A 2-1/2-DIMENSIONAL NUMERICAL SOLUTION FOR THE ELECTROMAGNETIC SCATTERING USING A HYBRID TECHNIQUE

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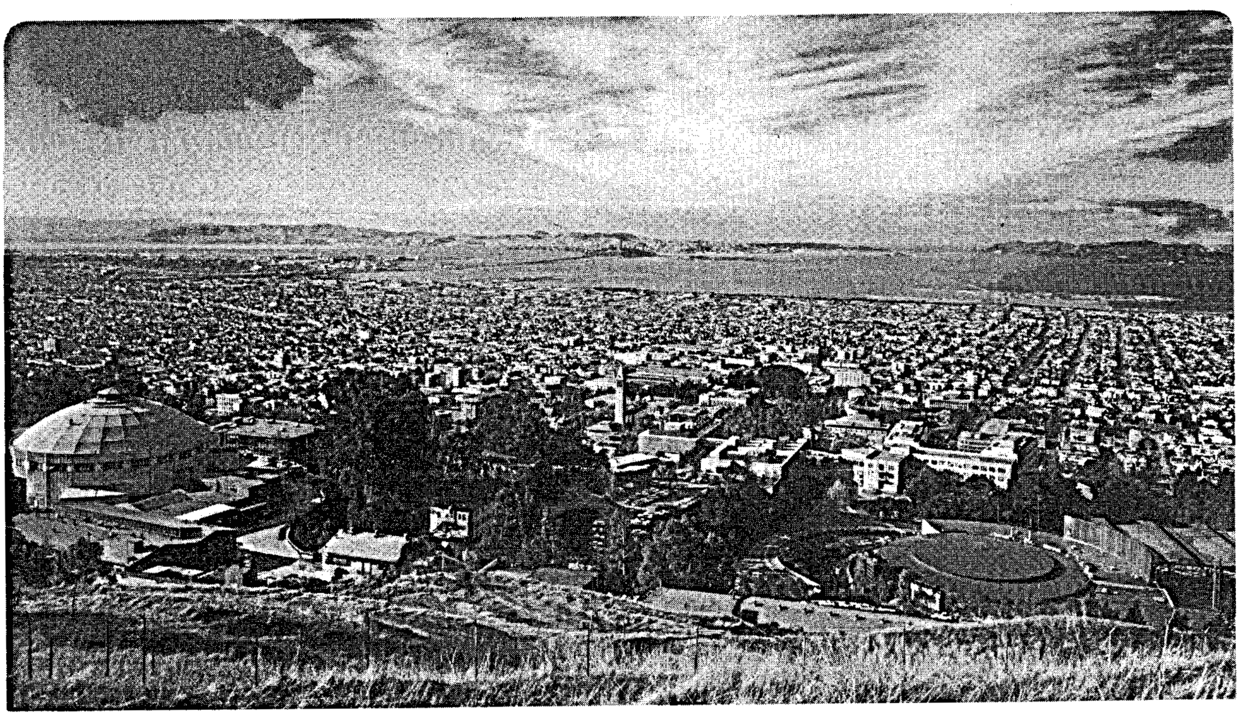

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Introduction

The electromagnetic(EM) method has been used for a wide variety of applied geophysical problems, beginning perhaps and achieving widest usage in mining exploration. A considerable number of techniques for both airborne and ground exploration have been developed and utilized in the search for conductive (sulfide) mineral deposits (Ward, 1967). Some of these techniques and methodologies have also been adapted to groundwater exploration, and more recently to geothermal, uranium and fossil fuel exploration. Since the introduction of the magnetotelluric technique in the 1950's and the large moment, controlled-source EM techniques in the 1970's, the electromagnetic method has been used increasingly for basic crustal investigations to depths of 10 km or more, such as in deep sedimentary basins, orogenic zones and at active plate margins.

An important applied problem studied at LBL is the use of EM techniques for geothermal reservoir exploration and delineation. We have used both controlled-source EM and magnetotelluric techniques at a number of hydro-thermal - geothermal prospects and reservoirs in Nevada (Wilt et al., 1982). Through this research we have been able to develop and demonstrate a number of techniques that provide high quality field data. The problem remains of how to interpret these data where complex geologic structures exist, and simple one-dimensional (layered earth) inversions cannot be safely applied. For these problems we rely on either numerical solutions or laboratory measurements made on carefully constructed scale models. Only a limited number of tank model results are available because of the difficulty of constructing models with the appropriate conductivities and geometries for each area investigated. Numerical solutions exist and are amenable to simple two-and three-dimensional models. The problem with many numerical techniques is the trade-off between accuracy and computation costs. Therefore, we have addressed the problem of developing faster numerical algorithms for EM interpretation without sacrificing accuracy.

