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Extending the Lee-Carter method to model the rotation of age patterns of mortality-decline for long-term projection

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Abstract

In developed countries, mortality decline is decelerating at younger ages and accelerating at old ages, which we call a "rotation". We expect that this rotation will also occur in developing countries as they attain high life expectancies. But the rotation is subtle and has proved difficult to handle in mortality models that include all age groups. Without taking it into account, however, long-term mortality projections will produce questionable results. Here we simplify the problem by focusing on the relative magnitude of death rates at two ages, 0 and 15–19, while making assumptions about changes in rates of decline at other ages. We extend the Lee-Carter method to incorporate this subtle rotation in projection. We suggest that the extended Lee-Carter method could provide plausible projections of the age pattern of mortality for populations that currently have very high life expectancies as well as others. Detailed examples are given using data from Japan and the US.

This paper addresses a practical problem faced by the United Nations Population Division: how to modify the Lee-Carter method to project mortality over a long time horizon to the year 2100 for 196 countries and areas. The Lee-Carter method is based on extrapolating the historical rates of mortality decline by age. During the period for which mortality estimates are available, mortality at younger ages and particularly for infants has declined very rapidly. Continuation of these rates of decline would lead to extremely low projected death rates 90 years from now, and would also alter the age pattern of mortality in childhood so that, for example, in some countries infant mortality would be lower than at other childhood ages. We cannot know for sure that such projected patterns will not actually occur. Yet analysis of the existing data suggests that age patterns of rates of mortality decline have been changing, and in particular that declines at younger ages have been slowing while declines at older ages have been accelerating. Changes of this sort could be called a "rotation" of the vector of age specific rates of decline (the Lee-Carter $b(x)$). If the rates of decline are

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rotating, then projections which ignore the rotation will lead to errors, particularly in the projected age patterns of future death rates.

Our task is to make forecasts of mortality a century into the future. The basic approach is to extrapolate based on patterns and trends in the age specific mortality data in recent decades. There are several difficulties that may appear over such a long horizon. First, a number of analysts have noted that the *b(x)* age schedule has not remained constant in the historical data. For example, Bongaarts (2005) wrote "Instead of being constant, rates of improvement in mortality have tended to decline over time at younger ages, while they have risen at older ages…." Bongaarts then proposed the "shifting logistic" method to make long run forecasts of adult mortality. However, he explained that this method cannot be used for mortality under age 25, so the problem remains. A second problem is that extrapolation over such a long time period using the Lee-Carter method can lead to age patterns that appear anomalous. For example, analysts have observed that differences across age in constant rates of decline, as reflected in the *b(x)* vector, lead to increasingly large proportional differences in the forecasts for death rates at adjacent ages. Such discontinuities or jaggedness in the forecasted age profile of mortality are inconsistent with our prior belief that the profile should vary smoothly and continuously across age. Girosi and King (2008) address this issue using Bayesian methods to impose smoothness, but in practice their projections, which typically also use covariate risk factors, are for medium horizons of a decade or two. The present authors note a third problem, that infant mortality declines more rapidly than at other young ages, and consequently forecasted levels may appear implausibly low relative to those ages. These last two difficulties are both based on prior ideas about how future mortality will look.

It is difficult to formulate and defend priors for the age shape of mortality schedules a century hence. A place to start is the theory of how evolutionary forces have shaped the age schedule of human mortality. Many traits of an organism influence its level and age-shape of mortality. Examples are the timing and sequencing of organ system development (including the immune system and the reproductive system); resources devoted to proofreading DNA replications; hormonal influences on behaviour such as risk taking; investment in body armour, weaponry, camouflage or speed to escape predators; parental care of offspring; capability to repair body damage with the risk that repair mechanisms may be hijacked by cancer; and so on. In addition, mutations with deleterious consequences for health and mortality at different ages (e.g. Alzheimer's, Huntington's) occur at conception and are deselected from the population at rates that are lower at older ages, influencing the age pattern of mortality. Evolutionary theory provides clues about the deep structure of mortality across the life span, an age structure that persists even after deaths from infectious disease have been largely eliminated.

In a seminal article, Hamilton (1966) implicitly differentiated the intrinsic rate of natural increase in Lotka's equation with respect to perturbations in the force of mortality, on the assumption that mortality would be lower at ages where it made more difference to reproductive fitness. He concluded that evolved mortality should be low and constant from birth to the age at reproductive maturity, contrary to expectations based on Fisher's (1930) concept of reproductive value. This result arises because dying at age 0 entails no greater loss in expected life time production than does dying just before reproductive maturity e.g. at 14, since survival to 15 depends on survival through every preceding year of life. However, as Hamilton realized, his analysis ignored post-birth investments in a child by the parents and others. Following reproductive maturity, Hamilton concluded that death rates would rise throughout the reproductive years as remaining future reproduction declined. After menopause mortality should rise rapidly without limit. But this part of the analysis also ignored the post-reproductive contributions made by older individuals to the reproductive success of their adult children through transfers of food, childcare, advice, and so on.

Articles by Lee (2008; 2003), and Chu, Chien and Lee (2008) built on Hamilton's analysis, but incorporated intergenerational resource flows. They concluded that mortality should decline strongly from birth to the age of sexual maturity, reflecting the rising cumulative value of resources invested in a child as age increases. They also concluded that mortality will reach its lowest point around the age of reproductive maturity in the mid to late teens, and then rise through the reproductive years and thereafter, as the value of all summed transfers to be made in the future diminishes. The age shape of mortality should be U shaped and should imply substantial post-reproductive survival. Infant mortality should be relatively high because the cumulated investment in an infant is very low, even when costs of pregnancy are included.

