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SEARCH FOR MULTIPION RESONANCES IN THE REACTION $\bar{p} + p \to 3\pi^+ + 3\pi^- + n\pi^0$

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ABSTRACT

We report here the study of the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + n\pi^0$ at 1.61 BeV/c ($E_{c.m.}$ = 2.290 BeV), with the aim of detecting multipion resonances in the final states.

The experiment was performed in the Lawrence Radiation Laboratory's 72-inch liquid hydrogen bubble chamber. The total number of six-prong events in the sample is 715. The events were measured with the Franckenstein measuring projector. The events were analyzed by using the PANG, KICK, and EXAMIN programs with IBM 704, 709, and 7090 computers.

The cross sections of various processes are found to be

$\sigma(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-) = 1.16 \pm 0.1$ mb,

$\sigma(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0) = 1.8 \pm 0.25$ mb,

$\sigma(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0) = 1.05 \pm 0.25$ mb.

The angular distributions are symmetrical for all three types of events.

The existence of the ω meson (T = 0 three-pion resonance at 780 MeV) is further confirmed. With the hypothesis of G-parity conservation in the decay process (strong decay), the spin and parity of the ω meson is confirmed as 1^{--} by the Dalitz-plot method. Even with the hypothesis of G-parity nonconservation in the decay process (electromagnetic decay),

the 1^{--} spin-parity assignment is still strongly suggested by the small values of the ratios of $R[(\omega \rightarrow 4\pi)/(\omega \rightarrow \pi^+\pi^-\pi^0)]$ and $R[(\omega \rightarrow \text{neutral})/(\omega \rightarrow \pi^+ + \pi^- + \pi^0)]$. We do not observe any T = 0 three-pion resonance at 550 MeV (η meson).

The neutral four-pion effective mass M_4 distribution shows a suggestive peak at 1.04 BeV.

The distribution of the two-pion effective mass M_2 of the $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-$ events shows a big difference between $|Q| = 2$ (for like-pion pairs) and $Q = 0$ (for unlike-pion pairs) at the low-value region of M_2. At this region the M_2 distribution of like pion pairs lies above that from phase-space calculations, and the one of unlike-pion pairs is well below. We tentatively attribute this effect to the Bose-Einstein effect on the pions.

The ratio $R[(\rho^{\pm} \rightarrow \pi^{\pm} + \eta$ followed by $\eta \rightarrow \pi^+\pi^-\pi^0)/(\rho^{\pm} \rightarrow \pi^{\pm}\pi^0)]$ is determined to be 1.2 ± 2.0%. This small ratio agrees with a 0^{-+} assignment for spin, parity and G parity of the η meson, but cannot rule out the 1^{--} possibility. Upper limits of some other decay rates of ρ and ω mesons are presented.

SEARCH FOR MULTIPION RESONANCES IN THE REACTION $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + n\pi^0$ *†

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June 13, 1962

I. INTRODUCTION

It is a little paradoxical that a search for more resonances or unstable particles can be introduced by a theory that tries to reduce the fundamental particles to three: the "Eightfold Way" Theory.[1]

In this theory Gell-Mann, using the Sakata Model[2] with only three fundamental particles p, n, Λ (and their antiparticles $\bar{p}$, $\bar{n}$, $\bar{\Lambda}$), and supposing that mesons are formed of fundamental baryons and their antiparticles interacting via a "gluon," predicts the existences of two sets of mesons, the pseudoscalar set of 0^- mesons (spin = 0, parity odd) and the vector set of 1^- mesons (spin 1, parity odd). Each set is divided into a singlet and an octet. He also conjectures the existence of the scalar 0^+ and 1^+ axial-vector mesons. These mesons are shown in Table I.

The decay of these proposed particles is governed by the conservation laws of strong (or electromagnetic) interactions. Here we are mostly interested in nonstrange particles (S = 0). Table II shows the prediction of the decays of these particles.

Though the mesons presented in Table II include all possible combinations of spin, isotopic spin, and parity with neither spin nor isotopic spin exceeding unity, they represent only half of the possible states because only one of the two possible G-parity assignments has been indicated. Thus Table II is not merely a listing of all possible states but rather a prediction of which of the possible states should be found. A very similar prediction is obtained by predicting that only those states which can be represented by a nucleon-antinucleon system can exist. The particles allowed by this rule include all those in Table II as well as two other 1^+ mesons, one with $I = 0$, $G = -1$ and the other with $I = 1$, $G = +1$ (see Appendix).

In the vector theory of strong interactions Sakurai predicts the existence of three vector mesons: one with isotopic spin 1 corresponding to the ρ and coupled to the isotopic spin current, and two with isotopic spin 0, of which the heavier, corresponding to B in the Gell-Mann notation, is coupled to hypercharge current and the lighter ω to the baryonic current.[3]

The χ^0 has also been predicted by many theorists.[4]

The existence of the ρ meson has much more theoretical basis due to the calculations by Chew et al., by Federbush et al., and especially by Fraser and Fulco on the isovector form factor of the nucleon.[5] The existence of the ω meson is suggested by Nambu to explain the isoscalar form factor of the nucleon.[6]

Here we propose to search for these mesons in the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + n\pi^0$ by calculating the effective mass of two, three, four, and five pions, to study their distribution to see if they show any striking peaks, and to see if any of these peaks can be identified with the proposed meson decays.

II. SELECTION OF EVENTS

The events that have been analyzed were from interactions of 1.61-BeV/c antiprotons in the 72-inch hydrogen bubble chamber. The design of the "separated" antiproton beam was patterned after the 1.17-BeV/c K beam of Eberhard, Good and Ticho.[7] The details of the beam have been described elsewhere.[8]

The reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + n\pi^0$ can be recognized by a negative beam track producing a six-prong event. Figure 1 shows a typical six-prong event.

Measurements were made only on events that occurred inside a specified fiducial volume in an edited film sample.[9] In this sample there were 16,586 interactions, of which 14,560 ± 300 were antiproton interactions, including 715 six-prong events.[9]

III. MEASUREMENT AND FITTING OF EVENTS

Of the 715 six-prong events 57 were rejected on the scanning table because they showed characteristics of a Dalitz pair, and 63 were rejected because they were unmeasurable. The remaining 595 events constitute an almost pure sample of $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + n\pi^0$. Of these events less than 1% are $\pi^- + p \rightarrow p + 3\pi^- + 2\pi^+ + n\pi^0$.[10] Likewise, George Kalbfleisch has shown that less than 1% of these events are annihilations involving kaons;[8] the biggest contamination came from events with four charged pions and a Dalitz pair. But these events can be eliminated after fitting because they show a negative missing energy or an imaginary missing mass. All 595 events were measured with the Franckenstein measuring projector for the 72-inch bubble chamber.[11] The track reconstructions were performed with the IBM computer program called "PANG."[12] This program computes the momentum, azimuthal angle, and dip angle for each track.

After track reconstruction, a least-squares fit of the events was performed with the IBM program KICK.[13] We tried to fit the events to the reactions

$$\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-, \tag{1}$$

$$\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0. \tag{2}$$

For each fit, a χ^2 function is computed which measures the goodness of the fit.

Normally KICK can handle only seven particles and can at most fit Reaction (1). We have to fit Reaction (2) by considering the antiproton plus proton as a body which decays into seven pions. (Before fitting the decay of a $\bar{p}p$ particle of zero momentum and zero energy into seven pions, we add the appropriate momentum and energy to the particle: momentum = momentum of $\bar{p}$; energy = energy of $\bar{p}$ + mass of proton).

This way we must ignore the uncertainty in the momentum and angle of the $\bar{p}$. But by making use of the known characteristics of the beam, we reduce the uncertainty in the momentum of $\bar{p}$ to less than 2%, and because the incident track is generally long, the measured momentum error is often less than 1.0%; also, the errors on the angles of the beam are small. To make a check we fit the Hypothesis (1) in the normal way and in the new way ($\bar{p}p$ body decaying into six pions). We find that all the fitted values are about the same and that the ratio of the average of the new χ^2 to the average of the old χ^2 is about 1.6, which agrees more or less with a theoretical estimate of 18/15.

An event is considered fitted to Reaction (1) when it has $\chi^2 \leq 30.0$ for this hypothesis. It is considered fitted to Reaction (2) when it has $\chi^2 \leq 5.1$ for this hypothesis and $\chi^2 > 30.0$ for Hypothesis (1).

For Reaction (2) (one constraint) a χ^2 of 5.1 corresponds to a confidence level of 3%. For Reaction (1) (four constraints), the same confidence level would correspond to a χ^2 of about 10, but — as we will see later — our experimental distribution of this reaction seems to be three times that expected; this gives us 30 as a reasonable cutoff value.

When an event does not fit either Reaction (1) or Reaction (2) and has a missing mass ≥ 270 MeV (minus 1 standard deviation) it may have two or more neutral pions missing. Such events were put in a third category called "8π events."

Every event that was rejected by PANG or by KICK or that had a measurement error greater than 50% on the momentum of any track was examined on the scanning table and sent back to be remeasured.

Of the 595 events measured,

153 fitted Hypothesis (1); let us call them "6π events";

239 fitted Hypothesis (2), and were called "7π events";

139 were classified as "8π events";

28 were found to have a negative energy or imaginary missing mass and could be attributed to a Dalitz pair associated with four-prong events

36 remaining were due to bad measurements.

The total number of Dalitz pairs associated with four prongs is thus 57 + 28 = 85; this agrees very well with the prediction (84) of a Lorentz-invariant statistical model using an interaction volume of $5(4\pi/3)(\hbar/m_{\pi}c)^3$, which gives a good charged-pion multiplicity at various energies.[14]

It is interesting to see that the χ^2 distribution for Hypothesis (2) (Fig. 2), which has only one constraint, can be compared somewhat to the theoretical curve, whereas the one for Hypothesis (1) (Fig. 3), which has four constraints, can be compared to the theoretical curve only if we multiply the scale of the latter by a factor of 3. (This could be due to underestimated uncertainties in the measured variables.)

On the other hand, the missing-mass distribution of both the 6π events (Fig. 4) and the 7π events (Fig. 5) look very good and very symmetrical. We believe that about 85% of the 139 8π events actually do have two missing pions, because the missing-mass distribution of these events follows, within statistics, the effective mass distribution of two charged pions coming from the same event (Fig. 6).

An examination of the missing-mass distribution as well as the χ^2 distribution convinces us that the 7π events contain a contamination of less than 17% of the "eight-body" annihilations and about 3% of the "six-body" annihilations. On the other hand, roughly 10% of the true seven-body events have been excluded from the sample and are grouped with the 6π and 8π events.

Further processing of the events was done by an IBM 709 program called EXAMIN.[15] The EXAMIN program consists basically of various

Fortran language subroutines which calculate the various quantities of physical interest desired by the experimenter. For example, for an event it calculates the momentum and the c.m. angle (relative to beam) of each particle, and the cosines of the angle between all pairs of final particles in the $\bar{p}p$ center-of-mass system. It also calculates the effective mass of 2, 3, 4, or 5 pions with all possible combinations of charges.

The effective mass is given by the equation

$$M_n = \left[\left(\sum_1^n E_i\right)^2 - \left| \sum_1^n \vec{p}_i \right|^2 \right]^{1/2} ,$$

where n indicates the number of pions included in the effective mass, and E_i and $\vec{p}_i$ are the energy and momentum respectively of the ith particle. We calculated for each value of M_n an uncertainty δM_n by using the variance-covariance matrix of the fitted track variables, which is evaluated by KICK.

All the important quantities are stored in one magnetic tape called a summary tape.

Many small Fortran 709 IBM programs then use this tape to pick out the right quantities, make comparisons, or make histograms or ideograms, or even make further calculations if necessary.

IV. PRESENTATION AND DISCUSSION OF RESULTS

A. Cross Section, Momentum Distribution, and Angular Distribution

Cross Section

To determine the cross sections of 6π, 7π, and 8π events, we make use of the known antiproton-proton total cross section at 1.61 BeV/c,[16]

$$\sigma_{total} = 96 \pm 3 \text{ mb}.$$

We estimate about 3 mb for the cross section of elastic scattering at very small angles, where the recoil proton track is too short to be detected by our scanners. Thus a cross section of 93 ± 3 mb corresponds to $14,556 \pm 300$ interactions in our sample, of which 715 - 85 = 630 are six-prong events. Because of rejection of events due to difficulty of measurement, we have to apply a correction factor of $630/(153 + 239 + 139) = 1.19$ to the number of 6π, 7π, and 8π events.

To estimate the scanning efficiency for six-prong events, we make an independent second scan of about one-half of our sample. By comparing the results of this scan with the results of the first scan we find a remarkable efficiency of 99% for each separate scan.

The cross sections are then found to be

$$\sigma(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-) = 1.16 \pm .1 \text{ mb},$$

$$\sigma(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0) = 1.80 \pm .25 \text{ mb},$$

$$\sigma(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0) = 1.05 \pm .25 \text{ mb}.$$

Table III shows the cross sections of antiproton-proton processes at 1.61 BeV/c.

Momentum Distribution

Figure 7 shows the momentum distribution in the $\bar{p}p$ center-of-mass system, respectively for π^+, π^-, and both π^+ and π^-, for the 6π events. The curves represent the Lorentz-invariant phase-space calculations. We observe that distributions of π^+ and π^- mesons are alike, as predicted by CP invariance,[19] and that both of them agree very well with the phase-space calculation.

Figures 8 and 9 show the momentum distributions in the $\bar{p}p$ center-of-mass system for the charged pions only in the 7π events and in the 8π events. Here we do not yet have the phase-space calculation, but we observed that the π^+-meson distribution agrees with the π^--meson distribution, and neither shows any significant peaks.

Figure 10 shows the c. m. momentum distribution of the neutral π of the 7π events. This distribution agrees very well with the distribution of charged pions from the same event (Fig. 8). If a real 6π event were mistaken as a 7π event, the fake π^0 would have a small momentum in the laboratory system; in the $\bar{p}p$ c. m. system its momentum would be about 100 MeV/c or less. As shown in Fig. 10, we have 14 events with $p_{\pi^0} \leq 100$ MeV/c. From the c. m. momentum distribution of charged pions of 7π events (Fig. 8) we can estimate nine true 7π events (52/6) with $p_{\pi^0} \leq 100$ MeV/c. This gives us an estimation of about 14 - 9 = 5 real 6π events that are mistaken for 7π events.

Angular Distribution

The c. m. angular distributions for π^+ and π^- mesons of 6π, 7π, and 8π events are plotted respectively in Figs. 11, 12, and 13. As expected on the basis of C and CP,[19] the distributions of π^+ appear to be equal, within statistics, to the reflections of the angular distribution of the π^-'s. For the three types of events, we have reflected the π^- distributions about cos $\theta = 0$, added them to the π^+ distributions, and plotted them in the bottom histograms in Figs. 11, 12, and 13. All these distributions look very symmetrical and become more and more isotropic as the number of pions increases. These results are in contrast with the asymmetry found by Maglić, Kalbfleisch, and Stevenson in the angular distribution of $\pi^{\pm}$ mesons coming from the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$ at the same antiproton energy.[20] One explanation could come from the Koba and Takeda theory,[21] which assumes that about two annihilation pions are emitted from the antiproton and proton clouds. Those pions would be responsible for the asymmetry in the angular distribution. In the 6π, 7π, and 8π events the ratio of "cloud" pions over "core" pions is smaller than in the 4π and 5π events ($\bar{p} + p \rightarrow 2\pi^+ + 2\pi^-$ and $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$), and the asymmetry can be masked. It is true also that the numbers of 6π, 7π, and 8π events are much smaller than the numbers of 4π and 5π events (see Table III), so that we cannot observe a small asymmetry.

Figure 14 (a) shows the angular distribution of π^0 from the 7π events. CP predicts this distribution to be symmetrical,[19] but our results seem to show a small forward-backward asymmetry and can be fitted to a $(1 + a \cos \theta)$ distribution with $a = 0.26 \pm .11$. Although it is only 2.3 standard deviations from $a = 0$, we have looked at all possible biases:

(a) It is possible that some real 7π events with backward π^0 have been misclassified as 6π events.

(b) Another bias can come from the real 8π background ($\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0$) in our 7π events ($\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0$). Let us use the name "dipi" for the real $2\pi^0$ system and "fake π^0" for that calculated when it is fitted to a 7π hypothesis.

In the lab system we have $\vec{p}_{(\text{fake } \pi^0)} \approx \vec{p}_{(\text{dipi})} \equiv \vec{p}$, but $E_{(\text{fake})}$ tends to be smaller than E_{dipi}. When the dipi is transformed to the $\bar{p}p$ c.m. system its momentum becomes

$$\begin{cases} p'_{\parallel} = \gamma p_{\parallel} - \eta E_{dipi}, \\ p'_{\perp} = p_{\perp} . \end{cases}$$

The c.m. momentum of the "fake π^0" would be

$$\begin{cases} p' = \gamma p - \eta E_{fake\ \pi^0} \\ p' = p . \end{cases}$$

Since $E_{fake\ \pi^0} < E_{dipi}$, this makes the component in the $\bar{p}$ direction of the "fake π^0" too big in the $\bar{p}p$ c.m. system, and therefore produces a fake forward peak.

To check the first bias hypothesis we plot the π^0 angular distribution of those 15 events which were ambiguous between "6π - 7π" (15 events with $\chi^2 \leq 30.0$ for the 6π hypothesis and $\chi^2 \leq 5.1$ for the 7π hypotheses). This angular distribution is isotropic and cannot produce the asymmetry observed.

To check the second bias hypothesis we replot in Fig. 14 (b) the angular distribution of the 211 7π events, excluding the 28 7π-8π ambiguous events (events with $\chi^2 \leq 5.1$ for the 7π hypothesis but with a missing mass ≥ 270 MeV). This more carefully selected 7π distribution is much less asymmetrical and can be considered as isotropic within statistics. [A best fit gives $1 + (0.15 \pm .12) \cos\theta$.]. We conclude that the small forward-backward asymmetry in the angular distribution of π^0 was due to the 8π background in the 7π events (17%). A π^0 angular distribution of real 7π events would be isotropic, in agreement with CP prediction. We have also made sure that the association of the "7π-8π" ambiguous events does not change our other results appreciably.

B. Effective Mass Distribution of Three Pions

Existence of the I = 0 Three-Pion Resonance

For each event we have evaluated the three-pion effective mass,

$$M_3 = [(E_1 + E_1 + E_3)^2 - (\vec{P}_1 + \vec{P}_2 + \vec{P}_3)^2]^{1/2}.$$

The 6π and the 8π events can yield M_3 combinations with charge $|Q| = 1$ (that is, $\pi^\pm \pi^\pm \pi^\mp$) and $|Q| = 3$ (that is, $\pi^\pm \pi^\pm \pi^\pm$). In the 8π events we could also form another combination of M_3 with $|Q| = 1$ (that is, $\pi^\pm \pi^0 \pi^0$). The M_3 distributions for these triplets do not show any significant peak. These data, however, are less useful than for 7π events for which we can form combinations with $|Q| = 0, 1, 2$ and make comparisons between their distributions.

Each 7π event can yield 33 triplets corresponding to the charge states

$Q = 0,$	$\pi^+ \pi^- \pi^0$	(9 combinations);
$\|Q\| = 1,$	$\pi^\pm \pi^\pm \pi^\mp$	(18 combinations);
$\|Q\| = 2,$	$\pi^\pm \pi^\pm \pi^0$	(6 combinations).

We calculated for each value of M_3 an uncertainty δM_3 by using the variance-covariance matrix of the fitted track variables, which is evaluated by KICK. The M_3 resolution[22] is $\Gamma_{resol}/2 = 9.0$ MeV. However, because of systematic errors known to exist in our track reconstruction, our estimate of $\Gamma_{resol}/2$ probably should be increased by $\sqrt{3}$ to $\Gamma_{resol}/2 = 16$ MeV.

Figure 15 is a histogram of the M_3 distribution for the 239 7π events. The distributions (a) and (b) are for the charge combinations $|Q| = 1$ and 2 respectively. To show the difference between the neutral M_3 distribution and that for $|Q| \geq 1$, we have replotted at the bottom of Fig. 15 both the neutral distribution and 9/24 of the sum of the $|Q| = 1$ and $|Q| = 2$ distributions. This latter distribution we use as an estimate of the background. The neutral distribution, (c), shows a peak at 780 MeV that contains 79 pion triplets above the background of 238. Figure 16 is a histogram of the neutral distribution around the 780-MeV region.

The peak at 780 MeV can be interpreted as a resonance of isotopic spin I = 0, with a half width Γ of about 15 MeV. The isotopic spin, the energy, and the half width of this resonance agree very well with the ω meson found by Maglić et al., who analyzed the four-prong annihilations in the same experiment.[23] We must also conclude, as they did, that the half width of the experimental peak is so close to our resolution that the true width of the peak is less than 15 MeV and could be zero.

Of the 239 7π events, 79 ± 18 (or 33 ± 8%) of these proceed via the reaction $\bar{p} + p \rightarrow \omega + 2\pi^+ + 2\pi^-$, $(\omega \rightarrow \pi^+\pi^-\pi^0)$, for which the cross section is 0.6 ± .15 mb. It is interesting to compare this yield with the 130 ± 20 events (or 12 ± 3%) of the 5π events $(\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$, studied by Maglić et al.[23]) that proceed via the reaction $\bar{p} + p \rightarrow \omega + \pi^+ + \pi^-$.

Spin and Parity of the ω Meson

To determine the spin and parity of this resonance (or unstable particle) we used a Dalitz plot.[24] We used events of the center of the peak (760 to 800 MeV) and events in a control region (822 to 878 MeV) of the same charge state. About 27% of the triplets in the peak region belong to the resonance. Figures 17 and 18 show the Dalitz plot of the peak-region events and the control-region events, respectively. Unit area on a Dalitz plot is proportional to the corresponding Lorentz-invariant phase space, so that the density of plotted points is proportional to the square of the matrix element. It is easily shown that the size of the figure is proportional to $T_1 + T_2 + T_3 = Q = M - (2M_{\pi\pm} + M_{\pi^0})$. Because of the finite width of the peak and the control region, Q varies from event to event, so we use normalized variables, T_i/Q.

At first we will take the hypothesis of G-parity conservation in the decay of the ω meson (strong interaction). In this hypothesis, the G parity of the resonance must be the same as that of three pions, i. e., negative. Then there are three possible three-pion resonances with T = 0, $J \leqslant 1$: the vector meson (1^{--}), the pseudoscalar meson (0^{--}), and the axial meson (1^{+-}) (the first superscript indicates the parity, the second, G parity). Table IV shows the three possible hypotheses, with their characteristics. The meaning of the angular momenta ℓ and L is as follows: the matrix element is analyzed in terms of a single pion plus a dipion; the pions of the dipion are assigned a momentum q and an angular momentum L (in the dipion rest frame); then another pair of variables, p and ℓ, describes the remaining pion in the rest frame.[25]

In Fig. 19b are plotted the curves of density of the population of events on the Dalitz plot corresponding to the three possible types of mesons. These curves have been calculated by Stevenson et al.[25] In Fig. 19a we plot the numbers of events per unit area of the Dalitz plot for the peak region and for the control region versus the distance from the center of the plot. The number of events per unit area of the control region is normalized to be equal to the estimated background in the peak region. Fig. 19b shows the difference of the two sets of points in Fig. 19a. We assume that this difference represents the number of events per unit area of the resonance.[26] These data agree very well with the curve predicted by a matrix element of a vector meson (1^{--}), and not at all with the prediction of a pseudoscalar (0^{--}) or axial vector meson (1^{+-}).

Because the width of the resonance is very small, Duerr and Heisenberg suggest the possibility of electromagnetic decay with nonconservation of G parity.[27] With the present data,[23, 25] they can already eliminate many types of mesons except the 1^{++}, 0^{-+}, and 1^{--} mesons. But as we will see in the next section, the small value of the ratio $R(\omega \rightarrow 4\pi/\omega \rightarrow \pi^+\pi^-\pi^0)$ and

$R(\omega \not\to \text{neutral } \pi^+\pi^-\pi^0)$ will eliminate 1^{++} and favor strongly the 1^{--} spin-parity and G-parity interpretation for the ω meson.

