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# THE ECONOMIC THRESHOLD WITH A STOCHASTIC PEST POPULATION: A REAL OPTIONS APPROACH \*

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## ABSTRACT

Using real options, this paper formulates an optimal stopping model for applying pest control measures when the density of a pest population varies randomly. A delay between successive pesticide applications is introduced to analyze the farmer's expected marginal cost of reentry. This model is applied to the control of a foliar pest of apples via a pesticide, and solved numerically. A sensitivity analysis shows that the pest density that should trigger pesticide use can vary significantly with the pest density volatility. Incorporating pest randomness into simple decision rules may thus help better manage the chemicals applied to soils and crops.

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While integrated pest management (IPM) has been gaining ground in an effort to reduce the reliance on chemicals with harmful environmental side effects, the concepts of economic thresholds (ET) and economic injury level (EIL) are still “the backbone of progressive concepts in (...) insect pest management” (Poston, Pedigo, and Welch). These concepts were introduced by entomologists to provide practical decision rules for using pesticides in a context of complex interactions between plants and pests, which are typically very difficult to model. Stern et al. gave the most widely accepted definition of economic threshold: it is the “density at which control measures should be initiated to prevent an increasing pest population from reaching the economic injury level.” The ET is thus an operating rule intimately tied to the EIL, which is “the lowest population density that will cause economic damage”, where the later is the amount of injury that justifies the cost of artificial control measures.

Although quite useful, these definitions needed some clarification regarding the amount of injury that should trigger some pest control action. Headley formalized Hillebrandt’s marginal analysis of pest control, but he redefined the economic threshold as the level to which a pest should be reduced in order for the marginal revenue from the application of a pesticide to just equal its marginal cost. Hall and Norgaard expanded Headley’s model and focused on the optimal timing of a pesticide application.

However, economic models of pest control have mostly remained theoretical tools of limited practical value. On one hand, economic models have been criticized for being too stylized. Some attempts to better describe the dynamics of pest and host interaction, while providing some insights in the factors affecting optimal control strategies, have

shown that more realistic mathematical models can quickly become almost untractable (See for example Shoemaker). On the other hand, the economic threshold as defined by economists has not been used widely in practice (Moffit et al.) because it requires the simultaneous calculation of optimal spraying time and pesticide dosage, whereas information needed for these calculations is usually unavailable. Moreover, farmers are often advised by pesticide manufacturers to use a prescribed dosage. Hence, they face the problem of determining an action threshold (Edwards and Heath), i.e. the critical pest density at which a specific action, such as spraying a pesticide, should be taken.

Another reason for the resistance to the adoption of the economic threshold is that it relies on perfect information while there is often considerable uncertainty regarding the evolution of a pest population or the long-term effects of the application of a pesticide. There have been some attempts to address the problem of uncertainty related to pest control. Feder investigated, qualitatively, the effect of uncertainty on the dosage of a pesticide by a risk-averse farmer; he considered uncertainty in the rate of damage per pest, in the size of the pest population, and in the efficacy of the pesticide. Moffit et al. examined the impact of pest population uncertainty on the economic threshold, but in a static framework. Later, Moffit investigated uncertainty in the initial rate of pest infestation to derive more efficient action-thresholds-type-rules for risk-averse farmers. Mangel coupled a genetic model with a model of pest population dynamic in a multi-seasonal setting to take into account the build-up of insect resistance to a pesticide. Using a continuous-time formulation, he showed how to incorporate pest age structure, the random arrival rate of pests, and the stochastic effect of a pesticide on pest growth rate.

However, his stochastic model requires biological information that is often unavailable, and its numerical resolution appears quite challenging. For practical uses, a compromise has to be found between the realism of a model and its usability. To date, the impact of uncertainty in the evolution of the pest population on the decision to use pest control measures does not appear to have been solved satisfactorily.

The potential adverse health effects of pesticides, not only for the public, but also for farm workers, also raises some legitimate concerns. In their analysis of the economics of reentry regulations for pesticides, Lichtenberg et al., were the first to provide a methodology for analyzing the tradeoffs between the adverse effects from pesticide exposure on the health of farm workers, and lost grower revenues due to restrictions on pesticide use. They showed that these tradeoffs could be substantial in the case of parathion use in apples. They considered, however, a deterministic setting where a grower can choose to apply once a standard dose of pesticide at a negligible cost.

There are a number of other important issues in pest management, such as increasing pest resistance (see for example Regev et al.), issues of coordination between adjacent farms, inter-seasonal pest dynamics, or environmental side-effects of pesticides. To decrease the reliance on chemicals to fight crop-damaging pests, IPM has been promoted to diversify the pest-control portfolio with a variety of biological and managerial strategies. For a survey of economic and biological issues pertaining to pest control, see Carlson and Wetzstein, or Fernandez-Cornejo et al.

In this paper, we explore how the randomness of the density of a pest population can impact the use of a pesticide, and we provide a framework to incorporate this feature in

the determination of simple action rules like the economic threshold, in the spirit of Moffit et al. We also explicitly model a delay between successive pest control measures and before harvest in order to examine the expected marginal cost of reentry regulations. Although we focus on pesticides, the same approach could be used for a variety of pest control measures. We propose to view the possibility to spray a pesticide as a “real option,” i.e. similar to a financial option but applied to a non-financial instrument. See Dixit and Pindyck for an introduction to the theory of real options, or Trigeorgis for examples of applications. We use a continuous-time framework to conveniently analyze the impact of the pest density volatility on the timing of application of a pesticide.

In the next section, we formulate the farmer’s decision of applying a pesticide as an optimal sequential stopping problem and we show how to solve it using stochastic dynamic programming. We first consider a fixed pesticide dosage but we extend this model to allow for the selection of pesticide dosage. We then apply this model to the European Red Mite, a foliar pest of apples. We find that taking into account the randomness of the mite density can have an impact on pesticide use. The last section offers concluding comments and some possible extensions of our model. The numerical method used to solve our model is summarized in Appendix A, and the algorithm to calculate the expected marginal cost for the grower of lengthening the reentry period is presented in Appendix B.

## The Model

We consider a risk-neutral farmer whose objective is to maximize discounted net revenue from a crop threatened by a single pest. Let  $Q(t)$  and  $X(t)$  denote, respectively, the plant biomass of the crop and the pest density at instant  $t$ . The initial plant biomass  $Q_0$  and pest density  $X_0$  are given and we assume that the evolution of the plant biomass can be described by:

$$(1) \frac{dQ}{dt} = G(t, X)$$

where  $G(t, X)$  is a continuously differentiable function, increasing in  $t$  and decreasing in  $X$ , which models crop growth. We also suppose that the pest density,  $X$ , follows a diffusion process with growth rate  $r_n(t, X)$  and infinitesimal variance  $\sigma_n^2(t, X)$ , so that an increment in  $X$  can be written (Dixit and Pindyck):

$$(2) dX = r_n(t, X)dt + \sigma_n(t, X)dz$$

The subscript “n” indicates that the parameters of the diffusion process followed by  $X$  may be influenced by applications of the pesticide.<sup>1</sup> In the above,  $dz$  is the increment of a standard Wiener process (Dixit and Pindyck). Note that the absence of  $Q$  as an argument of  $r_n(t, X)$  may overstate pest growth if the crop is the pest’s only food source.

To fight the pest problem, the farmer can apply up to  $N$  times a specified dose of pesticide at a total cost  $K$  per application per unit area, starting at time  $t = 0$ . For notational convenience, we number successive pesticide applications in decreasing order, from  $N$  to 1. For now, we also assume that the farmer follows the recommendation of the manufacturer for pesticide dosage, and that there is a single recommended dosage as

opposed to a range of values. We postulate that applying the pesticide will cause the instantaneous death of a fraction  $M$  of the pest population. Hence, if the pesticide is applied at time  $t$ , the pest density just after spraying, denoted by  $X_t^+$ , and the pest density just before spraying, denoted by  $X_t^-$ , are related by:

$$(3) X_t^+ = (1 - M)X_t^-$$

In fact, pests do not die instantly and the impact of a pesticide depends on weather conditions and pest resistance. To simplify our model, we ignore these issues and focus on the effect of stochastic pest growth on the timing of application.