This excursion into evolutionary theory supports a simple prior for mortality in the distant future: it will remain U-shaped, with relatively high infant mortality, a minimum in the teens, and rising mortality thereafter. The forecast method we propose in this paper incorporates these priors by assuming that the *b(x)* coefficients from ages 0 to 65 converge to a single constant value. Above age 65 or 70 the *b(x)* coefficients decline with age in any event, guaranteeing that mortality will continue to rise steadily with age. Once the *b(x)* have converged to a constant value for ages 0–64, the proportional age pattern for those ages will remain fixed at the U-shape it has at that point. These modifications are intended to address the point raised by Girosi and King (2008:39): "Almost no matter what one's prior is for a reasonable age profile, Lee-Carter forecasts made sufficiently far into the future will eventually violate it."

Over the years we have made various efforts to model a rotation based on systematic analysis of the historical data, but these efforts have failed, in part because of the complexities of modelling changes in *b(x)* at all ages, and in part because rotations are not consistent across different countries and different time periods. Here we take a simpler approach, focusing on two specific age groups, 0 and 15–19, for reasons to be specified soon, and we make various assumptions about the patterns of changes at other ages. The rotation we suggest makes little or no difference to the projections for most countries, other than those countries that already have life expectancies above 80. For the US, for example, the differences are quite small, whereas for Japan they are larger.

The Lee-Carter method (Lee and Carter 1992) has been applied to middle-term (around 50 years) mortality projections for almost all the countries with reliable data and in normal socioeconomic conditions, and has provided satisfactory results (Lee and Miller 2001;

Tuljapurkar, Li and Boe 2000). Since its inception, various extensions and modifications have been proposed to improve the performance of the original Lee-Carter method for short and medium term projections (see review by Booth 2006; Girosi and King 2008; Lee 2000; Shang, Booth and Hyndman 2011; Soneji and King 2011), but all of them assume that *b(x)* remains constant over time which is unlikely to hold for more than a number of decades into the future.

A subtle trend of historical mortality change among the low-mortality countries, is that the decline of infant and child mortality was decelerating, and the reduction of old-age mortality was accelerating (Horiuchi and Wilmoth 1995; Kannisto et al. 1994; Li and Gerland 2011). This trend implies that the age-specific rates of mortality-decline are rotating over time. This change in *b(x)* is not utilised by any existing Lee-Carter approaches as far as we know, perhaps because it is too subtle to model. Without modelling these changes, however, longer-term projections (50 to 100 years or longer) could imply questionable age patterns of mortality, especially for low-mortality countries where the subtle trends are more significant. To avoid this, Li and Gerland (2011) introduced a robust rotation in the Lee-Carter *b(x)* and called it the Lee-Carter method with robust rotation (LC_RR), which is subjective and may entail unnecessarily strong modifications. In this paper, we improve the rotation model by providing a more objective basis for it and by making the rotation occur continuously over time rather than abruptly. We call the proposed method the Lee-Carter method extended to model the rotation (LC_ER).

Let the death rate at age *x* and time *t* be *m(x,t)*, and let *a(x)* be the latest observed value of *log[m(x,t)]* or the over-time average of *log[m(x,t)]*. The Lee-Carter method uses the singular-value decomposition of *{log[m(x,t)]-a(x)}* to obtain

$$\log[m(x,t)]=a(x)+b(x)k(t)+\varepsilon(x,t). \quad (1)$$

In order to produce projections that are non-divergent between the two sexes, Li and Lee (2005) suggested using a sexes combined *b(x)* and *k(t)* for projection, which is adopted in this paper. In (1), *b(x)* represents the age pattern of the average mortality rates of decline over-time, and *k(t)* describes the cross-age pace of mortality decline at time *t*. The main reason for the success of the Lee-Carter method for midterm projections remains in that the Lee-Carter method captured the age pattern of historical mortality-decline rates using *b(x)*, and for populations in normal situations these rates should be stable over time.

We apply the Lee-Carter method to the 20 populations with the highest life expectancy in 2008[1] and reliable mortality data during 1950–2010[2]. The resulting ratios of the death rate at age 0 to ages 15–19 (*m(0)/m(15–19))* for the two-sexes-combined projected to year 2098, are shown by the black bars in Figure 1. The lowest and highest life expectancies of these 20

[1]In this paper, period 2005–2010 is simplified as 2008, which is a rounding up of the middle point 2007.5; and years ending with 3 and 8 are used to represent the corresponding 5-year periods.

[2]Death rates of abridged life tables in 5-year period were obtained on December 2011 from National Statistic Offices and the University of California at Berkeley (USA) and Max Planck Institute for Demographic Research (Germany). 2012. “Human Mortality Database (HMD).” Available at www.mortality.org or www.humanmortality.de. These death rates are computed from the 5-year-period deaths and exposures. The use of 5-years period data is more parsimonious, and allows us to compute more robust *b(x)* distributions even for smaller populations.

populations in 2008 were 78.3 and 82.7 years, respectively, with a median of about 80.5 years.

Following our earlier theoretical discussion, we expect that the risk of death at age 0 should be significantly higher than that at ages 15–19. We note that the median of *m(0)/m(15–19)* among the 20 populations in 2008 was about 11. However, this ratio in Figure 1 is problematic for 5 populations with a value lower than 1 in 2098: China Hong Kong, Finland, Ireland, Singapore, and Spain. And this ratio is questionable for another 7 populations with values lower than 2: Austria, Belgium, Italy, Japan, New Zealand, Norway, and Sweden. At a minimum, the projected ratios for these populations are far outside the range of observed experience.

The main reason for the problem resides in the shapes of *b(x)*, as shown in Figure 2[3]. It can be seen that the values of *b(x)* at infant and child ages are much higher than those at adolescent and adult ages, leading to projected reductions of death rates at infant and child ages that are much bigger than those at adolescent and adult ages, causing the problems shown in Figure 1.