Geologic models in which the electric parameters are invariant with strike constitute an important class of targets for electromagnetic exploration. A numerical solution for this class of models was obtained using the finite element method (Lee, 1978). In this technique the entire model is represented by a mesh composed of volume elements, each of which is assumed to have constant electrical properties. Mainly due to the large number of elements, the computing costs are usually prohibitive. Another disadvantage of the technique is the lack of accuracy in the numerical solution for models in which the discontinuity of lateral conductivity distribution is located close to the surface of the earth.

To overcome these limitations we have developed a new, efficient numerical solution based on the hybrid technique (Lee et al., 1981), a technique that makes use of both the finite element and the integral equation techniques. The finite element method is used for the solution internal to a anomalous conductivity structure embedded in a layered earth and the integral equation is used for the external layer-boundary value problem. The solution obtained in this manner tends to be more accurate than the one obtained by the finite element method alone. The major improvement with this technique is in the computing speed; often an order of magnitude faster than the finite element solution.

Formulation of Numerical Integral Equations

If a two-dimensional (infinite strike length) conductor exists in the lower half-space of an otherwise layered earth (Figure 1), one may approximate the electromagnetic variational integral as the sum (Lee, 1978)

$$I(E) \doteq \sum_{i=1}^{N} I_i \left\{ E(\eta_i) \right\}, \tag{1}$$

where η_i is the i-th discrete wave number in the strike direction, and

$$I_i \left\{ E(\eta_i) \right\} = \frac{1}{L} \iint_S \Bigg[\frac{k^2}{2\omega^2 \mu_o} \left(-E_x^2 + E_y^2 - E_z^2 \right) - \frac{1}{2\omega^2 \mu_o} \Bigg\{ \left(j\eta_i E_z - \frac{\partial E_y}{\partial z} \right)^2 - \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right)^2 + \left(\frac{\partial E_y}{\partial x} - j\eta_i E_x \right)^2 \Bigg\} \Bigg] dxdz . \tag{2}$$

In equation (2), E is the electric field, k is the wave number

$$k^2 = \omega^2 \mu_o \varepsilon - j\sigma\omega\mu_o ,$$

and L is the half strike length of the conductor characterizing the periodicity of the two-dimensional structure. Using the finite element method (Zienkiewicz, 1977) equation (2) may be evaluated as

$$I_i\left\{E(n_i)\right\} = E^T K E .$$

Following the variational principle, this reduces to a set of simultaneous equations

$$K E = 0 ,$$

which in turn may be partitioned into

$$\begin{pmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{pmatrix}\begin{pmatrix} E_i \\ E_b \end{pmatrix} = 0 ,$$

the upper portion of which suggests

$$E_i = -K_{ii}^{-1} \; K_{ib} \; E_b . \qquad (3)$$

Here the subscripts i and b indicate "internal" and "boundary", respectively.

The field equations on the surface ∂V can also be derived independently from the finite element equation. The result is an integro-differential equation governing the tangential electric field and the rotation of the electric fields as r approaches the surface ∂V:

$$\Omega(r)E(r) - E_p(r) = \int_{\partial V}\left\{ G^{EJ}(r/r') \cdot nxH(r') - G^{EM}(r/r') \cdot nxE(r') \right\} ds \qquad (4)$$

where $\Omega(r)$ is the normalized angle at r subtended by the volume to be integrated in that vicinity, and subscript "p" refers to the incident electric field at r that would exist in the absence of the inhomogeneity. $G^{EJ}(r/r')$ and $G^{EM}(r/r')$ are tensor electric Green's functions due to electric and magnetic current sources at r'. For a two-dimensional earth, Fourier transform of equation (4) in the strike direction for discrete harmonics η_i yields

$$\Omega(\rho)E(\rho,\eta_i) - E_p(\rho,\eta_i) = \int_\ell \Big\{ G^{EJ}(\rho/\rho',\eta_i) \cdot nxH(\rho',\eta_i) - G^{EM}(\rho/\rho',\eta_i) \cdot nxE(\rho',\eta_i) \Big\} d\ell \tag{5}$$

where ρ and ρ' are position vectors defined on the two-dimensional cross-section S.

