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Note

A note on pre-play communication ☆

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ABSTRACT

Consider a finite two-player game with one round of communication. Restrict players to a subset of “monotonic” strategies. The paper justifies this restriction. The paper provides sufficient conditions under which the strategies of the restricted game that survive iterative deletion of weakly dominated strategies favor the agent who can communicate.

1. Introduction

This paper adds to the literature studying whether costless pre-play communication can influence outcomes. The setting is standard. Start with a finite, two-player (underlying) game. Create a new game in which one player (the Sender) has the opportunity to send a costless message to the other player (the Receiver) prior to playing the underlying game. The question is whether the opportunity to communicate changes outcomes.

Common sense and experimental evidence suggest that the ability to communicate can change outcomes. Intuitively, communication may help rational players coordinate when the underlying game has multiple, Pareto-ranked equilibria. If the Sender can credibly communicate her intentions, then communication may enable her to select her most preferred equilibrium.

The main result gives conditions under which the unique prediction of the game with pre-play communication is the equilibrium in the underlying game that gives the Sender her highest payoff. This conclusion is familiar, but this paper derives it as a consequence of novel and restrictive conditions.

It is difficult to guarantee that communication influences outcomes because adding communication inserts a new coordination problem into the original strategic setting. Whenever there is an equilibrium in which the speaker can communicate her intentions through the message she sends, there is another equilibrium that also communicates her intentions but in which the connection between message and action is different. In addition to these equilibria, there will be another in which messages are ignored.

This paper deals with the problem by imposing assumptions on the way that messages are interpreted. I assume that messages and actions are ordered. I construct the game with pre-play communication as usual, but assume that it is common knowledge that the Receiver will interpret messages in a systematic way. Specifically, the Sender believes that the

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Receiver will play a strategy that is (weakly) monotonic in the Sender's message. That is, the Sender thinks that the Receiver will use only strategies that respond to higher messages with (weakly) higher actions. This restriction reduces the game with pre-play communication to a new game in which the Receiver can only use monotonic strategies. Finally, I solve the reduced game using iterative deletion of weakly dominated strategies. Both steps in this procedure are necessary for a selection result. It is straightforward to check that the restriction to monotonic strategies does not eliminate any pure-strategy Nash equilibrium predictions. Further, given an equilibrium of the underlying game that survives iterative deletion of weakly dominated strategies, one can construct an equilibrium for the game with pre-play communication in which the Sender randomizes uniformly over all messages and, after any message, the players use the equilibrium strategies of the underlying game. These strategies survive iterative deletion of weakly dominated strategies. Consequently, my results require the combination of a restriction on strategies (monotonicity) and an equilibrium refinement (deletion of weakly dominated strategies) to make a selection.

One stylized intuition for the result that communication influences behavior goes as follows. Consider a situation in which the Receiver ignores the Sender's message. Imagine the Sender deviates from the equilibrium by using a novel message. If the Sender can make a suggestion that is consistent with the incentives of both players, then the Receiver "should" believe it. For example, if the Sender and Receiver are playing the battle of the sexes, then the Receiver's favorite equilibrium would be destabilized if the Sender can credibly communicate her intention to play the strategy consistent with her favorite equilibrium. There are at least two things missing from this stylized intuition. First, there may not be any unused messages (for example, the Receiver may expect the Sender to randomize uniformly over all available messages). Second, even if there is an unused message, the formal description of the game does not provide any guidance about how to interpret the message. The monotonicity restriction imposes (a small amount) of structure on how the Receiver interprets unexpected messages. Iterative weak dominance formalizes the "forward-induction" intuition that a rational Receiver interprets unexpected messages in a systematic way. If the Receiver does so, then the Sender can use communication to her advantage.

Farrell (1988) allows the Sender to make a suggestion about which strategies to use prior to playing a game. He observes that the Sender can select her preferred outcome when the Receiver must follow suggestions that satisfy consistency restrictions.[1] Informally, a suggestion is consistent if the Receiver believes that the Sender would follow the suggestion, then it would be in the Sender's best interest to follow it. It is therefore consistent to suggest that agents should coordinate on a Nash equilibrium and if one player is given the opportunity to make a suggestion, she would suggest the equilibrium that she most prefers. Aumann (1990) argues that communication need not be credible when the Sender's preferences over the Receiver's actions do not depend on the action the Sender intends to take. These games arise, for example, when there are positive spillovers so that increasing the Receiver's action always benefits the Sender. My results are consistent with Aumann's discussion. I show in Section 4 that the monotonicity restriction and iterative weak dominance do not select the Sender's favorite outcome in Aumann's game. My central result requires that the underlying game satisfy a self-signaling property: conditional on playing a particular action in the underlying game, the Sender obtains the highest payoff when the Receiver best responds. The self-signaling condition explicitly guarantees that the action of the Receiver that the Sender prefers most depends non-trivially on the Sender's intended action and rules out Aumann's example.

Farrell (1988) mentions the possibility that the timing of announcements may influence the credibility of cheap talk and suggests that Aumann's concern is more relevant when the Sender communicates after moving. Furthermore, experimental evidence (for example, Charness, 2000) suggests that the credibility of pre-play communication depends on whether the Sender talks before or after making her choice of strategy. Motivated in part by these observations, Schlag and Vida (2013) demonstrate that in generic 2×2 games if the Sender can select the language and communicates prior to moving, then she obtains her favorite outcome, while this result fails if the Sender communicates after moving.[2] Schlag and Vida give one agent the power to select a partition of Player 1's strategy space (called the language). Messages are elements of the language. A language is credible if given any message ($M \subset S_1$), there exists a Nash Equilibrium of the underlying game in which the Sender selects a strategy with support contained in M.