It is clear that in order to project long-term mortality changes, modelling the rotation of *b(x)* is necessary. But there are difficulties in modelling and projecting the changes in *b(x)* at all ages and for all countries. In fact, *b(x)* describes the first-order differences of *log[m(x,t)]* at all ages. Consequently, modelling the over-time change of *b(x)* is equivalent to modelling the second-order differences of *log[m(x,t)]* at all ages. Because taking differences magnifies random fluctuations, higher-order differences include stronger random fluctuations than those in a lower-order differential. Thus, methodologically, the rotations of *b(x)* are much more difficult to model than is *b(x)* itself. Moreover, rotations are observed only in a few low-mortality countries, making the empirical basis of modelling and projecting the rotations at all ages very weak, if not unusable.

Facing this reality and the demands of long-term population projections, we turn our focus to the ratio of death rates between some key ages. Among the cross-age relationships between the death rates at different pairs of ages, which one would be most likely to identify anomalous situations? We believe it is ages 0 and 15–19. At infant, child, and young-adult ages, during which the change of the death rate is the most complicated, the death rate is highest at age 0 due to the additional risks of birth, congenital problems, and more fundamentally the theoretical evolutionary reasons given earlier. It is expected to be the lowest at ages 15–19 for the evolutionary reasons given earlier, and consistent with observed historical and contemporary age patterns; see also (Patton et al. 2009). Thus, if the ratio of the forecasted death rates *m(0)/m(15–19)* should happen to be lower than 1, for example, we would recognize it as anomalous. A risk of death at age 0 lower than at ages 15–19 has not been observed in history, and we do not expect to occur in the future. On the other hand, if this ratio is observed, or controlled, to be within expected bounds, then we would also expect the cross-age relationships between the death rates at other ages to be normal, in the

[3]There are two ways to compute the total-person-years in the open age group, the first assuming that the population in this age group is stationary, and the second assuming the death rate in this age group obeys a Logistic model converging to 1. The HMD used the second, and we also use it to extend the age group to 130 years, at which death rates are close to 1.

context of the Lee-Carter method. Thus, we focus on the ratio of *m(0)/m(15–19)*, and make various assumptions about the patterns of changes at other ages.

From the perspective of projection, the focus on mortality at age 0 is also driven by the ever more intractable difficulty of reducing infant mortality, especially the early neonatal mortality once most exogenous causes of deaths are eliminated through improved obstetric practice and neonatal care, the alleviation of congenital anomalies and prenatal conditions remain a challenge (Galley and Woods 1999; Liu et al. 2012; Rao, Adair and Kinfu 2011).

METHODS

A solution

We first propose a sufficient solution, which is to modify the shape of the *b(x)* curves that are estimated from historical data, by smoothing their values at adolescent and adult ages (15–65) to equal the average value for this age range, and then reducing the values at infant and child ages (0–14) to this average, as is shown in Figure 3. Age 65 is often taken as the onset of old age in demographic studies. In Figure 2 we see that in the years following age 65, the *b(x)* schedule begins to decline, although this is sometimes delayed until the mid-70s. Choosing somewhat different bounds to this age range, such as 20 and 70, would make little difference, because *b(x)* changes least between youth and old age as seen in Figure 2. Since the values of *b(x)* sum to 1, reducing the values of *b(x)* at younger ages will naturally raise the values at older ages, implying a rotation. Using the *b(x)* in Figure 3 to replace the *b(x)* in (1) implies an instantaneous rotation, and provides a sufficient solution that makes the projected decline rate of *m(x)* identical at all ages younger than 65, meaning that the projected *m(0)/m(15–19)* will be kept constant at the level of the starting year. This solution is sufficient, because keeping *m(0)/m(15–19)* at its last observed level solves the problem of the projected levels being too low. But this solution is not necessary, because *m(0)/m(15–19)* has declined in history and is expected to do so in the future. In LC_RR, we introduced a subjective value to *m(0)* for the long-run, which often leads the rotated *b(0)* to be smaller than the rotated *b(15–19)*. According to the discussion here, the solution of LC_RR may be excessive.

As a second step, we push the sufficient solution to the direction of necessary, by rotating the *b(x)* smoothly, starting from the Lee-Carter values and reaching ultimately the values as are shown in Figure 3. For this reason, we call the values of *b(x)* in Figure 3 the ultimate values. These ultimate values are obtained by assuming mortality decline decelerates at younger ages and accelerates at old ages, based on empirical evidence and theoretical discussion. We know least about the long run pattern of adult-mortality change, and we assume the future pattern is a smoothed version of the historical decline rates.

Denote the projected two-sex combined life expectancy at time t by $e_o(t)$, which is projected by the Lee-Carter method, and the Lee-Carter age pattern of rates of mortality-decline by $b_o(x)$, where the subscript stands for the original. Further, let the ultimate age pattern of mortality-decline rate be$b_u(x)$, and the level of life expectancy at which the rotation finishes be e_o^u, for which the computation will be indicated soon. Furthermore, let the extended Lee-

Carter age pattern of mortality-decline at time *t* be *B(x,t)*, and denote the linear-weight function (of the original and the ultimate *b(x)*) and smooth-weight function as

$$w(t)=\frac{[e_o(t)-80]}{(e_o^u-80)}, \quad w_s(t)=\{0.5[1+\sin[\frac{\pi}{2}(2\cdot w(t)-1)]]\}^p. \qquad (2)$$

Then, the *B(x,t)* is written as:

$$B(x,t)=\begin{cases} b_o(x), & e_o(t)<80, \\ (1-w_s(t))b_o(x)+w_s(t)b_u(x), & 80\le e_o(t)<e_o^u, \\ b_u(x), & e_o^u\le e_o(t). \end{cases} \qquad (3)$$

In (2), the linear-weight function changes from 0 (when $e_o(t)$ = 80) to 1 (when $e_o(t)=e_o^u$), but its derivative does not exist at 0 and 1, which leads to discontinuous change in the rate of mortality decline at the starting and ending points of the rotation. The smooth-weight function makes the change in the rate of mortality decline continuous in projection. The power to the smooth-weight function, *p*, takes values between 0 and 1, which makes the rotation faster at starting times and slower at ending times. In this paper, *p* = 0.5 is taken as the default.

