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Moral Hazard, Adverse Selection, and Mortgage Markets

by

Barney Paul Hartman-Glaser

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Business Administration

in the

Graduate Division
of the
University of California, Berkeley

Committee in charge:
Professor Nancy Wallace, Co-chair
Professor Alexei Tchisty, Co-chair
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Spring 2011

Moral Hazard, Adverse Selection, and Mortgage Markets

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Barney Paul Hartman-Glaser

Abstract

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Doctor of Philosophy in Business Administration

University of California, Berkeley

Professor Nancy Wallace, Co-chair

Professor Alexei Tchisty, Co-chair

This dissertation considers problems of adverse selection and moral hazard in secondary mortgage markets. Chapters 2 and 3 consider moral hazard and adverse selection respectively. While Chapter 4 investigates the predictions of the model presented in Chapter 2 using data from the commercial mortgage backed securities market.

Chapter 2 derives the optimal design of mortgage backed securities (MBS) in a dynamic setting with moral hazard. A mortgage underwriter with limited liability can engage in costly effort to screen for low risk borrowers and can sell loans to a secondary market. Secondary market investors cannot observe the effort of the mortgage underwriter, but they can make their payments to the underwriter conditional on the mortgage defaults. The optimal contract between the underwriter and the investors involves a single payment to the underwriter after a waiting period. Unlike static models that focus on underwriter retention as a means of providing incentives, the model shows that the *timing* of payments to the underwriter is the key incentive mechanism. Moreover, the maturity of the optimal contract can be short even though the mortgages are long-lived. The model also gives a new reason for mortgage pooling: selling pooled mortgages is more efficient than selling mortgages individually because pooling allows investors to learn about underwriter effort more quickly, an *information enhancement effect*. The model also allows an evaluation of standard contracts and shows that the “first loss piece” is a very close approximation to the optimal contract.

Chapter 3 considers a repeated security issuance game with reputation concerns. Each period, an issuer can choose to securitize an asset and publicly report its quality. However, potential investors cannot directly observe the quality of the asset and a lemons problem ensues. The issuer can credibly signal the asset’s quality by retaining a portion of the asset. Incomplete information about issuer type induces reputation concerns which provide credibility to the issuer’s report of asset quality. A mixed strategy equilibrium obtains with the following 3 properties: (i) the issuer misreports asset quality at least part of the time, (ii) perceived asset quality is a U-shaped function of the issuer’s reputation, and (iii) the issuer retains less of the asset when she has a higher reputation.

Chapter 4 documents empirical evidence that subordination levels for commercial mortgage backed securities (CMBS) depend on issuer reputation in a manner consistent with the model of Chapter 3. Specifically, issuer retention is negatively correlated with issuer reputation. New measures for both issuer reputation and retention are considered.

To my father, Barney G. Glaser
an inspirational scholar, and
my wife, Tiffany M. Shih,
my academic and emotional rock.

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Chapter 1

Introduction

Financial economists have worried about problems of asymmetric information, such as moral hazard (MH) and adverse selection (AS), in financial markets for decades. Recent events, like the subprime default crisis, seem to indicate that these problems are particularly pronounced in mortgage markets. At the same time, mortgage markets have a structure that poses new challenges for the both the theoretical and empirical literatures dealing with MH and AS. Moreover, mortgage markets are some of the largest financial markets in existence and are essential to the efficiency of the real economy. Thus, a deeper understanding of the specific manifestations and implications of MH and AS for mortgage markets is in order. In the following chapters I will consider 2 important examples of how these issues may be intrinsically different from those considered in the past literature. I will also present new empirical evidence from the CMBS market.

In Chapter 2, I consider the problem of providing incentives to a mortgage underwriter, in which effort has long term consequences and must be exerted over many tasks at once.¹ A mortgage underwriter with limited liability can engage in costly effort to screen for low risk borrowers and can sell loans to a secondary market. Secondary market investors cannot observe the effort of the mortgage underwriter, but they can make their payments to the underwriter conditional on the mortgage defaults. The optimal contract between the underwriter and the investors involves a single payment to the underwriter after a waiting period. Unlike static models that focus on underwriter retention as a means of providing incentives, the model shows that the *timing* of payments to the underwriter is the key incentive mechanism. Moreover, the maturity of the optimal contract can be short even though the mortgages are long-lived. The model also gives a new reason for mortgage pooling: selling pooled mortgages is more efficient than selling mortgages individually because pooling allows investors to learn about underwriter effort more quickly, an *information enhancement effect*. The model also allows an evaluation of standard contracts and shows that the “first loss piece” is a very close approximation to the optimal contract.

¹Chapter 2 draws on the co-authored article “Optimal Securitization with Moral Hazard” with Alexei Tchisty and Tomazs Piskorski. At the time of submission of this dissertation, the article had not been published.

In Chapter 3, I consider an adverse selection problem faced by investors when an issuer can both signal her private information through partial retention and build a reputation for honest reporting. The model can be characterized as a repeated security issuance game with reputation concerns. Each period, an issuer can choose to securitize an asset and publicly report its quality. However, potential investors cannot directly observe the quality of the asset and a lemons problem ensues. The issuer can credibly signal the asset's quality by retaining a portion of the asset. Incomplete information about issuer type induces reputation concerns which provide credibility to the issuer's report of asset quality. A mixed strategy equilibrium obtains with the following 3 properties: (i) the issuer misreports asset quality at least part of the time, (ii) perceived asset quality is a U-shaped function of the issuer's reputation, and (iii) the issuer retains less of the asset when she has a higher reputation.

Chapter 4 documents empirical evidence that subordination levels for commercial mortgage backed securities (CMBS) depend on issuer reputation in a manner consistent with the model of Chapter 3. Specifically, issuer retention is negatively correlated with issuer reputation. New measures for both issuer reputation and retention are considered.

Note that in Chapter 2, I use the pronoun "we" to indicate that the work was done by multiple people whereas in Chapter 3 I used the pronoun "I," as that work is entirely my own. Also note that although some symbols are present in both Chapters 2 and 3, their meaning may differ across chapters.

Chapter 2

Optimal securitization with moral hazard

Mortgage underwriters face a dilemma: either to implement high underwriting standards and underwrite only high quality mortgages or relax underwriting standards in order to save on expenses. For example, an underwriter can collect as much information as possible about each mortgage applicant and fund only the most credit-worthy borrowers. Alternatively, an underwriter could collect no information at all and simply make loans to every mortgage applicant. Clearly, the second approach, while less costly in terms of underwriting expenses, will result in higher default risks for the underwritten mortgages. Moreover, mortgage underwriters typically wish to sell their loans in a secondary market rather than hold loans in their portfolio. Investors do not observe the underwriter's effort and consequently do not observe the quality of the mortgages they are buying.¹

The recent mortgage crisis has brought a lot of attention to the potential agency conflict arising from the separation of loan's originator and the bearer of the loan's default risk. Policy-makers and market observers have emphasized that the originators of loans and the underwriters of mortgage-backed securities might have lacked proper incentives to act in the best interests of investors, and as a consequence this possible misalignment of incentives might have importantly contributed to the mortgage default crisis. Ultimately, these concerns resulted in a number of policy proposals, beginning on June 2009 when the Obama Administration released its "Financial Regulatory Reform" proposal which calls for loan originators or sponsors to retain a part of the credit risk of securitized assets.² However, beside a general notion that aligning incentives requires the participants in securitization

¹Recent empirical evidence in Keys, Mukherjee, Seru, and Vig (2008) suggests that securitization might have adversely affected the screening incentives of lenders. For an good description of the market for securitized subprime loans, see Ashcraft and Schuermann (2009).

²More precisely, the proposal requires originators or sponsors to retain a material portion (generally 5%) of the credit risk of securitized assets and prohibits "hedging or otherwise transferring" the retained risk. See "Financial Regulatory Reform, A New Foundation: Rebuilding Financial Regulation and Supervision," Department of the Treasury, pages 44-45, at http://www.financialstability.gov/docs/regs/FinalReport_web.pdf

process to hold an economic interest in the credit risk of securitized assets (or “skin in the game”), little is known about how to design these provisions efficiently, especially fully recognizing the long term duration of assets in question. Consequently, this question has been the subject of significant and ongoing discussions among the Administration, Congress, regulators and market participants.³ Our paper aims to inform this debate by examining how the design of mortgage backed securities (MBS) can efficiently address this agency conflict.

We consider an optimal contracting problem between a mortgage underwriter and secondary-market investors. At the origination date, the underwriter can choose to undertake costly effort that results in low expected default rates for the underwritten mortgages. If the underwriter chooses to shirk, the mortgages will have a high expected default rate. Thus, the effort technology of our model results in mortgage performance that occurs through time, rather than on a single date as in the previous literature. In addition to costly hidden effort, we include a motivation to securitize by assuming that by selling mortgages, rather than holding them in her portfolio, the mortgage underwriter can exploit new investment opportunities, i.e., underwrite more mortgages. We model this feature by assuming that the underwriter is impatient, as in DeMarzo and Duffie (1999), so that the underwriter has a higher discount rate than the investors.⁴ Investors do not observe the actions of the underwriter, however the timing of mortgage defaults is publicly observable and contractible.

We derive the optimal contract between the underwriter and the investors that implements costly effort and maximizes the expected payoff for the underwriter, provided the investors are making non-negative profits in expectation. We do not make restrictive assumptions on the form of the contract. Instead, we include all possible payment schedules between the investors and the underwriter in the space of admissible contracts, so long as they depend only on the realization of mortgage defaults and provide limited liability to the underwriter. This setup, which allows information to be revealed over time, allows us to address a central issues in the market for MBS. Namely, how does the fact that mortgage performance occurs through time affect the contracting problem between the investors and the underwriter? Moreover, how much time is needed before the investors can completely pay off the underwriter while maintaining incentive compatibility?

Despite the apparent complexity of the contracting problem, the optimal contract takes a simple form: The investors receive the entire pool of mortgages at time zero and make a single lump sum transfer to the underwriter after a waiting period provided no default occurs. If a single default occurs during the waiting period, the investors keep the mortgages, but no payments are made to the underwriter.

The timing and structure of the optimal contract arise from a trade-off between two forces. Delaying payment to the underwriter results in a more precise signal on the underwriter’s action, which lowers the cost of incentive provision. On the other hand, delaying payment is costly due to the relative impatience of the underwriter. By making the payment

³See American Securitization Forum (2009).

⁴An alternative motivation for the relative impatience of underwriters are regulatory capital requirements which induce a preference for cash over risky assets.

for mortgages contingent upon an initial period of no default, the investors can provide incentives for underwriters to underwrite low risk mortgages since high risk mortgages will be more likely to default during the initial waiting period. However, delaying payments past the initial waiting period is suboptimal since the underwriter is impatient.

Interestingly, the optimal contract calls for the underwriter to *pool* mortgages rather than sell each mortgage individually. By observing the timing of a single default, the investors learn about the quality of the remaining mortgages. As a result, the investors can infer the quality of the mortgages sooner by observing the entire pool rather than a single mortgage at a time, which we call the *information enhancement effect*. By making payment contingent upon the performance of the entire pool rather than each individual mortgage, it is possible to speed up payment to the underwriter while maintaining incentive compatibility. This result is not driven by any benefits from diversification.

Our findings are in contrast with the previous literature on security design with asymmetric information that primarily focuses on a static setting, e.g., DeMarzo (2005). We show that the timing of payments plays an important role when the information about the underlying assets is revealed over time. In the dynamic setting of this paper, the optimal contract is about *when* the underwriter is paid, rather than what piece of the underlying assets it retains.

Our paper relates most closely to the literature on optimal security design and asset backed securities (ABS). One approach of this literature is to treat the security design problem faced by issuers as a standard capital structure problem as characterized by Myers and Majluf (1984) giving rise to a pecking order theory of asset backed securities. Nachman and Noe (1994) present a rigorous frame work for when a given a security design minimizes mispricing due to asymmetric information, showing standard debt is optimal over a very broad class of security design problems. Building on the pecking order intuition, Riddiough (1997a) shows that an informed issuer can increase her proceeds from securitization by creating multiple securities, or tranches, with differing levels of exposure to the issuer's private information. Moreover, pooling assets that are not perfectly correlated can provide some diversification benefits and thus reduce the lemons discount.

Another approach to the optimal design of asset backed securities considers the role of costly signaling. The basic intuition of this approach is that an issuer of ABS can signal her private information by retaining a fraction of the issued security as in Leland and Pyle (1977). Building on this intuition, DeMarzo and Duffie (1999) presents a model of security design where an issuer minimizes the cost of signaling her private information by choosing a security design. Applying the security design framework of DeMarzo and Duffie (1999), DeMarzo (2005) explains the pooling and tranching structure of ABS. In his model, an issuer of ABS can signal her private information about a pool of assets by retaining a fraction of a security which is highly sensitive to that information. This signaling mechanism explains the multi-class, or tranced structure, of ABS.

Other studies of asset backed securities have focused on different types of asymmetric information. For example, Axelson (2007) considers a setting in which *investors* have superior information about the distribution of asset cash flows. The author gives conditions

under which pooling may be an optimal response to investor private information and for which single asset sale is preferred.

The literature on security design and ABS presented above utilizes mostly one period models of securitization. In contrast to the previous literature, we take a different approach by modeling mortgages which can default after some time has elapsed. This additional aspect allows us to show that the timing of payments from mortgage securitization can be a key incentive mechanism and that the duration of the optimal contract can be short while the duration of the mortgages is long.

Another important difference between our model and the literature is that there is little previous work on costly hidden actions in underwriting practices. An exception is Innes (1990), who considers a one period moral hazard model of security design. Other work that considers a security design in the context of a moral hazard framework include DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007), which both consider long term contracts for repeated agency conflicts in which actions have short term consequences. In our setting, the agent may only take actions at a single point in time, however those actions have long term consequences.

The moral hazard problem in underwriting practices is likely to be important in private securitization markets where both the quality of assets and the operations of issuers are extremely difficult to verify. Indeed, some empirical studies, such as Mian and Sufi (2009), suggest that “mis-priced agency conflicts” may have played a crucial role in the current mortgage crisis. In addition, evidence presented in Keys, Mukherjee, Seru, and Vig (2008) suggests that securitization of subprime loans led to lax lending standards, especially when there is “soft” information about borrowers which determines default risk but is not easily verifiable by investors.

Our paper also closely relates to the literature on “multi-task” Principal-Agent problems, for example Bond and Gomes (2009) and Laux (2001), and to the literature on Principal-Agent problems where actions have persistent effects, for example Abreu, Milgrom, and Pearce (1991) and Sannikov and Skrzypacz (2010). The problem of providing incentives to a mortgage underwriter can be viewed as a multitask contracting problem since the underwriter must evaluate multiple loans. Bond and Gomes (2009) analyzes a very broad class of “multi-task” Principal-Agent problems and finds that optimal contracts for multi-task problems take the form of cutoff rules analogous to the optimal contract in our setting. Our results differ from this literature in that we emphasize the delayed nature of information revelation inherent in the specific problem of providing incentives to mortgage underwriters. It is this feature which provides a contribution to the literature on agency problems in which actions have persistent effects, as in Abreu, Milgrom, and Pearce (1991). In this literature, increasing the lag between when an action is taken and output is observed, can *increase* efficiency when discount rates are small. This effect arises due to a statistical inference effect that is similar to our information enhancement effect. In our setting, the lag between actions and outcomes is stochastic and has a distribution that depends on hidden actions. Therefore, the key parameters which drive efficiency are the *difference* between expected information lags and the *difference* in discount rates. Moreover, contract efficiency

increases when either difference decreases. Although we specifically frame the contracting problem in terms of a mortgage underwriter and investors in mortgage backed securities, the model could apply to other problems in which an agent takes multiple hidden actions that have persistent consequences, for example a CEO deciding the allocation of capital to multiple long lived projects within a single firm.

2.1 The Model

2.1.1 Preferences, Technology, and Information

Time is infinite, continuous, and indexed by t . A risk-neutral agent (the underwriter⁵) originates N mortgages that she wants to sell to a risk-neutral principal (the investors) immediately after origination. The underwriter has the constant discount rate of γ and the investors have the constant discount rate of r . We assume $\gamma > r$. This assumption could proxy for a preference for cash or additional investment opportunities of the underwriter DeMarzo and Duffie (1999).

The underwriter may undertake an action $e \in \{0, 1\}$ at cost $C \cdot e$ at the origination of the pool of mortgages ($t=0$). This action is hidden from the investors and hence not contractible. Each mortgage generates constant coupon u until it defaults. Individual mortgages default according to an exponential random variable with parameter $\lambda \in \{\lambda_H, \lambda_L\}$ such that $\lambda = \lambda_L$ if $e = 1$ and $\lambda = \lambda_H$ if $e = 0$ and $\lambda_H > \lambda_L$. Upon default, all assets pay a lump sum recovery of $R < u/r$. All defaults are mutually independent conditional on effort. It may seem overly simplistic to assume that low underwriter effort leads to only high risk mortgages. A more realistic assumption is that low effort leads to a mixture of both high risk and low risk mortgages. Such a setup would complicate the analysis as the mixture of two exponential distributions is not itself exponential. However, as we argue below, it does not add richness to the model to assume that low effort leads to a mixture of mortgage risk types.

A contract consists of transfers from the investors to the underwriter depending on mortgage defaults. Specifically, let D_t denote the total number of defaults that have occurred by time t and \mathcal{F}_t the filtration generated by D_t . It will also be convenient to define the following sequence of stopping times

$$\tau_n = \inf\{t : D_t \geq n\}.$$

for $n \geq 0$. Also let $\tau_0 = 0$ for convenience. Formally, a contract is an \mathcal{F}_t -measurable process X_t giving the cumulative transfer to the underwriter by time t so that dX_t denotes the instantaneous transfer to the underwriter at time t . Readers unfamiliar with this notation can think of X_t as a function of time t and all previous default times $\tau_n \leq t$ so that $dX_t = x_n(t, \tau_0, \dots, \tau_n)dt$ for some family of functions $\{x_n(\cdot)\}_{n \leq N}$ such that X_t is adapted to \mathcal{F}_t . We

⁵Although we refer to the agent in our model as the underwriter, our setting applies equally well to other actors than mortgage underwriters. The defining characteristic of the agent in our model is that she can undertake costly hidden action to screen out high risk mortgages or assets.

restrict our attention to contracts that satisfy the limited liability constraint $dX_t \geq 0$ and are absolutely integrable. The underwriter thus has the following utility for a given contract X_t and effort e

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e \right] - Ce.$$

All integrals will be Stieljes integrals.⁶

2.1.2 Optimal Contracts

We assume that implementing high effort is optimal, hence the investors' problem is to maximize profits subject to delivering a contract that provides incentives to expend effort and a certain promised level of utility to the underwriter. We call a such contracts *incentive compatible*. Once we restrict our attention to incentive compatible contracts, the value the investors place on holding the mortgages is fixed since the contract cannot affect the distribution of mortgage defaults other than to guarantee that the underwriter only originates low risk mortgages. Hence, the investors maximize profits by choosing the incentive compatible contract with the lowest expected cost under their discount rate. In other words, the investor chooses the least costly incentive compatible contract. We state this formally in the following definition.

Definition 1. Given a promised utility a_0 to the underwriter, a contract X_t is *optimal* if it solves the following problem

$$b(a_0) = \min_{dX_t \geq 0} E \left[\int_0^\infty e^{-rt} dX_t | e = 1 \right] \quad (\text{I})$$

such that

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] - C. \quad (\text{IC})$$

and

$$a_0 \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] - C \quad (\text{PC})$$

It is important to note that Definition 1 is equivalent to a definition where we hold the cost to the investor fixed and maximize the utility of the underwriter. As an alternative to Definition 1, we could fix the cost paid by the investor b and find the contract which maximizes the underwriters initial utility a_0 . In the analysis that follows, we will find a one-to-one relationship between the cost paid by the investors and the value delivered to the underwriter; for any level of initial promised utility of the underwriter, we know the cost paid by the investors under the optimal contract and vice versa.

⁶Readers unfamiliar with Stieljes integrals may simply view this integral for an arbitrary integrand g as $\int g(t)dX_t = \int g(t)x(t)dt$ where $x(t)$ is the time t change in $X(t)$. For a reference see Carter and Van Brunt (2000).

2.1.3 Solution

The contracting problem stated thus far amounts to solving an infinite dimensional optimization problem which at first glance seems quite complicated. In this section we will give a heuristic argument that transforms, subject to verification, our contracting problem to a simple problem of solving two equations for two unknowns by making a series of intuitive guesses about the form of the optimal contract. This argument follows from the observation that the optimal contract should feature the most front-loaded payment schedule which maintains incentive compatibility. It does not, however, constitute a rigorous proof of the main result. The proof requires the slightly more sophisticated, yet still quite simple, observation that a useful characterization of the class of incentive compatible contracts obtains by relating the underwriter's expected value for the contract under high and low effort via a linear approximation of the Radon-Nikodym derivative of the respective probability measures.

We start by giving a heuristic derivation of the optimal contract when $N = 2$. This base case provides the basic intuition we will use throughout our solution. Some payment must be contingent on τ_1 and τ_2 to provide incentives to exert effort. If all payment occurs regardless of the realization of τ_1 and τ_2 , then there is no incentive for the underwriter to exert effort. Specifically, the contract should reward the underwriter when τ_1 and τ_2 are relatively larger and punish the underwriter when τ_1 and τ_2 are relatively smaller since τ_1 and τ_2 are more likely to be large when the underwriter exerts effort. At the same time, the optimal contract should completely pay off the underwriter as quickly as possible to exploit the difference in discount rates of the investor and underwriter.

Given the intuition above, it is clear that the optimal contract will balance providing incentives with front loading payment. Observe that we can always take an arbitrary incentive compatible contract and move some payment that occurs later to an earlier date. To maintain incentive compatibility, we must then move some payment that occurs earlier and move it to a later date. Doing so repeatedly should move all payment to a single date strictly later than $t = 0$. It is not yet clear that this process will result in reducing the cost of the contract, but we use it as a starting point. To that end, we focus on contracts of the form $dX_t = 0$ for $t \neq t_0$ and $dX_{t_0} = \mathbb{I}((\tau_1, \tau_2) \in \mathcal{A})y_0$ for some $t_0 \geq 0$ and set of events \mathcal{A} chosen such that X_t is \mathcal{F}_t -measurable.⁷ We further suppose that both the participation and incentive compatibility constraints bind.⁸ We then have the following two equations

$$e^{-\gamma t_0} y_0 P((\tau_1, \tau_2) \in \mathcal{A} | e = 1) = a_0 + C, \quad (2.1)$$

$$e^{-\gamma t_0} y_0 P((\tau_1, \tau_2) \in \mathcal{A} | e = 0) = a_0, \quad (2.2)$$

where $P(\cdot | e)$ denotes probability conditional on effort. Such a contract will cost the investors

$$e^{-r t_0} y_0 P((\tau_1, \tau_2) \in \mathcal{A} | e = 1) = e^{-(r-\gamma)t_0} (a_0 + C), \quad (2.3)$$

⁷ $\mathbb{I}(\cdot)$ is the indicator function.

⁸In fact, the participation constraint may be slack at the optimal contract. However for the purpose of this informal derivation the assumption that the participation constraint binds will turn out to be innocuous.

which is increasing in t_0 and independent of \mathcal{A} . This implies that the optimal contract of this form will choose the set \mathcal{A} to minimize t_0 subject to satisfying equations (2.1) and (2.2). To do so, we choose \mathcal{A} such that $\mathbb{I}((\tau_1, \tau_2) \in \mathcal{A})$ depends *only* on τ_1 since any dependence on τ_2 will require an increase in t_0 to maintain incentive compatibility. We will return to this point after stating the optimal contract. Moreover, we guess that \mathcal{A} should take on as simple structure as possible. So let $\mathcal{A} = \{\tau_1 \geq t_0\}$, then equations (2.1) and (2.2) reduce to the following

$$e^{-(\gamma+2\lambda_L)t_0}y_0 = a_0 + C, \quad (2.4)$$

$$e^{-(\gamma+2\lambda_H)t_0}y_0 = a_0. \quad (2.5)$$

We can easily solve (2.4) and (2.5) for t_0 and y_0 . We formally state the optimal contract (which provides incentives for effort) in the following proposition.

Proposition 1. *Let*

$$\hat{a}_0 = \max \left\{ a_0, \frac{C(\gamma - r)}{N(\lambda_H - \lambda_L)} \right\} \quad (2.6)$$

An optimal contract X_t that implements high effort is given by

$$dX_t = \begin{cases} 0 & \text{if } t \neq t_0, \\ y_0 \mathbb{I}(\tau_1 \geq t_0) & \text{if } t = t_0, \end{cases} \quad (2.7)$$

where

$$t_0 = \frac{1}{N(\lambda_H - \lambda_L)} \log \left(\frac{\hat{a}_0 + C}{\hat{a}_0} \right), \quad (2.8)$$

$$y_0 = \left(\frac{\hat{a}_0 + C}{\hat{a}_0} \right)^{\frac{\gamma + N\lambda_L}{N(\lambda_H - \lambda_L)}} (\hat{a}_0 + C). \quad (2.9)$$

Moreover

$$b(a_0) = (\hat{a}_0 + C) \left(\frac{\hat{a}_0 + C}{\hat{a}_0} \right)^{\frac{\gamma - r}{N(\lambda_H - \lambda_L)}}. \quad (2.10)$$

The contract calls for no transfers from the investors to the underwriter to take place until the time t_0 given by equation (2.8). If the first default time $\tau_1 \geq t_0$ then the contract calls for a payment of y_0 given by equation (2.9) at time t_0 . Equation (2.8) is the product of two terms. The first term is the inverse of the difference between the arrival intensity of τ_1 given low effort and the arrival intensity of t_0 given high effort. The second term is the difference in the logs of the present value (gross of the cost of effort) of the contract from high effort and low effort respectively. Thus, t_0 is set so that the expected present value of a transfer of y_0 at t_0 under the underwriter's discount rate is exactly equal to $\hat{a}_0 + C$ under high effort, and \hat{a}_0 under low effort.

The cost of the contract to the investors is given by $b(a_0)$ in equation (2.10) and is the product of two terms. The first term is the expected present value of transfers under the

discount rate of the underwriter. The second term adjusts this expected present value to the discount rate of the investors. Note that when $a_0 < \hat{a}_0$, the optimal contract delivers a payoff to the investor greater than necessary to satisfy the participation constraint. This is because for small a_0 , the time required to make the participation constraint bind, while using a contract of the proposed form, would be very long, and thus destroy some social surplus.

The intuition behind Proposition 1 lies in the tradeoff between two forces: the *cost of waiting effect*, and the *wealth transfer effect*. On the one hand, the cost of waiting effect arises because delaying payment is costly, in fact reduces total surplus, due to the difference in the discount rates of the underwriter and investors. On the other hand, the incentive compatible constraint implies a minimum sensitivity of contracted payments to mortgage performance which is greater when the contract conditions solely on more noisy, i.e. earlier, signals. At the same time, the limited liability constraint places a lower bound on these payments. This interaction leads to the wealth transfer effect: contracts that condition solely on early information imply the underwriter must receive a payoff greater than necessary to satisfy her participation constraint. The optimal contract then balances the cost of waiting effect with the wealth transfer effect. This trade-off results in a type of “cutoff” rule similar to that of the optimal contract in Bond and Gomes (2009): if the performance of the mortgages passes some threshold then the underwriter is compensated, otherwise the underwriter is punished.

The strategy of the proof of Proposition 1 is to find a weaker condition than (IC) which is more convenient to work with. We then show the proposed contract is optimal over the space of contracts satisfying this weaker condition and verify that it satisfies (IC) and (PC). Note that the underwriter’s expected value of the contract under low effort is related to the expected underwriter value of the contract under high effort via a change of measure. If we let \mathbb{Q} be the measure induced by low effort and \mathbb{P} be the measure induced by high effort, then an application of the monotone convergence theorem leads to the following equality:

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] = E \left[\int_0^\infty e^{-\gamma t} \pi(t) dX_t | e = 1 \right], \quad (2.11)$$

where $\tilde{\pi}_t = \frac{d\mathbb{Q}}{d\mathbb{P}}$ is the Radon-Nikodym derivative of \mathbb{Q} with respect to \mathbb{P} .⁹ Such a change of measure allows one to write the investors problem entirely in terms of conditional expectations with respect to $e = 1$. Unfortunately, direct calculation of $\pi(t)$ is not possible, however we do have the following inequality for an arbitrary incentive compatible contract

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] \geq E \left[\int_0^\infty e^{-\gamma t} e^{-N(\lambda_H - \lambda_L)t} dX_t | e = 1 \right] \quad (2.12)$$

$$\geq \frac{a_0}{a_0 + C} E \left[\int_0^\infty (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dX_t | e = 1 \right]. \quad (2.13)$$

⁹This is a slight abuse of notation, but the reader familiar with change of measure can think of π_t as the restriction of $\frac{d\mathbb{Q}}{d\mathbb{P}}$ to \mathcal{F}_t the filtration generated by D_t .

