PUNCH POT and PUNCH Z( ) S
       data.

54 - The default value 0 specifies that ray tracking is not to
       be terminated when retrograde motion is detected. The
       retrograde ray is ignored. When CONST(54)=1 ray tracking
       s terminated on retrograde motion.

55 - The self magnetic field is inversely proportional to the
       ray radius. CONST(55) is he small radius field cut off
       such that $B\phi \propto 1/(R+CONST(55))$.

56 - The self magnetic field radial correction factor CONST(55)
       will only be used for $Z > CONST(56)$. Default value is 0.

57 - Three types of phase plots are available according to the
       value of CONST(57).

       1   10   100   10i   110   111

       R-R'   Y-Y'   X-X'   X-X'   X-X'   X-X'

                                  R-R'   Y Y'   Y-Y'

                                                    R-R

-   - Normally every ray is output during ray tracking every
       CONST( ) cm. This parameter specifies output of only
       every N-th ray. Use to reduce the volume of output.
       The default value is 1.

5   - Printout of the transverse force terms of the equations of
       motion is made along with the field output generated by
       CONST(19), CONST( ) and FIELD data when CONST(59)=1.
       These terms are:
       $-E_x/P_.$, $-E_y/P_.$, $-E_z/P_z$, $v \times B|_x$, $v \times B|_.$, $v \times B|_z$, P'_X, P'_Y, P'_Z

60 - SEGMENT transfer back space in cm. During segmentation of
       a problem, ray transfer and left edge boundary conditions
       will be set CONST(60) cm to the left the terminating right
       boundary of the preceding segment.

61 - The beam envelope of the preceding cycle is made on the ray
       trace CalComp plot as a dashed curve if CONST(61)=1.

62 - Charge scale factor applied to the charge array, RH( ) for
       longitudinal positions Z greater than the longitudinal
       retrograde location beyond which rays are not tracked. Its
       effect is to prevent inhibition of ray tracking past a
       point of a high charge density such as a virtual cathode.
       Typically, CONST(62) ≈ 0.9.

63 - Output of the beam emittance E in cm-mrad for the x-yp and
       y-yp planes along with the normalized emittance EβY is
       generated for any location where the beam moments are
       calculated. These locations are specified by the MOMENT
       and/or CONST(42) data. The normalized beam ellipse tilt,
       r_{12} is also printed.

64 - Minimum beam energy EMIN, normally 0. Any ray with kinetic
       energy in eV less than EMIN will be stopped. (VELD( )=-3)

65 - Maximum beam energy EMAX, normally 10^{100}. Any ray with
       kinetic energy in eV greater than EMAX will be stopped
       (VELD( )=-3).

66 - 0, plot all ray groups together. Otherwise, generate
       separate phase plots for each ray species type.

68   –   SEGMENT expansion per major cycle. The longitudinal Z( )
         values of the SEGMENT data will be incremented by
         DZ=CONST(68) cm per major iteration of the problem.

70   –   If equal to 1, will cause a plot of the coil cross sections
         used in finding the magnetic field map during COILGEN input
         to appear on CalComp beam plots.

71   –   If equal to 1, will cause CalComp plot of the beam envelope
         on the beam plots.

72   –   Turn on Child Law output.
             -1 default, no output, only current generated
              0 find Run Table at ZMAX on each iteration
              1 starting surface printed for each iteration.

73   –   0 output generated by CONST(19), (42), (45) relative to
         ZSTART ray origin.
         1 output generated by CONST(19), (42), (45) relative to
         ZMIN. Default is 1. The origin relation holds for calls
         to segment and density.

74   –   Over-relaxation parameter for Childs Law generation.
         Normally 0.5.

75   –   Normally 0. If 1, specifies that the vector potential
         APHI( ) from which the external magnetic fields are
         calculated is to be accumulated as the sum of all COILGEN
         data sets encountered. If CONST(75)=0, then APHI( ) is
         initialized to zero before each COILGEN data set is
         processed.

76   -   **Normally** 0.   Specifies radius at which equipotentials will
           be found for CalComp plotting if CONST(48)>0.   Gives Z
           value at which equipotentials will be found if CONST(48)<0.

77   -   Normally 0.   If nonzero, then a check of the charge array
           RH( ) for charge exceeding the maximum allowed before
           solving Poisson's equation will be performed.   If a point
           has too much charge, it will be frozen at the value
           specified by the value of CONST(77).   If CHARGE LIMIT data
           has been used, then CONST(77) is the number of radial mesh
           points over which excess charge will be deposited.

78   -   Normally 4.   Minimum radial mesh number for which excess
           charge will be inwardly deposited, see CHARGE LIMIT data.

80   -   CONST(80), (81), (82) and (83) are used in solving
           Poisson's equation and determining the optimal value of its
           over-relaxation factor.   The initial over-relaxation
           factor, ω, CONST(15) as set by the data deck is used for
           the first CONST(80) cycles, after which the optimal as
           determined by EBQ will be used.   The last known value of
           this optimal factor is stored by EBQ as CONST(83) and
           CONST(80) set to zero at the conclusion of a Poisson
           equation solution, so any subsequent pass will start with
           this value of ω.

81   -   Calculation of the optimal over-relaxation factor will be
           made every CONST(81) cycles.   The default value is 10.

82 — Maximum value ω_m of the optimal ω allowed. This is
       really determined as an offset ω_o as
$$\omega_o = 2.0 - \omega_m$$

83 — Last optimal Poisson over-relaxation factor ω is stored as
       CONST(83) for passage to tape dumps and subsequent
       problems.

84 —

85 — Constants 84 and 85 are used during Childs Law starting and
       have the default values of 0.5 and 2.0. During Childs law
       starting the current actually run is under-relaxed by
       CONST(74). The ratio of this under-relaxed current I_r to
       its value on the previous iteration is forced to lie within
       the range specified by CONST(84) and (85).

$$CONST(84) < \frac{I_{rN}}{I_{rN-1}} > CONST(85)$$

       A quantity called FACTOR is printed as output. Factor is
       the ratio of I_r to the Childs Law current.

type code is also negative. Each of these RAYGEN data cards is preceeding by a CONST *1 MASS CHARGE S data card specifying the species rest mass and charge.

The data is shown in Figure 4.1. The plots are shown in Figure 4.2. The plot of the 1C, 20, and 30th ray includes the beam envelope identified by the tri_ gular symbol. Figure 4.2A, was generated by data line 32. The plot of all rays, Figure 4.2B was generated by data line 31 or 33.

DEFINE NAME
STOP
CALL NAME

Data input to EBQ can be saved in a special array (max. 100

numbers) under a NAME which can be used to refer to this data (max. 10

names). The general structure would be

```
EBQ data input
-
-
-
-
DEFINE NAME1
.
.
.
STOP
-
-
-
CALL NAME1
.
-
CALL NAMEj
-
-
END
```


The data bracketed between the DEFINE and STOP entries is not processed,

but merely read and stored. The two commands CALL NAME 1 in the above

example actually cause execution of the data known as "NAME1."

DENSITY ON
DENSITY OFF
DENSITY TYPE SJ SR

    CURDEN( ) S
    RODENJ( ) S

DENSITY UNIFORM J

    Density input of initial current distribution and radius for each
ray at origin--i.e., cathode. This data input selects the Neil
procedure of charge deposition. SJ is a current scaling factor which
multiplies each CURDEN value. SR is a radial scaling factor multiplying
each RODENJ value. Type selects the input format for the CURDEN and
RODENJ arries.

    TYPE = ON   Turns on Neil-charge deposition if previously turned
                 off.

    TYPE = OFF   Turns off Neil-charge deposition, except with Childs
                 law start.

    TYPE = -1   Turns off Neil-charge deposition in problem using
                 Childs law start.

    TYPE = 1   The DENSITY card is followed by a list of CURDEN
                 values in amperes/cm^2 terminated by an "S". There
                 should be 1 value for each ray to be run in the
                 problem. This list is then followed by a list of the
                 radial values (in cm's) for each ray.

                         DENSITY 1

                         CURDEN ------- S

                         RODEN ------- S

TYPE = 2   Specifies input of CURDEN-RODENJ pairs of numbers,
            1 pair for each ray to be run. The pair input is
            terminated by an "S".

                        DENSITY 2

                J,R,----J,R,--------J,R,S

TYPE = 0   Turns off the Neil-charge procedure. Charge is then
            deposited according to a linear deposition based on
            individual ray distance of approach to mesh lines.

    Consider a beam of six rays originating from a cathode with
approximately uniform current density as shown in the sketch below.
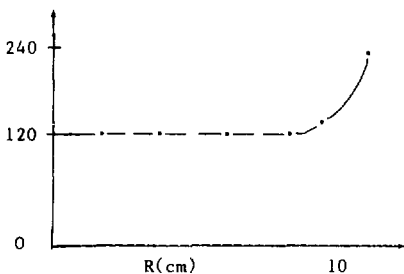
    The data input would be:

        DENSITY 1   120.

        1.   1.   1.   1.05   1.34   2.10   S

        1   3   5   7   9   11   S

EBQ can generate Neil-charge density data based on a uniform current density $J(amp/cm^2)$ by entering the single card:

DENSITY UNIFORM J

The Neil-charge array CURDEN( ) is set to J and the array RODENJ( ) is set to the radius of the rays in storage at this moment. The current carried by each ray is then set, based on the inter-ray separations for this uniform current density.

DENSITY UNIFORM

the value of J will be calculated from the total current for all rays.

DUMP IN
DUMP OUT

A problem may be continued for several reasons. One may wish to
iterate the orbit -- Poisson equation additional cycles to improve the
solution; or one may wish to propagate the trajectories further down the
beam line. Whatever the reason for continuing, it may be facilitated by
two types of dumps discussed in the data sections DUMP, RAYS 8, PUNCHB,
and CONST (32 and 33). When DUMP OUT is encountered in the data stream,
a binary file is written on tape 8.

The information dumped is 11 records in 1 file preserving the
following data:

```
NDUMP,NAME,NX
TITLE (8+3)
SIZE common block,NRAYS,ICOEF,CONST( )
MESH (NTOT)
U (NTOT)
RH (NTOT)
COEF(ICOEF)
POT(MAXPOT)
NCOILS,MAG( ),A( ),BOUNDARY( )
XORB,YORB,ZORB,PXORB,PYORB,CURNT,TORB,SORB(100)
SPECIAL THINGS( )
EOF
```

Where NDUMP is the value of CONST(32), NTOT is the number of points in
the boundary mesh, and ICOEF is the number of coupling coefficients
generated. This information can be read from tape 8 to re-initialize a
problem for continuation. The data would be, for example:

```
TITLE
CONST *32 1 S
DUMP IN
TRACK
END
```

Tape 8 is searched for a dump number CONST(32) written as NDUMP in the first record. The orbit arrays XORB( ), YORB( ) etc. transfer the initial conditions of the rays. If it is desired to continue the problem from some intermediate position, Z, the rays can be dumped to tape 8 by the PUNCHB Z data and read by the RAYS 8 data. See example at end of this report.

    DUMP CHARGE IN
    DUMP CHARGE OUT

These two data cards define a read or write of binary disc file 8 transferring the potential and charge arrays, U( ), RH( ) as two records. These records are identical with records 5 and 6 of the normal DUMP IN or DUMP OUT format.

    DUMP CHARGE IN SKIP

Causes the reading of a standard tape 8 file, skipping the first four records, transferring records 5 and 6 to the U( ) and RH( ) arrays and then skipping pass the file mark on that dump.

    DUMP IN X

This data, where X is the letteral X, will cause tape file 8 to be rewound and the first file dumped in regardless of the dump number found on that file.

    DUMP IN FILE N

This causes tape 8 to be rewound and files skipped to the Nth file, which will be read as the current problem.

ENDTRACK

When ray cycle convergence has been achieved a flag is set which prevents further cycles on tracking. All further data in this deck is skipped until an ENDTRACK card is encountered, when normal processing of the data resumes. If no ENDTRACK card is found, the data is skipped to the END card for this case, at which point the next case is read. Ray convergence is defined as achieved when the rms (root mean square) value of the radii at some specified Z value changes by less than a pre-assigned value on an iteration. The Z value is given by CONST(25) and has the default value of ZMAX, while the specified change is given by CONST(22) and has the default value of 0.01.
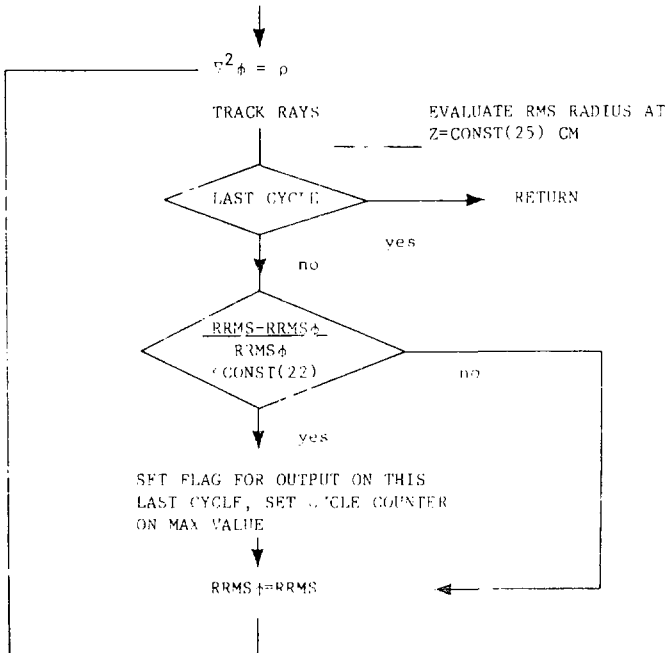
EQUIP V( ) S

EQUIP defines a list of up to 100 equipotentials values to be
plotted and/or printed according to the value of CONST(47) and
CONST(49). These equipotentials are plotted in addition to any
specified by the CONST(47) value. Let there be N values in the V( )
list. Let M=CONST(47), and let P_{MAX} and P_{MIN} be the maximum and
minimum potentials in the POT( ) array. The total number of
equipotentials is N+M with the values given by V( ) and $P_{MIN}+dP$,
$P_{MIN}+2dP...P_{MAX}$ where $dP=(PMAX-PMIN)$ M. Only those values and the
Z-axis (R=0) will be followed and plotted. The other values will be
ignored. Some other line, R=CONST(76), can be searched for
equipotentials if desired.

FIELD   Z( ) S

This card specifies field output for each ray at the Z array values during orbit tracking. A maximum of 20 such values may be given, although the actual limit of 20 refers to the sum of all orbit output Z values.

IDRAY   S

IDRAY   1   S

IDRAY   ID( )   S

When rays are generated by RAYGEN or read by RAYS data, an ID
number will be generated for each ray. This number is the ray number,
ID=1,2,3,......MAXRAY. Optionally, a user may assign a four digit ray
number to the rays of EBQ for identification purposes. These ID numbers
can be in the range of -999 $\leq$ ID $\leq$ 9999. This data is input via the
IDRAY 1D( ) S data and should preceed the RAYGEN or RAYS data.

Two special exceptions exist with the RAYS 5 and RAYS 6 input.
RAYS 5 input will have an additional input list if preceeded by an IDRAY
1 S data line, e.g.,

```
     IDRAY   1   S
     RAYS 5
     X( ) S
     Z( )
     Y( )
     E( )
     DX/DZ( )
     DY/DZ( )
     AMPS( )
     IDRAY( )
```

RAYS 6 input will assign the index N to IDRAY, i.e.,

```
     RAYS   6
     N   X   Z   E   DX/DZ   T   DY/DZ   Y
     .
     .
     .
```

will assign IDRAY(K) = N, where K = 1,2,....MAXRAY. When IDRAY S data
is read, with no parameter list, it turns off any special IDRAY options,
such as the read of an eighth array list during RAYS 5 input.

Currently, this option is only available on the cartesian version
of EBQ.

IGNOR ON

IGNOR OFF

IGNOR K( ) S

IGNOR S

Ignors rays K in calculation of moments and emittance (area).

INITIALIZE

INITIALIZE

      This initializes certain constants. Mainly used when running many

independent data cases.

      MAG=0   Turning off external magnetic fields
      MAXTRAJ=0   TRAJ OFF
      MEQUPLT=0   PLOT EQUIP OFF
      MPHZPLT=0   PLOT PHASE OFF
      MRAYPLT=0   PLOT RAY OFF
      NZSTOP=0   Turn off by reinitializing orbit integration
                     stop at Z values. Specified on FIELD, PUNCH,
                     OUTPUT and PLOT-PHASE cards.
      CONST( )=CONST( ) Initial default values
      NSCALEP=0   Cancel phase-plot scales
      NSCALEE=0   Cancel ray-plot scales
      NPOSOUT=0
      MAPON=1   Standard MAP OUTPUT TURNED ON
      SCLMAPS( )=1   Sets plot scales to unity

LAPLACE

LAPLACE SWITCH INITIALIZE

Causes solution of the electrostatic problem by solving Laplace's equation and printing a map of the potentials so obtained.

If the second parameter is SWITCH, then a call to subroutine LEDGE is made to switch potentials before solving Laplace's equation. The charge array (RH( )) is initialized to zero. If the third parameter is INITIALIZE, the potential array is set to CONST(30); otherwise it is continued from whatever values exist. The "Boundary" input initially sets RH( ) = 0 and U( ) = CONST(30). After LAPLACE, a map of the U( ) array is normally printed.

POISSON

POISSON SWITCH INITIALIZE LIMIT

Same as Laplace, except is solves POISSON'S equation by not initializing the charge array to zero. If the fourth parameter is the literal "LIMIT", the procedure described under CHARGE LIMIT will be applied before solving Poisson's equation. After POISSON, a map of the U( ) and RH( ) array is normally printed.

LEDGE N1 N2 NRSKIP ZLOC SWITCH IP1 IP2

The LEDGE control card allows certain specified potentials to be switched after the first cycle. Its use is necessary when strongly space charge depressed beams are considered. On the first cycle the vacuum fields would be used; on subsequent cycles, the vacuum fields plus space charge field are used, requiring modifications of non-conductive boundary conditions.

    LEDGE   N1   N2   0   0   SWITCH   IP1   IP2

will case the potential array, POT( ) defined on data input to be modified after the first RAY-POISSON cycle such that

    POT(I)   =   POT(N)

        I=IP1, IP2,   N = N1,N2

If NRSKIP or ZLOC are non-zero, then the potentials stored in POT(N), N=N1,N2 will be taken from the potential array U( ) as calculated in the previous cas: or cycle. ZLOC is the Z index, ZLOC=(Z-ZMIN)/DZMESH, while eve: NRSKIP points will be skipped--e.g., consider skipping every other value of potential at Z=ZLOC*DZMESH+ZMIN, then
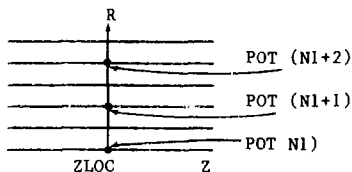
LEDGE N1 N2 2 ZLOC SWITCH IP1 IP2

If NP2 - NP1 + 1 = IP2 - IP2 + 1, then the potentials are swapped within
the POT( ) array. If NP2 - NP1 ≠ IP2 - IP2, then

$$POT(I) \ = \ POT(N), \quad I \ = \ IP1, \ IP2$$

and the values stored originally in POT(I) are lost.

<u>MAP TYPE</u>

<u>MAP SCALE TYPE SCL</u>

<u>MAP SECTION LSTART LEND LSKIP</u>

<u>MAP MESH LMAP1 LMAP2</u>

There are several arrays which can be mapped onto the output file.

These arrays are:

	TYPE	ARRAY	UNITS
MESH	1	MESH( ) Low order 30 bits	_____
POTENTIAL	2	U( )	Volts
CHARGE	3	RH( )	$Volts/cm^2$
COEF	4	COEF( )	_____
BMESH	5	MESH( ) High order 30 bits	_____
BR,BZ	11	_____	gauss
BX,BY	12	_____	gauss
VECPOT	13	AVEC( )	gauss-cm
ER,EZ	14	_____	Volts/cm
POTENTIAL	15	U( )	Volts
_____	OFF	Suppresses automatically generated maps.	

In order to generate a MAP of the mesh and potentials of the

problem

<div align="center">MAP 1 2</div>

In order to see the boundary deviations from the mesh intersections

(high order 30 bits of the MESH array) normalized to unity.

<div align="center">MAP 5</div>

To see the charge array used in solving Poisson's equation

<div align="center">MAP 3</div>

or to see this array multiplied by 10^6,

<div align="center">MAP SCALE 3 1.0E6</div>

The magnetic **external** fields are calculated from the vector potential
$A\phi(R,Z)$. A map of the B_r, B_z or A_ϕ value can be produced on a
grid which must be given following the map specification. Consider a
map of A_ϕ extending from R1 to R2 in steps of DR, and from Z1 to Z2 in
steps of DZ, the data would be

MAP 13

R1 R2 dR Z1 Z2 dZ

Similarly, a map of the electric fields E_R, E_Z or potential U can be
produced on a grid which must be given following the map specification.

MAP 14 15

R1 R2 dR Z1 Z2 dZ

Produces a map of E_R, E_Z (type 14) and a map of U (type 15).

MAP OFF

Will suppress the automatic MESH map after processing, the potential U
after calculations and the ρ, U after ray tracking.

MAP ON

turns back on the usual map options.

MAP SECTION LSTART LEND LSKIP

A maximum of five such entries can be made which cause all maps produced
by the MAP entries to be sectioned from LSTART to LEND in steps of
LSKIP--e.g., say a map is desired from L of 25 to 75 in steps 5 and 150
to 180 in steps of two. The data would be:

MAP SECTION 25 75 5

MAP SECTION 150 180 2

L is the Longitudinal Index, L=1+(Z-Zmin)/DZMESH.

A MAP SECTION data entry with no parameters restores the full map from 1
to LMAX1.

The extent of the boundary mesh map and electrode displacement
(beta) map can be specified by the

                    MAP MESH LMAP1 LMAP2

data.   The boundary mesh map would then extend radially from KMIN-KMAX
(RMIN-RMAX) for the longitudinal range of LMAP1-LMAP2.   The default
values are LMAP1=1, LMAP2=LMAX1.

MESH IR IZ IB IP IC BETA C1 C2 C3 C4 C5 C6

The mesh coupling coefficients for solving Poisson's equation and the various flags can be user specified to create a special point in the problem or fix up an error if one exists. The mesh point is defined by IR IZ as

$$I = (IZ-1)KMAX1+IR$$

where KMAX1 is the number of columns in the mesh. IB is the boundary type and IP the potential index. IC is the coefficient index and is normally printed by map in octal. BETA is the unit normalized mesh displacement, and C1-C6 are the star coefficients.

$$MESH(I) = BETA.AND.7777777770000000000B.OR.$$
$$LEFT(IB,21).OR.LEFT(IP,12).OR.IC$$

IB, IP, and IC can optionally be read as octal integers by following them with a B, such as 23B, etc. If only seven entries are used, then the coupling coefficients are not generated, only the MESH( ) type being re-defined. This situation occurs when the coupling coefficients have been calculated but the mesh type is to be changed. Should one desire to have the coupling coefficients calculated from the first seven entries, this can be accomplished by entering α as C1 and the Hollerith literal CALCULATE as C2.

MESH IR IZ IB IP IC β α CALCULATE

Representation of the boundary intersection by 30 bits yields results accurate to five significant figures, as shown in Table 1, page 26. Columns 2 and 3 show the bit value in base 10 and octal, while columns 4 and 5 show the effects of truncation of the word to the high order 30 bits.

MOMENT Z   ) S

Cal ulation of the moments of the ray distribution can be specified
at sever il Z values given by the parameter list Z( ). Z is in
centimet rs. A maximum of 20 such locations can be specified and count
along w n the Z value specified by FIELD Z( ), OUTPUT Z( ), PUNCH Z( ),
and DENS TY Z( ) value.

Mo nts can also be specified on a regular interval given by CONST
(42), su h that this calculation is performed every ΔZ=CONST(42) cm
along t beam path. If CONST(25) is not the default ZMAX value, then
the mom ts are additionally calculated at this location, also.

Th moment calculation generates one line of output giving the
iterati number, the average radius, the rms radius in cm, the average
P_r and e rms P_r value in eV/c.

OUTPUT Z( ) S

OUTPUT POLAR Z( ) S

Specifies a list of Z values for which the orbit output will be printed. A maximum of 20 such locations can be specified, including the locations given on FIELD, MOMENT, PUNCH and DENSITY cards. To turn off all special output, use the following

OUTPUT OFF

This will turn off phase plots, trajectory plots etc. and all output generated by OUTPUT, FIELD, MOMENT, PUNCH data.

If the second entry on the output card is the Hollerith literal "POISSON", then the other entries specify 4 mesh integer pairs for which the potential will be printed every CONST(18) Poisson cycles. The mesh coordinates are determined from the R,Z value as

$$IZ = (Z - ZMIN)/dZ + 1$$

$$IR = R/DR + 1$$

OUTPUT POISSON IR_1, IZ_1, IR_2 IZ_2 IR_3 IZ_3 IR_4 IZ_4

OUTPUT POLAR Z( ) S

Generates a polar ray output table at each Z value given in the Z( ) list along with the standard cartesian ray output table. The printed values are:

$$\lambda \quad \beta_r \quad \beta_\phi \quad \beta_z \quad P_r \quad P_\phi \quad P_{TOTAL} \quad \alpha \quad \psi$$

See the RAYS section of this report for a definition of the coordinate system.

PAGE

      Causes ejection of a page.

PLOT JDENSITY	Z( ) S
PLOT TRAJ	N( ) S
PLOT PHASE	Z( ) S
PLOT BOUNDARY	S
PLOT ENERGY	S
PLOT AMPS	S
PLOT PSI	S
PLOT EQUIP	V( ) S
PLOT SCALES TYPE	
PLOT DENSITY	Z( ) S

PLOT TRAJ N( ) S

PLOT TRAJ S

EBQ can produce several types of paper and/or CalComp plots. PLOT
TRAJ specifies a print of the R and Z values for rays if CONST(34) has a
unit value and produces a CalComp plot of those rays if CONST(34) has a
decade value. If CONST(34) has both a unit and a decade value,
(CONS 34)=11), then both options are selected. If CONST(34) is zero,
then n ther option is produced. The rays which are selected are given
by the N( ) array list. If ray 1, 2, 10, and 27 have been specified by
the PLOT RAJ card, then these rays will be plotted. A PLOT TRAJ S with
no parameter list will produce a plot for the last defined ray number
list.

Trajectory plots are produced at the end of a beam tracking when CONST(34)=10 or 11 and a PLOT TRAJ data entry preceded the TRACK command. Three different trajectory plot formats are possible. One plot will be made for each format used.

PLOT TRAJ S will produce a plot of all rays. PLOT TRAJ 1 2 10 ... S will produce a plot of the designated rays, and PLOT TRAJ I1-I2 I3-I4...S e.g., PLOT TRAJ 10-15 37-48 91-97 S will produce a plot of the sequential ray groups specified--rays I1 through I2, I3 through I4, etc. When using more than one of these formats, they should occur in the data stream in the following order:

```
PLOT TRAJ S
PLOT TRAJ I1 I2 ... S
PLOT TRAJ I1-I2 I3-I4 ... S
TRACK

PLOT PHASE Z( ) S
PLOT PHASE S
```

Phase plots are available at any z value specified by the Z( ) array list of the PLOT PHASE card. A maximum of 20 locations can be specified. Paper and/or CalComp plots are rendered according to the boolian true or false value of the units or decade position of CONST(37). The physical size of the CalComp plot is set by the FRAME values specified by CONST(38) and CONST(39) giving the CalComp plot dr/dZ and R axis length in inches. The special PLOT PHASE S with no Z( ) array parameters produces a phase plot at the end of a run for the values of the rays regardless of where their integration terminated. This is useful in making phase plots at constant energy.

Phase plots can be made in x-x', y-y', and or r-r' coordinate according to the value of CONST(57).

PLOT BOUNDARY

A PLOT BOUNDARY data card causes the boundary to be plotted showing the electrodes and coils used in generating an COILGEN field. The plot of the axial magnetic field is controlled by CONST(43) in the usual way.

PLOT ENERGY S

A plot of the beam energy versus R is produced by this entry calculated from the ray information at the end of the preceding TRACK command.

PLOT AMPS S

A plot of the beam current versus R is produced by this entry.

PLOT PSI S

A plot of the rotational beam angle ψ is generated by this entry calculated from the ray information at the end of the preceding TRACK command.

$$\psi = \text{ASIN} \quad \frac{P_y\cos\phi - P_x\sin\phi}{\left(P_x^2 + P_y^2 + P_z^2\right)^{1/2}} \quad ,$$

PLOT EQUIP V( ) S

A plot of up to 20 equipotential lines can be specified by the V( ) array list of the PLOT EQUIP data card. These equipotentials are drawn on the CalComp plot along with the electrode boundary, magnetic field $B_z(R=0)$ and other equipotential lines specified by the value of EQUIP V( ) and CONST(47). The physical size of the CalComp plot is set by the frame values specified by CONST(35) and CONST(36) giving the CalComp plot R and Z axis length in inches.

PLOT SCALES

The scales of any of the above described plots can be set by the PLOT SCALES data entries. The third parameter of such a card has one of eight values: 1) ' ' i.e., Blank, 2) TRAJ, 3) PHASE, 4) EQUIP, 5) AMPS, 6) PSI, 7) ENERGY, and 8) BFIELD. The blank entry zeros out (cancels) all previous scales specifications.

```
PLOT SCALES TRAJ                          $  MAX  1

PLOT SCALES PHASE R1 R2 PR1 PR2 Z         $  MAX 10

PLOT SCALES PHASE R1 R2                    $  SET JUST R SCALE

PLOT SCALES EQUIP                         $  MAX  1

PLOT SCALES AMPS R1 R2 I1 I2

PLOT SCALES PSI   R1 R2 Ψ1 Ψ2

PLOT SCALES ENERGY R1 R2 E1 E2

PLOT SCALES BFIELD BZMIN BZMAX

PLOT SCALES JDENSITY R1 R2 J1 J2
```

The **PLOT SCALES PHASE** data sets the scales for the phase plots at
the given Z value. If no Z entry is made, the plot scale data is used
as the default scale for all Z values not explicitly specified.

