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An Approximation Method for Regge Trajectories and Its Implications on the Spin-2 Particle of the Chew-Frautschi Diagram*

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The Chew-Frautschi diagram[1] suggests the possibility of a spin-2 particle of mass of about 50 m_π^2 on the Pomeranchuk trajectory. Two experimental groups[2,3] have found such a particle at 80 m_π^2, and at first glance it is tempting to assume that this particle belongs to the Pomeranchuk trajectory. In this letter we suggest a method of approximation consistent with all the generally accepted properties and apply this to the Pomeranchuk trajectory.[4] Our results show that this particle cannot belong to the Pomeranchuk trajectory unless one of the generally accepted assumptions is wrong or the conclusions drawn from the high-energy p-p scattering experiments[5,6] are not correct.

The following features are either proved or conjectured on general grounds:

(a) $\mathrm{Im}\,\alpha(t) = 0$ for $t < t_0$,

and $\mathrm{Im}\,\alpha(t) > 0$ for $t > t_0$ (where $t_0 = 4\,m_\pi^2$), (1)

where $\sqrt{t}$ is the center-of-mass energy and $\sqrt{t_0}$ is the two-particle threshold energy;

(b) $\lim_{t \to \infty} \alpha(t) = -1$, (2)

(c) $\operatorname{Im} \alpha(t) \underset{t \approx t_0}{\sim} (t - t_0)^{\lambda}$, where $\lambda = \alpha(t_0) + \frac{1}{2}$, (3)

(d) $$\operatorname{Re} \alpha(t) = \operatorname{Re} \alpha(\infty) + \frac{P}{\pi} \int_{t_0}^{\infty} \frac{\operatorname{Im} \alpha(t')\, dt'}{t' - t}, \qquad (4)$$

where from (a) and (d) $\operatorname{Re} \alpha(t)$ is monotonic for $t < t_0$;

and (e) $$\Gamma = \frac{\operatorname{Im} \alpha(t)}{\frac{d \operatorname{Re} \alpha(t)}{dt}\, t^{1/2}}, \qquad (5)$$

where Γ is the width of the particle.[7]

Statements (a), (b), and (c) are satisfied at least for the first Regge trajectory of a Yukawa potential.[8] In this case, $\operatorname{Im} \alpha$ has a maximum and its derivative vanishes at only one point. Gribov has shown that $\alpha(t)$ for a fermion trajectory has a left-hand cut;[9] therefore, assumptions (a) and (d) should be modified for a fermion. Assumptions (a), (b), and (d) are also satisfied for Coulomb potential;[10] (c) has been proved,[11] and (d) has already been conjectured[12] in the relativistic case.

Our aim is to find a plausible form for $\operatorname{Im} \alpha$ satisfying the conditions discussed. From experience with the Yukawa potential[8] we expect that for a boson trajectory, $\operatorname{Im} \alpha$ should have the general shape of Fig. 1.

This shape suggests a formula of the type

$$\operatorname{Im} \alpha(x) = \frac{c x^{\lambda}}{c_1 + (x - c_2)^2}. \qquad (6)$$

Such a formula can represent a large variety of curves ranging from a curve with a narrow peak like a δ function, to an extremely flat curve. Parameters c, λ, c_1, and c_2 are to be determined from the conditions and experimental information about the trajectory. More generally, we can take $\mathrm{Im}\,\alpha = x^{\lambda}\, N/D$, where N and D are polynomials in x with no positive roots and the degree of $x^{\lambda} \cdot N$ is less than that of D for convergence reasons. The number of parameters one can introduce depends on the available information about the trajectory. Using expression (6) for the $\mathrm{Im}\,\alpha\,(t')$, the integral in Eq. (4) can be evaluated[13] exactly. With the help of the four conditions

$$\alpha\,(0) = 1, \qquad \text{(see Ref. 1)}$$

$$\lambda = \alpha\,(t_0) + \frac{1}{2},$$

$$\mathrm{Re}\,\alpha\,(80) = 2,$$

and

$$\Gamma\,(80) = \left(\frac{\mathrm{Im}\,\alpha\,(t)}{\frac{d\,\mathrm{Re}\,\alpha\,(t)}{dt}\, t^{1/2}} \right)_{t=80} \leq 200 \text{ MeV},$$