The hybrid technique is initiated by transforming equation (5) into a numerical integral equation by rewriting the tangential magnetic field in terms of the tangential electric field by making use of numerical relation given by equation (3) and Maxwell's equation $\nabla xE = -j\omega\mu_o H$. The magnetic field extenal to the conductor may be computed by taking curl of equation (5), where $\Omega(\rho)$ becomes unity.

$$H(\rho,\eta_i) = H_p(\rho,\eta_i) + \int_\ell \Big\{ G^{HJ}(\rho/\rho',\eta_i) \cdot nxH(\rho',\eta_i) - G^{HM}(\rho/\rho',\eta_i) \cdot nxE(\rho',\eta_i) \Big\} d\ell \ . \tag{6}$$

After obtaining these solutions at ρ for a number of harmonics (η_i, i = 1, N; typically N = 15), inverse Fourier transform is carried out to yield solutions at r in the spatial domain.

Numerical Example

The algorithm has been coded on the CDC 7600 computer, and the code was tested against a simple model for which we have tank model results. The model is a vertical slab of resistivity 2.63 ohm-m, 12 m wide and 60 m long in the vertical extent. The slab is buried 10 m below the surface of the earth of 100 ohm-m resistivity. A vertical transmitter-receiver pair separated by 12 m is flown 20 m above the surface of the earth. The magnetic field computed at the receiver (H_x) is plotted at array center in ppm (Figure 2). The numerical solution is compared with tank model results obtained at the Richmond Field Station, University of California. At the same time a modified version of finite element solution is also plotted. The straight forward finite element method produces an electric field everywhere. Instead of taking the numerical derivatives of the electric field, we obtain a better result for the magnetic field by integrating the scattering current multiplied by the Green's function over the conductor. This is called the finite element - Green's function solution. The numerical results show good agreement for the 30 Hz response with the tank model result. With the frequency increased to 263 Hz both numerical solutions show smaller peak anomalies than the tank model results. For the in-phase component in particular, the hybrid solution differs by 100% from the tank model result, and the finite element - Green's function solution becomes somewhat unstable.

Acknowledgement

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FIGURE CAPTIONS

Figure 1 A conductor (V) buried in the lower half-space of a layered earth. Current and magnetic sources are outside the conductor whose surface is ∂V. S is the cross-section of V if it is two-dimensional.

Figure 2. A coaxial transmitter-receiver pair separated by 12 m is flown 20 m above the surface of the earth in which a vertical tabular conductor is embedded (top). The responses in ppm for H_x are plotted for 30 Hz (middle) and 263 Hz (bottom).