Lo (2009) shows that if the underlying game is both self-committing[3] and self-signaling, then the Sender receives her favorite equilibrium payoff in the unique outcome surviving iterative deletion of weakly dominated strategies consistent with her notion of credible communication. Lo therefore reaches the same conclusion as I do. The assumptions used in obtaining the conclusion make the results non-comparable. Lo does not impose strong monotonicity conditions on the underlying game. In this way, her result applies to more situations. I do not require that the game be self committing, so my result covers some games not covered by her result. Although we both use iterative dominance, Lo's notion of credibility

[1] Farrell and Rabin (1996) review literature on this topic and Rabin (1990) describes a related approach.

[2] Zultan (2013) presents a model of timing complementary to that of Schlag and Vida.

[3] Baliga and Morris (2002) define self-committing games to be those two-player games in which, every pure strategy of the Sender is a best response to the Receiver's best response to it. That is, in a self-committing game if the Sender promises to take an action and the Receiver believes the promise (and therefore best responds), it is in the Sender's best interest to behave as promised. The consistency condition in Farrell (1988) is an analogous property restricted to proposals (rather than games): the Sender's proposal to play a particular strategy in the underlying game is consistent if the strategy is a best response to the Receiver's best response to it.

is distinct from the monotonicity restriction that I require. Loosely, Lo assumes that messages have literal meanings and that the Receiver either interprets words literally or makes no inference at all.[4]

Farrell, Lo, Rabin, Schlag and Vida and I all make assumptions designed to capture the idea that the meaning that messages have outside of a strategic encounter could determine how they are interpreted within game. All of the approaches make the constraints sufficiently weak so that agents can still make decisions that are consistent with their best interests. My approach operates by assuming that players believe that certain interpretations are impossible. (The Receiver will never associate higher messages with the intention to play lower actions.) The other approaches assume that players will interpret messages in specific ways if the interpretation is consistent with rationality. Both approaches impose conditions that go beyond conventional equilibrium notions in order to generate meaningful communication. My approach yields a fairly weak conclusion in the sense that I impose strong assumptions on the underlying game in order to reach the conclusion that communication must lead to the Sender's favorite outcome. It provides an additional conclusion, however. The analysis not only selects an equilibrium outcome, it selects the messages that players use. In this way, the monotonicity assumption imposes a structure on the way that strategic players use cheap-talk messages. This property is broadly consistent with experience (even in strategic situations, the conventional meaning of messages constrains how players interpret the messages), but is not a consequence of the other approaches, which use the possibility of using conventional meanings of terms off the equilibrium path to select equilibria but provide no guidance about which messages players use in equilibrium.

An alternative approach to the study of credible pre-play communication is to use dynamic arguments. Players may use past interactions to arrive at common understanding of messages. This approach is intuitive and leads to useful formal results. Demichelis (2012), Demichelis and Weibull (2008), and Sobel (1993) are examples of papers that study evolutionarily stable outcomes in games with pre-play communication about intentions. These papers allow both players to communicate and establish conditions under which communication leads to efficiency in common-interest games. The evolutionary arguments are qualitatively different from the arguments in this paper. This paper pursues an alternative explanation for three reasons. The most important reason is that dynamic arguments that predict effective communication will not identify a particular relationship between messages and actions.[5] If a dynamic process converges to a situation where the message "left" is associated with a strategy labeled "L", then it could equally well converge to a situation where the message "left" is associated with a strategy labeled "R." In reality, however, words share common interpretations across different contexts. A second reason is dynamic arguments may force communication even without pre-play communication.[6] Hence they may not isolate the importance of communication. A third reason to study the one-shot game is that doing so may lead to insights that complement the dynamic stories. This reason is important if one believes that communication influences outcomes even in one-shot interactions.

Ben-Porath and Dekel (1992) demonstrate how equilibrium refinements combine with costly signaling (money burning) to select equilibria. They arrive at the same conclusion as Proposition 1 under less restrictive assumptions on the underlying game. In their model, the Sender's signaling technology allows her to reduce her payoffs. Combined with dominance arguments this means that certain first-round actions become unambiguous signals of the Sender's intentions without additional restrictions on the credibility of communication.

Two papers study pre-play communication using Basu and Weibull (1991)'s notion of curb retracts.[7] Blume (1998) considers two-player games with a fixed message space in which only one player can communicate. He assumes that there is a strict Nash Equilibrium that uniquely attains the highest payoff available to the player who can communicate. Blume gives conditions under which the payoffs associated with the communicating player's favorite equilibrium will be the only curb equilibrium payoffs. The conditions require that the signaler's preferred equilibrium is not "risky" (in a sense he makes precise) relative to the number of available messages. Blume predicts effective communication in some games where the self-signaling condition fails. Blume connects his solution concept to dynamic stability arguments. Blume (1998) examines curb sets in games in which all agents can communicate (they simultaneously send messages prior to playing the underlying game) and, possibly, more than two players. He assumes that the underlying game has a unique Pareto-efficient payoff. He shows that communication is effective if all players can communicate (without cost) and, associated with each (strict) equilibrium of the underlying game is a profile of messages such that players receive slightly more utility when they achieve the equilibrium by using messages that are exogenously associated with the equilibrium. In this setting, Blume shows that communication is effective in the unique curb retract and players use the associated messages to achieve the efficient payoff.[8] Hurkens (1996) extends the arguments of Ben-Porath and Dekel to games with more than two players. Hurkens studies

[4] She assumes that the language is sufficiently rich so that there is a word to describe any subset of the Sender's strategy set.