For health reasons, we assume that the last application of pesticide has to take place before time  $T$ ,  $d$  days before harvest. The crop is thus harvested on day  $T + d$ . To study the issue of workers reentry, we also postulate that there is a minimum delay  $\Delta$  between successive applications of the pesticide to protect the health of farm workers. This assumption implies that, if it takes place, the  $n^{\text{th}}$  application of pesticide happens at the latest at time  $t_{nF} = T - (n-1)\Delta$ .

Because the pest density varies randomly, future crop biomass and thus net revenue are uncertain until harvest time. In taking the decision to spray, the farmer has to balance expected pest damages with the cost of applying the pesticide plus the loss of flexibility which comes from using one of its possibilities to reduce the density of the pest population by spraying the pesticide. This is the motivation for using concepts from the theory of real options.



To find the critical pest density that will trigger spraying, we view the crop as an investment and the possibility to spray as an option which can be exercised to improve the expected value of this investment. We thus write the value in current dollars of the crop “asset” before pesticide application number  $n$ , denoted  $J_n$ , as the sum of the expected value of the crop at time  $t$  plus the value of the option to spray  $n$  times. We denote it by  $\varphi_n(t, X)$  if spraying can take place (i.e. if the last spray took place at least  $\Delta$  days ago), and by  $\tilde{\varphi}_n(t, X)$  otherwise. In the first case,  $J_n$  can thus be written:

$$(4) \quad J_n(t, X) = \varphi_n(t, X) + pe^{-\delta(T+d-t)} \left( Q(t) + E_t \left[ \int_t^{T+d} G(u, X) du \mid X(t) \right] \right)$$

In the above,  $p$  denotes the per unit harvest price, net of harvest cost,  $\delta$  is the instantaneous discount rate,  $E_t$  stands for the expectation at  $t$ , and  $\dot{Q} = G(u, X)$  is the rate of change of plant biomass at time  $u$  given a pest density of  $X(t)$  at  $t$ . If at time  $t$ , however, the last pesticide application took place at time  $t - s$  with  $0 \leq s < \Delta$ ,  $\varphi_n(t, X)$  should be replaced by  $\tilde{\varphi}_n(t, X)$  in Equation (4) above, with:

$$(5) \quad \tilde{\varphi}_n(t, X_t) = e^{-\delta(\Delta-s)} \int_0^{+\infty} \varphi_n(t-s+\Delta, y) f(y, t-s+\Delta; X_t, t) dy$$

This is just the expected value at time  $t$ , in current dollars, of the option term at time  $t-s+\Delta$ ;  $f(y, t-s+\Delta; X_t, t)$  designates the value of the density function at time  $t-s+\Delta$  for a pest density value of  $y$ , given that the pest density at time  $t$  is  $X_t$ . Since no more than  $N$  applications of the pesticide are allowed, there is no option term in the expression of  $J_0(t, X)$  and  $\tilde{\varphi}_0$  or  $\varphi_0$  is identically zero in Equation (4).

This framework allows us to cast the decision to spray for the  $n^{\text{th}}$  time ( $1 \leq n \leq N$ ) as an optimal stopping problem (Malliaris and Brock). From standard results in optimal stopping theory, the  $(t, X)$  space, with  $0 \leq t \leq t_{nF} = T - (n-1)\Delta$  and  $X \geq 0$ , is divided in two regions: in the “continuation region,” where the pest density is low, it is optimal to wait and monitor the evolution of the pest density, while in the “stopping region,” where the pest density is higher, it is optimal to apply the pesticide immediately. In the continuation region, if the option can be exercised,  $\varphi_n(t, X)$  must satisfy the Hamilton-Jacobi-Bellman equation (Mangel):

$$(6) \quad \delta\varphi_n(t, X) = \frac{\partial\varphi_n(t, X)}{\partial t} + r_n(t, X)\frac{\partial\varphi_n(t, X)}{\partial X} + \frac{\sigma_n^2(t, X)}{2} \frac{\partial^2\varphi_n(t, X)}{\partial X^2}$$

The “continuation region” is separated from the “stopping region” by a curve called a stopping frontier. In the case study below, we examine how this curve depends on  $\sigma_n(t, X)$ , the volatility of the pest density. The stopping frontier for application “number  $n$ ” of the pesticide, noted  $(t, X_n^*(t))$  for  $0 \leq t \leq T - (n-1)\Delta$ , can be characterized by the “value-matching” and “smooth-pasting” conditions (Dixit and Pindyck).<sup>2</sup> They require the value function and its first derivative with respect to  $X$  to be continuous across the stopping frontier:

$$(7) \quad J_n(t, X_n^*) = J_{n-1}(t, (1-M)X_n^*) - K$$

$$(8) \quad \frac{\partial J_n(t, X_n^*)}{\partial X} = \frac{\partial J_{n-1}(t, (1-M)X_n^*)}{\partial X}$$

Equation (7) also gives the expression of  $\varphi_n$  in the stopping region ( $X_n^*(t) \geq X$ ): indeed, since it is optimal to apply a dose of pesticide immediately in the stopping region,  $\varphi_n(t, X)$  equals  $\tilde{\varphi}_{n-1}(t, X)$  plus the expected value of the crop saved by spraying at  $t$  if the pest density is  $X$ , net of spraying costs:

$$(9) \quad \varphi_n(t, X) = \tilde{\varphi}_{n-1}(t, (1-M)X) + CS(t, X) - K$$

$CS(t, X)$  is the expected value of the crop saved by spraying at  $t$  if the pest density is  $X$ :

$$(10) \quad CS(t, X) = pe^{-\delta(T+d-t)} \left( E_t \left[ \int_t^{T+d} G(u, X) du | (1-M)X(t) \right] - E_t \left[ \int_t^{T+d} G(u, X) du | X(t) \right] \right)$$

To solve these boundary value problems, we need, in addition to Equations (5) to (10), a final condition on the boundary of the domain of variation of  $t$  and  $X$ . Recall that we require the option to spray  $n$  times to expire at time  $t_{nF} = T - (n-1)\Delta$ . From a simple arbitrage argument, the value of the option to spray  $n$  times at that time equals the sum of the expected net benefits from spraying plus the expected value of the option to spray another  $n-1$  times, if this sum is positive, and 0 otherwise. Hence:

$$(11) \quad \varphi_n(t_{nF}, X) = \text{Max}(0, \tilde{\varphi}_{n-1}(t_{nF}, (1-M)X) + CS(t_{nF}, X) - K)$$

Based on biological considerations, we could also consider a variety of boundary conditions such as a reflecting barrier to prevent the extinction of the pest population over the time interval considered (Cox and Miller), but this is left for further work.

### *A Special Case*

To prepare for the application to the European Red Mite where we focus on the impact of the volatility of the pest density on the decision to spray, consider that only two pesticide applications are allowed (i.e.  $N = 2$ ) and assume that:

$$(12) \quad \frac{dQ}{dt} = a(t) - bX$$

In Equation (12), the continuous function  $a(t)$  represents the rate of plant growth in the absence of pest. We now assume that  $X$  follows a geometric Brownian motion (GBM) with pest growth rate  $r$  and standard deviation coefficient  $\sigma$  (Dixit and Pindyck), so that:

$$(13) \quad dX = rXd t + \sigma X dz$$

For simplicity, we suppose that  $r$  and  $\sigma$  are not affected by spraying. The assumption that the pest density follows a geometric Brownian motion is appropriate to model pests that tend to grow exponentially but that are affected by random factors such as weather conditions and predators. These pests, such as mites, aphids, or trips, have multiple generations per year and thus pose a risk of outbreak.