PLOT JDENSITY Z( ) S

A plot of the local current density is R is made at the Z values
specified in the Z( ) list. This current density J_k is calculated for
each ray, k as

$$J_k = \frac{i_k}{2\pi R_k}$$

The scales of this plot can be set with the PLOT SCALES JDENSITY data.

PLOT DENSITY Z( ) S

A plot of charge density versus radius can be specified at any Z
location by the PLOT DENSITY Z( ) S data. The plots are produced on
paper and/or CalComp according to the value of CONST(44). Additionally,
density plots can be produced on a regular interval specified by
CONST(45).

POLAR

    Polar coordinates is the default geometry of EBQ. In order to
re-establish a polar geometry after the introduction of cartesian
geometry, by an XYZ data card, the POLAR data card must be used. This
data is to preceed the boundary data:

        TITLE

        POLAR

        BOUNDARY

        .

        .

        .

        .

        ETC.

POT V( ) S   POT NULL

POT FIT   POT INITIALIZE Z R1 R2 Z1 Z2 A B C SCALE

POT SWITCH   POT INITIALIZE R R1 R2 Z1 Z2 A B C SCALE

POT STORE   POT MAX N

    Input of the potentials for the various electrodes defining the boundary of the problem is accomplished via the POT data. The potentials are in volts. Electrode 1 has POT(1), electrode 2 has POT(2), etc. The data might be:

       POT   1.24ES 0 0 200 .0345 S

or

       POT

       1.24ES 0 0 200 .0345

       S

    If we want to change a potential from some previous value, then input just the array index preceeded by an * and the value of the potential.

       POT *4 205. S

This would change the fourth potential value to 205 volts.

    POT SWITCH causes a switch of the potentials defined by a previous LEDGE input such that the potentials of index IP1 - IP2 are switched with the potentials of indexes N1 - N2, see the LEDGE data section for a description of this data.

Sometimes it is necessary to enter a graded Dirichlet boundary.
This can be greatly aided by fitting the potentials of these boundary
points to a few potentials given as data input.

```
POT FIT 2 I1 I2 V1 V2 . . . . . . . . . . . . . S
POT FIT 3 I1 I2 V1 V2  V3 . . . . . . . . . . S
```

FIT as second entry on the pot specifies that potentials will be
entered into the POT( ) array by a polynomial fit. The degree is
specified by the third entry as either 2 for a linear interpolation or 3
for a quadratic interpolation, followed by a number of groups consisting
of four numbers for linear fits and five numbers for quadratic fits.

The first two members of each group specifies the POT( ) array
indices into which the fitted values will be stored, while the last two
or three members of each group give the specified potentials to which
the fit will be made. The end of input must be indicated by entry of S.

```
POT STORE   R  I1  I2  ZL  R1  R2  DR
POT STORE   Z  I1  I2  RL  Z1  Z2  DZ
```

The POT STORE data card causes storage of the potential $P(R,Z)$ into
the POT( ) array, elements I1-I2. The potential values $P(R,Z)$ are
selected either on a radial line at a constant z value or a longitudinal
line at a constant radius according to the third parameter of the data
card being R or Z. The positional values of the line being given by the
next four parameters:

R1,R2 are the minimum and maximum values of the radius and dR the step, while Z1,Z2 are the minimum and maximum values of Z and dZ the step. Note, that

$$I2-I1+1 \ = \ \frac{R2-R1}{DR} + 1 \qquad or \qquad \frac{Z2-Z1}{DZ} + 1$$

If I_2 is zero, then three special storage procedures will be used according to the value of I_1:

$$I_1 \ = \ 0 \qquad \text{i.e., POT STORE } \frac{R}{Z} \ 0 \ \ 0 \ . \ . \ . \ . \ .$$

The values to be stored will be added to the end of the POT( ) array.

$$I_1 \ = \ I \qquad \text{i.e., POT STORE } \frac{R}{Z} \ 9 \ \ 0 \ \ . \ . \ . \ . \ .$$

The values to be stored will begin at the I-th location, in this example, I = 9.

$$I_1 \ = \ LAST \qquad \text{i.e., POT STORE } \frac{R}{Z} \ LAST \ \ 0 \ \ . \ . \ . \ . \ .$$

The values to be stored will start at the last value, note that this will replace the current last entry in the POT( ) array.

POT INITIALIZE R R1 R2 Z1 Z2 A B C D SCALE

POT INITIALIZE Z R1 R2 Z1 Z2 A B C D SCALE

This data will cause initialization of a part of the scalar potential
array U( ) by a power series expansion. The U( ) array may thus be so
initalized after its limits have been set by the BOUNDARY data and
before attempting a solution of either Laplace's or Poisson's equation.
The subregion Z1-Z2, R1-R2 will be set to U(R,Z) according to a
polynomial for the R,Z values lying on the mesh points within this
domain. The initialization region may extend out to the limits of the
U( ) array. The POT INITIALIZE Z data sets radial equipotential lines
along the longitudinal direction as

$$U(R,Z) = \left\{ A + B \left(\frac{Z-Z1}{Z2-Z1}\right) + C \left(\frac{Z-Z1}{Z2-Z1}\right)^2 + D \left(\frac{Z-Z1}{Z2-Z1}\right)^3 \right\} \text{SCALE}$$

where R,Z are on the mesh points of the U( ) array within the bounds

$$Z1 \leq Z \leq Z2 \quad , \quad R1 \leq R \leq R2$$

The POT INITIALIZE R data sets longitudinal equipotential lines along
the radial direction as

$$U(R,Z) = \left\{ A + B \left(\frac{R-R1}{R2-R1}\right) + C \left(\frac{R-R1}{R2-R1}\right)^2 + D \left(\frac{R-R1}{R2-R1}\right)^3 \right\} \text{SCALE}$$

POT NULL

The POT NULL data card sets the number of potentials   the pot
array to 1. This is useful when transferrring cases from tape dumps in
one run, where the number of potentials has become large.

POT MAX N

This data card sets MAXPOT=N where MAXPOT is the number of entries
in the POT( ) array. This is useful when establishing boundary values
from several different problems. Consider input of rays at some Z value
from some ; evious problem. The data might be

```
READ RAYS AND STORE POTENTIALS
DUMP IN FILE 3
POT STORE R ⁵¹ 83 3 0 15 .5              $ SET POT(51-83)
PUNCHB 3 S                               $
TRACK BEAM                               $ STORE RAYS AT Z=3 CM
POT MAX 50                               $ LIMIT POT INPUT TO 50 ENTRIES
DUMP IN FILE 2                           $ POT(1-50), DOESN'T OVERWRITE
                                                    POT(51-83)
BOUNDARY MODIFY
        INCREMENT 51 81
        I 0 3 81 15 3 S                  $ SET GRADED DIRICHLET BOUNDARY
                                           AT Z=3
  .
  .
  .
  .
  .
RAYS S
  .

  .
  .
  .
```

PRINT POT   PRINT R FIELDS ZL R1 R2 DR

PRINT FIELDS R ZMIN ZMAX DZ   PRINT Z FIELDS RL Z1 Z2 DZ

PRINT RAYS   PRINT BOUNDARY

PRINT CONST

PRINT DENSITY

The PRINT data entry causes the type of output specified by the second parameter to be generated. The PRINT POT card prints a list of the POT array. This is useful when some of the array has been generated by the code or when the LEDGE data is acted upon by the TRACK option.

PRINT FIELDS generates a table of field values by calling the FIELD routine at R, with ZMIN $\leq$ Z $\leq$ ZMAX in steps of dZ. Output consists of Br, Bz, AVEC, Er, Ez and U.

PRINT Z FIELDS produces the same effect of PRINT FIELDS while PRINT R FIELDS generates a table of field values at Z=ZL with R1 $\leq$ R $\leq$ R2 in steps of DR.

PRINT RAYS generates a table of the initial ray coordinates x, y, z, P_x, P_y, P_z, AMPS, R, E, α and ψ.

PRINT CONST output the values of the parameters stored in the CONST( ) array.

PRINT DENSITY outputs the radius and current density arrays used in the Neil charge deposition procedure.

PUNCH Z( ) S   MAX 20

Punch the ray values at the specified Z locations onto output file 4 in standard format--i.e., for each ray, write

X Y Z PX PY PZ CURRENT

in Format (7E11.4). The units are cm, eV/c and amperes. The potentials along the R axis are punched to tape 4 along with the rays at the Z values specified by the PUNCH Z( ) S data, taking every CONST(53) value. The format is (5E15.7). If the Neil charge deposition was being used, then the initializing current density and radial profile is also punched.

PUNCHB Z( ) S

Binary dump of ray output on tape 9 for restart. This dump is read by "RAYS 8 NDUM" input. The dump number is set by CONST(33) and is incremented by 1 before the writing of each dump. This writes three records in one file. Records 1 and 2 are for ID and record 3 is the binary orbit data.

PUNCH END S or PUNCHB END S

Writes either the tape 4 or tape 9 dump with the rays data for the ending point of the trajectory calculations. This option should be used if the rays have stopped at different Z values, as on a curved STOP boundary.

PUNCH POT

PUNCH POT NRSKIP ZLOC

Writes to tape 4 the POT( ) array. I· NRSKIP and ZLOC are provided, then the potentials written to ta,e 4 are taken from the U( ) array along the Z=ZLOC line taking every NRSЛIP point.

RAYGEN TYPE N IB IA E EM RB Z F RP Φ Ψ IRANDM S

RAYGEN data specifies generation of N rays in a random, two dimen-
sional phase space with space charge depression of the beam energy. The
beam emittance EM is tilted by RP specifying either the slope of the
envelope of the r_{12} beam ellipse matrix correlation. The parameters
are:

TYPE =	1	RP = r_{12}	beam ellipse matrix correlation
=	2	RP =	Slope of the beam envelope, see below. The units are normally radians, but can be scaled by CONST(13).
=	−1, or −2		Same as above, except rays generated are added to the rays already existing.
=	3		Child's Law start with equal dR spacing
=	4		Child's Law start with rays of constant current
=	±5		Brillon flow generation beam current and ψ will be proportional to R.
=	±11 or ±12		Calls for generation of a Kapchinsky-Vladimirsky (K·V) distribution on a four dimensional shell, while type = ±1 or 2 calls for a generation in a four dimensional volume.

N = Number of rays to be generated

IB = Total beam current, Amperes, if zero, these rays will not be

      space charge depressed.

IA = Total current carried by the ray generated by this RAYGEN

E = Total beam energy, kinetic energy electron volts

EM = Beam emittance, cm-radians in units of CONST(13)

RB = Beam radius, cm

Z = Initial longitudinal coordinate, cm

F = Focal tilt produced by preceding focusing elements. Units

      of CONST(13), normally radians

RP = Transverse beam, see below

ϕ = Rotational angle of beam, polar angle in units of CONST(13),

      normally radians

ψ = Transverse angle ATAN (Rdϕ/dz) in units of CONST(13),

      normally radians

IRANDM = 0   Use next random number generator seed

        = 1   Use previously stored random number generator seed

        = 2   Read in random number generator seed. The seed is

               read as a separate data card or cards following the

               "S" of the RAYGEN card. The 'seeds' are five 16 digit

               integers and the random number generator number,

               followed by an "S" -- e.g.,

                    RAYGEN .... 2 S
                    1048015011   2236846573   4216793093
                    7792106907   9630191977
                    1 S

= M Ray generation is on a regular grid (not random) of M r'

values for each of J=N/M radial sections as shown in the

case of N=24 and M=6.
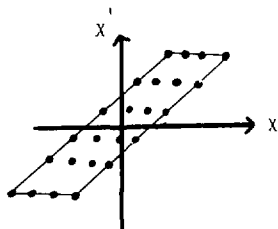
The beam phase can be tilted from two distinct sources: a) angular spread in the beam, and b) focusing action of preceding elements. Consider a beam of some angular divergence, X_M, passing through a drift distance S from a waist as shown below:

$$X_0 = X_m \sqrt{1 - r_{12}^2}$$
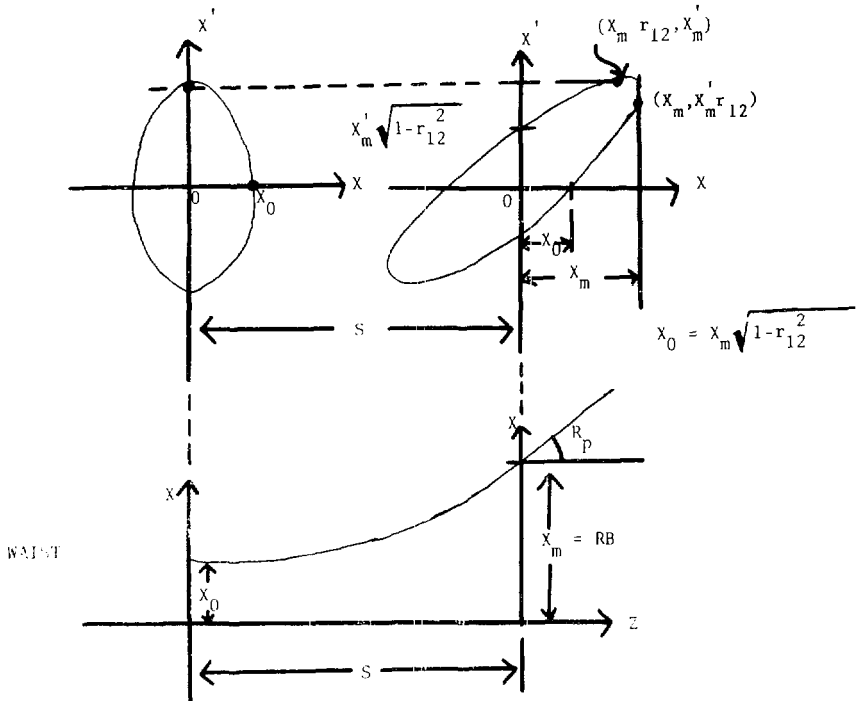
The angular divergence of the beam envelope is R_p. The value of R after the drift has increased from the angular divergence of the beam $X_o' = X_m'$. If the beam has no initial angular divergence, so EM=0, but has passed through a focusing region, the phase space will be tilted as shown below:

Focusing Region
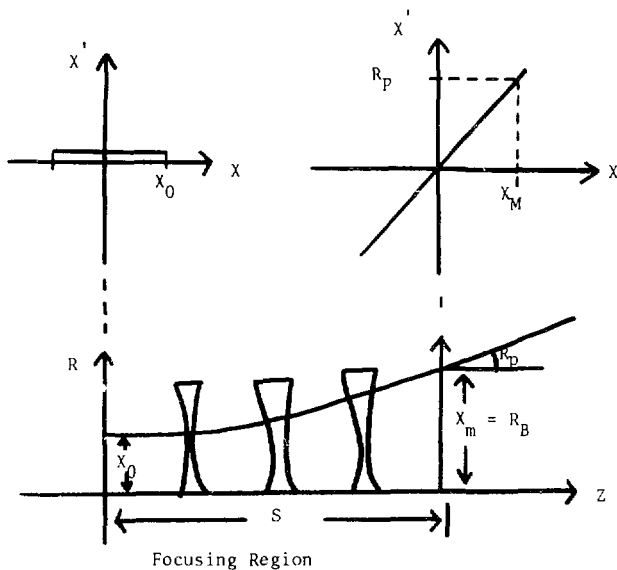
Consider generating a random phase space for a distribution tilted as shown. We know R_1, R_B, R_m'
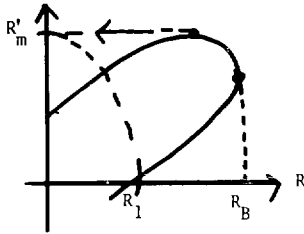
from a graph of the data. The emittance can be found by considering a drift backwards to where the ellipse is upright as shown. Then the input data values for r_{12}, E_m are found for type 1 data as

$$E_m = R_m' R_1$$

$$R_p = r_{12} = \left[1 - \left(\frac{R_1}{R_B}\right)^2\right]^{1/2}$$

where the beam emittance area is πE_m.

ϕ and ψ give the transverse beam rotations out of the plane, such that ψ is an additional transverse angle, added to Y' as

$$P_y = P \sin(\psi + Y')$$

$$P_x = P \cos(\psi + Y') \sin(x')$$

$$P_z = P \cos(\psi + Y') \cos(x')$$

and ϕ specifies a rotation in the x-y plane so

$$P_x = P_x \cos\phi - P_y \sin\phi$$

$$P_y = P_x \sin\phi + P_y \cos\phi$$

The energy of the rays generated by RAYGEN will be spaced charge depressed if the total beam current IB is non-zero. The current run by these rays is specified separately as IA. The magnitude of this energy depression is

$$E = E_{max} - D_{eo}\left[1 - \left(\frac{R}{R_B}\right)^2\right]^{1/2}$$

where D_{eo} is

$$D_{eo} = 25 \; I_B / (\epsilon_o \pi \; V)$$

R, V the ray radius and velocity in cm and cm/sec. When generating a beam close to some electrode, such as a grid, the space charge depression of the beam energy should be turned off.

CHILD'S LAW

        RAYGEN TYPE NRAYS KATHODE ZSTART VSTART MASK JLIM SMAX

        The ray generator may be used to start electrons by application of
Child's Law. The parameter list has a different interpretation than the
other types of RAYGEN data.

TYPE = 3   Rays generated with equal ΔR spacing
     = 4   Rays generated with ΔR spacing yielding approximately equal
            current for all rays.
NRAYS   Number of rays to be generated. Must be greater than four
            and $\leq$ 100.
KATHODE   Index pointer for potential of electrode from which emission
            will be allowed.
ZSTART   Starting conditions are determined from a surface starting
            at $(R,Z) = (0,ZSTART)$
VSTART   Potential V used in Child's Law on first iteration, if
            known. If VSTART is negative, it speficies the total beam
            current to be used on the first Child's law iteration.
MASK   Index pointer for potential of electrode allowed to mask
            electron emission.
JLIM   Maximum current density allowed, $amps/cm^2$. If Child's Law
            exceeds the value, JLIM will be used. This simulates a
            temperature limited cathode.

SMAX   Maximum length of starting surface cm. useful in problems

                involving a sharp corner at the end of the emission

                surface. If zero, SMAX is not used. If SMAX > 0 stop if

                surface length is >SMAX. If SMAX < 0 stop if cathode length

                > SMAX .

    The Child's Law start finds the current density J (amperes/cm^2)

for electron emission from the cathode electrode from the potential and

distance between the starting surface and cathode.

$$ J = \frac{4}{9} \varepsilon \sqrt{\frac{2e}{m}} \; \frac{(\theta_o - \theta)^{3/2}}{s^2} $$

    The value of the dielectric constant ε, charge e, and mass are

specified in the EBQ array CONST as the 8, 2 and 1 elements in units of

Farads/meter, relative charge and electron volts. θ_o is the potential

of the cathode, θ the potential of the starting surface, and s the

distance between the starting surface and the cathode. The emitting

surface is given by specifying its potential index, KATHODE.

    The current carried by an electron is found from

$$ I = 2\pi \, r \, dr \, J $$

where r is the radius of the electron and dr is the radial extension of

this ray. That is, half the difference of the radii of the rays on

either side. The starting energy of the electron is found as

$$E \; \approx \; e \, (\theta_o - \theta)$$

and the associated total momentum P is

$$P^2 C^2 \; \approx \; E^2 - E_o^2$$

P is resolved into three Cartesian components from the angles of radial and azimuthal slope, α and ψ.

The radial slope is determined from the assumption that the electrons are emitted normal to the starting surface. This surface is found by starting on the axis (R=0) at the given z value ZSTART and following the equipotential until:

1) Normal projection from this equipotential hits an electrode of potential index other than KATHODE or MASK,

or,

2) Leaves the boundary of the problem

or,

3) Requires more than 250 points.

If the surface is found, then from its known shape, the normal slope can be determined as

$\alpha = -\text{ATAN}(dZ/dR)$

The azimuthal slope of the trajectory is caused by a magnetic field threading the cathode surface. The magnitude of rotation imparted to the beam can be found from the conservation of angular momentum (Busch's theorem)

$$\dot{\theta} = \frac{e}{mcr^2} \left[r_c A(r_c) - rA(r) \right]$$

where r_c, $A(r_c)$ are the radius and vector potential of th ray at the cathode, and $r, A(r)$ are the radius and vector poten   l of this ray at the starting surface.

$$\psi = \text{ASIN } (^P\theta/P)$$

The three components of the momentum can now be fou

$$P_x = P \ (\cos\psi\sin\alpha\cos\theta - \sin\psi\sin\theta)$$

$$P_y = P \ (\cos\psi\sin\alpha\sin\theta + \sin\psi\cos\theta)$$

$$P_z = P \ \cos\psi\cos\alpha$$

The computational procedure is to

1) Follow the equipotential line originating at $r, z = 0, \text{ZSTART}$.
    This line defines the starting surface.

2) Calculate the radii of all the rays, r_R.

3) For each ray, interpolate from the starting surface and the ray
    radius, the values for the longitudinal distance, Z_k, cathode
    ray distance S_k, and ray slope $\tan\alpha_k$.

4) Find the local current density for each ray

$$J_k = cV^{3/2}/S_k^2$$

5) Find the current carried by each ray

$$i_k = 2\pi r_R dr_k J_k$$

6) Find the beam rotation induced by any magnetic field threading
    the cathode

$$P_\theta = \frac{e}{cr^2}(r_c A(r_c) - rA(r))$$

$$\psi_k = \text{ASIN}(P_\theta/P)$$

7) Deposit charge on the mesh between the starting surface and the
    cathode.

8) Find the momentum components of the ray, P_X, P_Y, P_Z from
    the angles and total momentum, $\alpha, \psi, 0$ and P.

9) Integrate each ray to the same mesh line.

10) Turn on the Neil charge deposition procedure.

RAYGEN 3 ON

RAYGEN 3 OFF

These options can be used with either the TYPE 3 or 4 Childs Law
start to turn on or off CHILDS law calculation of the beam.

<u>RAYS TYPE NSTART UNIFORM TOTALI</u>

<u>RAYS TYPE REPLACE   K( )   S</u>

<u>RAYS REMOVE K( )   S</u>

<u>RAYS TYPE NSTART</u>

Several different formats can be selected for specification of rays to be tracked by EBQ. The code converts each format into its standard form which is TYPE 1 consisting of specifying the three positional and three momentum components in Cartesian coordinates and the current. The units are cm, ev/c and amperes.

TYPE 1	X Y Z P_x P_y P_z I
TYPE 2	X Y Z E α ψ I
TYPE 3	R Z φ E α ψ I
TYPE 4	R Z φ P_r P_z P_ϕ I
TYPE 5	R( ) S
	Z( )
	φ( )
	F( )
	α( )
	ψ( )
	I( )
TYPE 6	N R Z E α I ψ φ
TYPE 7	Calls subroutine SPECIAL which must be user supplied.
TYPE 8	Reads Tape 9 Binary DUMP for ray coordinates.

The ray path length S is taken as zero if CONST(9)=0. If CONST(9) is non-zero, its value is used as Z for each ray and the Z value specified in the above data is taken as the path length S.

The input unit for angles θ, ψ, and α is determined by the value of CONST(13). If this value is 1 the angles are in radians, if its value is .01745 the input is in degrees, if its value is .001 the input is in milliradians, etc. That is, the value of CONST(13) multiplies the angles to convert to the standard radian value.

The interpretation and definition of the ray input parameters are:

    X, Y, Z ----------- Cartesian location in cm

    R, ϕ, Z ----------- Polar location in cm, radian, cm

    Pr, Pϕ, Pz -------- Polar momenta in ev/c

    Px, Py, Pz -------- Cartesian momenta in ev/c

    E ----------------- Kinetic energy in ev

    α ----------------- Radial angle ATAN(dR/dZ) in radians

    ψ ----------------- Transverse angle ATAN(Rdϕ/dZ) in radians

    I ----------------- Current in amperes
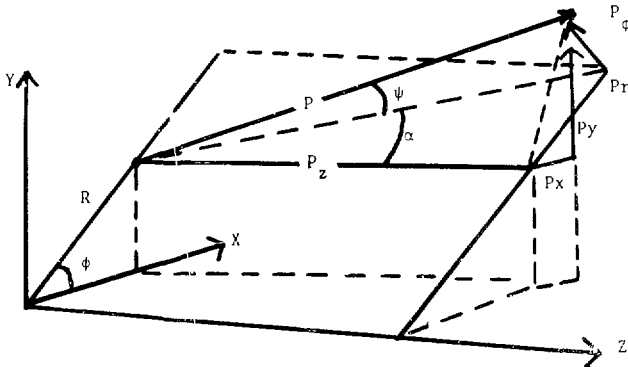
The total momentum P of a ray is determined from its energy. The required transformations between the various projections of momentum are:

$$Pz = P \cos \psi \cos \alpha$$

$$Pr = P \cos \psi \sin \alpha$$

$$P\phi = P \sin \psi$$

$$Px = Pr \cos\phi - P\phi \sin\phi$$

$$Py = Pr \sin\phi + P\phi \cos\phi$$

Following the "RAYS TYPE" card, up to 100 cards are read giving the coordinates of the rays. This data is read until either a blank card is encountered or a card saying END except for TYPE 5 and 8 which are somewhat special, for example:

```
RAYS      1
X Y Z PX PY PZ I
    .
    .
    .
    .
    .
END RAYS INPUT
```

TYPE 5 input consists of giving the like values of the coordinates for all rays together--i.e., all the R values until an S: is encountered. The number N of R values read determines the number of rays to be inputted; then N Z values will be read, followed by N φ values, etc. After the N I currents are read, an END cards should be used.

TYPE 8 input will cause a search of Tape 9 for a specified binary dump file which has previously been written. This dump file has the format structure described under the section "DUMP."

                    RAYS   8   NDUMP

Where NDUMP is the desired DUMP designation, some integer between 1 and ∞.

This data causes tape 9 to be rewound and searched for the designated dump number.

                    RAYS   8   X

This data, where X is the literal X, will cause tape 9 to be rewound and the first file read regardless of the dump number read on the file.

When a rectangular problem has been initiated by the inclusion of XYZ data, the RAYS 5 data will have the following interpretation

POLAR VALUE		XYZ VALUE
R( )	──────▶	X( )
Z( )		Z( )
∅( )	──────▶	Y( )
E( )		E( )
DR/DZ( )	──────▶	DX/DZ( )
Rd∅/DZ( )	──────▶	DY/DZ( )
AMPS( )		AMPS( )

More than one species of particles can be defined. This is useful in finding the space charge effects of multiple ion species in accelerating columns and heavy ion sources. The procedure for inputting more than one species is to read or generate each species after setting its mass and charge via the CONST(1) and CONST(2) input. These species will be tabulated separately in that each will have its own phase space plots, beam moments and emittance areas calculated. The species can be treated (printed and plotted) together if the value of CONST(66) is set to zero.

Example, consider input of protons, deuterons and charge five uraniums. The data would be

```
CONST *1  938E6  1  S           $ Protons mass and charge
RAYS 2  0                        $ INPUT, SAY 7 P+
   ⎰
   ⎱ X  Y  Z  E  α  ψ  J
END  PROTONS INPUT
CONST *1  1876E6  1  S           $ Deuterons mass and charge
RAYS 2  7                        $ INPUT, SAY 8 D+
   ⎰
   ⎱ X  Y  Z  E  α  ψ  I
END  D+
CONST *1  2.232E11  5  S         $ Uranium ion mass and charge
RAYS 2  15
      X  Y  Z  E  α  ψ  I
```

UNIFORM CURRENT DENSITY RAY INPUT

EBQ can calculate the current for rays from the inter-ray radial separation based on the assumption of a uniform current density by specifying the fourth parameter of the ray card as "UNIFOR." and given the total beam current as the fifth parameter. Any of the ray input types can be used with this procedure except type 8. This procedure will calculate the current for the rays so they do not have to be specified and load the Neil charge deposition arrays. Standard deposition will be used, however, unless the Neil procedure is turned on by another ON. Consider the input of six type 3 rays comprising a beam with uniform current density. The data would be.

```
RAYS 3 0 UNIFORM 12350
1  0  0  2.5E6  0  0  0  $ R Z PHI E ALPHA PSI I
5                         $ VALUES NOT GIVEN WII
10                        $ TAKEN FROM T E PRECEL
11                        $ DATA CARD.
12
12.7
END RAYS INPUT
```

If the Neil charge distribution is to be used during ray tracking, this data must be followed by DENSITY ON.

ADDING RAYS TO THOSE ALREADY SPECIFIED

Additional rays can always be added to those already in core store by giving a non-zero value to NSTART on the ray card. NSTART gives the ray index value after which the new rays will be stored.

Consider running 10 rays and then adding 7 additional rays to these
10 and running again with the 17 rays.

```
RAYS 3 0
     {
     {   DATA FOR 10 RAYS
     {
END INPUT OF RAYS
TRACK
RAYS 3 10

     {
     {   DATA FOR ADDITIONAL 7 RAYS
     {
END
TRACK  $ 17 RAYS
```

.

RAYS REMOVE K( ) S

Rays which have been previously defined or read, such as from a tape 8 dump or from the ray generator, can be removed by giving their number in the list K. To remove the second, third, and 19th rays, RAYS REMOVE 2 3 19 S.