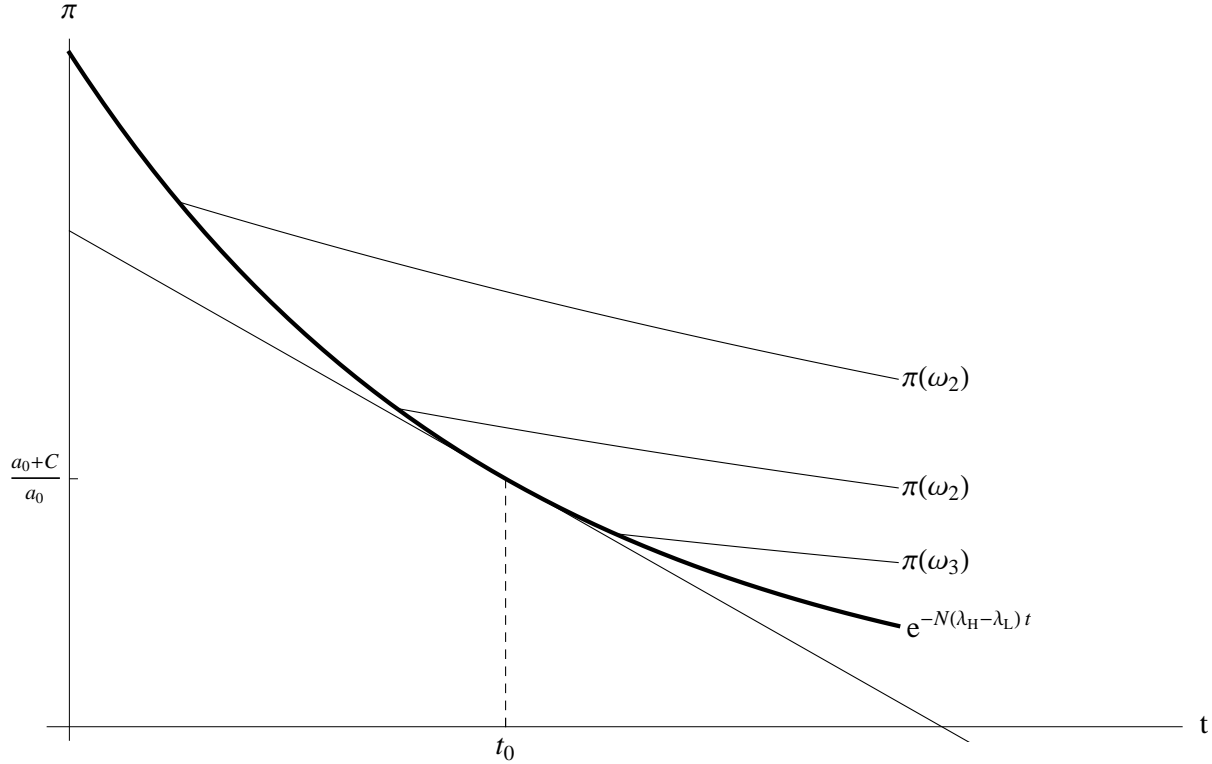


Figure 2.1: A plot of realizations of the Radon-Nikodym derivative $\pi(t)$ for different sample paths ω_1 , ω_2 , and ω_3 . The thick curve is the lower bound on the Radon-Nikodym for all possible sample paths. The straight line is a linear approximation to the lower bound. Since the lower bound is convex, it dominates the straight line and we can use the straight line to find an inequality relating the expectations of underwriter value under high and low effort. This inequality is a useful characterization of the set of incentive compatible contracts.

Inequality (2.12) arises from approximating $\pi(t)$ and is verified in the appendix. Inequality (2.13) arises from the fact that $e^{-N(\lambda_H - \lambda_L)t}$ is convex in t and hence can be bounded below by a linear function of t . Figure 2.1 gives graphical intuition of the argument that yields this inequality.

Inequality (2.13) allows us to approximate the incentive compatibility constraint in terms of conditional expectations with respect to $e = 1$. For the purpose of exposition, assume the participation constraint (PC) binds. This allows us to combine the incentive compatibility constraint (IC) and participation constraint (PC) (which binds) to get the following useful sufficient condition for an arbitrary contract to be incentive compatible

$$\frac{1}{a_0 + C} E \left[\int_0^\infty t e^{-\gamma t} dX_t | e = 1 \right] \geq t_0. \quad (2.14)$$

Inequality (2.14) shows that the minimum duration of any incentive compatible contract is exactly t_0 , which turns out to be the duration of the optimal contract. Hence, the proof of

Proposition 1 shows that optimal contracting problem we consider comes down to a duration minimization problem.

In order to find the minimum-duration contract we note that we can always improve on an arbitrary incentive compatible contract by delaying payment that occurs before t_0 and accelerating payment that occurs after t_0 until all payment occurs at t_0 . Doing so reduces the duration of the contract. Eventually we will have all payment occurring at time t_0 . The resulting contract is the optimal contract stated in Proposition 1.

To understand why the optimal contract only uses the information revealed by time t_0 as opposed to waiting to gather more information, it is useful to think of the investors' problem as a standard hypothesis testing problem in which there is a trade off between test power and the period of observation required to perform the test. At each point in time, the investors essentially test the null hypothesis H_0 : the underwriter chose $e = 1$ versus the alternative hypothesis H_1 : the underwriter chose $e = 0$. They then pay the underwriter based on the outcome of the test. However, they must choose the tests and payments to provide incentives to the underwriter while maintaining limited liability in the least costly manner.

For the purpose of exposition, let us consider the following alternative to the optimal contract

$$dX_t = \tilde{y}_0 \mathbb{I}(\text{accept } H_0 \text{ given } \{D_s\}_{s \leq \tilde{t}_0}, t = \tilde{t}_0),$$

where H_0 is accepted if $\{D_s\}_{s \leq \tilde{t}_0} \in \mathcal{A}$ for some $\mathcal{A} \subset \mathcal{F}_{\tilde{t}_0}$. Incentive compatibility implies that this contract must correspond to a likelihood ratio test which accepts the null hypothesis if

$$\frac{P(\{D_s\}_{s \leq \tilde{t}_0} \in \mathcal{A} | e = 1)}{P(\{D_s\}_{s \leq \tilde{t}_0} \in \mathcal{A} | e = 0)} = \frac{a_0 + c}{a_0}. \quad (2.15)$$

Equation (2.15) illustrates a bit of classic principal-agent intuition in the optimal contracting problem we consider. Incentive compatibility imposes a trade-off between the significance level and power of the likelihood ratio test used by the investors to determine payments to the underwriter. This implies that if the investors use a more powerful test than the test employed in the optimal contract, for instance a test which uses more information than the first default time, then the test must reject the null hypothesis at a lower significance level than the optimal contract. In other words, if the investors wish to condition payments to the underwriter on more information than used by the optimal contract, then the contract must be less strict in order to satisfy incentive compatibility. Accordingly, such a test will have to use a longer period of observation than the optimal contract, in other words $\tilde{t}_0 \geq t_0$. Since part of the surplus in the model arises from front loading payments to the underwriter, any alternative of the above form will be suboptimal. Hence, by adding a time dimension to the way information is revealed in the model, we emphasize the trade off between the power of the test employed in the contract, and the amount of observation time needed to perform the test and preserve incentive compatibility.

In our model, the trade-off between the cost of waiting and the benefit of waiting (better information quality) is driven by the limited liability of the underwriter. One could

implement high effort with contract with duration less than t_0 , that is by using a test which takes less time to implement than the optimal contract. However, using such a test requires the test be less powerful. Moreover, the most the agent can be punished for a rejection of the null hypothesis is a zero payoff. This in turn implies that a less powerful test must pay the underwriter more in the event that the null is not rejected in order to maintain incentive compatibility. Thus, such a contract will feature a slack participation constraint and will be suboptimal.

The contract given in Proposition 1 is slightly stark in that it severely punishes the underwriter if even one mortgage defaults prior to time t_0 . However, an alternative interpretation of Proposition 1 is that it provides an upper bound on the efficiency of an arbitrary incentive compatible contract. In this sense, the result is very useful for evaluating some standard alternative contracts, an exercise we take up in Section 2.2.

2.1.4 The benefits of pooling

One important feature of MBS is the process of “pooling.” In this process, an issuer of an MBS bundles together many mortgages to form a collateral pool. This security design contrasts with individual loan sale in which an issuer simply sells each loan separately. Individual loan sale means that the transfers corresponding to the sale of one loan cannot affect the transfers corresponding to the sale of another. Hence, in the context of our model, individual loan sales correspond to a contract which is the sum of N individual contracts, each of which depends on only one mortgage. Let W_t denote a contract which calls for individual mortgage sale, then

$$dW_t = \sum_{n=1}^N dX_t^n,$$

where X_t^n is the payment made to the underwriter for the sale of an *individual* mortgage. Since each mortgage payoff is independent and identically distributed after the underwriter chooses effort, it is natural to only consider individual loan sale contracts, which imply W_t is measurable with respect to the filtration generated by D_t . Thus, individual loan sale contracts are contained in the contract space we consider in the derivation of the optimal contract. This fact leads us to state the following important corollary to Proposition 1.

Corollary 1. *Pooling mortgages is more efficient than individual loan sale. Specifically, individual loan sale contracts are more costly to provide than the contract of Proposition 1.*

Notice that Corollary 1 does not depend on the number of mortgages, N , in the pool. Moreover, the benefits of pooling do not arise from risk diversification benefits. Other results in the literature, for example DeMarzo (2005), who considers an informed issuer selling multiple assets, attribute the benefits of pooling to the so called *risk diversification effect*.¹⁰ In his model, if some portion of the payoff from assets is unrelated to the private

¹⁰In Diamond (1984) a financial intermediary benefits from lending to multiple entrepreneurs due to the risk diversification effect.

information of the issuer, then under mild assumptions on the distribution of this residual risk, the issuer can create a security with less risky payoffs than the pure pass through pool, in other words a senior “tranche.” The issuer can then signal her private information by retaining a portion of the residual payoffs.

In our model, pooling is a consequence of providing incentives in the least costly manner. Returning to the intuition gained by viewing the contracting problem as a hypothesis testing problem, pooling in our model trades off the time it takes to implement the test with the loss in power from ignoring all defaults that occur after the first default. We call the decreased time required to implement the test the *information enhancement effect* of pooling. A similar concept is present, although in a static setting, in Laux (2001), in which a manager’s limited liability constraint can be relaxed by creating a contract that is contingent on multiple outcomes. The main difference between those results and our result is the channel by which pooling outcomes decreases the shadow price of limited liability. In our model the key is reducing the time necessary to implement a given test as opposed to minimizing the total punishment necessary to provide incentives.

The information enhancement effect in our setting also bears resemblance to the statistical inference effect of Abreu, Milgrom, and Pearce (1991). In that paper, increasing lags between initial actions and eventual outcomes increases the amount of information available to perform inference.

2.2 Standard contracts and the approximate optimality of the “first loss piece”

An interesting question to ask is how closely we can approximate the optimal contract using an alternative, and perhaps more standard, contract. To answer this question, we compare the optimal contract to two possible alternative contracts, one in which the underwriter retains a pure fraction of the mortgage pool, and one in which the underwriter retains a fraction of a “first loss piece.”

2.2.1 The optimal contract versus a fraction of the mortgage pool

First we consider contracts in which the underwriter retains an fraction of the pool of mortgages and receives a lump sum transfer at time $t = 0$. Note that the total cash flow from the pool of mortgages at time t is $u(N - D_t)dt + RdD_t$. Hence a contract which calls for the underwriter to receive a time zero cash payment of K and retain a fraction α of the pool of mortgages must take the following form

$$dX_t = \begin{cases} K & t = 0 \\ \alpha(u(N - D_t)dt + RdD_t) & t \geq 0 \end{cases} . \quad (2.16)$$

It will also be useful to compute the expected present value of the contract under the underwriter's discount rate given high effort and low effort.

$$\begin{aligned} E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] &= K + N\alpha \frac{u + \lambda_L R}{\gamma + \lambda_L} \\ E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] &= K + N\alpha \frac{u + \lambda_H R}{\gamma + \lambda_H} \end{aligned}$$

An optimal contract of the form given in (2.16) must make both the participation constraint and the incentive compatibility constraint bind. Hence we have the following system of equations

$$\begin{aligned} a_0 + C &= K + \alpha N \frac{u + \lambda_L R}{\gamma + \lambda_L} \\ a_0 &= K + \alpha N \frac{u + \lambda_H R}{\gamma + \lambda_H}. \end{aligned}$$

Which we can solve to get

$$\begin{aligned} \alpha &= \frac{C(\gamma + \lambda_H)(\gamma + \lambda_L)}{N(u - \gamma R)(\lambda_H - \lambda_L)} \\ K &= a_0 - \frac{C(u + \lambda_H R)(\gamma + \lambda_L)}{(u - \gamma R)(\lambda_H - \lambda_L)} \end{aligned}$$

The cost of providing such a contract, which we denote by $b^E(a_0)$, will then be

$$\begin{aligned} b^E(a_0) &= K + \alpha N \frac{u + \lambda_L R}{r + \lambda_L} \\ &= a_0 + C + \left(\frac{C(\gamma + \lambda_H)(\gamma - r)}{(u - \gamma R)(\lambda_H - \lambda_L)} \right) \left(\frac{u + \lambda_L R}{r + \lambda_L} \right) \end{aligned}$$

We assume that investors earn zero profits and calculate the implied promised value to the underwriter to get

$$a_0^E(N) = \left(N - \frac{C(\gamma + \lambda_H)(\gamma - r)}{(u - \gamma R)(\lambda_H - \lambda_L)} \right) \left(\frac{u + \lambda_L R}{r + \lambda_L} \right) - C.$$

Figure 2.2 shows the percentage gain in underwriter value for a range of values of the parameters λ_H and γ holding the other parameters fixed. The efficiency gain from using the optimal contract over the alternative contract increases as either γ increases or λ_H decreases. As γ increases, the gains from securitization increase, and hence any improvement in the efficiency of the structure of securitization will have a larger effect on efficiency. As λ_H decreases, the severity of the moral hazard problem increases since we assume the cost of effort stays constant. That is, effort becomes harder to infer since λ_H and λ_L are closer in

Comparing the optimal contract to a fraction of the mortgage pool

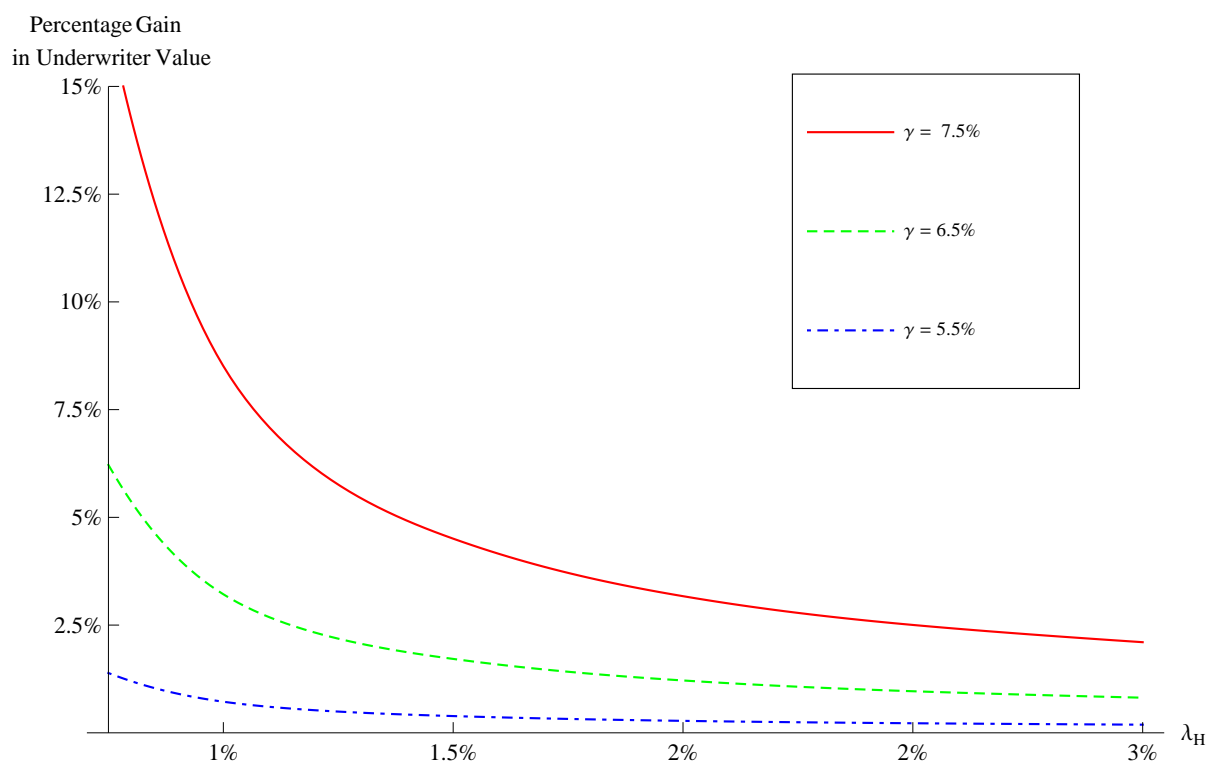


Figure 2.2: The percentage gain in underwriter value from using the optimal contract versus a contract in which the underwriter retains a fraction of the pool of mortgages. Parameter values : $r = 5\%$, $\lambda_L = .5\%$, $N = 100$, $R = (50\%) \frac{u}{r}$ and $c = (.25\%) \frac{Nu}{r}$

value, however the cost of effort stays the same, hence shirking becomes more attractive. As such, harsher incentives are needed to implement high effort and the optimal contract delivers larger efficiency gains over the alternative contract. The overall level of gains is high indicating that the equity-like contract is very inefficient. In fact, for some parameter values there does not exist an equity-like contract that yields positive profits to the investors and provides incentives for effort.

2.2.2 The optimal contract versus a “first loss piece”

The next alternative contract we consider is the so called “first loss piece.” In the most simple form of this structure, the mortgages are pooled and two tranches are sold to investors, a senior tranche, and a junior tranche or first loss piece. The underwriter retains a large enough junior tranche to maintain incentive compatibility. To define this contract we let Y_t and Z_t be the cumulative cash flow paid to the senior and junior tranches respectively by time t . The cash flow from the mortgages is distributed to the tranches according to the following rules

$$dY_t = (N - \max\{n, D_t\})udt + RdD_t \quad (2.17)$$

$$dZ_t = \max\{n - D_t, 0\}udt \quad (2.18)$$

for some $n < N$ which determines the size of the junior tranche.¹¹ We can design an incentive compatible contract using the above tranche structure as follows. The underwriter retains the junior tranche as well as receives the proceeds from the sale of the senior tranche. We call such contracts first loss piece contracts.

We state the value of the first loss piece and the optimal first loss piece contract in the following proposition

Proposition 2. *The value of the first loss piece of size n given the arbitrary discount factor δ and default intensity Λ is*

$$F(\delta, \Lambda, n) = \frac{u}{\delta} \left(n - \sum_{k=1}^n \frac{N! \Lambda^k}{(N-k)! \prod_{m=0}^{k-1} (\delta + (N-m)\Lambda)} \right) \quad (2.19)$$

The optimal first loss piece contract is given by n such that

$$n = \arg \min_m \{m : F(\gamma, \lambda_L, m) - F(\gamma, \lambda_H, m) \geq C\} \quad (2.20)$$

Proof. See Appendix. □

¹¹The reader may note that the first loss piece we have defined does not have claim to the recovery values of the first n defaulted mortgages. In reality, the first loss piece may indeed have such a claim. In our setting, paying the recovery value to the first loss piece would reduce its efficiency, though not substantially.

The value of the first loss piece given by equation (2.19) appears quite complicated but yields a simple decomposition. Consider a claim to a constant cash flow of udt until the k th default. We can express the underwriter's value for this claim as

$$E \left[\int_0^{\tau_k} ue^{-\gamma t} dt | e = i \right] = \frac{u}{\gamma} (1 - \delta_k)$$

where δ_k is some discount determined by the distribution of τ_k . We can directly calculate δ_k using the distribution of the order statistics of an independent identically distributed sample of exponential random variables with intensity λ_i . Summing the value of such claims for $k = 1$ to $k = n$ we are left with equation (2.19).

The intuition behind the optimality of the contract in Proposition 2 is the following. As the size of the first loss piece increases, the expected life of the first loss piece increases and hence the contract becomes less efficient. So the optimal contract features the smallest first loss piece such that the contract provides incentives to exert effort. We could improve on this contract by allowing the underwriter to sell a portion of the first loss piece to the investors, however such an improvement would be insubstantial for reasonable parameters.

Again, we can assume the investors are competitive and hence make zero profits. Figure 2.3 shows the gain in underwriter value from using the optimal contract versus the first loss piece structure. For parameter values with small gains from securitization and a small moral hazard problem, the first loss piece is a very good alternative to the optimal contract. This fact is robust to various pool sizes N . However, when γ is large or λ_H is small, the gain from using the optimal contract is substantial, for example more 12 basis points when $\gamma = 7.5$ and $\lambda_H = .525\%$. It should be noted that the agency problem must be very severe to generate a substantial gain in efficiency from using the optimal contract versus the first loss piece contract.

In light of Figure 2.3, we can conclude that in many instances the first loss piece is a very efficient incentive contract. The main reason for this conclusion is that the first loss piece, as presented above, accelerates payments relative to other contracts, such as the contract we consider in the previous section.

2.3 Extensions

2.3.1 An Initial Capital Constraint

In this section we consider the optimal contracting problem when the underwriter faces an initial capital constraint. This case arises when the underwriter does not have sufficient internal capital to originate the mortgages. The structure of the contract remains largely unchanged, except for the addition of a transfer at $t = 0$.

Suppose the underwriter requires K in initial capital at $t = 0$ to originate the mortgages. Specifically we add the following constraint

$$dX_0 \geq K \tag{2.21}$$

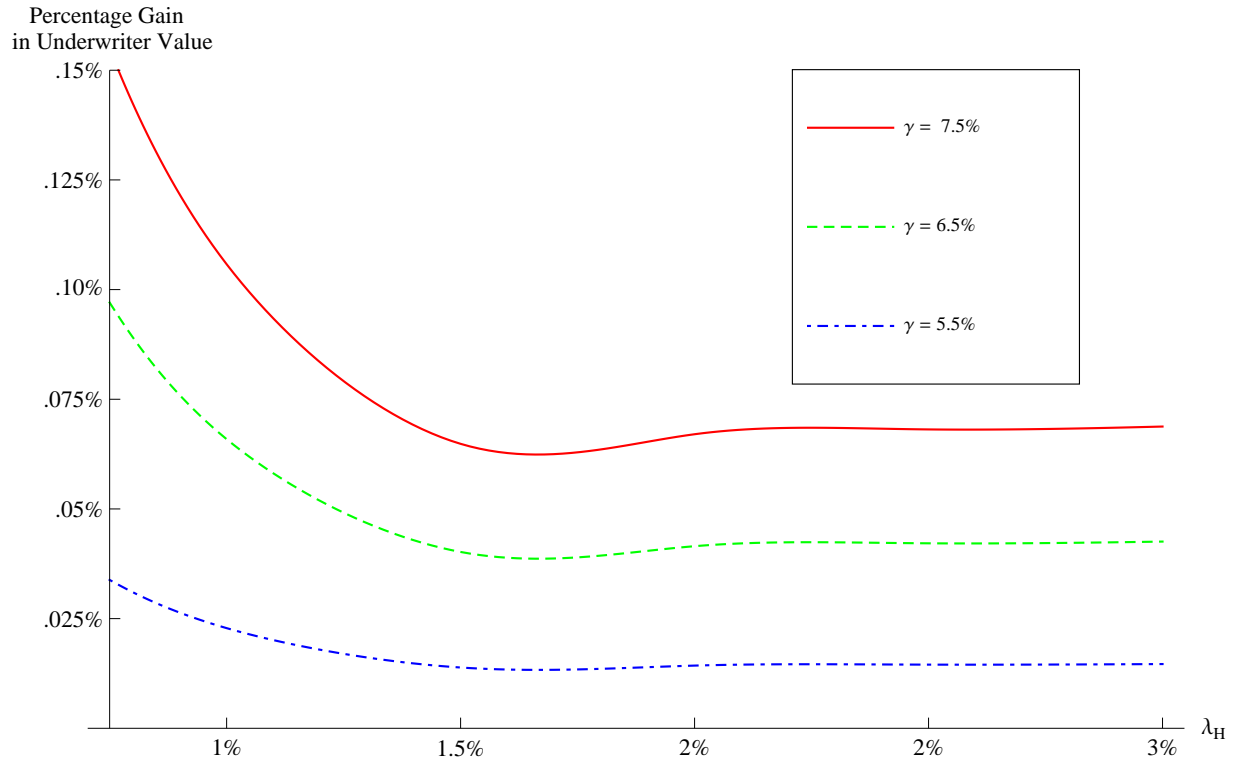


Figure 2.3: The percentage gain in underwriter value from using the optimal contract versus a contract in which the underwriter retains a fraction of the first loss piece. The slight non-monotonicity arises due to the fact that as $\lambda_H - \lambda_L$ increases, the size of the optimal first loss piece increase by increments of 1 rather than continuously. Thus as $\lambda_H - \lambda_L$ increases the loss in efficiency due to the fact that the first loss piece contract does not make the incentive compatibility constraint bind changes non-monotonically. Parameter values : $r = 5\%$, $\lambda_L = .5\%$, $N = 100$, $R = (50\%) \frac{u}{r}$ and $c = (.25\%) \frac{Nu}{r}$

to Definition 1. The following proposition states the solution to the optimal contracting problem.

Proposition 3. *Let a_0 be the promised value to the underwriter net of origination costs K . The optimal contract X_t that satisfies constraint (2.21) is given by $dX_0 = K$ and Proposition 1 for $t > 0$.*

Proof. See appendix □

The intuition behind Proposition 3 is essentially the same as for Proposition 1. After providing the initial required capital K , the contract makes all subsequent transfers dependent on the realization of the first default time to provide incentives to the underwriter to exert effort. Once a long enough period has passed before the first default time, it is optimal to completely pay off the underwriter due to the difference in discount rates between the investors and the underwriter.

2.3.2 Maturity of the optimal contract

One attractive feature of the optimal contract is its relatively short maturity. Even for high values of K (the upfront capital required to originate the mortgages) the waiting period required to provide incentives to the underwriter is short. Figure 2.4 shows t_0 as a function of λ_H for various levels of K holding other parameters constant. When λ_H is relatively close to λ_L and K is relatively large, the moral hazard problem is relatively more severe, since the inference problem is more difficult and the continuation value of the underwriter is smaller. However, even for $K = 99.5 \frac{N(u+\lambda_L R)}{r+\lambda_L}$ and $\lambda_H - \lambda_L = .5\%$, the maturity of the optimal contract is less than 2 years. When contrasted to the potentially infinitely lived mortgages of the model, this contract maturity is very short. This feature is appealing in practice since it implies that even when facing severe moral hazard problems, investors in MBS can enforce underwriter effort with a short-lived contract. In other words, even though a mortgage pool may last for 30 years, the underwriters position can be short-lived while still providing incentives to exert effort.

2.3.3 Partial Effort

The underwriter is endowed with an all-or-nothing effort technology in our model. In other words, the agent must choose a single effort level for each mortgage. Alternatively, we can consider a specification in which the underwriter can apply effort to some and not all of the mortgages. The optimal contract remains unchanged when we allow for such a deviation given a reasonable restriction on the cost of effort c detailed below. Specifically, suppose the underwriter can choose to apply effort to $n \leq N$ of the mortgages resulting in a pool of N mortgages in which n mortgages default according to an exponential distribution with parameter λ_L , and $N - n$ mortgages default according to an exponential distribution with parameter λ_H . We will refer to such a strategy as applying partial effort. We alter notation

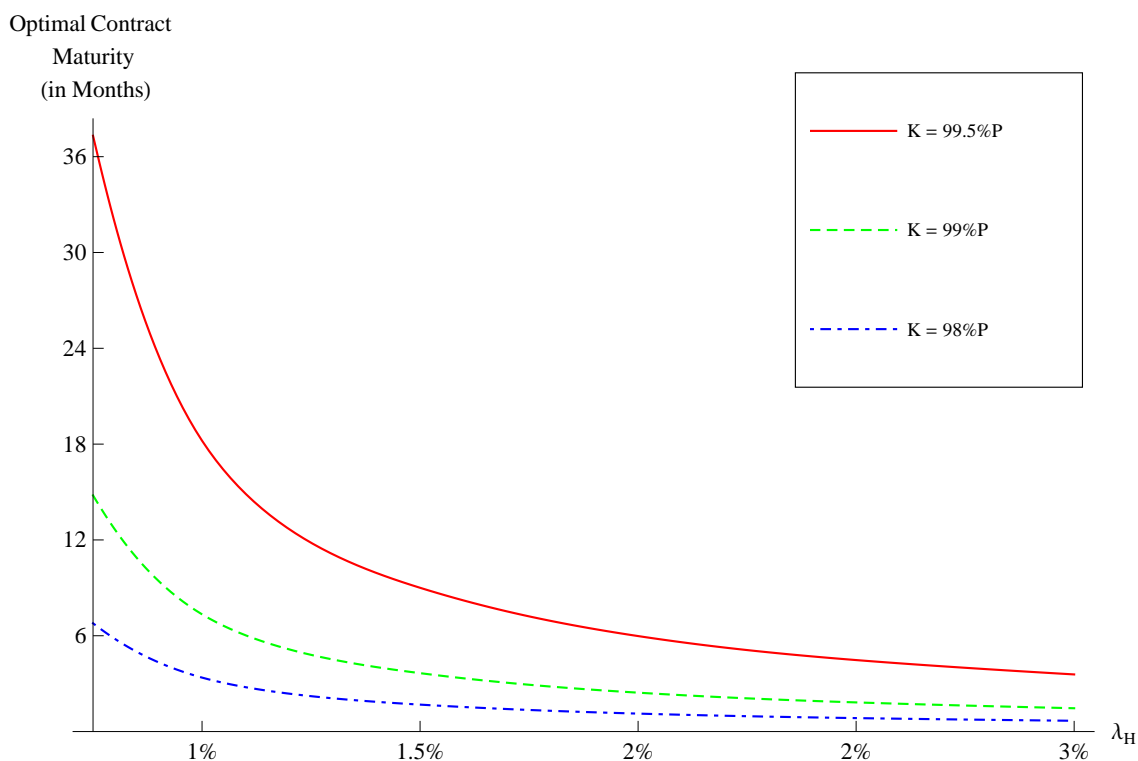


Figure 2.4: The maturity of the optimal contract t_0 for a range of parameters, where K is reported as a percentage price of the market value of the mortgage pool $P = \frac{N(u+\lambda_L R)}{r+\lambda_L}$. Parameter values $r = 5\%$, $\gamma = 6\%$, $\lambda_L = .5\%$, $N = 100$, $R = (50\%) \frac{u}{r}$ and $c = (.25\%) \frac{N u}{r}$.

to let a partial effort strategy be denoted $e = n$ so that $e = 0$ corresponds to zero effort and $e = N$ corresponds to full effort. The cost of applying a partial effort strategy is given by $c \cdot e$. We modify Definition 1 to include incentive compatibility constraints for each possible partial effort strategy as follows

Definition 2. Given a promised utility a_0 to the underwriter, a contract $\{x_n\}_{n=0}^N$ that implements $e = N$ is *optimal* if it solves the following problem

$$b(a_0) = \min_{dX_t \geq 0} E \left[\int_0^\infty e^{-\gamma t} dX_t | e = n \right] \quad (2.22)$$

such that

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e = m \right] - c \cdot m \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = N \right] - c \cdot N \quad \text{for all } m \quad (2.23)$$

and

$$a_0 \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = N \right] - c \cdot N. \quad (2.24)$$

Proposition 4. *The optimal contract is the same as that of Proposition 1.*

Proof. See appendix □

Proposition 4 relies on the fact that under the contract detailed in Proposition 1 and the flexible effort technology, the underwriter does not gain more by deviating to a strategy in which she applies effort to some and not all of the mortgages than a strategy in which she applies zero effort. This implies that the original optimal contract satisfies the additional incentive compatibility constraints ruling out partial effort deviations since it satisfies the original incentive compatibility constraint ruling out the zero effort strategy. Moreover, the space of contracts that satisfy (2.23) is contained in the set of contracts which satisfy (IC) since constraint (IC) is contained in (2.23). Since the contract of Proposition 1 satisfies the additional constraints induced by partial effort strategies and is optimal over the larger space of contracts satisfying constraint (2.23), it is optimal over the smaller space of contracts satisfying the additional constraints.