For this choice of diffusion process for  $X$ , Equation (6) becomes:

$$(14) \quad \delta\varphi_n(t, X) = \frac{\partial\varphi_n(t, X)}{\partial t} + rX \frac{\partial\varphi_n(t, X)}{\partial X} + \frac{\sigma^2 X^2}{2} \frac{\partial^2\varphi_n(t, X)}{\partial X^2}, n = 1, 2$$

The value function in the continuation region (before spraying, if spraying is allowed) is:

$$(15) \quad J_n(t, X) = \varphi_n(t, X) + pe^{-\delta(T+d-t)} \left( Q(t) + A(T+d) - A(t) - \frac{bX(t)}{r} (e^{r(T+d-t)} - 1) \right)$$

where  $A(t) = \int_0^t a(y)dy$ . Equation (13) implies that  $X$  is lognormally distributed so

Equation (5) becomes, for  $0 \leq s \leq \Delta$ :

$$(16) \quad \tilde{\varphi}_n(t, X_t) = e^{-\delta(\Delta-s)} \int_0^{+\infty} \varphi_n(t-s+\Delta, y) f_{LN}(y|X_t + [\Delta-s][r - \frac{\sigma^2}{2}], [\Delta-s]\sigma^2) dy$$

where  $f_{LN}(y|m, v^2)$  is the value at  $y$  of the probability density function of the lognormal distribution with parameters  $m$  and  $v^2$  (Casella and Berger, p. 628) and  $x_{t+s} = \text{Ln}(X_{t+s})$ .

Regrouping the continuity condition with Equation (7), we obtain, for  $X \geq X_n^*$ :

$$(17) \quad \varphi_n(t, X) = \tilde{\varphi}_{n-1}(t, (1-M)X) + pe^{-\delta(T+d-t)} \frac{bMX}{r} (e^{r(T+d-t)} - 1) - K$$

The smooth-pasting condition becomes:

$$(18) \quad \frac{\partial \varphi_n}{\partial X}(t, X_n^*) = \frac{\partial \tilde{\varphi}_{n-1}}{\partial X}(t, (1-M)X_n^*) + pe^{-\delta(T+d-t)} \frac{bM}{r} (e^{r(T+d-t)} - 1)$$

In the above,  $X_n^*$  is a function of  $t$  but we often omit its argument to lighten the notation.

Moreover, the terminal condition (Equation (11)), can be written, for  $X \geq 0$ :

$$(19) \quad \varphi_n(t_{nF}, X) = \text{Max} \left( 0, \tilde{\varphi}_{n-1}(t_{nF}, (1-M)X) + pe^{-\delta(T+d-t_{nF})} \frac{bMX}{r} (e^{r(T+d-t_{nF})} - 1) - K \right)$$

Equations (14), (16), (17), (18), and (19) uniquely define the stopping frontier  $(t, X_n^*)$ ; we note that they do not contain  $a(t)$ , the rate of plant growth in the absence of pest, so this term has no impact on the decision to spray. These equations have to be solved numerically. The numerical procedure used for simultaneously finding the stopping frontier and the option term,  $\varphi(\bullet, \bullet)$ , in the continuation region is outlined in Appendix A.

Note that our formulation, with continuous time and pest density, is relatively easy to solve compared to discrete time and state models for which the cost of a numerical solution increases exponentially with the number of states and time points considered.

### *Case of a Variable Pesticide Dose*

Let us now suppose that the farmer has the choice between several pesticide dosages. Then, at each point in time between 0 and T, the farmer has to select both the optimal pesticide dosage and the critical pest population above which spraying should take place. This can be done in two stages. Let us start with the last spray ( $n = 1$ ). First, for an arbitrary dose of pesticide, noted Y, the farmer uses the above formulation to calculate the stopping frontier  $(t, X_1^*(t, Y))$ . Note that both the pest mortality M, and the cost of pesticide application K are now functions of Y. When  $X_1^*(t, Y)$  is known for the values of Y of interest, the farmer calculates at each point in time between 0 and T the dosage  $Y^*$  that yields the largest expected profit. This procedure is then repeated successively for increasing values of n from 2 to N.

The resulting stopping frontiers are the stochastic counterparts of the “economic threshold” identified by Headley and Hall and Norgaard. This procedure is illustrated below for one pesticide application, but it can readily be extended to several applications. However, the information required to implement this approach is usually unavailable, so it is unlikely to be of practical use.

## **An Application to the European Red Mite**

### *Assumptions and Model Data*

We apply our model to the European red mite (ERM), which has become a problematic foliar pest of apples because of its resistance to many pesticides (Croft, Hoyt, and Westgard). Typically, in New York State, ERM eggs hatch during April and May. Preventive treatment, in the form of oil, can be applied during that period. The first adults appear in early June; they grow and reproduce until harvest time, in early October. In the absence of predators, ERM grows exponentially; Nyrop et al. report that the ERM daily growth rate varies between 0.025 and 0.125. From June to August, pesticides can be applied as a rescue treatment to limit mite damage. A common pesticide is Kelthane, which can be applied up to two times at a total cost (material and labor) of approximately \$30 per acre per application. We assume a mortality rate of 90% for the recommended dosage.<sup>3</sup>

The primary effect of mite infestation is to reduce yield through fruit size but it can also affect fruit appearance thus reducing the value of fresh fruits. A recent study by Francesconi et al. has shown that mites damage apples by reducing the CO<sub>2</sub> exchange rate of leaves. In general, quantifying the damage inflicted by foliar pests on perennial crops is difficult because of large variations between plants. For simplicity, we ignore appearance damage to focus only on crop weight loss and we adopt the relationship between average final fresh fruit weight  $W$  (in grams), and cumulative mite days (CMD) developed in Francesconi et al.:

$$(20) \quad W = 190 - 0.016 \text{ CMD}$$

In the above, CMD is the sum of all mite-days (MD) per leaf counted throughout the season. The number of mite-days between count  $i$  and count  $i+1$ , noted  $MD(i,i+1)$  is

$$(21) \quad MD(i,i+1) = 0.5 * [mpl(i) + mpl(i+1)] * days(i,i+1)$$

where  $mpl(j)$  is the average number of mites per leaf observed during count  $j$  and  $days(i,i+1)$  is the number of days between count  $i$  and count  $i+1$ .

Since treatment is commonly expressed in \$ per acre, we normalize this damage relationship to a standard crop per acre. Averaging good and bad years leads to an average crop of 18,000 lb./acre so that:

$$(22) \quad Q(lb) = 18,000 - 1.5 \text{ CMD}$$

From this relationship, we deduce that a reasonable value for the damage coefficient  $b$ , if our assumptions hold, is 1.5 lb. per (acre\*CMD). Equation (22) depends of course on the average number of leaves per apple: the more leaves there are per apple, the less important is the damage for a given number of mite days.

Another important model parameter is the price of a pound of apple. We consider only fresh fruit, which are much more valuable to the farmer than fruit destined to be processed. The price structure of fresh fruit depends on size, appearance, and variety. Half the farms in New York State, however, sell their crop to wholesalers who take care of packaging and distribution. In recent years, a bushel of apples has brought approximately \$6 to the farmer, net of harvesting. If we further assume that a bushel weights approximately 42 lb., we obtain a net farm price of \$ 0.15 per lb. of apples.<sup>4</sup>



Four more parameters are needed: the discount rate  $\delta$ ; the infinitesimal variance coefficient of the process of  $X$ , denoted  $\sigma^2$ ; the minimum delay between successive pesticide applications, which we denote  $\Delta$ ; and  $d$ , the minimum delay between harvest and the last pesticide application. For  $\delta$ , we consider values between 0.0001 per day (3.72% per year) and 0.0002 per day (7.57% per year). Since adequate time series data on mite density are not available, we use arbitrary but plausible values for  $\sigma^2$  ranging from 0.01 to 0.09 per day. Finally, we assume that  $\Delta = 7$  days and we suppose that  $d = 30$  days, based on the fact that, in New York State, most apple growers will not treat for red mites in the last thirty days before harvest. Table 1 summarizes the parameter values considered. The initial pest density,  $X_0$ , is not needed here; it comes into play only to calculate the probability of hitting a point on the stopping frontier.<sup>5</sup>

### *Results*

Details of the numerical procedures employed are presented in Appendix A. Results are summarized in Tables 2, 3a and 3b, and on Figures 1 to 8.