RAYS NTYPE REPLACE K( ) S

Rays can be replaced by the values following the RAYS NTYPE REPLACE card. The rays to be replaced are specified by their rays number in the list K. If there are N entries in this list, then N ray cards must follow in the format specified by NTYPE.

RAYS	1	REPLACE	2	3	19	S
X_2	Y_2	Z_2	PX_2	PY_2	PZ_2	I_2
X_3	Y_3	Z_3	PX_3	PY_3	PZ_3	I_3
X_{19}	Y_{19}	Z_{19}	PX_{19}	PY_{19}	PZ_{19}	I_{19}

The RAYS data does not require an END terminator when using the REMOVE or REPLACE options.

REWIND KTAPE NSKIP

Entire EBQ problems are dumped to tape 8 by DUMP OUT data and rays
are dumped to tape 9 by PUNCHB data. The dumps are sequentially numbered
from the values specified by CONST(32) are CONST(33) for tape 8 and 9
respectively. The files are not rewound. These files can be rewound by
input of the REWIND data as

REWIND 8

or

REWIND 9

Reading the tapes by input of DUMP IN or RAYS 8 will automatically
rewind the file and search it for the specified dump number, input this
dump and leave the file at this position. Input of DUMP OUT or PUNCHB
will then commence writing of the file from this position.

Entry of a third parameter will cause that number of files to be
skipped after the rewind of the file. Consider positioning the ray dump
tape, tape 9 past the fourth file mark. The input would be

REWIND 9 4

RKSTEP   DZ   Z ..... S

The Runga-Kutta integration step is normally set in the tenth
position of the CONST( ) array and has a default value of DZMESH.
For some problems it may be desirable to have a finer integration step
to properly pass through, say, a sharp magnetic fringe field. Up to 20
integration steps can be defined by the RKSTEP data. Pairs of DZ, Z
entries give the step and ending Z value of this step in cm. The default
CONST(10) step size is used for Z values greater than the largest Z given
in a RKSTEP list. Consider the following scheme:
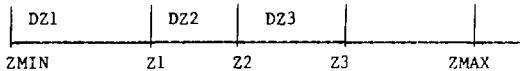
```
     | DZ1          | DZ2    | DZ3    |          |
     |_____|_____|_____|_____|_____
     ZMIN          Z1      Z2        Z3          ZMAX


     RKSTEP    DZ1    Z1    DZ2    Z2    DZ3    Z3    S
```

For $Z > Z3$, the step is the value given by CONST(10). The entry
of RKSTEP or RKSTEP S cancels the variable step, forcing use of the
CONST(10) value everywhere.

.

SEGMENT MINOR SEGERR   Z( )   S

SEGMENT MINOR SEGERR   S

SEGMENT S

Problems envolving long transport distances might have a tendency to self collapse with subsequent beam blow off when there is a large degree of cancellation between the beams self magnetic field and the beams own space charge forces. To allow solution for such problems, EBQ can segment the problem either at longitudinal locations specified by the user, or upon detection of charge density larger than some specified value. The ray orbits will be calculated only a partial (segmented) distance, with a number of minor Poisson-ray trace cycles, N, performed until either N exceeds MINOR or until the relative RMS beam radius at the segment transfer position is constant to within SEGERR.

$$\left| \left(\frac{R_{NEW} - R_{OLD}}{R_{NEW}} \right) \right| \quad < \quad SEGERR$$

The beam is then continued from this transfer point into the next segment. The segment transfer point is set by CONST(60) which specifies the distance (in cm) to the left of any segment boundary at which point the rays for the next interval will be initialized. This process of segmentation is continued until the end of the problem is reached, at which time we have completed one whole (major) Poisson-ray trace iteration.

The longitudinal locations at which the problem will be broken are given by the longitudinal z values in the data list Z( ).

Th:s list is terminated by a mandatory S and may have a maximum of
ten entries. The first Z value defines the starting longitudinal
position for segmentation. The rays will be tracked to this location
and segmentation initiated from there. A fictitious Neumann boundary
will be generated at segment ends and Poisson's equation solved only
over thi restricted range. The longitudinal boundary segment ends,
$Z(\ )$ will be increased at the end of each major iteration by the value
of CONST(68). Consider the problem below:
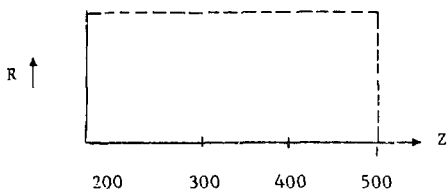
This segment data would be given as

SEGMENT   5   0.1   300   350   500   S

Forcing each major Poisson-ray race cycle to be calculated as
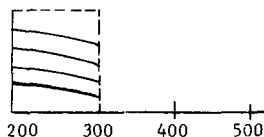
TRACK TO 300 CM
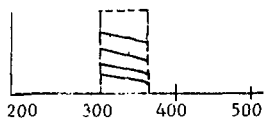
max of 5 cycles between
300 and 350 cm
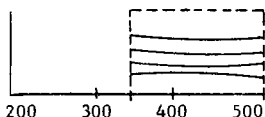
max of 5 cycles between
320 cm back spaced
point and 500 cm end.

With each minor segment iterated a maximum of 5 times (MINOR 5) or until
the relative change in the beams RMS radius is less than 10%.

SEGMENT -MINOR .....

The amount of output generated by SEGMENT can be reduced by using a
negative MINOR as flag for suppression of all output during the minor
cycles. The only output will be the final output and plots prior to the
continuation to the next major cycle. This output will be the same as
that generated by a TRACK BEAM at the end of each major cycle.

Segmentation can be turned off by input of either SEGMENT or
SEGMENT S with no parameter list.

It may be desirable to allow EBQ to decide on the locations to be
used for segmentation upon detection of the beam self collapse. This is
done by specifying the number of segment minor cycles and relative RMS
radial error with no longitudinal list, and specifying a non-zero
maximum charge density, CONST(77) in volts per cm^2. Any Z location
where the charge deposited exceeds this value will be treated as a
segment end. The input would be:

```
CONST  *77  75E3  S    $CHARGE DENSITY LIMIT
SEGMENT  5  .1  S      $ SET MINOR CYCLE AND ERROR LIMIT
TRACK
```

If the beam radius changes significantly along the problem, it may be necessary to specify the limiting charge density as a function of the longitudinal position via the CHARGE LIMIT ... input procedure. The value of CONST(77) must be non-zero, but the value is not used by the segmentation procedure. The limiting charge density being taken from the QLIMIT( ) list of the CHARGE LIMIT data. The data would be:

```
CONST *77-1  S              $NON ZERO VALUE
CHARGE LIMIT 1 1            $INPUT CHARGE LIMIT DENSITY
       QLIMIT ( ) S         $AS FUNCTION OF Z LOCATION
SEGMENT  5  .1  S           $SET CYCLE AND ERROR LIMITS
TRACK                       $TRACK BEAM
```

Any Z location where the average charge deposited exceeds the limiting value given by the QLIMIT( ) list will be treated as a segment end.

If the value of CONST(77) is negative, then the charge limits given by this constant or the CHARGE LIMIT data input will apply only to this segmentation procedure and will not invoke the search and subsequent limiting of the charge array before solution of Poisson's equation.

If minor = 0, then the segment data will set the limits of the Poisson equation solver so that the potential portion of the problem is run only over this limited range, $Z(1) \leq Z \leq Z(2)$. The rays can be limited by the value of CONST(26). Input of

$$SEGMENT \quad 0 \quad 0 \quad Z_1 \quad Z_2 \quad S$$

will cause Poisson's equation to be solved only over this Z interval $Z_1 \leq Z \leq Z_2$ with no fictitious Neuman boundaries. The orbits will be tracked out to the value specified by CONST(26).

SKIP

SKIP N

SKIP N MAX

SKIP causes skipping of data cases. This is useful when we have
several data cases in deck which are not all necessary to be run

```
SKIP 2
Case 1  title
.
.
.
.
.
END

Case 2 title
.
.
.
   RAYS
   END
.
.
.
END

Case 3 title
.
.
.
.
.
END
END
```

Only Case 3 will be run.

SKIP can be embedded in a data case, in this case, the data will be
skipped to the end of that case.

If we name multiple cases, and wish only to run a maximum of MAX case we use

SKIP N MAX

Say SKIP 5 2 will cause cases 1-5 to be skipped and case· 6 and 7 to be executed.

SLOPE $\theta(\ )$ S

The slope of the beam envelope can be forced to be within a given tolerance of the slope of the beam envelope of the preceding iteration so as to prevent a catostrophic beam collapse. The tolerance to be used on each iteration is specified by the slope data. The option of slope checking is turned on or off by setting CONST(60) to non-zero or zero.

Individual rays within the beam envelope are checked with a slope proportional to the ratio of their radius to the envelope radius. Should a ray be outside the range, its self magnetic field scale factor is adjusted so its slope is at the approximately positive or negatively deviated limit.

A maximum of fifteen values of θ can be specified for the first fifteen iterations. The units of θ is radians. The value of the self magnetic field factor used its printed at the end of each iteration for each ray at Z=ZMAX. Their normal value is 0.2.

STOP NAME

See DEFINE.

TIME

TIME OFF

TIME SWITCH   S

TIME SWITCH   CURNT( )   S

TIME   A   B   C   D   E   SCALE

     TIME without any parameters request a print of the CP time in seconds from the last TIME data entry without any parameters. When TIME has a parameter list, it refers to a time varying simulation. The time of a ray can be given on input with the RAYS data in place of the current position by preceeding the RAYS data with

<div align="center">TIME SWITCH S</div>

e.g., for RAYS 2 input, the data would be X Z Y E alpha, psi T, with T in seconds. The current of the rays will be set to the value of 10^{-16} amperes. If the current is to be given, then an array list should be included with the TIME SWITCH data, terminated by then mandatory S. When PUNCH Z( ) S data is executed, time will replace the current in the punched output written to tape 4. The current position will remain as the time until explicitly turned off by the

<div align="center">TIME OFF</div>

data. TIME OFF also will set the flag to kill the time function scaling of the electric potential and fields.

In order to simulate a time varying problem, such as deflection of a beam between two parallel plate electrodes, the local electric potential and fields U_{rz}, E_x, and E_z can be scaled by a power series function of the ray time

$$U_{rz} = U(R,Z) \; f(t)$$

$$E_x = dU/dx \; f(t)$$

$$E_z = dU/dz \; f(t)$$

This function is

$$f(T) = a + bT + cT^2 + dT^3 + eT^4$$

where the coefficients a, b, c, d, and e are given as

        TIME A B C D E SCALE

        a = A

        b = B*SCALE

        c = C*SCALE

        d = D*SCALE

        e = E*SCALE

Only A must be given, the default values for B, C, D, and E are zero and the default value for SCALE is 1. In order to turn off the time variation once set, use

        TIME OFF

Currently, this option is available only on the cartesian version of EBQ.

TITLE( )

Entry of one card, defining a title line for output with all CalComp plots.

TRACK

TRACK BEAM

TRACK CONTINUE ITERATION IOUTPUT

Causes tracking of rays. Output will be generated only on the last cycle. If output is desired on the first and last cycle, use the "continue" option where RH( ) charge array is not zeroed out -- i.e.,

        TRACK X 1
        TRACK CONTINUE 10

The calculation sequence is

        ITMAX = CONST(21) = DIN(3)
        EPS   = CONST(22)
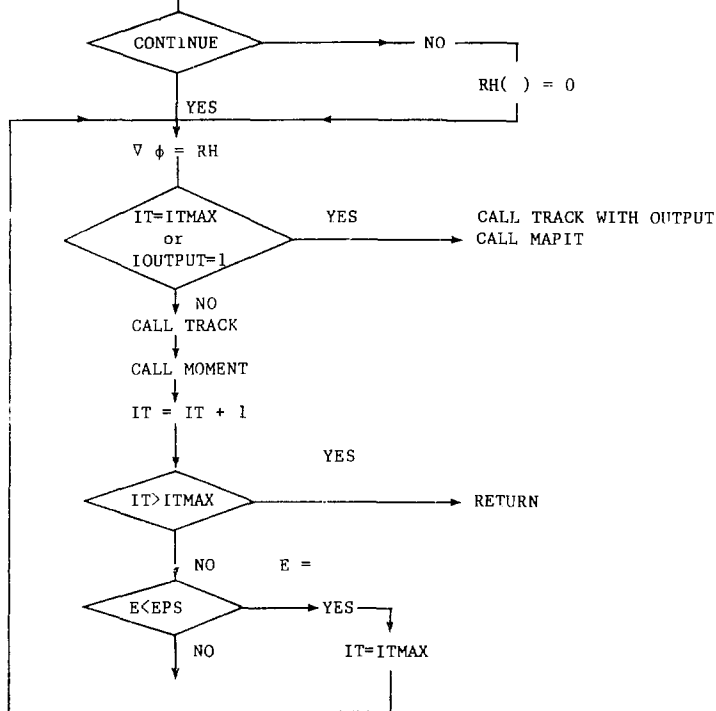
CONTINUE   -------------------- If = "CONTINUE," do not initialize
                                 ρ array

ITERATION --------------------   (CONST(21)) if ≠ 0, MAX number of
                                 ray trace cycles

IOUTPUT   -------------------- 0   Output only last cycle
                                 1   Output every cycle

    Sometimes one wishes to track the beam without disturbing the
charge or potential arrays. This can be accomplished by the

TRACK BEAM

data entry. The rays are tracked without updating the RH( ) array for
one cycle.

UNIT FACTOR SYMBOL

UNIT input changes the internal distance units of EBQ from CM to some new unit. FACTOR is the numerical value needed to multiply the new unit to convert it to cm. SYMBOL is a two character Hollorith name used to identify the internal unit in alpha-numeric output. The FACTOR values are not accumulative and are always relative to the original default value so that sequential UNIT cards do not interfere with each other. To change the internal units of EBQ to inches, the data input would be:

$$UNIT \quad 2.54 \quad IN$$

The unit conversion scales the constant array locations defining the distance units, locations 5, 8, 97, and 98.

```
CONST(5)  = CONST(5)/FACTOR  = Velocity of light
CONST(8)  = CONST(8)/FACTOR  = Distance/farad
CONST(97) = FACTOR
CONST(98) = CONST(98)/FACTOR = Ampere's law constant
```

The units conversion effect the values of distance, current density, velocity, magnetic vector potential and charge, as shown

	CM	UNIT
DISTANCE		
CURRENT DENSITY	$AMPS/CM^2$	$AMPS/UNIT^2$
VELOCITY	CM/SEC	$UNIT/SEC$
VECTOR POTENTIAL	$GAUSS-CM$	$GAUSS-UNIT$
CHARGE	$VOLTS/CM^2$	$VOLTS/UNIT^2$

WALL   R

WALL   R R_2 Z_1 Z_2

The WALL data entry instructs EBQ to include the self B_z magnetic field generated by the beam in addition to the self magnetic fields originating from the beams B_ϕ field component. This calculation requires specification of the beam pipe radius as this forms a boundary condition on the vector potential and return wall currents. Two models are included, a constant radius beam pipe of radius R,

<div align="center">WALL   R</div>

and a tapered beam pipe

<div align="center">WALL   R R_2 Z_1 Z_2</div>

where the wall radius r is R for z less than Z_1, R_2 for z greater than Z_2, and linearly interpolated for r in the range $Z_1 < z < Z_2$.

$$r = R + (R_2 - R) \left[\frac{z - z_1}{z_2 - z_1} \right]$$

XEQ

    This card causes a call to subroutine FLDINTL which scales any
magnetic fields, ray currents and other necessary bookkeeping. Such
calls are automatically produced by the appropriate data such as TRACK,
so the XEQ cards is only · eded when special data sequences are to be
used, such as PUNCH rays after CURRENT change. The use of XEQ card
never hurts, so when in doubt, use it.

                        CURRENT   .01

                        XEQ

                        PRINT RAYS

XYZ

    The XYZ data card request EBQ to generate a rectangular coordinate problem. This data must preceed the boundary data.

```
        TITLE
        XYZ
        BOUNDARY
            NX NZ XMIN XMAX ZMIN ZMAX
            ᵀ X Z .......... S
            .
            .
            .
        REST OF DATA ETC.
```

The polar R axis becomes the transverse cartesian X axis, the polar ϕ rotation axis becomes the cartesian Y axis; in fact, everywhere the polar EBQ uses an R or ϕ, you may consider the XYZ coordinates to be X,Y. The longitudinal position and independent variable of the orbit integration remains the Z coordinate. This restricts the application to basically paraxial problems.

TREATING RAYS WITH DIFFERENT Z STARTS

EBQ simultaneously tracks all rays together. Two exceptions are allowed, appropriate to a convex and concave starting surface. A convex starting surface will yield rays starting at different z values with the inner ray farther down stream and the outer rays farther up stream. EBQ will integrate the outer ray first, picking up the next ray in order, until all rays are together. The concave starting surface will yield rays starting at different Z values with the inner rays farther up stream than the outer rays. EBQ will integrate the inner ray; first picking up the next rays in order, until all rays are together.

Integration then proceeds in the usual way. When all rays are together, the Neil charge deposition procedure can be used if the input data requests this. Prior to all rays having reached the same z value, only standard charge deposition can be used with a switch to the Neil procedure when all rays have come to the same longitudinal position.

SELECTING MESH SIZES

There are three independent characteristic dimensions used in the simulation of a problem: 1) potential boundary mesh size, 2) external magnetic field vector potential mesh size, and 3) Runga-Kutta integration step size. Some care should be exercised in selecting the values of these dimensions.

The boundary mesh reflects the smallest resolution possible for the external electrodes and determines the frequency of storage for trajectory plotting and ordering the rays to find the enclosed current for calculation of the self-magnetic field and Gauss's law electric self-space charge field, if used. When rays frequently cross, a Z mesh size smaller than the crossing distance should be used. The self field (current enclosed) calculational frequency can be forced to occur every Nth Runga-Kutta integration step by making CONST(46)=N. Normally CONST(46)=0 so the enclosed current is calculated whenever the rays advance a longitudinal mesh spacing.

The vector potential mesh from which the external magnetic field is calculated should be chosen so its derivatives are sufficiently accurate (these derivatives are the magnetic field components). To facilitate this, the vector potential mesh can be sub-divided into ten regions as described under COILGEN. For example, when long solenoids are used, the fringing field regions typically will require a finer mesh than the far exterior or far interior regions in order to properly calculate B_z and B_r. The accuracy of these calculations can be readily verified by printing the fiels obtained from the data with the PRINT FIELDS option.

The Runga-Kutta integration step should be chosen so the rays do not make a large fraction of their wave length or experience a large change in external fields in one integration step. The user can check his step size by running at, say, half or twice his initial step to find if any significant change occurs in his results. If so, a different step should be used.

EBQ UNITS

EBQ code works in a standard set of units as shown in the table below. If the units are changed via the UNIT input, then the altered units are also indicated.

QUANTITY	STANDARD UNITS	ALTERED UNITS
DISTANCE	CM	UNIT
CURRENT	AMPERES	-----
ENERGY	eV	-----
MOMENTUM	eV/C	-----
VELOCITY	cm/sec	UNIT/sec
TIME	seconds	-----
MAGNETIC FIELD	Gauss	-----
VECTOR POTENTIAL	Gauss-cm	Gauss-UNIT
SCALAR POTENTIAL	Volts	-----
CHARGE	$Volts/cm^2$	$Volts/UNIT^2$
CURRENT DENSITY	$Amps/cm^2$	$Amps/UNIT^2$

The internal unit of length (centimeters) is defined by the value of four constants:

CONST(5)   Velocity of light 3×10^{10} cm/sec

CONST(8)   Meters/Farad 1.129×10^{11} meters/Farad

CONST(67) cm per unit length   1

CONST(68) Amperes law constant   0.2

The internal units can be changed by making the appropriate change in the value of these constants as is done by the UNIT FACTOR data input. FACTOR is the number of centimeters in the new unit, i.e., the number by which the new unit must be multiplied to yield its value in cm. The UNIT input adjusts the values of the CONST array so the internal units is the new unit.

The BOUNDARY input data is normally in cm but can be in any units desired with all R,Z values being multiplied by the seventh parameter of the second boundary card to convert them into the internal units being used. If the seventh parameter is missing, then no unit conversion is done so that the data must be in whatever the internal input unit is. The default value is cm.

The COILGEN input can be in any units desired with all R,Z values being multiplied by the eighth parameter of the COILGEN card to convert them into the standard internal units. If the eighth parameter is missing, no unit conversion of the R,Z input values is done. The default value is cm.

No provision is made to change the units of the output specifications or ray generator data, as only a few numbers are envolved. However, RAYS input can involve many cards, so that R,Z values can be converted from whatever their punched values are to the required internal units (default is cm) by the values of CONST(23) and CONST(24). The transformation is

$$R = R(\text{input}) \; CONST(24)$$

$$Z = Z(\text{input}) \; CONST(24) - CONST(23)$$

The Neil density input is normally in cm and amperes/cm^2. This data can be scaled on input by the third and fourth parameters of the DENSITY data card.

$$J = J(\text{input}) \; SJ$$
$$\rho = \rho(\text{input}) \; SR$$

If the value of SJ and or SR are missing, they are assumed to be unity.

All orbit (ray) input and output requests must be in the internal units. The boundary and coilgen data must be in the internal units or have been converted to the internal units by their individual unit scale factors. Consider the following three identical data cases, except for units.

```
FXR BALL (ALL INPUT AND OUTPUT IN CM)
  BOUNDARY
  41  26  0  10  0  6.25
  1  2.54  0  1  2.54  2.54     1  0.75  2.54
  1  0.75  2.75    1  0  2.75    1  0  0    1  2.54  0  S
  3  9  0    3  9  2.54    3  10  2.54  3  10  0    3  9  0  S
  4  0  6.25    4  10  6.25  S
  S
  POT  0  0    900E3    170E3  0  S
  COILGEN  2  0  0  10
     0  6.25  0.1  0.1
     100  100  S
     20  20  1    8  12  0.5  $ S10
     10  10  1    20  30  1   $ BOY
  RAYGEN  1  30  1.0E-6R2  100  .375  0.75 2.90 OR4  -3  S
  PLOT PHASE  6.25  S
  OUTPUT  3  4  6.25  S
  PLOT TRAJ S
  TRACK  X  1
  END
```

```
FXR BALL (UNITS INCHES, DATA IN CM)
UNIT   2.54 IN
BOUNDARY $ INPUT CM, CONVERTED TO INCHES BY 7-TH PARAMETER
41 26 0 10 0 6.25 0.3937
1 2.54 0    1 2.54 2.54    1 0.75 2.54
1 0.75 2.75    1 0 2.75    1 0 0    1 2.54 0 S
3 9 0    3 9 2.54    3 10 2.54    3 10 0    3 9 0 S
4 0 6.26    4 10 6.25   S
S
POT 0 0 900E3  1170E3 0 S
COILGEN 2 0 0 10 0 0 0.3937  $ CONVERTED IN BY 8-TH PARAM.
   0 6.25 0.1 0.1
   100 100 S
   20 20 1    8 12 0.5    $ S10
   10 10 1    20 30 1    $ BOY
RAYGEN 1 30 1.0E-6R2 100 .1476 .2953 1.142 OR4 -3 S
PLOT PHASE   2.4606 S
OUTPUT 1.1811 1.5748 2.4606 S
PLOT TRAJ S
TRACK X 1
END



FXR BALL (ALL INPUT IN INCHES)
UNIT 2.54 IN
BOUNDARY
41 26 0 3.937 0 2.4606
1 1 0    1 1 1    1 .2953 1
1 .2953 1.0827    1 0 1.0827    1 0 0 1 1 0 S
3 3.5433 0 3 3.5433 1.    3 3.937 1    3 3.937 0    3 3.5433 0 S
4 0 2.4606    4 3.937 2.4606 S
S
POT 0 0 900E3 1170E3 0 S
COILGEN 2 0 0 3.937
   0 2.4606 .03937 .03937
   100 100 S
   7.87 7.87    1 3.15 4.72 .1969
   3.94 3.84    1 7.87 11.81 .3937
RAYGEN 1 30 1.0E-6R2 100 .1476 .2953 1.142   OR4 -3 S
PLOT PHASE 2.4606 S
OUTPUT 1.1811 1.5748 2.4606 S
PLOT TRAJ S
TRACK X 1
END
```

RUNNING EBQ AT LBL

The EBQ program is available at LBL from GSS tape 12940. The
control cards necessary to execute the program are:

```
JOBNAME,5,200,170000.ACT NUMBER, AND USER NAME
FETCHGS,LGO=EBQ/V4,12940.
RFL,,140000.
LINK,X.
789 ← END OF RECORD
USER DATA FOR EBQ PROGRAM
.
.
.
.
.
END
END
6789 ← END OF FILE
```

EBQ currently uses 170K of small core and 140K (octal) of large core.

EXAMPLES OF THE USE OF THE EBQ CODE

In the following section there are ten examples illustrating different features of the EBQ code's input and output. The following table lists some of the features of the examples.

1) Simple electrostatic problem showing the data input and selected pages of the computer printout generated by EBQ.

2) Simple constant magnetic field problem with a matched orbit and discussion of the units of energy, momentum and field.

3) Design of an atomic beam spectrometer illustrating:

   a) boundary input in inches

   b) ray generation on a regular r-r' phase space grid

   c) boundary modification to add an electrode

   d) setting the Runga-Kutta integration step

   e) modifying the potential of an electrode to fine tune the device.

4) RTNS example illustrates the data for:

   a) running multiple ion species

   b) defining regions outside the boundary

   c) generating x-x' phase plots

5) Xe^{+1} source problem showing:

   a) ray orbit generation in a random phase space volume

   b) tracking beams with different current

   c) iterative solution to the space charge problem.

6) Continuation of problems

    a) starting from a previous dump

    b) graded Dirichlet incremental boundary input

    c) shifting the orbit origin

    d) dumping and reading ray data from case to case

    e) LEDGE data for setting the potential on problems left edge.

7) High current, relativistic electron beam problem

    a) Childs law start from cathode

    b) STOP PLOT data input

    c) stopping orbits at constant energy for phase plotting

    d) forcing a given phase space plot scale.

8) Use of EBQ code in long transport problems illustrating:

    a) Gauss's law approximation to the space charge

    b) magnetic field zoning and repeated fields

    c) RAYS 5 input with uniform current density generation.

9) Use of EBQ code in design of an X-ray tube illustrating:

    a) PLOT BOUNDARY data

    b) Global problem with expanded regions

    c) storing values from the potential map into the POT array

    d) removal of ray transferred from a tape dump.

10) Childs law electron source generating hollow beams via cathode masking.

Example 1

As a first example of the use of the EBQ code, consider the simulation of the motion of an electron initially at radius r_o at zero slope (dr/dz=0) in the electric field between two concentric infinitely long cylinde.s of radius a and b. The equation of motion is

$$m \ddot{r} = e E_r$$

The potential V between the cylinders define a scaling constant E_o. By Gauss's law, $E_r = E_o/r$ so that

$$V = \int_a^b E_r dr = E_o \ln(b/a)$$

Eliminating time as the independent variable, $d/dt = v \, d/dz$

$$r'' = \frac{e \, E_o}{m \, v^2} \quad \frac{1}{r}$$

integrating

$$\frac{dr}{dz} = \sqrt{\frac{2 \, E_o \, e}{m \, v^2}} \quad \sqrt{\ln(r/r_o)}$$

whose solution can be written in terms of the imaginary error function erfi(x) (see Jahnke, Emde, Losch, table of Higher Functions, 1960 page ...) with $-v^2 = \ln(r/r_o)$

$$z = \sqrt{\frac{m \, v^2}{2 \, e}} \int_{r_o}^{r} \sqrt{\ln \, r/r_o}$$

$$z = \sqrt{\frac{m\ v^2}{e\ E_0}} \quad 2\ r_0\ \text{erfi}(X)$$

where $X = \ln(b/r_0)$.

For example, a 5 keV electron starting at $r_0 = 20$ cm in a 100 volt potential between cylinders of 10 and 30 cm radius yields $E_0 = 100/\ln(3) = 91.0239$ and $X = 0.63676$ such that it hits the wall at

$$z = \sqrt{\frac{m\ v^2}{2\ e\ E_c}} \quad 2\ r_0\ \text{erfi}(0.6376) = 217.74\ \text{cm}$$

Figure 1.1 shows the problem geometry and Figure 1.2 gives the EBQ input data for this case.

The actual output from EBQ is shown for this example. Each data card read is immediately printed int the output stream. Each data type entry is preceeded by 45 dashes, this is useful when some abortive error is encountered before completion of the data input, in as much as the type of input last recognized by the input routines was identified by the dashes.

The boundary input shown in Figure 1.2 (lines 4-7, is printed into the output stream, page 1, and is followed by output listing the mesh size, number of boundary points, number of Poisson coefficients etc. Next follows a map of the type array, output pages 2-5, of which only page 2 is reproduced here. Note, the boundary mesh as described previously under BOUNDARY shows interior points as small integers and boundary points and electrodes as actual numbers concatenating the type-potential and coupling coefficient indexes. These are large ten digit octal numbers on the mesh map.

The output on page 6 shows the next output generated by read of the potentials, problem constants, and ray data input lines 8-19. The particle input is followed by a table giving the values of the particles cartesian position x, y, z (cm), cartesian momentum components p_x, p_y, p_z (eV/c), current (amperes), radius (cm), energy (eV), radial slope (ALPHA = dr/dz), and transverse slope (PSI = r d$\emptyset$/dz). Next follows a table of the average beam energy in eV, momentum in eV/c current in amperes, β (beta = v/c), γ (gamma = $1/\sqrt{1-\beta^2}$), and magnetic rigidity Bρ in Gauss-cm

The actual calculations begin with the TRACK command, line 23, Figure 1.2. The first output is a listing of the present values of the problem constants, the CONST( ) array. Poissons equation is then solved with output every ten iterations showing the sum of the residuals, maximum potential change of any mesh point, and the potentials of the four potential points selected for display by the OUTPUT POISSON data line 20, Figure 1.2. This problem converged in 53 iterations where the maximum change of any mesh point was 81 milivolts, page 7 of the output. Next follows output of the orbit data as requested by the data of the CONST( ) array and special output data preceeding the TRACK command.