We should mention that the $e_o(t)$ could also be projected by other methods, such as the double Logistic model (United Nations 2010) or the Bayesian probabilistic model (Chunn, Raftery and Gerland 2010). In (2), 80 years is roughly the median life expectancy of the 20 populations (mentioned above) in 2008, at which the problem occurs and therefore the rotation should start. It can be seen that when $e_o(t)$ is lower than 80 years, $B(x,t) = b_o(x)$. The rotation takes place when $80\le e_o(t)<e_o^u$. And when $e_o(t)$ reaches e_o^u, the rotation finishes at $B(x,t) = b_u(x)$.

Taking a smaller value of e_o^u would lead to a more rapid rotation, and a bigger *m(0)/m(15–19)* at the end of projection, which might be less questionable. On the other hand, using a bigger value of e_o^u would lead to a smoother rotation, and a smaller *m(0)/m(15–19)* at the end of projection, which might be more questionable. We next suggest an approach for choosing e_o^u.

Choosing e_o^u and the extent of the rotation

As discussed, we consider the ratio of infant mortality to the death rate for ages 15–19, and this ratio serves as a summary of the age-shape of mortality at younger ages. The level of *m(0)/m(15–19)* has declined historically, and we expect it to continue to decline in the future. However, inspection of its relation to the level of life expectancy in 20 populations as shown in Figure 4 suggests that it will eventually level off.

Computing the mean of the observed values in each integer interval of life expectancy, the decline and levelling-off trend is clearer. Modelling the mean values by an AR(1), the simplest model that describes such trends, indicates that the mean value of *m(0)/m(15–19)* will eventually converge to the level of 7.7, and would reach 7.8 when life expectancy reaches 100 years.

This ultimate value of *m(0)/m(15–19)* seems reasonable, but it should not be taken too literally. The ratio differs across populations in history so there is heterogeneity, and also other choices of models and periods could lead to different ultimate levels.

Given an ultimate level of *m(0)/m(15–19)* derived from fitting the AR(1), a value of e_o^u in (2) can be found to fit it exactly. Because the AR(1) ultimate values should not be taken as absolutely correct, we do not choose a value of e_o^u for each country, but instead search for a solution that is approximately right. We will apply a universal value of e_o^u to all the 20 populations, and we will choose this universal value to be the one that makes the mean of LC_ER ultimate (year 2098) *m(0)/m(15–19)* best fit the AR(1) ultimate mean. We found that, choosing $e_o^u=102$ makes the LC_ER ultimate mean *m(0)/m(15–19)* 7.8, which is the closest to 7.7 among the integer values of e_o^u. Thus, we choose 102 as the universal value of e_o^u, and apply it to all the 20 countries. Further, we recommend applying $e_o^u=102$ in (2) for all other countries and areas, including those for which the life expectancies were lower than 80 in 2008, assuming that their mortality change will be similar to one of the 20 populations above after their life expectancy reached 80 years. We should caution, however, that the LC_ER could produce questionable results when the changes of mortality in history were unusual. In such a situation, a different value of e_o^u could be chosen as a special case.

The extended Lee-Carter method

In this section, we suggest a strategy of initially projecting life expectancy using the standard LC, and then finding the adjusted values *K(t)* to fit the projected life expectancy using the rotational *B(x,t)* in (3). Thus, introducing the rotation does not change the projected values of life expectancy, but rather just changes the age pattern of mortality that generates those projected values.

Knowing the values of $b_u(x)$, e_o^u, and *B(x,t)*, the projected $e_o(t)$ by sex can be fitted by finding a value of *K(t)*, which will differ from that of the Lee-Carter *k(t)*. Thus, the Lee-Carter method is extended to:

$$\begin{aligned}\log[m_f(x,t)]=a_f(x)+B(x,t)K_f(t),\\ \log[m_m(x,t)]=a_m(x)+B(x,t)K_m(t).\end{aligned} \quad (4)$$

where subscript *f* or *m* refers to female or male, respectively.

The extended Lee-Carter method reduces to the Lee-Carter method when the projected $e_o(t)$ is smaller than 80 years. When the projected $e_o(t)$ exceeds 80 years, the extended Lee-Carter model will gradually depart from the Lee-Carter model, as the decline of death rates at younger ages decelerates and at older ages accelerates. Nonetheless, the extended Lee-Carter model projects life expectancies identical to the Lee-Carter method.

RESULTS

For the purpose of illustration, the median values of projected $e_o(t)$ are used and the data of Japan and the US are chosen. Since the two-sex combined $e_o(t)$ of Japan in 2008 was already 82.7 years, the rotation of *b(x)* starts in 2013. The rotation is projected to take place gradually as is shown in Figure 5. It can be seen that the *B(x,t)* declines at younger ages and rises at older ages, reflecting the subtle trend. As a comparison, it can be seen that LC_RR makes an instantaneous rotation that could be too large as will see below.

To keep the Lee-Carter projection of $e_o(t)$, the coherently projected (Li and Lee 2005) *k(t)* is changed to *K(t)*, which gradually departs from the Lee-Carter *k(t)*. The differences arise because *B(x,t)* differ from *b(x)*. In fact, the values of *K(t)* are sex-specific because so are the Lee-Carter projections of $e_o(t)$, but the sex differentials in *K(t)* are small.

Turning to the projections of Japan's age-specific death rates using various methods, Figure 6 shows three age patterns in 2098. Among these, the LC_ER is the most similar to the most recent observed pattern at younger ages. The LC_RR yields the smallest reductions of death rate at younger ages, which is perhaps too pessimistic and suggests an excessive rotation of *b(x)*.

Figure 7 focuses on the rate of decline of infant mortality. The Lee-Carter method projects future levels as an extrapolation of the average historical rates of decline, with a minor difference arising from using the sexes-combined average for projection in order to avoid divergence between the two sexes (Li and Lee 2005). Evidently there is a downward trend in the historical rate of decline. LC_ER projects a continuous path of decline, while LC_RR projects an instantaneous drop.

