Title
Targeted Couponing in Online Auctions

Journal
Information Systems Research, 28(3)

ISSN
1047-7047

Authors
Choudhary, Vidyanand
Shivendu, Shivendu

Publication Date
2017-09-01

DOI
10.1287/isre.2017.0688

Peer reviewed

Targeted Couponing in Online Auctions

Vidyanand Choudhary and Shivendu Shivendu

Forthcoming in *Information Systems Research*

Abstract

In order to study the role of targeted couponing in auctions, we develop a stylized model in which bidders have heterogeneous valuations and participation costs wherein their entry probabilities are endogenous. Couponing impacts the seller's profit in two ways: (i) impact on bidders' entry probability including negative externalities for the bidder who does not receive a coupon and (ii) value extraction. We find that targeting a coupon to the low-valuation bidder can be optimal for the firm even if it leads to a reduction in the joint entry probability of the two bidders because of the benefit from value extraction. A novel result is that in the context of auctions it can be optimal for the seller to issue targeted coupons to the high-valuation bidder. We also find that an increase in the bidders' valuation or reduction in the participation cost can lead to lower profit for the seller. This result is driven by the non-monotonicity of the joint entry probability of the two bidders and the seller profits being non-monotone functions of bidders' valuations and participation costs.

Keywords: Online auction, couponing, second-price auction, targeted couponing, participation cost, entry probability

1. Introduction

Academic researchers have studied the impact of couponing and targeted couponing on consumers, sellers and social welfare in the retail context. The analytics and big data technology that has enabled targeted couponing in the retail context also makes it feasible for online auctions. There is some evidence to suggest that online auction platforms are starting to offer coupons. For example, *eBay* provides a third-party hosted couponing solution through MyStoreRewards "to offer cash back and coupons to buyers."[1] *Aucser.com* allows online auction sellers to create a working coupon on the fly. The coupon can be either a dollar value, or a percentage of the winning bid, and it can be set to expire on a certain date, or after certain number of uses. *Aucser.com* also allows sellers to target the coupons and track redemptions. Similarly, *SkyAuction.com* gives coupons to bidders and the winning bidder receives a $50 discount for auctions in certain time periods. However, there is scant research on the implications of targeted couponing in the context of online auctions. In contrast, there is a rich literature on the role of targeted couponing in the retail context. In this paper we seek to bridge this gap between literature on targeted couponing in the retail context and online auctions.

What is the rationale for couponing in the retail setting and is that rationale applicable in an auction context? While the literature on couponing in the retail context has focused largely on the role of couponing as a second degree price discrimination mechanism (Narasimhan, 1984; Levedahl, 1984; Sweeney, 1984; Varian, 1989), some researchers have also examined the role of targeted couponing in the retail context (Shaffer & Zhang, 1995).[2] In the retail context, couponing is used to expand the market by either allowing lower valuation consumers or consumers who purchase from other sellers to purchase the product. Also, couponing in the retail context, does not create any externalities on consumers who do not receive the coupon.

[1] http://workshops.ebay.com/thread.jspa?threadID=130000733

[2] For the role of coupons as a price discrimination mechanism in a distribution channel, see Gerstner et al (1994).

The auction context is different in three important ways: (i) couponing does not lead to market expansion, (ii) couponing creates negative externalities on consumers who do not receive the coupon as they have to compete with coupon-bearing consumers to win the auction and (iii) bidders incur a cost to participate in the auction (participation cost) because they have to learn the rules of the auction, wait for auction to be over before realizing whether they win the item etc. This cost of participation is different from that in the retail context where the literature has focused on the cost of clipping coupons which is tied to the use of coupons. Whereas, in the auction context, the participation costs are not linked to the usage of coupons but are related to participation in the auction. These differences between the retail and auction contexts may lead to different couponing strategies. For example, since couponing leads to negative externalities in auctions, couponing low-valuations consumer may not be profit enhancing because it hurts high-valuation consumers. This may change the conditions under which targeted couponing is optimal and may create new opportunities for couponing that leverage the externalities.

In this paper we study the role of coupons in auctions and focus on the following research questions. What are the conditions that support targeted couponing in online auctions? To whom should the seller target the coupons? What factors determine the face value of the coupon? How does couponing impact the strategies of the bidders? What are the differences in targeted couponing in auctions from prior literature on couponing in retail contexts?

In order to study these questions, we develop an analytical model with two types of bidders who are heterogeneous in their valuations and participation costs. The seller has information about bidder types but is uncertain about their participation costs and can leverage information about bidder types to issue targeted coupons. As discussed before, bidders incur a participation cost because they have to learn the rules of the auction, and wait for the auction to be over before realizing whether they win the item etc. Since higher valuation bidders on average place a higher value on their time, in our model, high-valuation bidders on average, face higher participation costs. Furthermore, in our conceptualization, each bidder takes into account his[3] participation cost, the probability of winning the auction, and the expected surplus if he wins. The bidders enter the auction when their expected surplus is non-negative. The seller can influence bidders' decision to enter the auction by issuing coupons and her couponing strategy depends on the relative values of three parameters: the valuations of the low- and high-valuation bidders and the relative participation cost of the low-valuation bidder.