Output page 8 shows the ray trajectory output generated during tracking. The normal output consists of a print of the initial ray coordinates at the start of the problem, here at z = 0, and the final ray coordinates at the end of the problem. Since all rays hit the outer cylinder electrode in this case, the last position reached by the rays

was 248 cm where the final conditions are printed. Output at three
special longitudinal z positions was generated by the data line 19.
Notice, that rays lost to the boundary are not printed after their
loss. The velocities printed at the end of the page for the rays are
zero as the rays hit an electrode.

Page 9 shows the beam trace paper plot of the format indicated
below:

```
                ——————— r(cm)——————————→
                ——————— r index——————————→
   z(cm)                    xx
      |                     xx
      |                      xx
      |                       xx
      |                        xx
      |                         xx
      |                          xx
      ▼                           yx
```
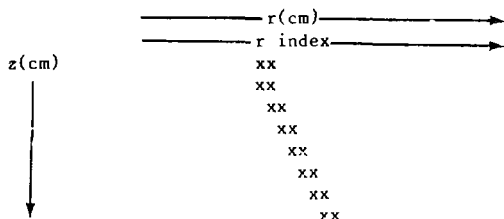
where the print of x simply occurs at each mesh point where the value of
the charge array RH( ) mesh point is non-zero. If the beam current is
zero, then every RH( ) mesh point is zero and there would be no beam
trace plot, i.e., it would be blank; for this reason, I suggest that the
user simulating a zero current beam uses actually a small infinitesimal
but finite beam current.

Since the value of CONST(4) had been set to have a unit value
component, a table of the R,Z points used in plotting the rays is
generated after the beam trace plot, page 9 - 10 of the output. Next
follows a map of the electric potential (Volts) and the charge RH( )
array (Volts/cm^2), page 11 - 18 of which only pages 11 and 15 are
reproduced here. The data case ends with a summary table giving the

number of calls and execution times of the main subroutines called in EBQ, page 19.

The CALCOMP plot of the boundary electrodes and the three rays traced in this problem is reproduced here in Figure I.3.
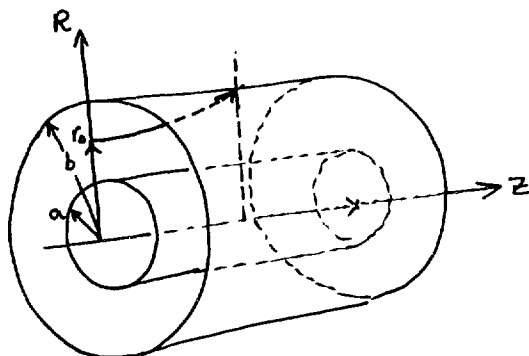
XBL 823-1941

Figure 1.1   Deflection of electron between two concentric cylinders.

```
ELECTRIC FIELD DEFLECTION BETWEEN TWO CONCENTRIC CYLINDERS 1
INITALIZE                                                   2
BOUNDARY                                                    3
   31 31 0 30 0 300                                         4
    1 10 0 1 10 300 S $ ----- INNER ELECTRODE              5
    2 30 0 2 30 300 S $ ----- OUTER ELECTRODE              6
    S                                                       7
POI 0 100 S                                                 8
CONST                                                       9
*34 11 $ ------ PRINT AND PLOT RAYS                        10
*63 0 $ ------ NO EMITTANCE CALCULATIONS                   11
   *71 0 $ DONT PLOT ENVELOP                               12
S                                                          13
RAYS 3 $ ----- R Z PHI E ALPHA PSI I                      14
 -14.81436 0 0 5.0E3 0 0 1.0E-6                            15
   19.91750 0 0 5.0E3 0 0 1.0E-6                           16
   25.14050 0 0 5.0E3 0 0 1.0E-6                           17
   END RAYS INPUT                                          18
OUTPUT 166.281 218.250 236.809 S                          19
OUTPUT POISSON 20 1 20 11 20 21 20 31 $ ---- IR IZ PAIRS  20
PLOT TRAJ S                                                22
TRACK                                                      23

END                                                       24
```

Figure 1.2   EBQ code data input for two concentric cylinder example.
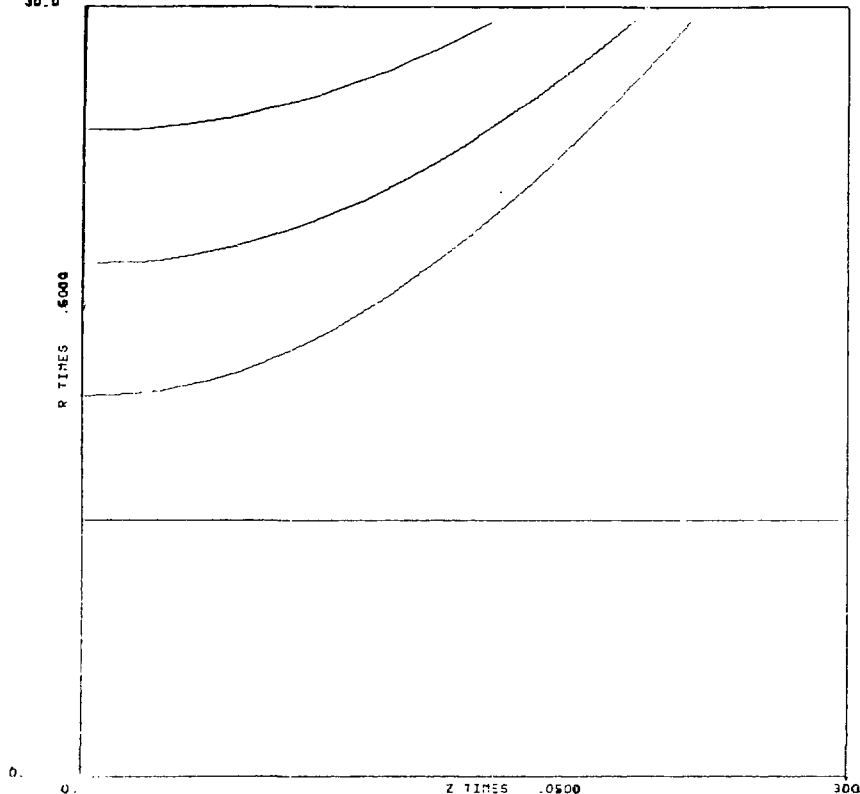
Figure 1.3   EBQ CALCOMP plot showing inner and outer electrode cylinders and the three deflected electron trajectories. The axial scale factor "z TIMES .05" and the radial scale factor "R TIMES .5" are the axis multipliers. This plot is half scale radially and 1/20 scale longitudinally. These factors are determined by RMAX, ZMAX-ZMIN and CONST(35) and CONST(36).

CASE NUMBER   1-------------------------------UCIDX00   7 24 OCT 78 13.09.15 ------------------------------CASE NUMBER   1
--- $ --
---   ELECTRIC FIELD DEFLECTION BETWEEN TWO CONCENTRIC CYLINDERS
---   TITLE X1UCID
---   INITALIZE
---   BOUNDARY
                                                   31 31 0 30 0 300
                                                    1 10 0 1 10 300 S $ ----- INNER ELECTRODE
                                                    2 30 0 2 30 300 S $ ----- OUTER ELECTRODE
                                                    S
POISSON OUTPUT INDICES ARE 218 435 652 869

BOUNDARY MESH SIZE= 1.000000 X 10.000000

THE NUMBER OF MESH POINTS IS= 961
        BOUNDARY POINTS= 180
            COEFICENTS= 1080
        BOUNDARY SEGMENTS=   3

ELECTRIC FIELD DEFLECTION BETWEEN TWO CONCENTRIC CYLINDERS

		1	2	3	4	5	6	7	8	9	10
		0.0000	10.0000	20.0000	30.0000	40.0000	50.0000	60.0000	70.0000	80.0000	90.0000
		31	52	93	124	15	136	217	248	279	310
31	30.0000	0070023135	0010020136	0010020137	0010020140	0010020141	0010020142	0010020143	0010020144	0010020145	0010020146
30	29.0000	0020000227	30	30	30	30	30	30	30	30	30
29	28.0000	0020000226	29	29	29	29	29	29	29	29	29
28	27.0000	0020000225	28	28	28	28	28	28	28	28	28
27	26.0000	0020000224	27	27	27	27	27	27	27	27	27
26	25.0000	0020000223	26	26	26	26	26	26	26	26	26
25	24.0000	0020000222	25	25	25	25	25	25	25	25	25
24	23.0000	0020000221	24	24	24	24	24	24	24	24	24
23	22.0000	0020000220	23	23	23	23	23	23	23	23	23
22	21.0000	0020000217	22	22	22	22	22	22	22	22	22
21	20.0000	0020000216	21	21	21	21	21	21	21	21	21
20	19.0000	0020000215	20	20	20	20	20	20	20	20	20
19	18.0000	0020000214	19	19	19	19	19	19	19	19	19
18	17.0000	0020000213	18	18	18	18	18	18	18	18	18
17	16.0000	0020000212	17	17	17	17	17	17	17	17	17
16	15.0000	0020000211	16	16	16	16	16	16	16	16	16
15	14.0000	0020000210	15	15	15	15	15	15	15	15	15
14	13.0000	0020000207	14	14	14	14	14	14	14	14	14
13	12.0000	0020000206	13	13	13	13	13	13	13	13	13
12	11.0000	0020000205	12	12	12	12	12	12	12	12	12
11	10.0000	0010010204	0010010230	0010010231	0010010232	0010010233	0010010234	0010010235	0010010236	0010010237	0010010240
10	9.0000	0020000203	10	10	10	10	10	10	10	10	10
9	8.0000	0020000202	9	9	9	9	9	9	9	9	9
8	7.0000	0020000201	8	8	8	8	8	8	8	8	8
7	6.0000	0020000200	7	7	7	7	7	7	7	7	7
6	5.0000	0020000177	6	6	6	6	6	6	6	6	6
5	4.0000	0020000176	5	5	5	5	5	5	5	5	5
4	3.0000	0020000175	4	4	4	4	4	4	4	4	4
3	2.0000	0020000174	3	3	3	3	3	3	3	3	3
2	1.0000	0020000173	2	2	2	2	2	2	2	2	2
1	0.0000	0100000040	0030000041	0030000042	0030000043	0030000044	0030000045	0030000046	0030000047	0030000050	0030000051

-218-

```
------------------------------------------   POT 0 100 S
------------------------------------------   CONST
                                             *34 11 $ ------ PRINT AND PLOT RAYS
                                             *35 -15. -15.
                                             *63 0 $ ------ NO EMITTANCE CALCULATIONS
                                              *71 0 $ OONT PLOT ENVELOP
                                             S
------------------------------------------   RAYS 3 $ ----- P Z PHI E ALPHA PSI I
                                              14.81436 C 0 5.0E3 0 0 1.0E-6
                                              19.91750 0 0 5.0E3 0 0 1.0F-6
                                              25.14850 0 0 5.0E3 0 0 1.0E-6
                                              ENO RAYS INPUT
```

N	X	Y	Z	PX	PY	PZ	AMPS	R	E	ALPHA	PSI
1	14.81436	0.00000	0.0000	0.	0.	7.16589F+04	1.00000E-06	14.81436	5.00000E+03	0.000000	0.000000
2	19.91750	0.00300	0.0000	0.	0.	7.16589E+04	1.00000E-06	19.91750	5.00000E+03	0.000000	0.000000
3	25.14950	0.00000	0.0000	0.	0.	7.16589E+04	1.00000E-06	25.14850	5.00000E+03	0.000000	0.000000

```
RAY GROUP 1 RAYS 1 THROUGH  3 --------- MASS=  5.110000E+05 EV   CHARGE -1  --------- ITERATION 0
AVE BEAM ENERGY  = 5.000000E+03              AVE GAMMA    = 1.009785E+00
TOTAL BEAM CURRENT= 3.000000E-06             AVE BETA     = 1.388739E-01
AVE BEAM MOMENTUM = 7.165891E+04             BRHO(GAUSS-CM)= 2.390287E+02
                                          OUTPUT 166.281 218.250 238.809 S
                                          OUTPUT POISSON 20 1 20 11 20 21 20 31 $ ---- IR IZ PAIRS
POISSON OUTPUT INDICES ARE  20 330 640 950
                                          PLOT TRAJ S
                                            TRACK
```

```
CONST( 1) 511000.0000      CONST( 26) 1.0000000000+100   CONST( 51) 1.0000000000E-04   CONST( 76) 0.
CONST( 2) -1.000000000     CONST( 27) 0.                 CONST( 52) 3.000000000        CONST( 77) 0.
CONST( 3) 1.000000000      CONST( 28) 0.                 CONST( 53) 1.000000000        CONST( 78) 0.
CONST( 4) 1.000000000      CONST( 29) 1.000000000        CONST( 54) 0.                 CONST( 79) 0.
CONST( 5) 2.997760000E+10  CONST( 30) 0.                 CONST( 55) 1.0000000000E-02   CONST( 80) 40.00000000
CONST( 6) -299.7921000     CONST( 31) 0.                 CONST( 56) 0.                 CONST( 81) 100.0000000
CONST( 7) -1.000000000     CONST( 32) 1.000000000        CONST( 57) 1.000000000        CONST( 82) 1.800000000
CONST( 8) 1.129400000E+11  CONST( 33) 0.                 CONST( 58) 1.000000000        CONST( 83) 1.900000000
CONST( 9) 0.               CONST( 34) 11.00000000        CONST( 59) 0.                 CONST( 84) 0.
CONST( 10) 10.00000000     CONST( 35) -15.00000000       CONST( 60) 0.                 CONST( 85) 0.
CONST( 11) 1.745329252E-02 CONST( 36) -15.00000000       CONST( 61) 1.000000000        CONST( 86) 0.
CONST( 12) 3.141592654     CONST( 37) 1.000000000        CONST( 62) .5000000000        CONST( 87) 0.
CONST( 13) 1.000000000     CONST( 38) 5.000000000        CONST( 63) 0.                 CONST( 88) 0.
CONST( 14) 1.000000000     CONST( 39) 5.000000000        CONST( 64) 0.                 CONST( 89) 0.
CONST( 15) 1.800000000     CONST( 40) 0.                 CONST( 65) 1.0000000000+100   CONST( 90) 305.0000000
CONST( 16) .1000000000     CONST( 41) 0.                 CONST( 66) 1.000000000        CONST( 91) 2.611210000E+11
CONST( 17) 200.0000000     CONST( 42) 0.                 CONST( 67) 0.                 CONST( 92) 2.000000000
CONST( 18) 10.00000000     CONST( 43) 1.000000000        CONST( 68) 0.                 CONST( 93) 2.000000000
CONST( 19) 300.0000000     CONST( 44) 0.                 CONST( 69) 0.                 CONST( 94) -1.129400000E+13
CONST( 20) 0.              CONST( 45) 0.                 CONST( 70) 1.000000000        CONST( 95) -1.797495927E+12
CONST( 21) 1.000000000     CONST( 46) 0.                 CONST( 71) 0.                 CONST( 96) 1.797495927E+12
CONST( 22) 1.000000000E-02 CONST( 47) 0.                 CONST( 72) -1.000000000       CONST( 97) 1.000000000
CONST( 23) 0.              CONST( 48) .5000000000        CONST( 73) 1.000000000        CONST( 98) .2000000000
CONST( 24) 1.000000000     CONST( 49) -1.000000000       CONST( 74) .5000000000        CONST( 99) 0.
CONST( 25) 300.0000000     CONST( 50) 0.                 CONST( 75) 0.                 CONST(100) 0.
```

CYCLE	SUM RESIDUALS	MAX RESIDUALS	I MAX RES	20 (19, 0)	330 (19, 10)	640 (19, 20)	950 (19, 30)
	OVER-RELAXATION FACTOR ALTERED	0	1.8000E+00				
0	5.2200E+03	1.8000E+02	930	0.0000	0.0000	0.0000	0.0000
10	1.3019E+04	7.6262E+01	920	0.0000	0.0000	0.0000	0.0000
20	6.4534E+03	5.7680E+01	911	61.5260	52.6249	52.6249	58.4339
30	8.5693E+02	6.6459E+00	911	58.5291	57.2571	57.2571	57.7412
	OVER-RELAXATION FACTOR ALTERED	40	1.4000E+00				
40	1.4331E+02	1.3482E+00	911	58.4360	58.2038	58.2038	58.2958
50	1.4993E+01	1.5684E-01	911	58.4253	58.3969	58.3969	58.4066
53	9.1926E+00	8.1105E-02	911	58.4133	58.4305	58.4305	58.4247

Z= 0.00000

RAY	E	X	Y	S	PX	PY	PZ	T	R	PHI
1	5.00000E+03	14.81436	0.00000	0.00000 0.	0.		7.16589E+04 0.		14.81436	0.00000
2	5.00000E+03	19.91750	0.00000	0.00000 0.	0.		7.16589E+04 0.		19.91750	0.00000
3	5.00000E+03	25.14850	0.00000	0.00000 0.	0.		7.16589E+04 0.		25.14850	0.00000

Z= 166.28100

RAY	E	X	Y	S	PX	PY	PZ	T	R	PHI
1	5.03615E+03	22.26061	0.00000	166.50356 6.18547E+03 0.			7.16523E+04 3.99461E-08		22.26061	0.00000
2	5.02265E+03	25.63285	0.00000	166.41357 4.86367E+03 0.			7.16566E+04 3.99434E-08		25.63285	0.00000
3	5.01573E+03	29.74425	0.00000	166.36723 3.97054E+03 0.			7.16622E+04 3.99402E-08		29.74425	0.00000

Z= 218.25000

RAY	E	X	Y	S	PX	PY	PZ	T	R	PHI
1	5.05463E+03	27.26656	0.00000	218.71390 7.57144E+03 0.			7.16523E+04 5.24316E-08		27.26656	0.00000
2	5.03583E+03	29.62405	0.00000	218.53625 6.10774E+03 0.			7.16566E+04 5.24277E-08		29.62405	0.00000

Z= 238.80900

RAY	E	X	Y	S	PX	PY	PZ	T	R	PHI
1	5.06182E+03	29.50811	0.00000	239.39478 8.04684E+03 0.			7.16523E+04 5.73710E-08		29.50811	0.00000

Z= 248.80900

RAY	E	X	Y	S	PX	PY	PZ	T	R	PHI
1	5.06240E+03	29.69528	0.00000	241.07192 8.08383E+03 0.			7.16523E+04 5.77714E-08		30.00000	0.00000
2	5.03627E+03	29.76611	0.00000	220.20896 6.14457E+03 0.			7.16566E+04 5.28281E-08		30.00000	0.00000
3	5.01602E+03	29.83660	0.00000	168.03645 4.00719E+03 0.			7.16622E+04 4.03406E-08		30.00000	0.00000

VELOCITY OF RAYS CM/SEC
0.   0.   0.

```
*ENV*
              0.2100   1.0200    2.000    13.0000  40.0000  50.0000  60.0000  70.0000  80.0000  90.0000 100.0000
              0                                         4        5        6        7        8        9       10
          --- ----- --- ----- --- --------- --------------------------------------------------------------------
    1   0.00000   I              ( (   ('      X X    I
    2  10.00000   I              X (   (       ( X
    3  20.00000   I              ( X   ( (     X (
    4  30.00000   I              X (   X (     X X    I
    5  40.00000   I              X X   ( X     ( (    I
    6  50.00000   I              X X   X X     X X    I
    7  60.00000   I              X X   ( X     ( (    I
    8  70.00000   I              X X   ( (     ( (    I
    9  80.00000   I              X X   X X     X X    I
   10  90.00000   I              X X   X X     X (    I
   11 100.00000   I              X X   X (     X X    I
   12 110.00000   I               X X   X X    X (    I
   13 120.00000   I               X X   X X    X X    I
   14 130.00000   I                X X   X X   X X    I
   15 140.00000   I                X X   X X    X X   I
   16 150.00000   I                X X   X X    X X   I
   17 160.00000   I                 X X   X X    X I
   18 170.00000   I                  X X  X X   I
   19 180.00000   I                  X X  X X   I
   20 190.00000   I                  X X X X    I
   21 200.00000   I                  X X X X X  I
   22 210.00000   I                   X X X X   I
   23 220.00000   I                    X X      I
   24 230.00000   I                    X I
   25 240.00000   I                      I
   26 250.00000   I                      I
   27 260.00000   I                      I
   28 270.00000   I                      I
   29 280.00000   I                      I
   30 290.00000   I                      I
   31 300.00000   I                      I
PLOT  1
```

PLOT RAY NUMBER   1

14.8144	14.8144	14.8144	14.8861	15.0092	15.1933	15.4375	15.7409	16.1024	16.5207
16.9946	17.5224	18.1023	18.7328	19.4122	20.1387	20.9108	21.7266	23.1442	24.0673
25.0285	26.0262	27.0591	28.3398	29.4456					

PLOT RAY NUMBER   2

19.9175	19.9175	19.9175	19.9710	20.0627	20.2000	20.3826	20.6103	20.8824	21.1984
21.5575	21.9589	22.4021	22.8862	23.4102	23.9733	24.5746	25.2132	26.3300	27.0619
27.8275	28.6262	29.4568	30.0000	30.0000					

PLOT RAY NUMBER   3

25.1485	25.1485	25.1485	25.1910	25.2638	25.3727	25.5177	25.6984	25.9147	26.1663
26.4529	26.7741	27.1295	27.5186	27.9409	28.3960	28.8833	29.4023	30.0000	30.0000
30.0000	30.0000	30.0000	30.0000	30.0000					

Z-AXIS(CM)

0.0000	0.0000	0.0000	20.0000	30.0000	40.0000	50.0000	60.0000	70.0000	80.0000
90.0000	100.0000	110.0000	120.0000	130.0000	140.0000	150.0000	160.0000	176.2810	186.2810
196.2810	206.2810	216.2810	228.2500	234.2500					

ENVELOPE AND Z-AXIS(Y)

25.1485	25.1485	25.1910	25.2638	25.3727	25.5177	25.6984	25.9147	26.1663	26.4529
26.7741	27.1295	27.5186	27.9409	28.3960	28.8433	29.4023	26.3300	27.0619	27.8276
28.6262	29.4568	28.3398	29.4456						

0.0000   10.0000   20.0000   30.0000   40.0000   50.0000   60.0000   70.0000   80.0000   90.0000
100.0030 110.0000 120.0000 130.0000 140.0000 150.0000 160.0000 170.0000 180.0000 190.0000
200.0000 210.0000 220.0000 230.0000

ELECTRIC FIELD DEFLECTION BETWEEN TWO CONCENTRIC CYLINDERS   U( )   SCALE= 1.00000E+00

		1	2	3	4	5	6	7	8	9	10
		0.00000	10.00000	20.00000	30.00000	40.00000	50.00000	60.00000	70.00000	80.00000	90.00000
31	30.0000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
30	29.0000	96.91	96.92	96.92	96.92	96.92	96.92	96.92	96.92	96.92	96.92
29	28.0000	93.72	93.72	93.72	93.72	93.72	93.72	93.72	93.72	93.72	93.72
28	27.0000	90.41	90.41	90.41	90.41	90.41	90.41	90.41	90.41	90.41	90.41
27	26.0000	86.97	86.97	86.97	86.97	86.97	86.97	86.97	86.97	86.97	86.97
26	25.0000	83.41	83.41	83.41	83.41	83.41	83.41	83.41	83.41	83.41	83.41
25	24.0000	79.68	79.68	79.68	79.68	79.68	79.68	79.68	79.68	79.68	79.68
24	23.0000	75.81	75.82	75.82	75.82	75.82	75.82	75.82	75.82	75.82	75.82
23	22.0000	71.76	71.76	71.76	71.76	71.76	71.76	71.76	71.76	71.76	71.76
22	21.0000	67.53	67.54	67.54	67.54	67.54	67.54	67.54	67.54	67.54	67.54
21	20.0000	63.09	63.09	63.09	63.09	63.09	63.09	63.09	63.09	63.09	63.09
20	19.0000	58.41	58.43	58.43	58.43	58.43	58.43	58.43	58.43	58.43	58.43
19	18.0000	53.50	53.50	53.49	53.49	53.49	53.49	53.49	53.49	53.49	53.49
18	17.0000	48.09	44.30	48.31	48.31	48.31	48.31	48.31	48.31	48.31	48.31
17	16.0000	42.77	.77	42.77	42.77	42.77	42.77	42.77	42.77	42.77	42.77
16	15.0000	36.40	30.61	36.92	36.92	36.92	36.92	36.92	36.92	36.92	36.92
15	14.0000	30.62	30.62	30.62	30.62	30.62	30.62	30.62	30.62	30.62	30.62
14	13.0000	23.90	23.90	23.90	23.90	23.90	23.90	23.90	23.90	23.90	23.90
13	12.0000	16.59	16.59	16.59	16.59	16.59	16.59	16.59	16.59	16.59	16.59
12	11.0000	8.65	8.70	8.72	8.70	8.70	8.70	8.70	8.70	8.70	8.70
11	10.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	9.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	8.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	7.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	6.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	5.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	4.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	3.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	2.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	1.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.0000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

ELECTRIC 'FIELD DEFLECTION BETWEEN TWO CONCENTRIC CYLINDERS   RH()   SCALE= 1.00000E+00

		1	2	3	4	5	6	7	8	9	10
		0.00000	10.00000	20.00000	30.00000	40.00000	50.00000	60.00000	70.00000	80.00000	90.00000
31	30.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30	29.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
29	28.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
28	27.0000	0.	0.	0.	0.	0.	0.	0.	0.	-2.744E-06	-7.233E-06
27	26.0000	-2.550E-06	-2.550E-06	-3.274E-06	-4.508E-06	-6.634E-06	-8.759E-06	-1.173E-05	-1.524E-05	-1.376E-05	-8.900E-06
26	25.0000	-1.442E-05	-1.462E-05	-1.386E-05	-1.258E-05	-1.067E-05	-8.161E-06	-5.067E-06	-1.421E-06	0.	0.
25	24.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
24	23.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
23	22.0000	0.	0.	0.	0.	0.	0.	0.	0.	-4.740E-06	-1.117E-05
22	21.0000	0.	0.	0.	-1.349E-06	-4.275E-06	-8.106E-06	-1.279E-05	-1.824E-05	-1.633E-05	-8.864E-06
21	20.0000	-1.989E-05	-1.983E-05	-2.009E-05	-2.017E-05	-1.713E-05	-1.309E-05	-8.165E-06	-2.432E-06	0.	0.
20	19.0000	-1.788E-06	-1.788E-06	-5.272E-07	0.	0.	0.	0.	0.	0.	0.
19	18.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
18	17.0000	0.	0.	0.	0.	0.	0.	0.	-2.745E-06	-1.706E-05	-2.527E-05
17	16.0000	0.	0.	0.	-2.645E-07	-5.493E-06	-1.226E-05	-2.033E-05	-2.407E-05	-1.253E-05	-1.373E-07
16	15.0000	-2.373E-05	-2.373E-05	-2.570E-05	-2.351E-05	-2.293E-05	-1.573E-05	-7.107E-06	0.	0.	0.
15	14.0000	-5.411E-06	-5.411E-06	3.324E-06	0.	0.	0.	0.	0.	0.	0.
14	13.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	12.0000	0.	0.	0.	0.	0.	0.	0.	0.	.	0.
12	11.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	10.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
10	9.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
9	8.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
8	7.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
7	6.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
6	5.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	4.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	3.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	2.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	1.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1	0.0000	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TIME = 1.2240 CP SEC 0236 DT = 1.3240   TIME

                                                END

SUMMARY OF NUMBER OF SUBROUTINE CALLS AND TIMES

```
SETUP     NUMBER OF CALLS =  0    TOTAL TIME =  0.000  SECONDS
ELM       NUMBER OF CALLS =  1    TOTAL TIME =   .721  SECONDS
BOUND     NUMBER OF CALLS =  1    TOTAL TIME =   .883  SECONDS
OUTPT     NUMBER OF CALLS =  4    TOTAL TIME =   .024  SECONDS
          NUMBER OF CALLS =  0    TOTAL TIME =  0.035  SECONDS
CRTLCN    NUMBER OF CALLS =  1    TOTAL TIME =   .018  SECONDS
BOUNDARY  NUMBER OF CALLS =  1    TOTAL TIME =  0.000  SECONDS
          NUMBER OF CALLS =  0    TOTAL TIME =  0.000  SECONDS
```

EXAMPLE 2

As a second, simple example of the use of the EBQ code for a problem for which an analytic solution exist, consider the motion of a 2.5 MeV electron traveling down a pipe in the presence of a constant 1000 gauss magnetic field. Two trajectories will be calculated, a helical one such that the particle will move at constant radius, and the other will be mis-matched so it widely oscillates in radius. The momentum of the rays are given as:

$$P = \sqrt{E^2 - E_o^2} = \sqrt{(2.5 + .511)^2 - .511^2} \times 10^6$$

$$P = 2.967 \times 10^6 \text{ eV/c}$$

The magnetic rigidity is B(guass) ρ(cm) = P(ergs)/e(esu)

$$B \rho \text{ (g-cm)} = 3335.7 \text{ P (MeV/c)}$$

The cyclotron radius is the transverse radius in this B field of 1000 gauss. If we start the electron with $P_z = P_x$, then:

$$P_z = P_x = P/\sqrt{2} = 2.0982 \text{ MeV/c}$$

$$B \rho = 6.99896 \times 10^3 \text{ g-cm}$$

$$R = 6.998 \text{ cm}$$

The wave length of a single oscillation is given by

$$X = X_o \sin \frac{2\pi z}{\lambda}$$

$$\frac{dx}{dz} = \frac{P_x}{P_z} = \frac{2\pi}{\lambda} X_o \cos \frac{2\pi z}{\lambda} = 1$$

so that $\lambda = 2\pi X_o = 43.944$ cm.