With the spin and parity 1^- we conclude that the ω meson can be the particle predicted by Nambu to explain the electromagnetic form factors of the proton and neutron,[6] and also expected in the vector-meson theory of Sakurai,[3] or as a member of an octet of vector mesons, according to the unitary symmetry theory.[1] Chew has pointed out that on dynamical grounds such a vector meson can exist as a bound state.[28]

C. Effective Mass Distribution of Four Pions

Motivation for a Search for a Four-Pion Resonance

Since the discovery of the two-pion and three-pion resonances[23, 29, 30] the search for a four-pion resonance has acquired much interest. The interest is threefold:

(a) Chew and Frautschi,[31] using the "Regge poles" theory, predict a possible resonance (or unstable particle) with spin 2 and other quantum numbers those of the "vacuum" (T = 0, parity even) at the region of 1 BeV. This particle could decay into two, four, or six pions. But the four-pion decay could possibly be favored because a two-pion decay would require a d wave, whereas a four-pion decay would need only two pion sets in p wave. (This particle was first theoretically predicted by Lovelace, but at 400 MeV.[31])

The four-pion resonance could also come from a decay of χ^0, the pseudoscalar meson with I = 0, formulated by many theoreticians.[1, 4]

(b) To the ω meson has been attributed a spin and parity 1^{--} (the first superscript refers to the parity and the second to G parity) if the decay is through strong interactions. But because of the small width of this meson (Γ/π not more than 12 MeV, and possibly zero[23]), Duerr and Heisenberg suggest the possibility of electromagnetic decay with violation of G parity.[27]

Four more states will then have to be considered: 0^{++}, 1^{-+}, 1^{++}, and 0^{-+} (we consider only states with spin ≤ 1). Duerr and Heisenberg eliminate the state 0^{++} and 1^{-+} because their three-pion decay would either not occur or be extremely weak.[27] For the three-pion decay of 1^{++} or 0^{-+} the Dalitz plot has uniform density, and these assignments cannot be eliminated with the present statistics on Dalitz plots. Therefore, the ω meson can still have one of the three spin and parity combinations 1^{--}, 0^{-+}, or 1^{++}. But Duerr and Heisenberg point out that these three states behave differently with respect to the four-pion decay.[27] For 1^{--} the four-pion decay is strongly forbidden, and therefore is completely negligible compared with the three-pion decay. For 0^{-+} the four-pion decay is an allowed transition, but reduced to small value by the fact that two D states and one P state are required for the outgoing waves.[27] For 1^{++} the four-pion decay is allowed and can be large.[27] Therefore, the very existence of a neutral four-pion resonance at 780 MeV would rule out the 1^{--} spin parity; its nonexistence would probably rule out the possibility of the 1^{++} spin parity, but not the possibility of the 0^{-+} spin parity.

It would also be interesting to see the decay of the ρ meson into four pions. This decay is allowed by strong interaction, but is not as favorable as the two-pion decay. Of special interest is the decay mode $\rho \rightarrow \pi + \eta$, with $\eta \rightarrow \pi^{+}\pi^{-}\pi^{0}$ (η being the I = 0 550 MeV three-pion resonance discovered by Pevsner et al.[30]) Because the G parity of ρ is +1 and that of π is -1, this decay mode of ρ is allowed if the G parity of η is -1 and forbidden if it is +1.

Search for Four-Pion Resonances

For all categories of events we have evaluated the four-body effective mass

$$M_4 = [(E_1 + E_2 + E_3 + E_4)^2 - (\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4)^2]^{1/2}$$

for each pion quadruplet.

For the 6π and 8π events we can get only the combinations $Q = 0$ (nine quadruplets for each event) and $|Q| = 2$ (six quadruplets for each event). For the 7π events we can also get the $|Q| = 1$ combination (18 quadruplets for each event).

For the 8π events we can also calculate the effective mass of two charged pions and two neutral pions by calculating the missing mass of the system consisting of the incoming antiproton, the proton target, and the four remaining visible charged pions:

$$M'_4 = [(E_{\bar{p}} + M_p - E_1 - E_2 - E_3 - E_4)^2 - (\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3 - \vec{p}_4)^2]^{1/2}.$$

For M'_4, we can form only the $Q = 0$ and $|Q| = 2$ combinations. We calculated for each value of M_4 or M'_4 an uncertainty δM_4 or $\delta M'_4$ by using the error matrix calculated by KICK. For the 8π events the half-width $\Gamma_{res}/2$ of the resolution function of M_4 is 13.5 MeV, and for M'_4 is 14 MeV. However, because of systematic errors known to exist in our track reconstruction, our estimate of $\Gamma_{res}/2$ probably should be increased by $\sqrt{3}$ to $\Gamma_{res}/2 = 23$ MeV for M_4 and $\Gamma_{res}/2 = 24$ MeV for M'_4.

For the 6π and the 7π events $\Gamma_{res}/2$ is a little smaller.

Figure 20a is the histogram of the M_4 distribution of the $Q = 0$ combination of the 6π events. The solid-line curve represents the background distribution estimated from the $|Q| = 2$ distribution of the same events (smooth curve drawn through $|Q| = 2$ distribution).

In Fig. 20, (b) and (c) are the histograms of the M_4 distribution of the 7π events, respectively with $Q = 0$ and $|Q| = 1$. We use the $|Q| = 1$ and $|Q| = 2$ distributions to estimate the background distribution (solid-line curves). In Fig. 20 (d) we renormalize the $|Q| = 1$ distribution of the 7π events and plot it against the neutral distribution of the same events.

None of these histograms shows any strong disaccord with the background distribution.

In Fig. 21 we plot separately (a) the histogram of the neutral distribution of $M_4(\pi^+\pi^-\pi^+\pi^-)$ and (b) the histogram of the neutral distribution of $M'_4(\pi^+\pi^-\pi^0\pi^0)$ of the 8π events. The solid-line curves represent the background distribution estimated from a smooth curve drawn through the sum of the distribution of $M_4(\pi^\pm\pi^\pm\pi^\pm\pi^\mp)$ and of $M'_4(\pi^\pm\pi^\pm\pi^0\pi^0)$ with $|Q| = 2$ of the same events (Fig. 22a). Figure 22(b) is the histogram of the sum of the neutral distributions of M_4 and M'_4.

The neutral M_4 distribution shows a suggestive but inconclusive peak at the region of 1.040 BeV. If this peak really exists, it may be a resonance with I = 0 or I = 1. It could come from a possible decay of the χ^0 meson (I = 0, 0^{-+}) or the particle predicted by Chew and Frautschi (I = 0, 2^{++}). The latter meson could also decay into two pions or two kaons.

Ratio ($\omega \to 4\pi/\omega \to \pi^+\pi^-\pi^0$) and Spin and Parity of the ω Meson

To estimate the ratio $R(\omega \to 4\pi/\omega \to \pi^+\pi^-\pi^0)$ we note that we have seen in the same sample of $\bar{p}p$ interactions 79 ± 18 interactions of the form $\bar{p} + p \to 2\pi^+ + 2\pi^- + \omega$, with $\omega \to \pi^+ + \pi^- + \pi^0$. If the ω produced by the preceding reaction were to decay by $\omega \to \pi^+ + \pi^- + \pi^0 + \pi^0$ we would see them in our 8π events. But the distribution of $M'_4(\pi^+\pi^-\pi^0\pi^0)$ (Fig. 21) does not show anything over the phase-space calculation at the region 780 ± 20 MeV. At this energy the background is about 26 pion quadruplets; therefore we can estimate a maximum of 10 pion quadruplets that could come from the decay of the ω, and the upper limit of the ratio of $R(\omega \to \pi^+\pi^-\pi^0\pi^0/\omega \to \pi^+\pi^-\pi^0)$ is about 12%.

If the ω mesons produced by the preceding reaction ($\bar{p} + p \to 2\pi^+ + 2\pi^- + \omega$) were to decay into $2\pi^+ + 2\pi^-$ we would see them in the reaction $\bar{p} + p \to 4\pi^+ + 4\pi^-$. We have only 4 ± 2 of the latter reactions.[9] This gives a maximum of 5% for $R(\omega \to \pi^+ + \pi^- \pi^+\pi^-/\omega \to \pi^+\pi^-\pi^0)$. We can then conclude that the ratio $R(\omega \to 4\pi/\omega \to \pi^+\pi^-\pi^0)$ is less than 17%, and can very possibly be zero.

If the ω produced by $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \omega$ were to decay in the neutral mode, it would show in the distribution of the missing mass of the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$. By $\omega \rightarrow$ neutral, we mean the decays $\omega \rightarrow 3\pi^0$, $\omega \rightarrow 2\gamma$, and $\omega \rightarrow \pi^0 + \gamma$. Looking at the latter distribution, J. Button et al. reported seeing no "peak" at the region of 780 MeV,[17] and using our value of 0.6 ± .15 mb for the cross section of the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \omega$ with $\omega \rightarrow \pi^+ + \pi^- + \pi^0$, they estimate $R(\omega \rightarrow \text{neutral}/\omega \rightarrow \pi^+\pi^-\pi^0) < 0.5$.

The small value of $R(\omega \rightarrow 4\pi/\omega \rightarrow \pi^+\pi^-\pi^0)$ agrees with a spin and parity assignment of 1^{--} and probably rules out the 1^{++} assignment, but does not rule out the possibility of 0^- for the spin and parity of the ω meson.[27]

The ratio of $R(\omega \rightarrow \text{neutral}/\omega \rightarrow \pi^+\pi^-\pi^0)$ is estimated by Duerr and Heisenberg to be larger than 3/2 for the 0^{-+} assignment and very small (10^{-4}) for the 1^{--} assignment.[27] Our values for the two ratios $R(\omega \rightarrow 4\pi/\omega \rightarrow \pi^+\pi^-\pi^0)$ and $R(\omega \rightarrow \text{neutral}/\omega \rightarrow \pi^+\pi^-\pi^0)$, which can be very small, agree with the 1^{--} assignment and disagree with the 0^{-+} assignment for the spin and parity of the ω meson. Since all other interpretations of spin and parity (with spin ≤ 1) can be ruled out by the present data,[25] we conclude that the spin and parity of the ω meson is most probably 1^{--}. This agrees with the conclusion reached by Stevenson et al.[25] Table V shows a summary of the experimental determination of spin, parity, and G parity of the ω meson.

Ratio $(\rho \rightarrow 4\pi)/(\rho \rightarrow 2\pi)$

To estimate the ratio of $R(\rho \rightarrow 4\pi)/(\rho \rightarrow 2\pi)$ we use some results from J. Button et al.[17] They find about 386 ρ^0 with $\rho^0 \rightarrow \pi^+\pi^-$ and about 274 $\rho^{\pm}$ with $\rho^{\pm} \rightarrow \pi^{\pm}\pi^0$ by analyzing the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$, from a smaller sample of the same $\bar{p}$ picture of our experiment. In our larger sample this would correspond to 482 ρ^0 with $\rho^0 \rightarrow \pi^+\pi^-$, and 329 $\rho^{\pm}$ with $\rho^{\pm} \rightarrow \pi^{\pm}\pi^0$. If the ρ mesons produced by the same mechanism decayed into $\rho^0 \rightarrow 2\pi^+ + 2\pi^-$

and $\rho^{\pm} \rightarrow \pi^0 + \pi^+ + \pi^- + \pi^{\pm}$ we would see them in the M_4 distributions of the 7π events (Fig. 20, b, c). In the region around 750 MeV in these distributions we see nothing exceeding phase-space predictions and we estimate a maximum of 2% for $R(\rho^0 \rightarrow \pi^+\pi^-\pi^+\pi^-/\rho^0 \rightarrow \pi^+\pi^-)$ and a maximum of 5% for $R(\rho^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^+\pi^-/\rho^{\pm} \rightarrow \pi^{\pm}\pi^0)$. A crude phase-space calculation predicts for $R(\rho \rightarrow 4\pi/\rho \rightarrow 2\pi)$ a value of $\lambda^2/4$, where $\lambda = \Omega/\Omega_0$, Ω being the interaction volume of the ρ meson and $\Omega_0 = (4\pi/3)(\hbar/m_\pi c)^3$. Thus an experimental ratio $R \leq 5\%$ would give $\lambda \leq 0.5$ for the ρ meson.

To estimate the ratio $R(\rho^{\pm} \rightarrow \pi^{\pm} + \eta,\ \eta \rightarrow \pi^+\pi^-\pi^0/\rho^{\pm} \rightarrow \pi^{\pm}\pi^0)$ we analyze carefully all the $|Q| = 1$ quadruplets with effective mass M_4 in the region 750 ± 50 MeV (41 quadruplets). In particular we compare the $Q = 0$ distribution of the effective mass of three pions $M_3(\pi^+\pi^-\pi^0)$ coming from these quadruplets with that for $|Q| \leq 1$, $(\pi^{\pm}\pi^{\pm}\pi^{\mp}$ and $\pi^{\pm}\pi^{\pm}\pi^0)$ (the latter is used here as an estimated background). In the region 548 ± 10 MeV we have 19 neutral triplets and 15 charged triplets. This enables us to estimate the number of $\rho^{\pm} \rightarrow \pi^{\pm} + \eta$ with $\eta \rightarrow \pi^+\pi^-\pi^0$ to be 4 ± 6, and the ratio $R(\rho^{\pm} \rightarrow \pi^{\pm} + \eta,\ \eta \rightarrow \pi^+\pi^-\pi^0/\rho^{\pm} \rightarrow \pi^{\pm}\pi^0)$ to be $1.2 \pm 2.0\%$. This result agrees very well with that of Rosenfeld et al., who find $R(\rho^+ \rightarrow \pi^+ + \eta,\ \eta \rightarrow \text{neutral}/\rho^+ \rightarrow \pi^+\pi^0) \leq 0.6\%$.[32] The actual data on the η meson seem to rule out all spin parity assignments except 1^{--} and 0^{-+}. The theoretical ratio $R(\rho \rightarrow \pi + \eta/\rho \rightarrow \pi\pi)$ is very small [i.e., proportional to $(e^2/\hbar c)^2$] for 0^{-+}. For 1^{--}, this ratio is not yet well determined (25% for a simple phase-space calculation,[31] 1% after Glashow and Sakurai[33]). We conclude that the small value of the ratio $R(\rho \rightarrow \pi + \eta/\rho \rightarrow \pi + \pi)$ agrees with the 0^{-+} assignment for the spin, parity, and G parity of the η meson; whether or not this can rule out the 1^{--} assignment depends on a more precise calculation.

Effective - Mass Distribution of Five Pions

The interest in a five-pion resonance is twofold:

(a) it could come from the decay of π' mesons, a scalar meson (0^{+-}) with isotopic spin I = 1;

(b) it would be interesting to know the ratio of $(\omega \to 5\pi/\omega \to \pi^+\pi^+\pi^0)$. The decay $(\omega \to 2\pi^+ + 2\pi^- + \pi^0)$ is allowed by strong interaction but is not as favorable as $(\omega \to \pi^+ + \pi^- + \pi^0)$, by phase-space considerations.

We have evaluated the five-pion effective mass for each type of event:

$$M_5 = [(E_1 + E_2 + E_3 + E_4 + E_5)^2 - (\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \vec{p}_5)^2]^{1/2}.$$

The 6π and 8π events can yield only the combination with total charge $|Q| = 1$. The distributions of these quintuplets are quite smooth. Each 7π event can yield 21 quintuplets corresponding to the charge states

$Q = 0,$	$\pi^+\pi^+\pi^-\pi^-\pi^0,$	(nine combinations);
$\lvert Q\rvert = 1,$	$\pi^\pm\pi^\pm\pi^\pm\pi^\mp\pi^\mp,$	(six combinations);
$\lvert Q\rvert = 2,$	$\pi^\pm\pi^\pm\pi^\pm\pi^\mp\pi^0$	(six combinations).

Figure 23(a, b, c) shows the histogram of the distributions of M_5 of 7π events with $Q = 0$, $|Q| = 1$, and $|Q| = 2$, respectively. The smooth curves represent the background distributions estimated from smooth curves drawn through the $|Q| = 2$ distributions. These distributions do not show any significant deviation from the estimated background.

This does not mean that the π' meson does not exist. Because in one event we can form many pion quintuplets, our efficiency of detecting new resonances is very poor. (For example, we can form 15 quintuplets with $Q = 0$ or $|Q| = 1$, but obviously, only one quintuplet can come from the π' meson).

To estimate the ratio $R(\omega \to 2\pi^+ + 2\pi^- + \pi^0/\omega \to \pi^+ + \pi^- + \pi^0)$ we note that Maglić et al.[22, 24] have reported seeing 130± 20 interactions of the form $\bar{p} + p \rightarrow \omega + \pi^+ + \pi^-$ with $\omega \rightarrow \pi^+ + \pi^- + \pi^0$ in the same sample of $\bar{p}p$ interaction. If the mesons produced by the preceding interaction decayed by $\omega \rightarrow 2\pi^+ + 2\pi^- + \pi^0$, we would see them in our 7π. But the distribution of $M_5(\pi^+\pi^+\pi^-\pi^-\pi^0)$ of the 7π events (Fig. 23a) does not show anything at the region 780 ± 20 MeV, and we can estimate a maximum of 1% for the ratio $R(\omega \rightarrow 2\pi^+ 2\pi^- 2\pi^0/\omega \rightarrow \pi^+\pi^-\pi^0)$. This result agrees very well with the prediction from a crude Lorentz-invariant phase-space calculation which gives a value of $\lambda^2/3000$ for this ratio, where $\lambda = \Omega/\Omega_0$, Ω being the interaction volume of the ω meson and $\Omega_0 = (4\pi/3)(\hbar/m_\pi C)^3$. For $R < \frac{1}{100}$, $\lambda < 5.4$, which is expected.

Effective Mass Distribution of Two Pions, and Angular Correlations

For every event we have evaluated the two-pion effective mass,

$$M_2 = [(E_1 + E_2)^2 - \left|\vec{p}_1 + \vec{p}_2\right|^2]^{1/2}.$$

The 6π and 8π events can yield only pion pairs with charge $Q = 0$ $(\pi^+\pi^-)$ and $|Q| = 2(\pi^\pm\pi^\pm)$ (nine and six combinations, respectively, per event). The 7π events can also yield pairs with charge $|Q| = 1(\pi^\pm\pi^0)$ (six combinations per event). We calculated for each value of M_2 an uncertainty δM_2 by using the error matrix propagated by KICK. After correction for the systematic errors known to exist in our track reconstruction we get, for the half width of the resolution function of M_2, $\Gamma_{res}/2 = 7$ MeV for a pair of visible pions ($\pi^+\pi^-$ or $\pi^\pm\pi^\pm$), and $\Gamma_{res}/2 = 14$ MeV for a pair consisting of one visible pion and one neutral pion $(\pi^\pm\pi^0)$.

The difference in the resolution function of M_2 comes from the fact that the momentum and direction of a charged--therefore "visible"--pion can be measured, i.e., be precisely known. The fitting process reduces the uncertainty only slightly. But the momentum and direction of a neutral pion are given only by the conservation of momentum and of energy in the fitting process with much bigger uncertainty (for example, $\Delta p/p \approx 2\%$ for a charged pion; it is 10% for a neutral pion).

As Goldhaber et al.[34] have done, we calculated for each pion pair the angle between the two pions in the center of mass of the antiproton-proton system:

$$\cos\theta_{12} = \vec{P}_1 \cdot \vec{P}_2 / |P_1| \cdot |P_2| .$$

We also calculated for each pion pair the decay angle of the pion in the rest frame of the dipion.

The 6π events ($\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-$) are the best ones to be investigated for a possible two-pion resonance. They cannot have the T = 0 three-pion resonance, and -- as we have seen -- they do not show any significant peak in their three- and four-pion effective-mass distributions. About one-third of the 7π events ($\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0$) would go by the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \omega$, and can complicate the interpretation of the effective-mass distribution of two pions of these events. Of the 8π events ($\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0$), as we discuss below, we expect a large fraction to go by the reaction $\bar{p} + p \rightarrow \pi^+ + \pi^- + 2\omega$.

Therefore in the following paragraphs we examine the 6π events in much greater detail than the 7π and 8π events.

<u>6π Events</u>

Figure 24 is the histogram distribution of the M_2 for the effective mass of pion pairs of the 6π events. In Fig. 24(a) we plot the Q = 0 combination, in Fig. 24(b), the |Q| = 2 combination. The solid curves represent the Lorentz-invariant phase-space calculation.[35]

In Fig. 25 is an alternative way of displaying effective-mass correlations. It shows the distribution of angles between pion pairs from 6π events as a function of $\cos\theta_{\pi\pi}$ for Q = 0 (or unlike pair, $\pi^+\pi^-$) and |Q| = 2 (or like pair, $\pi^\pm\pi^\pm$) respectively. The solid curves (identical except for normalization) correspond to calculations on the Lorentz-invariant phase-space model;[34] their slopes simply reflect conservation of momentum.

Figures 26, 27, and 28 are the decay-angle distributions of one pion in the dipion rest frame, (a) for $Q = 0$ and (b) for $|Q| = 2$. The three figures correspond to three regions of M_2, Fig. 26 for M_2 between 280 and 460 MeV, Fig. 27 for M_2 between 500 and 600 MeV, and Fig. 28 for M_2 between 600 and 800 MeV.

The most striking result seen in these 6π figures is in Fig. 24, namely, the big difference between the neutral M_2 distribution and the $|Q| = 2M_2$ distribution between 280 and 460 MeV. In this effective-mass range the $|Q| = 2$ distribution is above the phase space and the $Q = 0$ distribution is well below. The decay-angle distributions of those dipions (Fig. 26a and b) look the same for $Q = 0$ and $|Q| = 2$ pairs (unlike and like pairs). Both seem to peak at $\cos\phi = 0$ and decrease at $\cos\phi = \pm 1$. The alternative distribution of angles between all the pion pairs (Fig. 25a and b) also deviates strongly from the phase-space calculation. The angle between the unlike pions ($Q = 0$) is bigger than the value predicted by phase-space calculation. The phase-space calculation predicts a value of 1.7 for the ratio

γ = (no. of pion pairs with angle $> 90^\circ$/(no. of pion pairs with angle $< 90^\circ$), whereas we observe $\gamma = 2.45 \pm .1$ for unlike pions. The angle between the like pions ($Q = 2$) is found to be smaller than the value predicted by phase-space calculation, $\gamma = 0.92 \pm .07$. It is interesting to compare our results with the results of the analysis of four-prong events from the same experiment. Button et al. do not seem to observe this striking difference between the $Q = 0$ and $|Q| = 2$ distribution at the region of M_2 between 280 and 460 MeV.[17] Looking at the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ of the same experiment, Maglić et al.[20] report $\gamma = 1.56 \pm .08$ for like pions against $\gamma = 1.79$ predicted by phase space. But they attribute part of this effect to the asymmetry observed in the angular distribution of charged pions in the $\bar{p}p$ center of mass. It is also interesting to compare our results with those of Lee et al.,[36] who analyze the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ from $\bar{p}p$ annihilation at rest. They find a definite difference in the distribution of M_2 of the pion pair for like and unlike: low values of M_2 are enhanced for like pion pairs. Also they find a difference between the angular distributions of like- and unlike-pion pairs ($\gamma_{unlike} = 2.26 \pm .15$, $\gamma_{like} = 1.14 \pm .10$), even though they do not have any asymmetry in the angular distribution of pions in the $\bar{p}p$ center-of-mass system. Possibly the difference and similarity of results reflect the fact that the average energy available for each pion for our 6π event ($E_\pi = 2290/6 = 382$ MeV) is less than in the $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ event of Maglić et al. ($E_\pi = 2290/5 = 458$ MeV), but about the same as in the $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ at rest ($E_\pi = 1880/5 = 396$ MeV).

We cannot attribute the differences between the distributions of M_2 in the region 280 to 460 MeV and in the angular-correlation distribution between unlike ($Q = 0$) and like ($|Q| = 2$) pion pairs to the asymmetry of angular distribution of pions in the $\bar{p}p$ center of mass, because we do not observe any asymmetry (see Fig. 11); also, the c.m. angular distribution is the same for π^+ and π^-.

These differences seem to be best explained by the influence of the Bose-Einstein statistics for pions, as suggested by Goldhaber et al.[37] The Bose-Einstein effect acts like a weak attraction between like pions and a weak repulsion between unlike pions. This would favor small angles for the like-pion pair, and therefore enhance the lower part of their effective-mass distribution (Fig. 24b). This would also favor large angles for the unlike-pion pair and therefore depopulate the lower part of their effective-mass distribution (Fig. 24a). Goldhaber et al. have estimated the Bose-Einstein effect on the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$ with n = 0, 1, and 2.[37] The corresponding calculation for the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-$ is more complicated and has not been done. But with a maximum of three like pions instead of two like pions in the $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ reaction, we expect the Bose-Einstein effect to be greater in the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-$. And this seems to be verified by our results.

At the higher region of M_2(M_2 > 500 MeV) the $|Q| = 2$ distribution follows well the phase-space calculation, but the $Q = 0$ ($\pi^+\pi^-$) distribution disagrees completely with the phase-space calculation (Fig. 24 (a) and (b)). This effect is probably due in part to ρ mesons which can decay into $\pi^+\pi^-$ but not $\pi^\pm\pi^\pm$.

7π and 8π Events

Figure 29 shows histograms of distribution of the effective mass M_2 of the $Q = 0$, $|Q| = 1$, and $|Q| = 2$ pion pairs from the 7π events. Figure 30 shows distributions of the angle between the $Q = 0$, $|Q| = 1$, and $|Q| = 2$ pion pairs from the same events.

The effective-mass distributions still show a difference at the region of lower value of M_2 (280 to 460 MeV) between the like-pion pairs ($|Q| = 2$, $\pi^\pm\pi^\pm$) and the unlike-pion pairs ($Q = 0$, $\pi^+\pi^-$ or $|Q| = 1$, $\pi^\pm\pi^0$. Also at the higher regions of M_2 (500 to 800 MeV) the $|Q| = 2$ distribution agrees well with phase space and the neutral distribution seems to show an almost continuous enhancement above phase space.

Figure 31 shows histograms of the distribution of M_2 for $Q = 0$ and $|Q| = 2$ pion pairs from the 8π events. The solid curves are the Lorentz-invariant phase-space estimations. Figure 32 shows the distribution of angle between $Q = 0$ and $|Q| = 2$ pion pairs from the same events as a function of $\cos\theta_{\pi\pi}$.

Both the $Q = 0$ and $|Q| = 2$ effective-mass distribution agree with the phase-space estimation. Also the angle correlations are almost the same for unlike ($Q = 0$) and like ($|Q| = 2$) pairs ($\gamma_{unlike} = 1.55 \pm .07$, $\gamma_{like} = 1.40 \pm .08$).