Figures 1 and 2 illustrate how the stopping frontiers vary with  $\sigma$  for the median values of  $p$ ,  $b$ ,  $M$ ,  $r$ , and  $\delta$  presented in Table 1. The stopping frontiers on Figure 1 correspond to option to spray number 1, which expires after  $T = 90$  days, while those on Figure 2 represent option to spray number 2, which is valid for the first  $T - \Delta = 83$  days (option number 2 would be exercised first with our numbering system). These stopping frontiers are the stochastic counterpart of the “action threshold” (Edwards and Heath)

often used in practice. First, we observe that an increase in  $\sigma$ , everything else being the same, leads to spraying at higher pest densities: although pest density is expected to rise exponentially, the more it fluctuates randomly, the larger is the probability that it may go down to small values. This effect is not linked to the risk preference of the farmer, whom we assume to be risk neutral.<sup>6</sup> Taking into account random fluctuations in pest density could thus lead to better use of pesticides. This improvement, however, is likely to be practically significant only for pesticide applications after the first thirty days of the growing season when the pest volatility is small because initial stopping values can be quite small. The second application of pesticide, which typically takes place later in the growing season, is more likely to be affected. Quantifying the change in pesticide use would require a better knowledge of the volatility of the pest density and data on the pest densities at which the farmers have been applying pesticides.

Moreover, we see that these stopping frontiers curve towards a common point at time  $T$  for option 1. They appear to do the same at time  $T - \Delta$  for option 2, although their terminal points are distinct but close. We also observe that the critical mite density decreases near harvest time because the farmer has to account for the possibility of a mite outbreak during the  $d$  days before harvest where spraying is not allowed. A numerical investigation indicates that this effect becomes more pronounced when the length of the no-spraying period before harvest increases (i.e. when  $d$  is larger).

Figure 3 shows the second stopping frontiers for different values of the minimum delay between successive pesticide applications ( $\Delta$ ). Note that the vertical axis of Figure

3 is linear instead of logarithmic as for the other figures because the difference between the stopping frontiers displayed is small. We see that the farmer will spray sooner for higher values of  $\Delta$  as a preventive measure against the expected increase in mite density during the period  $\Delta$ .

Figure 4 shows the first stopping frontier corresponding to the economic threshold when the farmer can spray only once and has the choice between three doses of pesticide (80%, 90%, or 100% of a standard dose.) Parameters  $p$ ,  $b$ ,  $\delta$ , and  $r$  are at their median value. We postulate that pesticide efficacy,  $M$ , application cost,  $K$ , and the fraction of a standard pesticide dose,  $Y$ , are related by the relationships:  $M(Y) = 1 - e^{-2.3*Y}$  and  $K(Y) = 5 + 25 * Y$ . Note that when  $Y$  equals 1 (a standard dose of pesticide),  $M=0.9$  and  $K=\$30$ , the values of  $M$  and  $K$  used to generate Figures 1 and 2. For this case, the stopping frontier is a composite of the frontiers corresponding to  $Y \in \{0.8, 0.9, 1.0\}$ : until day 68, the farmer should apply a standard dose of pesticide ( $Y^*=1$ ); from day 69 to day 72,  $Y^*=0.9$ ; and  $Y^*=0.8$  thereafter. These results illustrate that allowing the farmer to select pesticide dosage could help reduce pesticide use if reliable information on pesticide efficacy were available, which is unfortunately seldom the case.

Some results of a sensitivity analysis of the impact on the stopping frontier of a change in  $p$ ,  $b$ ,  $M$ ,  $\delta$ , or  $r$ , are shown in Table 2 and on Figures 5 to 8. This analysis was conducted around the median parameter values presented in Table 1. First, we find that the value of the discount rate,  $\delta$ , has almost no influence on the decision to apply the pesticide, while changes in pesticide efficacy,  $M$ , have a limited practical impact on the

stopping frontiers (Figure 5). The damage coefficient  $b$ , and the per-pound price of apples  $p$ , which play symmetric roles in our formulation, are more important (Figure 6.) In fact, the critical parameter of this model is the pest growth rate,  $r$ , as illustrated on Figure 7. Everything else being the same, the pest density at which spraying should take place increases by more than an order of magnitude at the beginning of the growing season when  $r$  goes from 0.09 to 0.04. The difference between the stopping frontiers narrows as we get closer to harvest time.

Finally, Figure 8 illustrates how the stopping frontiers vary with the spraying cost per acre ( $K$ ). For the range we selected for  $K$ , the range of variation of the stopping frontiers is similar to that for the mite damage coefficient ( $d$ ).

Data collection efforts for implementing our methodology should thus concentrate on the pest growth rate, on its infinitesimal standard deviation, and on the damage coefficient. Consistent with the IPM philosophy, a better understanding of the pest biology and of the pest-host relationship should be pursued.

In addition, we computed the expected marginal cost (EMC), for the farmer, of increasing the minimum delay between successive pesticide applications to illustrate how our methodology could be used to analyze the reentry decision (Lichtenberg et al.) Since deriving an analytical expression for the EMC is cumbersome, it was simulated following the procedure outlined in Appendix B. Results are presented in Tables 3a and 3b for a simulation time step of  $DT=0.1$  days, 100 000 replications, and for different starting times and starting densities, defined as fractions ( $1/6, 1/2$ , and  $5/6$ ) of the critical mite density

for  $\Delta=7$  days. To have an indication of the convergence of the simulated EMCs, the standard error of the mean is shown in brackets below the corresponding EMC.

First, we observe that the simulated EMCs (in \$/acre) are generally quite small, although they start becoming significant towards the end of the period of mite damage (bottom right of Table 3b). It should be kept in mind, however, that the \$ amounts shown underestimate the true marginal cost of delaying reentry because the farmer usually has more than one pest to worry about. It is likely, however, that the EMC could become much larger for longer  $\Delta$ . Second, as expected, the EMCs increase with  $\Delta$ . Although the exact value of any EMC is non-negative (more rigidity is costly to the farmer) some simulated values can turn out to be small negative numbers as for the first EMC on the last line of Table 3a. This can happen, for example, if the day after the second option expired for  $\Delta=5$  days, the pest density increases just enough to justify using the second option for  $\Delta=4$  days, but later goes down to generate less damage than expected. The exploration of EMCs for longer values of  $\Delta$  and for other types of pests and pesticides is left for future research.

## **Conclusions**

In this paper, we propose to model the possibility to apply a pest control measure as a real option. We explicitly take into account the random variations of the density of a pest population, and we formulate an optimal pest control problem that allows for several pesticide applications as well as variable pesticide doses. We also explicitly model the

mandatory delay between consecutive pesticide applications. This formulation provides a convenient framework for analyzing the reentry decision and for incorporating pest density randomness in the determination of simple action rules. It has been known for some time (e.g. Arrow and Fisher), that ignoring randomness by replacing stochastic variables with their mean value can lead to decisions which differ greatly from the optimum. This result seems, however, to have been ignored so far in the determination of simple rules for pest control measures.

We illustrate this approach with the European red mite, a foliar pest of apples which has shown increasing resistance to pesticides. We find that optimal stopping frontiers can vary significantly with the volatility of the pest density, so modeling explicitly the randomness of mite density could lead to a better use of pesticides, and may help decrease pesticide use. This is an important finding in the context of increasing concerns about the environmental side-effects of pesticides.

The information requirements for implementing this approach are compatible with a well-designed IPM strategy where pest density is regularly monitored. Benefits of this approach over a conventional deterministic action rule are difficult to quantify, however, because they require information on the pest density volatility, which is not usually available, and the spraying habits of farmers.

Our model is highly stylized. Future research should consider time-dependent pest density diffusion parameters; uncertain pest growth rate and volatility; endogenous harvest date; damage control specification (e.g. see Hennessy or Fox and Weersink); farmer risk-aversion; and the quantification of reentry EMCs for different pesticides.