The physical system is shown in Figure 2.1. The EBQ code data input for this case is shown in Figure 2.2.

The data input defines a 41 x 26 mesh extending to 20 cm. radius, and longitudinally from 0 to 100 cm, line 4. The boundary generator initializes a Neuman boundary on the left, right and r=0 axis. A conductive cylinder is used as the outer boundary, defined by line 5. The potential is set to zero, line 8. Lines 10-13 set the problem constants which are to differ from the default values. The high pitch of the orbit requires a Rungga kutta integration step smaller than the default value of the longitudinal mesh size. Lines 12-13 turn off the calculation of phase space area and beam envelop, as these make no sense for this example.

The ray data is given by lines 15-18. The input type 1 specifies input of X, Y, Z, PX, PY, PZ, and current. The input is terminated by the END card, line 18. A constant B_z field is defined by the COILGEN data, line 19. A CALCOMP beam trace plot is requested, line 20 with the default value of CONST(34)=10, shown in Figure 2.3.

The particle started at R = 994 cm with $P_x = P_z = 2.0982 \times 10^6$ ev/c moves at constant radius while the ray started with R = 0, $P_x = P_z = 2.0982 \times 10^6$ ev/c oscillates with $0 > R \geq 13.998$ cm.
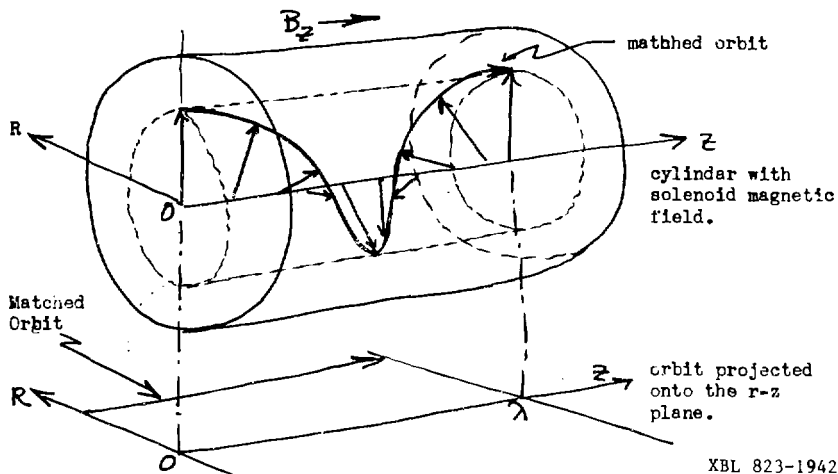
Figure 2.1   Constant B_z field, matched electron motion, showing three
dimensional orbit and its projection onto a plane.

```
TEST 2.5 MEV ELECTRCNS IN 1.000 KG FIELD          1
INITALIZE                                         2
BOUNDARY                                          3
   41 26 C 20 0 100                               4
              1 20 100 0 20 0                     5
   S                                              6
   S                                              7
PCT 0 S                                           8
CCNST                                             9
   *10 .C5                                        10
   *35 -10 -10 S ---- CALCCMP PLOT SIZE 10X10 CM  11
*63 C $ ------ NO EMITTANCE CALCULATICNS          12
   *71 0 $ CCNT PLOT ENVELOPE                     13
 S                                                14
RAYS 1                                            15
   C 6.99399 0 2.C982CE6 0 2.09820E6 1.0E-10      16
   0  0      0 2.C982CE6 0 2.09820UE6 1.0E-10     17
END RAYS INPUT                                    18
COILGEN 4 1.0E3                                   19
PLCT TRAJ S                                       20
TRACK                                             21
END                                               22
```

Figure 2.2   EBQ data input for the constant B_z matched motion
problem.

CELL   .5000 X 4.0000
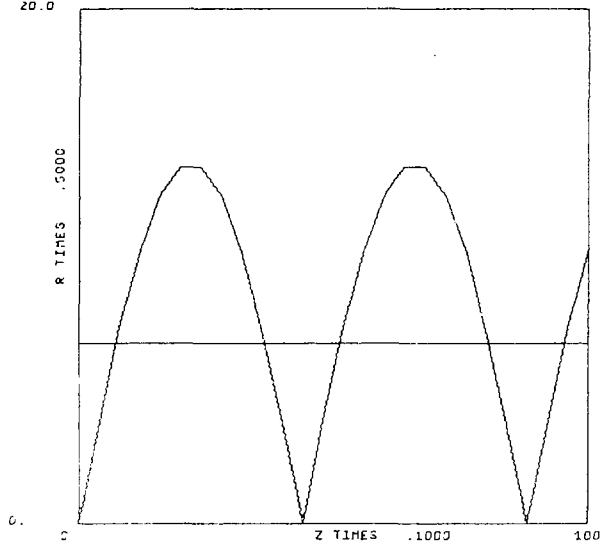
TEST 2.5 MEV ELECTRONS IN 1.000 KG FIELD
UCIDX01   31 AUG 78   ITER= 1   PLOT

XBL 323-1904

Figure 2.3   EBQ trajectory CALCOMP plot showing the matched and
             mis-match electron trajectories for example 2.

EXAMPLE 3

This example demonstrates the use of EBQ in the design of an atomic
beam spectrometer, originally performed with the EGUN program, UCID-
3860, (1974). The particular features of the EBQ code data input
exemplified in this example is the use of the BOUNDARY MODIFY to add an
electrode to an existing problem, the use of the RAYGEN input to
generate a particle distribution on a regular r-r' phase space grid, and
the input of boundary data in inches.

The physical problem considered is a H^- beam produced by colli-
sion of an incident H^o beam on the source electrode S at an energy of
about 1/4 eV by a charge exchange reaction. The space charge of the
H^- beam can be neglected. The simulations where performed for a two
(S and Q), three (S, F1 and Q), and four (S, F1, F2, and Q) electrode
structure shown in Figure 3.1. Figure 3.2 shows the EBQ data for the
two and three electrode case. The focusing provided by the 3-rd elec-
trode F is evident from the trajectory plots, Figures 3.3 and 3.4 and
yields considerably higher acceptance to the mass-spectrometer quadru-
pole Q than the two electrode case.

The first data case has only two electrodes, the source and the
quadrupole acting as an accelerating anode in this problem. The source
is given as potential index 1 and the quadrupole given as potential
index 3. The rays are generated by the RAYGEN data on a regular grid in
the r-r' phase space with 7 values of r' and $8 = 56/7$ values of r. The
beam current is set to the infinitesimal value of 10^{-9} amperes. The
RAYGEN data, line 19, has the following meaning:

RAYGEN

1	Type 1 data
56	Total of 56 rays to be generated
10^{-9}	Beam current for this generation
10^{-9}	Beam current in the total beam
0.25	Initial energy of the ions in eV
0.24	Phase space emittance, cm-rad.
0.6	Maximum beam radius, cm
3.81	Starting z location in cm
OR4	Four zeros for F, R', $\emptyset$, and ψ
-7	Minus specifies regular grid of 7r' values
S	Terminates ray data input

A phase plot is requested at Z = 11.42 cm, line 20, and a CALCOMP
plot of all trajectories is specified by the PLOT TRAJ S data, line 21.
The actual calculations are performed on execution of the TRACK command,
line 22.

The three electrode spectrometer is run using the same data defined
by the two electrode case preceding, but adding the focus electrode
(potential index 2) by the BOUNDARY MODIFY data, line 25-27. The TRACK
command, line 28, executes the same rays as where generated by the
RAYGEN data of the two electrode case.

The four electrode data is shown in Figure 3.5. The object of this
example is to show the data to obtain the tuning characteristics of the
focus electrode, F1. The problem is defined as in the two and three
electrode case by the BOUNDARY, INST, RAYGEN data. The MAP SECTION,
line 28 specifies that only every tenth longitudinal position will be
used in generating the potential and charge array maps so as to reduce
the volume of printed output. The EQUIP data, lines 22-25, specifies
the equipotentials to be plotted so as to better show the curvature of

the field lines near the source which provides the focusing of the device. The equipotential lines calculated from the minimum – maximum potentials have been suppressed by setting CONST(47) = 0, line 18. The case is executed by the TRACK data, line 31. The focus electrode is at -300 volts and the beam plot generated by this run is shown on the bottom of Figure 3.6.

The next group of data cards, lines 33-44 reduce the potential of the focus electrode, potential index 2, set a new title to identify the focus potential on the plot, and execute the data. The plots generated at focus potentials of -295, -290, and -285 volts are shown in Figure 3.6.

The RAYGEN data has generated 30 rays on regular grid, Figure 3.7, with each value of radius given five rays of different initial slope. These rays have been phase plotted on the r-r' phase plane at 12.7 and 27.0 cm. Figure 3.8 shows the r-r' phase plot at 27 cm for the case of the focus electrode at -295 volts. Rays C, H, N, S, Y, and) were emitted from the source initially at zero slope and have been connected in the figure as shown. The other rays of the same initial starting radius, but different initial slope, have been connected to the central ray so as to demonstrate the non-linearity of the forces typical of such problems.
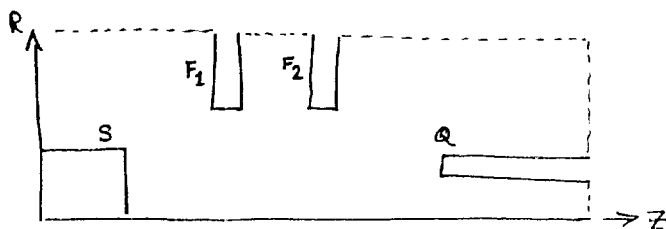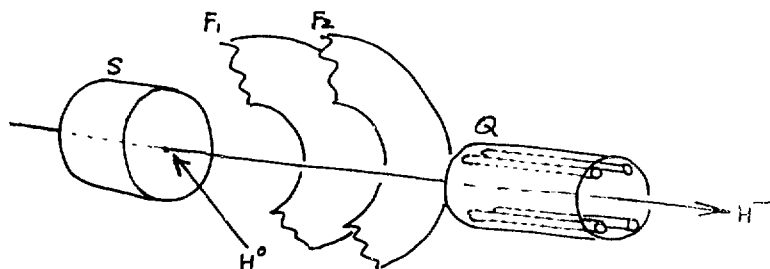
XBL 823-1943

Figure 3.1   Atomic beam spectrometer. Source electrode on which charge
              exchange occurs is indicated as S, F_1 and F_2 are the
              two focus electrodes introduced in the three and four
              electrode cases. Q is an electro-static quadrupole mass-
              spectrometer considered as cylindrically symmetric in this
              simulation.

```
ATOMIC BEAM APPARATUS -- TWO ELECTRODE CASE                                      1
TITLE  RUN WITH EGUN UCID-3860                                                   2
INITALIZE                                                                        3
BOUNDARY                                                                         4
   41 61 0 2 0 6 2.54      $ CONVERT TO CM, DATA INPUT IN INCHES                 5
   1 0 .25 1 .5 .25 1 .5 1.5 1 0 1.5 5 $ SOURCE                                  6
   3 .2 4.5 3 .2 6. 3 .5 6 3 .5 4.5 3 .2 4.5 5 $                                 7
   3 0 6 3 .2 6 S $ END CLAMP                                                    8
S                                                                               9
POT -300 -300 0 S                                                               10
CONST                                                                           11
   *1 93856 $ ----- H- IONS                                                     12
   *16 0.1 $ POISSON ERROR .1 VOLTS                                             13
   *35 8 6 $ SCALE 4 X 1                                                        14
   *47 20                                                                       15
S                                                                              16
MAP SECTION 1 121 10                                                           17
LAPLACE                                                                        18
RAYGEN 1 56 1.0E-9 1.0E-9 .25 .254 .635 3.81 084 -7 S                          19
PLOT PHASE 11.42 S $ ------ CM                                                 20
PLOT TRAJ S                                                                    21
TRACK X 1                                                                      22
END                                                                           23
ATOMIC BEAM APPARATUS --- THREE ELECTRODE CASE                                 24
BOUNDARY MODIFY       $ ------ ADD FOCUS ELECTRODE                             25
   2 2. 2.25 2 1 2.25 2 1 2.5 2 2 2.5 2 2 2.25 S $ FOCUS ELECTRODE             26
S                                                                             27
TRACK X 1                                                                     28
END                                                                          29
END                                                                          30
                                                                            31
                                                                            32
```

Figure 3.2   EBQ data for the two and three electrode cases of
             example 3.

Figure 3.3   Two electrode device showing ion trajectories and
equipotential lines. This figure should be compared with
the three electrode device shown in Figure 3.4.

CELL .1270 X   .2540
5.08

R TIMES 4.0000
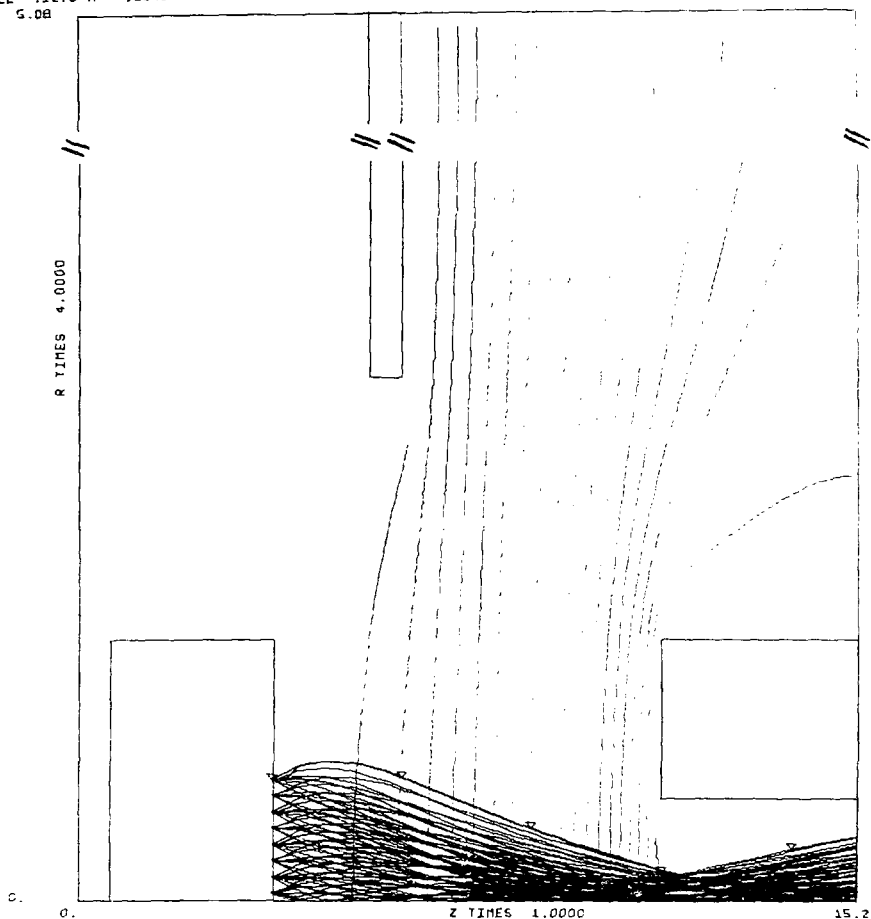
0.

TITLE   RUN WITH EGUN UCIO-3860
ATOMIC BEAM APPARATUS --- THREE ELECTRODE CASE

UCIOX00   24 OCT 78   ITER= 1   PLOT   4

XBL 823-1906

Figure 3.4 Three electrode device showing ion trajectories and
equipotential lines. The presence of the third electrode
has curved the equipotentials in the low energy region near
the source so as to provide focusing.

```
ATOMIC BEAM APPARATUS -- FOUR ELECTRODE CASE                              1
 TITLE                                                                    2
 INITALIZE                                                                3
 BOUNDARY                                                                 4
29 114 0 1.4 0 11.5 2.54 $ ----- INPUT INCHES, CONVERT TO CM             5
   1 0 .25 1 .5 .25 1 .5 1.0 1.0 1.0 1.0 .25 S $ SOURCE                  6
   2 1.4 3.5 2 1.0 3.5 2 1.0 3.75 2 1.4 3.75 2 1.4 3.5 S $ FOCUS 1       7
   3 1.4 5.25 3 1.0 5.25 3 1. 5.5 3 1.4 5.5 3 1.4 5.25 S $ FOCUS 2       8
   4 .2 11.5 4 .2 10.65 4 .5 10.65 4 .5 11.5 4 .2 11.5 S $ QUADRIPOL     9
 S                                                                       10
 POT -300 -3   -100 0 S                                                  11
 CONST                                                                   12
   *1 93856 $ HYDROGEN ION REST MASS                                     13
   *3 0.0 $   URN OFF GAUSS LAW AND SELF MAGNETIC FIELD                  14
   *16 .1 150 $ POISSON ERROR AND CYCLE LIMIT                            15
   *29 2 $ STORE EVERY OTHER POINT FOR PLOTTING                          16
   *35 -3.56 -29.2 $ CALCOMP PLOT SIZE IN CM                            17
   *47 0 $ NO EQUIPOTENTIAL LINES                                        18
 S                                                                       19
 LAPLACE                                                                 20
 PRINT FIELDS 0 0 29 0.5                                                 21
 EQUIP                                                                   22
   -295 -290  285 -280 -275 -270 -265 -260 -255 -250                    23
   -110 -105  100 -95 -90 -85 -90                                       24
 S                                                                       25
 RAYGEN 2 30  .0E-209 .25 .254 .635 2.54 0P4 -5 S                       26
 PLOT PHASE   2.7 27.9 S                                                 27
 MAP SECTION  116 10                                                    28
 RKSTEP .025  3.81 .0508 5.08 .127 6.35 .254 100 S                      29
 PLOT TRAJ S                                                             30
 TRACK X 1                                                               31
 END                                                                     32
 ATOMIC BEAM  APPARATUS --- FOUR ELECTRODE CASE                          33
 TITLE FOCUS  ELECTRODE -295 VOLTS                                       34
 POT #2 -295                                                            35
 PLOT TRAJ S                                                            36
 TRACK X 1                                                              37
 TITLE FOCUS  ELECTRODE -290 VOLTS                                       38
 POT #2 -290                                                            39
 TRACK X 1                                                              40
 TITLE FOCUS  ELECTRODE -285 VOLTS                                       41
 POT #2 -285                                                            42
 TRACK X 1                                                              43
 END                                                                    44
```

Figure 3.5   EBQ data for the 4 electrode case of example 3.
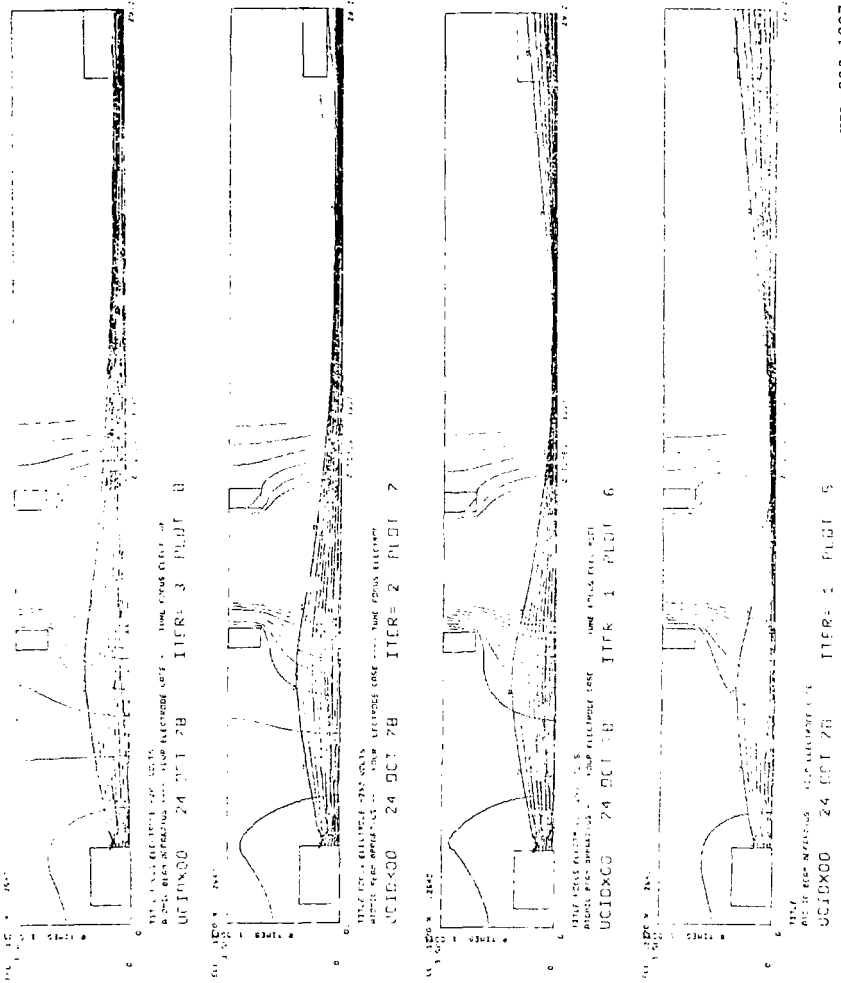
XBL 823-1907

Figure 3.6   Four electrode geometry showing trajectories and
equipotential lines for four different focusing electrode
potentials as run by the data Figure 3.5.
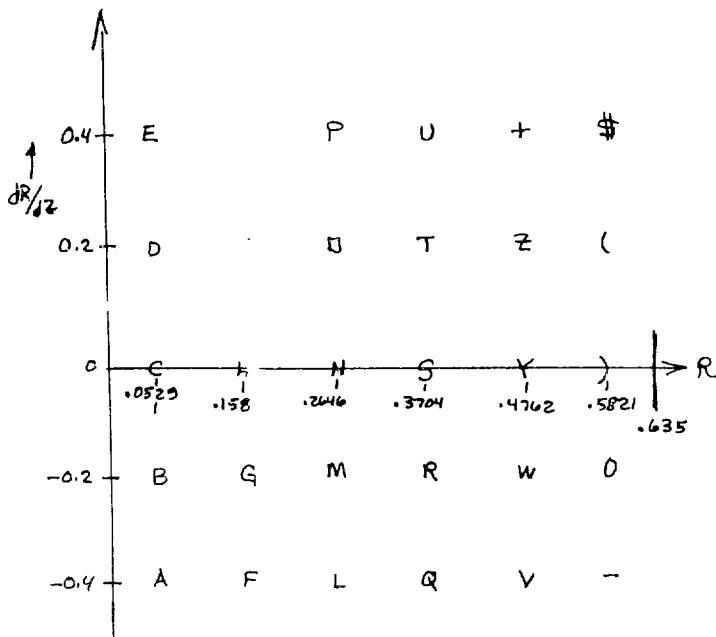
XBL 823-1908

Figure 3.7   Initial phas   space generated for the four electrode atomic
             beam example   The five rays at a given r are symmetrically
             centered on   ie ray of zero slope.

Figure 3.8

Phase space at z = 27 cm for the initial phase space shown in Figure 3.7. The ion positions are labled by alphanumeric indicators which have been connected here to show the curvature introduced by the device.

XBL 823-1909

EXAMPLE 4

This example shows the use of EBQ in running multiple ion species together in one problem. The phase space plots, beam moments and emittance can be individually calculated for each species present or calculated for the beam as a whole, depending on the value of CONST(66). In the following example, a beam of protons (H^+), deuterons (D^+) and singly charged molecular deuterium ions (D_2^+) were simultaneously tracked with space charge included for all species. The trajectories can be plotted in any desired combination as specified by the PLOT TRAJ cards.

PLOT TRAJ 10 20 30 S

request a plot of rays 10, 20, and 30, while

PLOT TRAJ 1-10, 11-20, 21-30 S

request a plot of the rays 1 through 10, 11 through 20, and 21 through 30 on a single CALCOMP plot.

The multiple species were generated by the RAYGEN data preceded by CONST data specifying the ions rest mass and charge. In Figure 4.1, the first RAYGEN data, lines 25-26, generates 10 deuterons. The second RAYGEN data, lines 27-28, adds 10 molecular deuterium ions to the first 10 rays since the type is negative, and the third RAYGEN data lines 29-30, adds 10 protons to the existing deuteron ions since the

type code is also negative. Each of these RAYGEN data cards is preceeding by a CONST *1 MASS CHARGE S data card specifying the species rest mass and charge.

The data is shown in Figure 4.1. The plots are shown in Figure 4.2. The plot of the 10, 20, and 30th ray includes the beam envelope identified by the tri_.gular symbol. Figure 4.2A, was generated by data line 32. The plot of all rays, Figure 4.2B was generated by data line 31 or 33.

```
MULTIPAL ION SPCIES RUN D+1, D2+L, AND H+1 ICNS TOGETHER      1
TITLE RTNS 2ND COLUMN WITH 150 MA OF D+                       2
  INITALIZE                                                   3
  BOUNDARY                                                    4
     13 111 0 6 0 55 $ ------ 0.5 X 0.5 CM CELLS              5
     OUTSIDE 5.5 5.5                                          6
     1 5 0 1 5 12 1 6 12 1 5 0 1 5 0 S                        7
     2 5 17 2 5 18 2 6 18 2 6 17 2 5 17 S                     8
     3 5 23 3 5 24 3 6 24 3 6 23 3 5 23 S                     9
     4 5 29 4 5 30 4 6 30 4 6 29 4 5 29 S                    10
     OUTSIDE 5 50                                            11
     5 4 36 5 4 55 5 6 55 5 6 36 5 4 36 S                    12
     S                                                       13
  POT 400000 327000 218000 109000 0 S                        14
  CONST                                                      15
    *1 1.876E9 $ ----- D+ REST MASS EV                       16
    *2 1 $ ----- CHARGE + 1                                  17
    *19 55 0 $ ----- TURN OEF OUTPUT                         18
    *34 11 $ CALCOMP AND PRINT BEAM LINE                     19
    *35 -6 5.5 $ CALCOMP PLOT SIZE                           20
    *47 50 $ NUMBER OF EQUIPOTENTIAL LINES                   21
    *55 .01 $ SELF FIELD RADIAL CUTOFF                       22
    *57 100 $ ------ X-XP PHASE PLOTS ONLY                   23
  S                                                          24
  CONST *1 1.876E9 1 S $ ------- D+1                          25
  RAYGEN 1 10 .05 .05 14320 .025 2.5 0 0 .32 0 0 0 S          26
  CONST *1 3.752E9 1 S $ ------ D2+1                          27
  RAYGEN -1 10 .05 .05 14320 .025 2.5 0 0 .92 0 0 0 S         28
  CONST *1 .938E9 1 S $-------- H+1                           29
  RAYGEN -1 10 .05 .05 14320 .025 2.5 0 0 .92 0 0 0 S         30
  PLOT TRAJ S                                                 31
  PLOT TRAJ 10 20 30 S                                        32
  PLOT TRAJ 1-10 11-20 21-30 S                                33
  MAP SECTION 1 110 11                                        34
  EQUIP                                                       35
    1000 2000 3000 4000 5000 6000 7000 8000 9000 1E4 5E4     36
    10E4 2E4 3E4 4E4 350E3 375E3 376E3 377E3 378E3 379E3 380E3  37
    381E3 382E3 383E3 384E3 385E3 386E3 387E3 388E3 389E3 390E3  38
    391E3 392E3 393E3 394E3 395E3                            39
    S                                                        40
  TRACK                                                      41
  END                                                        42
  END                                                        43
```

Figure 4.1 EBQ data for the multiple species RTNS run.

Figure 4.2 Trajectory plots showing plot of selected ions A) above
and all ions B) below. The equipotential lines in
the center of the plot were generated by CONST(47) while
the higher density lines at each end were generated from
the EQUIP data shown in figure 4.1.

EXAMPLE 5

This example demonstrates the use of EBQ to simulate a 750 kV
accelerating electrostatic focusing column following a 30 ma 20 keV
Xe^+ ion source. This column is part of the injector for the 50 MeV
proton linac at LBL. An iterative solution to the space charge problem
was found and compared to the first cycle based on the use of Gauss's
law. The effect of space charge is then demonstrated by setting the
beam current to zero and running one additional cycle with only single
particle motion in the external fields. The Xe source and quadrupole
focusing system are up stream from the column, the ions are charge
neutralized by the electrons generated by their propagation through the
residual gas to the left of the boundary, with the Xe ions pulled from a
pseudo plasma boundary waist at $z = 1$ cm. The typical emittance of such
a heavy ion source is 70 π cm-mr, Figure 5.1. The rays are generated in
an upright phase ellipse randomly populated in a four dimensional volume
by the RAYGEN data card, line 32, Figure 5.2.

The volume of output is reduced by taking every fifth point of the
potential and charge deposition maps as specified by the MAP SECTION
1 131 5 data card, line 33. Phase space plots in the X-X', Y-Y', and
R-R' planes are to be generated at $Z = 1$ and 25 cm, lines 24 and 34.

The first cycle of the calculation is based on a space charge
evaluation from Gauss's law and should be very close to an iterative
solution found from successive cycles of solving Poissons equation and
ray tracking. To verify this supposition, a trajectory and phase plot
is requested for the first cycle by lines 34-35. The iterative solution
is initiated from the first cycle by a continuation for a maximum of 5

additional cycles, line 37, TRACK CONTINUE 5. Note the similarity in
the Gauss's law and iterative solutions, Figure 5.3 and 5.4. The
importance of space charge can be readily demonstrated by reducing the
beam current to essentially zero and re-running, lines 40-42. The ray
trace plot is shown in Figure 5.5 for the zero current case.