We can assume that most of the 8π events go by the reaction $\bar{p} + p \rightarrow \pi^+ + \pi^- \omega + \omega$, and that if a four-pion resonance really existed, a big fraction of the 8π events would go by this resonance, $\bar{p} + p \rightarrow 4\pi + X^0$ and $X^0 \rightarrow 4\pi$. We base this assumption on the big difference between the value of the cross section of the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0$ ($1.05 \pm .2$ mb) and that of $\bar{p} + p \rightarrow 4\pi^+ + 4\pi^-$ ($.025 \pm .01$ mb) (Table III). A statistical calculation would give a ratio of 7 for the rate of the first reaction over the second one; we find here a ratio of 40 ± 10. This assumption can explain also the absence of ρ mesons in the 8π events.

V. SUMMARY AND CONCLUSION

Let us summarize a few of the interesting findings of this work. The cross sections of the three analyzed reactions are found to be

$$(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-) = 1.16 \pm .1 \text{ mb},$$
$$(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0 = 1.80 \pm .25 \text{ mb},$$
$$(\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0) = 1.05 \pm .25 \text{ mb}.$$

The difference between the cross section of the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + 2\pi^0$ and the reaction $\bar{p} + p \rightarrow 4\pi^+ + 4\pi^-$ (.025 ± .01 mb) (see Table III) suggests that the first one would go through resonances whose decay includes neutral pions. This can be the known three-pion resonance or a yet undetermined four-pion resonance. The angular distributions are symmetrical for all three types of events (Figs. 11, 12, 13). This result is in contrast with the asymmetry found by Maglić et al.[20] in the reactions $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$ at the same energy. But because the ratio of the number of cloud pions to the number of core pions is smaller, Koba-Takeda theory would predict a smaller asymmetry, which could be masked in our case.

The existence of the ω meson (I = 0 three-pion resonance at 780 MeV) is further confirmed. With the hypothesis of G-parity conservation in the decay process (strong decay), the ω spin and parity is found to be 1^{--} by the Dalitz-plot method. Even with the hypothesis of nonconservation of G parity in the decay process (electromagnetic decay), the 1^{--} spin and parity assignment is strongly suggested by the small values of the ratios $R(\omega \rightarrow 4\pi/\omega \rightarrow \pi^+ + \pi^- + \pi^0)$ and $R(\omega \rightarrow \text{neutral}/\omega \rightarrow \pi^+ + \pi^- + \pi^0)$. These results agree very well with those of Stevenson et al.[24]

Neither we nor Maglić et al. have observed in the $\bar{p}p$ annihilation the T = 0 three-pion resonance at 550 MeV reported by Pevsner et al.[30] and Bastien et al.[30]

The distribution of the neutral four-pion effective mass M_4 shows a suggestive but inconclusive peak at 1.04 BeV (Fig. 21).

The two-pion effective-mass distributions at 6π events show a big difference between the $|Q| = 2$ or like-pion pairs and the $Q = 0$ or unlike-pion pairs at the low-value region of M_2 (Fig. 24). At this region (M_2 between 280 and 460 MeV) the distribution for like-pion pairs is above that obtained from phase-space calculation, and the one for unlike pion pairs is well below. This result is similar to the result from the analysis of $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ at rest,[34] and different from that of $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$ at the same energy.[17] It may be that the similarity and difference may reflect the similarity and difference in the average energy available for one pion (382 MeV in our case, 396 MeV for 5π events at rest, and 458 MeV for 5π events at 1.61 BeV/c).

We tentatively attribute the difference of M_2 distributions in the low-value region, at least at present, to the Bose-Einstein effect on the pion as suggested by Goldhaber et al.[37] But no calculations have been done for the $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^-$ reaction.

The ratio $R(\rho \rightarrow \pi^{\pm} + \eta,\ \eta \rightarrow \pi^+\pi^-\pi^0/\rho^{\pm} \rightarrow \pi^{\pm}\pi^0)$ has been determined to be $1.2 \pm 2.0\%$. This small ratio agrees with the 0^{-+} assignment for spin, parity, and G parity of the η meson but cannot rule out the possibility of 1^{--} assignment.

Table VI gives the upper limits of some decay rates of the ρ and ω mesons determined in this experiment.

In this work, we concern ourselves mostly with the search for multipion resonances and the determination of their quantum numbers. We feel that the real behavior of the antiproton-proton annihilation process will not be understood until all the multipion resonances and their quantum numbers are known. Many theories on $\bar{p}p$ annihilations do not include the multipion resonance states. The latest statistical model that includes all the known resonances is given by Kalbfleisch.[38] It gives a good prediction of the rate of diverse $\bar{p}p$ annihilation processes. But it is semi-empirical and still needs a large interaction volume ($\Omega = 4\,\Omega_0$, where $\Omega_0 = (4\pi/3)(\hbar/m_\pi C^3)^2$. Also, no theory has ever included the spins of the proton and of the antiproton. On the experimental side Button and Maglić by studying the double scattering of antiprotons in hydrogen, have shown that antiprotons can be polarized.[39] The annihilation of polarized antiprotons on hydrogen has been studied in a preliminary way.[40] It would be interesting to push this study forward, because it may give new understanding of the antiproton-proton annihilation process.

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APPENDIX

G Parity and Strong Decay of Heavy Mesons With Zero Strangeness as Proposed by the "Eightfold Way" Theory

In the "eightfold way" theory, a heavy meson with strangeness S = 0 can be represented by a definite state of $\bar{N}N$ (or $\bar{\Lambda}\Lambda$). It has been shown that for such a state the G parity is given by $G = (-1)^{s + \ell + I}$, where s, ℓ, and I represent respectively the spin, angular momentum, and I spin of the $\bar{N}N$ state.[41] Table VII gives the characteristics of the $\bar{N}N$ states representing the mesons with S = 0 proposed in the eightfold way theory, and the resulting G parity.

The G parity of a state of n pions is given by $G = (-1)^n$. Since in strong interactions G parity is conserved, a heavy meson with G parity even can decay strongly into only an even number of pions (2, 4, 6, etc.), and a heavy meson with G parity odd can decay strongly into only an odd number of pions. The exceptions are that a χ^0 or A meson cannot decay into two pions because of Bose statistics and conservation of intrinsic parity; also the π' meson cannot decay into three pions for the same reasons.

References and Footnotes

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1. M. Gell-Mann, The Eightfold Way: A Theory of Strong Interaction Symmetry, California Institute of Technology Synchrotron Laboratory Report No. CTSL 20 (1961); Symmetries of Baryons and Mesons, Phys. Rev. (to be published).

2. S. Sakata, Progr. Theoret. Phys. (Kyoto) 16, 686 (1956).

3. J. J. Sakurai, Ann. Phys. 11 1 (1960); Nuovo cimento 16, 388 (1960). See also G. Breit, Phys. Rev. 120, 287 (1960); Y. Ne'eman, Nuclear Phys. (in press); A. Salam and J. C. Ward, Nuovo cimento (in press).

4. M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) 22, 715 (1959); Y. Ohnuki, in Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester (Interscience Publishers, New York, 1960); Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) Suppl. No. 11 (1959); J. Wess, Nuovo cimento 10, 15 (1960); Shigeo Minami, Heavy Mesons and Hard Core in Nuclear Force, Preprint, Department of Physics, Osaka City University, Osaka; H. P. Duerr, W. Heisenberg, H. Mitter, S. Schneider, and K. Yamaguchi, Naturforsch. 14a, 441 (1959).

5. G. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. 110, 265 (1958); P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. 112, 642 (1958); W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960); see also J. Bowcock, N. Cottingham, and D. Lurie, Nuovo cimento 16, 918 (1960).

6. Y. Nambu, Phys. Rev. 106, 1366 (1957).

7. P. Eberhard, M. L. Good, and H. K. Ticho, A Separated 1.17-BeV/c K^--Meson Beam, UCRL-8878, Aug. 1959.

8. J. Button, P. Eberhard, G. R. Kalbfleisch, J. E. Lannutti, G. R. Lynch, B. C. Maglić, M. L. Stevenson, and N. H. Xuong, Phys. Rev. 121, 1788 (1961); George R. Kalbfleisch, A Study of K Mesons in Antiproton-Proton Annihilation (Thesis), Lawrence Radiation Laboratory Report UCRL-9597, March 1961 (unpublished).

9. More complete data are given in the doctoral dissertation of Nguyen-huu Xuong, Lawrence Radiation Laboratory Report UCRL-10129, March 1962 (unpublished) of which this is a condensation.

10. We have seen no δ ray, with a diameter greater than 1 cm on the scanning table, connected with a six-prong event. These δ rays would correspond to electrons with momentum greater than 4.14 MeV/c, and cannot be produced by 1.61-BeV/c antiprotons. The probability for a pion in the beam to make such a δ ray before interaction is 0.16.

11. Hugh Bradner, Bubble Chambers, UCRL-9199, May 1960; J. V. Franck, "Franckenstein," A Semiautomatic Measuring Projector for Bubble Chamber Film, to be submitted to Rev. Sci. Instr.

12. William E. Humphrey, A Description of the PANG Program, Alvarez Group Memo 111, Sept. 18, 1959, and Memo 115, Oct. 25, 1959; A. H. Rosenfeld, Digital-Computer Analysis of Data from Hydrogen Bubble Chambers at Berkeley, In Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959 (CERN, Geneva, 1959).

13. Arthur H. Rosenfeld and James N. Snyder, Digital-Computer Analysis of Data from Bubble Chambers. IV. The Kinematic Analysis of Complete Events (UCRL-9098, Feb. 1960), Rev. Sci. Instr. 33, 181 (1962); J. P. Berge, F. T. Solmitz, and H. Taft, Digital-Computer Analysis of Data from Bubble Chambers. III. The Kinematical Analysis of Interaction Vertices (UCRL-9097, March 1960), Rev. Sci. Instr. 32, 538 (1961).

14. G. R. Lynch, Revs. Modern Phys. 33, 395 (1961). The process $\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^0$ is being studied. The cross section quoted here is just a preliminary result. Also Nguyen-huu Xuong, G. R. Lynch, and C. K. Hinrichs, Phys. Rev. 124, 575 (1961).

15. The EXAMIN program is described by D. Johnson of our programming staff in Alvarez Group Memo 271, March 1961.

16. T. Elioff, L. Agnew, O. Chamberlain, H. Steiner, C. Wiegand, and T. Y. Ypsilantis, Phys. Rev. Letters 3, 285 (1959); R. Armenteros, C. H. Coombes, B. Cork, G. R. Lambertson, and W. Wenzel, Phys. Rev. 119, 2068 (1960); C. Keith Hinrichs, Charge-Exchange Production of Antineutrons and Their Annihilation in Hydrogen (Thesis), Lawrence Radiation Laboratory Report UCRL-9589, March 1961 (unpublished).

17. J. Button, G. R. Kalbfeisch, G. R. Lynch, B. C. Maglić, A. H. Rosenfeld, and J. L. Stevenson, Pion-Pion Interaction in the Reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$, Phys. Rev. 126, (UCRL-9814, Dec. 1961) The cross sections are only preliminary results.

18. Robert Foulks (Lawrence Radiation Laboratory), private communication.

19. A. Pais, Phys. Rev. Letters 3, 242 (1959).

20. B. C. Maglić, G. R. Kalbfleisch, and M. L. Stevenson, Phys. Rev. Letters 7, 137 (1961).

21. Z. Koba and G. Takeda, Progr. Theoret. Phys. (Kyoto) 19, 269 (1958).

22. The M_3 resolution is defined as the half width at the half maximum of the Gaussian ideogram obtained by using the uncertainty δM_3 and a fixed central value.

23. B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters 7, 178 (1961); N. H. Xuong and G. R. Lynch, Phys. Rev. Letters 7, 327 (1961); A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Gessaroli, and C. Meltzer, in Proceedings of 1961 Conference on Elementary Particles, Aix en Provence; E. L. Hart, R. I. Louttit, D. Luers, T. W. Morris, W. J. Willis and S. S. Yamamoto, Multiple Meson Production in Proton-Proton Collisions at 2.85 BeV, submitted to Phys. Rev.

24. R. Dalitz, Phil. Mag. 44, 1068 (1953). See also E. Fabri, Nuovo cimento 11, 479 (1954).

25. M. L. Stevenson, L. W. Alvarez, B. C. Maglić, and A. H. Rosenfeld, Phys. Rev. 125, 687 (1962); see also N. H. Xuong and G. R. Lynch, Phys. Rev. Letters 7, 327 (1961).

26. We are aware of the difficulty of making this subtraction. Some of the triplets in the peak region which are assumed to be in the background may have the same quantum number as the ω meson. If this is true, the background subtraction should overemphasize the effect, no matter which spin and parity the particle has.

27. H. P. Duerr and W. Heisenberg (Max-Planck Institut, Munich), The Quantum Numbers of the ω Meson (preprint).

28. G. F. Chew, Phys. Rev. Letters 4, 142 (1960).

29. A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961); E. Pickup, D. K. Robinson, and E. W. Salant, Phys. Rev. Letters 7, 192 (1961); D. Carmony and R. Van de Walle, Phys. Rev. Letters 8, 73 (1962). These articles give a more complete reference to earlier publications on this subject.

30. A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Gessaroli, and C. Meltzer, Phys. Rev. Letters 7, 421 (1961); P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and M. B. Watson, Phys. Rev. Letters 8, 114 (1962); D. C. Carmony, A. Rosenfeld, and R. Van de Walle, Phys. Rev. Letters 8, 117 (1962).

31. G. F. Chew and Steven C. Frautschi, Phys. Rev. Letters 8, 41 (1962); Phys. Rev. Letters 7, 395 (1961); also, C. Lovelace, Imperial College, London (preprint).

32. A. Rosenfeld, D. Carmony, and R. Van de Walle, Phys. Rev. Letters 8, 293 (1962).

33. S. Glashow and J. Sakurai (University of Chicago), private communication.

34. G. Goldhaber, W. B. Fowler, S. Goldhaber, T. F. Hoang, T. E. Kalogeropoulos, and W. M. Powell, Phys. Rev. Letters 3, 181 (1959); see also T. H. Hoang and J. Young, Covariant Phase-Space Factors for Reactions Involving Four to Six Secondary Particles, Lawrence Radiation Laboratory Report UCRL-9050, Jan. 1960 (unpublished).

35. G. Kalbfleisch, Lorentz-Invariant Phase-Space Distribution for the Effective Mass of Two Particles, Alvarez Physics Note No. 234, November 17, 1960 (unpublished).

36. W. Lee, W. Chinowsky, G. Goldhaber, and S. Goldhaber, Bull. Am. Phys. Soc. 6, 522 (1961).

37. G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120, 300 (1960).

38. George R. Kalbfleisch, A Two-Parameter Statistical Model, Lawrence Radiation Laboratory Report UCRL-10024, Jan. 1962 (unpublished).

39. J. Button and B. Maglić, An Experimental Study of the Polarization and Magnetic Moment of the Antiproton, to be submitted to Phys. Rev. (UCRL-10092, Feb. 1962); J. Lannutti, G. Lynch, B. Maglić, M. L. Stevenson, and Nguyen-huu Xuong, Double Scattering of Antiprotons in Hydrogen, in Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960).

40. Nguyen-huu Xuong, Annihilation of Polarized Antiprotons in Hydrogen, Bull. Am. Phys. Soc. 7, 298 (1962).

41. T. D. Lee and C. N. Yang, Nuovo cimento 3, 749 (1956).

Table I. Mesons proposed in the "Eightfold Way" Theory

A pseudoscalar octet would be composed of three pions (π^+, π^0, π^-), two K mesons (K^+, K^0), two $\bar{K}$ ($\bar{K}^0$, K^-), and one χ^0.

Unitary spin	I	S	PS(0^-)	V(1^-)	S(0^+)	A(1^+)
Octet	1	0	π	ρ	π'	ρ'
	1/2	+1	K	M	K'	M'
	1/2	-1	$\bar{K}$	$\bar{M}$	$\bar{K}'$	$\bar{M}'$
	0	0	χ^0	ω	$\chi^{0'}$	ω'
Singlet	0	0	A	B	A'	B'

Table II. Prediction of decay of some proposed mesons with strangeness S = 0.[a]

Particle	G parity	Strong decay	Electromagnetic decay
ρ	+1	$\pi^+\pi^0, \pi^+\pi^-, \pi^-\pi^0$	
ω, B	-1	$\pi^+\pi^-\pi^0$	$\pi^0\gamma$
χ, A	+1	4π	$3\pi, \pi^+\pi^-\gamma, 2\gamma$
π'	-1	$5\pi, \bar{K}K, \chi^0\pi$	$2\pi\gamma, 4\pi$
ρ'	-1	3π	$\pi\gamma$
ω', B'	+1	4π	$\pi^+\pi^-\gamma, 3\pi$
χ', A'	+1	$\pi^+\pi^-, \pi^0\pi^0$	2γ

a. See Appendix

Table III. Cross sections of diverse processes of $\bar{p}p$ interactions at 1.61 BeV/c ($E_{c.m.}$ = 2.290 BeV).
(The total cross section is 96±3 mb from Ref. 3.)

Process	Reference	Cross section (mb)
Elastic		
$\bar{p} + p \to \bar{p} + p$	16	33 ± 3
Charge-exchange		
$\bar{p} + p \to \bar{n} + n$ (and) $\bar{n} + n + \pi^0$	16	7.8 ± .55
Inelastic		
$\bar{p} + p \to \bar{p} + p + \pi^0$	14	1.6 ± .30
$\to \bar{n} + p + \pi^-$	14	0.96 ± .22
$\to \bar{p} + n + \pi^+$	14	1.15 ± .30
Hyperon-antihyperon		
$\bar{p} + p \to \Lambda + \bar{\Lambda}$	8	0.057 ± .018
Annihilation involving K mesons and π mesons		
$\bar{p} + p \to K^0 + \bar{K}$	14	≤ .050
$\to K^+ + K^-$	14	0.055 ± .018
$\bar{p} + p \to K + \bar{K} + \pi$	8	0.74± .16
$\to K + \bar{K} + 2\pi$	8	1.95 ± .26
$\to K + \bar{K} + 3\pi$	8	2.2 ± .26
$\to K + \bar{K} + 4\pi$	8	0.37 ± .011
$\to K + \bar{K} + 5\pi$	8	$0^{+.02}_{-0}$
Annihilation involving π mesons		
$\bar{p} + p \to n\pi^0$, for $n \geq 2$	14	$0.3^{+.4}_{-.3}$
$\bar{p} + p \to \pi^+ + \pi^-$	14	0.1 ± .025
$\to \pi^+ + \pi^- + \pi^0$	14	2.5 ± 1.5
$\to \pi^+ + \pi^- + n\pi^0$ for $n \geq 2$	14	14.1 ± 3
$\to 2\pi^+ + 2\pi^-$	17	1.4 ± .3
Annihilation involving π mesons		
$\bar{p} + p \to 2\pi^+ + 2\pi^- + \pi^0$	17	10.4 ± 1.0
$\to 2\pi^+ + 2\pi^- + n\pi^0$, for $n \geq 2$	17	12.0 ± 1.5
$\to 3\pi^+ + 3\pi^-$		1.2 ± .1
$\to 3\pi^+ + 3\pi + \pi^0$		1.8 ± .25
$\to 3\pi^+ + 3\pi + 2\pi^0$		1.05 ± .25
$\to 4\pi^+ + 4\pi^-$	18	0.025 ± .01
$\to 4\pi^+ + 4\pi^- + \pi^0$	18	0.006 ± .006

Table IV. Possible three-pion resonances with T=0, J≤1, G parity.

Meson Type	J	ℓ	L	Matrix element: Simple example	Vanishes at (see Fig. 16)
V	1^{--}	1	1	$(\vec{p}_0 \times \vec{p}_+) + (\vec{p}_+ \times \vec{p}_-) + (\vec{p}_- \times \vec{p}_0)$	whole boundary
PS	0^{--}	1 and 3	1 and 3	$(E_- - E_0)(E_0 - E_+)(E_+ - E_0)$	straight lines
A	1^{+-}	0 and 2	1	$E_-(\vec{p}_0 - \vec{P}_+) + E_0(\vec{P}_+ - \vec{P}_-) + E_+(\vec{P}_- - \vec{P}_0)$	center, b, d, f

Table V. Summary of experimental determination of spin parity and G parity of ω meson (I=0, $M_{\pi^+\pi^-\pi^0}$ = 780 MeV).

Possible assignment (Spin ≤ 1, parity, G parity)	Eliminated by
0^{--}	Dalitz plot
0^{+-}	Parity conservation
$\boxed{1^{--}}$	
1^{+-}	Dalitz plot
0^{-+}	Dalitz plot and small ratio $R(\omega \rightarrow \text{neutral}/\omega \rightarrow \pi^+\pi^-\pi^0)$
0^{++}	Parity conservation
1^{-+}	Small ratio $R(\omega \rightarrow 4\pi/\omega \rightarrow \pi^+\pi^-\pi^0)$
1^{++}	Dalitz plot, small ratio $R(\omega \rightarrow 4\pi/\omega \rightarrow \pi^+\pi^-\pi^0)$ and small ratio $R(\omega \rightarrow \text{neutral}/\omega \rightarrow \pi^+\pi^-\pi^0)$

Table VI. Upper limits of some decay rates of the ω and ρ mesons

Ratio	Upper limit
$R(\rho^0 \to 2\pi^+ 2\pi^- / \rho^0 \to \pi^+\pi^-)$	0.02
$R(\rho^\pm \to \pi^\pm\pi^-\pi^+\pi^0 / \rho^\pm \to \pi^\pm\pi^0)$	0.05
$R(\omega \to \text{neutral} / \omega \to \pi^+\pi^-\pi^0)$	0.5
$R(\omega \to 4\pi / \omega \to \pi^+\pi^-\pi^0)$	0.17
$R(\omega \to 2\pi^+ 2\pi^- \pi^0 / \omega \to \pi^+\pi^-\pi^0)$	0.01

Table VII. Characteristics of $\bar{N}N$ states representing some proposed heavy mesons and their G parities

Meson	I spin	Spin and parity	$\bar{N}N$	G parity
π	1	0^-	$^1S_0$	-1
ρ	1	1^-	$^3S_1$	+1
ω, B	0	1^-	$^3S_1$	-1
K^0, A	0	0^-	$^1S_0$	+1
π'	1	0^+	$^3P_0$	-1
ρ' [a]	1	1^+	$^3P_1$	-1
ω', B' [(a)]	0	1^+	$^3P_1$	+1
$\chi^{0\prime}$, A'	0	0^+	$^3P_0$	+1

(a) ρ', ω' and B' can be represented by either the $^3P_1$ or $^1P_1$ state, but Gell-Mann (private communication) prefers the first state for "field theory" reasons.

Figure Legends

Fig. 1. A typical six-prong event.

Fig. 2. Comparison of χ^2 distribution for $\bar{p} + p \to 3\pi^+ + 3\pi^- + \pi^0$ events (one constraint), ——, with theoretical χ^2 distribution, ---- .

Fig. 3. Comparison of χ^2 distribution for $\bar{p} + p \to 3\pi^+ + 3\pi^-$ events (four constraints), ———, with theoretical χ^2 distribution, ----. Scale factor of 3 for the latter.

Fig. 4. Distribution of the square of the missing mass (MM^2) for "6π" events.

Fig. 5. Distribution of the square of the missing mass for "7π" events.

Fig. 6. Distribution of the missing mass of the "8π" event. The solid curve is drawn from the distribution of the effective mass of two charged pions from the same events.

Fig. 7. Center-of-mass momentum distribution for pions of "6π" events, $\bar{p} + p \to 3\pi^+ + 3\pi^-$. Top, π^+; center, π^-; bottom, π^+ and π^-. The solid curves represent the Lorentz-invariant phase-space calculation. (153 events.)

Fig. 8. Momentum distribution (c.m.) for charged pions of 7π events, $\bar{p} + p \to 3\pi^+ + 3\pi^- + \pi^0$. Top, π^+; center, π^-; bottom, π^+ and π^-. (239 events.)

Fig. 9. Momentum distribution (c.m.) for charged pions of 8π events, $\bar{p} + p \to 3\pi^+ + 3\pi^- + 2\pi^0$. Top, π^+; center, π^-; bottom, π^+ and π^-. (139 events.)

Fig. 10. Momentum distribution (c.m.) for neutral pions of 7π events.

Fig. 11. Distribution of cosine of c.m. production angle for individual charged pions of 6π events. Top, π^+; center, π^-; bottom, π^+ and reflected π^-. (153 events.)

Fig. 12. Distribution of cosine of c.m. production angle for individual charged pions of 7π events. Top, π^+; center, π^-; bottom, π^+ and reflected π^-. (239 events.)

Fig. 13. Distribution of cosine of c.m. production angle for individual charged pions of 8π events. Top, π^+; center, π^-; bottom, π^+ and reflected π^-. (139 events.)

Fig. 14. Angular distribution (c.m.) for neutral pions of 7π events (a) for 239 7π events, (b) same as (a) except that 28 ambiguous events have been excluded.

Fig. 15. Histograms of the distributions of the effective masses (M_3) of pion triplets for the 7π events, (a) for the distribution for the triplets with $|Q| = 1$ (239×18 triplets); (b) with $|Q| = 2$(239×6 triplets); (c), with $Q = 0$ (139×9 triplets). In (d) the combined distributions of (a) and (b) (shaded area) are compared with the (c) distribution. The same smooth curve has been drawn on (a), (b), and (c).

