Title

A DETERMINATION OP THE REFRACTIVE INDEX OF LIQUID DEUTERRIUM USING THE CERENKOV EFFECT

Authors

Ayres, David S.
Cormack, Allan M.
Greenberg, Arthur J.
et al.

Publication Date

1968-11-08

University of California

Ernest O. Lawrence Radiation Laboratory

A DETERMINATION OF THE REFRACTIVE INDEX OF LIQUID DEUTERIUM USING THE CERENKOV EFFECT

David S. Ayres, Allan M. Cormack, Arthur J. Greenberg,
Robert W. Kenney, David O. Caldwell, Virgil B. Elings,
William P. Hesse, and Rollin J. Morrison

November 8, 1968

DISCLAIMER

Submitted to Physica

UCRL-18567
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

A DETERMINATION OF THE REFRACTIVE INDEX OF LIQUID DEUTERIUM USING THE CERENKOV EFFECT

David S. Ayres, Allan M. Cormack, Arthur J. Greenberg,
Robert W. Kenney, David O. Caldwell, Virgil B. Elings,
William P. Hesse, and Rollin J. Morrison

November 8, 1968

A DETERMINATION OF THE REFRACTIVE INDEX OF LIQUID DEUTERIUM USING THE CERENKOV EFFECT*

David S. Ayres, Allan M. Cormack,† Arthur J. Greenberg, and Robert W. Kenney

Lawrence Radiation Laboratory
University of California, Berkeley, California

and

David O. Caldwell, Virgil B. Elings, William P. Hesse, and Rollin J. Morrison

Physics Department, University of California
Santa Barbara, California

November 8, 1968

ABSTRACT

We have found the refractive index of liquid orthodeuterium at 24.2°K and 3200 Å to be 1.1321 ± 0.0025. This is in agreement with the value calculated from the Lorentz-Lorenz relation, the Lorentz theory of the dielectric constant, and the measured dispersion of deuterium gas. Our measurements also yield a rough value for the difference in molecular polarizabilities of deuterium and hydrogen, $\Delta\alpha/\alpha = (2.3 \pm 1.9)\%$.

The measurement was a coincidental result of a particle-physics experiment in which Cerenkov counters were used with either liquid hydrogen or liquid deuterium as a radiating medium. The ratio of the refractive indices of the two substances was determined from the change in response of the counters when the radiating medium was changed.

A recent experiment to compare the lifetimes of π^+ and π^- mesons in a check of CPT invariance employed two differential Cerenkov counters which could be used with either liquid hydrogen or liquid deuterium as a radiating medium[1]). While the Cerenkov counters were being designed, we found that no experimental measurements of the refractive index of liquid deuterium had been published. The purpose of this note is to report the value of the index which was calculated from our data, by comparing the velocity acceptance of each counter when filled with liquid deuterium with the velocity acceptance observed using liquid hydrogen.

When a particle of velocity βc (c = velocity of light) traverses a transparent medium of refractive index n, Cerenkov light is produced at an angle θ to the direction of motion, where

$$\cos\theta = 1/n\beta \leqslant 1. \quad (1)$$

Our Cerenkov counters responded only to light within the angular range $8° \leqslant \theta \leqslant 14°$, from particles with $\beta = 0.92 \pm 0.01$ when filled with liquid hydrogen, and from particles with a somewhat lower velocity (corresponding to the same range of θ) when filled with liquid deuterium. By measuring a counter's efficiency as a function of incident beam momentum, a steep-sided momentum-response curve was obtained. This curve had a short flattop near a central momentum, with steeply sloping sides (30% change in efficiency for a 1% change in momentum) at a higher and lower momenta. By comparing the steep sides of the curve obtained from a counter filled with liquid deuterium with those obtained using liquid hydrogen, it was possible to find the ratio of the central velocities. If Eq. (1) is evaluated for both hydrogen (H) and deuterium

(D) as radiating media, we obtain $n_D/n_H = \beta_H/\beta_D$. Thus, the velocity ratio gives the index of refraction of liquid deuterium relative to the index for liquid hydrogen, which is known rather well[2,3]). The uncertainty in the measured ratios is mainly due to the fact that momentum-response curves obtained with liquid hydrogen and liquid deuterium differ slightly in shape.

Two independent measurements of n_D/n_H were made, one using each Cerenkov counter. The counters were sensitive to somewhat different portions of the spectrum of Cerenkov light, each spanning a range of about 1000 Å about central wavelengths of 2900 Å and 3450 Å. The absolute refractive index n_D was found using the index of liquid para-hydrogen[4]) measured by Diller[3]) at 5462 Å.

In order to extrapolate Diller's measurement to our wavelengths, the following procedure is used. The specific refraction, r_λ, is defined by the Lorentz-Lorenz relation,

$$r_\lambda = \frac{1}{\rho} \cdot \frac{n_\lambda^2 - 1}{n_\lambda^2 + 2}, \qquad (2)$$

and is evaluated at $\lambda = 5462$ Å from the measured index n_λ. The dispersion relation

$$r_\lambda = r_\infty + A/\lambda^2 + B/\lambda^4 \qquad (3)$$

is then applied to find r_∞. The constants A and B are assumed to be independent of density, and can be found from the dispersion of hydrogen gas[5]). Equation (3) is now used to find r_λ at the desired wavelength, and Eq. (2) is inverted to give n_λ. In this way we obtained the index of refraction of liquid hydrogen at 2900 Å and 3450 Å, and used these with our measured ratios to find the absolute refractive indices of liquid deuterium at the two wavelengths. These were then extrapolated to 3200 Å using Eqs. (2) and (3), the constants A and B being found from the measured dispersion of deuterium gas[6]. The weighted average of the two measurements gave finally: $n_D = 1.1321 \pm 0.0025$ at 3200 Å and $\rho = 0.1618$ g/cm^3. The density quoted is based on the assumption of pure orthodeuterium[4] at a temperature of 24.2°K, which was determined by platinum resistance thermometers immersed in the radiating medium. Table I summarizes the constants used for the calculation. No attempt has been made to estimate the uncertainty in the A and B, but they are probably much more accurate than available values of r_∞. The error given for n_D is due only to the uncertainties in our own measurement.

