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BOSE-EINSTEIN CORRELATIONS AND COLOR STRINGS

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Bose-Einstein Correlations and Color Strings

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Abstract. We show that correlations between like-sign bosons (the "Bose-Einstein Effect") arise naturally in string fragmentation models for e^+e^- annihilation. Via the area dependence of Wilson loop integrals, the correlation length in momentum space can be related to the string tension.

Introduction. Bose-Einstein (BE) correlations between identical bosons have been investigated in high energy physics for a long time[1]. The production of two identical bosons 1,2 from two particle sources is governed by an amplitude which is symmetrized with respect to exchange of the bosons 1,2, resulting in an enhanced probability of emission if the two bosons have similar momenta. BE correlations are usually measured in terms of a correlation coefficient R,

$$R = \sigma(d^2\sigma/dp_1dp_2) \,/\, (d\sigma/dp_1)(d\sigma/dp_2) \qquad (1).$$

Here σ denotes the total cross-section, $d\sigma/dp$ the single-particle inclusive cross section in the absence of correlations, and $d^2\sigma/dp_1dp_2$ the two-particle cross-section. BE correlations provide an important tool for the study the production dynamics[2]. In particular, it can be argued that (like in the astrophysical investigation of photon sources) $R = 1 + |\rho(q)|^2$, where $\rho(q)$ is the Fourier transform

$$\rho(q) \sim \int d^4x\, \rho(x) \exp(iqx) \qquad (2)$$

of the space-time distribution $\rho(x)$ of particle production points, with q denoting the four-momentum transfer p_1-p_2 (we set $\hbar=c=1$). In other words, R provides a measure of the distribution and lifetime of the boson sources.

This interpretation is somewhat questionable when applied to particle production in high energy collisions. Unlike in astrophysical applications the particle sources in high energy collisions typically move at relativistic velocities with respect to each other. This relativistic motion of sources is crucial for the proper interpretation of BE correlations in particle physics. Another difference is the possibility of coherent particle production, in which case the BE enhancement is absent[2]. Indeed, many models for hadron production in quark fragmentation, such as the bremsstrahlung model[3] or the Schwinger model[4] predict such a coherence. In dual models, the almost complete absence of BE enhancements for like-sign pions from e^+e^- annihilation is predicted[5] for a different reason, due to a cancellation of the positive BE correlation and a negative correlation arising from the cascade-like nature of the fragmentation process. In stochastic fragmentation models such as the LUND[6] model, interference effects such as BE correlations are of course absent. These predictions are in drastic disagreement with e^+e^- experiments, where BE enhancements are evident[1].

In this letter we present a different way to describe Bose-Einstein correlations, which addresses many of these problems[7]. Our approach is based on the concept of particle production in the decay of a quasi-onedimensional color force field - a picture exploited extensively e.g. in the LUND hadronization model[6]. We discuss new tests of the consistency of the string picture and its connection to lattice gauge theory.

Source motion in high-energy collisions. In order to point out some of the problems related to the interpretation of BE correlations according to eqn. (2), we will first briefly address the question of moving sources. In (2), it is assumed that all sources have identical characteristics. In this case, the coefficient R in (1) depends only on the momentum difference q, and the correlation length in q is determined by the distribution of particle sources. The situation changes, if different sources have different spectra: then the Bose-Einstein effect measures the distribution of sources weighted with the probability to find a particle 1 from one source and a particle 2 from another source nearby in phase space.

As discussed in detail by Bjorken[8], and explicitly demonstrated in the Lund hadronization model[6], particle production in high energy reactions exhibits a special space-time structure: on average, the production of a particle requires a certain time τ in the local restframe. Particle production points therefore scatter about a hyperbola $t^2-x^2 = \tau^2$. This property implies a large longitudinal extension $L \simeq 2(\beta\gamma)_{max}\tau \sim \sqrt{s}$ of the distribution of particle sources, and a strong correlation between particle production points ($x \simeq \beta\gamma\tau$) and -momentum ($p \simeq \beta\gamma m$). Particles from distant sources will typically exhibit large momentum differences; the probability to find particles from opposite "ends" of the event with a momentum difference of less then $1/L$ (resulting in significant interference effects) is negligible. The length scale measured by Bose-Einstein correlations is hence not L, but instead the distance in production points for which the momentum distributions still overlap. A quantitative formulation is particularly easy in terms of the rapidity $y = (1/2)\log\{(E+p_{||})/(E-p_{||})\}$, where E is the energy of a particle and $p_{||}$ its momentum parallel to the event axis. For a given particle source decaying isotropically in its restframe, the rapidity distribution of the decay products is well described by a gaussian distribution with a width of $\sigma \simeq 0.7$ units in rapidity, centered at the rapidity of the source. The particle distributions from two sources separated by Δy overlap reasonably well in rapidity and hence in phase space if $\Delta y \leq 1$, resulting in a BE correlation length in q^2 of the order $1/\tau^2$, or a apparent spread of particle sources of order $\tau \ll L$. In particular, this means that a) in high energy reactions the source distribution will not appear significantly elongated in the direction of the event axis, and b) that the measured distribution is independent of the center of mass energy.