As is mentioned above, life expectancy of Japan in 2008 was already 82.7 years, the highest among all the countries; and hence the rotation takes place throughout the entire projection period. For countries with lower life expectancy, the rotation should occur later and hence the projection would differ less from the standard Lee-Carter. The US is taken as an example for later rotations, although its life expectancy was already 78 years, ranking 40th among the 196 countries and areas in the world in 2008. The sexes-combined life expectancy is projected by the Lee-Carter method to reach 80 years in 2028, when LC_ER projects the rotation to start. Since the rotation begins later, *B(x, 2048)* differs from $b_o(x)$only slightly, and *B(x,2098)* is still far from the ultimate $b_u(x)$ as can be seen in Figure 8.

The rotation is not projected to occur until 2028, much later than that of Japan as can be seen in Figure 9.

As a result, the long-term (2098) LC_ER projection yields death rates at younger ages that are slightly higher than the LC rates. But for middle-term (2048), the results of LC_ER are almost identical to those of LC, as can be seen in Figure 10.

CONCLUSION

In low-mortality countries, mortality decline is decelerating at younger ages and accelerating at older ages, corresponding to a rotation of the *b(x)* in the Lee-Carter method. This rotation is expected to appear also in developing countries, when infant and child mortality drop to low levels and further reductions become difficult, and when resources for reducing old age mortality become increasingly available. But this rotation, even in low-mortality countries, is still too subtle to model and to project in ways that are entirely data driven and include all ages. Without modelling this rotation, however, long-term projections will show anomalous results, as we have discussed.

Facing this reality and the demands of long-term population projections for all the countries, and guided by some qualitative conclusions from evolutionary life history theory, we turn our focus to the ratio of *m(0)/m(15–19)*. Using data on death rates of the 20 populations with the lowest mortality level in 2005–2010, we found a declining trend in the ratio of *m(0)/m(15–19)*, and we expect other populations with higher levels of mortality would follow this trend when their mortality level declines in the future.

To utilize this declining trend we proposed a model in which, when life expectancy increases to a threshold level, the *b(x)* at younger ages will decline gradually to make the projected ratio of *m(0)/m(15–19)* follow the declining trend. Because the values of *b(x)* sum to 1, reducing the values of *b(x)* at younger ages will automatically raise its values at older ages, generating the expected gradual rotation. This method has the advantage of preserving the basic Lee-Carter method for most countries and most time periods, while introducing a subtle rotation in the projected age pattern of mortality decline when high levels of life expectancy are achieved and over long projection horizons. We judge these outcomes to be reasonable, and we have illustrated the results using the cases of Japan and the United States.

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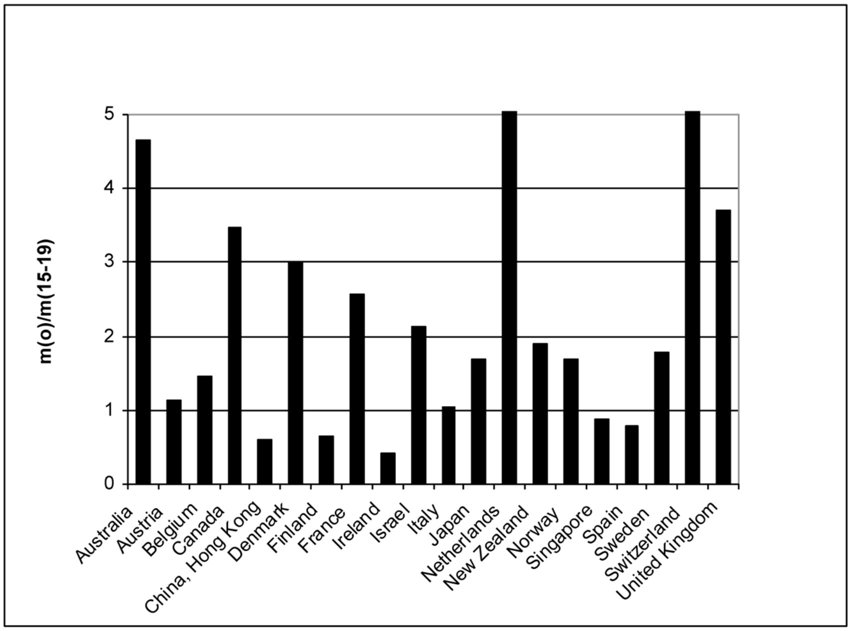

Figure 1.
The values of m(0)/m(15–19), two-sex, year 2098, projected by standard Lee-Carter method using data for 1950–2010

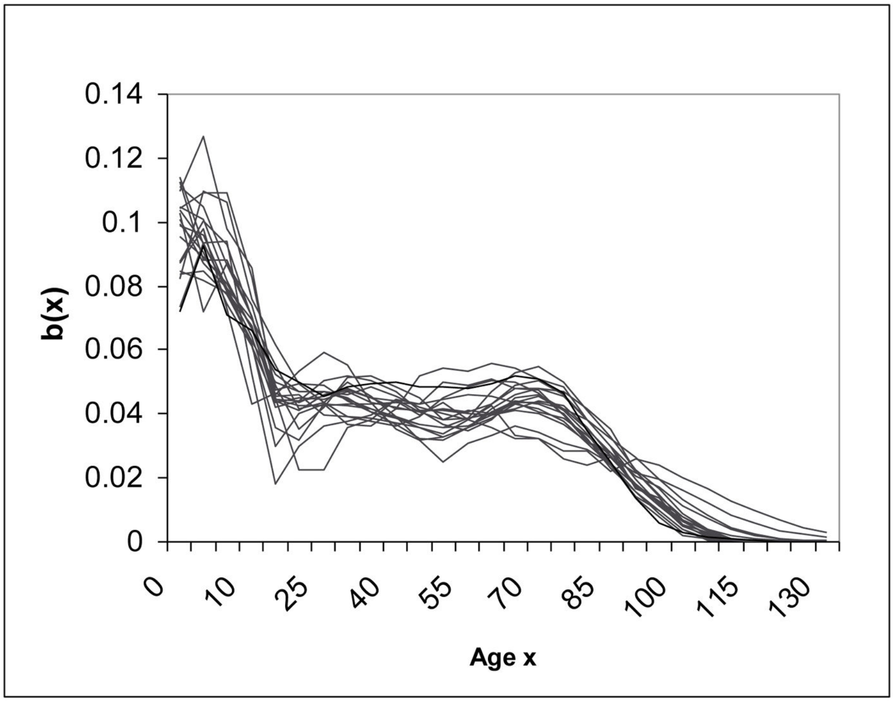

Figure 2.
The Lee-Carter b(x) of the 20 low-mortality populations based on 1950–2010 death rates