2.3.4 Adverse selection

Throughout the above analysis, we have focused on a moral hazard setting in which the underwriter makes a hidden effort choice that affects the risk of the mortgages she sells to the investors. In this section, we show how our model can be altered to address an adverse selection problem in which the underwriter is endowed with mortgages with a given default risk and wishes to sell them to secondary market investors. This setting is similar to other papers in the literature, notably DeMarzo (2005). Our main result is that the optimal contract for an underwriter with low risk mortgages remains qualitatively unchanged.

In a standard adverse selection model of asset backed security design, the issuer, in our case the underwriter, has private information about the assets she wishes to sell. We model this by assuming that the underwriter has N mortgages to sell, all of which are either low risk or high risk where mortgage cash flows are given in section 2.1.1. We look for a separating equilibrium in which the underwriter can signal the quality of her mortgages by choosing a contract. Specifically, we look for a pair of contracts (X^H, X^L) such that an underwriter with high risk mortgages chooses the contract X^H , and an underwriter with low risk mortgages chooses the contract X^L . As such, we can view the choice of contract as a signal of pool quality. Formally, we define a *separating equilibrium* as follows.

Definition 3. A separating equilibrium is a pair (X^H, X^L) such that:

1. The underwriter chooses X^H when she has λ_H type mortgages:

$$X^H \in \arg \max_X E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda_H \right] \quad (2.25)$$

2. The underwriter chooses X^L when she has λ_L type mortgages:

$$X^L \in \arg \max_X E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda_L \right]. \quad (2.26)$$

3. Investors earn zero expected profits:

$$E \left[\int_0^\infty e^{-rt} dX_t^H | X^H \right] = \frac{N(u + R\lambda_H)}{r + \lambda_H}, \quad (2.27)$$

$$E \left[\int_0^\infty e^{-rt} dX_t^L | X^L \right] = \frac{N(u + R\lambda_L)}{r + \lambda_L}. \quad (2.28)$$

Since all gains from securitization arise by front loading payment to the underwriter, and investors are competitive and earn zero profits, we choose

$$dX_t^H = \begin{cases} \frac{N(u + \lambda_H)}{r + \lambda_H} & \text{for } t = 0 \\ 0 & \text{otherwise} \end{cases}. \quad (2.29)$$

Of course separating equilibria with different X^H may exist, however, such equilibria will be Pareto dominated by equilibria with X^H described by equation (2.29).

In its current form, Definition 3 is not directly related to the contracting problem given in Definition 1. However we can draw a relation between the two definitions by finding the *least cost separating equilibrium*, which is the equilibrium which satisfies Definition 3 and maximizes the expected payment to the underwriter when she has type λ_L mortgages. The

least cost separating equilibrium thus solves the following problem:

$$\begin{aligned} & \max_X E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda_L \right] \\ \text{s.t. } & E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda_H \right] \leq \frac{N(u + \lambda_H)}{r + \lambda_H}, \\ & E \left[\int_0^\infty e^{-rt} dX_t | \lambda_L \right] \leq \frac{N(u + \lambda_L)}{r + \lambda_L}. \end{aligned} \quad (2.30)$$

Problem (2.30) is the dual problem of (I) and thus we can apply Proposition 1 to get the following proposition.

Proposition 5. *The least cost separating equilibrium is given by $dX_0^H = N \frac{u + \lambda_H R}{r + \lambda_H} = a_0^H$ and $dX_t^H = 0$ for $t > 0$, and*

$$dX_t^L = \begin{cases} 0 & \text{if } t \neq t_0, \\ y_0 \mathbb{I}(t_0 \leq \tau_1) & \text{if } t = t_0, \end{cases}$$

where

$$\begin{aligned} t_0 &= \frac{1}{N(\lambda_H - \lambda_L)} \frac{a_0^L}{a_0^H} \\ y_0 &= e^{(\gamma + N\lambda_L)t_0} a_0^L, \end{aligned}$$

where

$$a_0^L = \left(N \frac{u + \lambda_H R}{r + \lambda_H} \right)^{1/(1+\eta)} (a_0^H)^{\eta/(1+\eta)} \quad (2.31)$$

and

$$\eta = \frac{\gamma - r}{N(\lambda_H - \lambda_L)}. \quad (2.32)$$

Proof. Follows directly from Proposition 1. □

In a static framework, such as that of DeMarzo (2005), the signal space is limited to the fraction the underwriter chooses to retain of some security backed by the mortgages. In our setting, the signal space is much richer since it includes any payment profile through time that is adapted to the information filtration generated by the cumulative default process of mortgages. Accordingly, the most efficient signal in our model uses timing as a central feature rather than the fraction of some high risk tranche retained by the underwriter.

2.4 Conclusion

This paper studies a model of mortgage securitization in a moral hazard setting that highlights an important aspect of contracting in mortgage markets, namely that information

is revealed over time. We find that the optimal contract is a lump sum payment from the investors to the underwriter conditional on a period of no defaults. In addition, we evaluate two alternative contracts and find that a “first loss piece” style contract comes very close to achieving the same level of efficiency as the optimal contract.

If we view the contracting problem as essentially a hypothesis testing problem, a natural trade-off arises between the power of the test used in the contract and the amount of time needed to implement the test. This intuition gives rise to three new findings. First, the timing of payments to the underwriter is a key mechanism providing incentive to the mortgage underwriter to exert effort. Second, the optimal contract maturity can be short while the mortgages are long lived. Finally, mortgage pooling, the process whereby mortgages are bundled to create a collateral pool, follows from an *information enhancement effect*: by conditioning all payments on an aggregate signal taken from the entire pool of mortgages, rather than observing each mortgage individually, the optimal contract achieves the best possible trade-off between the testing power and the testing time.

One could raise a concern that the optimal contract provides incentives to both parties to manipulate the default process. For example the underwriter could have an incentive to postpone default times until she receives payment. Similarly, investors could have an incentive to bribe a single borrower to default early in order to avoid payments to the underwriter. One possible solution to this problem would be to use a third party servicer, thereby limiting direct access to borrowers rendering any such manipulation more costly. In addition, directly affecting default times through side payments would constitute fraud, making manipulation subject to significant legal risk. More importantly, we note that in the presence of moderate manipulation costs, the first loss piece, which closely approximates the optimal contract, does not create incentives for the investors or underwriter to manipulate mortgage defaults because a single mortgage default has only limited contractual consequences. In sum, although manipulation of the default process could be a concern, we believe it is not of first order importance since simple legal and practical institutions can prevent it with little loss of efficiency.

We consider three important extensions to the basic model. The main result is that the structure of the optimal contract is robust to altering the contracting problem in plausible ways. The first extension introduces an initial capital constraint, that is the underwriter lacks sufficient initial capital to originate the mortgages. The qualitative features of the contract remain unchanged after introducing this constraint. The only significant difference being a time zero transfer from the investors to the underwriter exactly equal to the capital required to originate the mortgages. After time zero, the optimal contract is identical to the optimal contract without the initial capital constraint except for the magnitude and timing of the one time lump sum transfer.

The next extension we consider is a flexible effort technology. It is possible that the underwriter could apply costly underwriting practices to some and not all of the mortgages. In this case, we identify a set of additional incentive compatibility constraints induced by the new effort technology. We then show, that the original optimal contract satisfies these new constraints and hence solves the optimal contracting problem of the more restricted space

of incentive compatible contracts.

In the third extension, we show how our result can be adapted to an adverse selection setting. In this setting, we view the choice of contract structure as a signal of the underwriter's private information. The problem then becomes finding the most efficient signal, and can easily be mapped into our original contracting problem.

The extensions we consider are certainly not exhaustive. For example, we could have considered risk averse underwriters or investors, correlation among mortgage defaults, time varying default rates, or a richer action set for the underwriter. These additional features would potentially change the specific lump sum form of the optimal contract and are all potential directions for future research. For example, time varying default rates may extend the duration of the optimal contract while a richer action set may provide insights into the equilibrium level of risk in mortgage pools by allowing for interior levels of effort to be optimal. However, the key economic forces of the model, specifically the relative impatience of the underwriter and the dynamic nature of information revelation, will remain unchanged and the optimal contract will balance front loading payments with providing incentives to exert effort.

Chapter 3

Reputation and signaling in asset sales

3.1 Introduction

In many important financial markets, issuer private information leads to a basic adverse selection problem. For example, an issuer of asset backed securities (ABS) may know more about underlying assets than do investors. In practice, such an issuer can reveal her private information either by retaining a fraction of the securities she issues (costly signaling) or by maintaining a reputation for honesty. Although there is a substantial literature investigating costly signaling and one investigating reputation, there is little work that considers the possible interactive effects of these two mechanisms. This paper presents a model of security issuance under asymmetric information that allows for both costly signaling and reputation effects. I consider the problem faced by an issuer selling assets to investors in a repeated game. In a given period, the issuer is endowed with a single asset. She perfectly observes the quality of this asset while potential investors do not. This asymmetry of information means that the issuer faces a lemons problem as in Akerlof (1970). The issuer may signal asset quality by retaining a portion of the asset. At the same time, the issuer may build a reputation for truthfully reporting asset quality through the performance of past assets.

Combining costly signaling and reputation admits three new findings. First, issuer retention can decrease with issuer reputation, indicating that costly signaling and reputation can act as substitutes. Second, the equilibrium relationship between retention and issuer reputation implies that a better reputation can decrease the probability that the issuer will truthfully reveal asset quality. Finally, equilibrium prices can be U-shaped in reputation. These results translate into an important empirical implication for ABS markets. The asset of the model can be thought of as the informationally sensitive portion of a securitization. Under this interpretation, the first result of the model implies that an ABS issuer with a worse track record will retain more of given issue. I test this implication of the model using data on subordination levels in the commercial mortgage backed securities (CMBS) market and find that issuers who have experienced a greater number of downgrades on past deals will retain more of the below investment grade principal of new deals.

The model is an infinite repetition of a securitization stage game. In each stage game, a risk neutral issuer is endowed with an asset to securitize for sale to investors. Nature chooses whether the asset is the good type or the bad type, where asset type denotes expected future cash flow. The asset yields cash flow one period after it is securitized. The cash flow distribution is intentionally simple in order to abstract from security design problems. There is a competitive market in which relatively patient risk neutral investors will buy a fraction of the asset. The issuer can perfectly observe the type of the asset, however this information is hidden from investors and non-verifiable at the time of securitization. The issuer may report or misreport the type of the asset to the investors in a prospectus. In addition, the issuer can signal asset type by retaining a fraction of the asset. Because the issuer has a higher discount rate than investors, a common assumption in the literature, such a signal is costly and hence credible.

Reputation concerns arise due to asymmetric information over *issuer* preferences for honesty. Specifically, the issuer could be of two possible types. The honest type issuer is committed to truthfully reporting the asset type in the prospectus. In contrast, the opportunistic type issuer will choose her reporting strategy by maximizing her payoffs. Both types optimally choose a fraction to retain. The issuer's reputation is the probability the investors place on the issuer being the honest type. By mimicking an honest issuer, i.e. truthfully reporting asset type in the prospectus, an opportunistic issuer can improve her reputation and thereby reduce the lemons discount on the fraction of the asset sold to investors.

Issuer type in the model can be viewed as a proxy for the issuer's preferences over accuracy. For example, the issuer may have separate lines of business that depend on reputation in an opaque fashion. This would be the case for an investment bank with many lines of business, all of which depend on its reputation for accuracy. Investors in one type of product issued by this bank, say CMBS, may not know the profitability and sensitivity to reputation of the bank's underwriting business. A bank with a highly profitable underwriting business would correspond to the honest type, while a bank with a less profitable underwriting business would correspond to the opportunistic type.

Combining costly signaling and reputation leads to multiple equilibria. The simplest class of equilibria, which I call separating equilibria, arises when the issuer perfectly reveals the quality of assets through either retention or public report. In a separating equilibrium, the ex post performance of an asset does not yield any new information about the type of the issuer, so reputation and price remain constant. A truth telling equilibrium, a special type of separating equilibrium, obtains when the issuer's public report is credible regardless of issuer retention. I show that a truth telling equilibrium exists if and only if the issuer is sufficiently patient.

The next class of equilibria, which I call mixed strategy equilibria, arises when the opportunistic issuer deviates from truthful reporting at least part of the time and does not perfectly reveal asset quality through retention. For certain parameter values, this type of equilibrium will Pareto dominate the separating equilibrium. In the discussion below, I characterize a particular mixed strategy equilibrium. In such an equilibrium, the issuer does

not perfectly reveal asset type and the reputation of the issuer fluctuates according to the ex post asset performance. Retention now becomes a signal of the opportunistic issuer's reporting strategy. The importance of retention necessarily varies with the reputation of the issuer since as issuer reputation increases, the strategy of the opportunistic issuer has a lower impact on price.

The mixed strategy equilibrium delivers a theoretical link between issuer reputation, and asset retention. As might be expected, retention decreases with issuer reputation. What is less obvious is that the opportunistic issuer will decrease the probability that she will truthfully report a bad type asset as she gains a better reputation. Ultimately, the opportunistic issuer will "cash in" on her reputation by reporting that a bad type asset is the good type and collect one period payoffs. This will occur even when the issuer retains a positive fraction of the asset. In addition, the mixed strategy equilibrium demonstrates that the price for reportedly good type assets may not depend monotonically on issuer reputation. For low levels of reputation, the issuer perfectly reveals asset type through retention alone, and prices will be equal to the full information case. For higher levels of reputation, the issuer's public report is more credible and prices for reportedly good type assets will be close to the full information value of a good type asset regardless of retention.

Finally, the model shows that although the addition of reputation to a costly signaling framework can increase issuer payoffs, it does not increase the probability of perfect information transfer. This result casts an interesting light on the claim that reputation is the core self-disciplining mechanism for ABS markets. If one ignores the possibility that issuers may signal asset quality through costly retention, reputation certainly provides some incentives for issuers of ABS to truthfully reveal their private information. However, including costly retention shows that reputation may actually diminish the issuer's incentives to truthfully reveal information. This feature of the model is appealing given that opportunistic behavior allegedly occurred in many ABS markets.

This chapter relates to the long literature studying the role of private information in causing distortions in markets Akerlof (1970). As early as Myers and Majluf (1984), this idea has been applied to financial markets. If entrepreneurs know more about investment opportunities than outside investors, then the irrelevance of capital structure Modigliani and Miller (1958) no longer holds and a particular security design may be more advantageous than another. This effect leads to the "pecking order" theory of capital structure, with managers issuing the least informationally sensitive securities (i.e. debt) first. Using a similar model to Myers and Majluf (1984), Nachman and Noe (1994) show that debt is the optimal security design to finance investment over a very broad set of payoff distributions. For the special case of asset backed securities, Riddiough (1997b) shows that a senior-subordinated security structure dominates whole asset sales when an issuer has valuable information about assets and liquidation motives are non-verifiable. However, the pecking order literature assumes an investment of fixed size, resulting in a pooling equilibrium in which no mechanism of information transmission exists between issuers and investors.

Another branch of the literature focuses on signaling mechanisms in security design. Leland and Pyle (1977) introduce the notion that retention can be a credible signal of private

information because it is costly, in their case because of reduced risk sharing. DeMarzo and Duffie (1999) build on this signaling mechanism and show how debt arises optimally by creating a more informationally sensitive security which the issuer can retain in order to signal her private information. The addition of a signaling mechanism leads to a separating equilibrium in which all information costs are borne through signaling rather than through prices as in pooling equilibria. DeMarzo (2005) uses the signaling theory of security design, to understand the benefits of pooling and tranching in the market for asset backed securities. Downing, Jaffee, and Wallace (2009) consider the case when issuers cannot choose partial retention and a market for lemons ensues. Unlike the previous literature on signaling and security design, the model I consider focuses on binary cash flow distributions for the sake of tractability. However, the key difference between this paper and previous work on signaling equilibria in security design is that I include dynamic reputation effects. As a result, issuers may choose to do some signaling, but such signaling will not lead to a perfect separating equilibrium all of the time. In that sense, this paper provides a theoretical rationale for a middle ground between separating and pooling equilibria.

The second literature to which this paper relates is that of reputation effects in repeated games. Intuitively, agents involved in repeated games may try to attain a reputation for a certain characteristic in early stages of the game if that characteristic improves payoffs in later stages. Kreps and Wilson (1982) and Milgrom and Roberts (1982) introduce the notion that imperfect or asymmetric information about player preferences can provide such a mechanism. By observing a given player's previous actions, other agents can form beliefs about that player's type. When a player may be one of two types, e.g. honest or opportunistic, such a learning process provides a means by which that player can gain a reputation. If having a reputation for being honest leads to higher equilibrium payoffs, then an opportunistic player may employ the equilibrium strategies of the honest type. Thus, reputation can act as an effective mechanism to encourage "desirable" characteristics. This literature often assumes that the desirable type plays a mechanical strategy and does not optimize. In the model below, I depart from this assumption by allowing the honest type to optimize over her retention strategy. To my knowledge, this is the first model to include a separate dimension of the strategy space over which the honest type player has as much flexibility as the opportunistic type player.

Some notable papers have applied the Kreps and Wilson (1982) concept of reputation to financial markets. Diamond (1989) shows that the possibility of acquiring a reputation for good investment opportunities can incentivize firms to choose safer investments by lowering the cost of borrowing for firms with good reputations. Diamond (1991) examines the choice between bank and public debt in the presence of reputation. John and Nachman (1985) analyze the role of reputation in mitigating the problem of underinvestment induced by risky debt in the presence of asymmetric information. They show that a reputation for "good" investment policies can lead to higher prices in bond markets, and hence firms will want to implement such an investment policy to attain a good reputation. Benabou and Laroque (1992) show that when private information is not ex post verifiable, insiders have an incentive to exploit a reputation for honesty in order to manipulate securities markets

for gain. These papers focus the efficacy of reputation to implement “good” behavior in financial markets without considering other mechanisms that may have similar effects. In contrast, I examine the interaction of reputation with costly signaling.

A very closely related paper is Mathis, McAndrews, and Rochet (2009), here after MMR, which considers a reputation building model of credit rating agencies. They show that the effectiveness of reputation concerns for imposing market discipline on rating agencies depends importantly on the parameters of the model. Under some circumstances, reputation may be enough to impose complete honesty on rating agencies. Although some aspects of my model resemble that of MMR, unlike MMR I allow the issuer to retain a portion of any security she issues, creating an additional mechanism for the credible revelation of private information. MMR only allow the stage game payoff to depend on reputation through a single equilibrium strategy: the probability the rating agency tells the truth. While rating agencies do not typically retain a meaningful stake in an issue that they have rated, securities issuers do. In this way, my model explores the interaction of reputation effects with a more formal mechanism like costly retention, an interaction which is understudied in the literature.

There is also an empirical literature which examines the relationship between issuer reputation and retention. Lin and Paravisini (2010) find the lead arrangers in the syndicated loan market make larger capital contributions (i.e. retain) more after exposed involvement in the accounting scandals of the early 2000’s. In this case, issuer (lead arranger) retention (capital contribution) is likely used to mitigate a moral hazard, rather than adverse selection, problem. Sufi (2007) finds a similar relationship between lead arranger reputation and capital contribution by using market share as a measure of reputation. He shows that lead arranger capital contribution weakly decreases in lead arranger market share although capital contribution never reaches zero. This relationship is likely do to a combination of moral hazard and adverse selection problems. Finally, Titman and Tsyplakov (2010), who investigate the link between mortgage originator performance, commercial mortgage spreads and default rates, and CMBS structure. They find originator’s with poor stock price performance originate lower quality mortgages which also end up in CMBS deals with more subordination (less AAA rated principal).

3.2 The model

3.2.1 Assets, agents and actions

The economy is populated by an issuer and a measure of competitive investors. The issuer has the constant per period discount factor $\gamma < 1$ and the investors have the per period discount factor 1. The difference in the discount rates of the issuer and investors represents the relative impatience of the issuer and drives the gains from trade in the model. The relative impatience of the issuer could arise for a variety of reasons, including capital requirements and access to additional investment opportunities.

Time is infinite and indexed by t . At fixed dates $t = 0, 1, 2, \dots$ the issuer is endowed

with a single asset which produces a cash flow X_{t+1} at the beginning of the next date and is one of two possible types. For convenience, I denote the type of the asset at time t by the process $A_t \in \{G, B\}$. The asset is the good type with probability λ and is the bad type otherwise. Good assets produce a cash flow of 1 while bad assets produce a cash flow ℓ such that $0 < \ell < 1$.¹ These asset cash flows imply perfect observability since after observing cash flows the type of the asset is known. Perfect observability will allow me to explicitly characterize strategies and value functions.

At the start of date t , the issuer may sell a fraction $q_t \in [0, 1]$ of the asset to investors. The investors observe the quantity q_t at each date. The issuer has an incentive to sell the asset since investors are relatively patient. In practice, it may be impossible for the issuer to choose $q \in (0, 1)$ due to regulatory restrictions as considered in Downing, Jaffee, and Wallace (2009). I consider this type of restricted signaling space as an extension to the basic model. In addition to choosing the level of retention, the issuer produces a *prospectus* which contains a report $a_t \in \{g, b\}$ indicating the type of the asset.

3.2.2 Issuer type, reputation and strategies

Following Kreps and Wilson (1982) reputation arises from incomplete information about issuer preferences. Specifically, the issuer can be of two possible types. The *honest* type issuer always provides a truthful report. Moreover, she chooses a quantity of the asset to issue that maximizes her expected proceeds from securitization and retained assets. In contrast, the *opportunistic* type issuer chooses both a report and quantity to maximize her expected proceeds from securitization and retained assets. The formal definition of the objective functions of both types of issuer is given in Definition 4. The reputation of the issuer is then summarized by the probability the investors place on the issuer being the honest type with an initial probability ϕ_0 . As time evolves, the investors update their beliefs about issuer type by observing the history of public information of the game, denoted \mathcal{H}_t . The investors' belief that the issuer is the honest type is thus given by

$$\phi_t = \mathbb{P}(\text{issuer is honest type} | \mathcal{H}_t).$$

I will assume the current quantity Q and the reputation ϕ_t contain all the relevant information for the beliefs of the investor about the current asset type given a public report. That is, I assume that ϕ_t is a Markov state variable for the history of the game. In principal, the investors' beliefs about the current asset type could depend on the entire history of the game and in particular the path of past quantities. Because such a dependence makes the notation overly cumbersome, I do not consider it in the main text. In Appendix ??, I allow investor beliefs to depend on the path of past quantities and reports and show that

¹Since the focus of the present paper is to jointly analyze the effect of costly signaling and reputation, I consider binary asset cash flows and abstract away from security design issues. For an analysis of security design under a rich set of cash flow distributions in a static setting see Nachman and Noe (1994) or DeMarzo and Duffie (1999).

restricting attention to investor beliefs which are Markov in reputation does not rule out important equilibria.

The market must price the asset based on the report of asset type, the issued quantity, and the reputation of the issuer. If the issuer reveals that the asset is the bad type, then investors believe the asset is the bad type with probability one, so the market will pay a price ℓ per unit for the offering. When the issuer reports that the asset is the good type, the market price per unit is given by the inverse demand curve, denoted $P(q, \phi) : [0, 1] \times [0, 1] \rightarrow [0, 1]$, which is the value investors place on the underlying asset of an ABS with quantity q and report g offered by an issuer with reputation ϕ . In other words, if an issuer with reputation ϕ offers an ABS with quantity q and report g , she will receive $qP(q, \phi)$ in proceeds from the sale of the ABS to investors. It is potentially costly for the issuer to choose $q < 1$ because she has a larger discount rate than the investors. Accordingly, the level of retention represents a credible signaling mechanism for an issuer with a good type asset. That is, investors should believe that an ABS with a relatively higher level of retention is backed by an asset of relatively higher quality. Thus, for each ϕ , the inverse demand curve $P(q, \phi)$ is downward sloping in q . This setup is a simplified version of the signaling mechanism of DeMarzo and Duffie (1999), with the exception that the demand curve may now depend on a report of asset quality and issuer reputation.

Both issuer types form strategies conditional on their current reputation and the type of the current asset. Before formally defining the issuer's strategy space, I make some convenient assumptions to simplify notation. Specifically, I assume that either type of issuer will always provide a truthful report when the asset is the good type and will always sell the entire asset when she reveals that the asset is the bad type.² Thus a *reporting strategy* is a function $\pi(\phi) : [0, 1] \rightarrow [0, 1]$ giving the probability of accurately reporting a bad asset's true type. A *quantity strategy* is a function $Q(\phi) : [0, 1] \rightarrow [0, 1]$ giving the fraction of the asset sold to investors when the issuer reports that the asset is the good type. Recall that the set of admissible strategies for the issuer depends on her type. The set of admissible strategies for the opportunistic type, denoted \mathcal{A}_O , is simply the set of all possible strategies pairs defined above, whereas the set of admissible strategies for the honest type issuer is given by $\mathcal{A}_H = \{(\pi, Q) \in \mathcal{A}_O | \pi(\phi) = 1\}$. The restriction that defines \mathcal{A}_H reflects the fact that the honest type is committed to truthfully revealing a bad type asset.

To recapitulate, the timing of the game is as follows. At each date $t = 0, 1, 2 \dots$ the investors and issuer play a securitization stage game. At the beginning of a given date t , the cash flow from the asset sold on the previous date is realized and the players update the reputation of the issuer. Second, the current asset type is revealed to the issuer and she chooses her report and retention strategies. Finally, investors buy the security at a price $qP(q, \phi)$ and the process starts anew. Figure 3.1 gives a time-line of the game with the

²This assumption is without loss of generality. The opportunistic type issuer would never report that a good type asset is the bad type in equilibrium since doing so would not increase her reputation or her instantaneous proceeds from asset sale. Thus, if the issuer reveals that the asset is the bad type, then her private value for the asset is always less than that of the investors, regardless of her quantity strategy, and selling the entire asset is optimal.

sequence of actions that occur within a given date t .

3.2.3 Equilibrium

At any given date t the issuer will maximize the discounted expected value of her proceeds from securitization plus the cash flow from retained assets. Formally an issuer with reputation ϕ has an instantaneous cash flow to an action (q, π) given by

$$U_t(q, \pi, P, \phi) = \lambda(qP + \gamma(1 - q)) + (1 - \lambda)[\pi\ell + (1 - \pi)(qP + \gamma\ell(1 - q))], \quad (3.1)$$

when facing the demand curve P at time t .

Definition 4. The quadruple (P, Q^H, π, Q^O) is an *equilibrium* if at all times t

1. The strategy of the honest type maximizes her payoffs:

$$Q^H \in \arg \max_{\mathbf{q}} E_t [\sum_{n=t}^{\infty} \gamma^{n-t} U_n(q_n, 1, P, \phi_n)],$$

2. The strategy of the opportunistic type maximizes her payoffs:

$$(Q^O, \pi) \in \arg \max_{\mathbf{q}, \pi} E_t [\sum_{n=t}^{\infty} \gamma^{n-t} U_n(q_n, \pi_n, P, \phi_n)],$$

3. ϕ_t is determined using Bayes rule whenever possible, and

4. Investors earn zero expected profits: $P(Q^i(\phi_t), \phi_t) = E[X_{t+1} | \phi_t, Q^i(\phi_t)]$ for $i \in \{O, H\}$.

5. An equilibrium is *separating* if $P(Q^H(\phi_t), \phi_t) = P(Q^O(\phi_t), \phi_t) = 1$.

To be clear, by using the term separating equilibrium, I am referring to equilibria that reveal the true type of the underlying *assets*, rather than the true type of the issuer. Indeed, it is impossible for the issuer to credibly reveal her type via a particular retention strategy in equilibrium as will become apparent shortly. I will often refer to a given equilibrium as the least cost separating equilibrium. By this, I mean the separating equilibrium which delivers the highest payoff to the issuer.

3.3 Solution

3.3.1 The game without reputation

To begin the analysis, I consider equilibria when the issuer is known to be the opportunistic type. When the issuer is revealed to be the opportunistic type by the history of the game, so that $\phi_t = 0$, Bayes Rule implies that $\phi_s = 0$ for all $s \geq t$. In other words, a reputation of zero is an absorbing state. Thus, equilibria in this state will serve as an important input to the solution of the general case.

Before considering the repeated game, it is useful to consider equilibria of the static game. The natural restriction of Definition 4 for the static game replaces conditions (1) and (2) with a one period maximization problem. The following proposition summarizes the equilibria of the static game without reputation.