Interference effects in string models. Fig. 1(a) illustrates particle production from a color string in 1+1 dimension. Through creation of new quark-antiquark pairs, the color force field breaks up and adjacent quarks and antiquarks recombine into mesons. Fig. 1(b) describes the production of the same final state, but the two identical bosons 1 and 2 have been exchanged. In a quantum mechanical description of the production process, the amplitudes corresponding to Figs. 1(a) and (b) will interfere. Present phenomenological models however assign probabilities, not amplitudes, to the diagrams and neglect interference. Given this caveat, one of main results of the LUND model is that in a color string model the (non-normalized) probability $d\Gamma_n$ to produce an n-particle state $\{p_i\}$, $i=1,n$ (with distinguishable particles) has the functional form[6]

$$d\Gamma_n = \{\Pi N dp_j \delta(p_j^2-m_j^2)\}\, \delta(\Sigma p_j - P) \exp(-bA_n) \qquad (3)$$

Here A_n is the total space-time area over which the color field extends. The Lorentz-invariant A_n can be readily expressed in terms of particle momenta[9]. P is the total energy-momentum of the state. The two phenomenological parameters N and b in this expression are related to the mean multiplicity and the correlation length in rapidity. The area dependence in (3) is reminiscent of the behavior of Wilson loop integrals[10]

$$M(\sigma) = \langle \exp(i\oint A^\mu dx_\mu)\rangle \qquad (4)$$

which in a confined theory should be

$$M(\sigma) = \exp\{i\xi A(\sigma)\} \qquad (5)$$

with $A(\sigma)$ denoting the area spanned by a closed quark loop σ. If we now interprete the quantity $d\Gamma_n$ as longitudinal phase space multiplying a squared matrixelement $|M_n|^2 = \exp(-bA)$, it is tempting to identify $M(\sigma)$ with M_n and to assume for ξ

$$Re(\xi) = \kappa, \quad Im(\xi) = b/2 \qquad (6).$$

The assumption on the real part of ξ corresponds to Wilson's result that the coefficient in front of the space-time area in (5) can be identified as the string tension κ. The corresponding imaginary part of ξ would imply in this interpretation that the string state decays. From phenomenological investigations we know that $\kappa \approx 0.2\ GeV^2$ and $b/\kappa^2 \approx 0.1 - 1\ GeV^{-2}$.

The production of two identical bosons (1,2), e.g. π's, is governed by the symmetrical matrix element

$$M = (1/\sqrt{2})(M_{12}+M_{21}) = (1/\sqrt{2})\{exp(i\xi A_{12}) + exp(i\xi A_{21})\} \qquad (7),$$

where A_{12} is the area of the color field for a given order of particles 1,2, and A_{21} is the area resulting from an exchange of the two particles. We note in particular that there is an area difference (and consequently a phase difference) between the matrix elements M_{12} and M_{21}:

$$\Delta A = |A_{21}-A_{12}| = \{(p_1p_2)^2-p_1{}^2p_2{}^2\}^{1/2} + \{(P_I q)^2-P_I{}^2q^2)\}^{1/2} \qquad (8).$$

Here p_1, p_2 are the two-momenta of the bosons and q is the (space-like) momentum difference. P_I is the momentum of the intermediate state I (Fig. 1). In case of two like-sign pions it is evidently necessary to have such an intermediate state. Using the matrix element (3) we obtain for the ratio R in (1)[11]

$$R = 1 + \cos(\kappa\Delta A)/\cosh(b\Delta A/2) \qquad (9).$$

In the limit q=0, (8) and (9) imply ΔA=0 and hence R=2, in agreement with the result obtained from the conventional interpretation (2) for completely incoherent sources. In contrast to (2), (9) predicts an additional dependence on the momentum of the system "in between" the two bosons.