the four parameters c_1, c_2, c, and λ can be determined numerically from four nonlinear equations. This calculation was carried out at the IBM 7090 computer of the Lawrence Radiation Laboratory. The resulting solution is given by Curve (1) of Fig. 2. The values obtained for the parameters are $c_1 = 259.66$, $c_2 = 55.243$, $c = 3.79$, and $\lambda = 1.533$. Parameter c_2 gives approximately the position of the maximum of the $\mathrm{Im}\,\alpha\,(t)$, and λ controls the slope of the $\mathrm{Re}\,\alpha\,(t)$ at $t = 0$, which turns out to be $2/5\ \mathrm{BeV}^{-2}$. The width Γ is about 180 MeV. Figure 3 shows $\mathrm{Im}\,\alpha\,(t)$ for this solution. The differential p-p cross section at high energies has been measured by two experimental groups,[5,6] and from this $\mathrm{Re}\,\alpha\,(t)$ for negative t is found. Their analysis is based on the assumption that at these energies only the contribution of the Pomeranchuk trajectory is significant. If we take the interpretation of these experimental data seriously we would arrive at the conclusion that the

1250 MeV particle cannot belong to the first Pomeranchuk trajectory. Using the general assumptions (a) through (e) and $\alpha(0) = 1$, we find that a trajectory in reasonable agreement with these experimental data reaches $\mathrm{Re}\,\alpha = 2$ at an energy much lower than 1250 MeV. Curve (2) is an example of such a trajectory.

It is interesting to note that if we relax the condition $\alpha(\infty) = -1$ and find a solution which coincides with a "visual least-squares fit" of the experimental data the solution would still predict a mass much lower than 1250 MeV (Curve 3). For Curve (3) $\alpha(\infty) = -3.25$.

From our result (Curve 2), A. Pignotti suggested that the spin-2 particle of the Pomeranchuk trajectory may be hidden under the ρ resonance.[14] There has also been some evidence of a resonance at 500 MeV[15] and a width between 50 and 100 MeV which would be also compatible with Curve (2).

When this paper was first written we left the question open as to which of the two alternatives (Curves 1 or 2) should represent the Pomeranchuk trajectory. Now in view of the new experimental high energy p-p cross-section data of Foley et al.,[16] who find $\alpha'(0) = 0.66\ \mathrm{BeV}^{-2}$, we feel that:

(1) Our Curve (1) is a better candidate for the Pomeranchuk trajectory.

(2) The interpretation of the experimental data of Diddens et al.[6] and Baker et al.[5] that the Pomeranchuk trajectory alone contributes at these energies is not reasonable.

(3) The 1250-MeV particle may indeed be the spin-2 particle of the Chew-Frautschi diagram.

From the way the experimental data have progressed so far it is almost certain that when the p-p cross sections are measured at still higher energies the experimental result for $\alpha'(0)$ will become even smaller.

It is our pleasure to thank Professor G. F. Chew for his interest and encouragement throughout the course of this work. One of us (I. A. S.) wishes to thank Dr. David Judd for his hospitality at the Lawrence Radiation Laboratory and Robert College of Istanbul for an "American Colleges Fellowship."

FOOTNOTES AND REFERENCES

* Work done under the auspices of the U. S. Atomic Energy Commission.

† American Colleges Fellow.

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4. G. Domokos has already used an approximation with a different Ansatz for $\mathrm{Im}\,\alpha(t)$ which gives a set of linear equations (CERN Proceedings). Although our Ansatz leads to nonlinear equations which are more difficult to solve we feel that it represents a more economical use of parameters.
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13. We would like to thank A. Pignotti for pointing out that the integration can be carried out if we apply Cauchy's theorem to the imaginary part of (t). The same integration has been evaluated by Domokos using Feynman's trick.