## References

- Arrow, K.J. and A.C. Fisher. "Environmental Preservation, Uncertainty, and Irreversibility." *Quarterly Journal of Economics*, 88 (1974): 312-319.
- Carlson, G.A. and M.E. Wetzstein. "Pesticide and Pest Management," in *Agriculture and Resource Economic*, D. Zilberman, G.A. Carlson, and J.A. Miranowski, Eds. (New York, NY: Oxford University Press, 1993), 268-318.
- Casella, G. and R.L. Berger. *Statistical Inference*. (Pacific Grove, California: Wadsworth & Brooks/Cole Advanced Books & Software).
- Cox, D.R. and H.D. Miller. *The Theory of Stochastic Processes*. Paperback edition, (London: Chapman & Hill, 1995.)
- Crank, J. *Free and Moving Boundary Problems*. (Oxford University Press, 1984.)
- Croft, B. A., S.C. Hoyt, and P.H. Westgard. "Spider Mite Management on Pome Fruit Revisited: Organotin and Acaricide Resistance Management." *Journal of Economical Entomology* 80 (1987): 304-311.
- Dixit, A. K., and R. S. Pindyck. *Investment under Uncertainty*. (Princeton, New Jersey: Princeton University Press, 1994.)
- Edwards, C. A. and G. W. Heath, Eds. *The Principles of Agricultural Entomology*. (Springfield, Illinois: Charles C. Thomas, 1964.)
- Feder, G. "Pesticides, Information, and Pest Management under Uncertainty." *American Journal of Agricultural Economics* 61 (1979): 97-103.
- Fernandez-Cornejo, J., S. Jans, and M. Smith. "Issues in the Economics of Pesticide Use in Agriculture: A Review of the Empirical Evidence" *Review of Agricultural Economics* 20 (1998): 462-488.

- Fox, G. and A. Weersink. "Damage Control and Increasing Returns," *American Journal of Agricultural Economics* 77 (February 1995): 33-39.
- Francesconi, A.H.D., A.N. Lakso, J.P. Nyrop, J. Barnard, and S.S. Denning. "Carbon Balance as a Physiological Basis for the Interactions of European Red Mite and Crop Load on Starkrimson Delicious Apple Trees." *J. Amer. Soc. Hort. Sci.* forthcoming.
- Gotsch, N. and U. Regev. "Fungicide Use under Risk in Swiss Wheat Production." *Agricultural Economics* 14 (1996): 1-9.
- Hall, D. C. and R. B. Norgaard. "On the Timing and Application of Pesticides." *American Journal of Agricultural Economics* 55.May (1973): 198-201.
- Headley, J. C. "Defining the Economic Threshold," in *Pest Control Strategies for the Future*, Agricultural Board Ed. (Washington, D.C., 1972), 100-108.
- Hennessy, D.A. "Damage Control and Increasing Returns: Further Results," *American Journal of Agricultural Economics* 79 (August 1997): 786-791.
- Hillebrandt, P. M. "The Economic Theory of the Use of Pesticides. Part 1: The Dosage Response Curve, the Rate of Application, and the Area to be Treated." *Journal of Agricultural Economics* 13 (1960): 464-472.
- Karlin, S. and H. M. Taylor. *A Second Course in Stochastic Processes* (Academic Press, 1981.)
- Lichtenberg, E., R. C. Spear, and D. Zilberman. "The Economics of Reentry Regulation of Pesticides," *American Journal of Agricultural Economics* 75 (November 1993): 946-958.
- Malliari, A.G. and W.A. Brock. *Stochastic Methods in Economics and Finance*. (New York: North-Holland, 1982.)



- Mangel, M. *Decision and Control in Uncertain Resource Systems*. (Orlando, Florida: Academic press, 1985.)
- Moffit, L. J. "Risk-Efficient Thresholds for Pest Control Decisions." *Journal of Agricultural Economics* 37 (1986): 69-75.
- Moffit, L. J., D. C. Hall, and C. D. Osteen. "Economic Threshold under Uncertainty with Application to the Corn Nematode Management," *Southern Journal of Agricultural Economics* (December 1984.)
- Nyrop, J.P., M.R. Binns, W. Van der Werf, and J. Kovach. "Cascading Tripartite Binomial Classification Plans to Monitor European Red Mite (Acari, Tetranychidae) through a Season; Development and Evaluation of a New Methodology for Pest Monitoring." *Experimental and Applied Acarology* 18 (1994): 123-153.
- Poston, F. L., L. P. Pedigo, and S. M. Welch. "Economic Injury Levels: Reality and Practicality." *Bulletin of the ESA*. (Spring 1983): 49-53.
- Regev, U., H. Shalit, and A.P. Gutierrez. "On the Optimal Allocation of Pesticides with Increasing Resistance: The Case of Alfalfa Weevil." *Journal of Environmental Economics and Management* 10 (1983): 86-100.
- Shoemaker, C. "Optimization of Agricultural Pest Management III: Results and Extensions of a Model." *Mathematical Biosciences* 18 (October 1973): 1-22.
- Stern, V. M., R. E. Smith, R. van den Bosch, and K. S. Hagen. "The Integrated Control Concept." *Hilgardia* 29 (1959): 81-101.
- Trigeorgis, L., *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, (Cambridge, MA: MIT Press, 1996).

Wilmott, P., S. Howison, and J. Dewynne. *The Mathematics of Financial Derivatives, A Student Introduction*. (Cambridge University Press, 1995.)

## Appendix A

To solve numerically the stochastic pest problem, we start by changing variables:

$$(A1) \quad \tau = (T - t) \frac{\sigma^2}{2}, \quad z = \text{Ln} \left( \frac{X}{X_{1F}^*} \right),$$

$$KU_n(\tau, z) e^{\alpha z + \beta \tau} = \varphi_n(t, X), \quad K\tilde{U}_n(\tau, z) e^{\alpha z + \beta \tau} = \tilde{\varphi}_n(t, X), \quad n = 1, 2$$

We choose  $\alpha$  and  $\beta$  to cancel out terms in  $U_n$  and  $\frac{\partial U_n}{\partial z}$  in the transform of Equation (14):

$$(A2) \quad \alpha = 0.5(1 - \mu), \quad \beta = -0.25(1 - \mu)^2 - \lambda$$

where  $\mu$  and  $\lambda$  are dimensionless parameters:

$$(A3) \quad \mu = \frac{2r}{\sigma^2}, \quad \lambda = \frac{2\delta}{\sigma^2}$$

Moreover,  $X_{1F}^*$  is the critical pest density at  $T$ , when there is only one more possibility to spray, for which the farmer is indifferent between spraying and not spraying because expected pest damages equal the cost of spraying. It equals:

$$(A4) \quad X_{1F}^* = \frac{rK}{pbM} \frac{e^{\delta d}}{e^{rd} - 1}$$

Introducing the new variables into Equations (14), (16), and (17), we obtain:

$$(A5) \quad \frac{\partial^2 U_n}{\partial z^2} = \frac{\partial U_n}{\partial \tau}, \quad -\infty \leq z \leq z_n^*, \quad (n-1)\Delta \frac{\sigma^2}{2} \leq \tau \leq T \frac{\sigma^2}{2}, \quad n = 1, 2$$

$$(A6) \quad \tilde{U}_{n-1}(\tau, z_n^* + \text{Ln}(1-M)) = \int_{-\infty}^{+\infty} U_{n-1}\left(\tau - \frac{\Delta\sigma^2}{2}, z_n^* + \text{Ln}(1-M) + \sigma\sqrt{\Delta}y\right)\phi(y)dy$$

$$(A7) \quad U_n(\tau, z_n^*) = U_{n-1}(\tau, z_n^* + \text{Ln}(1-M))e^{\alpha\text{Ln}(1-M)} + e^{(1-\alpha)z_n^* + \alpha^2\tau} \frac{e^{\text{rd} + (1-2\alpha)\tau} - 1}{e^{\text{rd}} - 1} - e^{-\alpha z_n^* - \beta\tau}$$

Equation(A7) is valid for  $z \geq z_n^*$ ,  $(n-1)\Delta \frac{\sigma^2}{2} \leq \tau \leq T \frac{\sigma^2}{2}$ ,  $n = 1, 2$ . In (A6),  $\phi$  is the

density of the standard normal. The terminal conditions (Equation (19)) can be written:

$$(A8) \quad U_1(0, z) = \text{Max}(0, e^{(1-\alpha)z} - e^{-\alpha z}), -\infty < z < +\infty$$

$$(A9) \quad U_2(\tau_{2F}, z) = \text{Max}(0, [I_{1-\alpha}(z + \text{Ln}(1-M)) - I_{-\alpha}(z + \text{Ln}(1-M))]e^{\alpha\text{Ln}(1-M)} + e^{(1-\alpha)z + \alpha^2\tau_{2F}} \frac{e^{\text{rd} + (1-2\alpha)\tau_{2F}} - 1}{e^{\text{rd}} - 1} - e^{-\alpha z - \beta\tau_{2F}}), -\infty < z < +\infty$$

with  $I_a(x) = e^{ax + \frac{a^2\sigma^2\Delta}{2}} (1 - \Phi(-a\sigma\sqrt{\Delta} - \frac{x}{\sigma\sqrt{\Delta}}))$ , and  $\Phi(y) = \int_{-\infty}^y \phi(u)du$  is the standard normal

cumulative distribution function. In the above:

$$(A10) \quad z_n^* = \text{Ln}\left(\frac{X^*}{X_{1F}^*}\right), \tau_{nF} = (n-1)\Delta \frac{\sigma^2}{2}$$