One novel result in the context of couponing in auction setting, that is different from prior literature on couponing in retail setting, is that, it can be optimal to target the low-valuation bidder even when he always enters the auction. This is different from the literature where coupons are typically used to entice consumers who may not otherwise purchase (Narasimhan, 1984). To understand this result, note that in online auctions, coupons impact seller's profit in two distinct ways: (i) value extraction from the bidder who wins the auction and (ii) impact on bidders' entry probabilities. Since a bidder who has a coupon will be willing to bid higher than his valuation, one role of coupons in auctions is to enhance value extraction from the winner of the auction. The value extraction role of coupons does not exist in couponing literature in the retail context. The second role of couponing is to influence bidders' entry probabilities, which are endogenous. The entry probability of each bidder depends on their expected surplus from participating in the auction and the seller can impact the expected surplus of the bidders through her couponing strategy -- the bidders who receive a coupon are more likely to enter whereas those who do not are less likely to enter. The couponing literature in the retail context has modeled the positive impact of coupons on purchase probability whereas in the auction context, coupons also have a negative externality on the entry probability of bidders who do not receive coupons. The optimal couponing strategy depends on the balance of the benefits and the costs of value extraction and bidders' entry probabilities.

Another interesting finding is that, in the context of online auctions it can be optimal for the seller to issue targeted coupons to the high-valuation bidder. In contrast, prior literature recommends issuing coupons to low-valuation consumers for price discrimination or to marginal consumers to poach them from

[3] The seller is she and bidder is he throughout the paper.

competitors (Narasimhan, 1984; Levedahl, 1984; Sweeney, 1984; Varian, 1989; Shaffer & Zhang, 1995; Fudenberg and Tirole, 2000). The strategy of targeting the high-valuation bidder is driven by the impact of coupons on the joint entry probability of both bidders. This occurs because in a two-bidder model, the seller obtains strictly positive profit only when both bidders enter the auction. Since couponing exhibits negative externalities, therefore, in order to maximize profit, the firm seeks to optimize the bidders' joint entry probability by targeting either the high-valuation or the low-valuation bidder depending on their entry probabilities. If the entry probability of the low-valuation bidder is low, then the seller seeks to optimize the joint entry probability by targeting the low-valuation bidder. On the other hand, when the entry probability of the high-valuation bidder is low, the firm targets that bidder to increase the joint entry probability.

Whereas one may expect seller's profit to increase with bidders' valuations and decrease with their participation costs, we find that the seller's profit is non-monotone in valuations and participation costs of the bidders. This non-monotonicity of the profit is driven by non-monotonicity in the joint entry probability of the two bidders. In other words, high valuation or low participation costs of one bidder boost his entry probability but will lower the joint entry probability when this bidder's entry probability is high enough.

Finally, we find that it is never optimal for the firm to target both bidders with coupons. As discussed before, coupons have two roles: value extraction and impact on bidder entry probability. The value extraction role does not generate any extra benefit by couponing both bidders compared to issuing coupon to only one bidder. Moreover, issuing coupons to both bidders provides lower marginal benefit from changes in joint entry probability because the coupons work at cross-purposes. However the cost of couponing increases significantly because the winning bidder always cashes his coupon even when he alone enters the auction. Thus the net profit impact of issuing coupons to both bidders is negative and hence it is not optimal to issue coupons to both bidders.

1.1 Literature Review

With the advent of personalization technologies, it has become rather commonplace for sellers to identify individual consumers and "tailor their promotional prices to consumers on a one-to-one basis"

(Shaffer & Zhang, 2002). For example, online content-enabled workflow solutions provider *LexisNexis* sells to different users at different prices (Shapiro & Varian, 1999). Online commercial transactions enable sellers to collect and analyze data at the individual buyer level to decipher each buyer's willingness to pay for a certain good and adopt personalized pricing strategies (Choudhary et al, 2005; Chen & Iyer, 2002). Researchers have also studied targeted couponing as a competitive strategy to poach rival firms' loyal customers (Shaffer & Zhang, 1995; Bester & Petrakis, 1996; Fudenberg & Tirole, 2000); and couponing to convert buyers into repeat loyal consumers (Fong & Liu, 2012).