14. We would like to mention that A. Pignotti, using assumptions similar to ours, has again arrived at similar conclusions; see A. Pignotti, Lawrence Radiation Laboratory Report UCRL-10600, December 1962.

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FIGURE CAPTIONS

Fig. 1. General shape of Im α for boson trajectory.

Fig. 2. Re α vs t.

Fig. 3. Im α vs t corresponding to Curve (1) of Fig. 2.

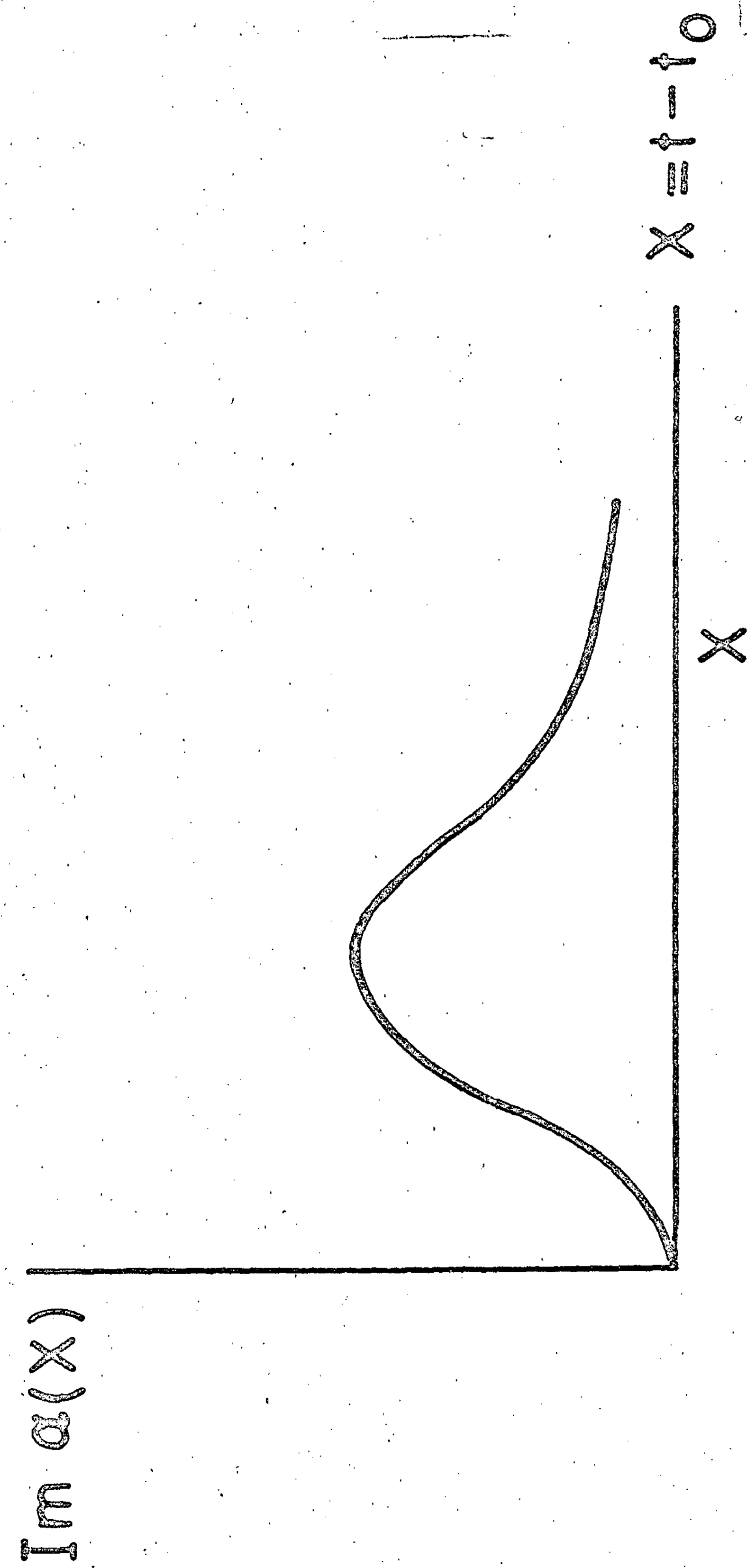

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Fig. 1.