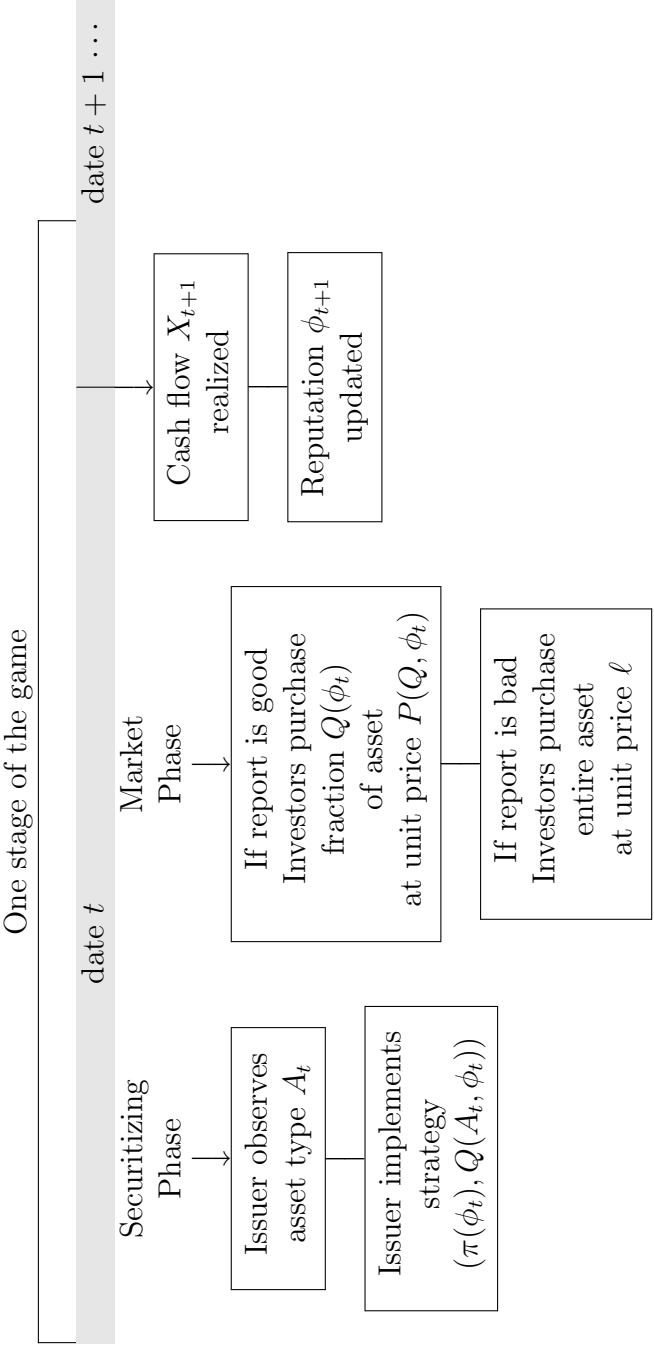


Figure 3.1: Timeline of the game. At the beginning of a given date the cash flow from the previous date's asset is realized and reputation is updated. Next, the issuer learns the type of the asset and implements a purchasing and securitization strategy (π, Q) . Finally the investors purchase the fraction of the asset Q at a price P .

Proposition 6. *Suppose $\phi_0 = 0$. Then, for the static game:*

- *A separating equilibrium is given by $Q = \tilde{q}$, $\pi = 1$, and*

$$P(q, 0) = \begin{cases} 1 & q \leq \tilde{q} \\ \ell & q > \tilde{q} \end{cases}$$

for all $\tilde{q} \leq \hat{q} = \frac{\ell(1-\gamma)}{(1-\gamma)\ell}$. The least cost separating equilibrium is $\tilde{q} = \hat{q}$.

- *A pooling equilibrium exists and is given by $Q = 1$, $\pi = 0$, and $P(q, 0) = \lambda + (1 - \lambda)\ell$ for all $q \in [0, 1]$ if and only if $\gamma \leq \lambda + (1 - \lambda)\ell$.*

Other equilibria, in particular those with mixed reporting strategies in which $0 < \pi < 1$, may also exist. However, as will become clear when considering repeated versions of the static equilibria, mixed strategy equilibria of the static game without reputation are Pareto dominated by the least cost separating equilibrium or the pooling equilibrium, depending on the parameterization of the model.

The least cost separating equilibrium follows from the classic signaling intuition. The quantity \hat{q} is defined so that even when the market responds with a price per share of one for the quantity \hat{q} , the issuer with a bad type asset is better off selling the entire asset for a price of ℓ . At the same time, the issuer with a good type asset strictly prefers selling the quantity \hat{q} at a price per unit of one to retaining the entire asset. Such a quantity \hat{q} exists because the relative impatience of the issuer implies that retaining a fraction of the asset is more costly for the issuer when she has a good type asset than when she has a bad type asset since the issuer is less patient than the investors.

The pooling equilibrium arises when the issuer's value for retaining a good type asset is less than the ex ante expected value of the asset to the investors. In a standard static signaling game, pooling equilibria are typically ruled out by the D1 refinement of Cho and Kreps (1987) which restricts off equilibrium beliefs. However, it is not clearly valid to apply this refinement to the repeated game. At the same time, the pooling equilibrium of the static game only exists for parameterizations of the model in which the lemons problem is not too "severe" and markets could function efficiently without any means of information transfer. Specifically, a repetition of the pooling equilibrium would lead to payoffs equal to the full information case. For the remainder of the paper I will assume that parameters are such that the pooling equilibrium does not exist:

Assumption 1. The parameters of the model do not admit a pooling equilibrium of the static game: $\lambda + (1 - \lambda)\ell \leq \gamma$.

I can now consider equilibria of the repeated game for $\phi_t = 0$. Since investors' strategies were assumed to be Markovian in ϕ , there is no mechanism to make the issuer's current payoffs depend on her past actions. And thus, public reports of asset quality are no more credible than in the static version of the game. This observation leads to the following proposition.

Proposition 7. *Suppose $\phi_t = 0$. Then $\phi_s = 0$ for all $s \geq t$, and a strategy pair (Q, π) and price schedule $P(q, 0)$ are an equilibrium if and only if they are an equilibrium of the static game.*

In principal, investors could play punishment strategies in which past quantities affect beliefs about current asset types even though reputation is fixed at zero. The result of such a strategy would be that a truth telling equilibrium may emerge even though $\phi_0 = 0$. This type of equilibrium behavior is sometimes thought of as resulting from “reputation”, however this is not the concept of reputation I consider here. For completeness, I consider the possibility of punishment strategies in Appendix ???. With punishment strategies, the set of possible equilibria for the repeated game with no reputation would include a truth telling equilibrium, provided the parameters satisfy a given restriction. It will turn out that this restriction is also a necessary condition for the existence of a truth telling equilibrium for the case with positive reputation. Thus, by assuming away punishment strategies, I have only eliminated truth telling equilibrium for the no reputation state. In particular, introducing punishment strategies does not allow for additional truth telling equilibria for the positive reputation state.

Propositions 6 and 7 imply there are multiple equilibria for the repeated game without reputation. Since the structure of equilibria with positive reputation will hinge on what equilibrium strategies obtain if the issuer’s reputation falls to zero, it is necessary to have a consistent means of selecting an equilibrium in this state. Again, applying the D1 refinement of Cho and Kreps (1987) is not clearly applicable in the repeated setting. Instead, I rely on the fact that conditional on parameters, a single equilibrium delivers the most value to the issuer. Since investors are competitive and always earn zero profits in expectation, such an equilibrium is Pareto dominant. Under Assumption 1, the least cost separating equilibrium of Proposition 6 delivers the highest equilibrium payoffs to the issuer. Thus, I will assume the least cost separating equilibrium obtains for the no reputation state.

3.3.2 Reputation dynamics and optimization

Now that I have fixed equilibrium for the repeated game in the no reputation state, I can proceed to solve for the dynamics of reputation in equilibrium. First, I make the following important observation.

Lemma 1. *The honest issuer and the opportunistic issuer always issue the same quantity, $Q^H(\phi) = Q^O(\phi)$ for all ϕ .*

The intuition behind Lemma 1 is the following. The opportunistic issuer and the honest issuer both value instantaneous payoffs from the securitization of a good asset identically. Moreover, the opportunistic issuer values a higher reputation weakly more than the honest issuer. Therefore, any quantity strategy which increases reputation will be at least as attractive to the opportunistic type as the honest type. This implies that the quantity issued is not a credible signal of *issuer* type and cannot contain any new information about the type

of the issuer. In particular, this means that reputation is only updated during the reputation updating phase, and not during the securitization phase. This will be a useful fact in the analysis since it means that the issuer need not take into account the effect of her quantity strategy on her future reputation. In addition it will allow me to simplify the analysis of the reputation updating process.

Given Lemma 1, it is straightforward to derive the dynamics of reputation in terms of the report and ex post performance of the asset. Let $f : \{g, b\} \times \{\ell, 1\} \times [0, 1] \rightarrow [0, 1]$ denote the reputation updating function. Using Bayes rule whenever possible, I have

$$f(g, 1, \phi) = \phi^S = \phi \quad (3.2)$$

$$f(g, \ell, \phi) = \phi^F = 0 \quad (3.3)$$

$$f(b, \ell, \phi) = \phi^B = \frac{\phi}{\phi + \pi(1 - \phi)}. \quad (3.4)$$

The optimization problem faced by the issuer can now be simplified given the reputation updating function and the fact that the opportunistic and honest type issuers always choose the same retention strategy. Since Lemma 1 implies that the honest type issuer and opportunistic type issuer play the same retention strategy, I drop the superscript and refer to a retention strategy as simply Q . Consider the opportunistic issuer's problem. Let $V(\phi|P)$ denote the value function of the opportunistic type issuer when facing the demand schedule P and $V_G(\phi|P)$ and $V_B(\phi|P)$ denote the value functions when the opportunistic type issuer faces the demand schedule P and is endowed with good asset or a bad asset respectively. Then $V(\phi|P) = \lambda V_G(\phi|P) + (1 - \lambda)V_B(\phi|P)$, and $V_G(\phi|P)$ and $V_B(\phi|P)$ satisfy the following system of Bellman equations

$$V_G(\phi|P) = \max_{(\pi, Q) \in \mathcal{A}_O} \{(\gamma(1 - Q) + QP(Q, \phi) + \gamma V(\phi^S|P))\}. \quad (3.5)$$

$$V_B(\phi|P) = \max_{(\pi, Q) \in \mathcal{A}_O} \{\pi(\ell + \gamma V(\phi^B)) + (1 - \pi)(\gamma(1 - Q)\ell + QP(Q, \phi) + \gamma V(0))\}. \quad (3.6)$$

3.3.3 Separating equilibria

For an equilibrium to be separating, it must be the case that by observing the quantity issued and the report given, the investors can perfectly infer the type of the asset. In general, such perfect inference can arise either because the quantity issued maps perfectly to the type of the asset, or the loss of continuation value from being exposed as the opportunistic type is so great that the issuer will never misreport a bad type asset. I refer to equilibria of this latter type as *truth telling*. Specifically an equilibrium is truth telling if $Q(\phi) > \hat{q}$ for some $\phi > 0$ and $\pi(\phi) = 1$ for all $\phi > 0$. The truth telling equilibrium is desirable in that it allows for the credible revelation of issuer private information with less issuer retention. Indeed, when a truth telling equilibrium exists with $Q(\phi) = 1$ for all $\phi > 0$, it delivers payoffs to the issuer equal to what she would receive in a first best setting. However, some restriction on parameters is needed for a truth telling equilibrium to obtain.

Proposition 8 (Folk Theorem). *Suppose $\phi_0 > 0$. There exists a truth telling equilibrium if and only if $\gamma \geq \frac{1}{\lambda + \ell}$.*

The restriction on parameters required for the existence of a truth telling equilibrium depends on the instantaneous gains to the issuer from misreporting a bad type asset and the loss in continuation value from being identified as the opportunistic type. Suppose the investors always believe the report of the issuer, then an opportunistic issuer may receive a price of 1 for a bad type asset for one period and then be known to be the opportunistic type thereafter. Such a deviation is profitable if and only if

$$\underbrace{1 - \ell}_{\text{gain in proceeds}} \leq \frac{\gamma \lambda (1 - \hat{q})(1 - \gamma)}{\underbrace{1 - \gamma}_{\text{loss in continuation value}}}. \quad (3.7)$$

This restriction simplifies to $\gamma \geq \frac{1}{\lambda + \ell}$. One interpretation of this restriction is that the issuer must be sufficiently patient so as to make a loss in continuation value severe enough to provide incentives to always accurately report a bad type asset. In this way, the conditions guaranteeing the existence of a truth telling equilibrium are similar to a classic folk theorem.³

When the issuer is impatient, the truth telling equilibrium can not be supported even with positive initial reputation. However, the repeated version of the least cost separating equilibrium of the game without reputation still obtains.

Proposition 9. *The least cost separating equilibrium of Proposition 6 where $P(q, \phi) = P(q, 0)$ for all ϕ is an equilibrium of the game for all $\phi_0 \in [0, 1)$.*

The cost to the issuer to credibly reveal her information about the quality of the asset to the investors thus depends importantly on the parameters of the model. When the issuer is relatively patient, or $1 - \lambda \ell > \lambda \gamma$, she can credibly reveal her information via her public report without cost, so long as she has strictly positive reputation. This is the truth telling equilibrium. If the issuer is relatively impatient, she can still credibly reveal her information, however, doing so requires her to issue a quantity strictly less than one, which is costly. Figure 3.2 shows a partition of the parameter space highlighting the region for which truth telling is supported in equilibrium. Region I corresponds to parameters for which the truth telling equilibrium are not supported, whereas in Region II, truth telling is supported. A natural question is whether higher equilibrium payoffs for the issuer may be supported in Region I by considering mixed reporting strategies. In other words, is there an equilibrium for parameters in Region I in which the issuer achieves higher payoffs and does not always perfectly reveal the type of the asset. I consider this possibility in the next subsection.

³Here the assumption that ϕ is a Markov state variable has some bite. If investor beliefs could depend on the path of past signals, truth telling would be supportable in equilibrium without positive reputation. Moreover the parameter restriction required would be slightly weaker.

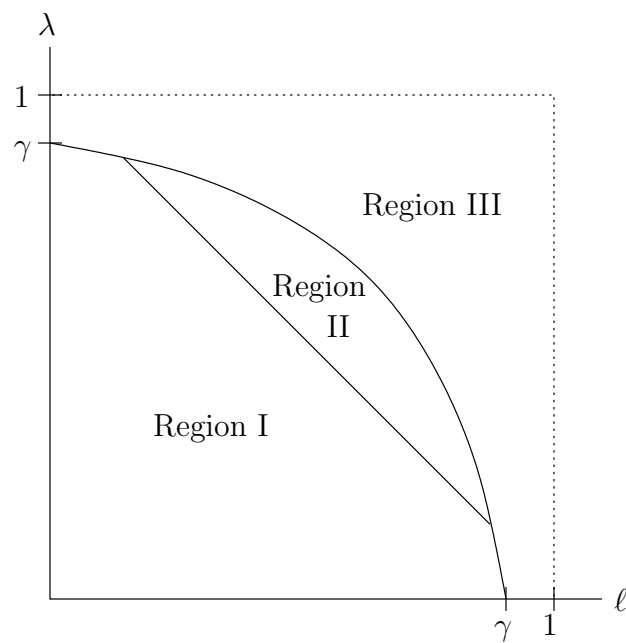


Figure 3.2: A partition of the parameter space of the model. Region I corresponds to parameters for which the truth telling equilibrium is not supported. Region II corresponds to parameters for which the truth telling equilibrium is supported. Region III corresponds to parameters for which Assumption 1 fails and hence is not considered.

3.3.4 Mixed strategy equilibria

In light of results of the previous subsection, I look for equilibria in which $0 < \pi(\phi) < 1$ for some $\phi \in [0, 1]$, or mixed strategy equilibria for parameters under which truth telling is not supported in equilibrium. Specifically, I impose the following assumption to rule out truth telling.

Assumption 2. The parameters of the model do not support the truth telling equilibrium:
 $\gamma < \frac{1}{\lambda + \ell}$.

In what follows, I will outline a procedure to construct a particular equilibrium, the details of which can be found in Appendix ???. The approach will be to assume the existence of an equilibrium in which $\pi(0) = 1$ and $\pi(1) = 0$. In other words, the opportunistic issuer will always be honest for low levels of reputation and always submit inaccurate reports for high levels of reputation. I also assume a particular functional form for the demand schedule $P(q, \phi)$. Doing so amounts to making assumptions about the off-equilibrium beliefs of the investors. Given, a demand schedule $P(q, \phi)$, I find $Q(\phi)$, the optimal quantity chosen by the issuer when faced with a good type asset. Next, I suppose the opportunistic issuer chooses the same quantity when faced with a bad asset for high enough levels of reputation, a necessary condition for the equilibrium to not be a separating equilibrium. Using the one-shot deviation principal for infinitely repeated games, I find an inequality relating the quantity strategy $Q(\phi)$ to the reporting strategy $\pi(\phi)$ and the value function of the opportunistic type issuer V . Finally, I construct the value function V using this inequality.

To begin the analysis, I assume a candidate equilibrium demand curve $P(q, \phi)$ is a step function of q of the following form

$$P(q, \phi) = \begin{cases} 1 & q \leq \hat{q} \\ p^*(\phi) & \hat{q} < q \leq q^*(\phi) \\ \ell & q > q^*(\phi) \end{cases} \quad (3.8)$$

where $p^*(\phi)$ and $q^*(\phi)$ are continuous in ϕ such that $p^*(0) = 1$ and $q^*(0) = \hat{q}$.⁴ The inverse demand curve P , depicted in Figure 3.3, is consistent with the investor beliefs that only an issuer with a good type asset would ever offer a quantity q less than \hat{q} , while an issuer with either type asset might offer a quantity q greater than \hat{q} but less than some level $q^*(\phi)$, and only an issuer with a bad type asset would choose a quantity q greater than $q^*(\phi)$. Given the demand curve $P(q, \phi)$, an issuer with a good asset and reputation ϕ will choose a quantity $Q(\phi) \in \{\hat{q}, q^*(\phi)\}$. To see this, observe that the issuer's proceeds are increasing over each subinterval of quantity given in the definition of the demand curve, while her continuation value is fixed. This argument implies that $q^*(\phi)$ is a natural candidate equilibrium quantity strategy given the inverse demand curve P . I denote the candidate equilibrium reporting

⁴By assuming that the opportunistic issuer plays the strategies that arise in the static signaling game when she is known to be the opportunistic type, I am explicitly forcing this derivation to yield an equilibrium consistent with Proposition 7 as issuer reputation decreases to zero.

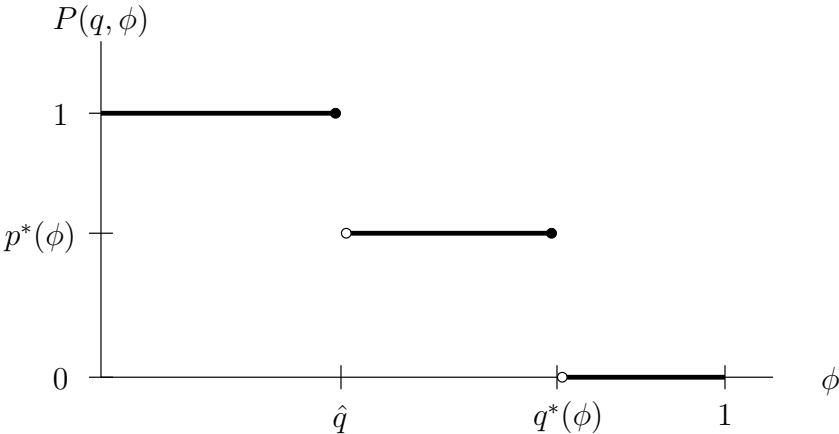


Figure 3.3: An example of the demand curve (3.8) for a given ϕ . For $q \leq \hat{q}$ investors believe the asset is the good type. For $\hat{q} < q \leq q^*(\phi)$, investors believe the asset is the good type with some probability and the bad type with some probability. For $q > q^*(\phi)$ the investors believe the asset is the bad type.

strategy as $\pi^*(\phi)$. In addition, I let the discounted loss in issuer value associated with a drop in reputation from ϕ to 0 be denoted $L(\phi) = \gamma(V(\phi) - V(0))$. Since the candidate equilibrium strategies are known at $\phi = 0$, it is trivial to calculate

$$V(0) = \frac{1}{1-\gamma}(\lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell), \quad (3.9)$$

and hence deriving the value function $V(\phi)$ is equivalent to deriving the discounted loss function $L(\phi)$.

Now that I have assumed a particular functional form for the demand schedule P , I can further simplify the maximization problem faced by the issuer. Specifically, I identify the following four conditions that must hold in any equilibrium in which the quantity strategy is $Q(\phi) = q^*(\phi)$, the reporting strategy is $\pi^*(\phi)$, and the demand schedule is given by equation (3.8),

$$q^*(\phi)p^*(\phi) - \hat{q} \geq \gamma(q^*(\phi) - \hat{q}) \quad (3.10)$$

$$q^*(\phi)p^*(\phi) - \ell \geq L(\phi^B) - (1 - q^*(\phi))\gamma\ell \quad (3.11)$$

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{\lambda + (1-\lambda)(1-\phi)(1-\pi^*(\phi))} \quad (3.12)$$

$$L(\phi) = \frac{\gamma}{1-\gamma\lambda} [q^*(\phi)(p^*(\phi) - \gamma\bar{x}) - (1-\gamma)(\lambda\hat{q} + (1-\lambda)\ell)], \quad (3.13)$$

where $\bar{x} = \lambda + (1-\lambda)\ell$. Inequality (3.10) states that an issuer with a good asset must weakly prefer the quantity strategy $Q(\phi) = q^*(\phi)$ to the strategy $Q(\phi) = \hat{q}$. Similarly, inequality (3.11) states that an issuer with a bad type asset must weakly prefer the retention strategy $Q(\phi) = q^*(\phi)$ and reporting strategy $\pi^*(\phi)$ to the strategy $\pi(\phi) = 1$. Equation (3.12) follows directly from the fact that investors earn zero profits in expectation, in other words condition (4) of Definition 4, and Bayes rule. This equation must hold for all ϕ , so that to characterize an equilibrium, it is enough to find the quantity-price pair $(q^*(\phi), p^*(\phi))$. Finally, equation (3.13) follows from the maximization problem described by equations (3.6) and (3.5).

The next step in constructing a candidate equilibrium is to divide the interval $\phi \in [0, 1]$ into subintervals over which the inequalities (3.10) and (3.11) either bind, or are slack. To that end, I assume there exists $\underline{\phi}$ and $\bar{\phi}$ such that $\pi^*(\phi) = 1$ and $q^*(\phi) = \hat{q}$ for $\phi \leq \underline{\phi}$ and $\pi^*(\phi) = 0$ and for $\phi \geq \bar{\phi}$. The one-shot deviation principal implies the inequality (3.11) must bind whenever $0 < \pi^*(\phi) < 1$, hence it must bind whenever $\underline{\phi} < \phi < \bar{\phi}$. I assume there exists $\hat{\phi}$ such that inequality (3.10) binds for $\phi \leq \hat{\phi}$ and $q^*(\phi) = 1$ for $\phi \geq \hat{\phi}$. It will turn out that $\hat{\phi} \leq \bar{\phi}$, so that the problem of deriving equilibrium strategies can be broken up into the four subintervals of reputation, $[0, \underline{\phi}]$, $(\underline{\phi}, \hat{\phi}]$, $(\hat{\phi}, \bar{\phi}]$, and $(\bar{\phi}, 1]$. Figure 3.4 shows the proposed decomposition of the interval with the corresponding constraints on the candidate equilibrium strategies π^* and q^* .

The assumption that inequality (3.10) binds for $\underline{\phi} < \phi < \hat{\phi}$ amounts to restricting attention to the corners of the space of incentive compatible strategies. Once inequality

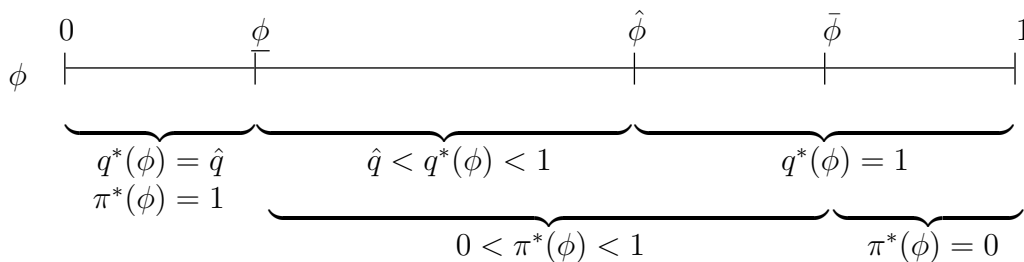


Figure 3.4: The interval $0 \leq \phi \leq 1$ can be decomposed into subintervals over which the candidate strategies are either known or the inequalities (3.10) and (3.11) bind. For $\phi < \hat{\phi}$, so that $\hat{q} < q^*(\phi) < 1$, inequality (3.10) binds. For $\hat{\phi} < \phi < \bar{\phi}$, so that $0 < \pi^*(\phi) < 1$, inequality (3.11) binds.

(3.11) binds, one can interpret inequality (3.10) as placing a lower bound on the set of possible equilibrium reporting strategies. For example, suppose ϕ^B is fixed and $L(\phi^B)$ is known, then the reporting strategy is a known function

$$\pi^*(\phi) = \frac{\phi(1 - \phi^B)}{\phi^B(1 - \phi)}. \quad (3.14)$$

In other words, for each level of reputation $\phi \in (0, \phi^B)$ there is a single reporting strategy $\pi^*(\phi)$ for which reporting a bad asset will result in an increase of reputation to ϕ^B . Moreover, since (3.11) binds and $L(\phi^B)$ is known, inequality (3.10) can be rearranged to get

$$\pi(\phi) \geq 1 - C \frac{1}{1 - \phi}, \quad (3.15)$$

where C is some constant which may depend on ϕ^B . Figure 3.5 plots equation (3.14) and inequality (3.15) and illustrates that by assuming inequality (3.10) binds, I am choosing the minimal admissible π^* for each ϕ^* .

With the partition of the unit interval of reputation described above and the four relations which must hold in equilibrium given by (3.10), (3.11), (3.12), and (3.13), the problem becomes one of solving a system of equations in a certain number of unknowns. The important caveat to this approach is that to solve the equations for equilibrium strategies at given level of reputation ϕ , I must first know the discounted loss $L(\phi^B)$ at the level of reputation which would arise from a truthful report of a bad type asset. Since $\phi^B \geq \phi$ for all ϕ , I can solve the problem by working downwards from $\phi = 1$.

For $\phi \in [\bar{\phi}, 1]$, the equilibrium strategies are assumed to be $q^*(\phi) = 1$ and $\pi^*(\phi) = 0$. This means the equilibrium price $p^*(\phi)$ and associated discounted loss function $L(\phi)$ follow directly from equations (3.12) and (3.13). For convenience, let $L_1(\phi)$ denote the solution to (3.13) when $q^*(\phi) = 1$ and $\pi^*(\phi) = 0$.

For $\phi \in [\hat{\phi}, \bar{\phi})$, the equilibrium retention strategy is assumed to be $q^*(\phi) = 1$, however the equilibrium price must be calculated. I assume for the time being that $\phi^B \geq \bar{\phi}$ for all

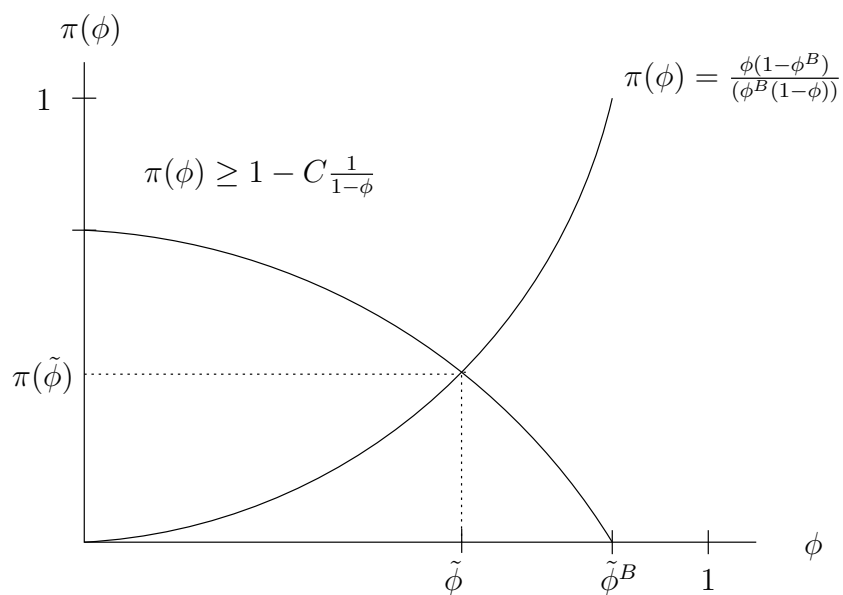


Figure 3.5: Plot of inequality (3.15) and $\pi(\phi) = \frac{\phi(1-\phi^B)}{\phi^B(1-\phi)}$. The downward sloping curve represents the lower bound on $\pi(\phi)$ imposed by (3.15). The region above this curve is the set of all pairs $(\phi, \pi(\phi))$ such that the opportunistic issuer at least weakly prefers the quantity $q^*(\phi)$ at price per unit $p^*(\phi)$ implied by the strategy $\pi(\phi)$ and reputation ϕ . The upward sloping curve is given by the definition of $\pi(\phi)$. The portion of the upward sloping curve which lies above the downward sloping curve is the set of all pairs $(\phi, \pi(\phi))$ such that inequality (3.15) is satisfied and inequality (3.11) binds, and this set may contain many points supportable by an equilibrium. To simplify the analysis, I choose the point at which the two curves intersect. This point gives the lowest value of π such that both inequality (3.15) and the definition of π are satisfied.

$\hat{\phi} \leq \phi < \bar{\phi}$.⁵ Thus, for $\phi \in [\hat{\phi}, \bar{\phi})$, the equilibrium price $p^*(\phi)$ of a reportedly good asset solves the equation

$$p^*(\phi) = L_1 \left(\left(\frac{(1-\lambda)(p^*(\phi) - \ell)}{p^*(\phi) - \ell - \lambda(1-\ell)} \right) \frac{1}{\phi} \right). \quad (3.16)$$

Equation (3.16) follows from substituting equation (3.12) and the reputation updating function into inequality (3.11) (which must bind). Again for convenience, let $L_2(\phi)$ denote the solution to equation (3.13) when $p^*(\phi)$ solves equation (3.16) and $q^*(\phi) = 1$.