Using r_∞ from our deuterium measurement, we can obtain the molecular polarizability of liquid deuterium, a quantity of some theoretical interest[7, 8]. The Lorentz theory of the dielectric constant[9] predicts

$$r_\infty = \frac{4\pi}{3} \frac{N_0}{M} \alpha, \tag{4}$$

where N_0 is Avogadro's number, M is the molecular weight, and α is the molecular polarizability. Comparing our results to $\alpha(H_2)$ calculated from Diller's measurements, we obtain

$$\frac{\alpha(H_2) - \alpha(D_2)}{\alpha(H_2)} \equiv \Delta\alpha/\alpha = (2.3 \pm 1.9) \times 10^{-2}.$$

This difference is consistent with the more accurate determinations of $\Delta\alpha/\alpha$ for hydrogen and deuterium gas[10].

Because relatively accurate measurements have been made on gaseous deuterium, the result of an indirect calculation of the refractive index of liquid deuterium from Eqs. (2) and (3) may well be more precise than extrapolation of our direct measurement. This type of indirect calculation has been carried out for liquid hydrogen and experimentally verified to high precision in Refs. 2 and 3. The constant r_∞ in Eq. (3) can be found either from Eq. (4)--using $\Delta\alpha/\alpha$ from Ref. 10 and $\alpha(D_2) = \alpha(H_2) + (\Delta\alpha/\alpha)\alpha(H_2)$--or from the Clausius-Mossotti relation, $r_\infty = \frac{1}{\rho} \cdot \frac{\epsilon - 1}{\epsilon + 2}$, where ϵ is the experimentally determined dielectric constant of liquid deuterium[11, 12]. The agreement between values of r_∞ found by the two methods has been used to estimate its uncertainty, and the results are given under "r_∞(calculated)" in Table I. The value of r_∞ found in this way can be used in Eq. (3) to calculate r_λ, where A and B are obtained from the dispersion of deuterium gas[6]. The refractive index n_λ is then calculated from the Lorentz-Lorenz relation, Eq. (2).

We thank Professor Burton J. Moyer and Professor A. Carl Helmholz for support and encouragement throughout the experiment. Mr. Dwain E. Diller kindly provided a preprint of Ref. 3, and suggested the method used to calculate the expected value of the refractive index. Mr. Edwin F. McLaughlin assisted with our initial search for an experimental measurement of the refractive index of liquid deuterium.

FOOTNOTES AND REFERENCES

*Work done under the auspices of the U. S. Atomic Energy Commission.

†On leave from the Physics Department, Tufts University, Medford, Mass.

1) D. S. Ayres, A. M. Cormack, A. J. Greenberg, R. W. Kenney, D. O. Caldwell, V. B. Elings, W. P. Hesse, and R. J. Morrison, Phys. Rev. Letters 21 (1968) 261.

2) R. J. Corruccini, Refractive Index and Dispersion of Liquid Hydrogen, National Bureau of Standards Technical Note 323 (1965).

3) D. E. Diller, The Refractive Index of Gaseous and Liquid Hydrogen, J. Chem. Phys. (to be published).

4) The ortho-para conversion process was accelerated in both hydrogen and deuterium radiating media by the use of a nickel oxide, silica-gel catalyst. Thus the data used for our calculations were obtained from e-hydrogen (99.8% para) and e-deuterium (97.8% ortho).

5) J. Koch, Ark. Mat. Astr. Fys. 8 (1912) No. 20.

6) T. Larsen, Z. Physik 100 (1936) 543.

7) R. P. Bell, Trans. Faraday Soc. 38 (1942) 422.

8) E. Ishiguro, T. Arai, M. Mizushima, and M. Kotani, Proc. Phys. Soc. (London) A, 65 (1952) 178.

9) H. A. Lorentz, Ann. Physik. Chem. Wied. 9 (1880) 641.

10) See for example, L. J. F. Hermans, J. J. de Groot, H. F. P. Knapp, and J. J. M. Beenakker, Physica 31 (1965) 1567.

11) A. Van Itterbeek and J. Spaepen, Physica 9 (1942) 339.

12) Dudley B. Chelton and Douglas B. Mann, Cryogenic Data Book, Lawrence Radiation Laboratory Report UCRL-3421, May 1956 (unpublished).

Table I

Quantities used in the calculations

Parameter	Liquid deuterium	Liquid hydrogen
r_∞ (cm^3/g)* (calculated)	0.495 ±0.003	1.0039
r_∞ (cm^3/g) (from our measurements)	0.4909±0.0096	---
A (× 10^3) (λ in microns)	3.737	8.005
B (× 10^5) (λ in microns)	3.12	6.76
ρ(g/cm^3)	0.1618	0.07007
T (°K)	24.2	20.9
n at 3200 Å	1.1321±0.0025	1.11701
Raw Data		
n_D/n_H at 2900 Å	1.0096±0.0045	
n_D/n_H at 3450 Å	1.0145±0.0024	

*) The uncertainty quoted for liquid deuterium reflects the difference in values obtained using different methods of estimation. The number quoted for liquid hydrogen is calculated from Ref. 3.