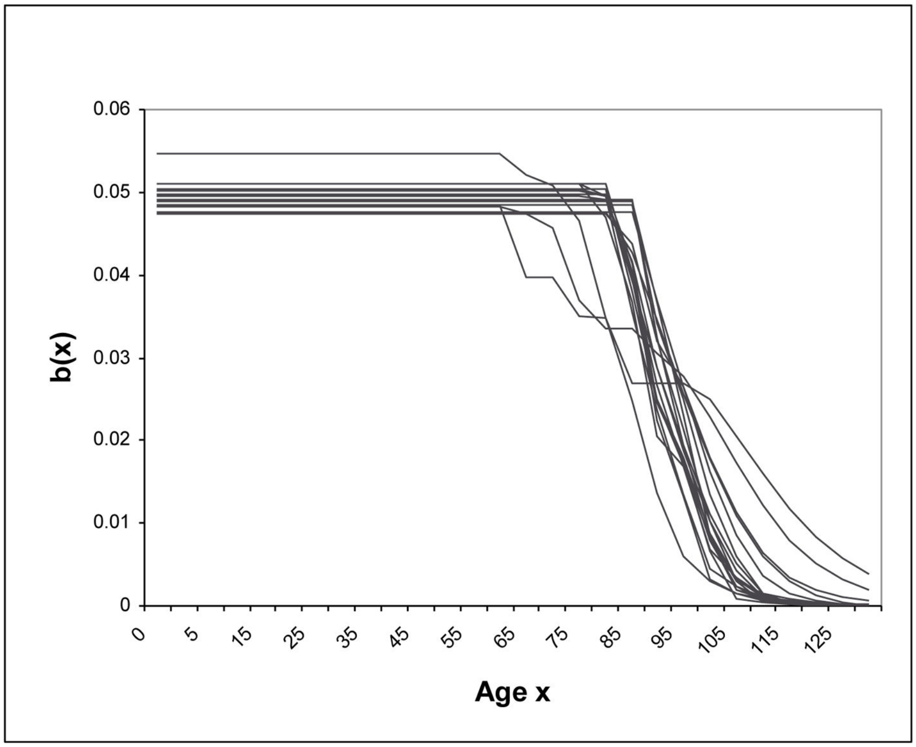

Figure 3.
The ultimate b(x) of the 20 low-mortality populations

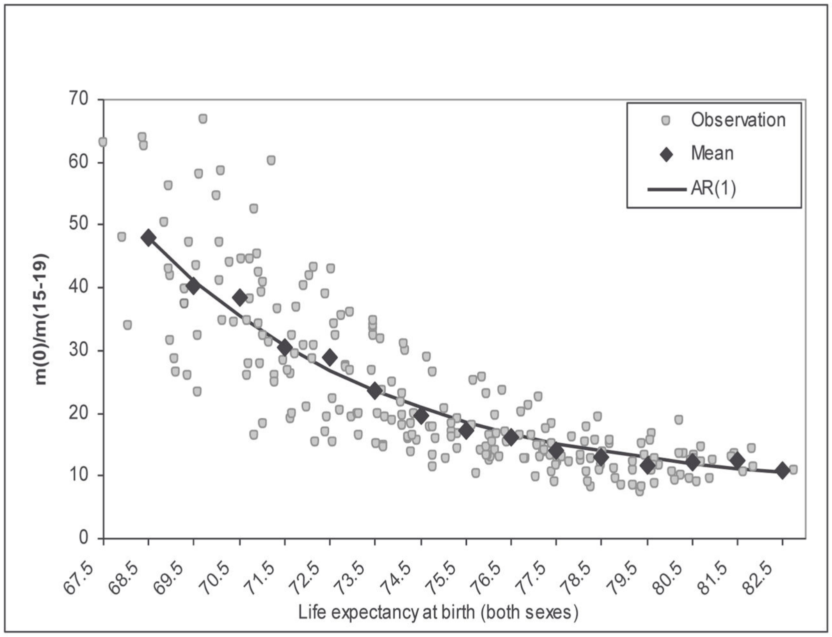

Figure 4.
Historical values of m(0)/m(15–19) of the 20 low-mortality populations

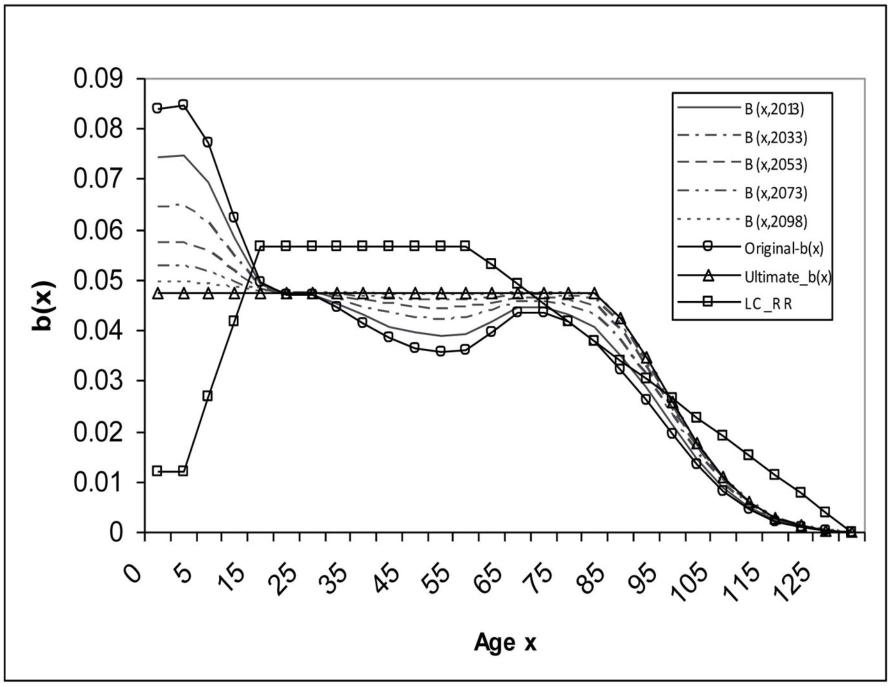