Finally, I characterize the discounted loss function $L(\phi)$, reporting strategy π^* , and quantity strategy $q^*(\phi)$ of the opportunistic type issuer for $\underline{\phi} \leq \phi < \hat{\phi}$. To do so, I construct a decreasing sequence starting at $\hat{\phi}$ such that each element of the sequence is the level of reputation for which the decision to truthfully report a bad type asset would lead to an increase in reputation to the preceding element of the sequence. Formally, let $\phi(n)$ be a sequence given by

$$\phi(0) = \hat{\phi} \quad (3.17)$$

$$\phi^B(n) = \phi(n-1) \quad (3.18)$$

I can combine (3.11) and (3.10) to get

$$q^*(\phi)p^*(\phi) + \gamma(1 - q^*(\phi)\bar{x}) = \lambda(\hat{q} + \gamma(1 - \hat{q})) + (1 - \lambda)(\ell + L(\phi^B)). \quad (3.19)$$

Equation 3.19 can be thought of as a combined incentive compatibility constraint. It states that the expected one period proceeds from playing the strategy $(q^*(\phi), \pi^*(\phi))$ is exactly equal to the expected loss from doing so. Equations (3.19) and (3.13), along with the definition of the sequence $\phi(n)$, imply

$$L(\phi(n)) = \beta^n L_2(\hat{\phi}) \quad (3.20)$$

where $\beta = \frac{\gamma(1-\lambda)}{1-\gamma\lambda}$. Now suppose that the values $\phi(k)$ are known for $k \leq n-1$. Then the strategy pair $(q^*(\phi(n)), \pi^*(\phi(n)))$ and the level of reputation $\phi(n)$ solve the following three equations

$$q^*(\phi(n))p^*(\phi(n)) - \hat{q} = \gamma(q^*(\phi(n)) - \hat{q}) \quad (3.21)$$

$$q^*(\phi(n))p^*(\phi(n)) - \ell = \beta^{n-1} L_2(\hat{\phi}) - (1 - q^*(\phi(n)))\gamma\ell \quad (3.22)$$

$$p^*(\phi(n)) = \ell + \frac{\lambda(1-\ell)\phi(n-1)}{\lambda\phi(n-1) - (1-\lambda)\phi(n)}. \quad (3.23)$$

Equation (3.21) defines the price quantity pairs such that the issuer with a good type asset is indifferent between issuing the quantity q^* at a price per unit p^* and issuing the quantity \hat{q} at a price per unit ℓ . Equation (3.22) defines the price quantity pairs such that the issuer

⁵This amounts to a parameter restriction, detailed in Appendix ???. This assumption can be relaxed, although with a considerable amount of extra algebra.

with a bad type is indifferent between issuing the quantity q^* at a price per unit p^* resulting in a reputation of zero, and issuing the quantity one, at the price per unit ℓ , resulting in a reputation of $\phi(n-1)$. Figure 3.6 illustrates the solution to the system of equations (3.21)-(3.23). Equations (3.21) and (3.22) both define p^* as a function of q^* . It is straightforward to show that these two functions satisfy a single crossing property and thus admit a unique solution for the quantity price pair $(q^*(\phi(n)), p^*(\phi(n)))$, as demonstrated by Figure 3.6(a). Similarly, equation (3.23) defines p^* as a function of ϕ . Again, it is straightforward to show that this function is decreasing in ϕ . Thus by setting the right hand side of equation (3.23) equal to the solution for $p^*(\phi(n))$ from equations (3.21) and (3.22), a unique solution for $\phi(n)$ obtains, as demonstrated by Figure 3.6(b).

The above argument shows how to characterize the candidate equilibrium on a sequence of levels of reputation defined by (3.18) up to the solution of $L(\phi)$ for $\phi \geq \bar{\phi}$. Applying the reverse of the sequence argument to values of $\phi(n) < \phi < \phi(n-1)$ yields a characterization of the candidate equilibrium for levels of reputation not lying in the above sequence. Namely, for each $\phi \in (\phi(n), \phi(n-1))$ there exists a $\tilde{\phi} \in (\hat{\phi}, 1]$ such that an issuer with current reputation ϕ will have reputation $\tilde{\phi}$ after choosing to report a bad type asset n times in a row.

The complete derivation of this mixed strategy equilibrium is detailed in the Appendix B. The key step is to show that the system of equations given above has a solution for each sub-interval described above. Once this is shown, it is simple to verify that the proposed strategies are indeed an equilibrium by using the single deviation principal for repeated games. I close this section with a statement of existence of a mixed strategy equilibrium in the following proposition.

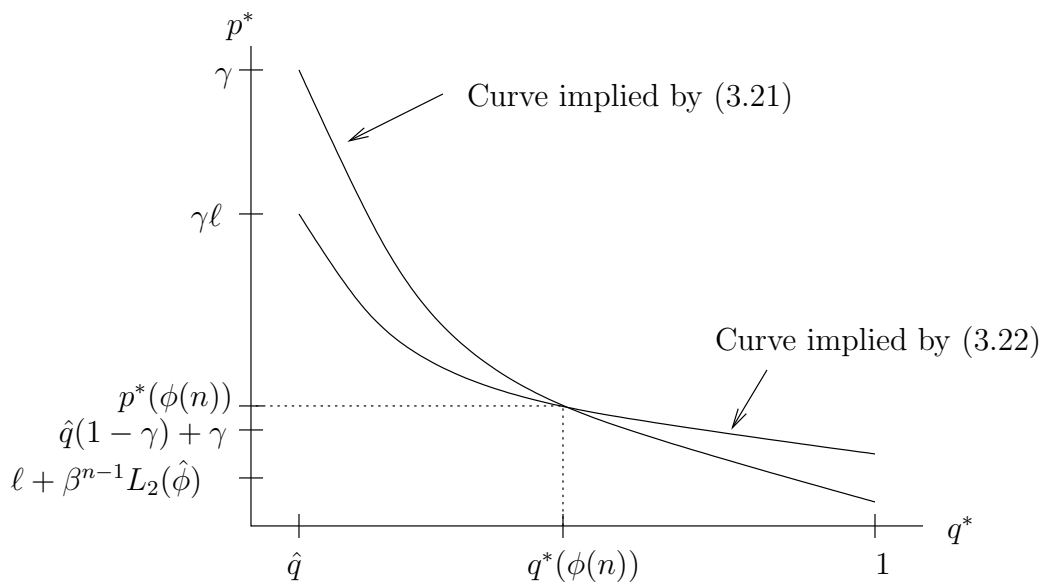
Proposition 10. *Suppose Assumptions 1 and 2 hold. There exist thresholds $\underline{\phi}$, $\hat{\phi}$, $\bar{\phi}$, and an equilibrium in which the opportunistic issuer*

- *plays the separating equilibrium strategies for low levels of reputation: $Q(\phi) = \hat{q}$ and $\pi(\phi) = 1$ for $\phi \leq \underline{\phi}$,*
- *sells a larger portion of the asset than the separating quantity and misreports a bad type asset with positive probability for mid-range levels of reputation: $\hat{q} < Q(\phi) < 1$ for $\underline{\phi} < \phi < \hat{\phi}$, and $0 < \pi(\phi) < 1$ for $\underline{\phi} < \phi < \bar{\phi}$,*
- *sells the entire asset and always chooses to misreport a bad type asset for high levels of reputation: $Q(\phi) = 1$ for $\phi \geq \hat{\phi}$, and $\pi(\phi) = 0$ for $\phi \geq \bar{\phi}$.*

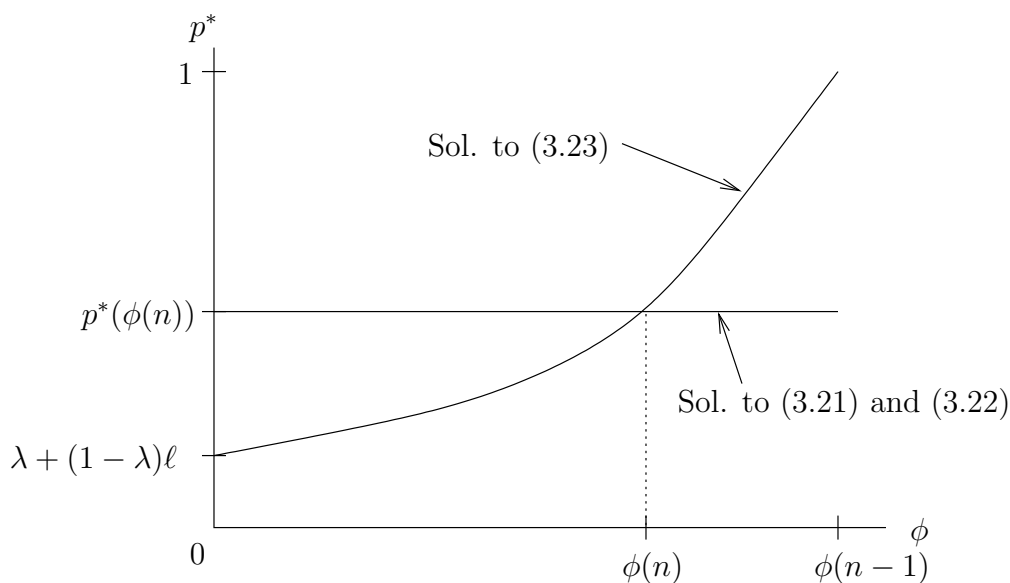
Moreover, $Q(\phi)$ is weakly increasing ϕ and $\pi(\phi)$ is weakly decreasing in ϕ .

3.3.5 Analysis of the mixed strategy equilibrium

In this section, I consider implications of the existence of a mixed strategy equilibrium of the game. In particular, I discuss the following questions. What selection criteria would



(a) Equations (3.21) and (3.22) in price quantity space. The equations both define p^* as a function of q^* and satisfy a single crossing property which allows for a unique solution $(q^*(\phi(n)), p^*(\phi(n)))$.



(b) The solution to (3.21) and (3.22) versus (3.23). Equation (3.23) defines p^* as a downward sloping function of ϕ , thus giving a unique solution $(\phi(n), p^*(\phi(n)))$.

Figure 3.6: The solution to the system of equations (3.21)-(3.22). The single crossings shown in panels (a) and (a) imply that the solution is unique.

I need to predict that a mixed strategy equilibrium of the game would obtain in the data? What do the strategy and price functions for the particular equilibrium derived above imply about how information is revealed in the model? And finally, is this equilibrium unique among mixed strategy equilibria, and if so, what are the useful comparative statics? These questions lead to empirical implications, some of which I will examine in Chapter 4.

The model admits multiple equilibria. This means that in order to understand the empirical implications of the model, one must have a consistent method of selecting among different equilibria. For static signaling games, the D1 refinement of Cho and Kreps (1987) provides such a selection criterion. In my dynamic setting it is not clear how to apply the D1 refinement, so I take the following simple approach. Observe that by construction, the mixed strategy equilibrium delivers at least as much per period payoff as the separating equilibrium when reputation alone is not sufficient to implement full market discipline. This observation leads directly to the following corollary of Proposition 10.

Corollary 2. *Suppose Assumptions 1 and 2 hold. The mixed strategy equilibrium delivers weakly greater value to both issuer types for all levels of reputation.*

I now move on to discussing the properties of the mixed strategy equilibrium. The equilibrium quantity issued in the mixed strategy equilibrium q^* , is an increasing function of reputation. Figure 3.7 plots q^* versus reputation ϕ . For $\phi \leq \underline{\phi}$, reputation is too low to make misreporting a bad type asset attractive to the opportunistic underwriter, so $Q = \hat{q}$ and the equilibrium reduces to the static signaling equilibrium of Proposition 6. For $\underline{\phi} \leq \phi \leq \hat{\phi}$ the equilibrium quantity must make the opportunistic issuer indifferent, conditional on facing a good type asset, between signaling the true type of the asset by choosing the quantity \hat{q} versus issuing q^* . This implies that q^* must be a strictly increasing function of ϕ for $\underline{\phi} \leq \phi \leq \bar{\phi}$ since the equilibrium price p^* is a decreasing function of ϕ over this interval. The equilibrium quantity issued reaches one at $\hat{\phi}$. The shape of q^* leads directly to the following implication.

Implication 1. *Suppose Assumptions 1 and 2 hold. Issuer reputation and retention should be negatively correlated both conditionally given the issuer's report of asset quality and unconditionally.*

Implication 1 is useful in that it provides a testable empirical prediction. If both signaling and reputation are available as a means of credible information transfer and the market conditions are such that reputation alone cannot enforce, then the amount of signaling employed by the issuer should be decreasing in her reputation, conditional on her public report of asset quality. At the same time, the implication holds unconditionally in that although the issuer's retention does not depend on her reputation when she reveals that the asset is the bad type, a negative, though less so, correlation between her reputation and retention still obtains.

The next interesting implication concerns the flow of information in equilibrium. In a standard static signaling model, the quantity issued is a perfect signal of asset quality. However, in the equilibrium presented above, quantity is a signal of issuer reporting strategy.

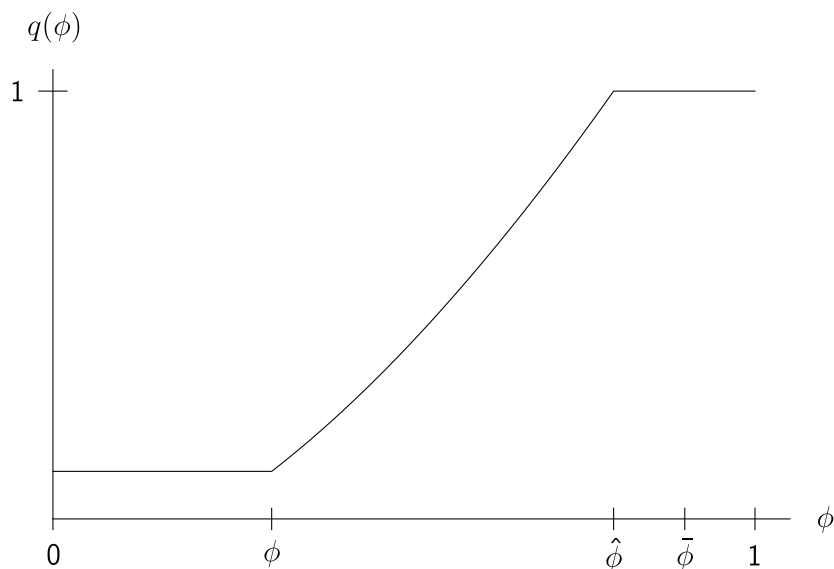


Figure 3.7: A plot of $q^*(\phi)$ versus ϕ . For $\phi \leq \underline{\phi}$, the issuer simply issues the quantity from the least cost separating equilibrium. For $\underline{\phi} \leq \phi \leq \hat{\phi}$, the issuer chooses an increasing quantity in her reputation. For $\phi > \hat{\phi}$ the issuer chooses to sell the entire asset.

This interpretation of Q means that given a level of reputation ϕ , the investors can infer the reporting strategy $\pi(\phi)$ of the opportunistic issuer by observing issuer retention. When investors observe a relatively large Q , they can infer that the probability the issuer will misreport a bad asset is high, and will price assets accordingly. Hence, the opportunistic issuer faces a trade-off between choosing a perfect signal of asset quality, which implies no lemons discount, and choosing an imperfect signal of asset quality, which gives the issuer the ability to sell a larger fraction of the asset to the investors. This trade-off importantly depends on the current level of issuer reputation. Figure 3.8 shows the equilibrium reporting strategy of the opportunistic issuer. For $\phi \leq \hat{\phi}$, an increase in reputation increases instantaneous gains to misreporting a bad type asset, and hence the issuer increases the probability she will indeed misreport. The issuer offsets the price impact of this increase by retaining less of the asset. The shape of π leads to the following implication.

Implication 2. *Suppose Assumptions 1 and 2 hold. An asymmetry of information should persist ex post for high levels of reputation. That is, investors cannot infer the issuer's private information from retention alone.*

Implication 2 implies that observing asset performance after issuance provides investors with valuable information. This is important in the context of allegations of fraudulent activity of issuer's. If investors could perfectly infer an issuer's private information from observing issued quantity, there would be no scope for fraud. However, since the mixed

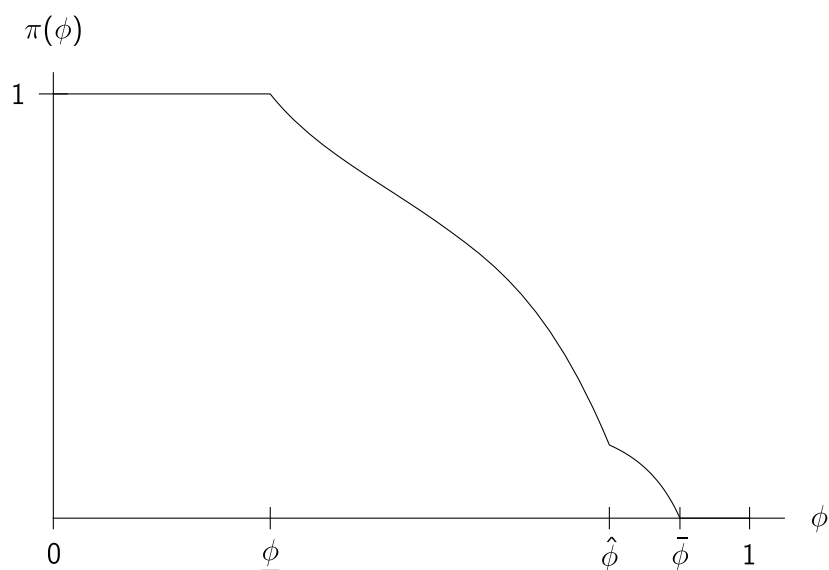


Figure 3.8: Plots of $\pi(\phi)$ versus ϕ . For $\phi \leq \underline{\phi}$ the issuer simply plays the least cost separating equilibrium and never misreports a bad type asset. For $\underline{\phi} \leq \phi \leq \bar{\phi}$ the issuer increases the probability she will misreport a bad type asset as her reputation increases. The kink at $\hat{\phi}$ arises from the constraint that quantity cannot be greater than one. Finally for $\phi > \bar{\phi}$ the issuer always misreports a bad type asset.

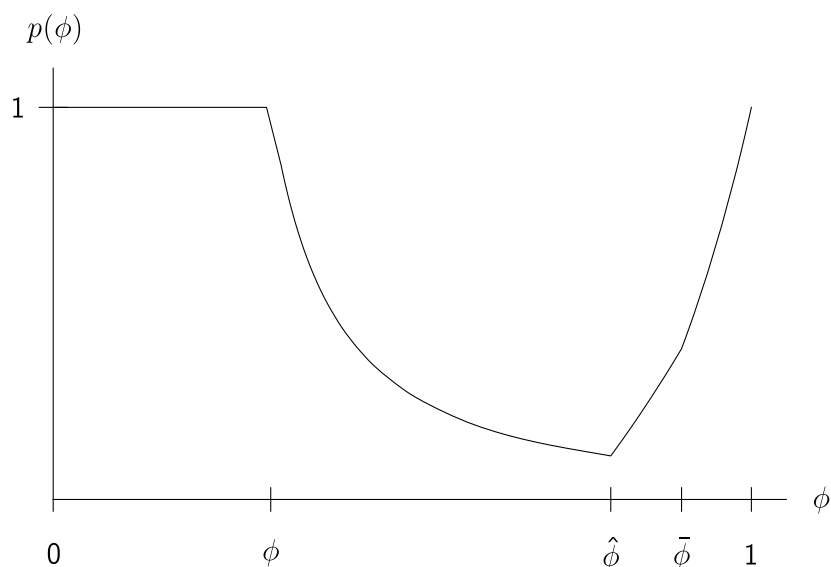


Figure 3.9: A plot of $p^*(\phi)$ versus ϕ . For $\phi \leq \underline{\phi}$, the opportunistic issuer never reports that a bad type asset is the bad type, hence the equilibrium price for a reportedly good type asset must be one. For $\underline{\phi} \leq \phi \leq \bar{\phi}$, the equilibrium price is a decreasing function of reputation. For relatively low (high) levels of reputation, the potential gain from misreporting a bad type asset is relatively small (large) since any probability that the issuer misreports a bad type asset impacts prices more (less) at low (high) levels of reputation.

strategy equilibrium implies that an ex post asymmetry of information is possible, the issuer knowingly misreports asset type and fraud is feasible.

The fact that an asymmetry of information can persist post issuance leads to an interesting observation about the investors beliefs on asset quality as it relates to issuer reputation. The opportunistic issuer's equilibrium reporting strategy implies that p^* is a U-shaped function of ϕ . Figure 3.9 plots the equilibrium price function. For $\phi \leq \underline{\phi}$, the opportunistic issuer never reports that a bad type asset is the good type, hence the equilibrium price for a reportedly good type asset must be one. For $\underline{\phi} \leq \phi \leq \bar{\phi}$, the equilibrium price is a decreasing function of reputation. For relatively low (high) levels of reputation, the potential gain from misreporting a bad type asset is relatively small (large) since any probability that the issuer misreports a bad type asset impacts prices more (less) at low (high) levels of reputation. Hence, in equilibrium the investors place a low probability on the issuer misreporting a bad type asset for low levels of reputation and the equilibrium price is high. For $\phi \geq \bar{\phi}$ the investors know that an opportunistic issuer will always misreport a bad type asset, but by definition the probability that investors are facing an opportunistic issuer decreases as ϕ increases, so equilibrium price increases with ϕ for $\phi \geq \bar{\phi}$. The shape of p^* leads to the following implication.

Implication 3. *The investors' beliefs about the quality of the asset is u-shaped in the issuer's reputation, conditional on the report of asset quality.*

The final aspect of the mixed strategy equilibrium which requires some discussion is the upper threshold $\bar{\phi}$ that represents the level of reputation past which the opportunistic issuer will “cash in” on her reputation. For $\phi \geq \bar{\phi}$ equilibrium proceeds from securitization will be high regardless of the opportunistic issuer's reporting strategy, so that if the opportunistic issuer achieves a reputation $\phi \geq \bar{\phi}$, she strictly prefers to report all assets as the good type. This outcome of the model is somewhat paradoxical when considering the benefits of reputation effects for providing incentives for truth telling. Once an opportunistic issuer has a high enough reputation, her optimal strategy under the mixed strategy equilibrium calls for a complete lack of reporting discipline.

3.4 Extensions

3.4.1 Binary signal space

In this section I consider the case when the issuer may not divide the asset and hence must sell the entire asset to investors or retain the asset. I maintain the definition equilibrium, with the additional restriction of the strategy space that $Q \in \{0, 1\}$. Assumption 1 implies that when the issuer has zero reputation she will only sell bad type assets since there is no mechanism that allows the separation of good type assets from bad and the pooled price is lower than the issuer's value for retaining the asset. Thus, the only equilibrium of the game when reputation reaches zero is for the issuer to retain all good assets and sell all bad assets. This means that losing reputation in this setting results in a greater loss of value than when the issuer can signal her private information through costly information regardless of her reputation. As a result, the indivisibility of assets acts as a commitment mechanism allowing the loss of reputation to be a more powerful incentive mechanism.

First I consider the existence of a truth telling equilibrium in this new setting.

Proposition 11. *A truth telling equilibrium exists if and only if $\gamma \geq \frac{\lambda}{1-\ell}$.*

Comparing Proposition 11 with Proposition 8, it is clear that a looser restriction on parameters is required to implement truth telling when the issuer's quantity choice is restricted to all or nothing. This is precisely because reputation is a more powerful incentive mechanism in this new setting and hence the issuer can credibly commit to truthfully reporting asset type for a wider range of parameters.

It remains to characterize an equilibrium for parametrizations which do not allow the existence of a truth telling equilibrium, i.e. when $\gamma < \frac{\lambda}{1-\ell}$. The method for constructing is essentially the same as in Section 3.3.4. First, I assume a particular form for the demand schedule

$$P(\phi, q) = \begin{cases} 1 & \text{for } q = 0 \\ p^*(\phi) & \text{for } q = 1. \end{cases} \quad (3.24)$$

Then the analogs to equations (3.10), (3.11), and (3.13) in this setting are

$$p^*(\phi) \geq \gamma \tag{3.25}$$

$$p^*(\phi) \geq \ell + \gamma(V(\phi^B) - V(0)) \tag{3.26}$$

$$L(\phi) = \frac{\gamma}{1 - \gamma\lambda}(p^*(\phi) - (1 - \gamma)V(0)) \tag{3.27}$$

for $Q = 1$. Inequality (3.25) states that the issuer must prefer to sell a good type asset at price $p^*(\phi)$ rather than retain it. Inequality (3.26) states that the issuer must prefer to misreport a bad type asset and sell it at price $p^*(\phi)$ rather than accurately report it and sell it at price ℓ . Equation (3.27) simply follows from equations (3.5) and (3.6). Inequalities (3.25) and (3.26) only apply to levels of reputation for which there is a positive probability the issuer misreports a bad type asset and still chooses to sell a good type asset. I assume there exists a subinterval for which (3.26) binds, so that the issuer misreports the bad type asset with probability strictly between zero and one. Then finding equilibrium strategies reduces to a problem of solving a system of equations in a certain number of unknowns. The following proposition summarizes the resulting mixed strategy equilibrium.

Proposition 12. *Suppose $1 - \ell > \gamma\lambda$. There exists $\underline{\phi}'$ and $\bar{\phi}'$ and an equilibrium in which the opportunistic issuer*

- *retains good assets and sells bad assets for low levels of reputation, that is*

$$Q(\phi) = 0 \text{ for } \phi \leq \underline{\phi}',$$

- *sells both asset types and misreports a bad type asset with positive probability for mid-range levels of reputation, that is*

$$Q(\phi) = 1, \text{ and } 0 < \pi(\phi) < 1 \text{ for } \underline{\phi}' < \phi < \bar{\phi}',$$

- *sells both asset types and always misreports a bad type asset for high levels of reputation, that is*

$$Q(\phi) = 1 \text{ and } \pi(\phi) = 0 \text{ for } \phi \geq \bar{\phi}'.$$

Moreover, $\pi(\phi)$ is weakly decreasing.

Restricting of quantities to the set $\{0, 1\}$ substantially changes the equilibrium reporting strategy relative to the general case. Figure 3.10 compares the equilibrium reporting strategies for Propositions 10 and 12. The first difference is that there is a discontinuity in the reporting strategy for the indivisible asset case. This discontinuity exists because quantities cannot continuously adjust with reputation to make the issuer indifferent between retaining a good asset and selling it in its entirety. Notably, the indivisibility of assets causes the issuer to refrain from sending inaccurate reports at levels of reputation for which she would do so if she could retain part of the asset. In other words, $\underline{\phi} < \underline{\phi}'$. Similarly, the issuer will send

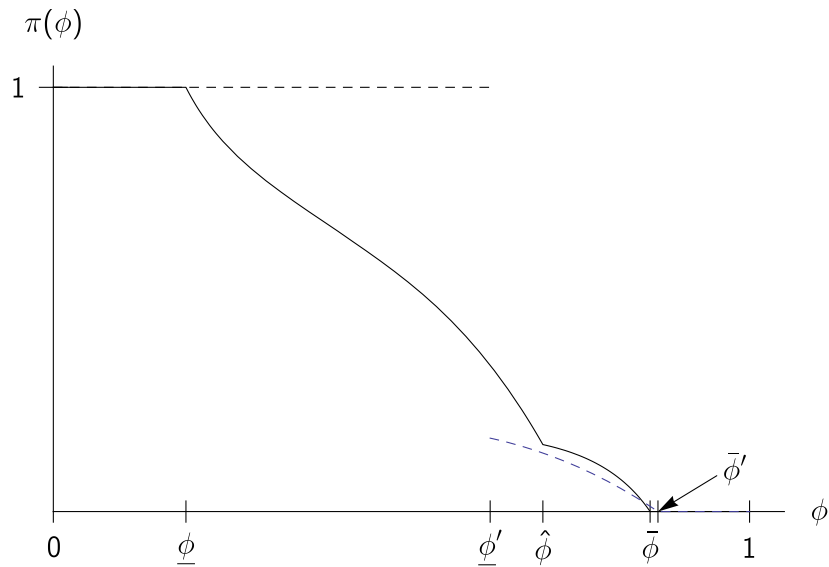


Figure 3.10: A comparison of $\pi(\phi)$ for the divisible and indivisible asset settings. The solid curve is the reporting strategy for the divisible setting, while the dashed curve is the reporting strategy for the indivisible setting. $\underline{\phi} < \underline{\phi}'$ implies that the indivisibility of assets causes the issuer to refrain from sending inaccurate reports at levels of reputation for which she would do so if she could retain part of the asset. Similarly, $\bar{\phi} < \bar{\phi}'$ implies the issuer will send an inaccurate report with probability 1 for lower levels of reputation when she can retain a part of the asset than when the asset is indivisible.

an inaccurate report with probability 1 for lower levels of reputation when she can retain a part of the asset than when the asset is indivisible. In other words, $\bar{\phi} < \bar{\phi}'$.

The differences between reporting strategies described above have important implications for policies which aim to reduce the probability that the issuer sends an inaccurate report. Disallowing issuer retention is actually one possible mechanism to decrease the amount of lying in equilibrium. The drawback to such a restriction is that issuer payoffs will decrease. In addition, for low levels of reputation, the market for ABS will be a market for lemons.⁶ Thus the issuer, conditional on having a low initial reputation, receives a lower value for the game when the asset is indivisible than when the asset is perfectly divisible. However, this means that the cost of a loss in reputation is potentially greater when the issuer is not free to choose quantity.

If the opportunistic issuer starts with a high enough reputation, she will send an inaccurate report at least part of the time. In this case the relationship between the probability of sending an inaccurate report and the divisibility of assets is not clear. For intermediate levels of reputation the issuer will want to sell only part of the asset to investors but is constrained to sell the entire asset, and thus may lie with greater frequency. However for high levels of reputation, the threat of losing reputation becomes more powerful for the indivisible asset case because the market will revert to a market for lemons when the issuer loses her reputation. As a result, the issuer has less of an incentive to send an inaccurate report for the indivisible asset case when she has a high reputation.

3.4.2 Risky assets

In this section I consider a richer specification for the cash flow of assets. As above, I assume there are two types of assets: good and bad. Both asset types produce a cash flow in the next period contained in the set $\{x^l, x^h\}$ with $x^l < x^h$. Good assets have a probability p_G of producing a cash flow x^h such that

$$p_G x^h + (1 - p_G) x^l = 1$$

while bad assets have a probability p_B of producing x^h such that

$$p_B x^h + (1 - p_B) x^l = \ell.$$

These asset cash flows mean that the expected cash flow of the asset conditional on its type remains unchanged.