[5] Demichelis and Weibull (2008) use a meaning correspondence and assume that players have a (weak) preference for sending messages that are consistent with the correspondence. These assumptions guarantee that there will be unused messages in equilibrium, but they do not guarantee a particular relationship between messages and actions in equilibrium.

[6] Pre-play communication still changes the interaction in an essential way. A strict equilibrium in the underlying game will not be strict in a game with pre-play communication. Consequently pre-play communication may destabilize certain outcomes.

[7] A retract is a subset of the set of mixed strategies that is a cartesian product of closed, convex, nonempty sets. A retract satisfies the closed-under-rational-behavior property (curb) if all best responses to any strategy profile in the retract are also in the retract. A curb retract is a minimal retract with respect to the curb property.

[8] In private communication, Péter Vida has pointed out that Kalai and Samet (1984)'s notion of persistence, which is closely related to curb retracts, selects the Stackelberg outcome for a class of games that include the standard battle-of-the-sexes game. The mechanism behind the selection appears to share features of the evolutionary arguments. Blume (1994) applies persistence to cheap-talk games about private information.

n player games in which a nonempty subset of players can burn money (the signalers). He assumes that the signalers have common interests in the sense that the base game has a Nash Equilibrium strategy profile that gives each signaler her highest possible payoff. He shows that the money-burning game has a unique curb retract. In this retract signalers receive their highest possible payoff.

Baliga and Morris (2002) study the role of cheap-talk communication in simple two-player games with one-sided incomplete information. Depending on the realization of uncertainty, the underlying stage game is either dominance solvable or has two, Pareto-ranked, pure-strategy equilibria. Baliga and Morris identify conditions under which, without communication, the only equilibrium outcome of the incomplete-information game is inefficient. They show that adding pre-play communication may enable the informed player to reveal her type and coordinate on the efficient outcome. They also show that this result does not hold in games with positive spillovers. Baliga and Morris provide sufficient conditions under which adding communication can change the set of perfect Bayesian equilibrium payoffs in a game with incomplete information. I provide sufficient conditions under which adding communication can change the set of payoffs that survive iterated deletion of weakly dominated strategies in a game with complete information. In Baliga–Morris, the change is that communication enlarges the equilibrium set. In this paper, the change is that adding communication reduces the equilibrium set. In each paper, self signaling is one of the sufficient conditions.[9]

Finally, Crawford (2003) derives credible communication (and deception) in a model in which some Senders believe Receivers take messages literally and some Receivers believe that Senders communicate honestly.[10]

Section 2 describes the model. Section 3 presents the main result. Section 4 examines Aumann's example in detail and demonstrates that the conclusions of the main result do not hold for this example. Section 5 is a discussion of the result. Appendix A contains some definitions and describes a result that support the use of iterative deletion of weakly dominated strategies as the solution concept. The (simple) proofs are in Appendix B.

2. The model

Let $G = (S, u)$ denote a finite 2 player game,[11] where $S = S_1 \times S_2$ is the strategy space (S_i finite for $i = 1, 2$), and $u : S \to \mathbb{R}^2$ is the payoff function. Let $S_{-i} \equiv S_j$, $j \neq i$; let s_{-i} denote a representative element of S_{-i}. A set W is a restriction of S if $W = W_1 \times W_2$ and $\emptyset \neq W_i \subset S_i$ for $i = 1, 2$. Assume that the payoffs in the game are generic (no two payoffs are the same) so that best responses to pure strategies are unique. Assume that Player i has N_i pure strategies for $i = 1$ and 2 and write $S_i = \{s_i(1), \ldots, s_i(N_i)\}$. Let $b^i : S_{-i} \to S_i$ be the best response function of Player i ($b^i(\cdot)$ is a function when payoffs are generic). I make several assumptions on G.

Definition 1 *(SS).* The game G is self signaling for Player i if for all $s_i \in S_i$, $u_i(s_i, b^j(s_i)) > u_i(s_i, s_j)$ for all $s_j \in S_j$, $s_j \neq b^j(s_i)$ ($j \neq i$).

Farrell (1993) introduces this idea.[12] The condition states that for every action, Player i's utility is highest when Player j responds optimally to Player i's action.

The remaining definitions require strategy spaces to be ordered. I will write $s_i(k'') > (\geq)\, s_i(k')$ when $k'' > (\geq)\, k'$.

Definition 2 *(ID).* Suppose $s'_j, s''_j \in S_j$ and $s'_j \leq s''_j$. The game G satisfies the Interval Dominance Condition for Player i if for all $k'' > k'$, $u_i(s_i(k'), s''_j) > (=)\, u_i(s_i(k'), s'_j)$ implies $u_i(s_i(k''), s''_j) > (=)\, u_i(s_i(k''), s'_j)$ whenever $u_i(s_i(k'), s''_j) > (=)\, u_i(s_i(k'), s_j)$ for all $s'_j \leq s_j \leq s''_j$.