Fig. 16. Histograms of the distribution of neutral M_3 for 7π events around the 780-MeV region. The resonance curve contains 79 ω mesons.

Fig. 17. Dalitz plot for 238 events from the center of the peak region (760 to 800 MeV), $27 \pm 8\%$ of which are due to ω mesons.
$Q = M_3 - (2M_{\pi^\pm} + M_{\pi^0}) \approx 365$ MeV.

Fig. 18. Dalitz plot for 238 events from the control region (822 to 878 MeV).
$Q = M_3 - (2M_{\pi^\pm} + M_{\pi^0}) \approx 435$ MeV.

Fig. 19. The density of the population of events on the Dalitz plot for five regions of approximately equal area as a function of the average distance of this region from the center of the Dalitz plot. Both scales are in arbitrary units. (a) Events in the energy region of the ω meson and in a control region. In the ω peak region there are 238 triplets: 79 omega and 159 background triplets. The number of events per unit area of the

Fig. 19. (continued) control region is normalized to be equal to the estimated background in the peak region. (b) Difference between the two sets of points in (a) (79 triplets). The three curves correspond to the predictions for this distribution on the assumptions that the particle has spin and parity 1^{--}, 1^{+-}, or 0^{--}.

Fig. 20. Histograms of the distributions of the effective masses M_4 of pion quadruplets; (a) is for distribution for quadruplets of 6π events with $Q = 0$ (153×9 quadruplets), (b) and (c) for distribution for quadruplets of 7π events with $Q = 0$ and $|Q| = 1$ respectively (239×9 and 239×18 quadruplets). In (d) the (c) distribution (shaded area) is compared with the (b) distribution.

Fig. 21. Histograms of the distributions of the effective masses of neutral pion quadruplets of 8π events; (a) is for distribution of $M_4(\pi^+\pi^+\pi^-\pi^-)$, ($139 \times 9$ quadruplets), (b) is for distribution of $M_4(\pi^+\pi^-\pi^0\pi^0)$ (139×9 quadruplets). The same smooth curve has been drawn on (a) and (b).

Fig. 22. Histograms of the distributions of the effective masses of pions quadruplets of 8π events; (a) is for distribution of quadruplets with $|Q| = 2$(139×6 quadruplets), (b) is for quadruplets with $Q = 0$ (139×18 quadruplets). The same smooth curve has been drawn on (a) and (b).

Fig. 23. Histograms of the distributions of the effective masses of pion quintuplets of 7π events; (a) with $Q = 0$ (239×9 quintuplets) (b) with $|Q| = 1$ (239×6 quintuplets); (c) with $|Q| = 2$(239×6 quintuplet.) The same smooth curve has been drawn on (a), (b), and (c).

Fig. 24. Histograms of the distributions of the effective masses of pion pairs of 6π events; (a) with $Q = 0$ (153×9 pairs); (b) with $|Q| = 2$ (153×6 pairs). The solid curves are from the Lorentz-invariant phase-space calculation.

Fig. 25. Distribution of angle between pion pairs from 6π events as a function of $\cos\theta_{\pi\pi}$; (a) for pairs $(\pi^+\pi^-)$ with Q = 0 (153 × 9 pairs); (b) for pairs $(\pi^\pm\pi^\pm)$ with $|Q| = 2$ (153 × 6 pairs). The solid curves correspond to the Lorentz-invariant phase-space calculation.

Fig. 26. Distribution of decay angle of one pion in the rest frame of a dipion from 6π events with effective mass M_2 between 280 and 460 MeV (a) for π^+ from a $\pi^+\pi^-$ dipion (b) for $\pi^\pm$ from a $\pi^\pm\pi^\pm$ dipion (symmetrized).

Fig. 27. Distribution of decay angle of one pion in the rest frame of a dipion from 6π events with effective mass M_2 between 500 and 600 MeV (a) for π^+ from a $\pi^+\pi^-$ dipion (b) for $\pi^\pm$ from a $\pi^\pm\pi^\pm$ dipion (symmetrized).

Fig. 28. Distribution of decay angle of one pion in the rest frame of a dipion from 6π events with effective mass M_2 between 600 and 800 MeV (a) for π^+ from a $\pi^+\pi^-$ dipion (b) for $\pi^\pm$ from a $\pi^\pm\pi^\pm$ dipion (symmetrized).

Fig. 29. Histograms of the distribution of the effective mass of pion pairs of 7π events (a) with Q = 0 (239 × 9 pairs); (b) with $|Q| = 1$ (239 × 6 pairs); (c) with $|Q| = 2$ (239 × 6 pairs). The solid curves correspond to a Lorentz-invariant phase-space calculation.

Fig. 30. Distribution of angle between pion pairs from 7π events as a function of $\cos\theta_{\pi\pi}$ (a) Q = 0 pairs $(\pi^+\pi^-)$ (239 × 9 pairs); (b) for $|Q| = 1$ pair $(\pi^\pm\pi^0)$ (239 × 6 pairs); (c) for $|Q| = 2$ pairs $(\pi^\pm\pi^\pm)$ (239 × 6 pairs).

Fig. 31. Histogram of the distribution of the effective mass of pion pairs of 8π events (a) with Q = 0 (139 × 9 pairs); (b) with Q = 2 (139 × 6 pairs). The solid curve is an estimation of Lorentz-invariant phase-space prediction.

Fig. 32. Distribution of angle between pion pairs from 8π events as a function of $\cos\theta_{\pi\pi}$ (a) for Q = 0 pairs $(\pi^+\pi^-)$ (139 × 9 pairs), (b) for $|Q| = 2$ pairs $(\pi^\pm\pi^\pm)$ (139 × 6 pairs).

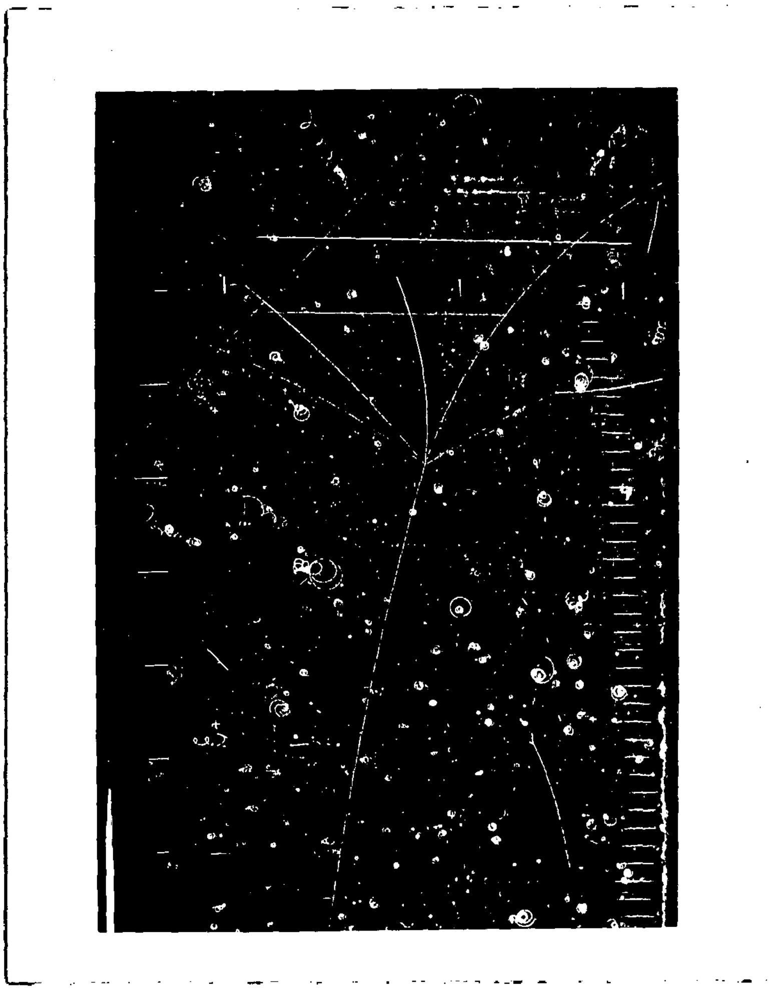
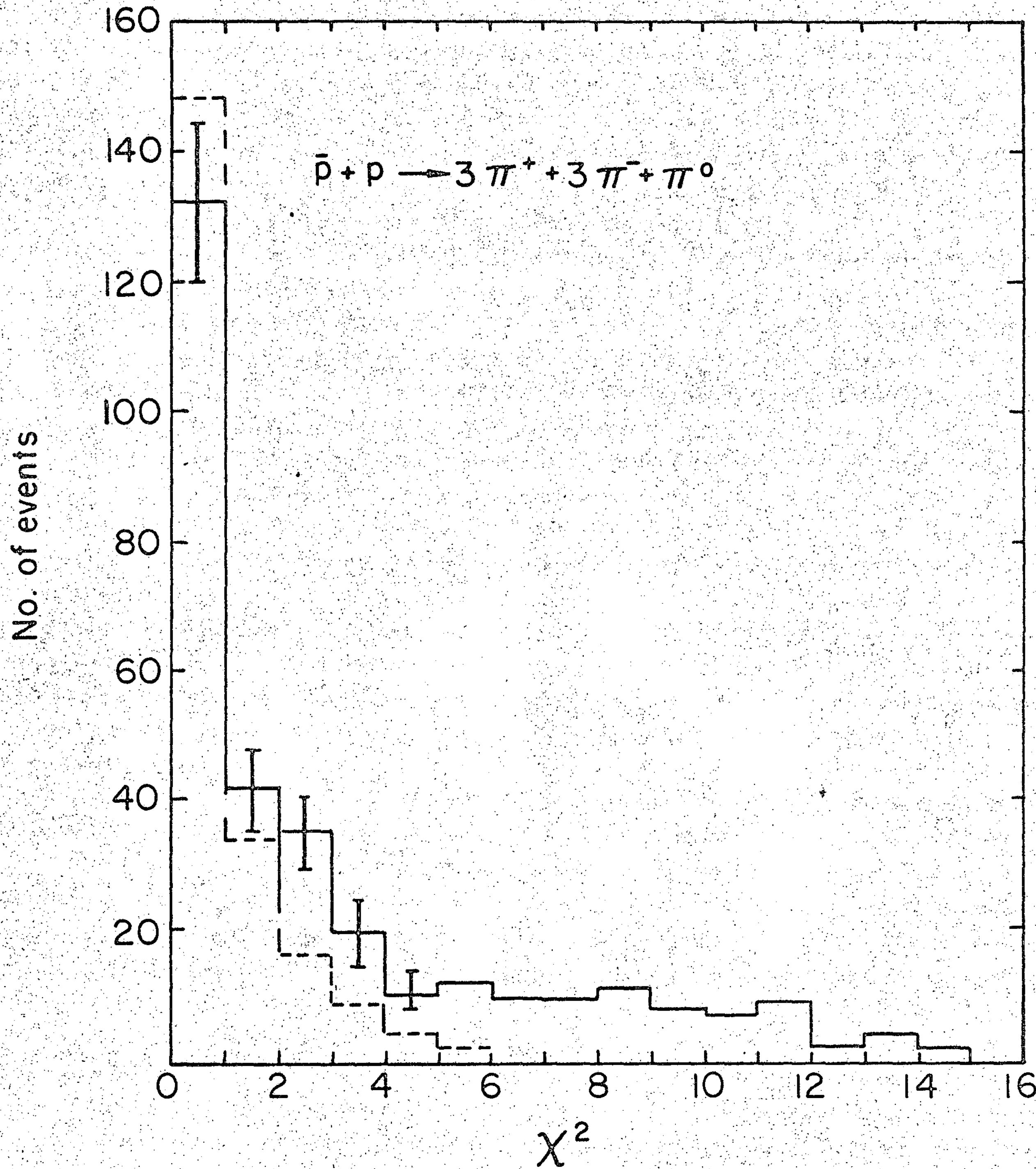
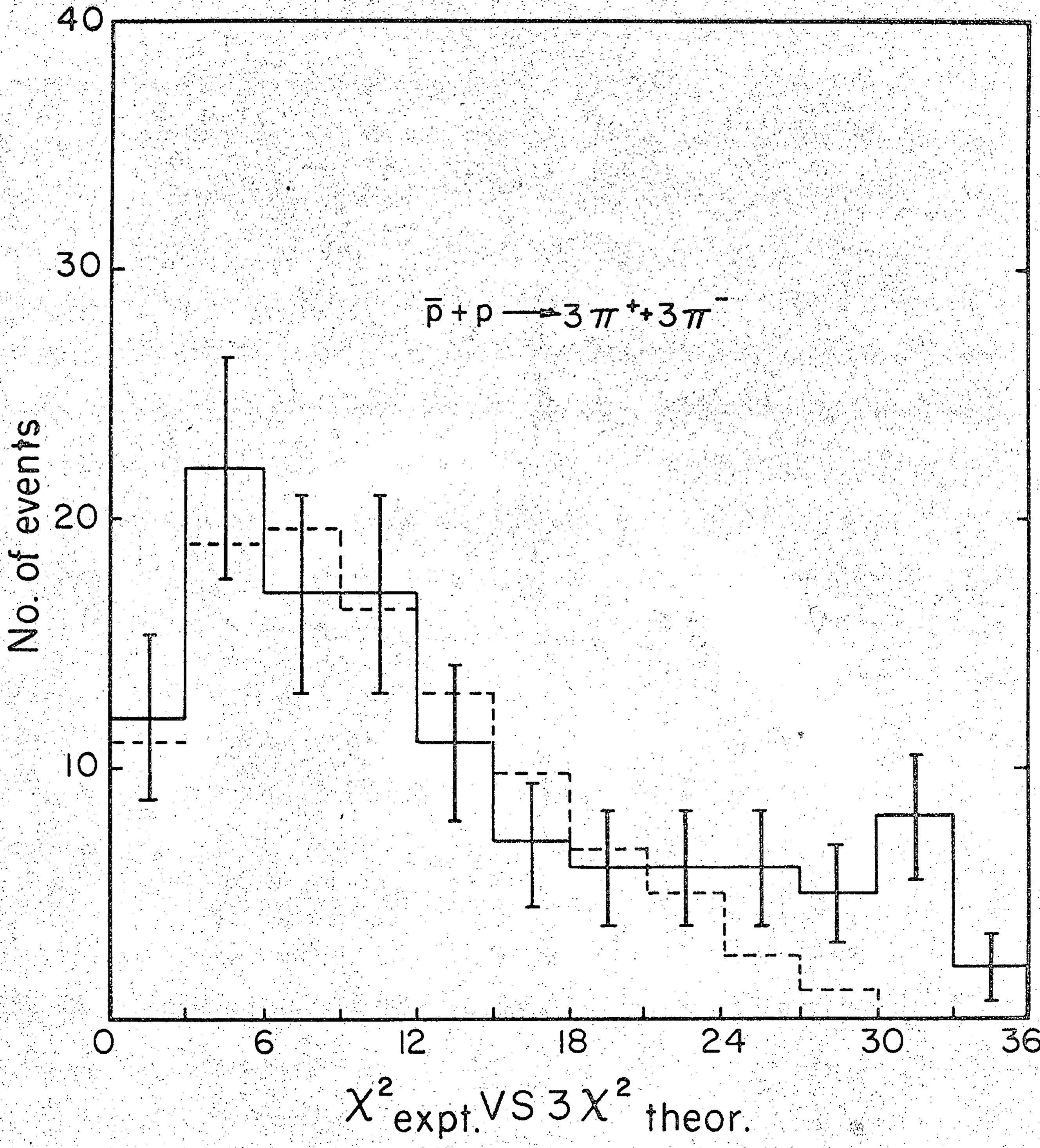