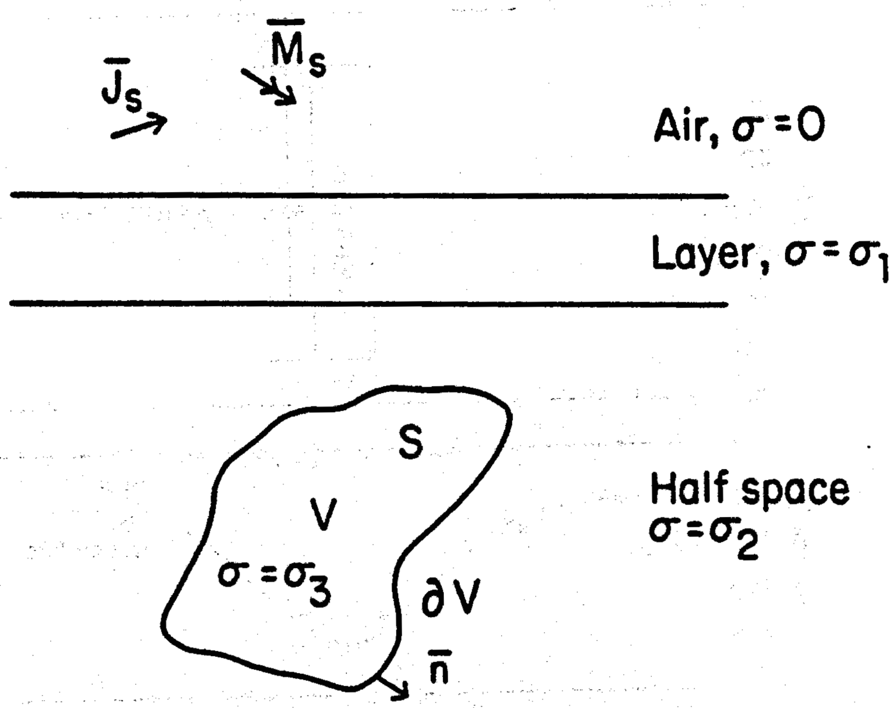

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Fig. 1

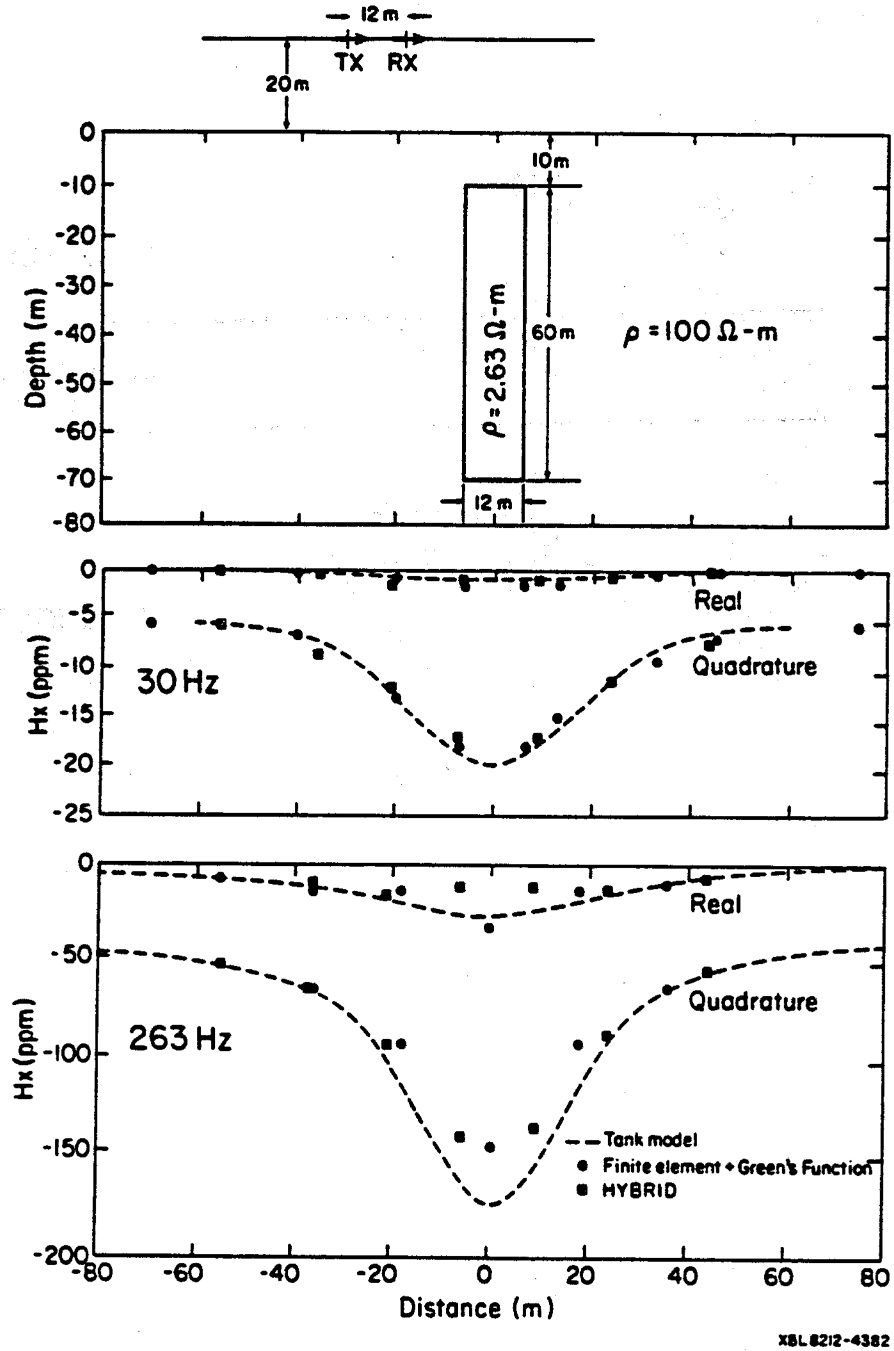
12 m
TX RX
20m
10m
60m
12 m
ρ = 2.63 Ω-m
ρ = 100 Ω-m
Depth (m)
Hx (ppm)
30 Hz
Real
Quadrature
263 Hz
Tank model
Finite element + Green's Function
HYBRID
Distance (m)
XBL 8212-4382

Fig. 2