Quah and Strulovici (2009) introduce the interval dominance condition. The condition is weaker than familiar single-crossing and increasing-differences conditions and guarantees that Player i's best response is an increasing function of Player j's strategy.

Definition 3 *(IM).* The game G is increasing to the maximum for Player i if for all $s_i \in S_i$, there is a $s^*_j = s^*_j(s_i)$ such that $u_i(s_i, s'_j) < u_i(s_i, s''_j) < u_i(s_i, s^*_j)$ for $s'_j < s''_j \leq s^*_j(s_i)$.

The increasing to the maximum condition states that when Player i intends to play s_i she prefers Player j to take higher actions up to the action she most prefers ($s^*_j(s_i)$). The results of the paper continue to hold under an alternative assumption:

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9 Several other papers use cheap-talk communication to select equilibria in two-player games with one-sided incomplete information and some form of common interests. Examples of selection results can be found, for example, in Blume et al. (1993), Canning (1992), Demichelis and Weibull (2008), Sobel (1993), Wärneryd (1993), who present dynamic arguments, and Chen et al. (2008) and Farrell (1993) who use static arguments. Blume (1996) presents a static model in which words have exogenously given meaning which is reflected in equilibrium.

10 Ellingsen and Östling (2010) provide experimental evidence for this theory.

11 The basic definitions extend easily to the $I > 2$ player games. Conclusions in these settings are weaker.

12 Myerson (1983) uses a similar condition.

Definition 4 *(DM).* The game G is decreasing from the maximum for Player i if for all $s_i \in S_i$, there is a $s_j^*(s_i)$ such that $u_i(s_i, s_j'') > u_i(s_i, s_j') > u_i(s_i, s_j^*(s_i))$ for $s_i'' > s_i' \geq s_j^*(s_i)$.

If both (IM) and (DM) hold, G is single peaked (SP) for Player i. It is straightforward to show that if every pure strategy s_j^* of Player j solves $\max_{s_j \in S_j} u_i(s_i, s_j)$ for some s_i, then (ID) implies (SP) in generic games.[13] From G one obtains a game with pre-play communication, G^*. The strategy set for Player 1 in G^* is $S_1^* \equiv \{1, \dots, N_1\} \times S_1$ and the strategy set for Player 2 is $S_2^* \equiv S_2^{N_1}$. A representative strategy for Player 1, $(n; l)$ consists of an announcement n and a strategy $s_1(l) \in S_1$. A representative strategy for Player 2, $(k_1, \dots, k_{N_1})$ consists of a strategy in S_2 for each possible announcement of Player 1. The payoff functions of G^* are $u_i^*\left((n,l),(k_1,\dots,k_{N_1})\right) = u_i(s_1(l), s_2(k_n))$. Informally, in G^*, Player 1 first makes an announcement, which can be interpreted as the action she intends to play in G and then the players play G.

The monotone reduction G_M^* is the game obtained from G^* by restricting Player 2 to monotonic strategies. The strategy set for Player 1 in G_M^* is S_1^* (Player 1's strategy set in G^*). The strategy set for Player 2 in G_M^* is the subset of S_2^M consisting monotonic strategies. That is, $S_2^M = \{k = (k_1, \dots, k_{N_1}) \in S_2^* : j' > j \text{ implies } k_{j'} \geq k_j\}$. The payoff functions in G_M^* are the restriction of payoff functions of G^* to $S_1^* \times S_2^M$. Restricting to monotonic strategies changes the game because the restriction removes feasible strategies. The justification for the assumption is that if words have exogenous meanings, it is reasonable to assume that players agree that these exogenous meanings influence interpretations in strategic settings. In this model, I make the idea operational by linking the exogenously given order on strategies and the exogenously given order on messages to the (typically endogenously determined) connection between messages and strategies. Saying that a strategy has a higher index than another strategy is a property of the description of a game; saying that a higher message induces a higher response is a property that one deduces in equilibrium. The monotonicity restriction forces the Receiver to interpret messages in a way that is consistent with the exogenously given orders. Notice that given any strategy of Player 2, it is always possible to relabel the messages so that the strategy is monotonic. (The relabeling defines higher messages to be the ones that lead to higher actions.) The power of the restriction comes because it eliminates a kind of strategic uncertainty: Players understand that the message "ten" will not induce a lower action than the message "one." The monotonicity restriction does not prevent agents from using language strategically, but the results demonstrate that it sometimes eliminates coordination failure that typically arises in cheap-talk models. I discuss the implications of the assumption below.

3. The result

The main result gives sufficient conditions for pre-play communication to select Player 1's favorite Nash Equilibrium. The solution concept is iterative deletion of weakly dominated strategies. It is well known that outcome of the process of iteratively deleting weakly dominated strategies may depend on the order in which strategies are deleted. For Proposition 1, I use a particular order (described in the Appendix). In order to justify this procedure, I rely on a result of Marx and Swinkels (1997).

Proposition 1. *Let G be a finite, generic, two-player game that is single peaked and self signaling for Player 1 and satisfies Interval Dominance for Player 2. Any outcome that survives iterative deletion of weakly dominated strategies induces Player 1's most preferred Nash Equilibrium in G.*

The appendix contains a proof of Proposition 1. The proof exhibits a particular order of deleting dominated strategies. For this order Player 1 is left with strategies in which she communicates her intentions accurately and Player 2 responds optimally. Hence Player 1 can take the action that maximizes $u_1(x, b^2(x))$, which by (SS) induces a Nash equilibrium. Having established the result for some order, I invoke a result of Marx and Swinkels to conclude that the Player 1 obtains her preferred Nash Equilibrium for any order of deletion of weakly dominated strategies.