The next step is to characterize the reputation updating function in this new setting. Lemma 1 still applies since its proof does not depend on the distribution of asset cash flows.

⁶Downing, Jaffee, and Wallace (2009) show that the “to be announced” MBS market is indeed a market for lemons. This finding follows from the indivisibility of assets and the anonymity of issuers. The implication of the current model is that such a finding can persist in a setting with reputation effects. This does not depend on the stylized type of reputation I consider here and would obtain even when punishment strategies are allowed since the parameters are such that punishment strategies are not sufficient to support a truth telling equilibrium.

So once again, reputation is not updated during the securitization phase and her current quantity choice does not affect her current reputation. The equilibrium reputation updating function is thus given by Bayes rule as follows

$$f(g, x^h, \phi) = \phi^S = \frac{\phi}{1 + (1 - \phi)(1 - \pi(\phi))^{\frac{1-\lambda}{\lambda} \frac{p_B}{p_G}}} \quad (3.28)$$

$$f(g, x^l, \phi) = \phi^F = \frac{\phi}{1 + (1 - \phi)(1 - \pi(\phi))^{\frac{1-\lambda}{\lambda} \frac{1-p_B}{1-p_G}}} \text{ if } p_G < 1 \text{ and } 0 \text{ otherwise} \quad (3.29)$$

$$f(b, x, \phi) = \phi^B = \frac{\phi}{\phi + (1 - \phi)\pi(\phi)} \quad (3.30)$$

The first observation is that the separating equilibrium of Proposition 9 still holds. In that equilibrium, the assets ex post cash flows do not affect the future play of the game. This is precisely because the issuer credibly reveals her private information about the asset's expected cash flow via her quantity choice. Therefore, no asymmetry of information about the current asset persists once the current period quantity choice is observed. As a result, earlier statements about the inability of the separating equilibrium of Proposition 9 to explain the existence of fraud do not depend on the stylized asset cash flow distribution of the previous section.

Since the assets expected cash flows have not changed, the existence of a pooling equilibrium remain unchanged as well. Specifically, Assumption 1 still implies that pooling equilibria do not exist. That assumption states that the issuer would rather retain a good type asset then sell it at a price equal to the ex ante expected cash flow of the asset.

The next task is to characterize the conditions under which a truth telling equilibrium exists for the game with risky asset cash flows. First consider the case when $p_G = 1$. This will imply that investors will know that the issuer is the opportunistic type when a cash flow of x^l follows a report that the asset is the good type. In this case, the restriction on parameters needed for the existence of a truth telling equilibrium again places an upper bound on the discount rate of the issuer, however now the bound also depends on the probability that the bad type asset produces the cash flow x^h as is stated in the following proposition.

Proposition 13. *There exists a truth telling equilibrium if and only if $\gamma \geq \frac{1}{\lambda(1-p_B)+\ell}$*

An important feature of Proposition 13 is that the restriction required for the existence of a truth telling equilibrium is more strict for the case of risky asset cash flows than for the setting of the previous section. This is because there is still a chance an opportunistic agent may not get “caught” misreporting the bad type asset. Thus, the issuer has less of an incentive to accurately report a bad type asset when asset cash flows are risky. In reality asset cash flows are indeed risky, making the empirical observability of truth telling equilibrium less likely.

If $p_G < 1$, then a cash flow of x^l does not reveal an inaccurate report. This again decreases the issuer's incentive to accurately report the bad type asset since even if the asset

yields the cash flow x^l , the investors cannot be sure that the issuer is indeed the bad type. This leads directly to the following negative result.

Proposition 14. *If $p_G < 1$, a truth telling equilibrium does not exist .*

The intuition behind Proposition 14 is as follows. Suppose that a truth telling equilibrium did indeed exist and $p_G < 1$. Then the equilibrium reputation updating function given in equations (3.28) - (3.30) imply that $\phi^S = \phi^F = \phi^B = \phi$. Thus, a cash flow of x^l does not affect reputation and the issuer would never lose any reputation from misreporting bad type assets. At the same time, if investors believe that the issuer is following the truth telling strategy, then they must price a reportedly good type asset accordingly, leading to one period gains for the issuer from misreporting a bad type.

The non-existence of a truth telling equilibrium for the case of risky asset cash flows suggests that other equilibria, such as the mixed strategy equilibrium of the previous section, may exist. Unfortunately, this setting will not admit a solution technique for a mixed strategy equilibrium like the one presented in the previous section. To see this, assume that the demand schedule is given by equation (3.8). Then $(q^*(\phi), \pi^*(\phi))$ is a natural candidate equilibrium strategy pair. However, the analog to equation (3.11) is

$$q^*(\phi)p^*(\phi) - \ell \geq \gamma(p_B(V(\phi^B) - V(\phi^S)) + (1 - p_B)(V(\phi^B) - V(\phi^F))) - (1 - q^*(\phi))\gamma\ell \quad (3.31)$$

which depends on the value function evaluated at three different levels of reputation. Hence more constraints on equilibrium strategies would be needed to pin down a particular mixed strategy equilibrium.

3.4.3 Path dependent beliefs

In this section, I consider investor beliefs that may depend on the entire history of the game, rather than just current reputation and current quantity. The investors' demand curve at time t is then given by a function $P_t(q, \mathcal{H}_t) : [0, 1] \times \mathbb{H}_t \rightarrow [0, 1]$ where \mathbb{H}_t is the set of all possible histories of the game up to time t . The issuer then implements a reporting strategy $\pi_t(\mathcal{H}_t) : \mathbb{H}_t \rightarrow [0, 1]$ and a quantity strategy $Q_t(\mathcal{H}_t) : \mathbb{H}_t \rightarrow [0, 1]$. I maintain the assumptions that the issuer always reports a good type asset as good, and always issues the quantity one when reporting the asset is the bad type.⁷ The definition of equilibrium in this setting replaces conditions (4) and (5) of Definition 4 with

5. Investors earn zero expected profits: $P_t(Q^i(\mathcal{H}_t), \mathcal{H}_t) = E[X_{t+1} | \mathcal{H}_t, Q^i(\mathcal{H}_t)]$ for $i \in \{H, O\}$

6. An equilibrium is separating if $P_t(Q^i(\mathcal{H}), \mathcal{H}_t) = 1$.

⁷These assumptions are without loss of generality as a variant of Lemma 1 holds in this more general setting.

The first result is that an equilibrium satisfying Definition 4 will satisfy the definition of equilibrium in this more general setting.

Proposition 15. *Suppose the quadruple $(\hat{P}, \hat{Q}^H, \hat{\pi}, \hat{Q}^O)$ satisfies Definition 4, then there exists an equilibrium of the game with history dependent strategies such that $P(q, \mathcal{H}_t) = \hat{P}(q, \phi)$, $Q_t^H(\mathcal{H}_t) = \hat{Q}^H(\phi)$, $\pi_t(\mathcal{H}_t) = \hat{\pi}(\phi)$, and $Q_t^O(\mathcal{H}_t) = \hat{Q}^O(\phi)$ where $\phi = \mathbb{P}(\text{Issuer is Honest Type}|\mathcal{H}_t)$*

Proof. Observe that if strategies are defined as in the proposition and

$$\phi = \mathbb{P}(\text{Issuer is Honest Type}|\mathcal{H}_t),$$

then conditions (4)-(5) of Definition 4 imply conditions (4)-(5) of this section. \square

The above proposition is not surprising. It simply states that stationary Markov equilibria are also equilibria under the more general definition. It is of greater interest to determine whether there are equilibria under the more general definition that do not obtain under Definition 4. I show that there are truth telling equilibria under the general definition that do not exist under Definition 4, however the parameter restriction required for their existence coincides exactly with that of Proposition 8.

Proposition 16. *Suppose $\phi_0 = 0$. Then there exists a truth telling equilibrium with $Q_t(\mathcal{H}_t) > \hat{q}$ for some \mathcal{H}_t if and only if $\gamma \geq \frac{1}{\lambda + \ell}$.*

Proof. Note that since $\phi_t = 0$ for all $t \geq 0$ by Bayes rule, hence it is enough to consider the strategies of the opportunistic type issuer. First suppose there exists a truth telling equilibrium given by $Q_t(\mathcal{H}_t)$ and $\pi_t = 1$. Let \tilde{Q} be defined as follows

$$\tilde{Q} = \sup\{Q_t(\mathcal{H}_t) | t \geq 0 \text{ and } \mathcal{H}_t \in \mathbb{H}_t\}. \quad (3.32)$$

Note that $\tilde{Q} > \hat{q}$. For all $\epsilon > 0$, there exist (t, \mathcal{H}_t) such that $\tilde{Q} - Q_t(\mathcal{H}_t) < \epsilon$ by the definition of supremum. Let (t, \mathcal{H}_t) be such a pair. The one-shot deviation principal implies that

$$Q_t(\mathcal{H}_t) + (1 - Q_t(\mathcal{H}_t))\gamma\ell - \ell \leq L_t \quad (3.33)$$

where L_t is the discounted loss faced by the issuer after misreporting a bad type asset given the history \mathcal{H}_t . By the definition of \tilde{Q} , I have

$$L_t \leq \gamma \left(\frac{\lambda(\tilde{Q} + (1 - \tilde{Q}))\gamma + (1 - \lambda)\ell}{1 - \gamma} - \frac{\lambda(\hat{q} + (1 - \hat{q}\gamma) + (1 - \lambda)\ell)}{1 - \gamma} \right) = \gamma\lambda(\tilde{Q} - \hat{q}). \quad (3.34)$$

But

$$Q_t(\mathcal{H}_t) + (1 - Q_t(\mathcal{H}_t))\gamma\ell - \ell = (1 - \gamma\ell)(\tilde{Q} - \epsilon - \hat{q}). \quad (3.35)$$

This implies that for all $\epsilon > 0$

$$(1 - \gamma\ell)(\tilde{Q} - \epsilon - \hat{q}) \leq \gamma\lambda(\tilde{Q} - \hat{q}) \quad (3.36)$$

which implies $\gamma \geq \frac{1}{\lambda + \ell}$ since $\tilde{Q} > \hat{q}$.

Now suppose $\gamma \geq \frac{1}{\lambda + \ell}$. I'll show that a truth telling equilibrium is given by $Q_t = 1$, $\pi_t = 1$, $P(q, H_t) = 1$ if no misreports have been made and

$$P(q, H_t) = \begin{cases} 1 & \text{if } q \leq \hat{q} \\ \ell & \text{if } q > \hat{q} \end{cases}$$

otherwise. Note that the continuation value of the issuer just depends on whether or not she has made a misreport. Let L be the discounted loss in continuation value faced by the issuer if she misreports a bad type asset then

$$L = \gamma \left(\frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} - \frac{\lambda(\hat{q} + (1 - \hat{q})\gamma + (1 - \lambda)\ell)}{1 - \gamma} \right) = \gamma\lambda(1 - \hat{q}). \quad (3.37)$$

To see that the above strategies indeed constitute an equilibrium observe that

$$\begin{aligned} L = \gamma\lambda(1 - \hat{q}) &= \gamma\lambda \frac{1 - \ell}{1 - \gamma\ell} \\ &\geq 1 - \ell. \end{aligned}$$

Thus the discounted loss in continuation value is greater than or equal to the one-shot gains from misreporting a bad type asset and the above strategies constitute a truth telling equilibrium for $\phi_0 = 0$. \square

Proposition 16 shows that although allowing for path dependent strategies does mean that a truth telling equilibrium is supportable without the type of reputation considered in this paper, the restriction on the parameters required for truth telling does not change.

3.5 Conclusion

In this chapter, I have presented a model of an issuer of a generic asset which unifies signaling and reputation effects. In my model, a lemons problem arises due to asymmetric information about the quality of assets. Partial retention of an asset by the issuer is a credible signal of asset quality since the issuer is impatient relative to investors causing retention to be costly. Imperfect information over issuer preferences induces a market reputation for the issuer. A high reputation can increase payoffs for the issuer by reducing the fraction of the asset the issuer retains in equilibrium while decreasing the lemons discount relative to an identically structured sale by an issuer with a low reputation. Reputation effects do not imply that an issuer will be more likely to perfectly reveal her information.

The implications of my model call into question the benefits of reputation as a substitute for regulation and oversight for imposing market discipline. Although conceptually appealing, the assertion that issuers of ABS, for example, will behave in the best interests of the wider markets as a consequence of protecting their reputations misses an important

point. The benefit of having a good reputation may be due to the ability to “cash in” on a high reputation in the future. In the case of my model, an opportunistic issuer will cash in on a good reputation by misreporting bad type assets. In contrast, signaling in the absence of reputation can force issuers to reveal the true full information value of any assets underlying ABS at the cost of reducing equilibrium payoffs.

While I mostly discuss the model by referring to asset backed securities, the theory developed in this paper applies equally well to other important financing problems. The key features of the model are that the issuer has valuable private information, a means by which to signal that information in a single period, and a means by which to gain a reputation for accurate reports. For example, the model could refer to a venture capitalist raising funds from limited partners. She may need to maintain a larger stake as general partner if she does not yet have a good track record of matching investment projects with stated fund goals. Similarly, a private equity firm may need to put up a larger amount of capital to implement a leveraged take over if that firm does not have a long history of accurate analysis of target firm prospects. Finally, both a private equity firm and venture capitalist may at some point find it advantageous to exploit a good reputation for one period gains.

I do not make specific policy suggestions since the model is silent on the effects of different equilibrium strategies on markets for underlying assets and collateral, like the primary mortgage market. However, depending on the goals of a regulator, the model offers the following advice. If the policy goal is to ensure accurate information disclosure, then a regulator should adopt a policy which encourages perfect signaling of asset quality. Providing a legal means for issuers to publicly disclose their holdings and commit to hold them to maturity could increase accurate information transmission. In contrast, if the policy goal is maximizing payoffs for issuers then reputation and signaling should both be facilitated, i.e. through a centralized repository of past deal performance and a credible means to reveal issuer holdings.

Chapter 4

CMBS issuer reputation and retention

4.1 Introduction

The theoretical results presented in the previous chapter lead to important empirical predictions that can and should be tested. In this chapter I will examine Implication 1 of the previous chapter, namely are issuer reputation and retention negatively correlated. This task brings up two main challenges. First, I must choose a particular market in which to make this inquiry, and as the title of this chapter suggests this will be the commercial mortgage backed securities (CMBS) market. Second, I must define consistent ways of measuring both issuer reputation and retention. The main result of this exercise is that issuer reputation and retention are indeed negatively correlated, controlling for other issuer characteristics, deal characteristics, and time trends.

The (CMBS) market represents a unique laboratory to study the relationship between reputation and signaling via retention for two main reasons. First, it is concentrated among a small number of issuers so that a given issuer has the opportunity to develop a reputation. Second, unexpectedly poor performance in this market occurred before the most recent financial crisis, leading to potential updates to some issuers' reputation among investors.

In order to conduct the analysis I need measurements of both issuer reputation and retention. To measure issuer reputation, I track the number of downgrades an issuer has had on past deals. This measure is motivated by the model: issuer reputation should be updated according to past performance. This differs from previous empirical work, for example Lin and Paravisini (2010) or Titman and Tsyplakov (2010) TS, which use market based proxies for reputation like market share or stock price performance. To measure retention I use a particular feature of the CMBS market. CMBS deals often contain an unrated tranche which is *likely* retained by the issuer for some time. Retention is then the proportion of the below investment grade principal which is unrated. To my knowledge, there has not been significant empirical attempts to test signal based theories of security design using an actual measure of issuer retention (signaling). Thus, my measure of issuer retention is a potentially unique and valuable tool for empirical work going forward.

One further difficulty is that Implication 1 is conditional on the issuers report of asset quality, which is difficult to observe for the econometrician. However, it will be more difficult to detect this effect in the data *without* conditioning on the report of asset quality. Hence, if an unconditional analysis suggests that the effect is present, the result should only be made stronger by including information about the issuers report of asset quality.

This negative correlation between issuer reputation and retention documented in this chapter is similar in spirit to effects discovered by Lin and Paravisini (2010) and Sufi (2007) in the syndicated loan market. Lin and Paravisini show that an exogenous shock to reputation, namely exposure to the accounting fraud scandals of the early 2000's, causes an increase in lead arranger share of a syndicated loan. They argue that this means that a change in reputation can affect the structure of incentive contracts used to mitigate moral hazard problems induced by the unobservable costly monitoring technology of the lead arranger. In the current setting, monitoring is not a primary concern due to the specific nature of ABS in which managers have no control rights once a security is issued, and as a result the motivating friction is adverse selection rather than moral hazard. Sufi finds that lead arrange capital contribution decreases in market share, although never reaches zero. This result is likely due to adverse selection problems decreasing with reputation (market share) while some capital contribution is always required to combat moral hazard.

Another related article is Titman and Tsyplakov (2010), who investigate the link between mortgage originator performance, commercial mortgage spreads and default rates, and CMBS structure. They find originator's with poor stock price performance originate lower quality mortgages which also end up in CMBS deals with more subordination (less AAA rated principal). While the focus of their paper is on mortgage originators, the focus of the empirical work in this paper is on CMBS issuers. Moreover, I find that by measuring issuer reputation in terms of real past performance, rather than in terms of a market based proxy, there is no affect of issuer reputation on the size of the AAA tranche. To the extent that a CMBS issuer is often the same entity as a mortgage originator, this result indicates that Titman and Tsyplakov's market based measure of originator reputation is substantively different than my performance based measure

4.2 Institutional Background

In the CMBS market, commercial mortgages are pooled together and placed in a passive trust by a sponsoring entity, usually an investment bank. Bonds of differing seniority and maturities, often called *tranches* or *classes*, are then sold by the sponsor to various investors. I will refer to a series of tranches all backed by the same mortgage pool as a *deal*. The structure of a typical CMBS deal is similar to the perhaps more familiar residential mortgage backed security (RMBS), in that pre-specified rules govern the distribution of principal and interest payments to the classes.

Before the bonds are marketed to investors, but after the characteristics of the pool of mortgages are fixed, a rating agency will assign a rating to at least some of the classes.

Ratings are assigned given the hard information provided by the sponsor, for example mortgage characteristics such as loan-to-value ratio, weighted average coupon, and geographic location, but could also take into account any soft information the sponsor chooses to reveal. The ratings agency might indicate to the sponsor that certain classes must be given more credit support, i.e. have a larger amount of principal with lower seniority in the deal, in order to obtain a certain rating. Once ratings have been assigned, the sponsor creates a prospectus that contains both hard and soft information as well as the ratings assigned to each class. This prospectus is available to any investor interested in purchasing a class from a given deal.

The sponsoring entity of a particular CMBS deal plays the role of the issuer in my model. Although some sponsors originate their own mortgages, other sponsors simply purchase mortgages from originators. In any case, sponsors will typically hold an inventory of mortgages waiting to be securitized. At this point, even sponsors who have played no role in the loan origination process have the opportunity to gain valuable, and potentially non-verifiable, information about the probability that a given mortgage will default and what recovery rates might be given default.

Although CMBS have existed since the mid 1980s, the market truly began to expand in 1995. The yearly issuance of private label (deals not issued by one of the government sponsored entities) expanded from around \$15 billion in 1995 to more than \$230 billion in 2007.¹ With this expansion, the CMBS market became a major source of capital for the commercial real estate market.

4.3 Data and Measurement

The data come from the Lewtan Technologies ABSnet dataset, Bloomberg, and Commercial Mortgage Alert and consist of structure and performance data for 703 CMBS deals issued between 1989 and 2008. Descriptive statistics for the dataset are displayed in Table 4.1. Two important features of the data to note are the mean deal size of \$1.43 billion and the mean weighted average loan to value (WALTV) of 66%. The mean deal size indicates that the average CMBS deal is quite large and that the data comprise around one trillion dollars of total issuance. Thus, although the CMBS market is smaller than the prime and subprime RMBS markets, it is still quite large. The mean WALTV indicates that commercial mortgages tend to be less leveraged than subprime residential mortgages. Although an exact characterization of the overall size of the CMBS market is not readily available, comparing the aggregate issuance contained in these data with other sources² indicate that the data represent a nearly exhaustive sample. My analysis can only be run on a subsample of 457 deals due to missing data for some covariates. Characteristics of this subsample match those of the total sample well, implying my subsample is representative of the larger sample.

¹Source: www.CMalert.com

²For instance, www.CMalert.com, or www.CMBS.com.

Table 4.1: Summary statistics for pool and deal characteristics. Percentage AAA is the proportion of principal rated AAA at issuance. Deal size is the total face value of all tranches at issuance in \$ billions. Weighted Average Coupon is the weighted average coupon rate of the mortgages in the pool. Weighted Average LTV is the weighted average loan-to-value ratio in the pool. Debt Service Ratio is the weighted average ratio of debt service payments to net operating income. Percentage office is the percentage of mortgages collateralized by office space. Conduit Dummy indicates deals comprised of conduit loans. # Loans is the number of loans in the pool (in hundreds). # Deals is the number of deals issued by the issuer prior to the current observation deal.

	Variable	N	Mean	Median	Standard Deviation
	Percent AAA	696	0.765	0.827	0.199
Percent of Below Investment Grade Unrated		703	0.346	0.313	0.278
Total Issuer Downgrades		703	0.551	0.000	1.132
Issuer time in Sample		703	5.147	4.528	4.016
Weighted Average Loan Spread to Treasury		659	0.019	0.018	0.011
Weighted Average Loan-to-Value		636	0.635	0.683	0.154
Debt Service Coverage Ratio		590	1.639	1.490	0.561
Percent Office		595	0.293	0.266	0.180
Percent Hotel		521	0.108	0.078	0.146
Percent Industrial		547	0.087	0.073	0.069
Number of Loans in Deal (in 100s)		645	2.657	2.090	2.059
Average Loan Size in Deal (in \$Mils)		645	31.670	7.310	125.883
Dummy for Conduit Deal		645	0.766	.	.

Table 4.2 provides an example of a deal structure, using the deal “Morgan Stanley Capital Inc. 2003-TOP11”. I will discuss signal score shortly. This deal structure is representative of the deals in the dataset and consists of 21 tranches, with 12 tranches making up 97% of the securitized principal rated above investment grade at issuance. Tranches receive principal payments in alphabetical order such that each tranche receives its scheduled principal payment before any junior tranches receive principal. Tranches X-1 and X-2 are “interest only” and receive interest based on a schedule of notional values detailed in the prospectus with an initial notional equal to the initial balance listed in the table. Tranches R-I through R-III are reserve tranches and only receive principal in the event that all other tranches are retired. The tranches rated below investment grade are the most informationally sensitive portion of the deal and can be thought of as the “asset” in the model.

The primary implication of the model is that issuers can substitute between costly signaling and reputation as means to increase the price of their marketed securities. Two principal empirical challenges emerge from this implication. First, although market participants may observe the identity of the eventual owner of each tranche, I do not. As such it is difficult to measure the fraction of the asset the issuer retains. Second, there is no established measure of the market reputation of a given issuer.

To deal with the first challenge, I appeal to the following stylized conditions of the CMBS market. Many deals contain a certain tranche (or number of tranches) which do not receive a rating. These tranches are referred to as “stipulations” or “stips” and are usually retained by the issuer. To measure the percentage of retention in manner which is comparable across deals, I divide the principal balance of the unrated tranches by the total principal balance of all tranches rated below investment grade and call this variable Signal Score. In this way I am measuring the fraction of the informationally sensitive portion of the deal, or the asset of the model, retained by the issuer. To the extent that Signal Score does not accurately measure issuer retention, it is not clear that the error in measurement should be correlated with an issuers reputation. Thus measurement error for issuer retention should bias against finding a result.

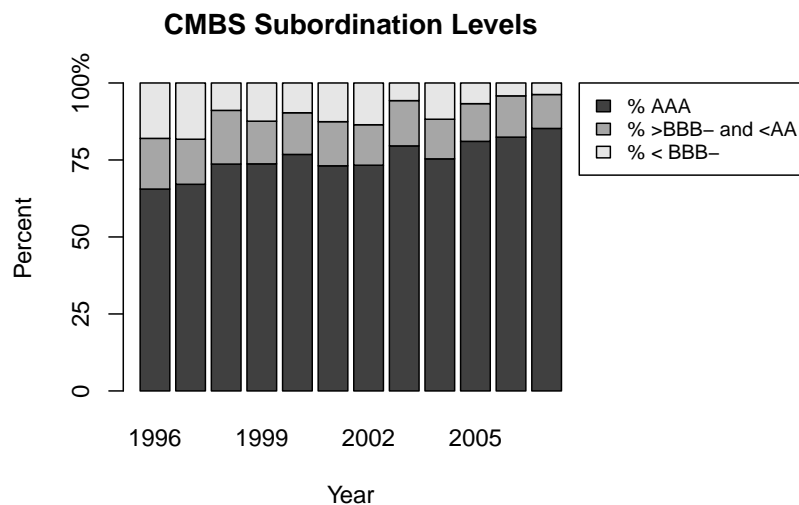
During the sample period, the CMBS market was going through many structural changes with respect to the average levels of subordination within deals. Figure 4.1(a) show the yearly average subordination levels for 1996 through 2008. Of particular note is the large increase in the percentage of *AAA* during that period, from around 56% in 1993 to around 80% in 2008, with a similar trend in the percentage rated above investment grade.³ Since many deal balances exceed \$1 billion, this represents a large dollar-amount increase in *AAA* CMBS tranche issuance. In addition, the average signal score increased over this period, as demonstrated by Figure 4.1(b). This means that the increase in *AAA* and above investment grade was at the expense of the tranches rated just below investment grade. These trends suggest that CMBS market characteristics varied substantially over time. To account for this apparently systematic time series variation, I will include either a time trend or quarterly

³This increase could be due to an improvement in underlying securitized collateral or a liberalization of rating standards. For evidence supporting the latter explanation see Stanton and Wallace (2010).

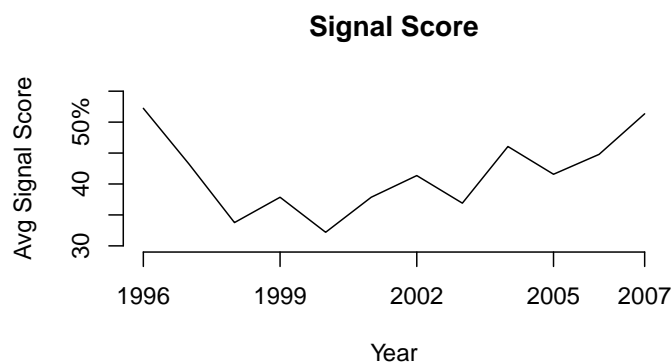
Table 4.2: The structure of the deal Morgan Stanley Capital Inc. 2003-TOP11. This deal consists of 21 tranches, with 12 tranches making up 97% of the securitized principal rated above investment grade at issuance. Tranches receive principal payments in alphabetical order such that each tranche receives its scheduled principal payment before any junior tranches receive principal. Tranches X-1 and X-2 are “interest only” and receive interest based on a schedule of notional values detailed in the prospectus with an initial notional equal to the initial balance listed in the table. Tranches R-I through R-III are reserve tranches and only receive principal in the event that all other tranches are retired. Totals do not include interest only tranches, as those balances are only nominal and the tranches do not receive principal payments from the mortgage pool. Signal Score is the ratio of unrated (NR) principal to total principal rated below investment grade

	Tranche Name	Initial Rating (S&P)	Initial Balance (\$ Mil.)	Subordination Pct.
Inv. Grade	A-1	AAA	150.00	12.0%
	A-2	AAA	175.00	12.0%
	A-3	AAA	165.11	12.0%
	A-4	AAA	561.38	12.0%
	X-1	AAA	1194.88	12.0%
	X-2	AAA	1099.30	12.0%
	B	AA	31.37	9.4%
	C	A	32.86	6.6%
	D	A-	13.44	5.5%
	E	BBB+	14.94	4.3%
	F	BBB	7.47	3.6%
	G	BBB-	7.47	3.0%
Below Inv. Grade	H	BB-	11.95	2.0%
	J	B+	2.99	1.8%
	K	B	2.99	1.5%
	L	B-	2.99	1.3%
	M	CCC+	2.99	1.0%
	N	NR	11.95	0%
	R-I	NR	0	
	R-II	NR	0	
R-III	NR	0		
			Total: 1194.88	
			Signal Score: 36.3%	

Figure 4.1: The yearly sample average subordination levels and signals scores for 1996-2008.



(a) CMBS Subordination Levels at issuance over time. Note how the percentage of principal rated AAA is increasing over the sample period, from about 60% in 1996 to nearly 80% in 2008, while at the same time the percentage of principal rated between AAA and BBB- stays somewhat constant. This indicates that either the perceived credit quality of the underlying mortgages was increasing, or rating standards were decreasing.



(b) Average Signal Score over time. The signal score is increasing over the sample period. This indicates that at the same time that the proportion of principal rated below investment grade was decreasing, the ratings within that portion of the capital structure were shifting downwards. So not only was there less credit support for the above investment grade classes, the credit support was also rated lower.

fixed effects in all regressions attempting to explain signal score.

The next empirical challenge consists of measuring market reputation. This problem can be thought of in two parts. First, I must obtain some measure of initial market reputation. Second, I must obtain some measure of changes in reputation. Since differences in initial issuer only varies cross-sectionally, I can control for initial issuer reputation by including issuer fixed effects. To address the second part, I measure the degree to which a given issuer has had bonds from past deals downgraded. The reasoning for equating a downgrade in a past deal to a loss in reputation is as follows. If a given issuer has had many downgrades on past deals, then the assets underlying those deals performed worse than expected, and the market will decrease the reputation of that issuer. Specifically I create a map from letter grade ratings to the $[0, 1]$ interval and measure a downgrade as the difference between the prior numeric rating and the newly assigned numeric rating of a bond. For example, if a bond is downgraded from *AAA* to *NR*, my measure for the downgrade would be 1. For each deal, I then sum the downgrades attributable to the issuer that occurred prior to the closing date of the deal.