Fig. 1



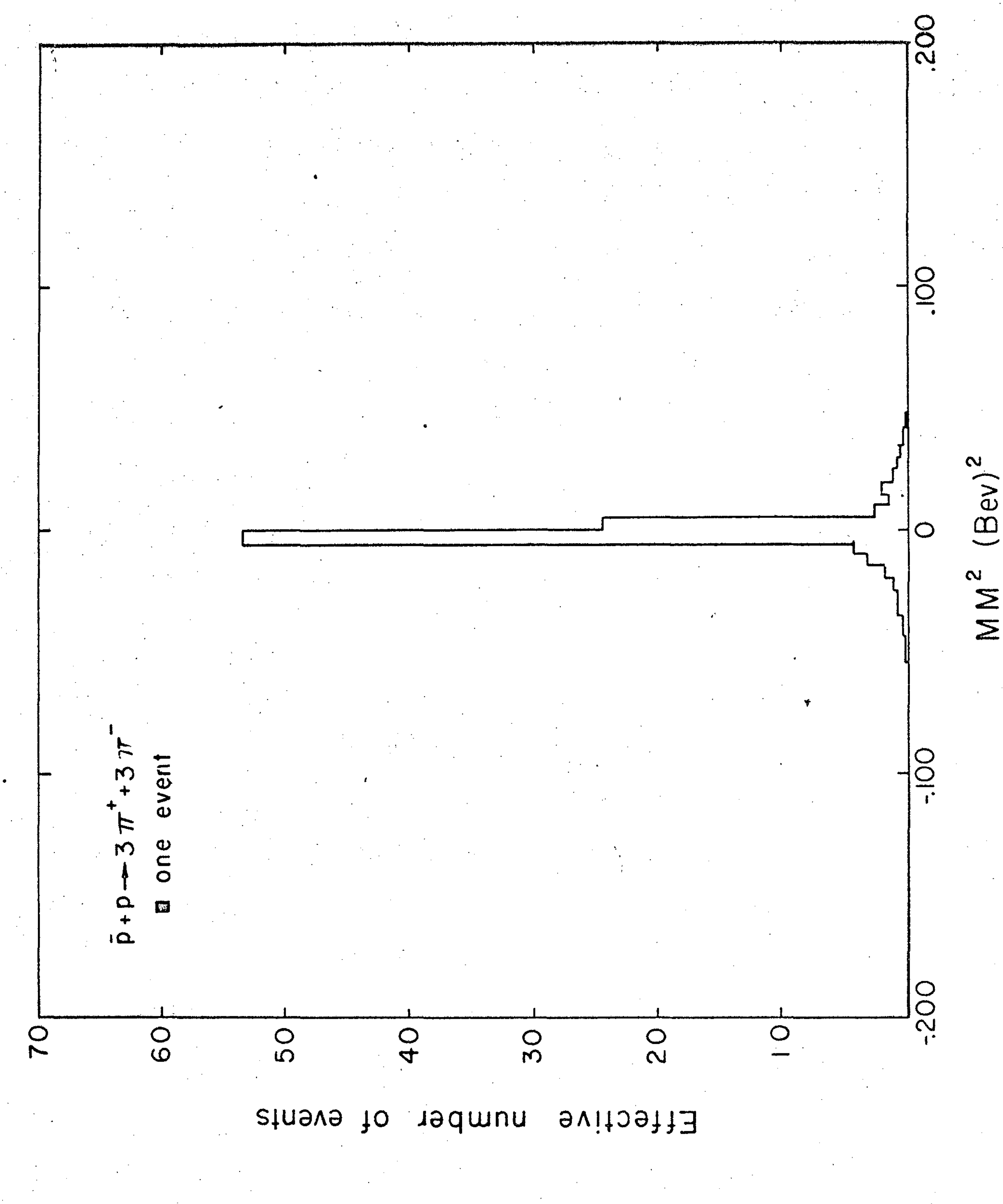
MU-26564

Fig. 2



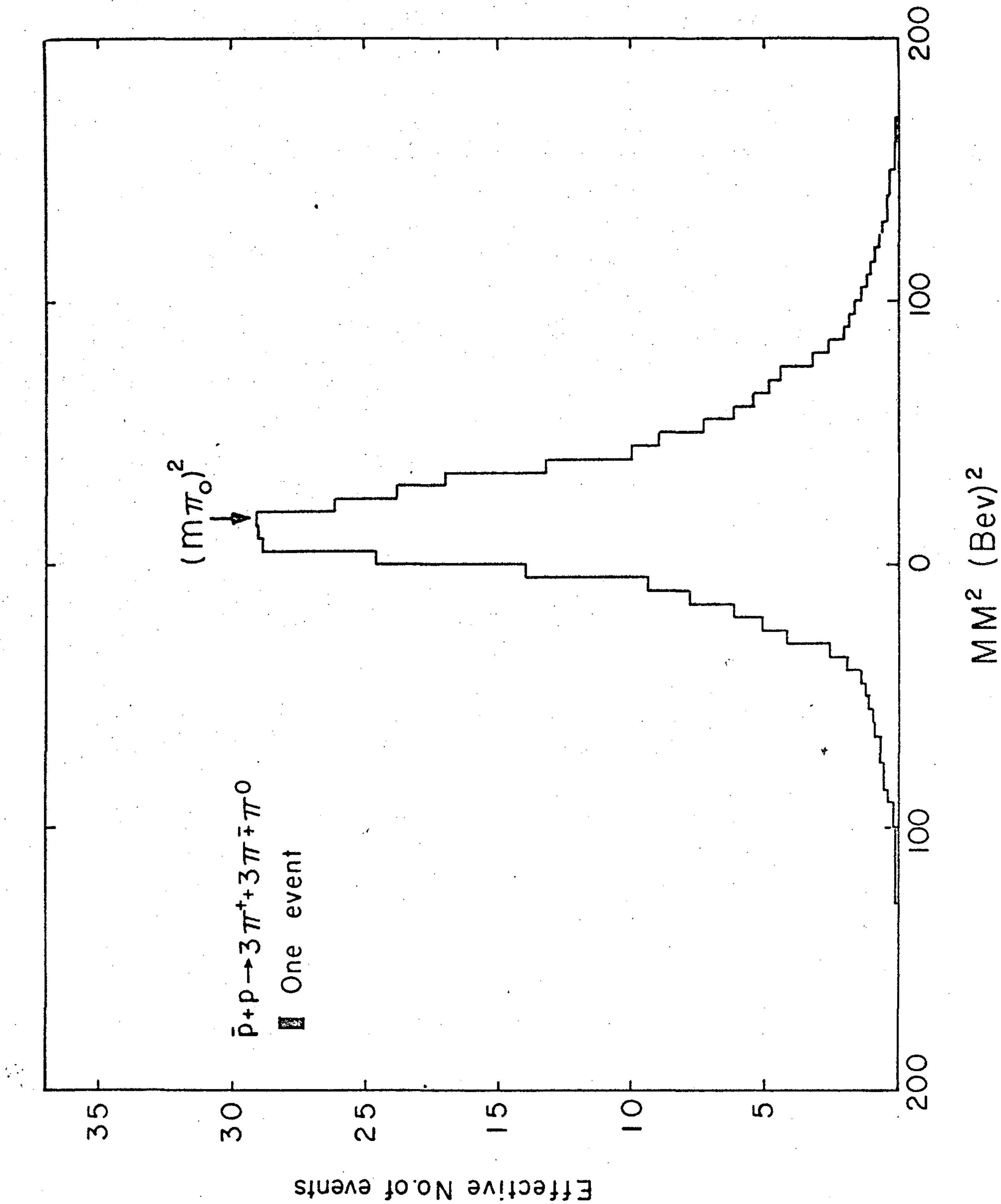
MU-26563

Fig. 3



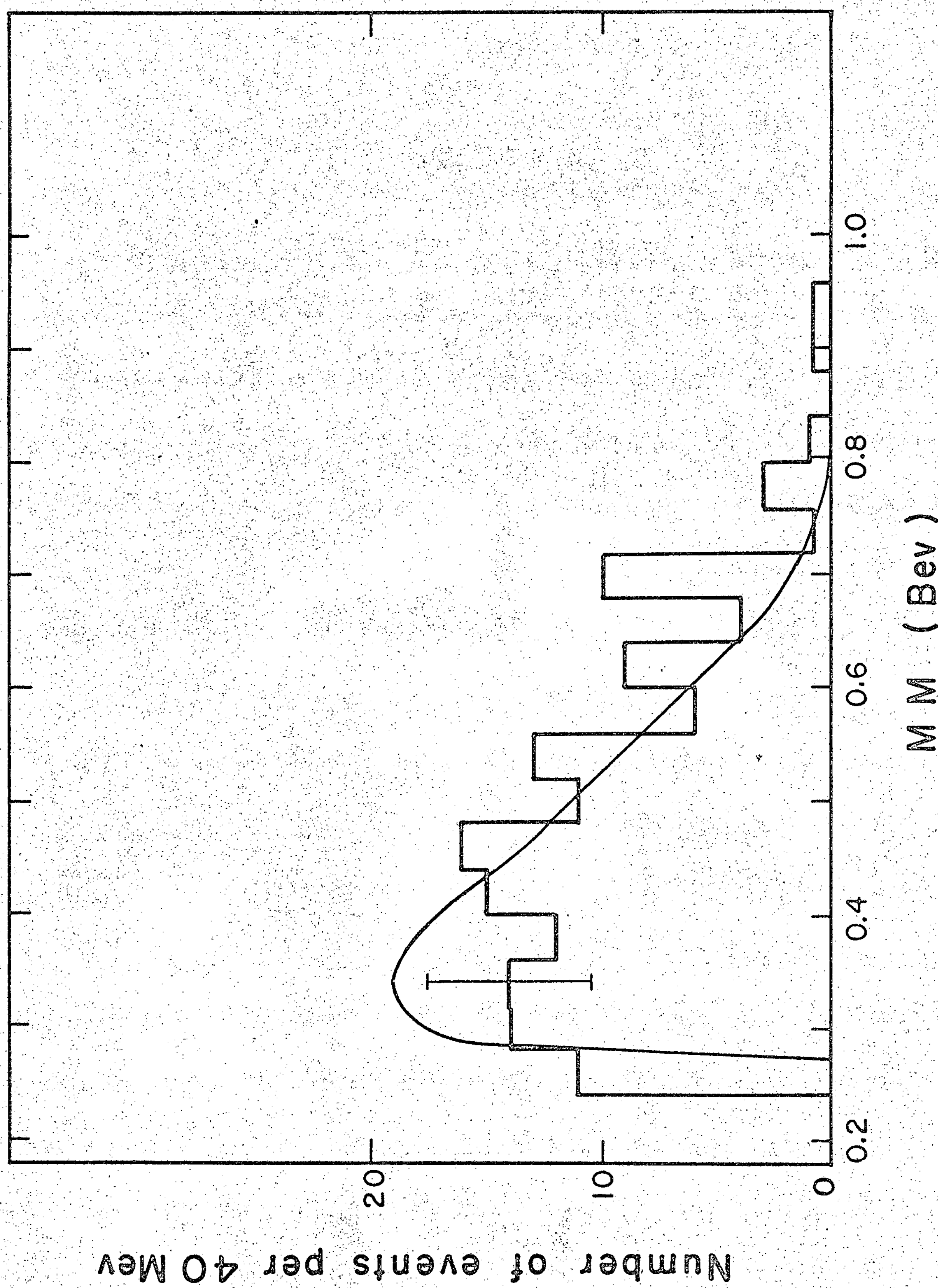
MU-26566

Fig. 4



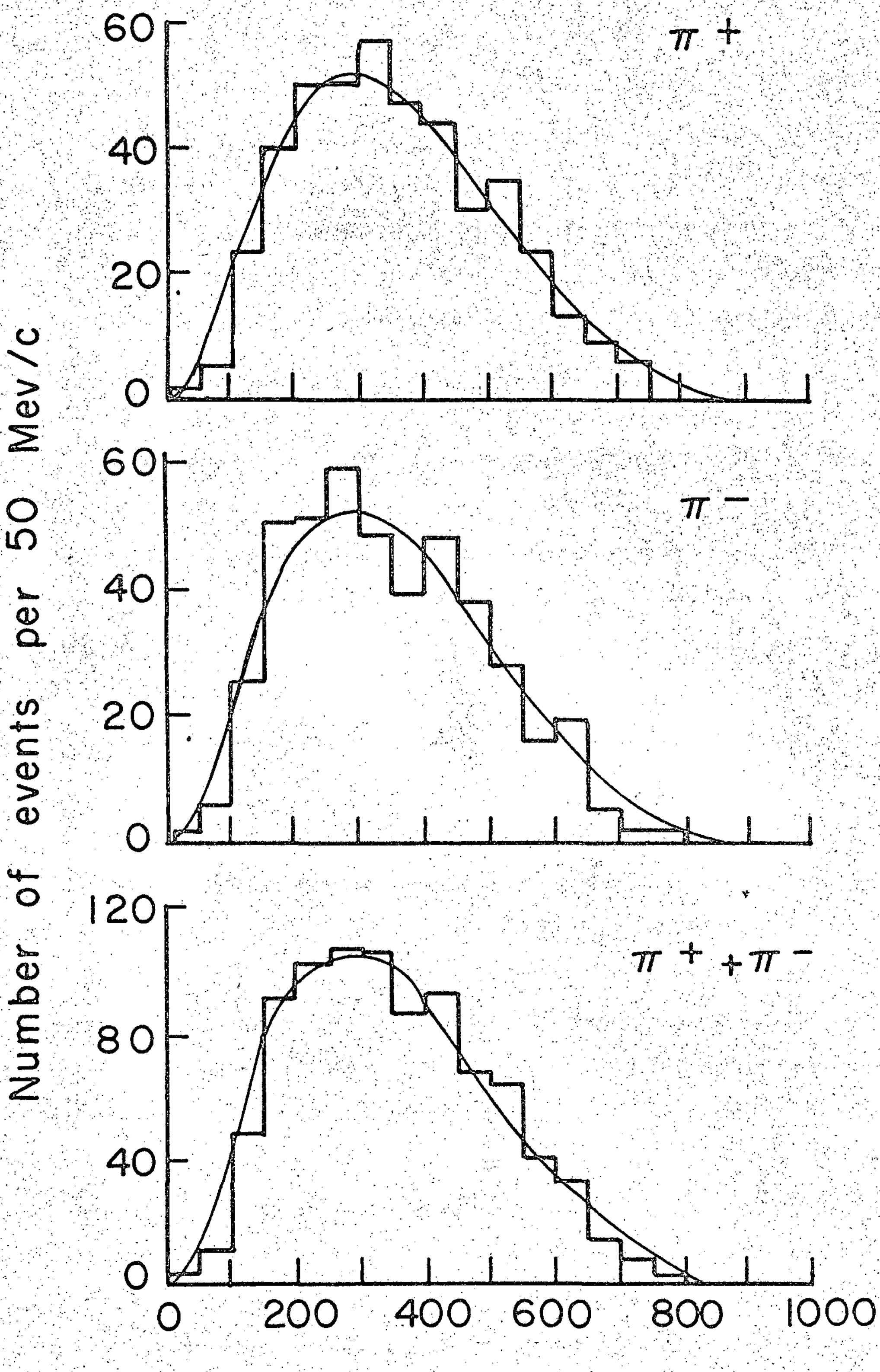
MU-26565

Fig. 5



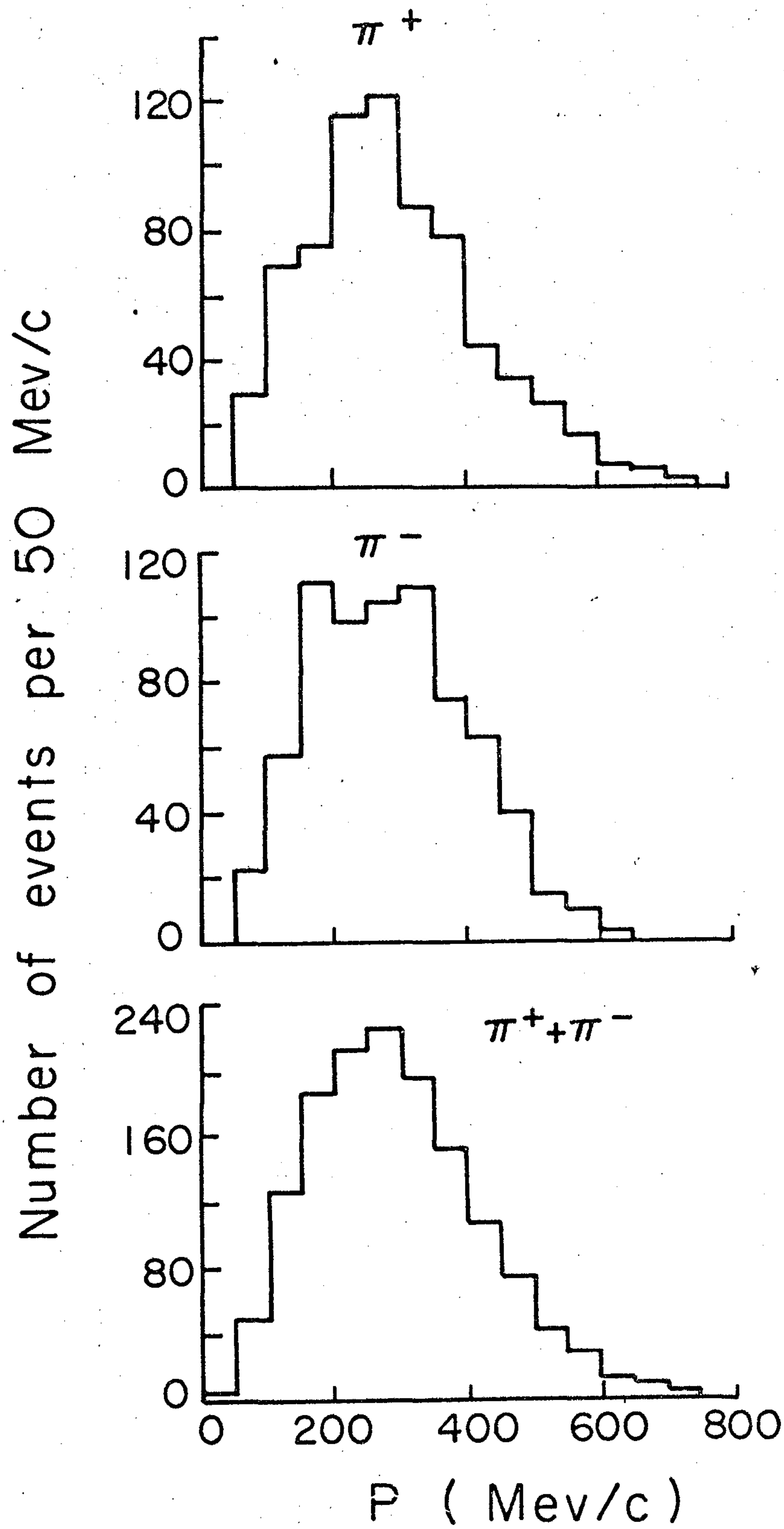
MU-26598

Fig. 6



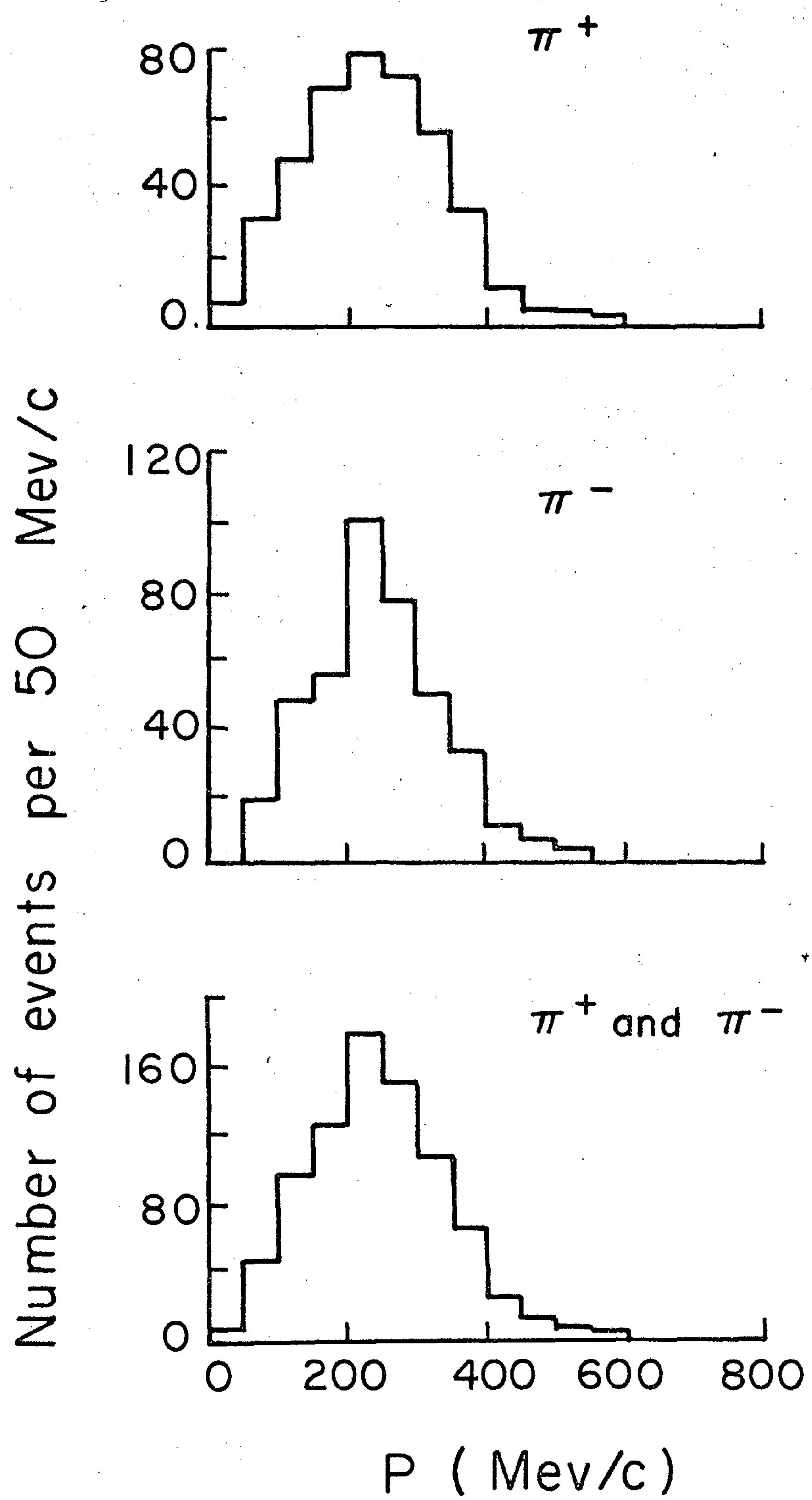
MU-26599

Fig. 7



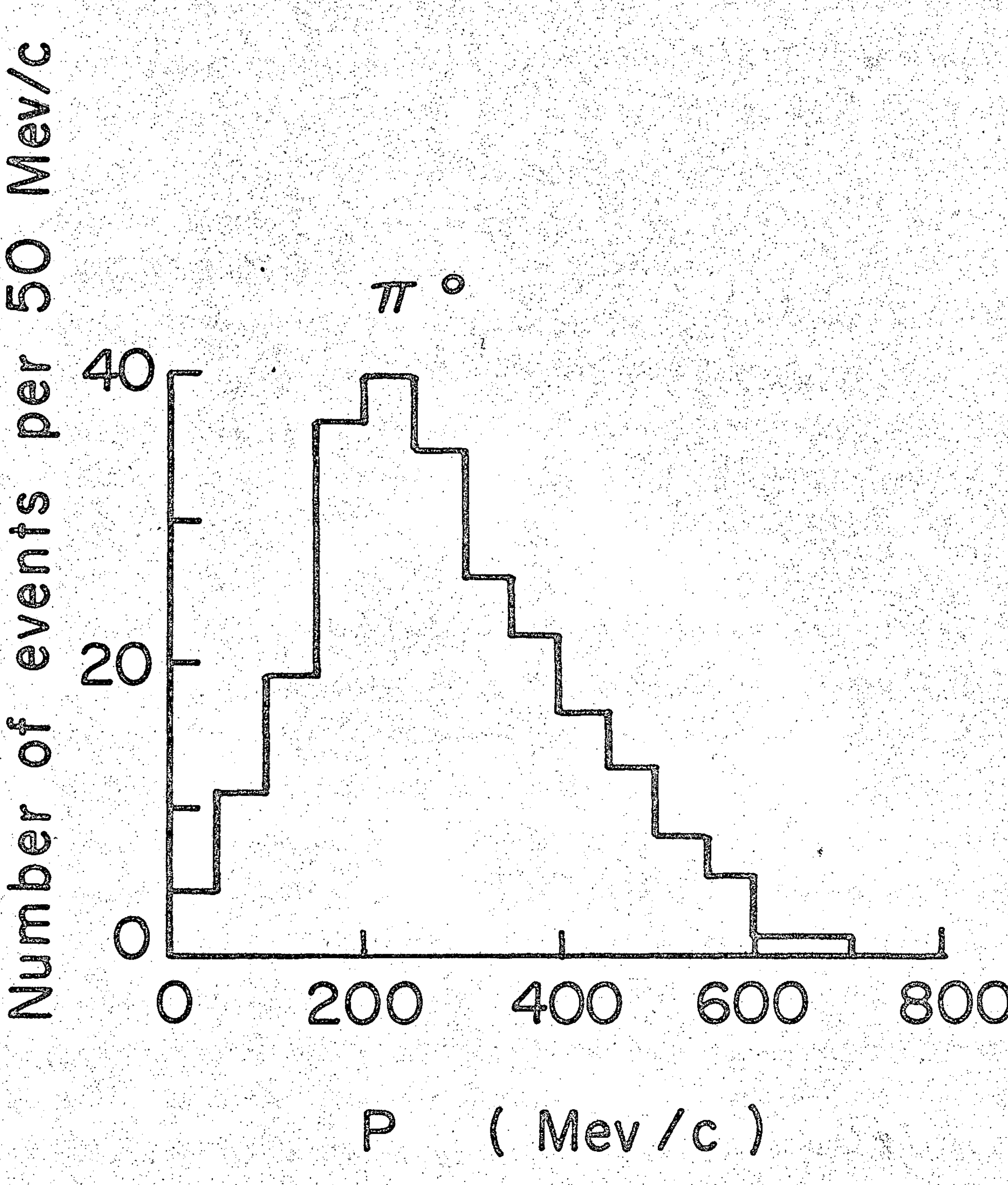
MU-26600

Fig. 8



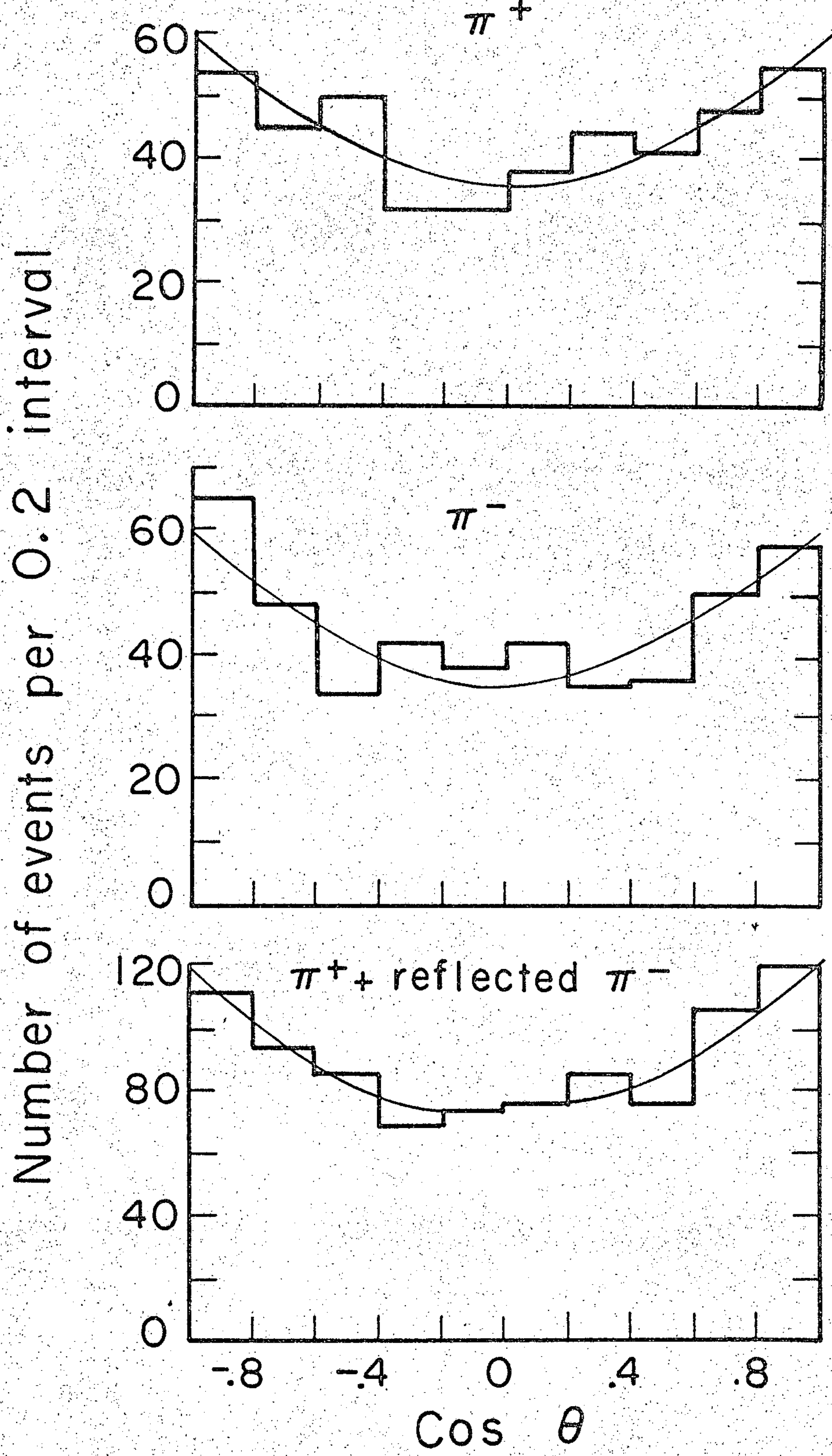
MU-26601