The construction is iterative. First one observes that Player 2 will never take an action greater than $b^2(x_{N_1})$. This follows from (ID), which guarantees that Player 2's best responses are increasing in the action of Player 1. The next step demonstrates that Player 1 will always send the highest message if she intends to take the highest action. This follows because Player 1's payoff (given that she plays x_{N_1}) is increasing in Player 2's respond, and Player 2's response is increasing in Player 1's message. Continuing by induction, it is possible to place bounds of Player 2's responses to lower messages and conclude that Player 1 will send relatively high messages if she intends to play a relatively high action. The induction leads to the conclusion that the lowest message perfectly reveals Player 1's intention to play her lowest action. Finally, (SS) implies that Player 1 uses a different message for each intended action and that the message will therefore perfectly coordinate the actions in the underlying game.

[13] To prove that (ID) implies (IM), let $\hat{s}_j(s_i) \equiv s_j(\hat{k}(s_i))$ solve $\max_{s_j \in S_j} u_i(s_i, s_j)$. In order to reach a contradiction, assume $u_i(s_i, s_j(k+1)) < u_i(s_i, s_j(k))$ for $k < \hat{k}(s_i)$ (by genericity, we can assume that the inequality is strict). By assumption, we know that $s_j(k+1)$ is Player i's favorite response for some s_i. Let $s_j(k+1) = \hat{s}_j^*(s_i')$. By (ID), $s_i' < s_i$. By definition, $u_i(s_i', s_j(k+1)) > u_i(s_i', s_j(k))$. Hence (ID) also implies that $u_i(s_i, s_j(k+1)) > u_i(s_i, s_j(k))$, which is a contradiction.

I have described how Player 1 achieves her favorite pure-strategy equilibrium. The next result notes that Player 1 can achieve her highest Nash Equilibrium payoff in a pure-strategy equilibrium.

Proposition 2. *Let G be a finite two-player game that is self signaling for Player 1. There exists a pure-strategy Nash Equilibrium of G in which Player 1 achieves her maximum Nash Equilibrium payoff.*

Given the literature on the effectiveness of pre-play communication, the conclusion is quite weak because the sufficient conditions are so strong. The literature suggests several mechanisms that could lead to credible communication in games. My result identifies conditions under which a common understanding of the meaning of words (leading to the monotonicity restriction) leads to credible communication.

The proposition uses three different kinds of assumption. First, it requires players to use monotonic strategies. Second, it invokes a strong solution concept (iterative deletion of weakly dominated strategies). Third, it places strong assumptions on the payoffs in the underlying game. The first two assumptions describe how I go from a game to a prediction. The third kind of assumption identifies when the solution procedure is effective.

The refinement involves two separate steps. The restriction to monotonic strategies is powerful and not standard. The requirement that the prediction must survive iterative deletion of weakly dominated strategies places high demands on the cognitive ability of the players, but is a general solution concept that is a useful benchmark for the analysis of strategic environments. It is also likely that the same results would hold if one used alternative equilibrium refinement arguments.[14]

Even if one accepts the solution concept, the restriction to monotonic strategies requires comment. The restriction to monotonic strategies is one way to eliminate the multiple equilibria problem that arises in models of cheap-talk. It is well known that adding cheap talk to a game cannot reduce the set of predictions. Hence any attempt to argue that pre-play communication influences behavior requires some type of refinement argument. Farrell (1988), Lo (2009), Rabin (1990), and Schlag and Vida (2013) make strong assumptions in order to study the effectiveness of communication. The approach that I propose is different. It is a mild restriction for two reasons. Restricting to monotonic strategies does not reduce the set of pure-strategy Nash Equilibrium payoffs to the communication game. This property holds without any restrictions on the underlying game. Given any pure-strategy Nash Equilibrium, one can construct an equivalent Nash equilibrium in monotonic strategies by relabeling the messages. To be precise, imagine that Player 1 uses the strategy (n, l) and Player 2 uses the strategy $(k_1, \dots, k_{N_1})$ in equilibrium. Let π be a permutation of the set $\{1, \dots, N_1\}$ such that $k_{\pi(i)} \le k_{\pi(j)}$ if and only if $i < j$. Now order the strategies using π (so that $\pi(1) < \pi(2) < \dots < \pi(N_1)$). In the relabeled game, the equilibrium strategy profile is $(\pi(n), l)$; $((k_{\pi(1)}, \dots, k_{\pi(N_1)})$.