$z_n^*$  and  $\tau_{nF}$  are respectively the transform of the critical value and the expiration time of the  $n^{\text{th}}$  option to spray. Finally, the transform of Equation (18) (the smooth-pasting condition), is the derivative of Equation(A7) with respect to  $z$  along the stopping frontier. For solving our problem numerically, we also impose that the value of the option vanishes when  $X$  goes to zero, which leads to:

$$(A11) \quad U(\tau, -\infty) = 0, (n-1)\Delta \frac{\sigma^2}{2} \leq \tau \leq T \frac{\sigma^2}{2}, n = 1, 2$$

This assumption is compatible with the specification of a geometric Brownian motion for  $X$ , but it has no impact on the stopping frontier (Wilmott, Howison, and Dewynne). Also note that the same approach can be followed when the pest growth rate  $r$ , and the volatility of the pest density vary with time. A change of variables leads to Equation (A5) with similar boundary conditions. See Wilmott, Howison, and Dewynne for details.

To evaluate the integral in Equation (A6), we fit third order splines to the integrand over the interval  $(-4,4)$ , which we then integrate. For this purpose, we use the Fortran IMSL library routines *dbsnak*, *db sint*, *dbscpp*, and *dppitg* with 20 splines.

To solve the problem described by Equations (A5) to (A11), we consider two methods: an implicit method based on the Projected Successive Over-Relaxation algorithm (PSOR), and a boundary tracking method. Both methods rely on defining, in  $(\tau, z)$  space, equally spaced nodes at which we solve Equation (A5) below the unknown stopping boundary, subject to initial conditions (A8) and (A9), to (A7) at and above the unknown stopping boundary, and to the smooth pasting condition or to boundary conditions (A11) for  $z = -\infty$ .

Since we cannot numerically deal with boundaries at infinity, we replace them with boundaries that are “sufficiently far.” We denote by  $\Delta z$  and  $\Delta \tau$  respectively the increments in  $z$  and  $\tau$ , and by  $n_{\min}$  ( $<0$ ) and  $n_{\max}$  ( $>0$ ) the lower and upper node numbers along the  $z$ -axis. Multiplying the  $z$ -coordinate of a node by  $\Delta z$  gives the corresponding  $z$  value, and likewise for its  $\tau$ -coordinate. Hence, we define  $U^{i,j} \equiv U(i * \Delta \tau, j * \Delta z)$ . To solve

Equation (A5), it is usual to report  $\Delta z$  and  $\varepsilon = \frac{\Delta\tau}{(\Delta z)^2}$ , which should be less than 0.5 for

explicit finite-difference schemes (Wilmott, Howison, and Dewynne).

- Implicit Method. This method relies on a Crank-Nicholson scheme coupled with the projected SOR method (see § 8.7 and § 9.4 of Wilmott, Howison, and Dewynne). The PSOR method transforms a free boundary problem to a fixed boundary problem; the stopping frontier is inferred afterwards.

We tried maximum values of +600 for  $n_{\max}$  and -1600 for  $n_{\min}$ , and values as small as 0.01 for  $\Delta z$ . Since a Crank-Nicholson scheme imposes no restriction on  $\varepsilon$ , we varied  $\varepsilon$  between 0.25 and 5.0. We found that the quality of the results depends on the value of  $d$  (see Table 1): for small values of  $d$ , we could easily compute the whole stopping frontier, but as  $d$  increased, we started “loosing” the stopping frontier for small values of  $t$ . For  $d=30$ , the value of interest here, we could not get the lower third of the stopping frontier. We thus resorted to using a boundary tracking method.

- Boundary Tracking Method

This method keeps track directly of the stopping frontier. Equation (A8) gives the initial location of the stopping frontier. Suppose now that at time  $\tau=i\Delta\tau$ , we know that the boundary is at node  $n_b$ . We want to locate the boundary at time step  $i+1$ . Since the boundary is continuous, it will not move by more than  $\Delta z$  in one time step if  $\Delta\tau$  is small enough. Then, we have to consider three cases at time step  $i+1$ : 1) the boundary goes up by  $\Delta z$ ; 2) it stays at the same  $z$  level; or 3) it goes down by  $\Delta z$ . For all cases, we start by calculating  $U^{i+1,j}$  for  $n_{\min} \leq j \leq n_b-1$  from an explicit finite-difference version of (A5):

$$(A12) \quad U^{i+1,j} = \varepsilon U^{i,j+1} + (1 - 2\varepsilon)U^{i,j} + \varepsilon U^{i,j-1}$$

For this scheme to converge to the desired solution, we must have:  $0 \leq \varepsilon < 0.5$  (Wilmott, Howison, and Dewynne).

In case 1, we calculate  $U^{i+1,n_b}$  from (A12), and we get  $U^{i+1,j}$  for  $n_b+1 \leq j \leq n_b+3$  from (A7). We then compute  $\omega_1$ , the absolute difference between  $DU^{i+1,n_b+1}$  (see (A13) below) and the smooth-pasting condition at  $\tau = (i+1)\Delta\tau$  and  $z = (j+1)\Delta z$ . This gives us a measure of how well the smooth-pasting condition is approximated, since  $DU^{i+1,n_b+1}$  is a finite-difference approximation of the derivative of  $U$  at  $z=(n_b+1)\Delta z$ .  $DU^{i,j}$  is defined by:

$$(A13) \quad DU^{i,j} \equiv \frac{U^{i,j+1} - U^{i,j-1}}{2\Delta\tau}$$

In case 2, we compute  $U^{i+1,j}$  for  $n_b \leq j \leq n_b+2$  from (A7), and then  $\omega_2$ , the absolute difference between  $DU^{i+1,n_b}$  and the smooth-pasting condition at  $\tau = (i+1)\Delta\tau$  and  $z = j\Delta z$ .

In case 3, we calculate  $U^{i+1,j}$  from (A7) for  $n_b \leq j \leq n_b+1$ .  $\omega_3$  is the absolute difference between  $DU^{i+1,n_b-1}$  and the smooth-pasting condition at  $\tau = (i+1)\Delta\tau$  and  $z = (j-1)\Delta z$ .

The lowest value of  $\omega_i$  tells us what case we are in. We calculate points above the stopping frontier (in the stopping region) from Equation(A7) as needed. We iterate until  $\tau = 0.5T\sigma^2$ . For more on boundary tracking methods, see Crank.

Results shown on Figures 1 to 6 were calculated for parameter values  $n_{\min} = -1600$ ,  $n_{\max} = 400$ ,  $\Delta z = 0.01$ , and  $\varepsilon = 0.45$ . We varied these parameters to insure that satisfactory convergence was reached from a practical point of view. Whenever possible, we compared results obtained by this method and by the PSOR method. We found that

there was good agreement between the two methods. The algorithms described above were programmed in Fortran 90 on a personal computer with an Intel Pentium 350 MHz processor. The stopping frontiers for two applications, using the boundary tracking method, were calculated in approximately two hours.

## Appendix B

In this appendix we summarize the procedure used to simulate the expected marginal cost of increasing  $\Delta$  by one day. It was used to generate the results reported in Tables 3a and 3b with a time step  $DT = 0.1$  days and 100 000 replications. First, we calculate expected damages (in \$ at harvest time) for the stopping frontiers corresponding to a given  $\Delta$ . The algorithm goes as follows: after initializing pest damage, pest density (denoted by  $X$ ), starting time, and the number of options to spray (2 in this case), we loop on time. For each time step, we first calculate the change in pest density from  $dX = r X DT + \sigma X \sqrt{DT} z$ , where  $z$  is a drawing from a standard normal distribution. We then add  $dX$  to  $X$  and calculate the increment in pest damage, which equals  $p b X DT$ . We then consider two cases:

- If there are still 2 options to spray, then if the current pest density is greater than the critical pest density for stopping frontier 2, we apply a standard dose of pesticide, change  $X$  to  $X(1-M)$ , and add  $Ke^{\delta(T+d-time)}$  to the damages. Otherwise, we do nothing.