Fig. 9



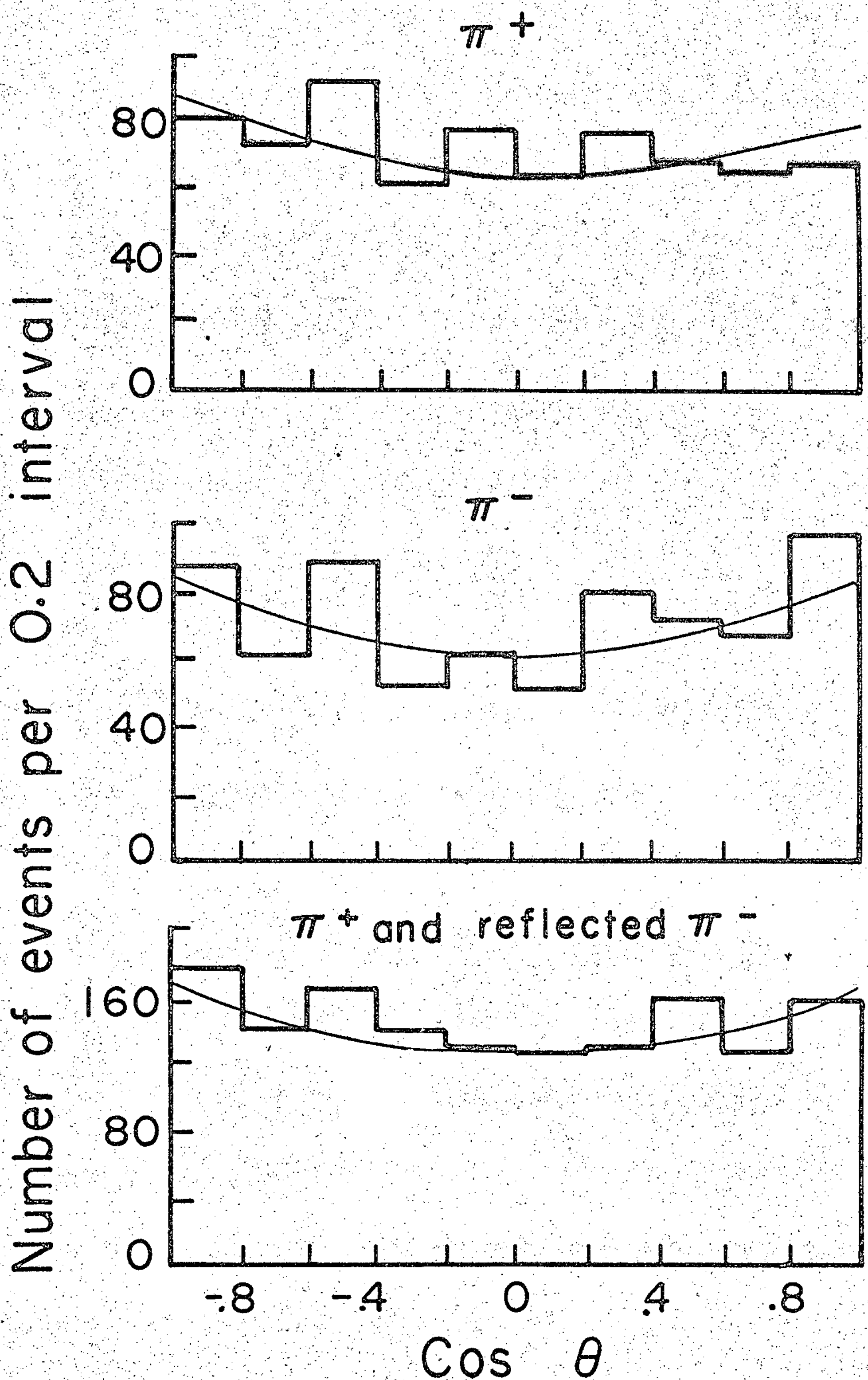
MU-26602

Fig. 10



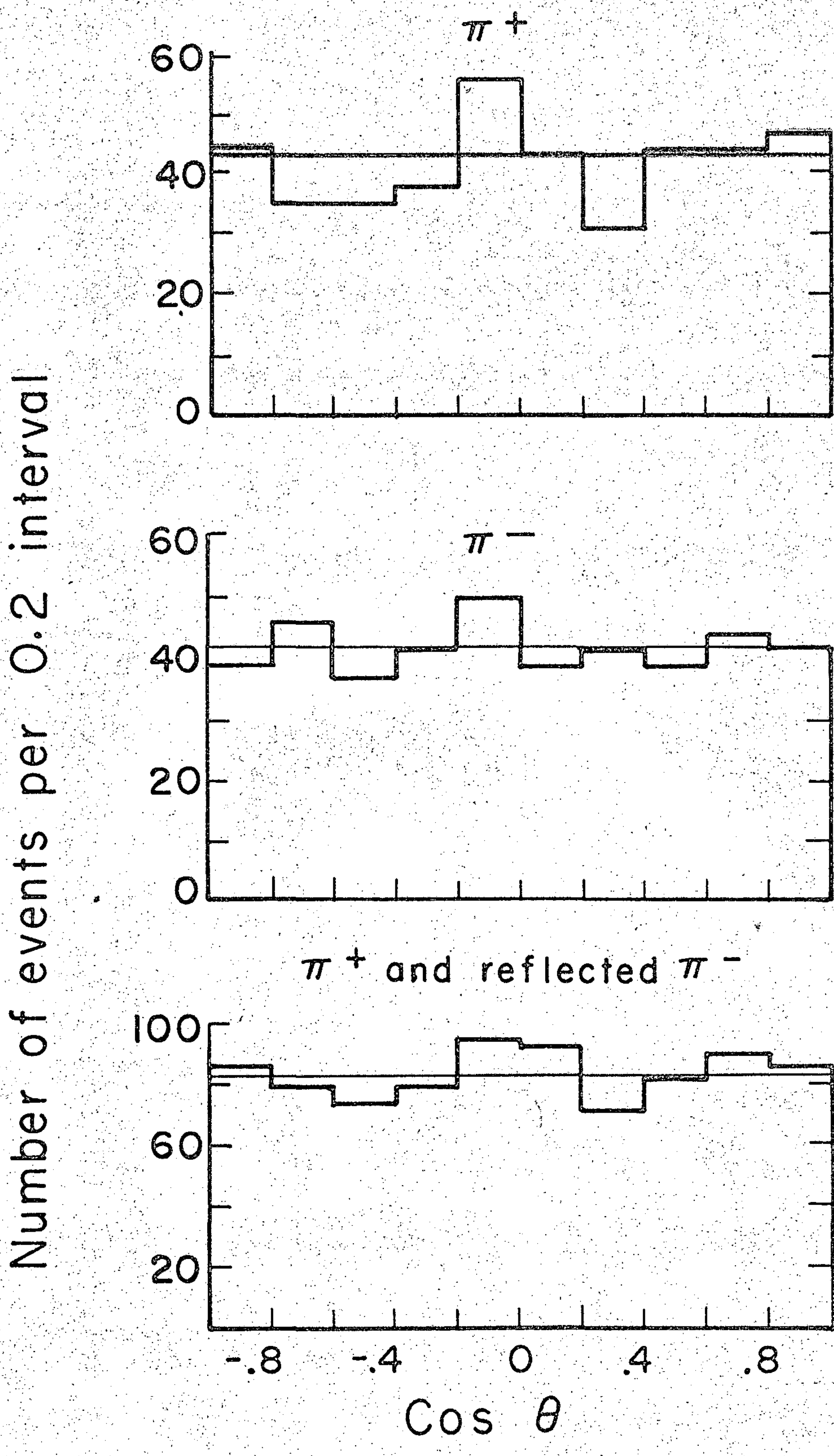
MU-26603

Fig. 11



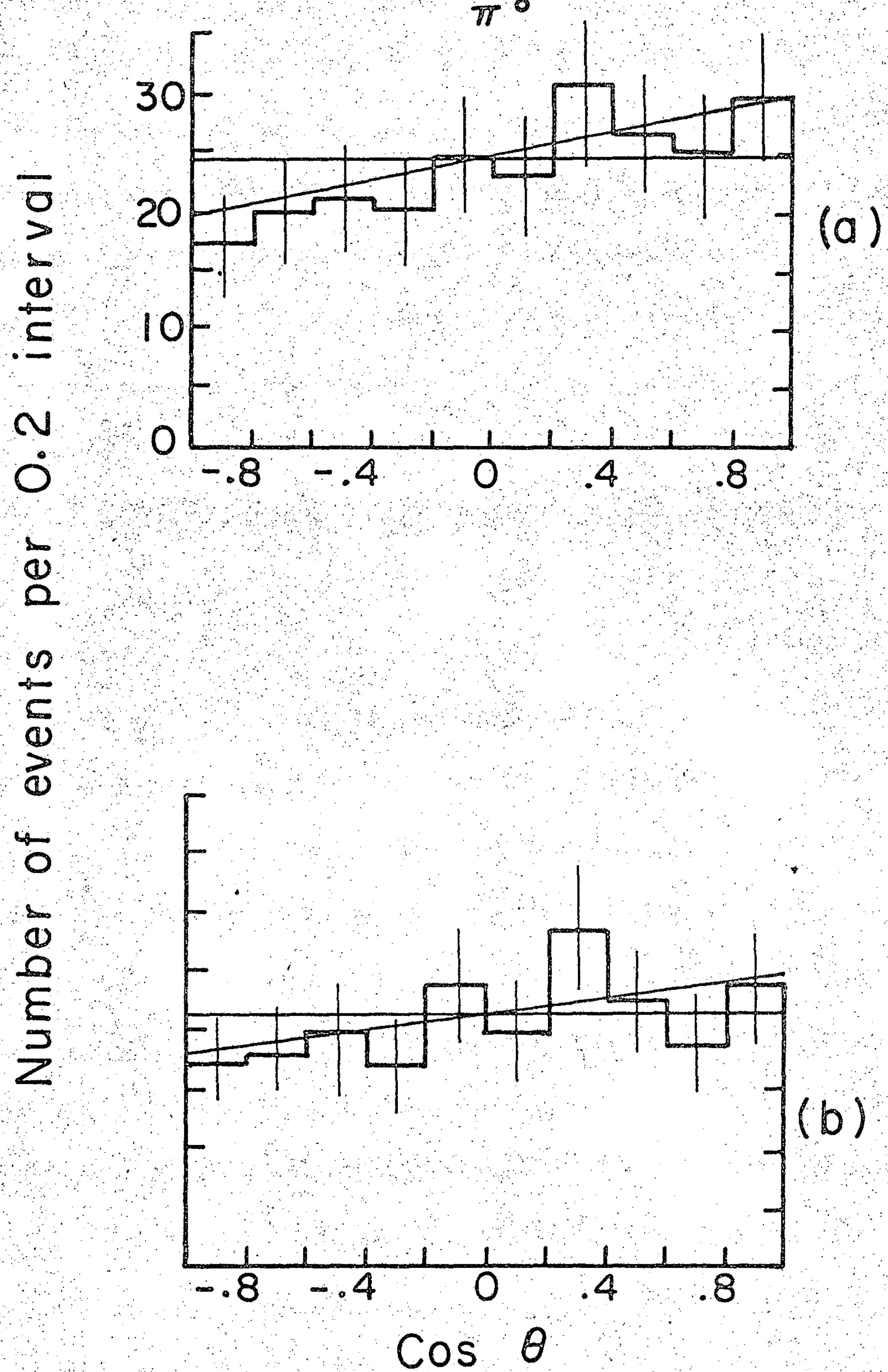
MU-26604

Fig. 12



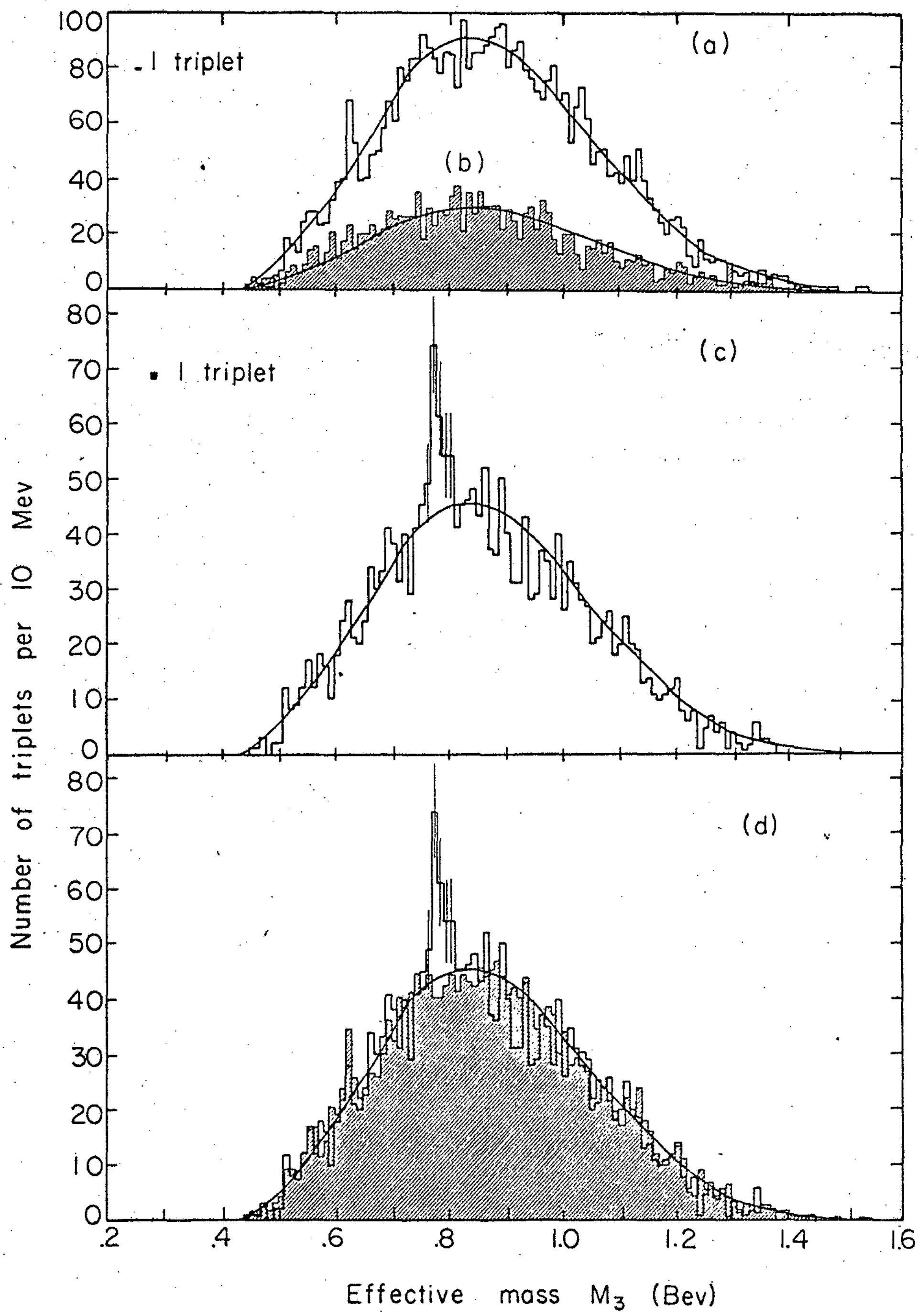
MU-26605

Fig. 13



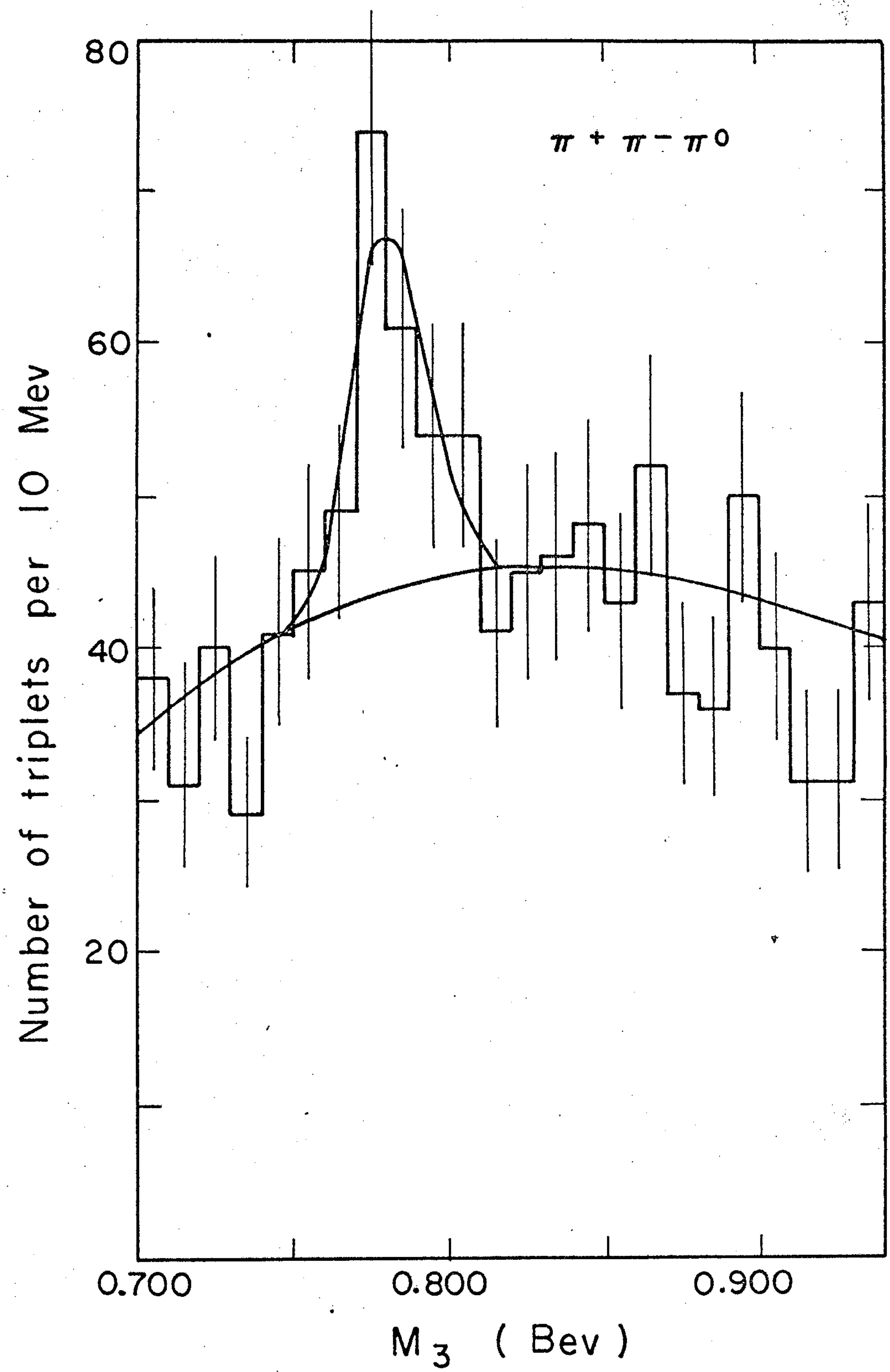
MU-26606

Fig. 14



MUB-1008

Fig. 15



MU-26607

Fig. 16

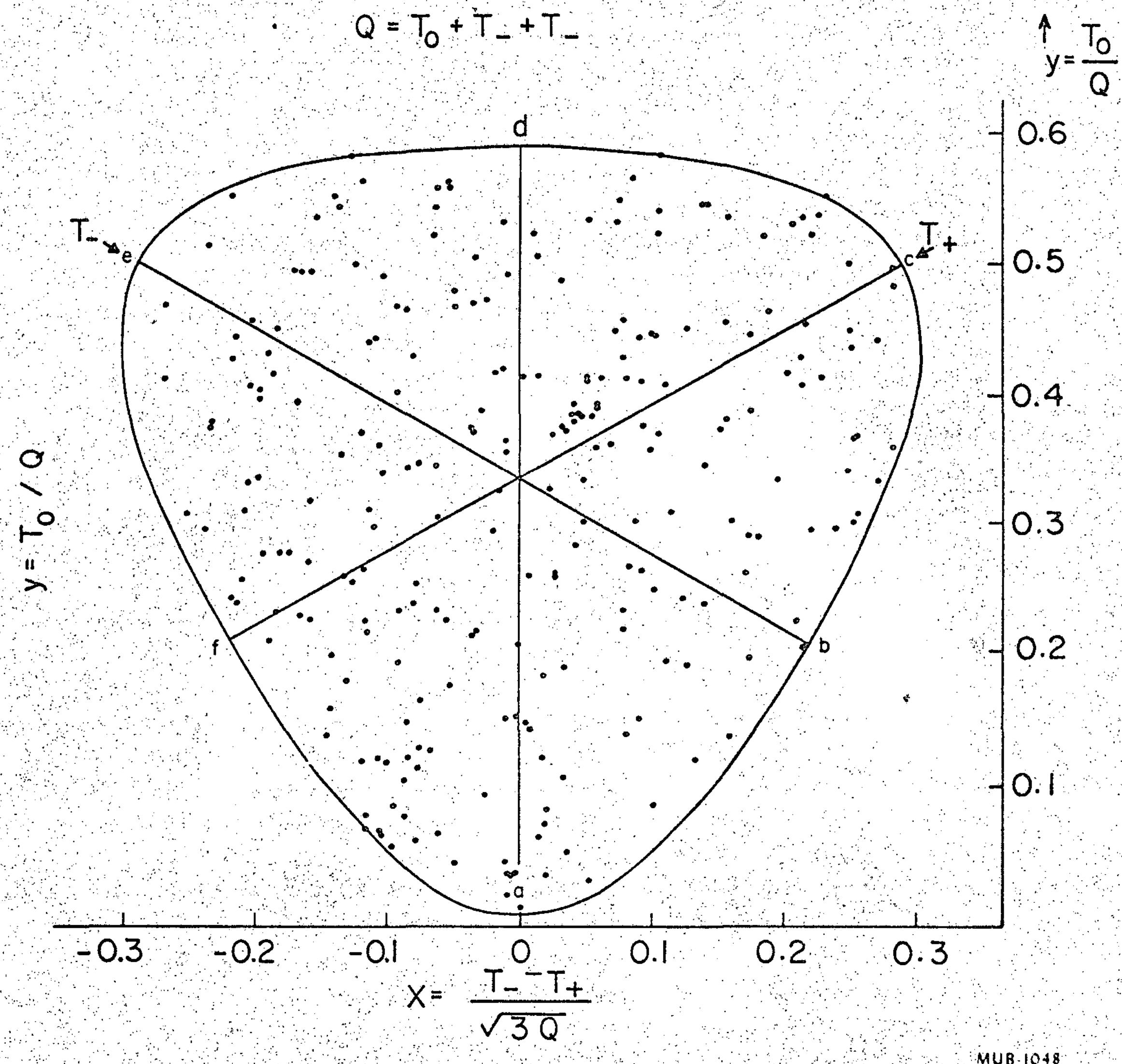
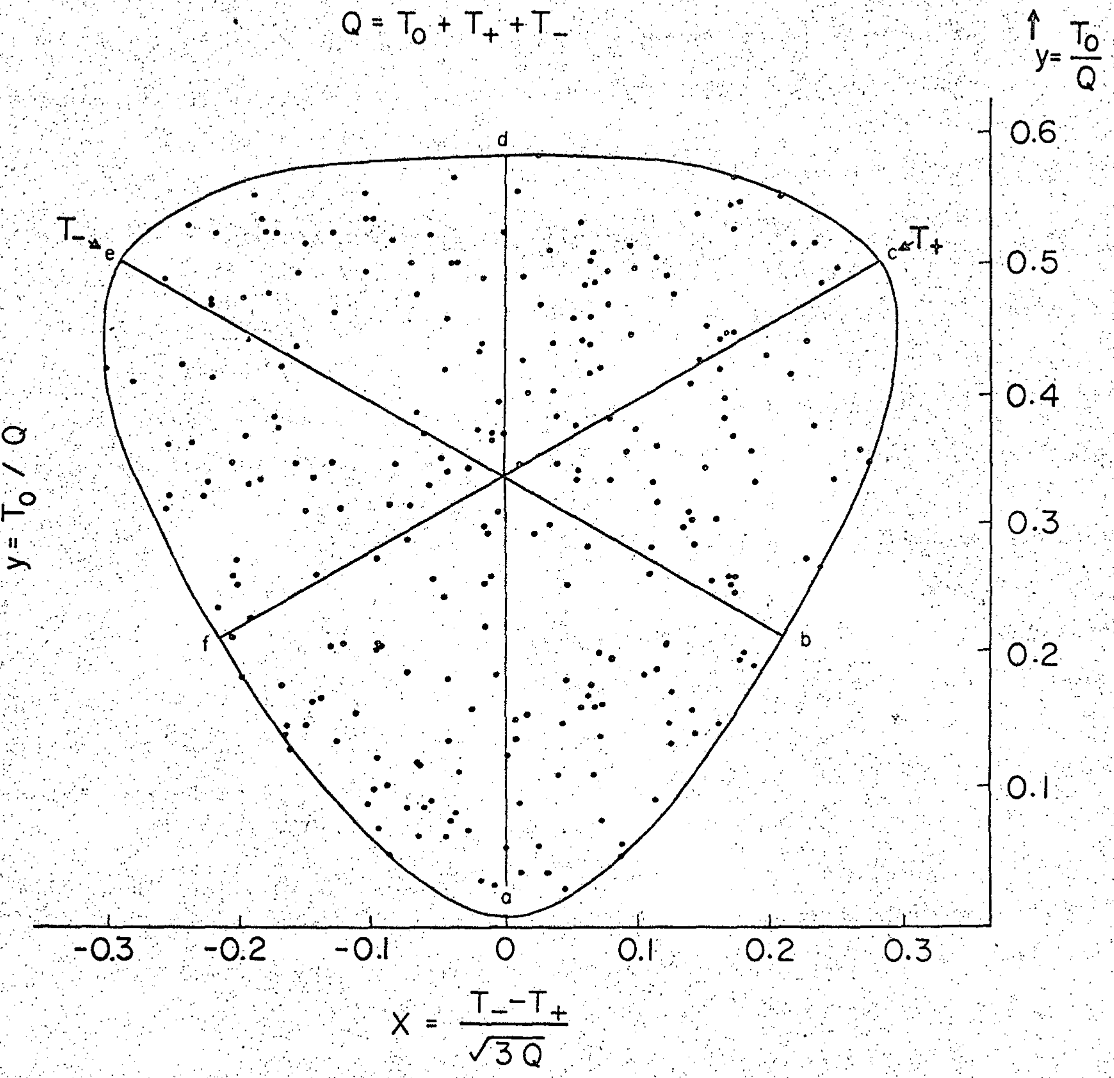
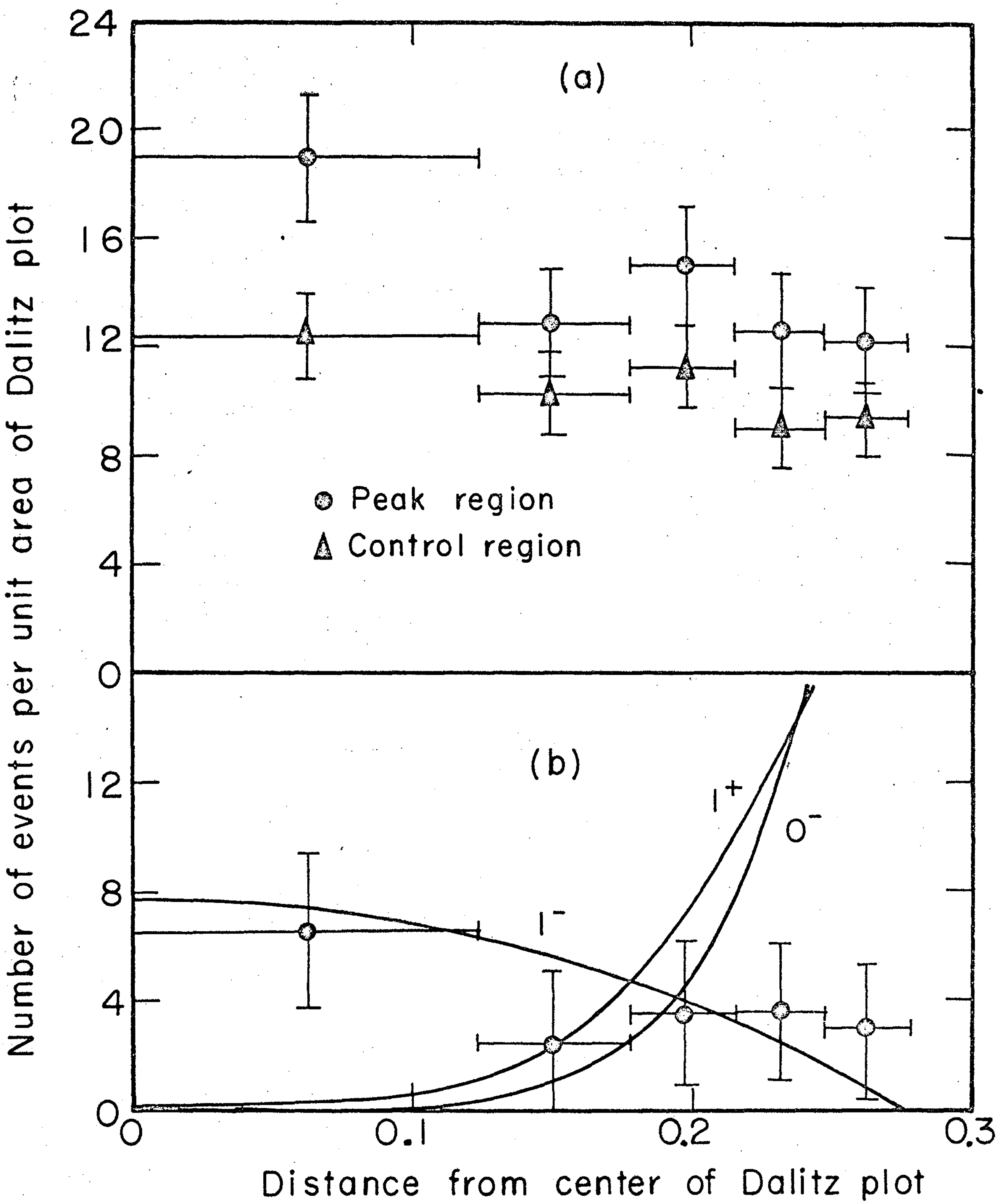