4.4 Results

I now estimate the relationship between reputation and signaling using the proxies constructed above. Formally, I estimate the tobit regression

$$S_{ij} = \beta_0 + \beta_1 q_t + \beta_2 \text{Downgrades}_{jt} + \theta X_i + \alpha_j + \epsilon_i \quad (4.1)$$

where S_i is signal score of deal i , q_t is either the time in years since the start of the sample or a time fixed effect, Downgrades_{jt} is the prior downgrades attributed to the issuer of the deal, α_j is an issuer level fixed effect and X is a vector of controls for the characteristics of the underlying mortgage pool and other deal level characteristics. Specifically, I include days since the start of the sample, days since the issuer entered the sample, pool weighted average spread to treasury, pool weighted average loan-to-value, weighted average debt service coverage ratio, the percentage of mortgages collateralized by office, hotel and industrial properties, the number of loans in the pool, the average loan size in the pool, a dummy for deals comprised of conduit loans,⁴ and controls for the geographic composition of the mortgage pool. See Table 4.1 gives summary statistics for the vector X of controls. Since S is confined to the interval $[0, 1]$, I estimate the coefficients of equation (4.1) using a tobit regression.

Models (1)-(5) of Table 4.3 show the results of the estimation of (4.1) for various vectors of controls. The main coefficient of interest is that on Total Issuer Downgrades, which is positive and significant for all specifications. This means that an increase in bad performance, i.e. downgrades, on past deals is associated with an increase in the amount of costly signaling employed by the issuer. The dynamic model of this paper is consistent with this fact. In

⁴Conduit loans are commercial mortgages originated to be securitized.

contrast, the previous literature on security design in a costly signaling framework is silent as to the effect of past performance on current issues, except through an exogenously specified change in beliefs about the distribution of the quality of underlying assets.

The estimates of the coefficients on the variables measuring observable pool characteristics are also of some interest. The variables with significant coefficients, other than Total Downgrades by Issuer, are WALT_V, Debt Service Coverage Ratio, and property type. The large positive WALT_V and the corresponding negative coefficient on WALT_V squared suggest that the effect of observable credit quality on the signal score is highly non-linear. For very low levels of WALT_V less signaling is employed. This effect is likely due to strong fundamental measures of credit quality being associated with less uncertainty about pool quality. In other words, the lemons problem is smaller for pools with very strong fundamentals since it is very unlikely that a low quality pool would attain very high fundamentals. For mid-range values of WALT_V, the pool should be of relatively high credit quality, this corresponds to the case in the model when the issuer makes the public report that the mortgages are of high quality and hence more retention is employed. For very high values of WALT_V, less signaling is again being employed. This corresponds to the case in the model in which the issuer reveals that the mortgages are of low quality and no signaling is necessary. The negative coefficient on Debt Service Coverage Ratio indicates that deals of higher observable credit quality require less retention.

The property type variables could proxy for the amount of diversification across loans in the pool. A pool with a high degree of concentration in a certain characteristic is more sensitive to the issuer's private information, *ceteris paribus*. For example, if a pool is predominately made up of mortgages on office space, then the performance of that pool is highly sensitive to the issuer's private information about the office space market. As a result, the issuer will need to retain more of a such a deal to signal its quality.

The results of Table 4.3 suggest that subordination levels within the lowest rated portion of a deal are related to issuer reputation. This affect could be due to a change in investor beliefs about the distribution of the underlying assets available to a given issuer rather than to the substitutability of reputation and signaling. To address this concern, I replicate the regressions of Table 4.3 with the percentage of the deal rated AAA as the dependent variable. Table 4.4 shows the results for this analysis. The coefficient on Total Issuer Downgrades is no longer significant, indicating that my measure of issuer reputation is not related to the over-all level of subordination within a deal. Consequently, the results of Table 4.4 are not likely driven by a change in investor beliefs about the quality of available assets.

4.5 Conclusion

Taken as a whole, the results of this chapter suggest there is a strong link between an issuer's past performance and current deal structure. Specifically, an issuer of CMBS can tradeoff between using her reputation and retaining a portion of an issue to increase the price of her offerings. This result indicates that empirical work that attempts to either explain

Table 4.3: The empirical relationship between signaling and reputation. The table displays results of a tobit estimation of (4.1) with various controls. The dependent variable is the percentage of below investment grade principal which is not rated. The main coefficient of interest is that on Total Issuer Downgrades, which is positive and significant for all specifications. This result indicates that an increase in downgrades on past deals is associated with an increase in the amount of costly signaling, consistent with the main prediction of the model. Standard errors in parenthesis are clustered at the Issuer level. All models have χ^2 statistics that imply significance at the .001 level.

	Percentage below investment grade not rated				
	(1)	(2)	(3)	(4)	(5)
Constant	0.346*** (0.000631)	2.104 (1.775)	0.810 (1.878)	1.532** (0.608)	-1.790 (1.250)
Days Since Start of Sample		-0.134 (0.123)	-0.154 (0.126)		
Total Issuer Downgrades	0.0634*** (0.00641)	0.0469*** (0.0117)	0.0283*** (0.00734)	0.0218*** (0.00817)	0.0212** (0.0106)
Issuer Days in Sample		0.140 (0.124)	0.170 (0.128)	-0.152 (0.111)	0.135 (0.131)
Weighted Average Spread		-1.425 (2.479)	-1.040 (2.076)	-1.703 (5.411)	-2.099 (4.059)
Weighted Average Spread Squared		-5.034 (34.23)	-2.527 (23.03)	-11.17 (53.95)	18.42 (39.39)
Weighted Average LTV		0.693 (0.823)	5.858** (2.887)	0.463 (0.665)	5.850** (2.838)
Weighted Average LTV Squared		-0.584 (0.898)	-4.904** (2.224)	-0.404 (0.788)	-4.960** (2.144)
Debt Service Coverage Ratio		-0.104 (0.0834)	-0.115*** (0.0433)	-0.0994 (0.0828)	-0.109** (0.0510)
Percentage Office			0.194 (0.131)		0.292** (0.121)
Percentage Hotel			0.131 (0.176)		0.0897 (0.212)
Percentage Industrial			0.162 (0.207)		0.204 (0.170)
Conduit Deal Dummy		-0.00325 (0.0698)	-0.0566 (0.0817)	0.0538 (0.0890)	-0.0196 (0.0776)
Number of Loans		0.0121** (0.00512)	0.00758 (0.00621)	0.00482 (0.00596)	0.00130 (0.00715)
Average Loan Size		-0.000347 (0.000599)	0.000880 (0.000655)	-0.000339 (0.000610)	0.000671 (0.000725)
Geographic Controls	no	no	yes	no	yes
Issuer Fixed Effects	yes	yes	yes	yes	yes
Quarter Fixed Effects	no	no	no	yes	yes
Observations	696	564	457	564	457

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4.4: The empirical relationship between Percent AAA and reputation. The small number of observations for models (2) - (5) is due to data availability and will be changed in future versions of the paper. The table displays results of a tobit estimation of (4.1) with various controls and the dependent variable Percent AAA. The main coefficient of interest is that on Total Issuer Downgrades, which is not significant any specification except for model (4). Standard errors in parenthesis are clustered at the Issuer level. All models have χ^2 statistics that imply significance at the .001 level.

	Percentage AAA				
	(1)	(2)	(3)	(4)	(5)
Constant	0.862*** (0.0000998)	-3.085*** (0.794)	-1.266** (0.521)	1.191*** (0.208)	1.033** (0.475)
Days Since Start of Sample		0.286*** (0.0588)	0.181*** (0.0480)		
Total Issuer Downgrades	0.0236*** (0.00641)	-0.00979 (0.00705)	0.00265 (0.00454)	-0.0143** (0.00711)	0.00392 (0.00431)
Issuer Days in Sample		-0.271*** (0.0592)	-0.169*** (0.0479)	-0.0711** (0.0332)	-0.00578 (0.0448)
Weighted Average Spread		-1.018 (1.463)	-1.659** (0.742)	0.240 (2.676)	2.252 (1.670)
Weighted Average Spread Squared		8.296 (20.51)	-0.933 (11.05)	3.618 (29.64)	-26.28 (16.84)
Weighted Average LTV		-0.339 (0.359)	-1.068 (1.080)	-0.319 (0.270)	-0.822 (0.968)
Weighted Average LTV Squared		0.296 (0.428)	0.830 (0.878)	0.280 (0.343)	0.653 (0.785)
Debt Service Coverage Ratio		0.0226 (0.0171)	0.0104 (0.0122)	0.0243 (0.0205)	0.0143 (0.0149)
Percentage Office			-0.0848 (0.0599)		-0.124* (0.0675)
Percentage Hotel			-0.151** (0.0742)		-0.146* (0.0835)
Percentage Industrial			0.00907 (0.126)		0.0376 (0.105)
Conduit Deal Dummy		0.161*** (0.0426)	0.0758* (0.0418)	0.170*** (0.0395)	0.0756** (0.0353)
Number of Loans		-0.00101 (0.00258)	0.00141 (0.00304)	-0.000550 (0.00199)	0.00138 (0.00291)
Average Loan Size		-0.0000804 (0.000223)	0.000922** (0.000447)	-0.0000193 (0.000227)	0.000899* (0.000458)
Geographic Controls	no	no	yes	no	yes
Issuer Fixed Effects	yes	yes	yes	yes	yes
Quarter Fixed Effects	no	no	no	yes	yes
Observations	696	564	457	564	457

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

market prices of ABS or evaluate policy measures aimed at increasing the efficiency of these markets should condition on some measure of issuer reputation.

Aside from the results, a principal contribution of this chapter is to introduce two new measures: a measure of CMBS issuer reputation, and a measure of CMBS issuer retention. The reputation measure uses performance, rather than market, based information in that I say that an issuer with more downgrades on past deals has a lower reputation for honesty, all else equal. The retention measure looks at capital structure in order to proxy for retention by assuming that the issuer retains unrated tranches and that the informationally sensitive portion of the deal is made up of the tranches rated below investment grade. Thus, issuer retention is the percentage of below investment grade principal that is unrated. This measure can be useful in other empirical work, especially in the understudied area of testing signaling based theories of security design.

Some implications of the model of Chapter 3 were not explored in this chapter and are left for future work. For instance, the model predicts that prices should be U-shaped in issuer reputation and that issuers will be less honest as their reputations improve, conditional on being opportunistic. Unfortunately, price data for CMBS markets is not widely available, and so further proxies must be formed. However, one could also examine the predictive power of issuer retention on subsequent deal performance to address the second implication.

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Appendix A

Proofs for Chapter Two

Proof of Proposition 1. Let X_t^* denote the proposed contract. First observe that X_t^* satisfies constraints (PC) since

$$\begin{aligned}
 E \left[\int_0^\infty e^{-\gamma t} dX_t^* | e = 1 \right] &= E \left[e^{-\gamma t_0} \mathbb{I}(t_0 \leq \tau_1) y_0 | e = 1 \right] \\
 &= E \left[e^{-\gamma t_0} \mathbb{I}(t_0 \leq \tau_1) e^{(\gamma + N\lambda_L)t_0} (\hat{a}_0 + C) | e = 1 \right] \\
 &= P(t_0 \leq \tau_1 | e = 1) e^{N\lambda_L t_0} (\hat{a}_0 + C) \\
 &= \hat{a}_0 + C \\
 &\geq a_0 + C.
 \end{aligned}$$

Next observe that the proposed contract satisfies constraint (IC) since

$$\begin{aligned}
 E \left[\int_0^\infty e^{-\gamma t} dX_t^* | e = 0 \right] &= E \left[e^{-\gamma t_0} \mathbb{I}(t_0 \leq \tau_1) y_0 | e = 0 \right] \\
 &= E \left[e^{-\gamma t_0} \mathbb{I}(t_0 \leq \tau_1) e^{(\gamma + N\lambda_L)t_0} (\hat{a}_0 + C) | e = 0 \right] \\
 &= P(t_0 \leq \tau_1 | e = 0) e^{N\lambda_L t_0} (\hat{a}_0 + C) \\
 &= e^{-N(\lambda_H - \lambda_L)t_0} (\hat{a}_0 + C) = \hat{a}_0.
 \end{aligned}$$

Now suppose X_t is an arbitrary incentive compatible contract. We will first find a convenient decomposition of X_t . For $n = 0 \dots N$ let $x^n(\mathbf{s}, t) = X_t \mathbb{I}(\omega_n)$ where $\omega_n \in \{s_0 = \tau_0, \dots, s_n = \tau_n, t \leq \tau_{n+1}\}$ for $n = 1, \dots, N$. For convenience let $\tau_0 = 0$ and $\tau_{N+1} = \infty$. Note that $x^n(\mathbf{s}, t) : \{0\} \times \mathbb{R}^{n+1} \times [s_n, \infty) \rightarrow \mathbb{R}^+$ is not a random variable, rather it is a function of the vector \mathbf{s} and time which denotes the payment at time t given the first n defaults occurred at times given by the vector \mathbf{s} and the $(n+1)$ th default has not yet occurred. This decomposition allows us to write

$$X_t = \sum_{n=0}^N x^n(\tau_0, \dots, \tau_n) \mathbb{I}(\tau_n < t \leq \tau_{n+1}, t). \tag{A.1}$$

First we consider $n = 0$. We have

$$\begin{aligned}
E \left[\int_0^{\tau_1} e^{-\gamma t} dx^0(t) | e = 0 \right] &= \int_0^\infty e^{-(\gamma + N\lambda_H)t} d^0 x(t) \\
&= \int_0^\infty e^{N(\lambda_L - \lambda_H)t} e^{-(\gamma + N\lambda_L)t} dx^0(t) \\
&\geq e^{-N(\lambda_H - \lambda_L)t_0} \int_0^\infty (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-(\gamma + N\lambda_L)t} dx^0(t) \\
&= \frac{\hat{a}_0}{\hat{a}_0 + C} E \left[\int_0^{\tau_1} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dx^0(t) | e = 1 \right],
\end{aligned} \tag{A.2}$$

where the second to last step follows from the fact that $e^{-N(\lambda_H - \lambda_L)t} \geq e^{-N(\lambda_H - \lambda_L)t_0} (N(\lambda_H - \lambda_L)(t_0 - t) + 1)$ since $e^{-N(\lambda_H - \lambda_L)t}$ is convex.

Before proceeding, we must find the joint distribution of τ_1, \dots, τ_n and the conditional distribution of τ_{n+1} given τ_1, \dots, τ_n given $\lambda = \lambda_i$ for $i \in \{L, H\}$. By equation (2.2.2) of David and Nagaraja (2003) the joint probability density function of τ_1, \dots, τ_n is

$$f_{\tau_1, \dots, \tau_n}(s_1, \dots, s_n) = \frac{\lambda_i^n N!}{(N - k)!} \exp \left(- \sum_{k=1}^n \lambda_i s_k \right).$$

By equation (2.2.5) of David and Nagaraja (2003), the increment $\tau_{n+1} - \tau_n$ is independent of τ_1, \dots, τ_n and is distributed as a exponential random variable with parameter $(N - n)\lambda_i$. For $n = 1, \dots, N$, let $\mathcal{A}_n = \{\mathbf{s} \in \mathbb{R}^{(n+1)+} | 0 = s_0 \leq s_1 \leq \dots \leq s_n\}$ and $d\mathcal{A}_n = ds_n \dots ds_1$ so that the set \mathcal{A}_n denotes the set of possible outcomes of (τ_0, \dots, τ_n) . We now calculate the time $t = 0$ expectation of the present value of the payment attributable to $x^n(\tau_0, \dots, \tau_n, t) \mathbb{I}(\tau_n <$

$t \leq \tau_{n+1}$). We have

$$\begin{aligned}
& E \left[\int_{\tau_n}^{\tau_{n+1}} e^{-\gamma t} dx^n(\tau_0, \dots, \tau_n, t) | e = 0 \right] \\
&= E \left[E \left[\int_{\tau_n}^{\tau_{n+1}} e^{-\gamma t} dx^n(\tau_0, \dots, \tau_n, t) | \tau_1, \dots, \tau_n, e = 0 \right] | e = 0 \right] \\
&= E \left[(N - n) \lambda_H e^{\gamma \tau_n} \int_{\tau_n}^{\infty} e^{-(\gamma + (N - n) \lambda_H)(t - \tau_n)} dx^n(\tau_0, \dots, \tau_n, t) | e = 0 \right] \\
&= \frac{\lambda_H^{n+1} N!}{(N - n - 1)!} \int_{\mathcal{A}_n} e^{\gamma \tau_n} \int_{s_n}^{\infty} \exp \left(-(\gamma + (N - n) \lambda_H)(t - s_n) - \sum_{k=1}^n \lambda_H s_k \right) dx^n(\mathbf{s}, t) d\mathcal{A}_n \\
&= \frac{\lambda_H^{n+1} N!}{(N - n - 1)!} \int_{\mathcal{A}_n} \int_{s_n}^{\infty} \exp \left(-(\gamma + N \lambda_H)t + (N - n) \lambda_H s_n + \sum_{k=1}^n \lambda_H(t - s_k) \right) dx^n(\mathbf{s}, t) d\mathcal{A}_n \\
&\geq \frac{\lambda_L^{n+1} N!}{(N - n - 1)!} \int_{\mathcal{A}_n} \int_{s_n}^{\infty} \exp \left(-(\gamma + N \lambda_H)t + (N - n) \lambda_H s_n + \sum_{k=1}^n \lambda_H(t - s_k) \right) dx^n(\mathbf{s}, t) d\mathcal{A}_n \\
&\geq \frac{\lambda_L^{n+1} N!}{(N - n - 1)!} \int_{\mathcal{A}_n} \int_{s_n}^{\infty} \exp \left(-(\gamma + N \lambda_H)t + (N - n) \lambda_L s_n + \sum_{k=1}^n \lambda_L(t - s_k) \right) dx^n(\mathbf{s}, t) d\mathcal{A}_n \\
&= \frac{\lambda_L^{n+1} N!}{(N - n - 1)!} \int_{\mathcal{A}_n} \int_{s_n}^{\infty} e^{-N(\lambda_H - \lambda_L)t} \exp \left(-(\gamma + N \lambda_L)t + (N - n) \lambda_L s_n + \sum_{k=1}^n \lambda_L(t - s_k) \right) dx^n(\mathbf{s}, t) d\mathcal{A}_n \\
&\geq e^{-N(\lambda_H - \lambda_L)t_0} \times \\
&\frac{\lambda_L^{n+1} N!}{(N - n - 1)!} \int_{\mathcal{A}_n} \int_{s_n}^{\infty} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) \exp \left(-(\gamma + N \lambda_L)t + (N - n) \lambda_L s_n + \sum_{k=1}^n \lambda_L(t - s_k) \right) dx^n(\mathbf{s}, t) d\mathcal{A}_n \\
&= \frac{\hat{a}_0}{\hat{a}_0 + C} E \left[\int_{\tau_n}^{\tau_{n+1}} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dx^n(\tau_0, \dots, \tau_n, t) | e = 1 \right]. \tag{A.3}
\end{aligned}$$

The second to last step follows from the fact that $e^{-N(\lambda_H - \lambda_L)t}$ is convex. The last step follows from noting that the last term in the integrand is exactly the probability density function used in the first step replacing λ_H with λ_L and using the definition of t_0 .

Note that by construction we have

$$dX_t = \sum_{n=0}^N dx^n(\tau_0, \dots, \tau_n, t) \mathbb{I}(\tau_n < t \leq \tau_{n+1}),$$

hence inequalities (A.2) and (A.3) imply

$$\begin{aligned}
E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] &= \sum_{n=0}^N E \left[\int_{\tau_n}^{\tau_{n+1}} e^{-\gamma t} dx^n(\tau_0, \dots, \tau_n, t) | e = 0 \right] \\
&\geq \sum_{n=0}^N \frac{\hat{a}_0}{\hat{a}_0 + C} E \left[\int_{\tau_n}^{\tau_{n+1}} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dx^n(\tau_0, \dots, \tau_n, t) | e = 1 \right] \\
&\geq \frac{\hat{a}_0}{\hat{a}_0 + C} E \left[\int_0^\infty (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dX_t | e = 1 \right]. \quad (\text{A.4})
\end{aligned}$$

Combining constraint (IC) with inequality (A.4) we have

$$\left(1 - \frac{\hat{a}_0}{\hat{a}_0 + C} \right) E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] - C \geq \frac{\hat{a}_0}{\hat{a}_0 + C} E \left[\int_0^\infty N(\lambda_H - \lambda_L)(t_0 - t) e^{-\gamma t} dX_t | e = 1 \right]. \quad (\text{A.5})$$

For convenience let $E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] = \tilde{a}_0 + C$. Rearranging (A.5) and canceling terms when possible, we are left with the following inequality

$$E \left[\int_0^\infty (t - t_0) e^{-\gamma t} dX_t | e = 1 \right] \geq -\frac{C(\tilde{a}_0 - \hat{a}_0)}{\hat{a}_0 N(\lambda_H - \lambda_L)}. \quad (\text{A.6})$$

Now consider the cost of this contract to the investors. Using a similar argument as above and the fact that $e^{-(\gamma-r)t}$ is convex we have

$$E \left[\int_0^\infty e^{-rt} dX_t | e = 1 \right] \geq e^{(\gamma-r)t_0} \left[E \left[\int_0^\infty (\gamma - r)(t - t_0) e^{-\gamma t} dX_t | e = 1 \right] + E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] \right] \quad (\text{A.7})$$

$$\geq e^{(\gamma-r)t_0} \left[-\frac{C(\tilde{a}_0 - \hat{a}_0)(\gamma - r)}{\hat{a}_0 N(\lambda_H - \lambda_L)} + \tilde{a}_0 + C \right] \quad (\text{A.8})$$

$$= e^{(\gamma-r)t_0} \left[\left(1 - \frac{C(\gamma - r)}{\hat{a}_0 N(\lambda_H - \lambda_L)} \right) (\tilde{a}_0 - \hat{a}_0) + \hat{a}_0 + C \right]. \quad (\text{A.9})$$

Now note that if $a_0 \geq \frac{C(\gamma-r)}{N(\lambda_H - \lambda_L)}$ then $\hat{a}_0 = a_0$ by definition and thus (PC) requires $\tilde{a}_0 \geq a_0$. Alternatively, if $a_0 < \frac{C(\gamma-r)}{N(\lambda_H - \lambda_L)}$, then $\hat{a}_0 = \frac{C(\gamma-r)}{N(\lambda_H - \lambda_L)}$ by definition. In either case we have

$$E \left[\int_0^\infty e^{-rt} dX_t | e = 1 \right] \geq e^{(\gamma-r)t_0} (\hat{a}_0 + C). \quad (\text{A.10})$$

But $e^{-(\gamma-r)t_0}(\hat{a}_0 + C)$ is the cost to the investors of the proposed contract. So we have shown that the proposed contract costs less (or the same) to the investors than any alternative contract that satisfies (IC) and (PC), hence the proposed contract is optimal. \square

Proof of Proposition 2. Before we verify equation (2.19) we prove the following combinatorial identity.

Lemma 2. For all $a, b > 0$ real numbers and $n \geq 0$ an integer

$$\sum_{m=0}^n \binom{n}{m} \frac{(-1)^{n-m}}{a-mb} = \frac{n!b^n}{\prod_{m=0}^n (a-mb)}. \quad (\text{A.11})$$

Proof. We proceed by induction on n . For $n = 0$ the identity is trivial. Now assume the identity holds for $n - 1$ so that

$$\sum_{m=0}^{n-1} \binom{n-1}{m} \frac{(-1)^{n-1-m}}{a-mb} = \frac{(n-1)!b^{n-1}}{\prod_{m=0}^{n-1} (a-mb)}. \quad (\text{A.12})$$

We have

$$\begin{aligned} \sum_{m=0}^n \binom{n}{m} \frac{(-1)^{n-m}}{a-mb} &= \frac{(-1)^n}{a} + \frac{1}{a-nb} + \sum_{m=1}^{n-1} \binom{n}{m} \frac{(-1)^{n-m}}{a-mb} \\ &= \frac{(-1)^n}{a} + \frac{1}{a-nb} + \sum_{m=1}^{n-1} \left(\binom{n-1}{m} + \binom{n-1}{m-1} \right) \frac{(-1)^{n-m}}{a-mb} \quad \text{by Pascal's Rule} \\ &= \sum_{m=1}^n \binom{n-1}{m-1} \frac{(-1)^{n-m}}{a-mb} - \sum_{m=0}^{n-1} \binom{n-1}{m} \frac{(-1)^{n-1-m}}{a-mb} \\ &= \sum_{m=0}^{n-1} \binom{n-1}{m} \frac{(-1)^{n-1-m}}{(a-b)-mb} - \sum_{m=0}^{n-1} \binom{n-1}{m} \frac{(-1)^{n-1-m}}{a-mb} \\ &= \frac{(n-1)!b^{n-1}}{\prod_{m=0}^{n-1} ((a-b)-mb)} - \frac{(n-1)!b^{n-1}}{\prod_{m=0}^{n-1} (a-mb)} \quad \text{by identity for } n-1 \\ &= \frac{(n-1)!b^{n-1}}{\prod_{m=1}^n (a-mb)} - \frac{(n-1)!b^{n-1}}{\prod_{m=0}^{n-1} (a-mb)} \\ &= \frac{(n-1)!b^{n-1}(a-(a-nb))}{\prod_{m=0}^n (a-mb)} \\ &= \frac{n!b^n}{\prod_{m=0}^n (a-mb)}. \end{aligned}$$

□

We now verify equation (2.19).¹ We have

$$\begin{aligned}
F(\delta, \Lambda, n) &= E \left[\sum_{k=1}^n \int_0^{\tau_k} u e^{-\delta t} dt | e = i \right] \\
&= \frac{u}{\delta} E \left[\sum_{k=1}^n (1 - e^{-\delta \tau_k}) | e = i \right] \\
&= \frac{u}{\delta} \sum_{k=1}^n \frac{N!}{(k-1)!(N-k)!} \int_0^\infty \Lambda (1 - e^{-\Lambda t})^{k-1} e^{-(N-k)\Lambda t} e^{-\Lambda t} (1 - e^{-\delta t}) dt \\
&= \frac{u}{\delta} \sum_{k=1}^n \left(1 - \frac{N!}{(k-1)!(N-k)!} \int_0^\infty \Lambda (1 - e^{-\Lambda t})^{k-1} e^{-(\delta + (N-k+1)\Lambda)t} dt \right) \\
&= \frac{u}{\delta} \sum_{k=1}^n \left(1 - \frac{N!}{(k-1)!(N-k)!} \sum_{m=0}^{k-1} \binom{k-1}{m} \int_0^\infty \Lambda e^{-(\delta + (N-m)\Lambda)t} dt \right) \\
&= \frac{u}{\delta} \sum_{k=1}^n \left(1 - \frac{N!}{(k-1)!(N-k)!} \sum_{m=0}^{k-1} \binom{k-1}{m} \frac{\Lambda (-1)^{k-1-m}}{\delta + (N-m)\Lambda} \right) \\
&= \frac{u}{\delta} \left(n - \sum_{k=1}^n \frac{N! \Lambda^k}{(N-k)! \prod_{m=0}^{k-1} (\delta + (N-m)\Lambda)} \right).
\end{aligned}$$

Finally we show that the contract given by equations (2.20) is optimal among all incentive compatible first loss piece contracts. Let the price of the senior tranche be denoted $S(n)$. Over this restricted contract space, the investors problem becomes

$$\begin{aligned}
&\min_n \{S(n) + F(r, \lambda_L, n)\} \\
&\text{s.t. } S(n) + F(\gamma, \lambda_L, n) \geq a_0 + C \\
&\quad S(n) + F(\gamma, \lambda_H, n) \geq a_0,
\end{aligned}$$

¹We will use the probability density function of τ_k for $k = 1, \dots, N$ given $\lambda = \Lambda$. τ_k is just the k th order statistic of a sample of N independent identically distributed exponential random variables with parameter $\Lambda \in \{\lambda_L, \lambda_H\}$. Let F and f denote the cumulative distribution function and probability density function respectively of an exponential random variable with parameter Λ . By equation (2.1.2) of David and Nagaraja (2003) the probability density function of τ_k is

$$\begin{aligned}
f_{\tau_k}(t) &= \frac{N!}{(k-1)!(N-k)!} F(t)^{k-1} (1 - F(t))^{N-k} f(t) \\
&= \frac{N! \Lambda}{(k-1)!(N-k)!} \Lambda (1 - e^{-\Lambda t})^{k-1} e^{-(N-k)\Lambda t} e^{-\Lambda t}.
\end{aligned}$$

which simplifies to

$$\begin{aligned} & \min_n \{F(r, \lambda_L, n)\} \\ & \text{s.t. } F(\gamma, \lambda_L, n) - F(\gamma, \lambda_H, n) \geq C. \end{aligned} \tag{A.13}$$

Since $F(r, \lambda_L, n)$ is increasing in n , the solution of problem (A.13) is given by the smallest n such that $F(\gamma, \lambda_L, n) - F(\gamma, \lambda_H, n) \geq C$, which is the desired result. \square

Proof of Proposition 3. Let \hat{X}_t be defined as

$$d\hat{X}_t = dX_t - \mathbb{I}(t=0)K.$$

We can rewrite the optimal contracting problem as

$$b(a_0) = \min_{d\hat{X}_t \geq 0} \left\{ E \left[\int_0^\infty e^{-rt} d\hat{X}_t | e = 1 \right] \right\},$$

such that

$$\begin{aligned} E \left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e = 0 \right] &\leq E \left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e = 1 \right] - C, \\ a_0 &\leq E \left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e = 1 \right] - C. \end{aligned}$$

The above problem is identical to Definition 1, thus the solution follows directly from Proposition 1. \square

Proof of Proposition 4. Suppose X_t is the contract detailed in Proposition 1. We have

$$\begin{aligned} E \left[\int_0^\infty e^{-\gamma t} dX_t | e = m \right] - c \cdot m &= P(\tau_1 \geq t_0 | e = m) e^{-\gamma t_0} y_0 - c \cdot m \\ &= e^{-(m\lambda_L + (N-m)\lambda_H)t_0} e^{(N\lambda_L)t_0} (a_0 + c \cdot N) - c \cdot m \\ &= e^{-(N-m)(\lambda_H - \lambda_L)t_0} (a_0 + c \cdot N) - c \cdot m \\ &= \left(\frac{a_0 + c \cdot n}{a_0} \right)^{-\frac{N-m}{N}} (a_0 + c \cdot n) - c \cdot m \\ &= a_0 \left(\left(\frac{a_0 + c \cdot N}{a_0} \right)^{\frac{m}{N}} - \frac{c \cdot m}{a_0} \right) \\ &< a_0 \left(\frac{m}{N} \left(\frac{a_0 + c \cdot N}{a_0} - 1 \right) + 1 - \frac{c \cdot m}{a_0} \right) \\ &= a_0 \\ &= E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right]. \end{aligned}$$

So that $e = 0$ is the most profitable deviation from $e = N$ given X_t . This implies that only the original incentive compatibility constraint binds. Hence, the result follows from Proposition 1

□

Appendix B

Proofs for Chapter Three

Proof of Proposition ??. Note that since $\phi_t = 0$ for all $t \geq 0$ by Bayes rule, hence it is enough to consider the strategies of the opportunistic type issuer. First suppose there exists a truth telling equilibrium given by $Q_t(\mathcal{H}_t)$ and $\pi_t = 1$. Let \tilde{Q} be defined as follows

$$\tilde{Q} = \sup\{Q_t(\mathcal{H}_t) | t \geq 0 \text{ and } \mathcal{H}_t \in \mathbb{H}_t\}. \quad (\text{B.1})$$

Note that $\tilde{Q} > \hat{q}$. For all $\epsilon > 0$, there exist (t, \mathcal{H}_t) such that $\tilde{Q} - Q_t(\mathcal{H}_t) < \epsilon$ by the definition of supremum. Let (t, \mathcal{H}_t) be such a pair. The one-shot deviation principal implies that

$$Q_t(\mathcal{H}_t) + (1 - Q_t(\mathcal{H}_t))\gamma\ell - \ell \leq L_t \quad (\text{B.2})$$

where L_t is the discounted loss faced by the issuer after misreporting a bad type asset given the history \mathcal{H}_t . By the definition of \tilde{Q} , I have

$$L_t \leq \gamma \left(\frac{\lambda(\tilde{Q} + (1 - \tilde{Q}))\gamma + (1 - \lambda)\ell}{1 - \gamma} - \frac{\lambda(\hat{q} + (1 - \hat{q})\gamma) + (1 - \lambda)\ell}{1 - \gamma} \right) = \gamma\lambda(\tilde{Q} - \hat{q}). \quad (\text{B.3})$$

But

$$Q_t(\mathcal{H}_t) + (1 - Q_t(\mathcal{H}_t))\gamma\ell - \ell = (1 - \gamma\ell)(\tilde{Q} - \epsilon - \hat{q}). \quad (\text{B.4})$$

This implies that for all $\epsilon > 0$

$$(1 - \gamma\ell)(\tilde{Q} - \epsilon - \hat{q}) \leq \gamma\lambda(\tilde{Q} - \hat{q}) \quad (\text{B.5})$$

which implies $\gamma \geq \frac{1}{\lambda + \ell}$ since $\tilde{Q} > \hat{q}$.