Note that the second option to spray expires at time  $T-\Delta$ .

- If there is only 1 option to spray, then if at least  $\Delta$  days passed since the first spray and if the current pest density is greater than the critical pest density for stopping frontier 1, we apply a standard dose of pesticide, change  $X$  to  $X(1-M)$ , and add  $Ke^{\delta(T+d-time)}$  to the damages. Otherwise, we do nothing. The first option to spray expires at time  $T$ .
- Otherwise, we just continue.

Then, we increment time by  $DT$  and loop until time equals  $T+d$ . We repeat this procedure after replacing  $\Delta$  with  $\Delta+1$ , with the stopping frontiers corresponding to  $\Delta+1$ , and take the difference of the damages obtained. We iterate this procedure to obtain a large enough sample of differences in damages. Its average value estimates the expected marginal cost of increasing  $\Delta$  by one day. The sample standard error divided by the square root of the number of replications is the standard error associated with the calculated sample average.



**Table 1: Summary of Model Parameters.**

Parameter	Unit	Values Considered	Parameter Meaning
p	\$/lb.	0.10, 0.15, 0.20	Price/lb. of apples.
b	lb./( $\text{acre} \cdot \text{CMD}$ )	1.0, 1.5, 2.0	Mite damage coefficient.
M	- -	0.8, 0.9, 0.99	Pesticide efficacy.
K	\$/acre	30	Cost of a pesticide application.
$\delta$	/day	$10^{-4}$ , $1.5 \cdot 10^{-4}$ , $2 \cdot 10^{-4}$	Farmer's discount rate.
T	days	90	Period of mite damage.
$\Delta$	days	7	Minimum delay between successive pesticide applications
d	days	30	Minimum delay between harvest and the last pesticide application.
r	/day	0.04, 0.065, 0.09	Mite density growth rate.
$\sigma$	$1/\sqrt{\text{day}}$	0.1, 0.2, 0.3	Mite density infinitesimal standard deviation.

Note: this table regroups all of the variables needed to generate the stopping frontiers. Parameters p, K, and  $\delta$  are usually well known economic variables.  $\Delta$  and d are often imposed by regulations. The other variables (b, M, T, r, and  $\sigma$ ) are biological parameters. T is fairly well known, whereas b and M are usually not known with much precision, but this is not a big drawback according to our sensitivity analysis. Parameters r and  $\sigma$  could be obtained by maximum likelihood from a time series of pest densities.

**Table 2: Sensitivity Analysis of X\*.**

Parameter		Time (Days)			
Name	Value	20	40	60	80
Reference		1.902	1.054	0.621	0.376
b	1.0	2.853	1.581	0.931	0.564
	2.0	1.426	0.790	0.465	0.282
$\delta$	1.00E-04	1.899	1.042	0.613	0.368
	2.00E-04	1.904	1.056	0.628	0.381
M	0.80	2.140	1.186	0.698	0.423
	0.99	1.729	0.958	0.564	0.342
r	0.040	3.103	2.059	1.466	1.107
	0.090	1.099	0.489	0.207	0.073
$\sigma$	0.1	1.119	0.412	0.156	0.057
	0.3	3.231	2.467	1.883	1.481
K	20	1.268	0.703	0.414	0.248
	40	2.536	1.406	0.827	0.497

Note: Parameters other than the ones indicated in the first column are at their median

Value. For their definition and units, see Table 1.

**Table 3a: Expected Marginal Cost (EMC) of increasing  $\Delta$  from 4 to 5 days.**

t (time days)	$\frac{1}{6}X_{\Delta=7}^*(t)$	EMC (\$/acre)	$\frac{1}{2}X_{\Delta=7}^*(t)$	EMC (\$/acre)	$\frac{5}{6}X_{\Delta=7}^*(t)$	EMC (\$/acre)
20	0.060	0.014 (0.008)	0.179	0.014 (0.005)	0.298	-0.001 (0.003)
40	0.097	0.038 (0.010)	0.292	0.007 (0.007)	0.487	0.007 (0.004)
60	0.162	0.056 (0.011)	0.487	0.055 (0.010)	0.811	0.004 (0.007)
80	0.242	-0.005 (0.001)	0.726	0.118 (0.018)	1.210	0.153 (0.019)

**Table 3b: Expected Marginal Cost (EMC) of increasing  $\Delta$  from 6 to 7 days.**

t (time days)	$\frac{1}{6}X_{\Delta=7}^*(t)$	EMC (\$/acre)	$\frac{1}{2}X_{\Delta=7}^*(t)$	EMC (\$/acre)	$\frac{5}{6}X_{\Delta=7}^*(t)$	EMC (\$/acre)
20	0.060	0.030 (0.008)	0.179	0.012 (0.007)	0.298	0.009 (0.003)
40	0.097	0.053 (0.010)	0.292	0.025 (0.007)	0.487	0.007 (0.005)
60	0.162	0.064 (0.012)	0.487	0.088 (0.011)	0.811	0.024 (0.007)
80	0.242	0.001 (0.007)	0.726	0.176 (0.020)	1.210	0.318 (0.019)

Note: A number in brackets below an EMC is the corresponding standard error of the mean  $s/\sqrt{n}$ , where s is the sample standard error and n is the number of replications.