At first, the prediction of BE effects in string models seems surprising, given that these are modeled after the Schwinger mechanism[4] of coherent production. There is however an important difference: for the Schwinger mechanism of confinement, the field couples directly and locally to a boson, whereas the quarks forming a boson in string models are created at two different space-time points. A description of such a process is not gauge invariant unless one introduces an exponential of the line integral of the gauge field between the two points, resulting in (4).

Eqn. (3) applies to a 1+1 dimensional world with massless quarks. If quark masses and transverse momenta are introduced, the creation of quark pairs in an external color field can be understood as a tunneling process[6]; hence the amplitude (3) is modified by additional (real) WKB factors $\exp\{-(\pi/2\kappa)(m_q^2 + \vec{p}_{Tq}^{\,2})\}$ for each new quark pair. Here m_q represents the quark mass and $\vec{p}_{Tq}$ the quark transverse momentum, which can be expressed in terms of particle transverse momenta $\vec{p}_T$ as $\vec{p}_{Tq} = \Sigma\, \vec{p}_T$, where the sum runs over all particles between one end of the string and the production point of the given quark. Exchanging two particles with $\Delta\vec{p}_T = \vec{p}_{T1} - \vec{p}_{T2} \neq 0$ will result in changes in the quark transverse momenta. Correspondingly, the $\cosh(b\Delta A/2)$-term in (9) changes into $\cosh(b\Delta A/2+\delta)$, $\delta = (\pi/2\kappa)\Delta(\Sigma\, \vec{p}_{Tq}^{\,2})$. In the limit of only one intermediate particle I, δ is approximately given by $(\pi/2\kappa)\Delta\vec{p}_T^{\,2}$. Obviously, maximum interference is obtained for particle pairs with a small difference both in longitudinal and transverse momentum.

The formalism given above applies when all particles produced in the "decay" of the color string are stable. In reality, a large fraction of the primary particles are resonances with lifetimes and decay pathlengths which are comparable to the corresponding time and length scales in string decay. To account for this, each amplitude given would have to include the decay amplitudes and phase space factors for resonance decays, and the total amplitude is the sum of all amplitudes leading to identical sets of observable stable particles. Unfortunately, the implementation of such a scheme is very difficult. In order to be able to compare (9) with experimental data, we tried three different simplifying assumptions:

a) Only charged or neutral pions are produced. Models like this have been used for a long time and successfully account for the gros features of particle production in high-energy reactions.

b) The usual mixture of stable and instable particles, including strange particles and baryons is assumed. The correlation coefficient (9) is then calculated using the final stable particles, ignoring the fact that they result from resonance decays.

c) Decay products of a resonance of four-momentum k, mass M and width Γ contribute to the BE effect only if $kq \leq \Gamma M$, since otherwise the amplitude after the exchange of particles is strongly suppressed by the Breit-Wigner shape of the resonance[12].

Detailed model predictions were obtained using a Monte-Carlo technique, by weighting events with the ratio of the squared sum of amplitudes and the sum of the squares of amplitudes, including the transverse-momentum dependence. Only 2-jet events were generated, neglecting the possibility of hard-gluon emission. Our predictions for the two-pion correlation coefficient R as a function of $Q = |q|$ are shown in Fig. 2, in comparison with data from the PEP4 experiment[13]. Assumptions a) and b) for the treatment of resonances give very similar results and are not shown separately.

Assumption c) results in a strong reduction of the BE correlation for non-zero Q, in particular since pions from η or ω decays fail to fullfill $kq \lesssim \Gamma M$ for Q -values in excess of a few MeV (in analogy to the experiment, pions from k^0 decays were not included in the calculation of R). For typical values of κ and b, the coefficient in front of ΔA in the cos-term in (9) is much larger than the corresponding coefficient in the cosh-term, hence the correlation length in Q is governed by the value of the string tension κ and is rather insensitive to b. Whereas alternatives a,b) reproduce the data surprisingly well (for $\kappa \approx 0.2 - 0.3\ GeV^2$), c) underestimates the R significantly. In lack of a possibility to implement resonance production in the matrix element (3), we can only speculate that approximation c), which is based on the assumption of independently produced resonances, is not adequate for our situation. In agreement with the experiment[13], we find that the correlation length in Q does not change drastically depending on the relative orientation of the $\vec{q}$-vector and the event axis; in the classical interpretation (2), this means that the source appears more or less spherical. We note that the effect predicted in ref. 5 - the competition of the positive BE correlation and a negative correlation - is present to some degree; since however the typical scale of the latter is much larger in q, it gives rise to a sloped background, from which the real BE effect is easily separated.