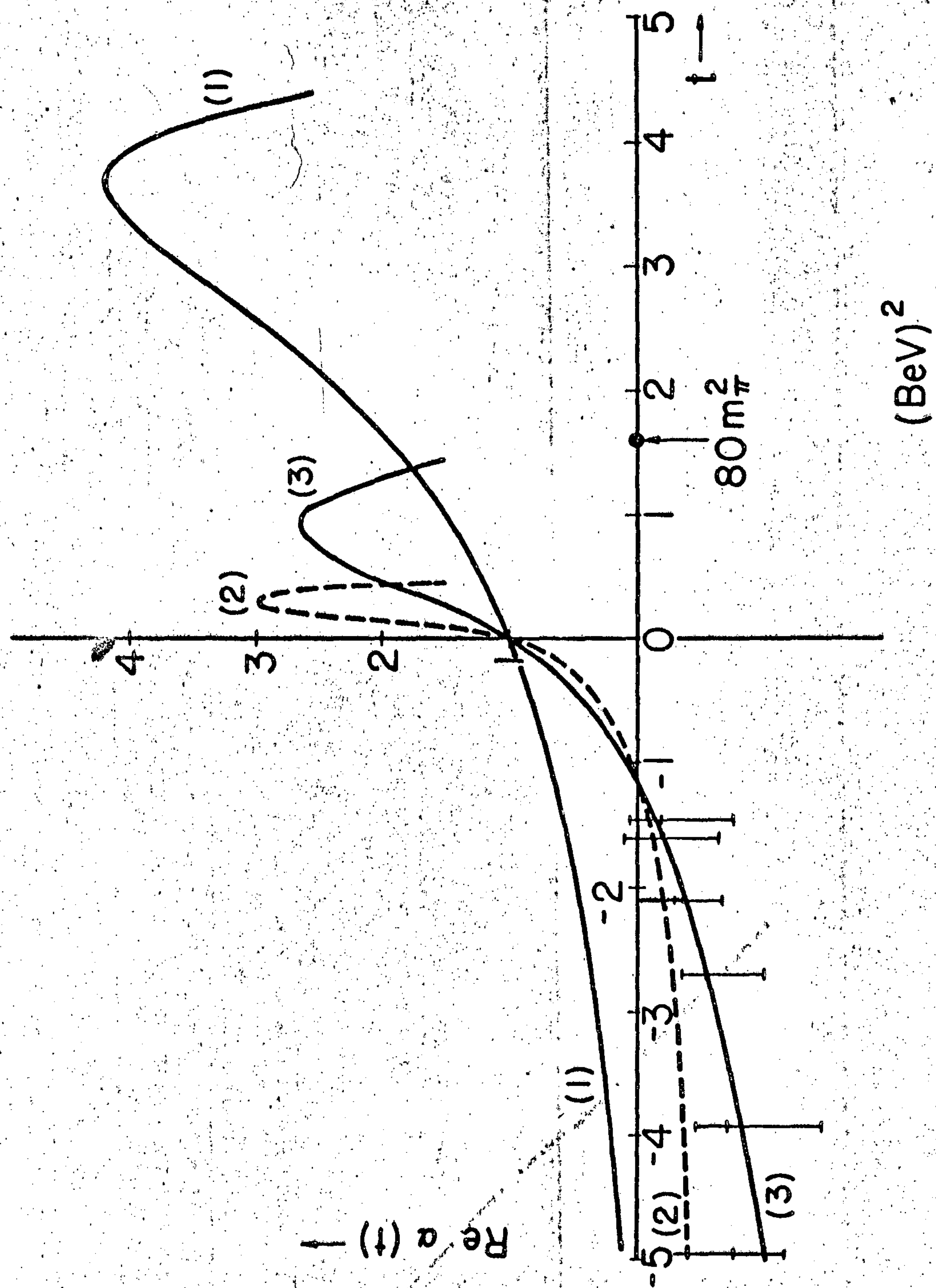

Fig. 2.