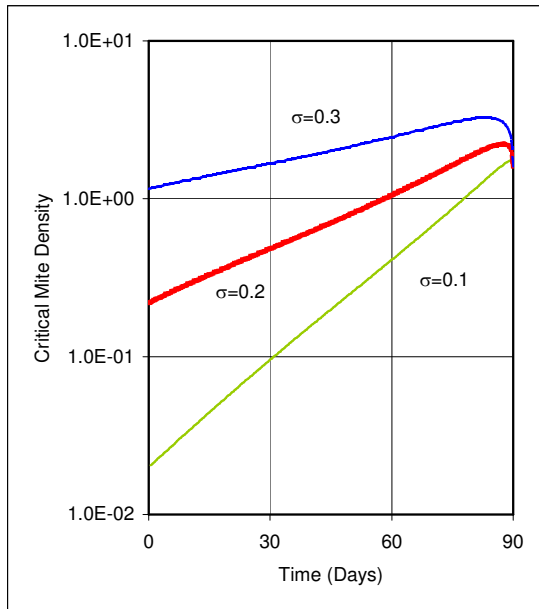


Figure 1: First Stopping Frontiers vs.  $\sigma$

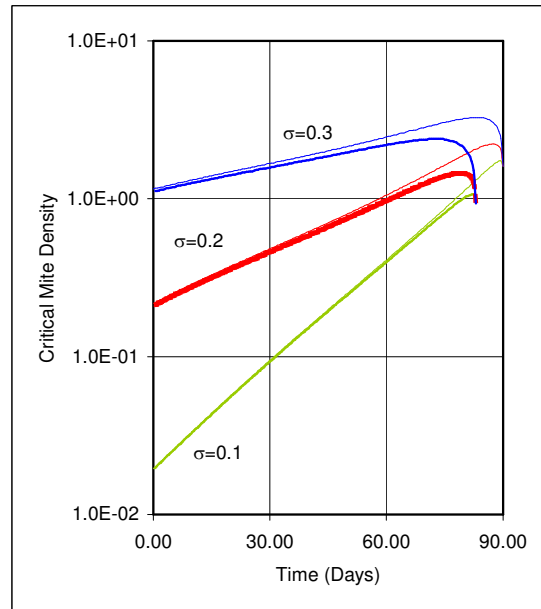


Figure 2: Second Stopping Frontiers vs.  $\sigma$

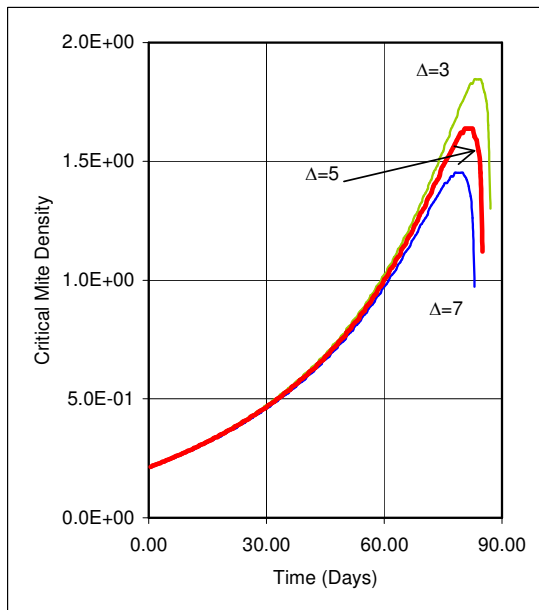


Figure 3: Second Stopping Frontiers vs.  $\Delta$

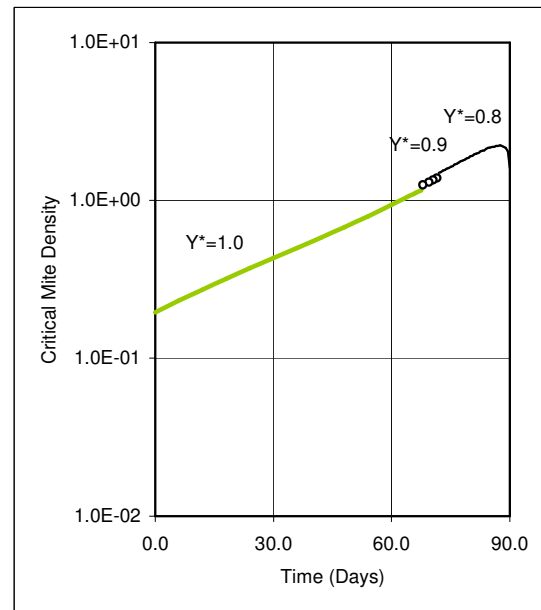


Figure 4: Economic Threshold for three Doses

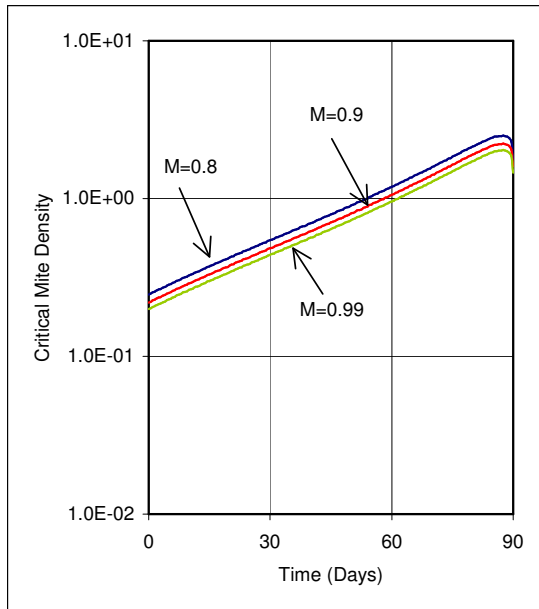


Figure 5: Sensitivity to pesticide efficacy ( $M$ )

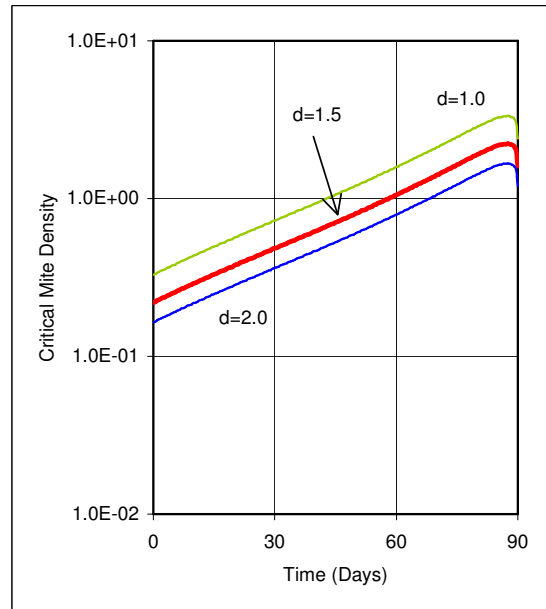


Figure 6: Sensitivity to the mite damage coefficient ( $d$ ).

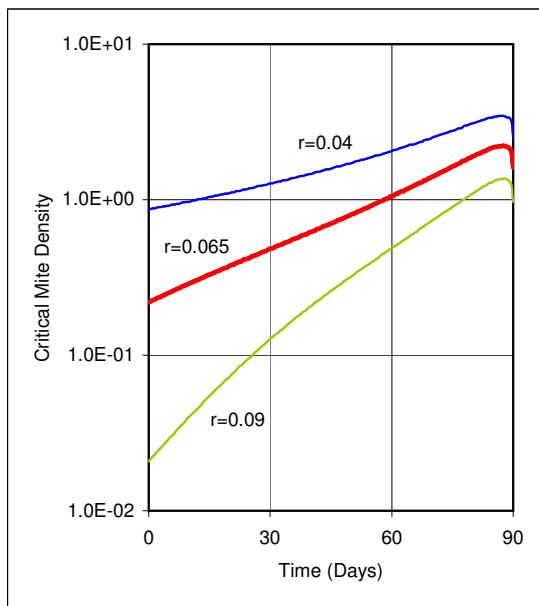


Figure 7: Sensitivity to the pest growth rate ( $r$ ).

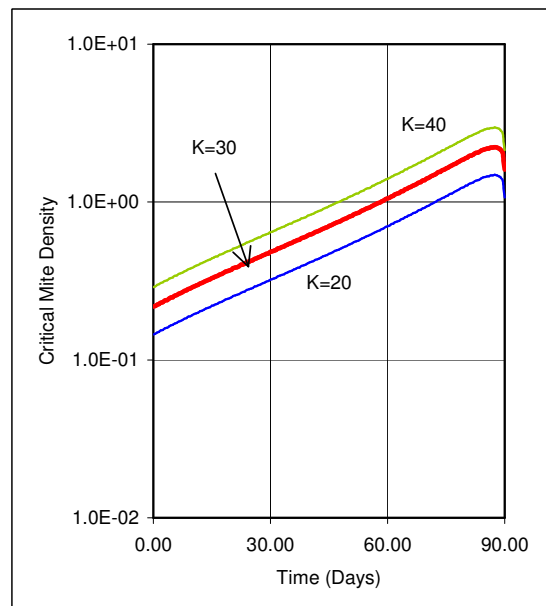


Figure 8: Sensitivity to the spraying cost/acre ( $K$ )

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<sup>1</sup> Diffusion processes have been found to model reasonably well many physical, biological, economic, and social phenomena. Examples include security price fluctuations in a perfect market, variations of population growth, or changes in species numbers subject to competition. For more examples and for a good introduction to stochastic processes, see Karlin and Taylor.

<sup>2</sup> In the following, we omit the argument  $t$  when there is no ambiguity to lighten our notation.

<sup>3</sup> Our biological data were kindly provided by Professor Jan Nyrop, an entomologist with the Cornell experimentation station in Geneva, NY.

<sup>4</sup> Our economic data were kindly provided by Professor Gerald B. White, from the Agricultural, Resource, and Managerial Economics Department at Cornell University.

<sup>5</sup> This is because the diffusion to spray has to balance expected pest damages, which depend only on the current pest density and model parameters  $r$  and  $\sigma$ , with the cost of spraying the pesticide. This cost should include the associated loss of flexibility since spraying is only allowed a finite number of times. Thus, the initial pest density is not needed. This is characteristic of Markov processes in stopping problems (see Malliaris and Brock).

<sup>6</sup> As remarked by an anonymous referee, we could expect, if we took risk aversion into account, that the stopping frontiers would not be shifted up as much when the volatility of the pest population density increases. The reduction in pesticide use would thus decrease with increasing farmer risk aversion. The impact of risk aversion still appears to be

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debated in the literature, however. While the conventional view holds that pesticides are risk-reducing, Moffit showed that average pesticide use may or may not be greater in the presence of risk-aversion when farmers decide to spray based on a threshold rule. Moreover, Gotsh and Regev found that fungicides for wheat producers in Switzerland can have a risk-increasing effect on revenue.