Now suppose $\gamma \geq \frac{1}{\lambda + \ell}$. I'll show that a truth telling equilibrium is given by $Q_t = 1$, $\pi_t = 1$, $P(q, H_t) = 1$ if no misreports have been made and

$$P(q, H_t) = \begin{cases} 1 & \text{if } q \leq \hat{q} \\ \ell & \text{if } q > \hat{q} \end{cases}$$

otherwise. Note that the continuation value of the issuer just depends on whether or not she has made a misreport. Let L be the discounted loss in continuation value faced by the issuer if she misreports a bad type asset then

$$L = \gamma \left(\frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} - \frac{\lambda(\hat{q} + (1 - \hat{q})\gamma + (1 - \lambda)\ell)}{1 - \gamma} \right) = \gamma\lambda(1 - \hat{q}). \quad (\text{B.6})$$

To see that the above strategies indeed constitute an equilibrium observe that

$$\begin{aligned} L &= \gamma\lambda(1 - \hat{q}) = \gamma\lambda \frac{1 - \ell}{1 - \gamma\ell} \\ &\geq 1 - \ell. \end{aligned}$$

Thus the discounted loss in continuation value is greater than or equal to the one-shot gains from misreporting a bad type asset and the above strategies constitute a truth telling equilibrium for $\phi_0 = 0$. \square

Proof of Proposition 6.

Separating Equilibrium: Let $\tilde{q} \leq \frac{\ell(1-\gamma)}{1-\gamma\ell}$ and

$$P(q, 0) = \begin{cases} 1 & \text{for } q \leq \tilde{q} \\ \ell & \text{for } q > \tilde{q} \end{cases},$$

then

$$\begin{aligned} qP(q, 0) + (1 - q)\gamma\ell &= \gamma\ell + (1 - \gamma\ell)q \leq \ell \\ qP(q, 0) + (1 - q)\gamma &\leq \tilde{q}P(q, 0) + (1 - \tilde{q})\gamma \\ \tilde{q}P(q, 0) + (1 - \tilde{q})\gamma &> \gamma. \end{aligned}$$

for all $q \leq \tilde{q}$. Thus $\tilde{q} = \arg \max_q \{qP(q, 0) + (1 - q)\gamma\}$ and

$$(1, 1) \in \arg \max_{q, \pi} \{\pi\ell + (1 - \pi)qP(q, 0) + (1 - q)\gamma\ell\}$$

Thus the issuer chooses to issue \tilde{q} when she has a good type asset and 1 when she has a bad type asset. Thus conditions (1) and (2) of Definition 4 are satisfied. This implies that $E[X|\tilde{Q}, g] = 1$ and condition (4) is satisfied, and the strategies proposed in Proposition 6 constitute a separating equilibrium.

Pooling Equilibrium Let $\gamma \leq \lambda + (1 - \lambda)\ell$ and $P(q, 0) = \lambda + (1 - \lambda)\ell$ for all q . Then $1 = \arg \max_q \{qP(q, 0) + (1 - q)\gamma\}$ and

$$(1, 0) \in \arg \max_{q, \pi} \{\pi\ell + (1 - \pi)qP(q, 0) + (1 - q)\gamma\ell\}.$$

Thus the issuer always chooses to issue a quantity 1 and report that the asset is the good type. This implies that $E[X|\tilde{Q}, g] = \lambda$ and condition (4) is satisfied, and the strategies proposed in Proposition 6 constitute a pooling equilibrium.

Now let $\gamma > \lambda + (1 - \lambda)\ell$ and let \tilde{Q} be the quantity chosen by the issuer in a pooling equilibrium. Then $P(\tilde{q}, 0) = \lambda + (1 - \lambda)\ell\gamma < \gamma = 0 \cdot P(q, 0) + (1 - 0)\gamma$ and \tilde{q} cannot be an equilibrium quantity strategy for the issuer when she has a good type asset, a contradiction. Thus a pooling equilibrium does not exist. □

Proof of Proposition 7. First note that Bayes rule implies that $\phi = 0$ is an absorbing state, since

$$\mathbb{P}(\text{Issuer is honest}|\mathcal{H}_t) = \frac{\mathbb{P}(\text{Issuer is honest} \cap \mathcal{H}_t)}{\mathbb{P}(\mathcal{H}_t)} = 0$$

so that $\phi_t = 0$ implies that $\phi_s = 0$ for all $s \geq t$.

Now suppose (Q, π) and $P(q, 0)$ is an equilibrium of the static game and not an equilibrium of the repeated game at time t with $\phi_t = 0$. Let \hat{V} be the continuation value the opportunistic issuer receives for playing the strategy (Q, π) given the demand curve $P(q, 0)$. Then at least one of the following must be true:

- $Q \notin \arg \max_q \{qP(q, 0) + (1 - q)\gamma + \hat{V}\}$
- $(Q, \pi) \notin \arg \max_{q, \pi} \{\pi\ell + (1 - \pi)(qP(q, 0) + (1 - q)\gamma\ell) + \hat{V}\}$
- $P(Q, 0) \neq E[X_{t+1}|Q, g]$,

any of which implies that (Q, π) and $P(q, 0)$ is not an equilibrium of the static game, a contradiction. □

Proof of Lemma 1. For convenience let $W(\phi)$ and $V(\phi)$ denote the value the honest and opportunistic issuers place on reputation ϕ respectively. Note that $V(0) = W(0)$ since $\phi = 0$ is an absorbing state and both issuer types play the separating equilibrium at $\phi = 0$. Suppose there exists $\hat{\phi}$ and equilibrium strategies Q^H and Q^O such that $Q^H(\hat{\phi}) \neq Q^O(\hat{\phi})$, then the investors would believe the issuer is the honest type upon observing a quantity equal to $Q^H(\hat{\phi})$, and would believe she is the opportunistic type upon observing a level of retention equal to $Q^O(\hat{\phi})$ when the issuer has a prior reputation $\hat{\phi}$. This implies that $P(Q^H(\hat{\phi}), \phi) = 1$. The definition of equilibrium strategies for the honest issuer thus implies that

$$Q^H(\hat{\phi}) + \gamma(1 - Q^H(\hat{\phi})) + \gamma W(1) \geq Q^O(\hat{\phi})P(Q^O(\hat{\phi}), \hat{\phi}) + \gamma(1 - Q^O(\hat{\phi})) + \gamma W(0)$$

It is straightforward to shown that $\pi(1) = 0$, which implies that $V(1) > W(1)$ so that

$$Q^H(\hat{\phi}) + \gamma(1 - Q^H(\hat{\phi})) + \gamma V(1) > Q^O(\hat{\phi})P(Q^O(\hat{\phi}), \hat{\phi}) + \gamma(1 - Q^O(\hat{\phi})) + \gamma V(0).$$

Hence, the opportunistic issuer can profitably deviate to $Q^H(\hat{\phi})$ and the strategies Q^H and Q^O cannot be played in equilibrium. \square

Proof of Proposition 8. First suppose there exists a truth telling equilibrium given by $Q(\phi)$ and $\pi(\phi) = 1$, such that $Q(\phi) > \hat{q}$ for some ϕ . Note that $\pi(\phi) = 1$ implies $\phi^B = \phi^S = \phi$, so the one-shot deviation principal then states that for all ϕ such that $Q(\phi) > \hat{q}$

$$Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell \leq \gamma(V(\phi) - V(0)). \quad (\text{B.7})$$

Next note that

$$\begin{aligned} Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell &= \hat{q} + (1 - \hat{q})\gamma\ell - \ell + (1 - \gamma\ell)(Q(\phi) - \hat{q}) \\ &= (1 - \gamma\ell)(Q(\phi) - \hat{q}) \end{aligned}$$

since $\hat{q} + (1 - \hat{q})\gamma\ell = \ell$. Finally, I have

$$\gamma(V(\phi) - V(0)) = \gamma\lambda(1 - p_B)(Q(\phi) - \hat{q}).$$

So it must be the case that $1 - \gamma\ell \leq \gamma\lambda$ since $Q(\phi) > \hat{q}$, which is what I needed to show.

Now suppose that $\gamma \geq \frac{1}{\lambda + \ell}$. I show that $\pi(\phi) = Q(\phi) = 1$ for all $\phi > 0$ defines an equilibrium. To check that these strategies constitute an equilibrium, note that they imply

$$V(\phi) = \frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} \quad (\text{B.8})$$

for all $\phi > 0$ and $\phi^B = \phi^S = \phi$. This in turn implies

$$\begin{aligned} \gamma V(\phi) - V(0) &= \gamma\lambda \frac{(1 - \hat{q})(1 - \gamma)}{1 - \gamma} \\ &= \gamma\lambda \frac{1 - \ell}{1 - \gamma\ell} \\ &\geq (1 - \ell), \end{aligned}$$

so that the issuer does not have a profitable one-shot deviation, and the equilibrium is verified. \square

Proof of Proposition 9. The case for $\phi_0 = 0$ is given in Proposition 7. Let $\tilde{V}(\phi)$ denote the value the issuer receives by playing the strategy $(\hat{q}, 1)$ forever. Note $\tilde{V}(\phi) = \hat{V}$ for all ϕ where \hat{V} is defined in the proof of Proposition 7, thus the proof applies to $\phi_0 > 0$ as well. \square

Proof of Proposition 10. First I provide the solution to the method of construction given in the text. Let

$$\bar{\phi} = \frac{1}{1-\lambda} \left(1 - \frac{\lambda(1-\gamma\lambda)}{\gamma(1-\gamma\lambda(1-\hat{q}))} \right) \quad (\text{B.9})$$

$$\hat{\phi} = \hat{\phi}^B \left(\hat{\phi}^B - \frac{1-\gamma}{1-\lambda} \right) \quad (\text{B.10})$$

where

$$\hat{\phi}^B = \frac{(1-\ell) - \lambda(1-\gamma\ell)}{(1-\lambda)(1-\ell)}. \quad (\text{B.11})$$

For the remainder of the proof, I assume

$$\frac{1-\gamma\ell}{1-\ell} < \frac{1-\gamma\lambda}{\gamma(1-\gamma\lambda(1-\hat{q}))}$$

so that $\hat{\phi}^B > \bar{\phi}$. This assumption could be relaxed with only minor changes to the proof, but doing so would make the calculation of $L(\phi)$ for $\phi \in [\hat{\phi}, \bar{\phi})$ needlessly laborious. It is straightforward to show that $\hat{\phi}$ solves equations (3.10)-(3.12) when $q^*(\phi) = 1$, $\pi^*(\phi) = 0$, and (3.11) binds. Similarly, it is straightforward to show that $\hat{\phi}$ solves (3.10)-(3.12) when $q^*(\phi) = 1$ and (3.10) binds.

For $\phi \geq \bar{\phi}$, $\pi^*(\phi) = 0$ and $q^*(\phi) = 1$, so

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{1-(1-\lambda)\phi}$$

$$L_1(\phi) = \frac{\gamma\lambda(1-\ell)}{1-\gamma\lambda} \left(\frac{1}{1-(1-\lambda)\phi} - \frac{\gamma(1-\ell)}{1-\gamma\ell} \right).$$

For $\phi \in [\hat{\phi}, \bar{\phi})$, $q^*(\phi) = 1$ and the solution to (3.16)

$$\pi^*(\phi) = 1 - \frac{\lambda}{1-\lambda} \left(\frac{(1-\ell)}{L_1(\zeta^{-1}(\phi))} - 1 \right) \frac{1}{1-\phi}, \quad (\text{B.12})$$

where

$$\zeta(\phi) = \frac{1}{1-\lambda} \left(1 - \frac{\lambda(1-\ell)}{L_1(\phi)} \right) \phi. \quad (\text{B.13})$$

Note that ζ has a unique inverse for $\hat{\phi} \leq \phi \leq \bar{\phi}$. Using (B.12), I can calculate $p^*(\phi)$ and the function L_2 for $\hat{\phi} \leq \phi \leq \bar{\phi}$:

$$p^*(\phi) = \ell + L_1(\zeta^{-1}(\phi)) \quad (\text{B.14})$$

$$L_2(\phi) = \frac{\gamma}{1-\gamma\lambda} \left(L_1(\zeta^{-1}(\phi)) - \frac{\gamma(1-\ell)^2}{1-\gamma\ell} \right). \quad (\text{B.15})$$

Next I consider $\underline{\phi} \leq \phi < \hat{\phi}$. Let $\psi_n(\phi) = \prod_{k=0}^n \delta_k(\phi)$ where $\delta_0(\phi) = \phi$, and

$$\delta_n(\tilde{\phi}) = 1 - \left(\frac{\lambda}{1-\lambda} \right) \left(\frac{(1-\gamma)\beta^{n-1}L(\tilde{\phi})}{\gamma(1-\ell)^2\hat{q} + (\gamma-\ell)\beta^{n-1}L(\tilde{\phi})} \right) \quad (\text{B.16})$$

for $n \geq 0$. Then the solution to equations (3.21)-(3.23) is

$$q^*(\phi) = \hat{q} + \frac{\beta^{n-1}}{\gamma(1-\ell)} L(\psi_n^{-1}(\phi)), \quad (\text{B.17})$$

$$\pi^*(\phi) = 1 - \left(\frac{\lambda}{1-\lambda} \right) \left(\frac{(1-\gamma)\beta^{n-1}L(\psi_n^{-1}(\phi))}{\gamma(1-\ell)^2\hat{q} + (\gamma-\ell)\beta^{n-1}L(\psi_n^{-1}(\phi))} \right) \frac{1}{1-\phi}, \quad (\text{B.18})$$

$$p^*(\phi) = 1 - \frac{(1-\gamma)\beta^{n-1}L(\psi_n^{-1}(\phi))}{\gamma(1-\ell)\hat{q} + \beta^{n-1}L(\psi_n^{-1}(\phi))}, \quad (\text{B.19})$$

$$L(\phi) = \beta^n L(\psi_n^{-1}(\phi)) \quad (\text{B.20})$$

where $\phi(k) = \psi_k(\hat{\phi})$ and n is such that $\phi(n) \leq \phi < \phi(n-1)$.

To show that the proposed strategies constitute an equilibrium, I must verify that conditions (1)-(4) of Definition 4 hold. First note condition (4), that investors earn zero expected profits in equilibrium, follows by construction. To show that conditions (1)-(3) hold, I repeatedly apply the ‘‘one-shot deviation principle.’’ To see that the principle applies in this case, note that the game has perfect monitoring so that Proposition 2.2.1 of Mailath and Samuelson (2006) applies. Hence, to show that no profitable deviation exists for the opportunistic issuer, it is enough to examine her single deviation payoffs.

Observe that by construction the function V is the value delivered to the opportunistic issuer by playing the strategies given. Also note that $\hat{q}(1-\gamma) + \gamma > \ell$ and $\ell = \hat{q}(1-\gamma\ell) + \gamma\ell$, thus to show that the opportunistic issuer never has a profitable one shot deviation it is sufficient to show that

$$q^*(\phi)(p^*(\phi) - \gamma) \geq \hat{q}(1-\gamma) \quad (\text{3.10})$$

$$q^*(\phi)(p^*(\phi) - \gamma\ell) \geq (1-\gamma)\ell + \gamma(V(\phi^B) - V(0)), \quad (\text{3.11})$$

where

$$\phi^B = \frac{\phi}{\phi + \phi(1-\phi)}.$$

Also by Lemma 1, I need only consider deviations for the opportunistic issuer, since the honest issuer will always choose the same quantity. Finally, I must show that the proposed equilibrium prices and strategies yield zero expected profits to the investors. The fact that equilibrium beliefs are given by Bayes’ rule whenever possible is by construction.

The proof proceeds in three steps by first showing that no profitable one shot deviation exists for the opportunistic issuer over the three subintervals of reputation $[\hat{\phi}, 1]$, $[\underline{\phi}, \hat{\phi})$, and $[0, \underline{\phi})$.

Step 1: $\phi \geq \hat{\phi}$ First note that $\zeta(\phi)$ is increasing for $\phi \geq \hat{\phi}$ since $L(\phi)$ is increasing in ϕ . This implies that $p^*(\phi)$ is increasing for $\hat{\phi} \leq \phi \leq \bar{\phi}$. More over $p^*(\phi)$ is increasing for $\phi \geq \bar{\phi}$ since for such ϕ

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{1-(1-\lambda)\phi}.$$

Note that $q^*(\phi) = 1$ so that

$$\begin{aligned} p^*(\phi) - \gamma &\geq p^*(\hat{\phi}) - \gamma \\ &= \ell + \frac{\lambda(1-\ell)}{\lambda + (1-\lambda)(1-\pi^*(\hat{\phi}))(1-\hat{\phi})} - \gamma \\ &= \ell + L(\zeta^{-1}(\hat{\phi})) - \gamma = \hat{q}(1-\gamma) \end{aligned}$$

by the definition $\hat{\phi}$. Thus, the opportunistic issuer does not have a profitable one shot deviation when faced with a good asset. Next note that $\phi^B = 1$ for all $\phi \geq \bar{\phi}$

$$\begin{aligned} p^*(\phi) - \gamma\ell &\geq p^*(\bar{\phi}) - \gamma\ell \\ &= \ell(1-\gamma) + \frac{\lambda(1-\ell)}{1-(1-\lambda)\bar{\phi}} \\ &= (1-\gamma)\ell + \gamma(V(1) - V(0)) \end{aligned}$$

by the definition of $\bar{\phi}$. For $\hat{\phi} \leq \phi < \bar{\phi}$

$$\begin{aligned} p^*(\phi) - \gamma\ell &= \ell(1-\gamma) + \frac{\lambda(1-\ell)}{\lambda + (1-\lambda)(1-\pi^*(\phi))(1-\phi)} \\ &= \ell(1-\gamma) + L(\zeta^{-1}(\phi)) \\ &= (1-\gamma)\ell + L(\phi^B) \end{aligned}$$

by the definition of $\zeta(\phi)$. Thus, the opportunistic issuer does not have a profitable deviation when faced with a bad asset.

Step 2: $\underline{\phi} \leq \phi < \hat{\phi}$

By construction I have

$$p^*(\phi) = \gamma + \frac{\hat{q}(1-\gamma)}{q^*(\phi)}$$

so that $q^*(\phi)(p^*(\phi) - \gamma) = \hat{q}(1-\gamma)$, which implies that the opportunistic issuer does not have a profitable deviation when faced with a good type asset.

Now consider the case when the opportunistic issuer has a bad type asset. I'll show that $q^*(\phi)(p^*(\phi) - \gamma\ell) = (1-\gamma)\ell + \gamma(V(\phi^B) - V(0))$ for all $\underline{\phi} \leq \phi < \bar{\phi}$. Let $I_n = (\phi_n, \phi_{n-1}]$

where ϕ_n is the sequence defined by equations (3.17) and (3.18). The argument proceeds by induction on n . For I_1 , I have

$$\begin{aligned} q^*(\phi)(p^*(\phi) - \gamma\ell) &= \hat{q}(1 - \gamma) + \gamma(1 - \ell)q^*(\phi) \\ &= \hat{q}(1 - \gamma\ell) + L(\psi_1^{-1}(\phi)) \\ &= (1 - \gamma)\ell + \gamma(V(\psi_1^{-1}(\phi)) - V(0)) \end{aligned}$$

and

$$\begin{aligned} \psi_1^{-1}(\phi) &= \frac{\phi}{\delta_1(\psi_1^{-1}(\phi))} \\ &= \frac{\phi}{\phi + \pi(\phi)(1 - \phi)} = \phi^B \end{aligned}$$

so that $q^*(\phi)(p^*(\phi) - \gamma\ell) = (1 - \gamma)\ell + \gamma(V(\phi^B) - V(0))$ for all $\phi \in I_1$. Now assume that the equality holds for all $\phi \in I_k$ and all $k \leq n - 1$.¹ This assumption implies

$$\begin{aligned} L(\phi) &= \frac{\gamma}{1 - \gamma\lambda}(\lambda(p^*(\phi)q^*(\phi) + \gamma(1 - q^*(\phi))) \\ &= +(1 - \lambda)(p^*(\phi)q^*(\phi) + \gamma(1 - q^*(\phi))\ell) - (1 - \gamma)V(0) \\ &= \frac{\gamma}{1 - \gamma\lambda}(\lambda(\hat{q} + \gamma(1 - \hat{q})) + (1 - \lambda)(\ell + L(\phi^B)) - (1 - \gamma)V(0)) \\ &= \frac{\gamma(1 - \lambda)}{1 - \gamma\lambda}L(\phi^B) = \beta L(\phi^B) \end{aligned}$$

for all $\phi \in I_k$ and $k \leq n_1$. Thus $L(\phi) = \beta^{n-1}L(\psi_{n-1}^{-1}(\phi))$ for all $\phi \in I_{n-1}$. For $\phi \in I_n$, note that

$$\begin{aligned} \phi^B &= \frac{\phi}{\phi + \pi(\phi)(1 - \phi)} \\ &= \frac{\phi}{\delta_n(\psi_n^{-1}(\phi))} \\ &= \psi_{n-1}(\psi_n^{-1}(\phi)) \end{aligned}$$

by the definition of π , δ_n and ψ_n , so that $\phi^B \in I_{n-1}$ and $L(\phi^B) = \beta^{n-1}L(\psi_{n-1}^{-1}(\phi^B))$. But $\psi_{n-1}^{-1}(\phi^B) = \psi_{n-1}^{-1}(\psi_{n-1}(\psi_n^{-1}(\phi))) = \psi_n^{-1}(\phi)$ so that

$$\begin{aligned} (1 - \gamma)\ell + L(\phi^B) &= (1 - \gamma)\ell + \beta^{n-1}L(\psi_n^{-1}(\phi)) \\ &= q^*(\phi)(p^*(\phi) - \gamma\ell). \end{aligned}$$

by the definition of q^* and p^* . Then, by induction, $q^*(\phi)(p^*(\phi) - \gamma\ell) = (1 - \gamma)\ell + L(\phi^B)$ for all $\phi \in I_n$ for all $n \geq 1$, and the opportunistic issuer does not have a profitable deviation when faced with a bad type asset for $\underline{\phi} \leq \phi \leq \bar{\phi}$.

¹This proof utilizes *complete induction* so the base case is unnecessary, but is left for clarity.

Step 3: $\phi < \underline{\phi}$ The fact that the opportunistic issuer does not have a profitable one shot deviation for $\phi < \underline{\phi}$ follows directly from Proposition 9. □

Proof of Proposition 12. First I construct a candidate equilibrium. As in Section 3.3.4, I assume there exists $\underline{\phi}'$ and $\bar{\phi}'$ so that (3.26) binds for $\underline{\phi}' \leq \phi \leq \bar{\phi}'$ and $\pi^*(\phi) = 0$ for $\phi \geq \bar{\phi}'$. Thus for $\phi \geq \bar{\phi}'$ it is trivial to compute the equilibrium loss and price functions

$$p^*(\phi) = \ell + \frac{\lambda(1-\ell)}{1-(1-\lambda)\phi} \quad (\text{B.21})$$

$$L(\phi) = \frac{\gamma}{1-\gamma\lambda} \left(\frac{\lambda(1-\ell)}{1-(1-\lambda)\phi} - \lambda(\gamma-\ell) \right) \quad (\text{B.22})$$

The next task is to calculate π^* , and subsequently p^* and L , for $\underline{\phi}' \leq \phi \leq \bar{\phi}'$. Let $\psi_n(\phi) = \prod_{k=0}^n \delta_k(\phi)$ where $\delta_0(\phi) = \phi$ and

$$\delta_n(\phi) = 1 - \frac{\lambda}{1-\lambda} \left(\frac{1-\ell}{\left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\phi) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k} - 1 \right) \quad (\text{B.23})$$

for $n \geq 1$. Let a sequence be given by $\phi(n) = \psi_n(\bar{\phi}')$, note that now the first element of the sequence is $\bar{\phi}'$ rather than $\hat{\phi}$, since it is no longer necessary to consider $0 < Q < 1$. Then a candidate equilibrium is given by

$$\pi^*(\phi) = 1 - \frac{\lambda}{1-\lambda} \left(\frac{1-\ell}{\left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\psi_n^{-1}(\phi)) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k} - 1 \right) \frac{1}{1-\phi} \quad (\text{B.24})$$

$$p^*(\phi) = \ell + \left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\psi_n^{-1}(\phi)) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k. \quad (\text{B.25})$$

$$L(\phi) = \left(\frac{\gamma}{1-\gamma\lambda}\right)^n L(\psi_n^{-1}(\phi)) - \lambda(\gamma-\ell) \sum_{k=1}^n \left(\frac{\gamma}{1-\gamma\lambda}\right)^k \quad (\text{B.26})$$

where n is such that $\max\{\phi_n, \underline{\phi}'\} \leq \phi \leq \phi_{n-1}$ and

$$\underline{\phi}' = \min\{\phi | p^*(\phi) \geq \gamma\}.$$

The rest of the proof proceeds almost identically to the proof of Proposition 10 □

Proof of Proposition 13. First suppose there exists a truth telling equilibrium given by $Q(\phi)$ and $\phi(\phi) = 1$ such that $Q(\phi) > \hat{q}$ for some ϕ . Note that $\pi(\phi) = 1$ implies $\phi^B = \phi^S = \phi$, so the one-shot deviation principal then states that for all $\phi > 0$ such that $Q(\phi) > \hat{q}$

$$Q(\phi) + (1-Q(\phi))\gamma\ell - \ell \leq \gamma(1-p_B)(V(\phi) - V(0)). \quad (\text{B.27})$$

Next note that

$$\begin{aligned} Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell &= \hat{q} + (1 - \hat{q})\gamma\ell - \ell + (1 - \gamma\ell)(Q(\phi) - \hat{q}) \\ &= (1 - \gamma\ell)(Q(\phi) - \hat{q}) \end{aligned}$$

since $\hat{q} + (1 - \hat{q})\gamma\ell = \ell$. Finally, I have

$$\gamma(1 - p_B)(V(\phi) - V(0)) = \gamma\lambda(1 - p_B)(Q(\phi) - \hat{q}).$$

So it must be the case that $1 - \gamma\ell \leq (1 - p_B)\gamma\lambda$ since $Q(\phi) > \hat{q}$, which is what I needed to show.

Now suppose that $\gamma \geq \frac{1}{(1-p_B)\lambda+\ell}$. I show that $\pi(\phi) = Q(\phi) = 1$ for all $\phi > 0$ defines an equilibrium. To check these strategies constitute an equilibrium, note that they imply

$$V(\phi) = \frac{\lambda + (1 - \lambda)\ell}{1 - \gamma} \quad (\text{B.28})$$

for all $\phi > 0$ and $\phi^B = \phi^S = \phi$. This in turn implies

$$\begin{aligned} \gamma(1 - p_B)(V(\phi) - V(0)) &= \gamma\lambda(1 - p_B)\frac{(1 - \hat{q})(1 - \gamma)}{1 - \gamma} \\ &= \gamma\lambda(1 - p_B)\frac{1 - \ell}{1 - \gamma\ell} \\ &\geq (1 - \ell), \end{aligned}$$

so that the issuer does not have a profitable one-shot deviation, and the equilibrium is verified. \square

Proof of Proposition 14. Suppose that there exists a truth telling equilibrium given by $Q(\phi)$ and $\pi(\phi) = 1$ such that $Q(\phi) > \hat{q}$ for some ϕ . Note that $\pi(\phi) = 1$ and $p_G < 1$ imply $\phi^B = \phi^S = \phi^F = \phi$, so the one-shot deviation principal then states that for all $\phi > 0$ that $Q(\phi) > \hat{q}$

$$Q(\phi) + (1 - Q(\phi))\gamma\ell - \ell \leq \gamma(1 - p_B)(V(\phi) - V(\phi)) = 0. \quad (\text{B.29})$$

a contradiction since $Q(\phi) + (1 - Q(\phi))\gamma > \hat{q} + (1 - \hat{q})\gamma\ell = \ell$. \square