Unless specifically inhibited by CONST(61) = 0, EBQ will draw a
dashed line marking the beam envelope projection of the previous cycle.
This makes comparison of ⌐ue 30 ma and 0 current beam profile easy,
Figure 5.5
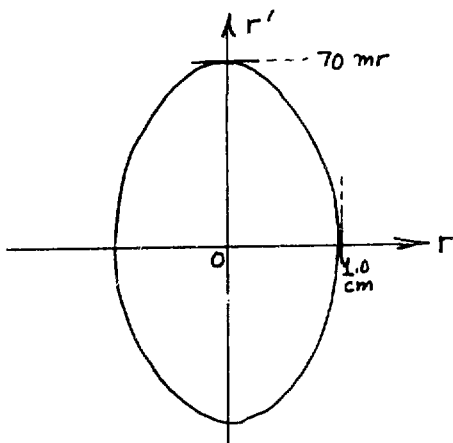
XBL 823-1944

Figure 5.1   Initial beam phase as given on the RAYGEN data card. The
             emittance of .070 cm-rad requires a 70 mr initial
             divergence for the 1 cm beam radius. The rays are randomly
             populated inside this area.

```
20 KV XE+1 ESCAR COLUMN                                              1
TITLE                                                               2
INITIALIZE                                                          3
BOUNDARY                                                            4
    26 131 0 5 0 26                                                 5
    1 2.6 0 1 2.6 2.8 1 1.2 2.8 1 1.2 3.0 1 5 4.6 S                 6
    2 5 8 2 1.2 6.4 2 1.2 6.6 2 5 8.2 S                             7
    3 5 11 3 1.4 10.2 3 1.4 10.4 3 5 11.2 S                         8
    4 5 14.4 4 1.6 14 4 1.5 14.2 4 5 14.6 S                         9
    5 5 17.2 5 1.8 17.0 5 1.8 17.2 5 5 17.4 S                       10
    6 5 19.8 6 1.8 19.6 6 1.8 19.8 6 5 20 S                         11
    7 5 22.6 7 2 22.4 7 2 22.6 7 5 22.8 S                           12
S                                                                  13
POT 500E3 416.7E3 333.3E3 250E3 166.7E3 83.3E3 0 S                 14
CONST                                                              15
    *1 1.22JE11 & ----- XE REST MASS                               16
    *2 1 $ ----- + 1 CHARGE                                        17
    *19 26 & OUTPUT EVERY 26 CM                                    18
    *22 .01 $ ----- RAY CONVERGENCE FACTOR                         19
    *34 1G -10 -13 $ ----- CALCOMP, RAXIS, ZAXIS                   20
    *42 2 % ----- CALCUMLATE MOMENTS EVERY 2 CM                    21
    *47 75 % ------ EQUIPOTENTIAL LINES EVERY 10 KV                22
    *55 .01  $ ------ SELF MAGNETIC FIELD CUT OFF                  23
    *57 100 $ PHASE PLOT ONLY X-XP                                 24
S                                                                  25
EQUIP                                                             26
    15000 14000 13000 12J00 11000 9000 8000 7000                  27
    6000 5000 4000 3000 2000 1000 500                             28
    749000 748000 747000 746000 745000                            29
    744000 743000 742000 741000 S                                 30
PRINT POT                                                         31
RAYGEN 1 50 .03 .03 20000 .070 1 1 0 0 0 0 0 S                    32
MAP SECTION 1 131 5                                               33
PLOT PHASE 1 25 S                                                 34
PLOT TRAJ S                                                       35
TRACK X 1 % ------ PLOT GAUSS-S LAW SOLUTION                      36
TRACK CONTINUE 5 % SOLVES AFTER 3 ON ITERATION 4                  37
ENDTRACK                                                         38
$ ----- NOW RUN AT INFINTESIMAL CURRENT                          39
CURRENT 1.0E-10                                                  40
PLOT TRAJ S                                                      41
TRACK X 1                                                        42
END                                                             43
END                                                             //
```

Figure 5.2   EBQ data for the Xe^{+1} source problem.
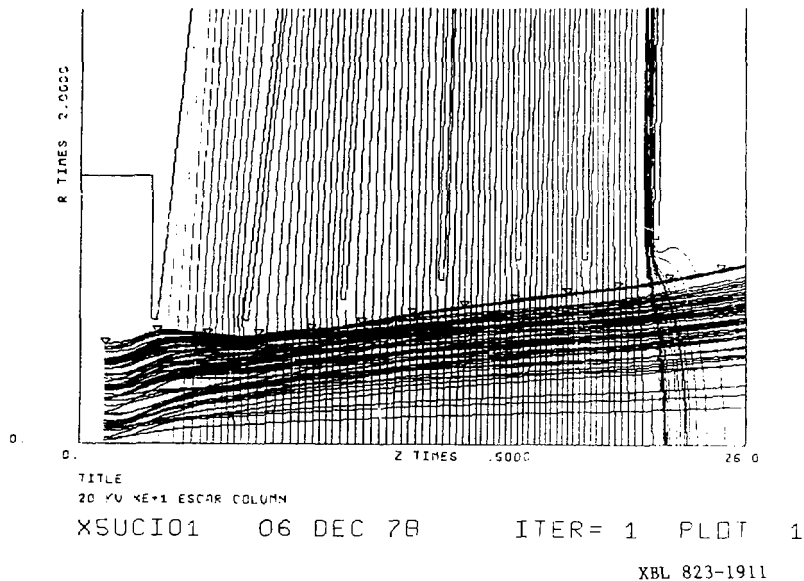
Figure 5.3 Plot on ion trajectories on the first iteration where space charge was calculated from Gauss's Law.
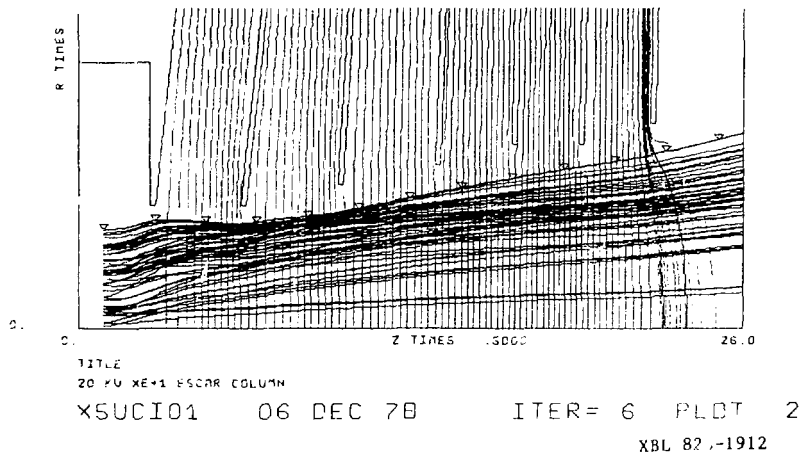
Figure 5.4 Final iterative solution of ion trajectories after six cycles. The space charge is now calculated from the solution of Poisson's equation.

CELL .2000 X .2000   I=3 00000E-12
5.00

R TIMES 2.0000

C.

C.   Z TIMES .5000   26.0
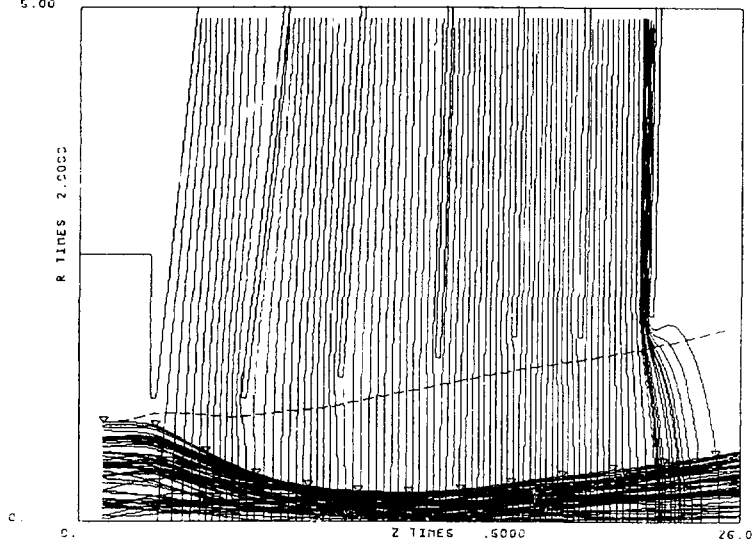
TITLE
20 KV XE+1 ESCAR COLUMN

XSUCI01   06 DEC 78   ITER= 7   PLOT   3

XBL 823-1913

Figure 5.5   The seventh iteration of the Xe^{+1} source showing the ion
              trajectories run without space charge. Note the plot of
              the envelope of the sixth iteration showing the space
              charge beam extent, dashed curve.

EXAMPLE 6

This example considers the continuation of problems from one case
to another. Problems can be continued for a variety of reasons which
fall into two broad classes, those in a single run, and those of a
subsequent run. This continuation example consists of three cases:

A) A long transport system demonstrating the continuation of a
   problem inside a single data case.

B) Continuation of a problem in separate runs where the entire
   problem is dumped to tape (or disk) and re-read.

C) Continuing a problem (within one run or otherwise) where the
   rays are dumped after an iterative solution is found which
   modifies the Laplace solution to the potential problem.
   The rays must then be started from an interior point of the
   problem to avoid the fictitious Neumann boundary of the
   problems right edge.

A) The first example of a continuation problem considers the transport
of a 11.1 ampere beam of U238, 50 meters in a constant solenoid field.
The transverse beam size is initially 10 cm and the beam is confined to
a vacuum pipe of 15 cm radius. The emittance of this beam is 1.94
cm-rad with about 49 kV of space charge depression. An unrealistically
high value of the longitudinal solenoid magnetic field was chosen to
reduce the ion wave length so many oscillations could be observed in the
modiest 50 meters of beam transport length.

The large beam emittance means that the rays frequently cross so
application of Gauss's law to find the space charge electric radial
field based on the enclosed current requires that the current inside

each ray be frequently calculated. In this run CONST(46) was set to
unity so the enclosed current was found every Runga-Kutta integration
step. A plot of the RMS beam radius was desired, so the moments of
the beam distributions was calculated every 50 cm as requested by
CONST(42) = 50. The 50 meter flight path was divided into five sections
of 1000 cm each, Figure 6.1.

r(cm)
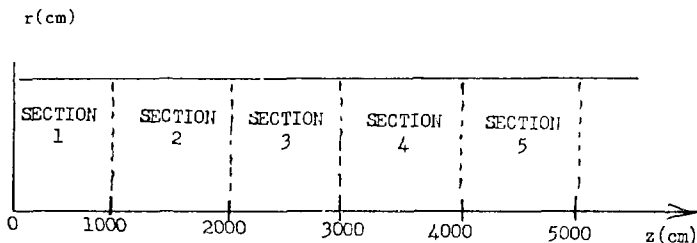
Figure 6.1

XBL 823-1945

The data is shown in Figure 6.2. The CONST *23 and the PUNCHB 1000
S data is global and crosses the problem boundary, producing a tape 9
ray dump at the end of each section of the problem. These dumps are
automatically sequenced, as dumps 1, 2, 3, 4, and 5. These dumps are
then read by the following section by RAYS 8 NDUMP, where NDUMP is the
dump number appropriate to the start of that section. All other data
being the same as in the first section, only the ray distribution
changing.

Figure 6.3 shows the ray trace for the five sections into which
this problem has been divided.

B)   Next, consider the continuation of a previous run, say continuing
the problem to improve the solution convergence from a previous run
dumped to tape 8. The left column of the control cards and data in
Figure 6.4 show the data generating the tape 8 dump after four
ray-Poisson cycles, the right column shows the data which reads this
tape and runs an additional two ray-Poisson cycles.

In this example, one file was written to tape 8 with a dump number
1 specified by the value of CONST(32). line 14. When the problem was to
be continued, the value of CONST(32) had to be set to this desired value
before DUMP IN. At this point, all necessary arrays have been read, and
tracking can be initiated. The TRACK card must specify continue to
prevent zeroing of the charge array, since the values of the potential
array and the charge array are related by the solution of Poisson's
equation.

C)   Next, consider the continuation of a beam from some intermediate z
position past the end of the initial problem after an iterative solution
of the ray-Poisson equation has been obtained.

The Poisson ray trace cycles have altered the boundary potential
values from the Laplace solution by the beam space charge depression.
It therefore becomes necessary to set the potential on the left side of
the continuation boundary as a graded Dirichlet boundary. This example
might be essentially the same as A), the U238 beam transport example,
except that the longitudinal transfer point is interior to the problem
at, say, 8 meters rather than the boundary end of 10 meters, Figures 6.5
and 6.6.

When the transfer takes place, the potentials at 8 meters become
the left edge of the continuation prob 'm, as shown by the use of the
increment option in the boundary data. The points in the POT( ) array
are stored from the scalar potential array U( ) in the previous problem,
shown on the left. The continuation is shown on the right of the
control card and data below. The left column is the conceptual data for
initializing the problem and propagating the rays, say to 10 meters.
The right column shows a following run which takes the rays from a 8
meter dump and runs them an additional ten meters to 18 meters, Figure
6.5.

Consider the data on the left, Figure 6.5. Line 9' specifics the
boundary mesh interval size as 0.5 cm radial x 20.0 cm longitudinally.
The boundary might have been that shown in Figure 6.6. The left edge
will be used as a starting edge of the continuation problem and should
have a potential associated with each radial interval so that it appears
as a graded Neumann boundary supporting the space charge depression of
the beam. Say we assign potentials 4-24 to this edge by

```
BOUNDARY
21  51  0  10  0  1000
INCREMENT  4  24
I  0  0  24  10.  0  S
.
.
.
```

The initial potential array (line 11L) reserves space for these 21
values and 21 values to be stored from the potential U map at 8 meters.
These values will then be switched in the subsequent run, line 15R, 16R
right (R).

The dump numbers written to Tape 8 are specified on line 13L and 14L. Line 16L sets a pointer so that the ray values are dumped to Tape 9 on the final iteration at 800 cm. Line 20 L stores the potential at 800 cm (ZLOC = 40) in the pot( ) array positions 25-45. Line 21L dumps out all the pertinent arrays defining th problem.

The continuation run represented by the data on the right (lines - R) set the dump numbers to be read on Tape 8 and 9, lines 9R, 10R. The DUMP IN data initialize the arrays at the values written at the end of the previous problem including the particle load at 10 meters. The particle load is redefined from the rays dumped at 8 meters by line 14R. The z value of the rays is $800 + CONST(23) \simeq 0$ initializing this load to the left hand boundary, a graded Dirichlet set by the switch options, lines 15R and 16R. These rays are then tracked an additional 10 meters.

```
$ ------------------------------------------------------------ ---
TRACK U238 IONS 0-10 METERS                                    1
TITLE                                                          2
INITALIZE                                                      3
MAP OFF                                                        4
BOUNDARY                                                       5
   31 101 0 15 0 1000 $ ----- CELL 0.5 X 10.0 CM              6
   1 15 0 1 15 1100 S $ ---- BEAM PIPE R=15 CM                7
   S                                                           8
POT 0 0 S                                                      9
CONST                                                          10
   *1 223.2954E9 $ ----- U238 RESTMASS                        11
   *2 1 $ ------ CHARGE= +1                                   12
   *10 1.0 $ ----- 1 CM STEP                                  13
     $ ----- PKSTEP=1 CM 50 RAYS  23.147 SECONDS             14
   *10 5.                                                      15
   *19 1000 $ OUTPUT EVERY 10 METERS                          16
   *35 -15.0 -22.0 $ ----- CALCOMP PLOT SIZE, CM              17
   *35 -7.5 -15.0 $ ------ CALCOM PLOT SIZE                   18
   *42 50 $ CALCULATE MOMENTS EVERY 50 CM                     19
   *46 1 $ ----- CALC. ENCLOSED CURRENT EVERY P K STEP        20
   S                                                           21
COILGEN 4 248274.4 $ ------ CONSTANT BZ FIELD                 22
   $                                                           23
   $  BETA**2GAMMA**3= 1.0E-4  RHO=30 CM                      24
   $    EPSILON 1.945608 CM*RAD   EN=2.30945E11 /CM           25
   $    N=50  R=10 CM  B=248274.4 GAUSS                       26
   $    SPACE CHARGE DEPRESSION  49.4329 KV  E=10.579 MEV    11.09626  27 PS
   $                                                           28
RAYGEN 2 50 11.09626 11.09626 10.656 1.945608 10 0 0 0 0 0 0 S  29
PUNCHB 1000 S $ ----- RAY DUMP TO TAPE 9 FOR CONTINUATION     30
PLOT TRAJ S                                                    31
TRACK                                                         32
PLOT ENERGY S                                                 33
PLOT PHASE S                                                  34
END                                                          35
```

Figure 6.2   U238 data for EBQ.

```
--------- CONTINUE 10-20 METERS                               36
CONST *23 -1000 S                                             37
RAYS 8 1 $ ----- READ TAPE 9 DUMP 1                           38
PLOT TRAJ S                                                   39
TRACK                                                         40
END                                                          41
------- CONTINUE 20-30 METERS                                 42
RAYS 8 2 $ READ TAPE 9 DUMP 2                                 43
PLOT TRAJ S                                                   44
TRACK                                                         45
END                                                         46
--------- CONTINUE 30-40 METERS                               47
RAYS 8 3 $ READ TAPE 9 DUMP 3                                 48
PLOT TRAJ S                                                   49
TRACK                                                         50
END                                                         51
--------- CONTINUE 40-50 METERS                               52
RAYS 8 4 $ READ TAPE 9 DUMP 4                                 53
PLOT TRAJ S                                                   54
TRACK                                                         55
END                                                         
```

Figure 6.2   (continued)

CELL .5000 × 10.000C

15.0

.5000

R TIMES

0.

0.   2 TIMES .0150   1.000E+03
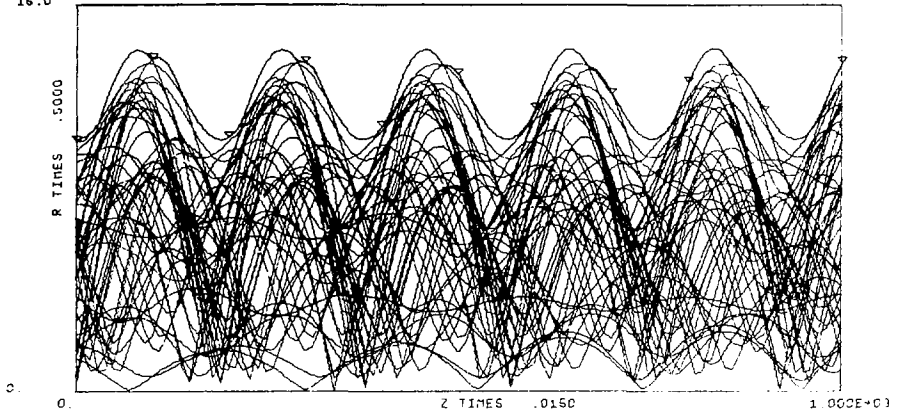
TITLE
TRACK U230 IONS 0-10 METERS

X6UCIO1   13 SEP 78   ITER= 1   PLOT 1

XBL 823-1914

Figure 6.3A

CELL .5000 × 10.000C

15.0

50.0

R TIMES

0

0   2 TIMES .0150   1.000E+03
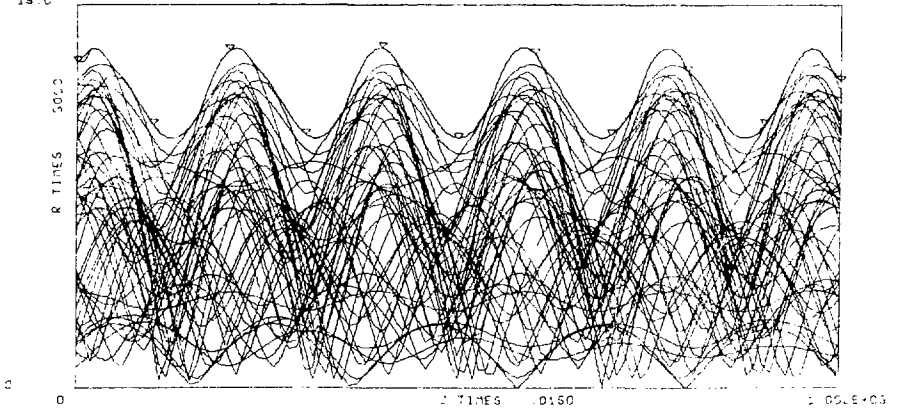
TITLE
--------- CONTINUE 10-20 METERS

X6UCIO1   13 SEP 78   ITER= 1   PLOT 2

XBL 823-1915

CELL .6000 X 10.0000
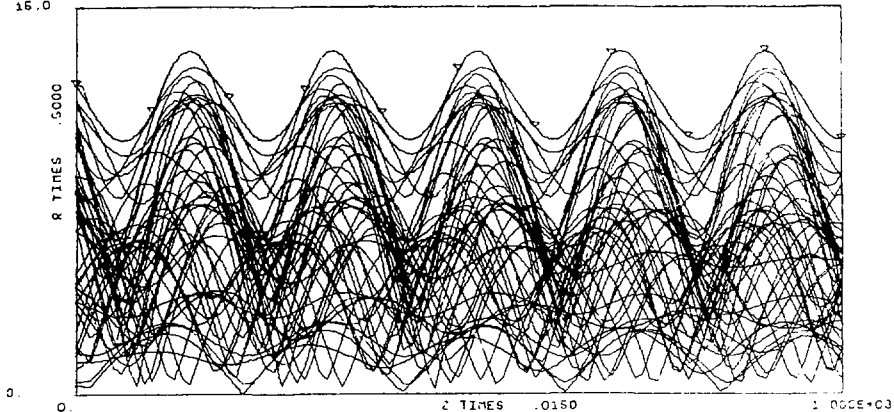
TITLE
------- CONTINUE 20-30 METERS

X6UCIO1   13 SEP 78   ITER= 1   PLOT   3

XBL 823-1916

Figure 6.3C
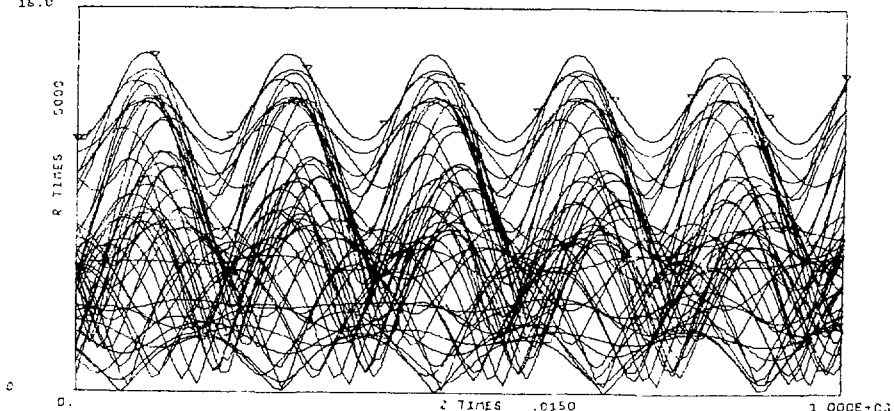
CELL .5000 X 10.0000

TITLE
------- CONTINUE 30-40 METERS

X6UCIO1   13 SEP 78   ITER= 1   PLOT   4

XBL 823-1917

CELL .5000 x 10.0000
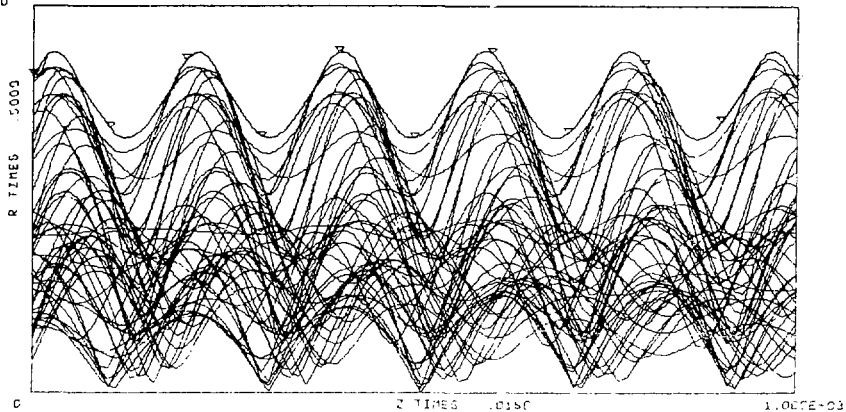
XBL 823-1918

Figure 6.3E Long transport of U238 ion beam in a constant solenoidal
magnetic field. The space charge is calculated from
application of Gauss's Law. Each section of the
calculation shows the ion motion over a 1000 cm flight
path. The ending ray distribution of a proceeding section
is the initial starting distribution for the next section.

```
JOB.........                              1   JOB.........
FETCHGS,LGO=EBQ/V3,12940.                 2   FETCHGS,LGO=EBQ/V3,12940.
RFL,,70000.                               3   FETCHMT,TAPE8,12345 INPUT DUMP
LINK,X.                                   4   RFL,,70000.
REQUEST,TAPE8,12345,W. OUTPUT DUMP        5   LINK,X.
EOR                                       6   EOR
EXAMPLE OF CYCLE CONTINUATION             7   CONTINUE PREVIOUS RUN 2 CYCLES
BOUNDARY                                  8   CONST *32 1 S $DUMP NUMBER 1
--------                                  9   DUMP IN
--------                                 10   TRACK CONTINUE 2
POT .....S                               11   END
CONST                                    12   END
  *21 4 $ N CYCLES                       13   EOF
  *32 1 $ DUMP NUMBER                    14
RAYS 1                                   15
--------                                 16
--------                                 17
END RAYS INPUT                           18
TRACK                                    19
DUMP OUT                                 20
END                                      21
END                                      22
EOF                                      23
```

Figure 6.4   Schematic EBQ data showing transfer of whole problem to
              intermediate disk or tape file for restart at a later time
              via the DUMP OUT and DUMP IN data.

```
JOB..........                          1   JOB..........
FETCHGS,LGO=EBQ/V3,12940.              2   FETCHGS,LGO=EBQ/V3,12940.
RFL,,70000.                            3   FETCHMT,TAPE8,12345 INPUT DUMP
LINK,X.                                4   RFL,,70000.
REQUEST,TAPE8,12345. OUTPUT DUMP       5   LINK,X.
EOR                                    6   EOR
TEM METER BEAM PROPAGATION             7   BEAM TRANSPORT, 8-18 METERS
BOUNDARY                               8   CONST
21 51 0 10 0 1000                      9     *32 11 $ BOUNDARY DUMP 11,DUMP1
..........                            10     *33 2  $ 8 METER RAY DUMP,DUMP2
POT 1 2 3 OR21 S                      11     *23 -800 $ SHIFT RAYS 8 METERS
CONST                                 12   S
  *32 11 $ BOUNDARY DUMP NO.          13   DUMP IN
  *33 2  $ ORBIT DUMP NO.             14   RAYS 8 2 $ READ 8 METER RAY DUMP
S                                     15   LEDGE 25 45 0 0 SWITCH 4 24
PUNCHB 800 $ 8 METER RAY DUMP         16   POT SWITCH
COILGEN                               17   TRACK X 5
..........                            18   END
TRACK X 5                             19   END
LEDGE 25 45 1 40 X 4 24               20   EOF
DUMP OUT                              21
END                                   22
END                                   23
EOF                                   24
```

Figure 6.5   Schematic EBQ data showing continuation of a problem with
               orbit transfer at intermediate position via PUNCHB and
               RAYS 8.   The boundary layer is shown in Figure 6.6.
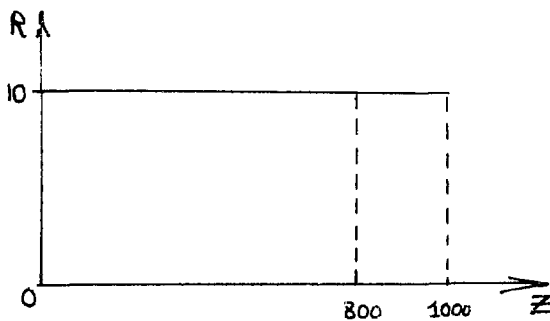
XBL 823-1919

Figure 6.6   Boundary layout showing orbit transfer and LEDGE cut for continuation data of Figure 6.5.

EXAMPLE 7

   This example shows the data to study a space charge limited plane
parallel diode. The initial conditions for the electrons emitted from
the cathode is determined by application of Child's law. The boundary
data sets up a cathode with an emission surface specified by potential
index 1, line 6 and line 40, Figure 7.1, terminated by potential index 5
associated with the continuation of the cathode electrode. The grid is
a finite set of concentric wire rings, lines 7-20. The expanded re ion
around the wire grid requires a reasonably fine mesh precluding running
the problem up to the actual anode, so the terminating potential is a
curved surface following the space charge depressed surface of a 250 kV
equipotential line determined in a previous run extending from the
cathode out to the actual anode, lines 21-26. These points on the
250 kV equipotential line were found after solution of Poisson's
equation in a previous problem by setting CONST(49) = 1, CONST(47) = 0,
EQUIP ⌐50E] S, which causes a printing of a table of R,Z value along the
specified equipotential lines in this region, a list of potentials to be
plotted close to the grid potential is specified via the EQUIP data,
line ⅗.

   The electrons are generated by the RAYGEN 3 data, line +0, calling
for 100 equispaced rays emanating from a equipotential surface crossing
the z axis at z = 0.6 cm. The initial potential to be used in applica-
tion of Child's law is given 25000 Volts. An iterative solution is
sought until the change in the total beam current is less than 0.01%
(1 amp in 10,000), line 32, or until 10 iterations have been completed,
line 47.