Also, under (ID) and concavity, a restriction to monotonic strategies does not prevent players from responding optimally. Player 2 always has a monotonic best response to a pure strategy choice of Player 1 because he can play a constant strategy. Moreover, if Player 2 believes that Player 1 is mixing in such a way that when Player 2 hears a higher message, then he believes that Player 1 is going to play higher strategies in the underlying game, then Player 2 has a monotonic best reply. Consider a mixed strategy in which Player 1 plays $s_1(i)$ with probability p_i and let $P_i = \sum_{k=1}^{i} p_k$.

$$\sum_{i=1}^{N_1} u_2(s_1(i), s_2'')p_i \ge \sum_{i=1}^{N_1} u_2(s_1(i), s_2')p_i$$

if and only if

$$\begin{aligned} u_2(s_1(N_1), s_2'') - u_2(s_1(N_1), s_2') \ge \sum_{i=1}^{N_1-1} \{ & \big(u_2(s_1(i+1), s_2'') - u_2(s_1(i+1), s_2')\big) \\ & - \big(u_2(s_1(i), s_2'') - u_2(s_1(i), s_2')\big)\} P_i. \end{aligned} \tag{1}$$

When $s_2'' > s_2'$, the coefficients of the sum in (1) are all non-negative by (ID). It follows that if Player 2 prefers s_2'' to s_2' when he expects Player 1 to mix according to p_i, then he will also prefer s_2'' to s_2' if Player 1 uses a mixed strategy that stochastically dominates p_i (that is a mixture q_i such that $Q_i = \sum_{k=1}^{i} q_k \le P_i$ for all $i = 1, \dots, N_1$). Hence if Player 1 uses strategies that associate higher messages to (stochastically) higher mixtures, then Player 2 will have a best response that is monotonic. Assume that $n'' > n'$ and $s_1'' > s_1'$. A similar argument guarantees that if a higher message induces stochastically higher responses, then if Player 1 prefers s_1'' to s_1' when she uses message n', then she will prefer s_1'' to s_1' when she uses message n''.

Now suppose that the payoff functions satisfy (SP). Maintaining the genericity assumption, this means that given any beliefs about the strategy choice of the opponent, each player will have at most two pure-strategy best replies and that if $s_i'' > s_i'$ are best replies, then the strategies are adjacent (there exists a k such that $s_i' = s_i(k)$ and $s_i'' = s_i(k+1)$). It follows that messages can be ordered so that any distribution over outcomes of G that can be obtained as an equilibrium

[14] The invariance properties of strategic stability (Kohlberg and Mertens, 1986) suggest that the conclusion of Proposition 1 would hold if the solution concept were strategic stability rather than iterative deletion of weakly dominated strategies.

of G^* can be obtained as an equilibrium of G^*_M after a relabeling of strategies. To do this, fix a mixed-strategy equilibrium of G^*. Order the responses that Player 2 makes by stochastic dominance. This is possible because the responses are pure or a mixture between adjacent actions. Relabel the messages to coincide with this order (the highest message induces the highest response, the second highest message induces the second highest response, and so on). The resulting strategies for Player 2 will be mixtures of monotonic strategies. If Player 1's payoffs satisfy internal dominance strictly, then her relabeled strategies will be monotonic in the sense that if she places positive probability on strategies (n'', k'') and (n', k') with $n'' > n'$, then $k'' > k'$. Hence for the class of underlying games that I consider, the restriction to monotonic strategies is both consistent with rational behavior and, taken by itself, insufficient to rule out predictions.

The two steps in my refinement process correspond to two features needed for communication to be effective. There must be some common understanding on the use of language. The common understanding prevents coordination failure by ruling out different ways to "say" the same thing. Without restrictions, Player 2 can interpret a low message as a sign of Player 1's intention to play low and a high message as a sign of Player 1's intention to play high **or** a high message as a sign of Player 1's intention to play low and a low message as a sign of Player 1's intention to play high. With the monotonicity restriction, only the first interpretation is possible. With the restrictions in place, dominance arguments permit Player 2 to draw additional inferences: that the highest type will not use the highest message, for example. These inferences make communication effective. Imposing monotonicity does not rule out strategic behavior. Nothing prevents Player 1 from sending an uninformative message about intentions. Equilibrium assumptions prevent Player 1 from systematically misleading Player 2.

I turn now to the third concern. Proposition 1 requires strong assumptions. The self-signaling property is what makes signaling in the monotonic game credible. The condition eliminates games in which Player 1's preferences over Player 2's actions are independent of Player 1's planned behavior in the underlying game. Hence the proposition does not apply to the kinds of game in which Aumann argued that cheap talk will be ineffective. In the next section, I confirm that conclusion of Proposition 1 does not apply in Aumann's example. The results of the paper plainly apply to simple coordination games or in games like the battle of the sexes. Specifically, if both players have n strategies in the underlying game, all off-diagonal payoffs are zero, and all diagonal payoffs are positive, then the game satisfies the conditions in Proposition 1. In this situation, there are multiple equilibria and communication permits the speaker to select her favorite payoff.

What if the strong conditions in Proposition 1 do not hold? The selection argument that I describe does not work. This does not mean that Player 1 will not receive her highest Nash Equilibrium payoff, which is always a possible prediction.

4. Aumann's example

Aumann's example is a 2 × 2 stag-hunt game that has two, Pareto-ranked equilibria and the property that Player 1 prefers Player 2 to take the action R independent of the action she intends to take. Farrell (1988) describes the game using the following payoff matrix:

	L	R
U	7, 7	8, 1
D	1, 8	9, 9

This game fails to satisfy the self-signaling condition because L is the best response to U, but if Row plays U she prefers Column to play R.[15] I will demonstrate that the conclusion of Proposition 1 does not hold.

If Player 1 can make a statement from a two-word language prior to playing this game, the game with communication takes the form:

	LL	LR	RL	RR
UU	7, 7	7, 7	8, 1	8, 1
UD	1, 8	1, 8	9, 9	9, 9
DU	7, 7	8, 1	7, 7	8, 1
DD	1, 8	9, 9	1, 8	9, 9

Where, the first element of the Column player's strategy is the response to the message "U" and the second element is the response to the message "D"; the first element in the Row player's strategy is the message (either U or D) and the second element is the strategy that Row actually plays.