Q = T0 + T− + T−
y = T0/Q
d
T−
e
T+
c
f
b
a
0.6
0.5
0.4
0.3
0.2
0.1
y = T0 / Q
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
X = (T− − T+)/√3 Q
MUB-1048

Fig. 17



Q = T₀ + T₊ + T₋
y = T₀/Q
d
T₋ e
c T₊
f
b
a
0.6
0.5
0.4
0.3
0.2
0.1
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
y = T₀ / Q
X = (T₋ − T₊)/√3 Q
MUB-1049

Fig. 18



MU-24568

Fig. 19

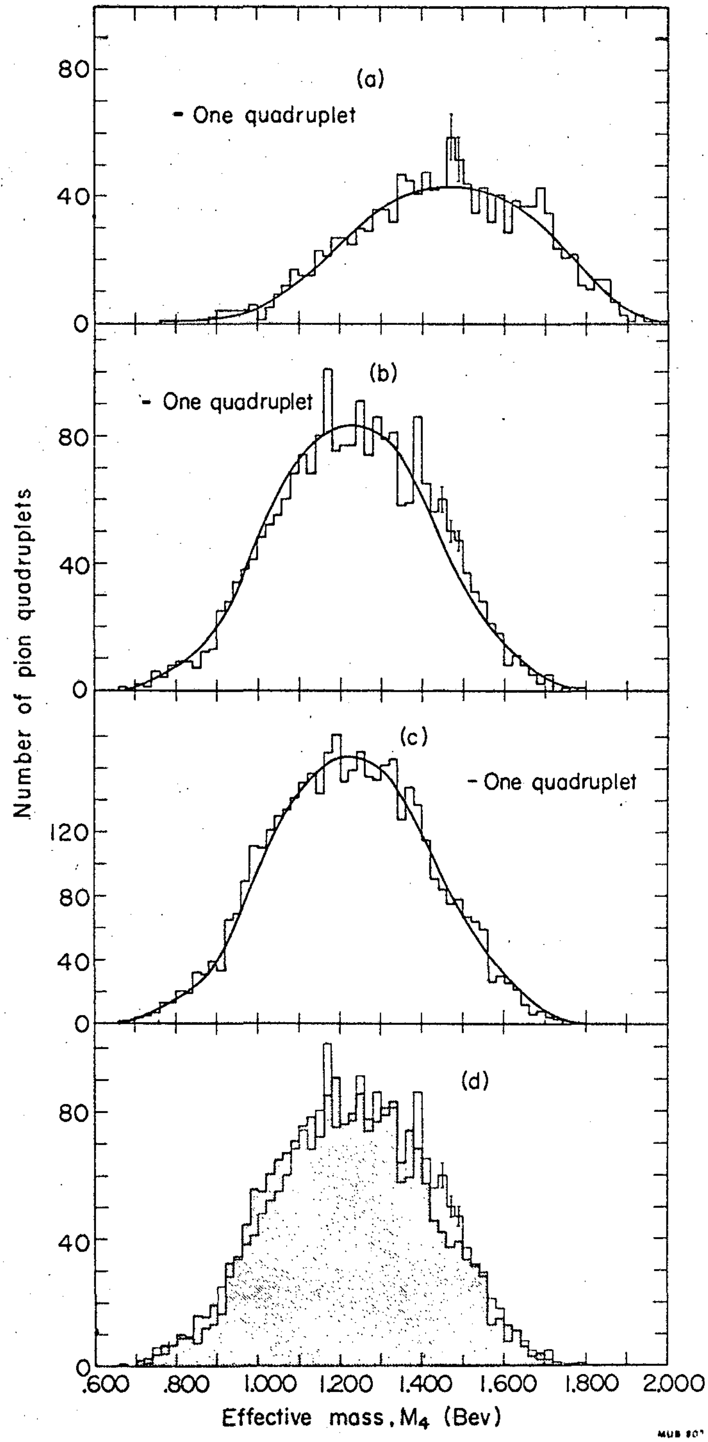

(a)
- One quadruplet
(b)
- One quadruplet
(c)
- One quadruplet
(d)
Number of pion quadruplets
0
40
80
120
.600
.800
1.000
1.200
1.400
1.600
1.800
2.000
Effective mass, M_4 (Bev)

Fig. 20

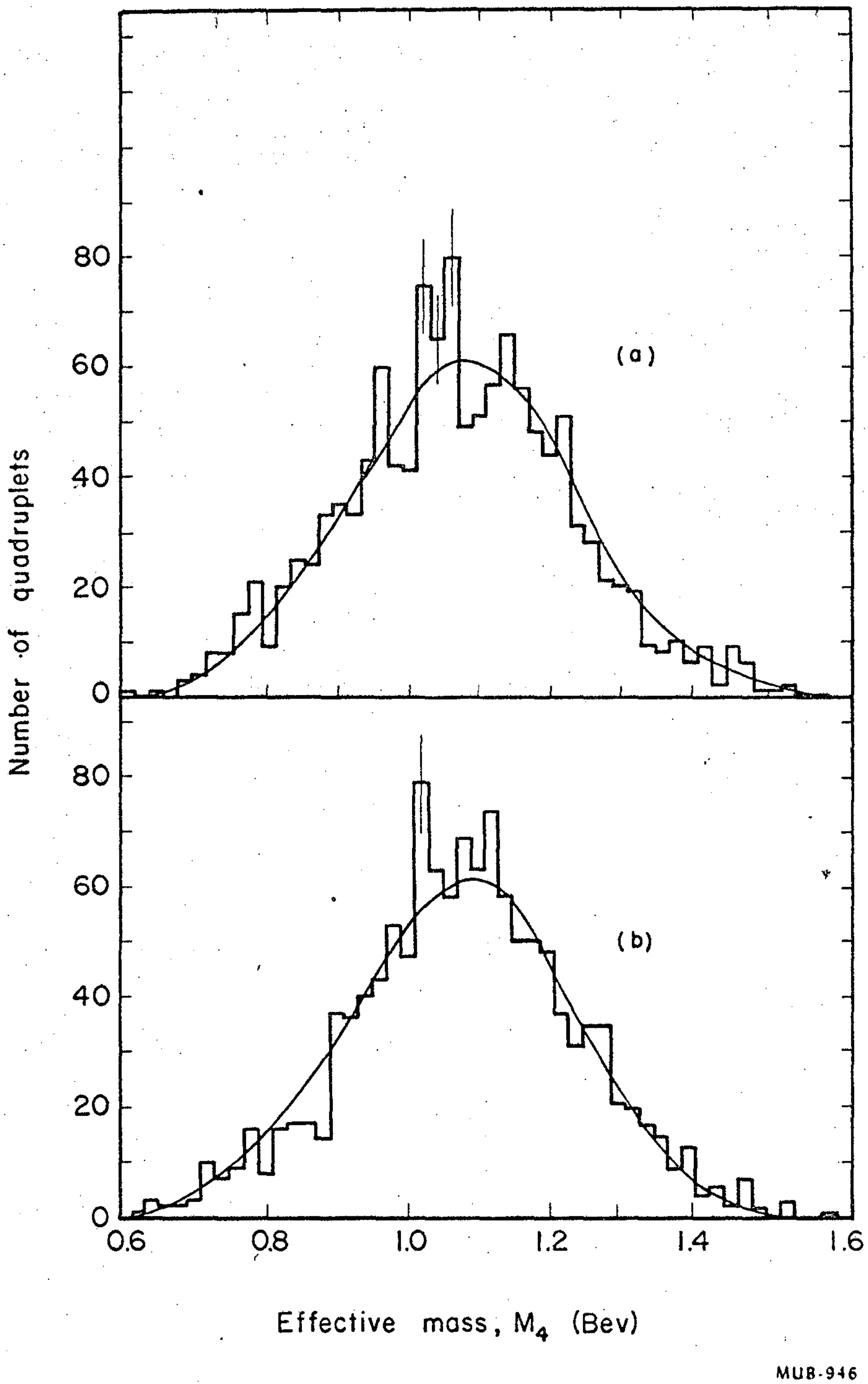
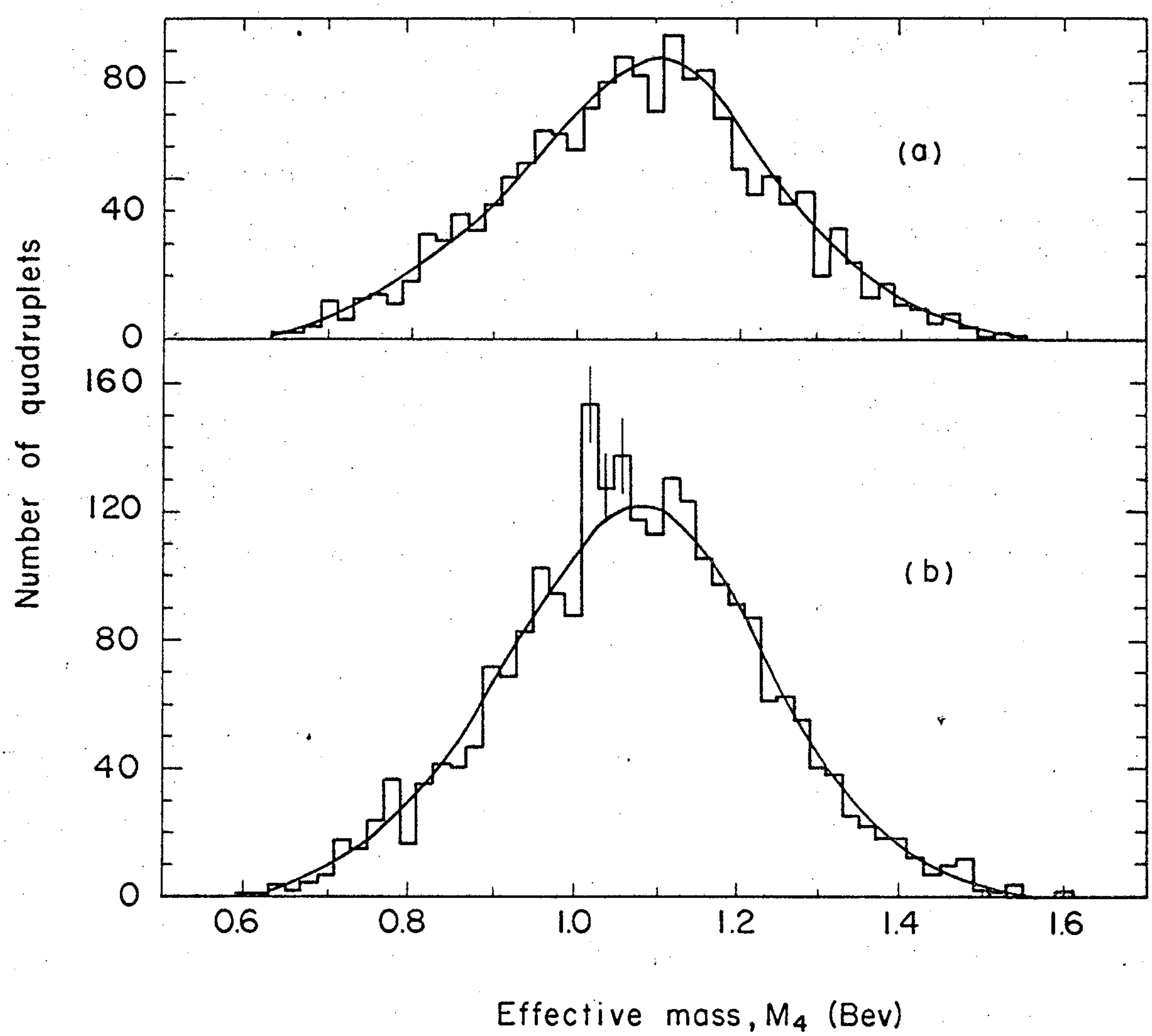

80
60
40
20
0
80
60
40
20
0
(a)
(b)
Number of quadruplets
0.6
0.8
1.0
1.2
1.4
1.6
Effective mass, M4 (Bev)
MUB-946

Fig. 21



MUB-947

Fig. 22

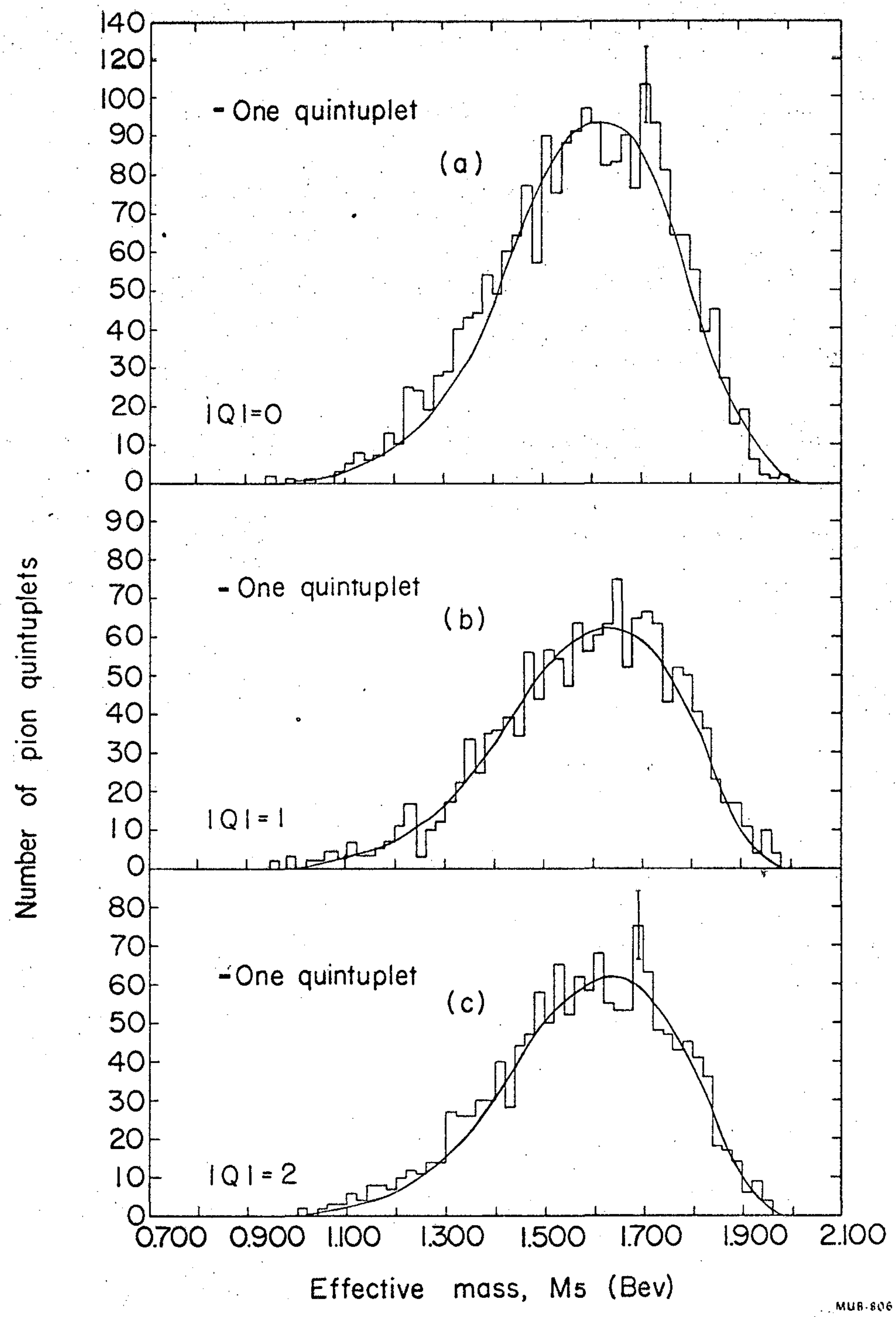
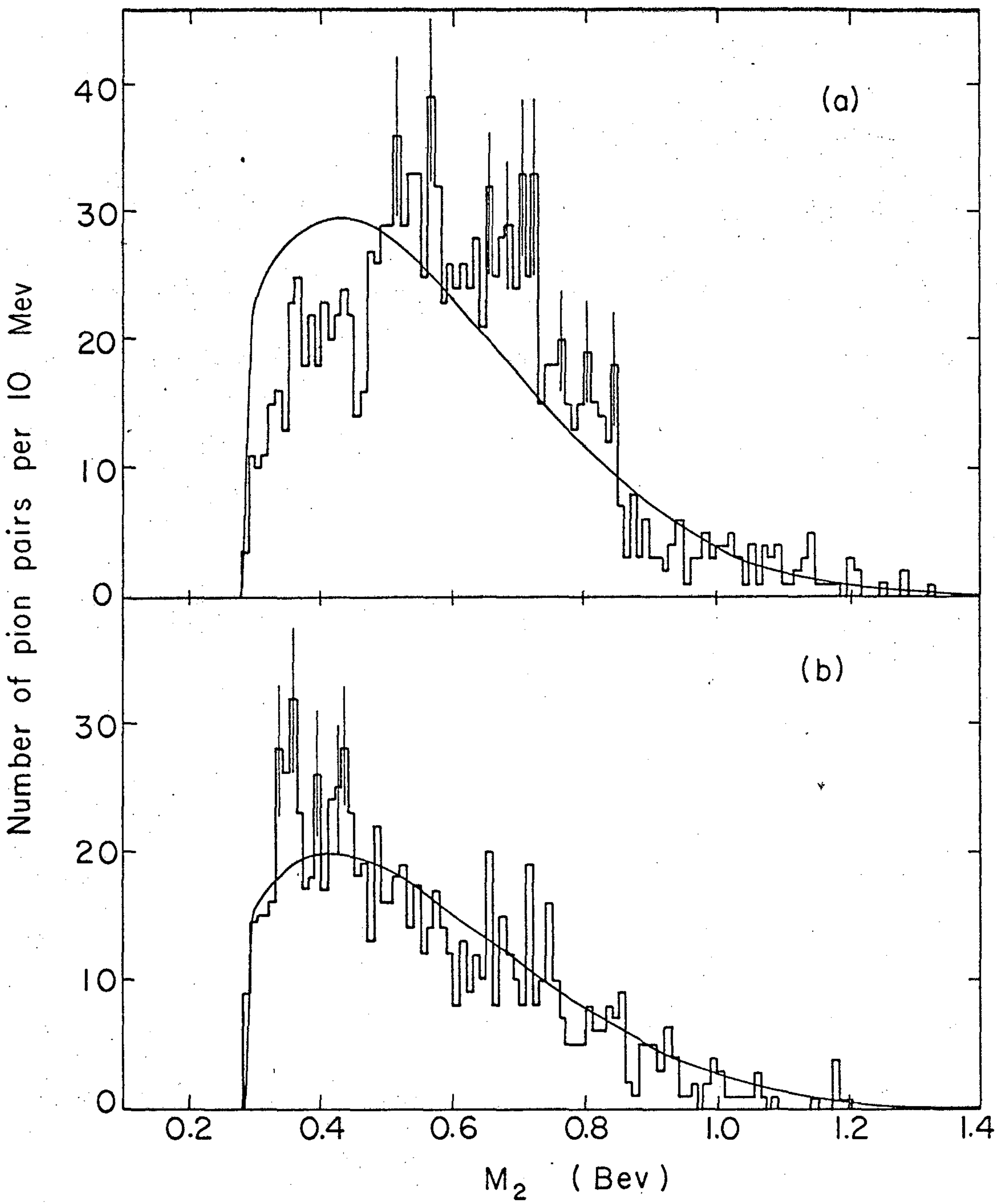
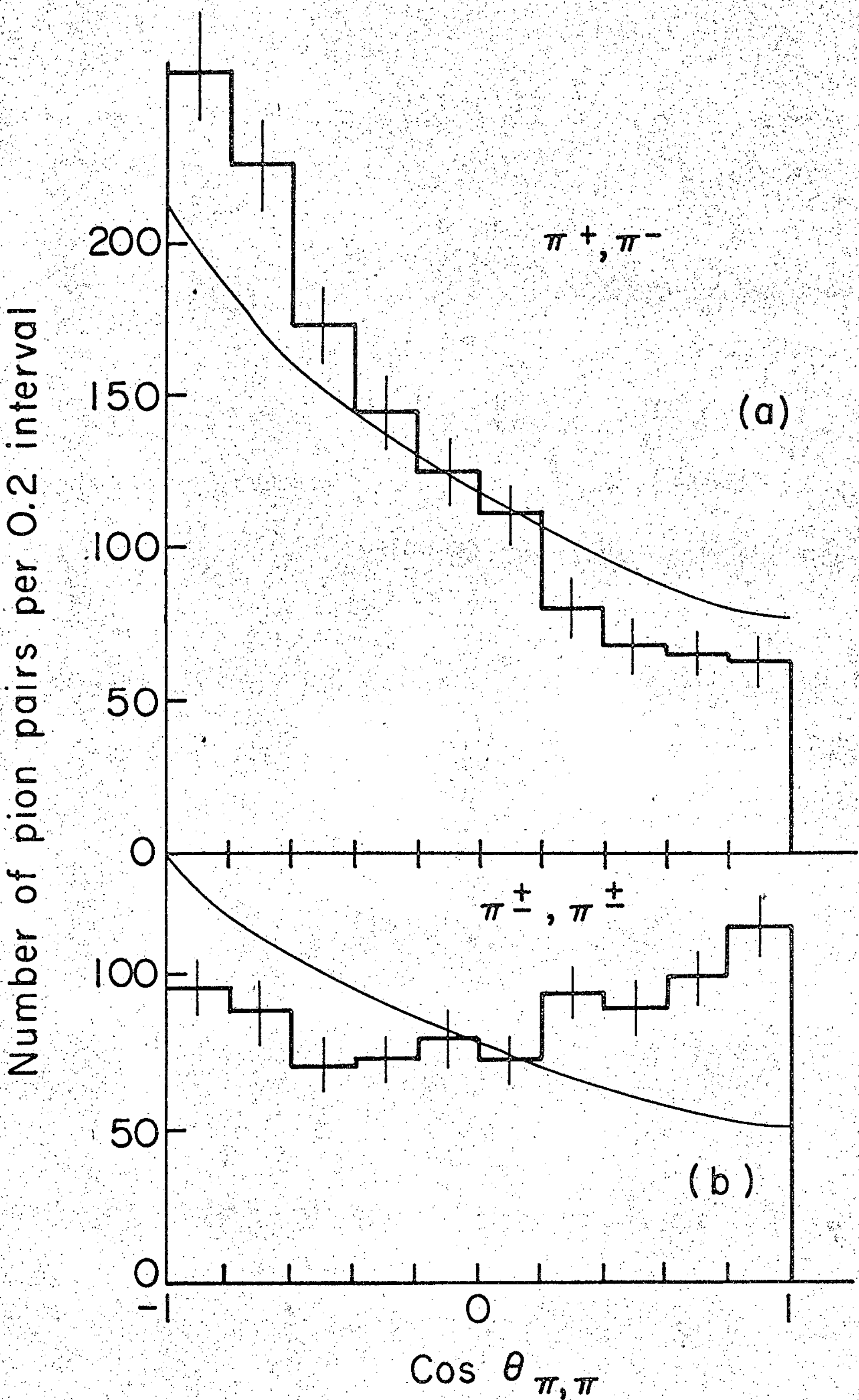

Number of pion quintuplets
- One quintuplet
(a)
|Q|=0
- One quintuplet
(b)
|Q|=1
- One quintuplet
(c)
|Q|=2
0.700 0.900 1.100 1.300 1.500 1.700 1.900 2.100
Effective mass, M5 (Bev)
MUB-806