Figure 7.2 shows the trajectories of these 100 electrons emanating from the equipotential lines at 0.6 cm, passing through a continuous plane grid at 1.5 cm and terminating on the 250 kV equipotential line. The phase space area occupied by the beam extracted from this diode is determined by the effect of the beam charge interior to its surface, radially defocusing the ions on this surface, Figure 7.3. Additionally, the construction of such a device with a mesh grid req⋅ for transparency and reduction of grid power dissipation pr⋅ ices a local radial focusing of the beam, filamenting the phase spa as shown in Figures 7.4-7.7. The grid would actually be construc⋅ d of a wire mesh which in the limit of a two dimensional axial simula ⋅on is modeled as concentric wire rings.

The orbits are terminated at an energy of 25⋅ ⋅eV, CONST(65), along the curved equipotential line, data lines 37. ⋅ phase plot, Figure 7.7 at this energy is produced by the PLOT PHASE S, line 49 after the TRACK data, line 47. This figure shows the phase space approximately on the 250 kV equipotential surface where all r⋅ ⋅s have the same energy.

The data required to modify the wire r ⋅g grid of Figures 7.4-7.7 into a continuous grid of Figures 7.2-7.3 s shown as case two of this example, lines 51-61. The boundary modi⋅ data does not initialize the problem, only alterations to the prece⋅ ⋅ng cases are included, as shown.

Note that the phase plot scales ⋅f Figures 7.3 and 7.5 are not the same. If the user desires the wir⋅ ⋅esh and continuous grid runs to have the same phase space plot sc⋅⋅es, he must set them explicitl⋅. The data lines shown as 42-44 should set the phase plot scales at the

longitudinal positions z = 0.75, 1.5 and 6.25 cm to the values shown, forcing these identical scales for the two cases.

```
GRID WIRE ANALYSIS ANODE AT 250 KV EQUIPOTENTIAL LINE            1
TITLE :ELL 0.25 X 0.25 CM PHASE SPACE FROM WIRE MESH GRID OF 1.0 CM  2
INITIALIZE                                                      3
BOUNDARY                                                        4
   61 26 0 15 0 6.25                                            5
   1 0 0 5 12.7 0 5 15 0 S                                      6
   2 0.5 1.5 S                                                  7
   2 1.5 1.5 S                                                  8
   2 2.5 1.5 S                                                  9
   2 3.5 1.5 S                                                 10
   2 4.5 1.5 S                                                 11
   2 5.5 1.5 S                                                 12
   2 6.5 1.5 S                                                 13
   2 7.5 1.5 S                                                 14
   2 8.5 1.5 S                                                 15
     9.5 1.5 S                                                 16
   2 10.5 1.5 S                                                17
   2 11.5 1.5 S                                                18
   2 12.5 1.5 S                                                19
   2 12.7 1.5 2 15.. 1.5 S                                     20
   STOP PLOT                                                   21
   3 0 6.088 3 1 6.057 3 2 5.965 3 3 5.815                     22
   3 4 5.615 3 5 5.375 3 6 5.108 3 7 4.827                     23
   3 8 4.544 3 9 4.273 3 10 4.019 3 11 3.792                   24
   3 12 3.624 3 13 3.603 3 14 3.672 3 15 3.741                 25
   S                                                           26
   S                                                           27
POT 0 85000 250000 0 0 0 S                                     28
CONST                                                          29
   *3 0 S ----- GUASS-S LAW OFF                                30
   *10 .05 S ---- RK STEP                                      31
   *22 1.0E-4 S ------ CHILDS LAW CONVERGENCE TU 1 AMPERE      32
   *35 -15 -12.5 S ------ CALCOMP PLOT SIZE IN CM              33
   *37 11 S ----- CALCCMP PHASE PLOTS                          34
   *47 20 S ----- NUMBER OF EQUIPOTENTIAL LINES                35
   *54 0 S ------ RETROGRADE MOTION                            36
   *64 0 250000 S ---- E-MIN  E-MAX                            37
S                                                              38
EQUIP 6CE3 62.5E3 65E3 67.5E3 70E3 72.5E3 75E3 77.5E3 80E3  S  39
RAYGEN 3 100 1 .6 25000                                        40
PLOT PHASE 0.75 1.5 S                                          41
PLOT SCALES PHASE 0 15  -.175 0 1.5                            42
PLOT SCALES PHASE 0 15  -0.25 0 6.25                           43
PLOT SCALES PHASE 0 15  -.05 .15 0.75                          44
PLOT TRAJ S                                                    45
RKSTEP .05 1.5 .25 10 S                                        46
TRACK X 10                                                     47
ENDTRACK                                                       48
PLOT PHASE S S ---- PHASE PLOT AT TERMINATION OF RAYS          49
END                                                            50
S -----------------------------------------------------------  51
GRID WIRE ANALYSIS. ANODE AT 250 KV EQUIPOENTIAL SURFACE       52
TITLE CELL 0.25 X 0.25 CH PHASE SPACE FROM CONTINUOUS GRID      53
BOUNDARY MODIFY                                                54
   2 0 1.5 2 12.7 1.5 2 15.0 1.5 S S ------ CONTINUOUS GRID    55
S                                                              56
RAYGEN 3 100 1 .6 25000                                        57
TRACK X 10                                                     58
ENDTRACK                                                       59
PLOT TRAJ S                                                    60
TRACK CONTINUE 1                                               61
END
```

Figure 7.1   EBQ data for high current grid wire analysis example.

TITLE CELL 0.25 X 0.25 CM, ORBITS TERMINATE AT 250 KEV
GRID WIRE ANALYSIS ANODE AT 250 KV EQUIPOTENTIAL
GRIDX03   21 JUL 78   ITER= 5   PLOT 4
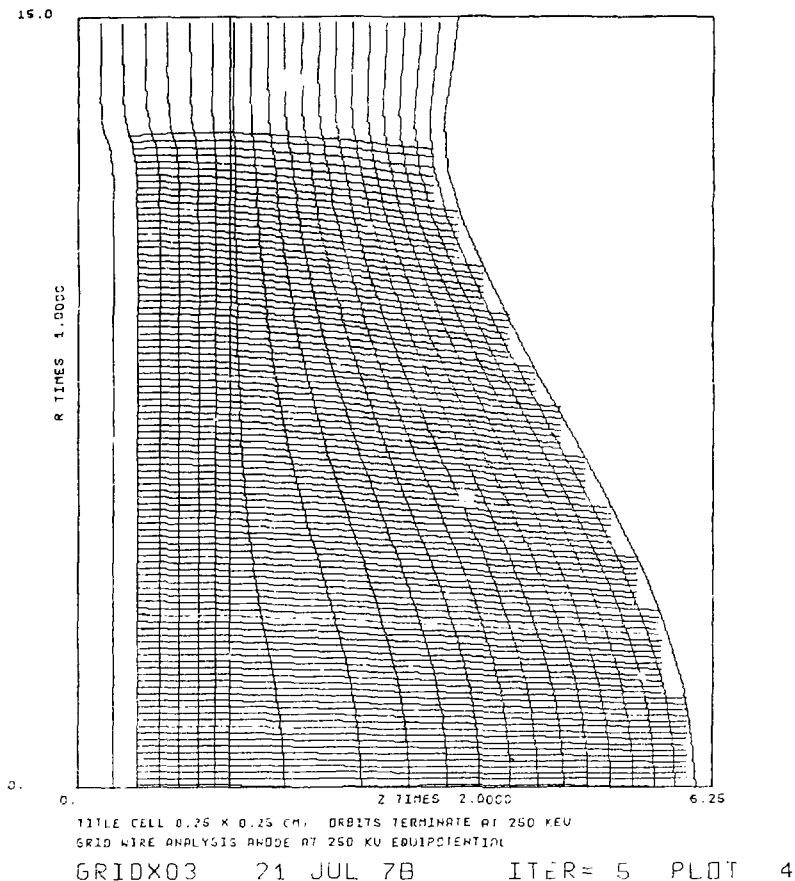
XBL 823-1920

Figure 7.2   R-Z plane showing equipotential lines and electron
             trajectories for a continuous grid extracting from a
             cathode emission surface extending to 12.7 cm radius.
             The electrons pass through a grid and terminate on a 250
             keV equipotential line.

XBL 823-1921

Figure 7.3   Phase space produced half way between cathode and
              continuous grid. Note the dominant effect is the space
              charge defocusing of the outer edge of this non-Pierce
              corrected structure.

TITLE CELL 0.25 X 0.25 CM, ORBITS TERMINATE AT 250 KEV
GRID WIRE ANALYSIS ANODE AT 250 KV EQUIPOTENTIAL

GRIDX01   20 JUL 78   ITER= 9   PLOT 4
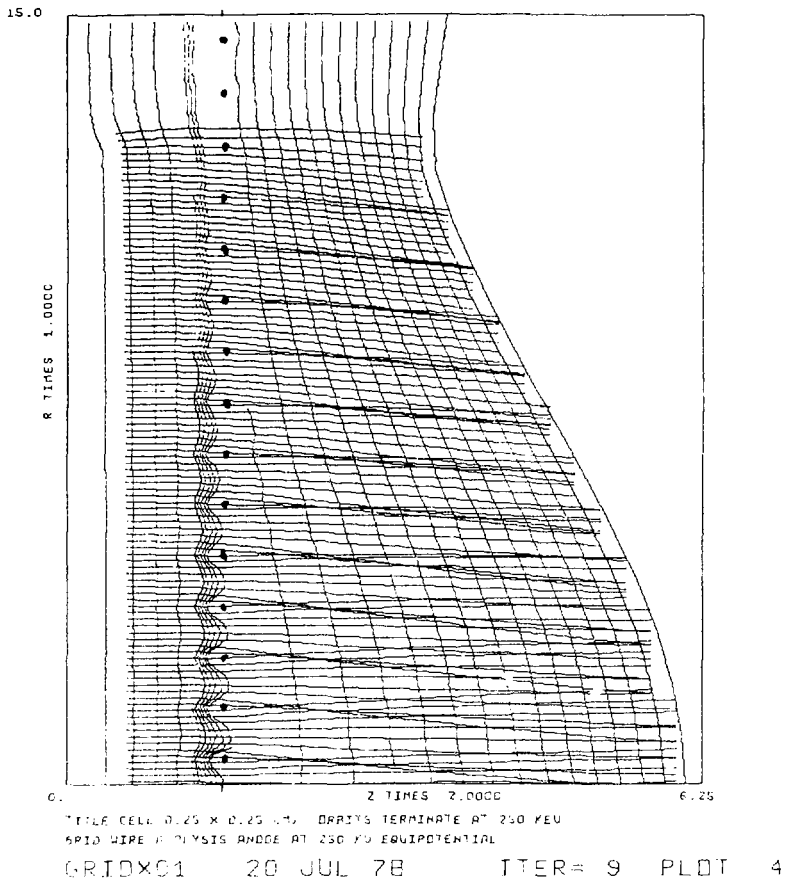
XBL 823-1922

<u>Figure 7.4</u>   R-Z plane showing equipotential lines and electron
trajectories for the same geometry of Figure 7.2 where the
continuous grid has been replaced by a set of concentric
rings as shown. Note the local einzel lens focusing of the
electron beam after passage through the grid electrode.

RAY GROUP 1   PR/PZ VS R   PHASE PLOT   Z= .7500
                                          I= 1201S

GRIDX01   20 JUL 78   ITER= 9   PLOT 1

TITLE CELL 0.25 X 0.25 CM, ORBITS TERMINATE AT 250 KEV
GRID WIRE ANALYSIS ANODE AT 250 KV EQUIPOTENTIAL
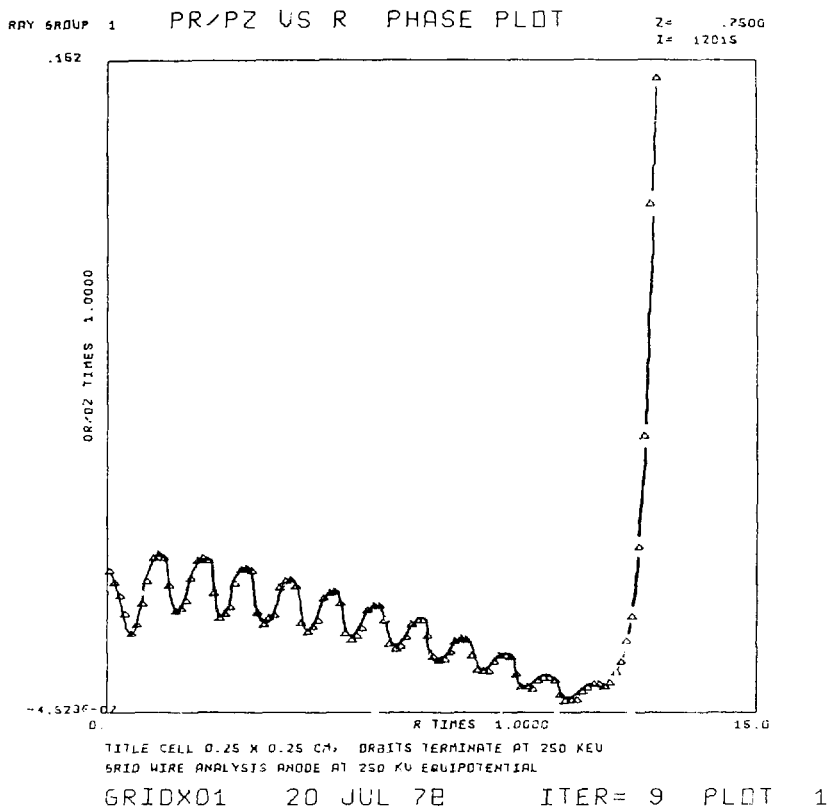
XBL 823-1923

Figure 7.5   Phase space produced half way between cathode and grid by
             local effect of finite grid wires prior to beam passing
             through the grid. Note the dominant effect is the space
             charge defocusing of the outer edge of this non-Pierce
             corrected structure.
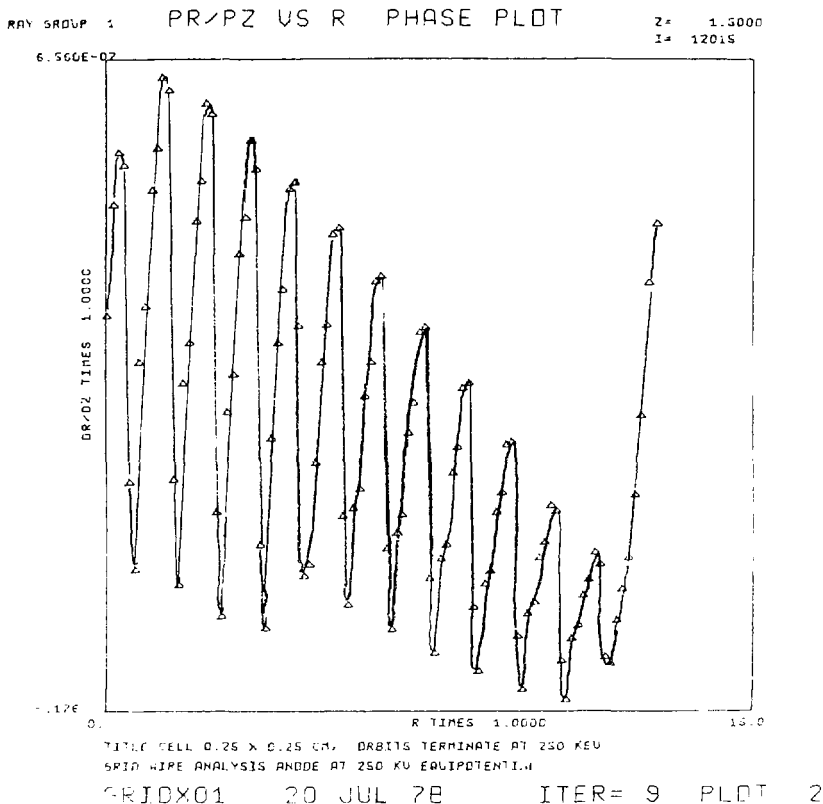
Figure 7.6   Phase space at the grid. The maximum beam energy is 85 keV as that is the potential set by the grid wires every cm in radius. The beam is filamented by the focusing of the 1 cm grid wires on approach to the grid plane. This einzel lens effect is comparable to the non-Pierce corrected space charge defocusing of the outer edge of the beam.

Figure 7.7   Phase space plot at the energy of ? ? k.V.   .?   . .?
             radius passed the grid region at lo. .+
             to the space charge depression u (in.?
             received a larger radial force produci..,
             ar·a contribution at the smaller· . ·\i.

EXAMPLE 8

    This example demonstrates a long transport system for a cesium ion
beam (100 meters) in a computationally economical way. Space charge
will be calculated from application of Gauss's law, the self magnetic
field is unimportant for these low energy ions.

    The problem has three distinct characteristic dimensions.
1)   Boundary mesh - used only for plotting, i.e., ions trajectories are
stored at each DZMESH interval for plotting. This mesh will not be used
to solve Poisson's equation, as the space change will be found from
Gauss's laws.
2)   Magnetic field array - the vector potential array is differentiated
to find the components of the external magnetic field B_x and B_y.
This array must be fine enough to allow accurate evaluation of these
fields in the sharp fringing field region of the short field lenses.
3)   Runga-Kutta integration step - this step must be smaller than
distances over which field changes are appreciable or rays cross, since
ray crossings effect the enclosed current used to find the space charge
fields.

    The magnetic field cell (0-300 cm) must be present over the entire
length of the problem since the solenoids provide the restoring force
against beam expansion from the beam emittance and space charge
divergence. The shortness of the field lenses require a relatively fine
mesh. It therefore becomes necessary to use the region--repeat
parameters of the COILGEN data to slide the 3 meter magnetic field cell
along the mesh as the beam propagates.

The data defining the basic problem is shown in Figure 8.1. MA·
OFF, Line 4 suppresses the boundary mesh map, (its quite long and
uninteresting), the potential, and the charge maps normally generated
after execution of TRACK. The various problem constants are defined by
lines 10-19, with the meanings given as comments following the $ on the
data cards. The magnetic field is generated from the data lines 20-27.
The field is rapidly varying in the coil region, an expanded grid is
used in the longitudinal location of each coil requiring the 0-300 cm
coil section to be divided into five regions. The positions, and grid
mesh size dr,dz of each region are defined by the data lines 22-24.
This entire 300 cm section is subjected to the 6000 point mesh
restriction of the APHI array. As the orbit calculation proceeds, the
field will be found by a z origin shift from this basic field section as
specified by REPEATS = 100, line 20.

The rays to be run are specified, lines 29-37. The current data
line is not used as the RAYS card, line 29 specified a total current of
0.25 amperes to be distributed amongst the rays according to their
radial positions, line 30, so as to yield a starting beam of uniform
current density.

This data is executed, line 39 producing the plot shown in Figure
8.2. The ten rays propagate from left to right, with the magnetic field
shown as the wildly oscillating curve. The scale of the magnetic field
is printed on the plot, BMIN, BMAX = -31801, 31801 Gauss. The zero line
for the field plot is marked by the short ticks on the left and right of
the plot, in this example, half way up as the field is symmetric. the

run is now repeated at essentially zero current, data lines 40-42, to demonstrate the importance of space charge for this beam, Figure 8.3. The dashed line of this figure is the space charged beam envelope of the preceding TRACK.

This first part of the data set used a mesh cell of 1.0 x 100 cm and a Runga-Kutta integration step of 2 cm. To better illustrate the magnetic field shape and orbit motion, an expanded region is now run. The accuracy of the calculations are not improved, as the mesh size in this example is unimportant, since Gauss's law is being used to evaluate the space charge forces and there are no external electric fields. The calculation of the current enclosed by the rays is performed each mesh interval, but this laminar beam does not cross. The data lines 45-49 set up an expanded section, from 0 -1000 cm with a mesh size of 1.0 x 10 cm. The beam current is reset to the initial 0.25 ampere value (line 51) and tracking initiated (line 52) Figure 8.1. The trajectory plot is shown in Figure 8.4. The current is then reduced to essentially zero and the beam tracked without space charge, Figure 8.5. The magnetic field is found from within the vector potential map generated for the preceeding case, since this map covers the physical space simulated in case 2. Compare the first 1000 cm section of Figures 8.2, 8.3 with Figures 8.4, and 8.5.

-277-

```
$ --------------------------------------------------------------
RUN CESIUM IONS IN LONG BROCKEN ALTERNATING POLARITY SOLENOID      1
TITLE                                                              2
INITIALIZE                                                         3
MAP OFF                                                            4
BOUNDARY                                                           5
11 101 0 10 0 10000 $ 10 KM DRIFT DISTANCE, CELL=1X100 CM          6
   1 10 0    1 10 10000 S $ ---- OUTER CYLINDAR, BEAM PIPE         7
   S                                                               8
POT 0 0 S                                                          9
CONST                                                              10
   *1 137E9 $ ----- CESIUM REST MASS                               11
   *10 1 $ ------ 1 CM RK INTEGRATION STEP                         12
   *10 2 $ ------ 2 CM INTEGRATION STEP                            13
   *14 5 $ ----- PAPER PLOT BEAM LINE GRAPH INTERVAL               14
   *14 2 $ ----- PAPER PLOT BEAM LINE GRAPH INTERVAL               15
   *19 1.0E100 $ TURN OFF ORBIT OUTPUT                             16
   *34 10 $ ------ CALCOMP TRAJECTORIES                            17
   *35  -10 -20 $ CALCOMP AXIS LENGTH IN CM(R AND Z)               18
   S                                                               19
$ ------- MAGNETIC FIELD INPUT HERE                                20
COILGEN 1 0 0 10. 5 100 $ ---- TYPE B1 B23 RMAG ZONES REPEATS      21
   0 30 120 160 260 300 $ ------ Z()                               22
   1.0R5 $ ------ DR()                                             23
   10 1 10 1 10 $ ----- DZ()                                       24
   1.0E6 -1.0E6 S $ ------ CURRENT IN AMPS                         25
   75 10 1  $SINGLE TURN Z=75 CM R=10 CM SCALE=1                   26
   225 10 1 $ SINGLE TURN COIL                                     27
$ -------- END MAGNETIC FIELD INPUT                                28
PRINT FIELDS 5 0 1000 10                                           29
RAYS 5 0 UNIFORM 0.25 $ ---- GENERATE RAYS UNIFORM CURRENT DENSITY, 30
   .25 .75 1.25 1.75 2.25 2.75 3.25 3.75 4.25 4.75 S $ ----- 10 RAYS 31
   0R10 $ Z()                                                      32
   0R10  $ PHI()                                                   33
   2.0E0R10 $ ---- 2 MEV IONS                                      34
   0R10 $ ----- APLHA                                              35
   0R10 $ PSI()                                                    36
   0R10 $ CURRENT CALCULATED FOR UNIFORM DENSITY                   37
END RAYS INPUT                                                     38
PLOT TRAJ S                                                        39
TRACK                                                              40
CURRENT 1.0E-20                                                    41
PLOT TRAJ S                                                        42
TRACK                                                              43
END                                                               44
$ --------------------------------------------------------------  45
RUN 1000 CM, PROVIDE PLOT SHOWING DETAILS OF MAGNETIC FIELD AND ION 46
BOUNDARY                                                           47
   1 101 0 10 0 1000 $ CELL=1X10 CM                                48
   1 10 0 1 10 1000 S                                              49
   S                                                               50
PLOT TRAJ S                                                        51
CURRENT 1.0 $ RESET CURRENT TO INITAL VALUE, 0.25 AMPS            52
TRACK X 1                                                          53
CURRENT 1.0E-10                                                    54
TRACK X 1                                                          55
END
END
```

Figure 8.1   EBQ data demonstrating the use of a repetitive magnetic
             field zone.

Figure 8.2 Plot of ion orbits and magnetic field for the cesium
ion example. Space charge is included by Gauss's law.

XBL 823-1926

Figure 8.3   Cesium ion example without space charge run as iteration
two following a space charge run. Note the beam envelope
(dashed curve) of the space charge cycle plotted for
comparison.

XBL 823-1927

XBL 823-1928

Figure 3.4 Expanded region of the cesium ion example showing more
detail of the magnetic field and orbit profiles. Space
charge is included via Gauss's law.

XRL 823-1929

Figure 8.5   Expanded region of the cesium ion example without space
            charge, compare with Figures 8.2, 8.3, and 8.4. Note the
            space change envelope plotted as dashed line.

EXAMPLE 9

Consider the design of an X-ray tube with a point source of
electrons. The details of the particle source region require running
the problem as a sequence of several sub-cases so as to reflect the
electro-static potential in the entire device from which is extracted
the potentials to be used as boundary values for the source region
problem. The electrons are then extracted from the emitter and run out
to a reasonable energy and injected from this source region in to the
rest of the device for transport to the target.

The data, Figure 9.2, divides the device, Figure 9.1A, into the
regions delineated in Figure 9.1B. Section 1 of this data, lines 1-25,
define the entire global problem comprising the wire cathode, focus
electrode, anode and transport regions. This problem is solved to find
the graded Dirichlet potentials to be used as boundary conditions in
case 2 and 3 which expand the region around the cathode emission point
to allow the required refinement of the mesh to simulate the high
electric fields found around a point source. Laplace's equation is
solved, line 17, or the global problem. Interpolated potentials along
selected lines within this global problem are then stored in the POT( )
array for use as boundary conditions in the following sections of this
data by the POT STORE data, lines 18-22.

Since no rays are tracked in this first section of this example,
a plot of the boundary with the equipotential lines must be produced
by specifically requesting it via the PLOT EQUIP data, line 24, or a
PLOT BOUNDARY data.

The number of points to be stored in the POT( ) array and the specification of the interpolation line to use is determined from the mesh size and boundaries of the subsequent cases. Consider the left edge boundary of the emitter case, case 2, lines 26-50. The 31 x 26 mesh covers the radial region from 0 to 0.6 cm in the longitudinal section extending from 0.7 to 1.7 cm, line 29. The 31 potential values to be used as the radial boundary on the left edge of the problem are stored in the pot array in locations 11-41 as specified by the POT STORE R 11 41 data, line 18, and inserted into the mesh by the INCREMENT code in the boundary data of case 2, line 32. The actual potentials used are interpolated from the U( ) array along the radial line (R) at $z = 0.7$ cm extending from 0 to 0.6 cm in 0.02 cm steps, lines 18 and 32-33. Similarly, for the other boundary potentials of the subsequent cases.

The rays are generated in case 2, line 45. The 25 rays are to be started at a constant longitudinal value, $z = 1.02$ cm inside a regular, upright phase space grid with five radial values and five angular values. The radial values are determined from the maximum beam radius, RBEAM = 0.07 cm, so R takes the values .007, .021, .035, .049 and .063 cm. The radial angular emission value is determined from the beam emittance specified as 0.1 (EM = RBEAM*RPMAX) as the five values -10/7, -5/7, 0, 5/7, 10/7 radians. The rays are to be traced and their values at the end, $z = 1.7$ cm, saved for continuance in the next case by writing a binary disk dump via the PUNCHB data, line 46. The rays are started at 2 eV in the very high electric field region around a point emitter, so the Runga-Kutta integration step must be adjusted so that the rays do not gain energy too rapidly. The RKSTEP data, line 49,

specifies a .001 cm step to 1.2 cm, and .0025 cm step to 1.4 cm, after which the value given by CONST(10) will be used.

Case 3 tracks these rays from the end of case 2 to the anode hole, data lines 52-77. The rays are read from the binary disk dump, line 46, by the RAYS 8 data, line 65. The last ray, ray 25, is to be removed, line 66.

The equipotential lines plotted in case 1 are turned off in this case by setting CONST(47) to zero and nulling the EQUIP array, lines 69 and 72. The rays are then tracked with phase plots taken at 2, 3, 4, 5 and 6 cm. The CALCOMP boundary and trajectory plots are shown as Figures 9.3-9.5.