[15] The game is not generic, but this does not influence the arguments. The game does satisfy the single-peaked and interval-dominance conditions.

The monotonicity restriction eliminates one of the middle strategies of the Column player:

	LL	LR	RR
UU	7, 7	7, 7	8, 1
UD	1, 8	1, 8	9, 9
DU	7, 7	8, 1	8, 1
DD	1, 8	9, 9	9, 9

In this game, there are two weakly dominated strategies for Row. If I delete them both, the game reduces to:

	LL	LR	RR
DU	7, 7	8, 1	8, 1
DD	1, 8	9, 9	9, 9

At this point, Column has a duplicate strategy, but the resulting game is equivalent to the original game. Deleting weakly dominated strategies in a different order may result in the deletion of either LR or RR (not both) to reach a 2×2 game equivalent to the original game.

The example illustrates two things. First, some version of the self-signaling condition is needed for my result. With self signaling, the proof of Proposition 1 demonstrates that the Row player will use the message U if she intends to play U: UU will never be weakly dominated. In the example, the Row player always prefers to induce the Column player to play R, consequently UU is weakly dominated (under monotonicity). Second, the argument does not eliminate the Row player's preferred equilibrium – it just is not sufficient to eliminate the other equilibrium outcome.

5. Discussion

There are some simple variations of the basic model that deserve mention.

Player 1 needs only one message for each pure strategy in G to communicate effectively. Adding more messages will not change the result. It is always possible to delete weakly dominated strategies in the communication game so that message credibly signals an intention to play a particular equilibrium strategy in the underlying game. That is, if in the k pure-strategy Nash equilibria of the underlying game, Player one plays $s_1(1) < \ldots < s_1(k)$, then for $j = 1, \ldots, k$, the strategy in the communication game $(N_1 - j, l)$ will eventually be deleted for all $l \neq N_1 - j$.

Removing messages may permit other outcomes to survive. In the extreme case in which only one message is available, communication cannot be effective. In general, if there are fewer messages then one cannot guarantee that Player 1 receives her highest payoff.

If both players can make pre-play statements, and do so in a fixed, finite, sequence, then one can impose the monotonicity restriction on strategies of the player who receives the final message. Suppose Player 1 makes the final statement. Consider the subgame determined by all but the last message. These subgames involve one round of communication. Assume that in the reduced game, Player 2's action in the underlying game is monotonic in the last message Player 1 sends. The arguments of this paper guarantee that in each of these subgames only Player 1's favorite action survives iterative deletion of weakly dominated strategies. Hence the history prior to arriving at one of these subgames is not relevant.

A weak test of my predictions would be to investigate whether the fraction of the time Player 1 obtains her highest Nash-equilibrium payoff depends significantly on exogenously assigned names of messages (names that evoke a connection between messages and actions presumably would trigger the common understandings modeled by the monotonicity restriction) and on the nature of the game (where lack of strategic complementarities or failure of self signaling should reduce the effectiveness of communication). It would also be useful to have more comparisons between outcomes with and without communication. In evaluating the evidence, it is important to note that my results predict both an outcome in the underlying game and a relationship between the Sender's equilibrium message and her action. My model predicts that people use cheap-talk messages to coordinate in systematic ways across games. As I discussed in the introduction, in other approaches the interpretation of messages may vary across equilibria.

Appendix A. Marx and Swinkels

Marx and Swinkels (1997) give conditions under which the outcome of iterative deletion of weakly dominated strategies does not depend on the order of deletion. Stating the result requires several definitions.

Definition 5. A game satisfies the transfer of decision maker indifference (TDI) property if $u_i(s) = u_i(s'_i, s_{-i})$ implies that $u_j(s) = u_j(s'_i, s_{-i})$ for all $i, j = 1, \ldots, I$, $s_i, s'_i \in S_i$, and $s_{-i} \in S_{-i}$.

Definition 6. Given a game $G = (S, u, I)$, a strategy s_i is weakly dominated relative to S for Player i if there exists $s'_i \in S_i$ such that $u_i(s'_i, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ with strict inequality for some $s_{-i} \in S_{-i}$.

Definition 7. Given a game $G = (S, u, I)$, a reduction of G by weak dominance is a game $\tilde{G} = (\tilde{S}, u)$ where there exists a nonnegative integer K such that $\tilde{S}$ can be obtained from S by letting $S^0 = S$, for each $k = 1, \ldots, K$, S^k is obtained from S^{k-1} by deleting strategies that are weakly dominated relative to S^{k-1} for some i, and $\tilde{S} = S^K$. The reduction is full if no strategies in $\tilde{S}$ are weakly dominated relative to $\tilde{S}$.

Definition 8. Let W and W' be restrictions of S. W is equivalent to a subset of W' if there exist one-to-one functions $m_i : W_i \to W_i'$ for $i = 1, \ldots, I$ such that $u(s) = u(m_1(s_1), \ldots, m_I(s_I))$ for all $s \in W$.

Definition 9. Let W be a restriction of S and let $s_i' \in S_i$. The strategy s_i' is redundant on W if there exists $s_i \in S_i$ such that $u_i(s_i, w_{-i}) = u_i(s_i', w_{-i})$ for all $w_{-i} \in W_{-i}$.