Fig. 23



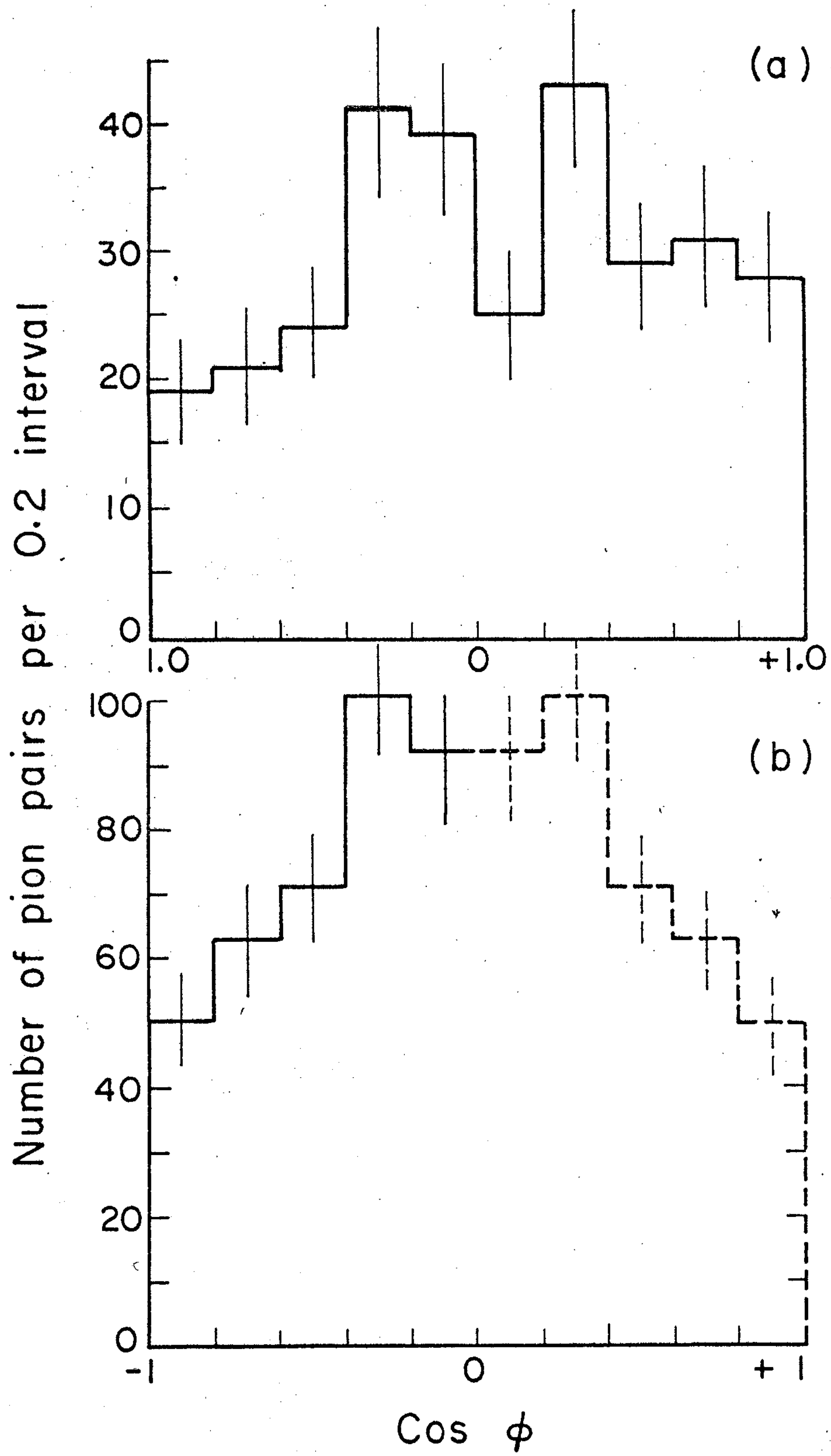
MUB-1050

Fig. 24



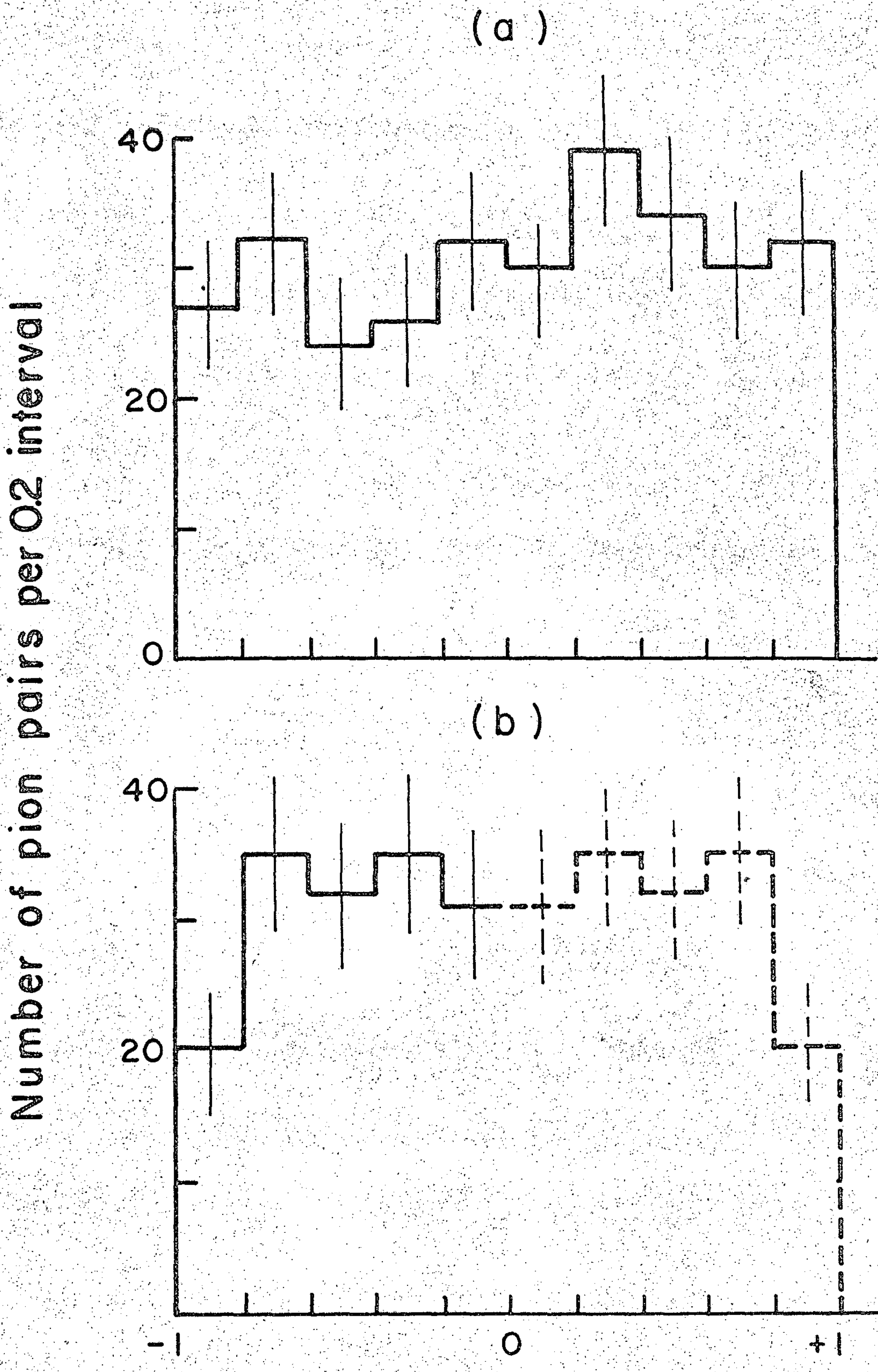
MU-26608

Fig. 25



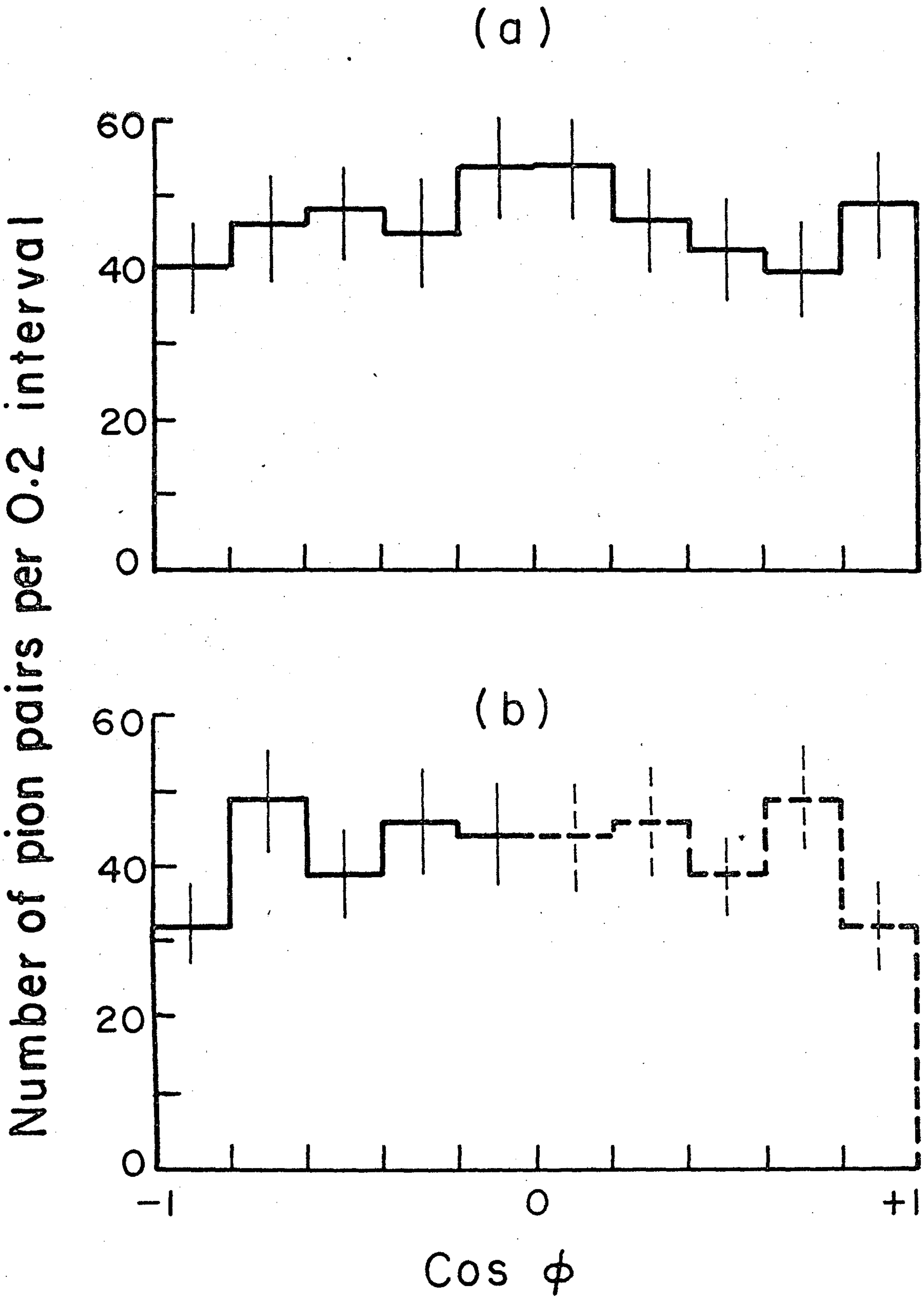
MU-26609

Fig. 26



MU-26610

Fig. 27



MU-26611

Fig. 28

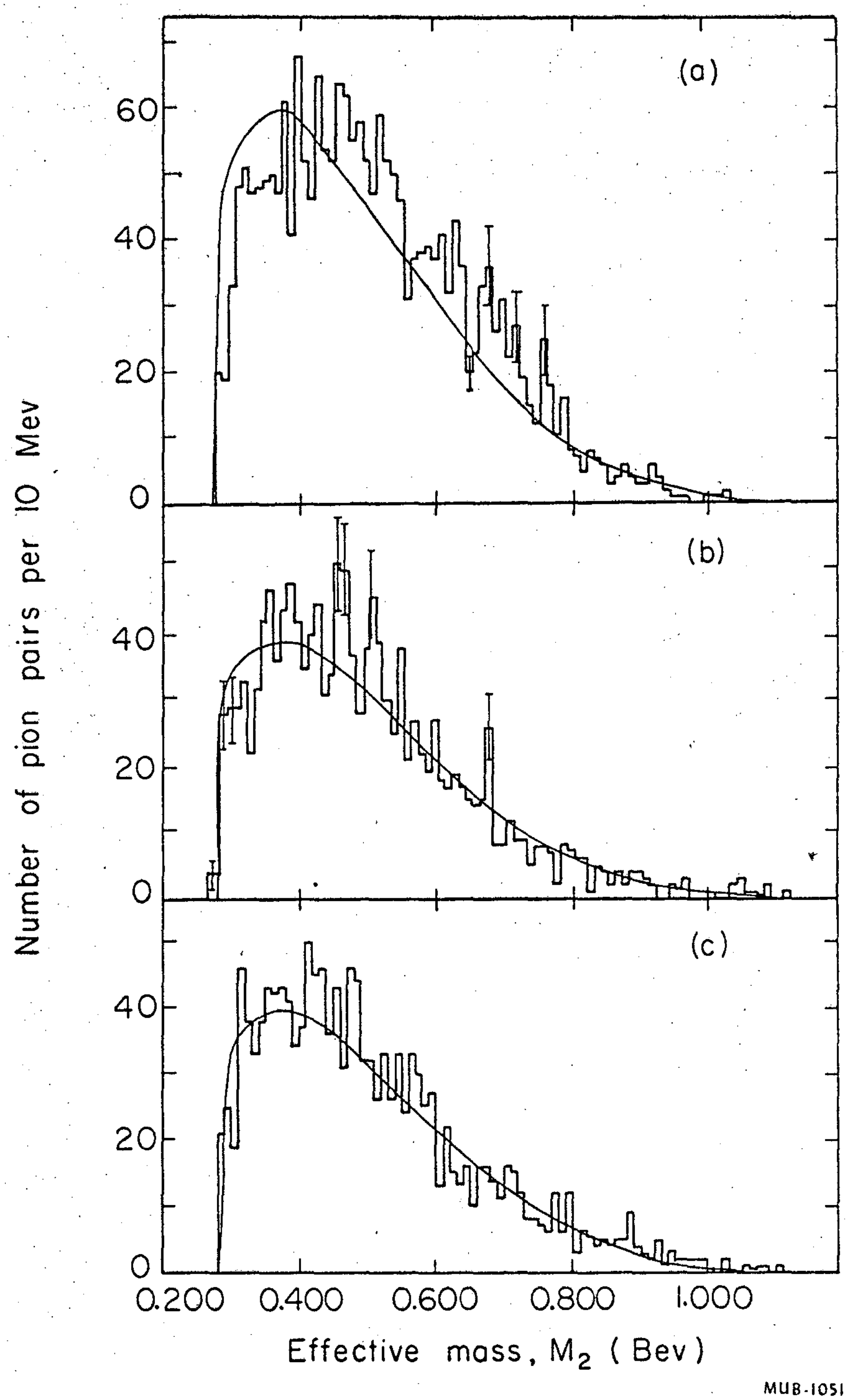
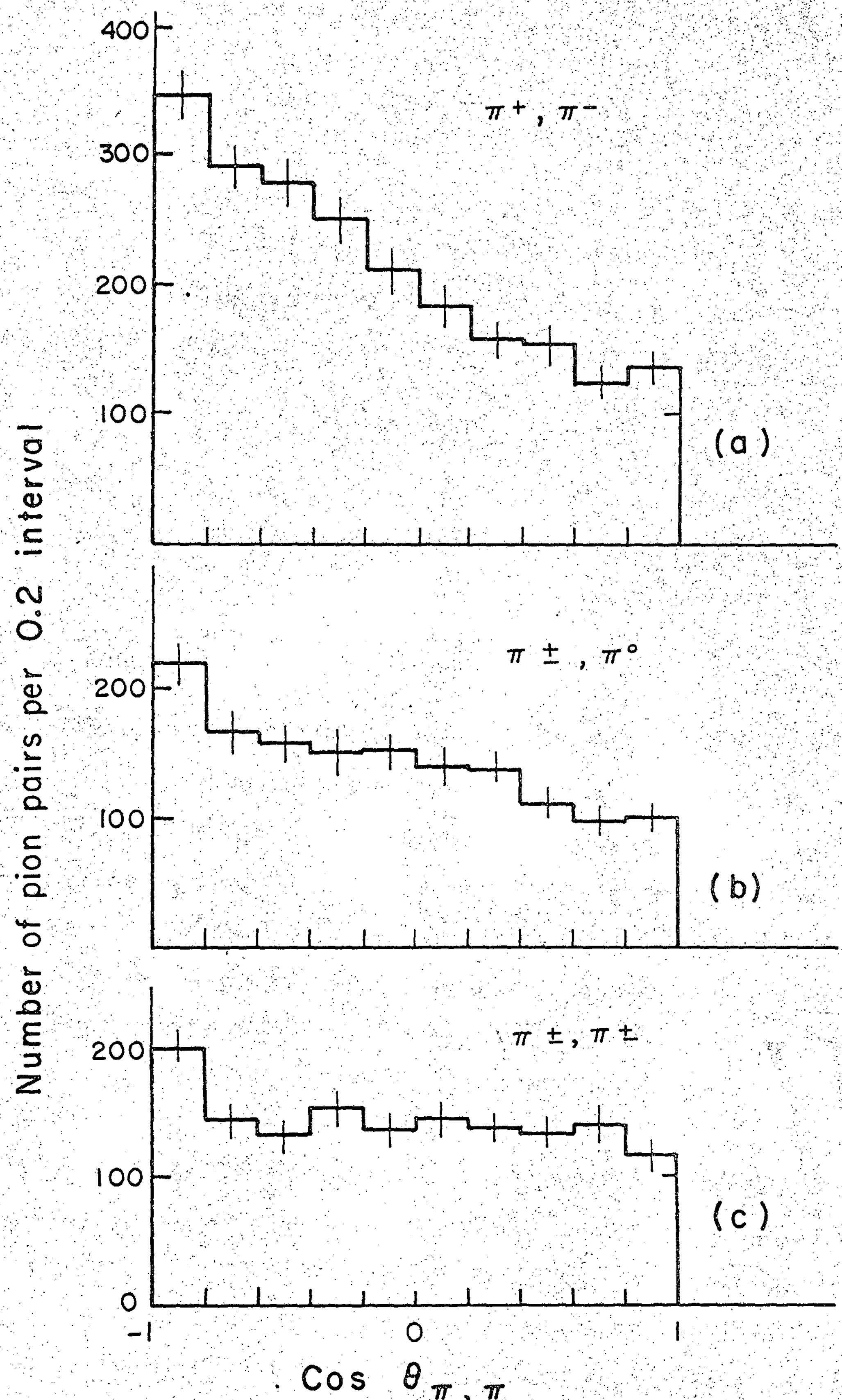
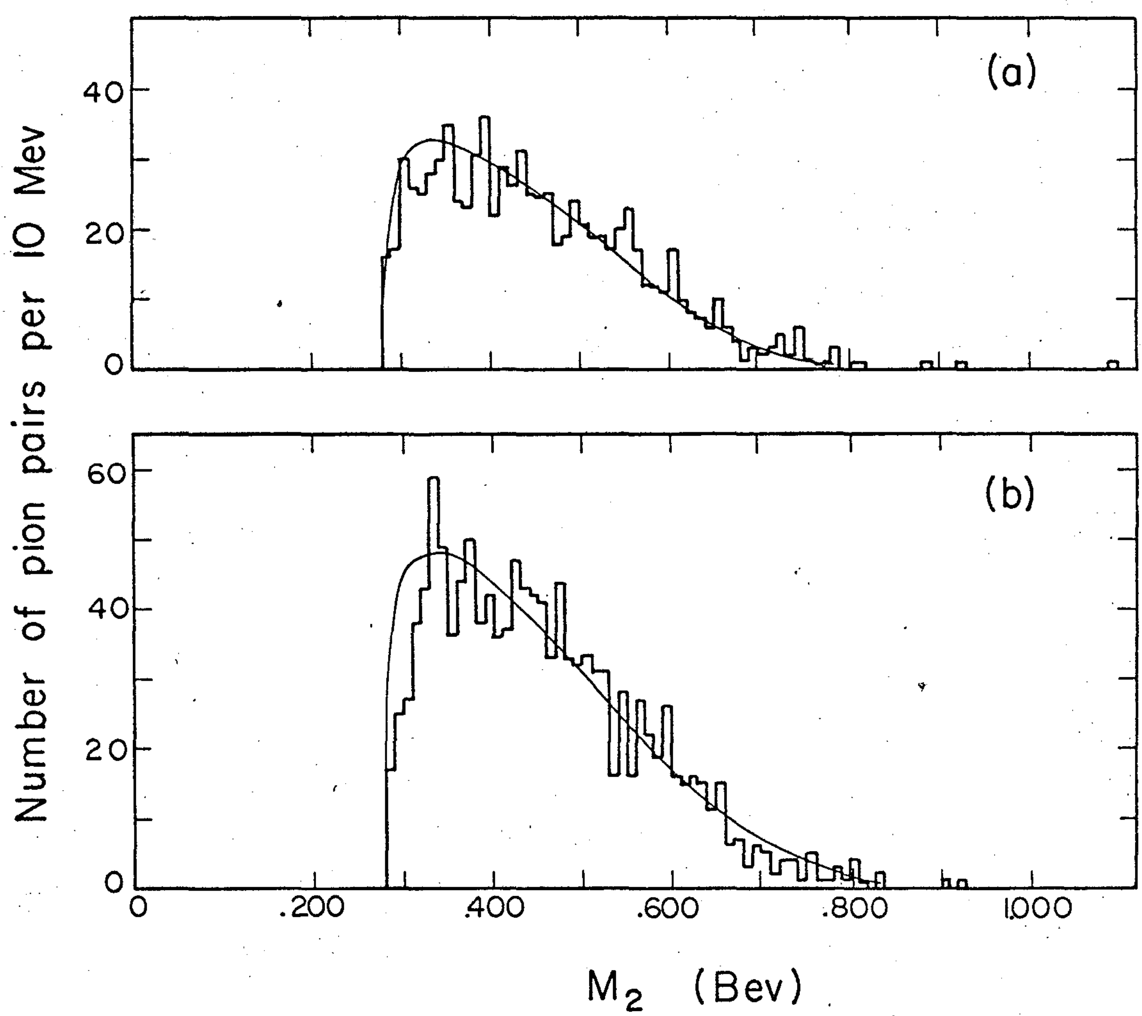

(a)
(b)
(c)
60
40
20
0
40
20
0
40
20
0
Number of pion pairs per 10 Mev
0.200
0.400
0.600
0.800
1.000
Effective mass, M_2 (Bev)
MUB-1051

Fig. 29



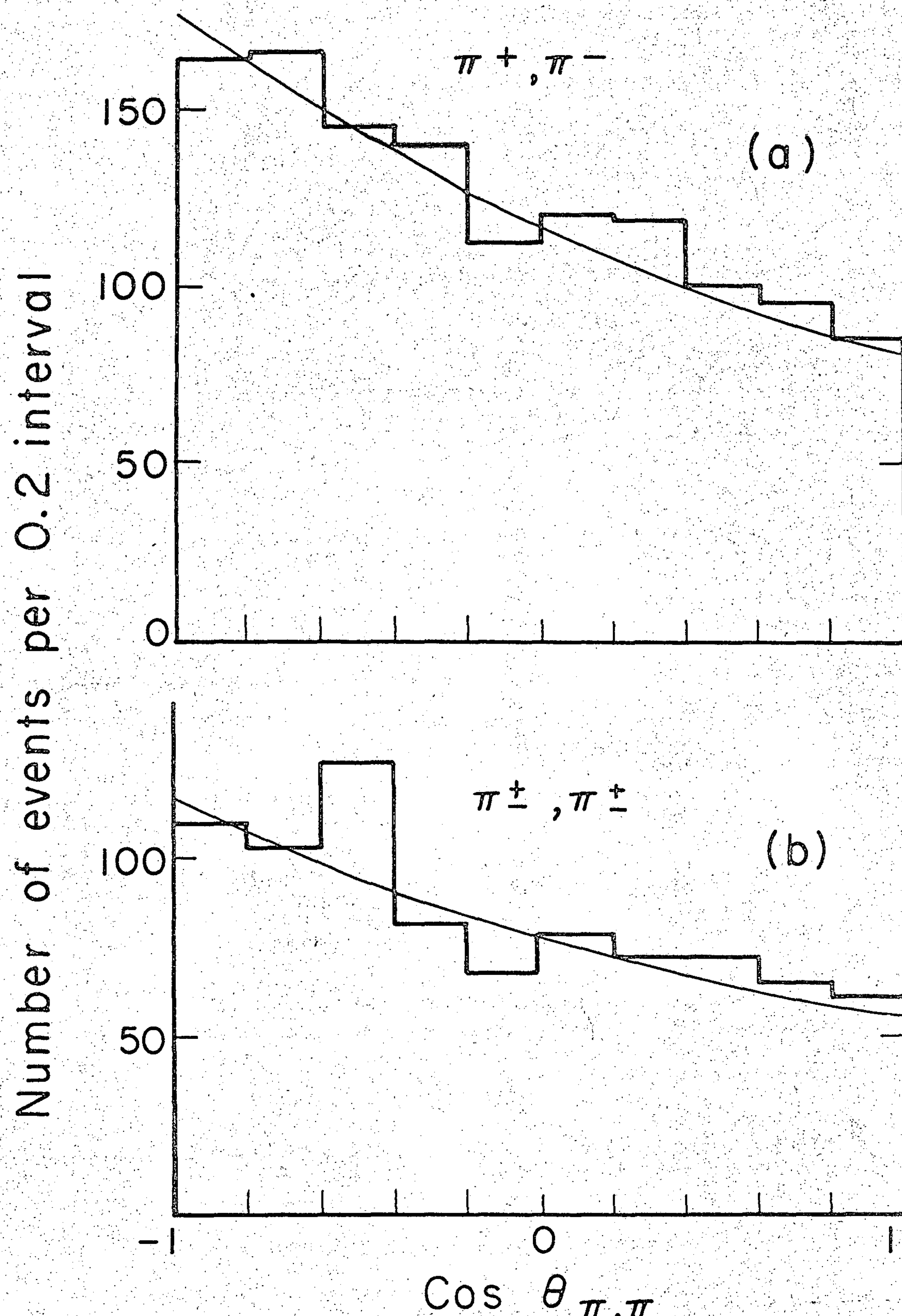
MU-26612

Fig. 30



MUB-1052

Fig. 31



MU-26613

Fig. 32

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