A)
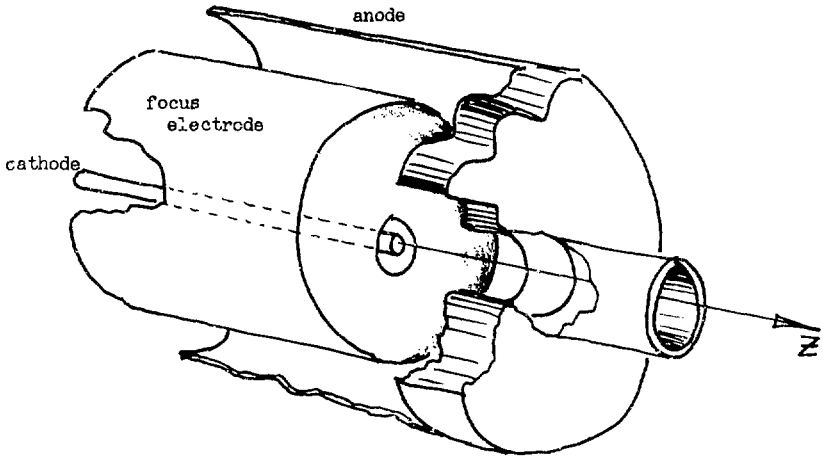
B)
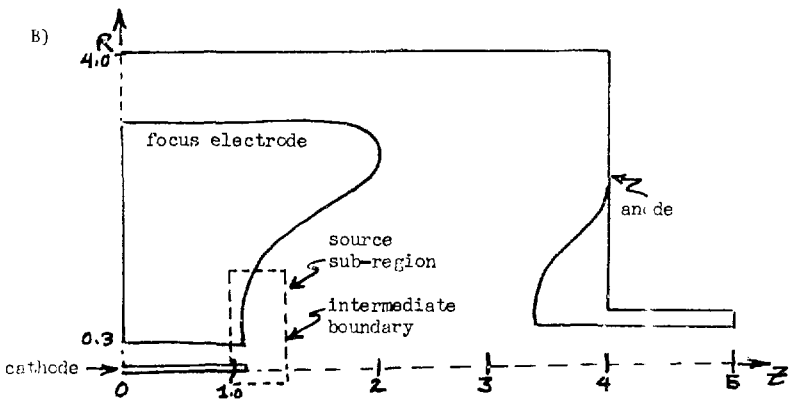
XBL 823-1930

Figure 9.1   X-ray tube electrodes A) and their EBQ bounda y diagram B).

```
GLOBAL PROBLEM BOUNDARY                                              1
INITALIZE                                                           2
BOUNDARY                                                            3
  21 66 0 4. 0 6.5 $ ------- CELL 0.2 X 0.1                         4
  1 0 0 1 0 1.0 S $  CATHODE NEEDLE                                 5
  2 .3 0 2 .3 1 2 .4 1.1 2 .5 1.15 2 2.5 2.45 2 2.85 2.75           6
  2 3.1 2.75 2 3.25 2.6 2 3.3 2.4 2 3.3 0 2 .3 0 S                  7
  3 4.0 3 4.5 3 .75 5 3 .75 6.5 3.0 6.5 3 4 6.5 3 4 5 S            8
  3 .5 6.5 3 .5 4 3 .6 3.8 3 .8 3.8 3 1 4 3 2 4.75 3 2.25 4.9 3 2.5 9
  3 6 S                                                             10
POT                                                                 11
  0 -100 60E3 0R191 S                                               12
CONST                                                               13
  *16 1.0 $ SOLVE POISSONS EQUATION TO 1 VOLT                       14
  *35 -20 -13 $ CALCOMP PLOT SIZE IN CM                             15
  *47 20 $ PLOT 20 EQUIPOTENTIALS ON CALCOMP PLOT                   16
  S                                                                 17
LAPLACE                                                             18
  POT STORE R  11  41  0.7 0.0 0.6 .02 $ LEFT EDGE PROB 2           19
  POT STORE Z  42  67  0.6 1.2 1.7 .04 $ TOP OF PROB 2              20
  POT STORE R  68  93  1.7 0.0 0.6 .02 $ RIGHT EDGE OF PROB 2       21
  POT STORE Z 131 150 0.6 1.7 3.8 .20 $  TOP OF PROB 3              22
  POT STORE R 151 182 1.7 0.0 0.6 .02 $ LEFT EDGE OF PROB 3         23
  PRINT POT                                                         24
  PLOT BOUNDARY-S EQUIP 10 20 50 100 200 500 1000 S                 25
  END                                                               26
$ ----------- BLOW UP CATHODE REGION -------------------------------27
EXPAND REGION AROUND CATHODE POINT                                  28
BOUNDARY                                                            29
  31 26 0 .6 .7 1.7 $ ------- CELL 0.02 X 0.04                      30
  1 .078 .7 1 .078 1.02 1 0 1.02 1 0 .7 1 .073 .7 S                 31
  2 .3 .7 2 .3 1.0  2 .35 1.07 2 .4 1.1 2 .6 1.2 S                  32
  INCREMENT 11 41 $ ------ LEFT EDGE OF CATHON FOCUS GAP            33
  I 0 .7 2 .3 .7 S                                                  34
  INCREMENT 42 67 $ ------ TOP OF PROBLEM                           35
  I .6 1.2 54 .6 1.7 S                                              36
  INCREMEN 68 99 $ ------- RIGHT EDGE OF PROBLEM                    37
  I 0 1.7 98 .6 1.7 S                                               38
  S                                                                 39
LAPLACE                                                             40
CONST                                                               41
  *10 .01 $ RUNGGA-KUTTA INTEGRATION SETP                           42
  *35 -12 -10 $ CALCOMP PLOT SIZE IN CM                             43
  *47 0                                                             44
  S                                                                 45
RAYGEN 2 25 1.0E-7 1.0E-7 2 .1 .07 1.02 0 0 0 0 -5 S                46
PUNCHB 1.7 S                                                        47
PLOT PHASE 1.7 S                                                    48
PLOT TRAJ S                                                         49
RKSTEP .001 1.2 .0025 1.4 S                                         50
TRACK X 1                                                           51
END
```

```
$ ------------- TRANSPORT ELECTRONS ON TO X-RAY TARGET ------------- 52
TRANSPORT RAY S                                                        53
BOUNDARY                                                               54
  31 25 0 .6 1.7 6.5 $ ------ CELL .02 X .2 CM                         55
  OUTSIDE .55 5.0                                                      56
  3 .6 3.8 3 .54 3.82 3 .51 3.9 3 .5 4.0 3 .5 6.5 3 .6 6.5 3 .6 3.8 S 57
  INCREMENT 151 182 $ ----- LEFT EDGE OF PROBLEM                      58
  I 3 1.7 182 .6 1.7 S                                                59
                                                                      60
  INCREMENT 131 150 $ ------ POT EDGE OF PROBLEM                      61
  131 .6 1.7 3 .6 3.8 S                                               61
  3 0 6.5 3 .6 6.5 S                                                  62
S                                                                     63
LAPLACE                                                               64
RAYS 8 1 $ ----- READ DUMP 1                                          65
RAYS REMOVE 25 S                                                      66
PRINT RAYS                                                            67
CONST                                                                 68
  *47 0                                                               69
*34 11 $ PRINT ORBITS TRANSPORT SECTION, WHY CALL CCHELF Z=1.7 CM     70
S                                                                     71
EQUIP S $ TURN OFF PREVIOUS SPECIAL EQUIPOTENTIAL PLOT LINE VALUES    72
PLOT TRAJ S                                                           73
PLOT PHASE 2 3 4 5 6 S                                                74
TRACK                                                                 75
END                                                                   76
END                                                                   77
```
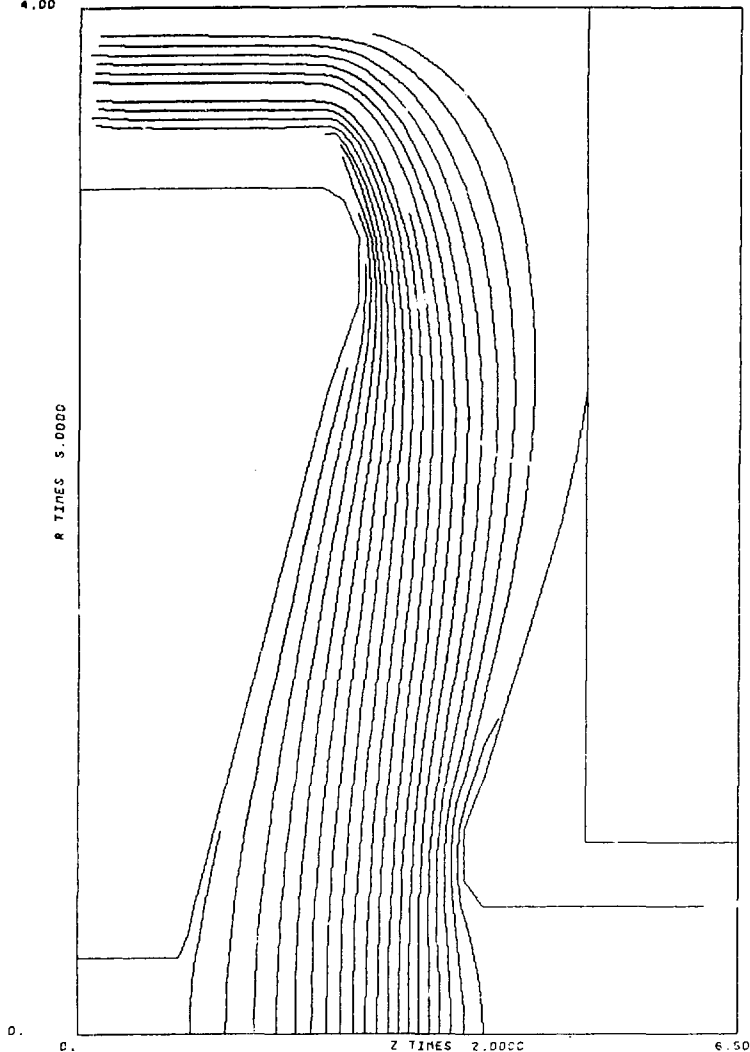
Figure 9.2   EBQ data for x-ray tube. The data divides the problem into
a "Global problem" from which potentials will be extracted
for use as boundary conditions for a second problem "Region
around cathode" used to find the electron orbits for
injection into a third problem "TRANSPORT RAYS."

XBL 823-1931

Figure 9.3   X-ray tube global boundary plot showing electrodes and
             equipotential lines. The boundary covers the entire
             problem region and encloses the cathode and transport
             sub-problem regions. The EBQ POT STORE data commands will
             extract interpolated potentials from this problem to be
             used as boundary conditions for the two subsequent regions,
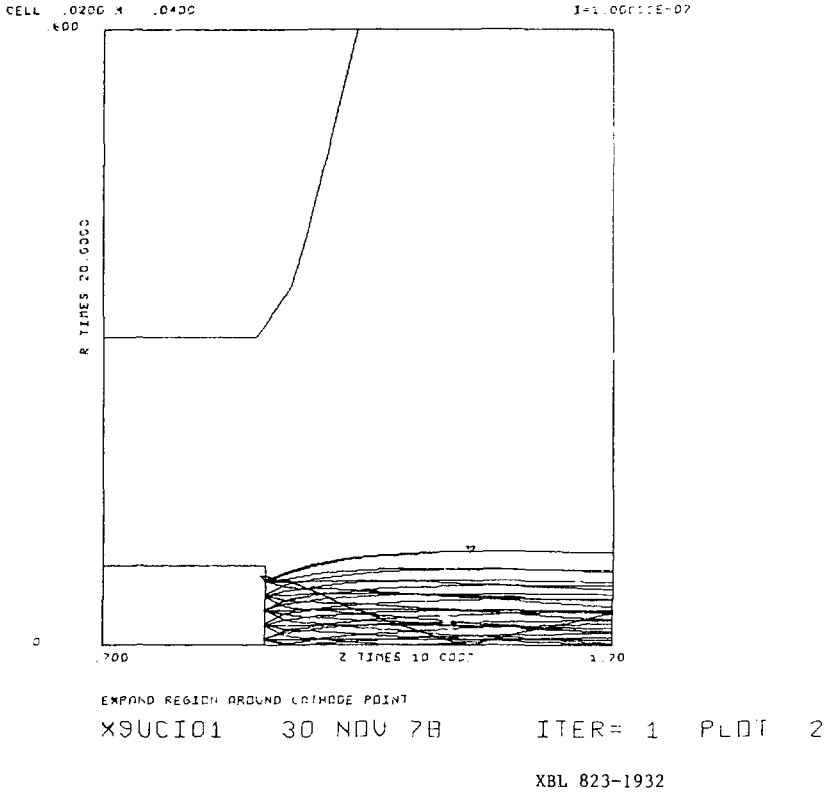   Figure 9.4 and 9.5.

Figure 9.4   EBQ trajectory plot for the sub-problem "expanded region
around the cathode" of the x-ray tube example. Note tip of
the focus electrode on the upper left portion of the plot
and the cathode electrode tip on the lower left.   The
electrons originate at z = 1 cm at essentially zero energy
and are accelerated to z = 1.7 cm, at which point they will
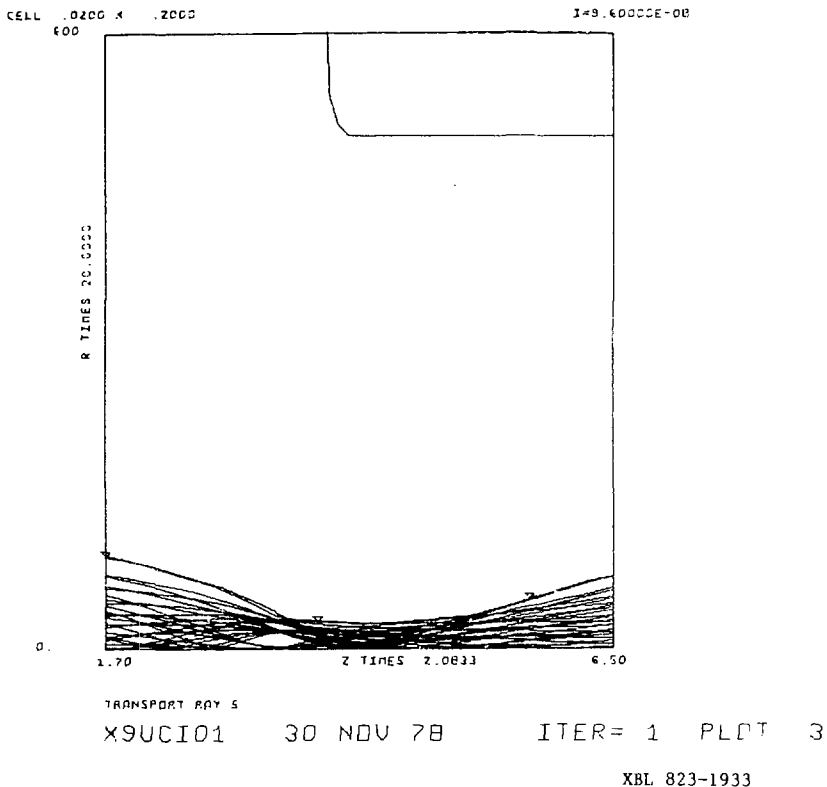be transferred to the next problem, Figure 9.5.

Figure 9.5   EBQ trajectory plot for the sub-problem "Transport Rays" of
the x-ray tube example. Note the anode tip shown in the
upper right of the plot. The trajectories begin at the
left edge as transferred from the previous problem, Figure
9.4, and continue to the right. This problem is at the
lower right portion of the Global problem shown in Figure
9.3

EXAMPLE 10

This example shows the use of a cathode masking with a Child's law start, to generate a hollow beam of electrons from a space charge limited diode. Consider two concentric plane disk electrodes, one a cathode with a ring treated for electron emission from 5 to 12.7 cm in radius, Figure 10.2. The Child's law start is defined by the RAYGEN data, line 15, Figure 10.1 specifies 50 equally space rays starting from a surface originating on the z axis at 0.6 cm. The rays will be distributed only over the radial interval of this surface which projects back to the cathode as defined by a potential index 1 surface, Figure 10.3. Note in the boundary data, the cathode electrode started as a potential 5 surface, from which no electron emission occurs. At 5 cm radius, the potential index of this surface changes to 1, from which electrons can be emitted. At a radius of 12.7 cm the potential index changes back to 5, terminating the emission surface. Both POT(1) and POT(5) have the potential of the cathode surface.

After iteratively converging the Poisson-ray trace cycles, the effect of the space charge is evident from the bows in the equipotential surface's away from the conductive electrode section Figure 10.4. The effect of the space charge is best displayed on a phase space plot, Figure 10.5. The beams self magnetic field generated a self focusing giving a general radial converging (inward slope) to the electrons, while the lack of electro-static space charge on the inside and outside of the beam (beam edges) causes a divergence of the outer edges of the beam away from its core, Figure 10.5.

As a further example of cathode masking, consider the case of a three concentric ring electron emitter shown in Figure 10.6. The data is shown in Figure 10.7 and shows two alternative ways of generating the cathode masking. The first method specifies a single boundary segment all at zero volts, but divided into sections with potential indexes 1, 5 and 10 (POT(1), POT(5), and POT(10) all zero volts) The RAYGEN data specifies emission from only potential index surfaces 1 and masking from potential index 5 surface, with termination of the cathode and mask at the encounter of any other potential index, such as index 10. The cathode generation (i.e., 10 is not index 1 or 5) is terminated at a radius of 6.5 cm in this example by the onset of any non cathodes or mask index in the mesh array. Recall, a potential index extends from the point of its occurrence (I,R,Z) in the mesh up to, but not including the next point (I,R,Z) in the mesh whose potential index is used.

The alternative method shown for the mesh generation of this three ring emission cathode demonstrates the redefinition of mesh points by subsequent electrode specification. Figure 10.7B shows the data where each boundary segment comprising the cathode electrode is given separately, using the potential index 1 for the electron emission sections, index 5 for the masks and index 10 for the cathode terminator section of the cathode electrode.

The ray and phase plots generated by this data is shown in Figures 10.8 and 10.9.

```
$ ------------------------------------------------------------
TEST HOLLOW CATHODE                                              1
BOUNDARY                                                         2
32 7 0 15.5 0 1.5                                                3
5 0 0 1 5 0 5 12.7 0 5 15.5 0 S                                  4
2 0 1.5 2 12.7 1.5 2 15.5 1.5 S                                  5
S                                                                6
POT 0 85.0E3 0 0 0 S                                             7
CONST                                                            8
*10 .1 $ RUNGGA-KUTTA INTEGRATION STEP, CM                      9
*35 -7.75 6.0 $ CALCOMP PLOT SIZE                              10
*37 11 $ CALCOMP AND PRINT PHASE PLOTS                         11
*47 20 $ 20 EQUIPOTENTIAL LINES                               12
S                                                              13
RAYGEN 3 50 1 .6                                               14
(RACK X 10                                                     15
ENDTRACK                                                       16
PLOT SCALES PHASE 0 15.5 -.15 0                                17
PLOT PHASE 1.5 S                                               18
PLOT TRAJ S                                                    19
TRACK CONTINUE 1                                               20
END                                                           21
$ ------------------------------------------------------------ 22
3 CONCENTRIC RING EMITTERS                                     23
INITALIZE                                                      24
BOUNDARY                                                       25
51 10 0 10.0 0 0.9                                             26
10 0 0    1 1.25 0     5 2.5 0    1 3.25 0    5 4.5 0          27
1 5.25 0    10 6.5 0    10 10 0   S $ ----- EMITTER AND MASKS   28
2 0 0.9   2 10 0.9 S $ ----- ANODE                             29
S                                                              30
POT 0 85.0E3 OR8 S                                             31
CONST                                                          32
*47 20 *35 -10 -9                                              33
S                                                              34
RAYGEN 3 50 1 .5 0 5 1.0E8                                     35
PLOT SCALES PHASE 0 10 -.15 .05                                36
PLOT PHASE 0.9 S                                               37
PLOT TRAJ S                                                    38
TRACK X 10                                                     39
END                                                           40
$ ------------------------------------------------------------ 41
3 CONCENTRIC RING EMITTERS DEFINED BY POTENTIAL OVERLAY REGIONS 42
BOUNDARY                                                       43
51 10  0 10  0 0.9                                             44
1 0 0     1 10 0     S $ EMISSION SURFACE                      45
10 0 0    10 1.25 0 S $ ---- INNER MASK LIMIT                  46
5 2.5 0    5 3.25 0 S $ ---- INNER RING MASK                   47
5 4.5 0    5 5.25 0 S $ ---- OUTER RING MASK                   48
10 6.5 0    10 10 0 S $ ----- OUTER MASK LIMIT                 49
2 0 .9 2 10 .9 S $ ANODE                                       50
S                                                              51
CONST                                                          52
*47 20 $ NUMBER OF EQUIPOTENTIALS ON TRAJ PLOTS               53
*35 -10 -9 $ CALCOMP PLOT SIZE 10 X 9 CM                       54
S                                                              55
RAYGEN 3 50 1 .5 100 5 1.0E8                                   56
PLOT TRAJ S
PLOT PHASE 0.9 S
TRACK X 10
END
```
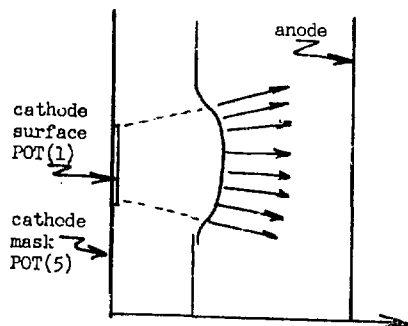
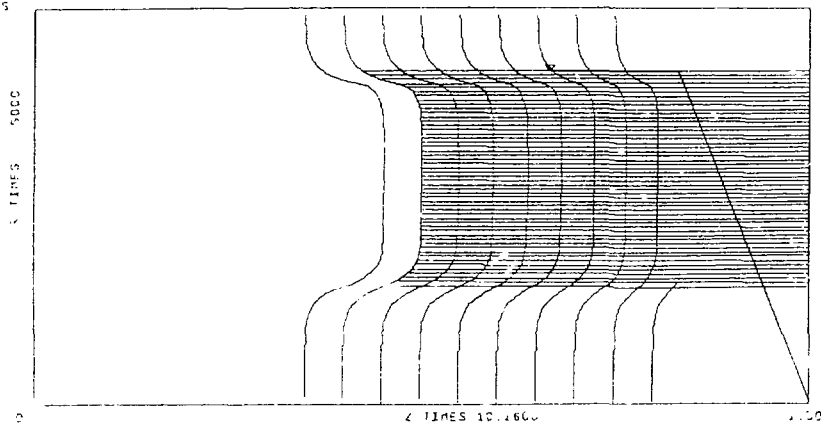Figure 10.1 EBQ data for Child's law cathode masking example.
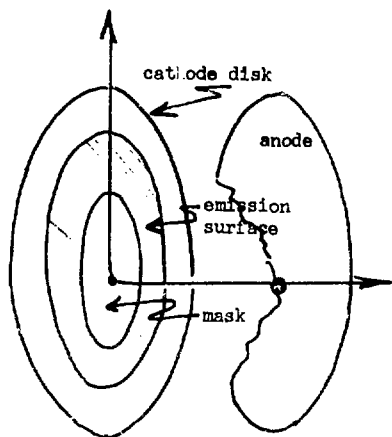
XBL 823-1934
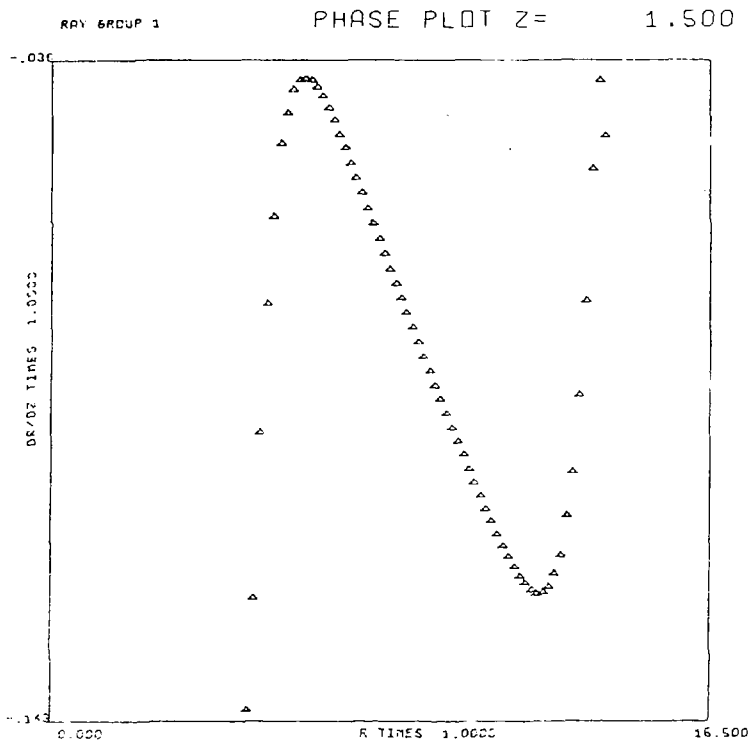
XBL 823-1935

Figure 10.2 Hollow cathode case.

Figure 10.3 EBQ simulation of
hollow cathode.
Note rays emanate
only from region of
potential 1 index.

XBL 823-1936

Figure 10.4 EBQ trajectory plot showing the Child's law electrons
beginning at the first mesh interval after the starting
surface. The curvature in the equipotential lines results
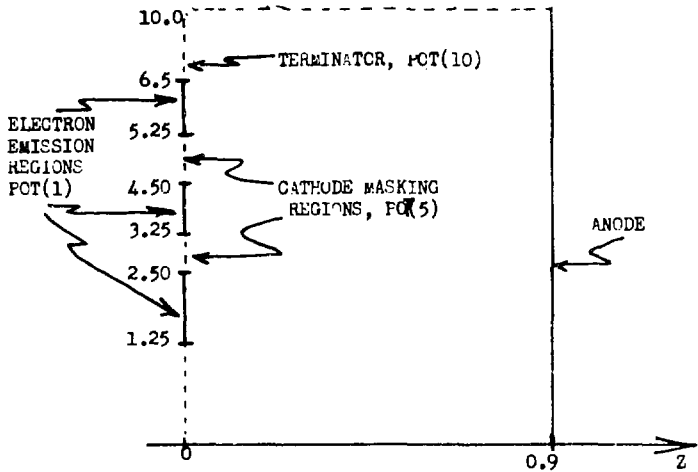from the beams space charge.

Figure 10.5 Phase space for the hollow cathode be shown in Figure
10.4. Note the space charge deflection of the electrons on
the inner and outer edge of the beam.

XBL 823-1938

Figure 10.6 Three concentric electron emission regions geometry
example, illustrating the EBQ cathode masking procedure
with Child's law starting.

```
3 CONCENTRIC RING EMITTERS
BOUNDARY
51 10 0 10 0 0.9
10 0 0 1 1.25 0 5 2.5 0 1 3.25 0 5 4.5 0 1 5.25 0 10 6.5 0 10 10 0 S
2 0 0.9 2 10 0.9 S $ ----- ANODE
S
POT 0 85.0E3 OR8 S
RAYGEN 3 50 1 0.5 0 5 10000000
TRACK X 10
ENDTRACK
PLOT TRAJ S
TRACK CONTINUE 1
END



3 CONCENTRIC RINGS DEFINED BY POT OVERLAY S
BOUNDARY
51 10 0 10 0 0.9
1 0 0 1 10 0 S $ ------ EMISSION
10 0 0 10 1.25 0 S $ ------- INNER LIMIT
10 6.5 0 10 10 0 S $ ----- OUTER LIMIT
5 2.5 0 5 3.25 0 S $ ------ INNER MASK
5 4.5 0 5 5.25 0 S $ ------ OUTER RING MASK
2 0 0.9 2 10 0.9 S $ ----- ANODE
S
POT 0 85000 OR8 S
RAYGEN 3 50 1 0.5 100 5 10000000 $ ---- RAYGEN TYPE N KATH Z ZLIM IMASK
TRACK X 10
END
```
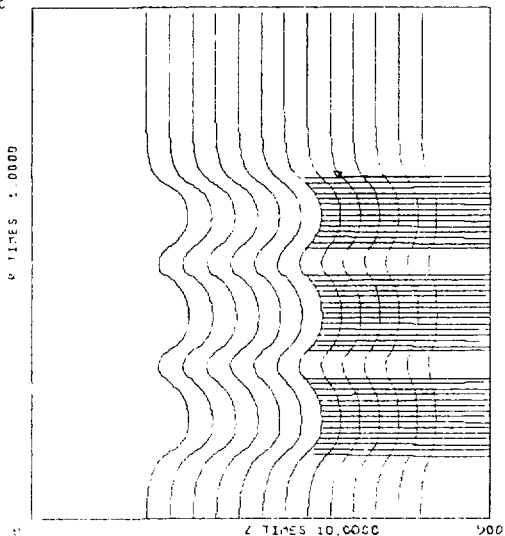
Figure 10.7   EBQ data for the three concentric ring emitters shown in
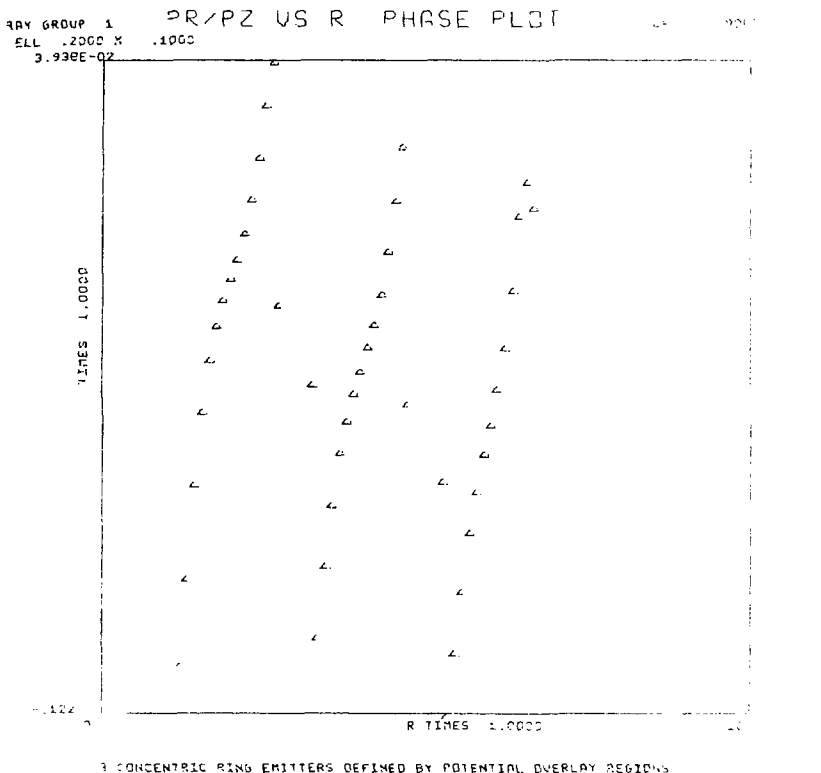Figure 10.6.   The boundary data is given in two alternate
ways.

CELL .2000 X .1000
10 C

XBL 823-1939

Figure 10.8   EBQ trajectory plot showing the Child's law emitted
              electron rings and the space charged altered equipotential
              surfaces for the data shown in Figure 10.7.

Figure 10.9 Phase space of the three concentric emitter case. The rays
are emitted normal to the equipotential surface of Figure
10.8 giving the particular M shaped phase space. Note the
dr/dz axis extends from -.122 to .039 radians, with the
asymmetry toward the converging side created by the
focusing effect of the beams self magnetic field. The
electrons on the six outer edges of the three beam rings
are defocused via the beams space charge, as shown.