Marx and Swinkels show that if a game satisfies the TDI property, then two full reductions by dominance are the same up to the additional or removal of redundant strategies and a renaming of strategies. The TDI property states that if (given the behavior of the other players) Player i is indifferent between two strategies, then all other players are indifferent between these strategies as well. The property plainly holds for generic games. Since messages are free, it also holds for G_M^* when G is generic.

Appendix B. Proofs

Proof of Proposition 1. The proof uses a particular order for deleting weakly dominated strategies. Thanks to the result of Marx and Swinkels (1997) the conclusion is essentially independent of the order selected. The proof uses the following procedure:

1. Set $k = 0$.
2. Delete all weakly dominated strategies of Player 2 greater than $(b^2(s_{N_1-k}), \ldots, b^2(s_{N_1-k}))$.
3. Delete all weakly dominated strategies of Player 1 of the form $(n, N_1 - k)$.
4. If $k < N_1 - 1$, add 1 to k and return to [2]; if $k = N_1$, set $l = 1$ and go to [5].
5. Delete all weakly dominated strategies of Player 2 less than $(b^2(s_l), \ldots, b^2(x_l))$.
6. Delete all weakly dominated strategies of Player 1 of the form (l, m) for $m > l$.
7. If $l < N_1 - 1$, add 1 to l and return to [5]; if $l = N_1$, go to [8].
8. Delete any remaining weakly dominated strategies of Player 2.
9. Delete any remaining weakly dominated strategies of Player 1.
10. If no strategies were deleted in [8] and [9] stop. Otherwise, return to [8].

I break the proof of Proposition 1 into several steps.

Lemma 1. *[Step 1.1] If $y \in S_2$ is greater than $b^2(s_{N_1})$, then y is strictly dominated in G.*

Proof of Lemma 1. It is sufficient to show that for all $s \in S_1$ and $y > b^2(s_{N_1})$,

$$u_2(s, b^2(s_{N_1})) > u_2(s, y). \tag{2}$$

Suppose that (2) holds for all $y \in (b^2(s_{N_1}), y')$ and all s, but that there exists an $s' \in S_1$ such that

$$u_2(s', b^2(s_{N_1})) \leq u_2(s', y'). \tag{3}$$

It follows from (2) and (3) that $u_2(s', y) \leq u_2(s', y')$ for all $y \in [b^2(s_{N_1}), y']$ and hence, by (ID), that $u_2(s_{N_1}, b^2(s_{N_1})) \leq u_2(s_{N_1}, y')$, which contradicts the definition of $b^2(\cdot)$. $\square$

On the basis of Step 1.1, delete all strategies that respond to any message with $y > b^2(s_{N_1})$.

Lemma 2. *[Step 1.2] Strategies $(n; N_1)$ for $n < N_1$ are weakly dominated for Player 1.*

Proof of Lemma 2. By (SS) and (SP), $u_1(s_{N_1}, y)$ is strictly increasing in y provided that $y \leq b^2(s_{N_1})$. The conclusion follows because Player 2 uses monotonic strategies. $\square$

On the basis of Step 1.2, delete all strategies for Player 1 of the form $(n; N_1)$ for $n < N_1$.
Now continue the process inductively.

Lemma 3. *[Step $k + 1.1$] $(y_1, \ldots, y_{N_1})$ is weakly dominated if there exists $j \leq N_1 - k$ such that $y_j > b^2(s_{N_1-k})$.*

Lemma 4. *[Step $k + 1.2$] $(n, N_1 - k)$ is weakly dominated if $n < N_1 - k$.*

Using Step $k + 1.1$, delete all strategies of Player 2 in which $y_j > b^2(s_{N_1-k})$ for $j \leq N_1 - k$. Using Step $k + 1.2$, delete all strategies of Player 1 of the form $(n; i)$ for $n < N_1 - k$ and $i \geq N_1 - k$.

Proof of Lemma 3. To prove Step $k + 1.1$, observe that Step $k.1$ deletes Player 1's strategies of the form $(n, N_1 - k + 1)$ for $n < N_1 - k + 1$. Hence Player 2 believes that Player 1's action is no higher than s_{N_1-k} given the message $j \leq N_1 - k$. Hence strategies of Player 2 that respond to messages $j \leq N_1 - k$ with actions greater than $b^2(s_{N_1-k})$ are weakly dominated. □

Proof of Lemma 4. To prove Step $k + 1.2$, observe from Step $k + 1.1$ that Player 2 does not respond to messages less than $N_1 - k$ with actions greater than $b^2(s_{N_1-k})$. By (SP), $u_1(n, l)$ is strictly increasing in n for $n \leq N_1 - k$. Consequently Step $k + 1.2$ follows. □

Proof of Proposition 2. Let (σ_1, σ_2) be a mixed-strategy equilibrium of G. Suppose that σ_1 places positive probability on $s_1 \in S_1$. It follows that Player 1's equilibrium utility in G is equal to

$$u_1(s_1, \sigma_2) = \sum_{s_2 \in S_2} \sigma_2(s_2) u_1(s_1, s_2) \leq u_1(s_1, b(s_1)), \tag{4}$$

where the inequality follows from self signaling. Hence the payoff Player 1 obtains from any mixed-strategy equilibrium in G is no greater than what she would receive in her most preferred pure strategy equilibrium.[16] □

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[16] The inequality in (4) is strict provided that G is self signaling for Player 1 and σ_2 is non-degenerate.