Summary. BE interference between identical bosons arises naturally in string models of parton fragmentation. Ignoring possible effects due to narrow resonances, we find very good agreement with experimental data, in support of our assumption that the overall phase of the production matrix element is governed by a Wilson loop integral.

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Figure captions

Fig. 1 a) Space-time diagram for particle production in e^+e^- annihilation, based on the string picture[6], where energy and momentum of a particle are given by the space resp. time difference of the production points of its quarks, multiplied by the string tension κ. A denotes the space-time area of the color field enclosed by the quark loop (antiquarks are represented as quarks moving backward in time). b) As a), but particles 1 and 2 are exchanged, resulting in a change of the area A of the color field by ΔA.

Fig. 2 Comparison of model predictions for R with exp. data[13], as a function of $Q = |q|$. Full line: based on assumption a) or b), for $\kappa = 0.2$ GeV^2 and $b/\kappa^2 = 0.7$ GeV^{-2}. Dashed: using $\kappa = 0.3$ GeV^2. Dotted: based on assumption c), for $\kappa = 0.2$ GeV^2. The curves are very insensitive to the value of b/κ^2.

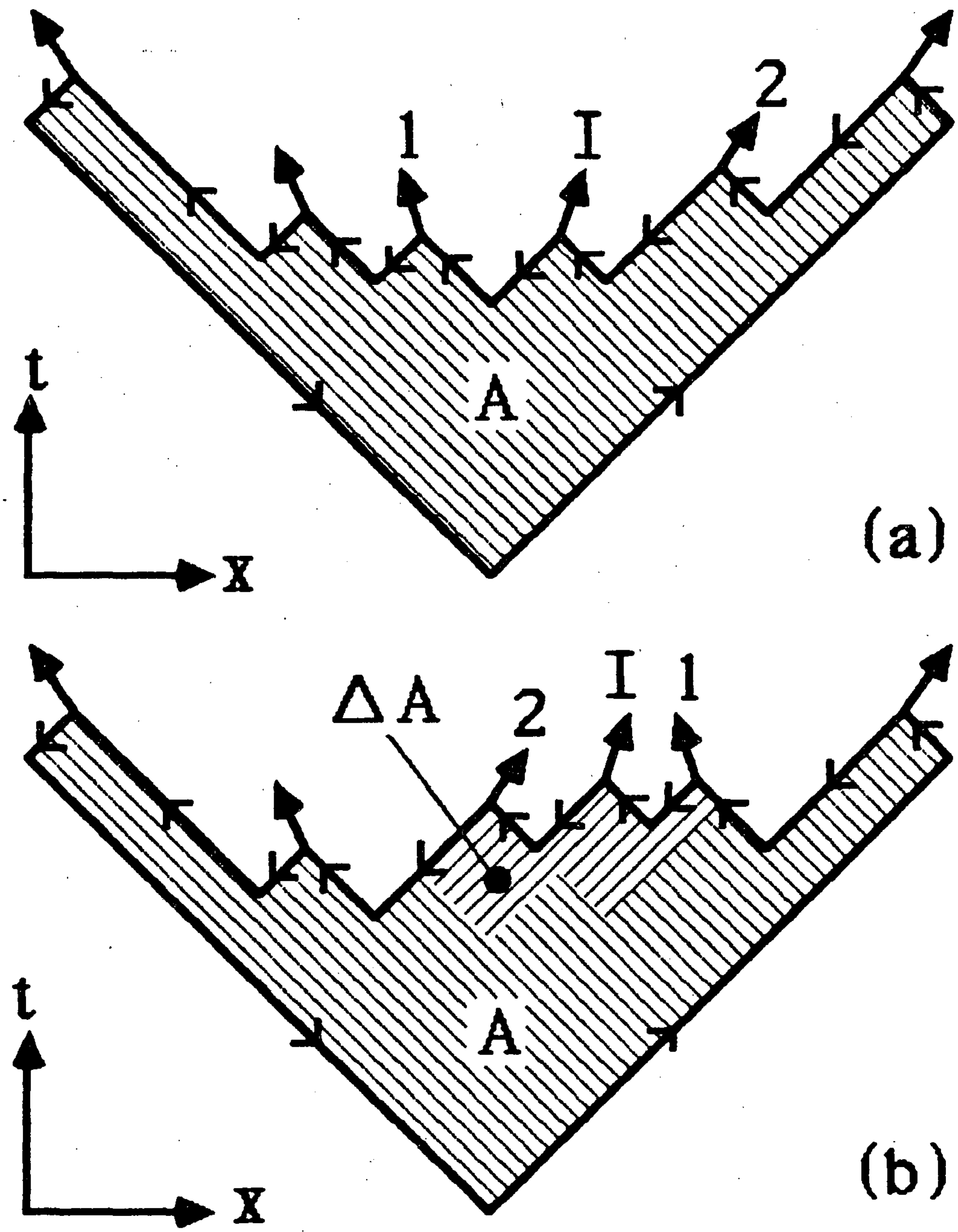

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Fig. 1

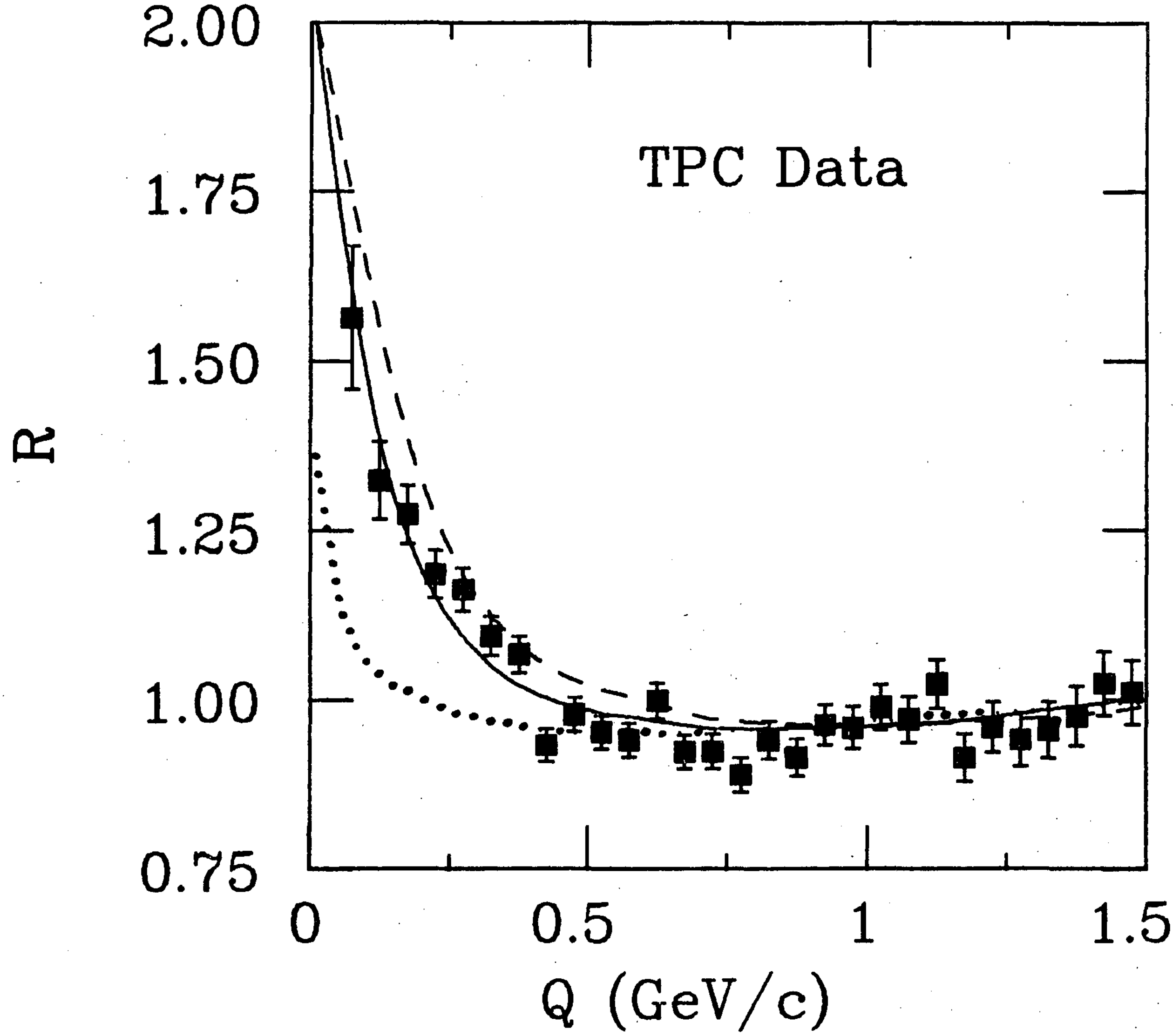

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Fig. 2

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