Figure 5.
The values of b(x), Japan

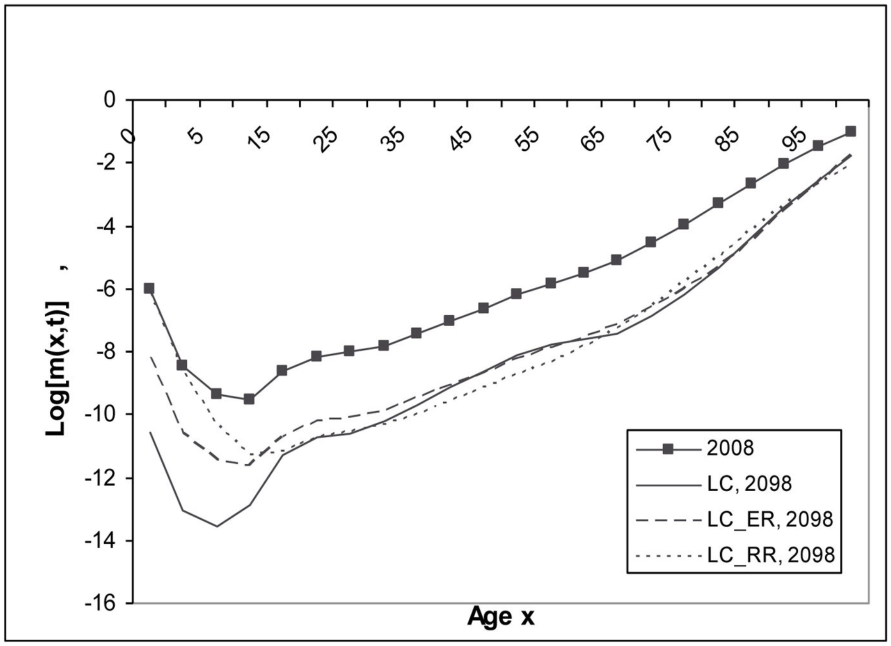

Figure 6.
Observed for projected death rates, Japanese females

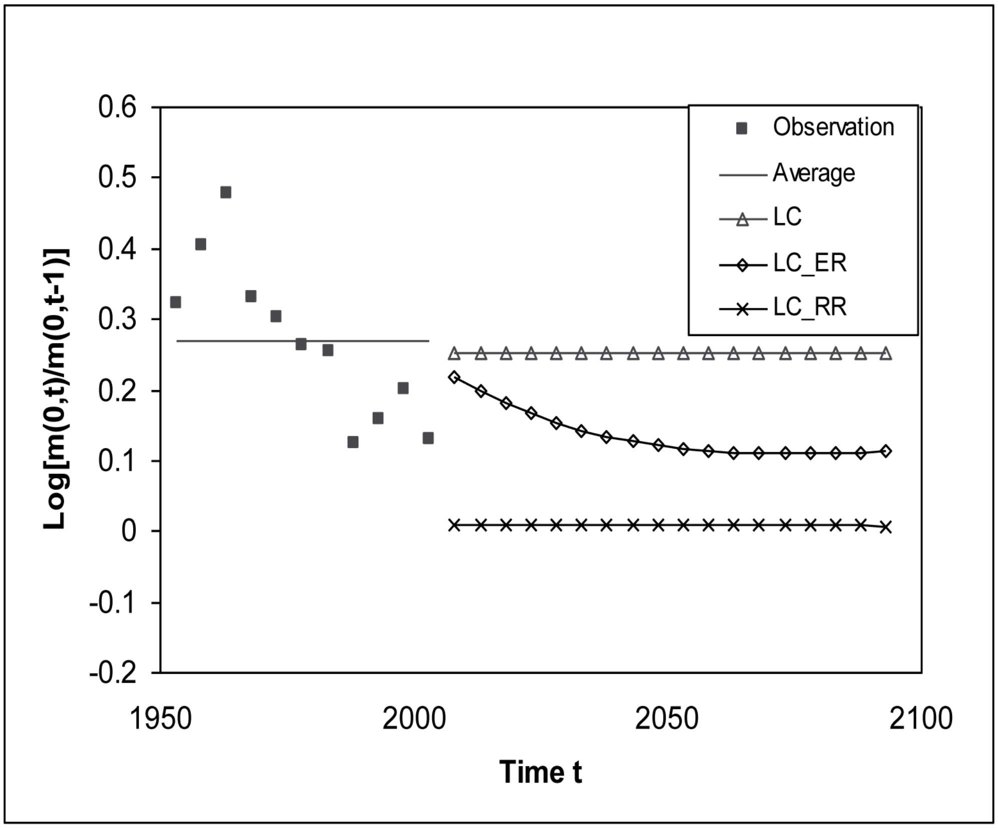

Figure 7.
The rate of decline of m(0), Japanese females