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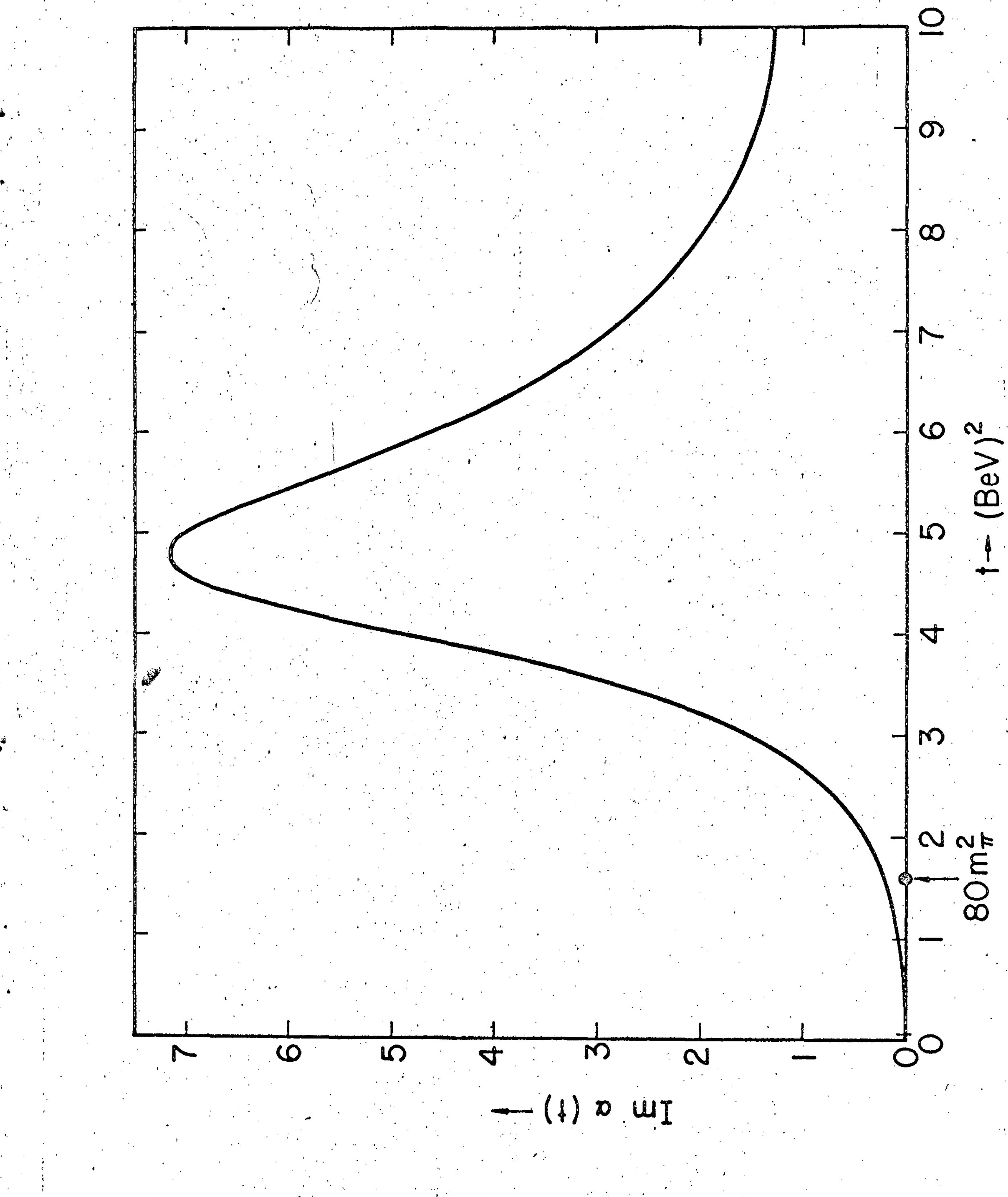

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Fig. 3.