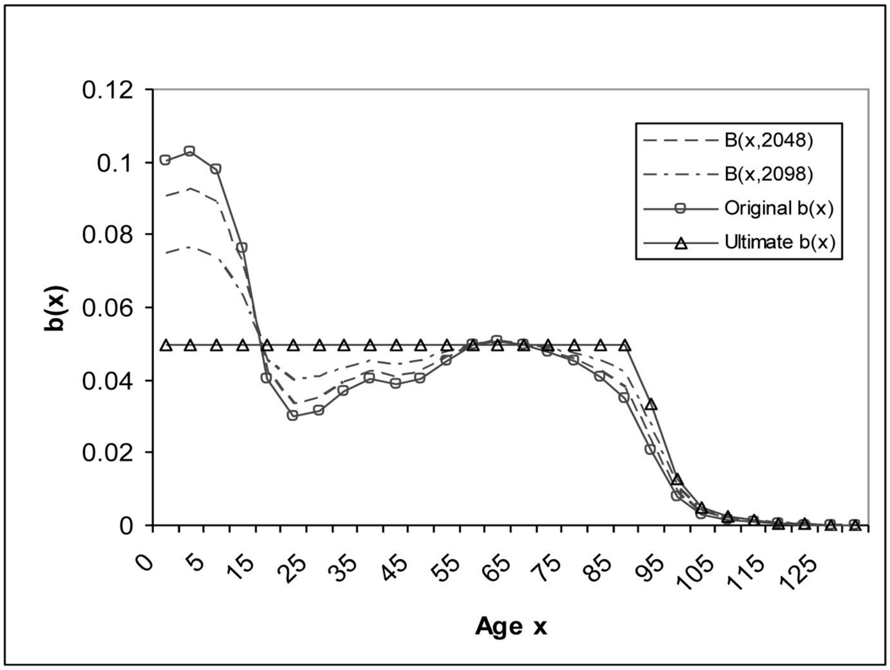

Figure 8.
Values of b(x), USA

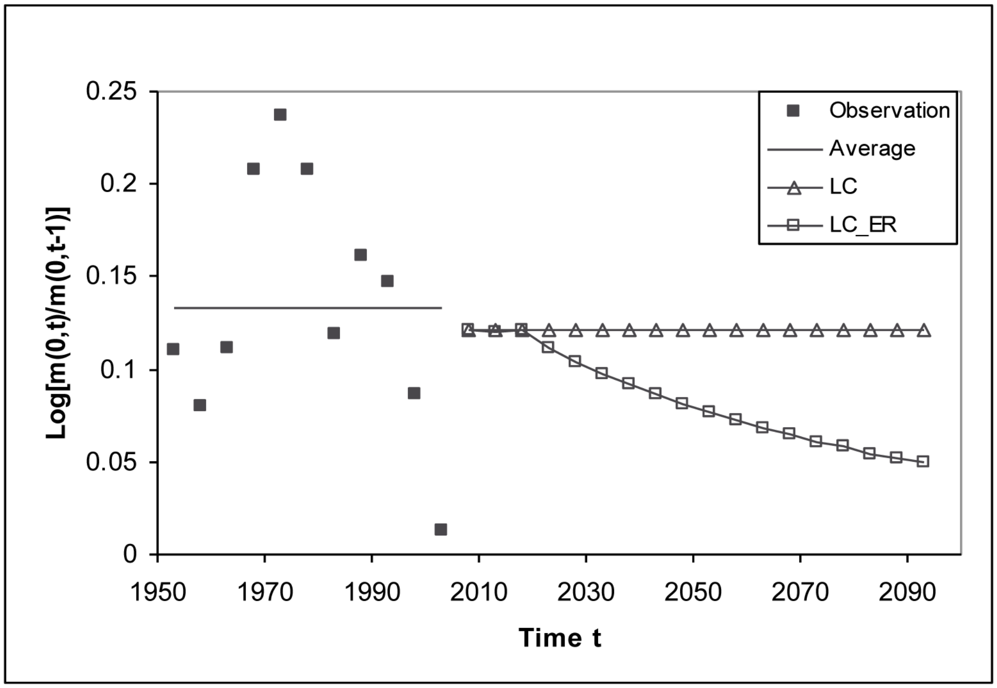

Figure 9.
The rate of decline of m(0), US females

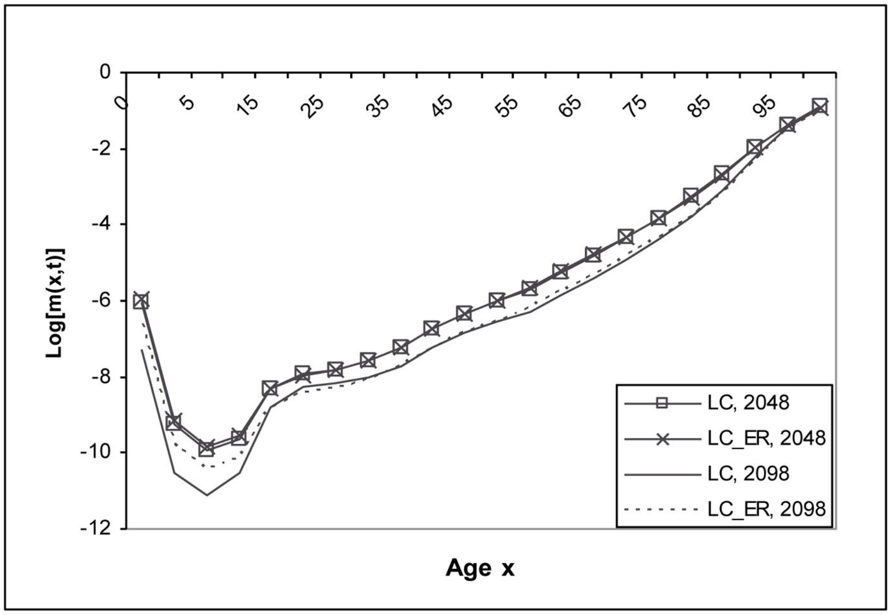

Figure 10.
Projected death rates, US females