Bapna et al. (2008) use data from Yankee and uniform-price multiunit auctions to predict bidders' willingness to pay (WTP) and report that their estimates are within 2% of bidders' revealed WTP. There is extensive empirical literature in Information Systems area on auctions. This research has studied differences between online and offline auctions (Overby & Jap, 2009), role of bidders and bidder characteristics (Ariely & Simonson, 2003; Bapna et al 2004), interdependence between different auctions (Bapna et al., 2009), and the impact of auction design on outcome and bidding behavior (Bapna et al., 2003; Gallien & Gupta, 2007; Goes et al., 2010). The role of seller search for a high-valuation buyer and buyer search for an appropriate product and low price are key to the growth of online auctions, and the search behavior impacts market outcome and efficiency (Kuruzovich et al., 2010).

There have been some notable exceptions where IS researchers have undertaken analytical investigations of online auctions, especially design of keyword auctions (Liu et al., 2010), design of online auctions (Liu & Chen 2006; Kannan, 2012; Kannan, 2010), and analysis of simultaneous use of online auctions and posted-prices (Etzion et al., 2006). These studies informed our research, although we restrict our attention to the role of targeted coupons in online auctions.

The remainder of this paper is organized as follows: In §2, we present the model, and in §3, we examine the benchmark case in which seller does not issue any coupons. In §4, we examine the seller's optimal targeted couponing strategy. We discuss our results and identify suitable theoretical and managerial implications in §5, and conclude in §6.

2. Model

We model the online auction as a second-price auction consistent with real-world online auctions on *eBay* (Roth and Ockenfels, 2002; Zeithammer, 2006). Our auction setting is similar to the Web-based auctions of Van Heck and Vervest (1998) in which one seller auctions a single item to many prospective buyers; much of the research in auctions has focused on this setting (see Klemperer (1999), for extensive literature survey of auction theory). Following the extant literature, in our model the seller has information about prospective buyers such that she has the ability to target coupons to different types of buyers.

We consider a *private-value* auction where each bidder knows his valuation for the item, which is independent of others' valuations. The private-value setting has been supported by extensive literature that has empirically studied online auction platforms like *eBay* (Hou & Rego, 2007; Ockenfels & Roth, 2006; Roth & Ockenfels, 2002; Zeithammer, 2006). The seller has no intrinsic value for the good, and therefore, her reservation price is zero. Following the literature, we assume that the cost of the good to the seller is sunk, and therefore, the seller's objective is to maximize revenue from the auction (Myerson, 1981).

There is empirical evidence to show that most of the online auctions have few bidders. Geldman (2003) examines a sample of 3,500 items across eBay categories and finds that 82% of the auctions have two or fewer bids although there may have been more than two potential bidders in the market. Since our context is online auctions, we develop a stylized model with two bidders: a high-valuation bidder with valuation $v_H \leq 1$ and a low-valuation bidder with valuation v_L where $v_L < v_H$. Our approach is similar to prior literature which has developed models with two bidders to study auctions (Budish and Takeyama, 2001; Gale and Hausch, 1994; Maskin and Riley, 1985; Milgrom and Weber, 1982b). Bidders' valuations for the product are unaffected by the valuations of other bidders and are common knowledge.

The bidders incur participation costs to participate in the auction. Prior literature has described several sources of participation costs. Etzion et al. (2006) describe two sources: the cost of monitoring the auction and making bids and the waiting cost for the end of the auction. Cao and Tian (2010) state that bidders incur costs to learn the rules of the auction, to make bids, and opportunity costs to attend the auction.

Bajari and Hortaçsu, (2003) studied eBay auctions and found evidence that bidders face participation costs in online auctions. One of the key determinants of the participation cost for a bidder is his opportunity cost of time. Therefore in our conceptualization, high-valuation bidders on average, face higher participation costs. This is consistent with literature, including Narasimhan (1982 and 1984) who models consumers with higher wage rates as facing higher opportunity cost of time. Etzion et al. (2006) also assume that participation cost is higher for high-type consumers.

Bidders' participation cost is their private information though the distribution is common knowledge. The high-valuation bidder incurs a participation cost of t_H which is drawn from U[0, 1]. Similarly, the low-valuation bidder's participation cost is t_L drawn from U[0, κ] where $\kappa \leq 1$. We refer to the parameter κ as the relative participation cost of the low-valuation bidder. Each bidder learns his true participation cost before entering the auction. The seller only knows the probability distributions from which the participation costs of the bidders are drawn. Each bidder computes his expected payoff from participating in the auction and enters the auction only if his expected payoff is non-negative.

The seller can auction the good with or without issuing coupons. When she adopts a couponing strategy, she can target the coupon to specific bidder(s) and the value of the coupon can be different for different types of bidders. A targeted couponing strategy involves the seller targeting a coupon of value c_L to low-valuation bidder and/or c_H to high-valuation bidder. The coupon has no value if the bidder does not win the auction. If the bidder with coupon wins the auction, then she pays the second highest bid amount less the value of the coupon. Note that coupon amount c_L and/or c_H may be greater than the second highest bid amount, resulting in net negative profit to the seller. The bidders are rational and can compute the seller's couponing strategy.

The summary of notations is in Appendix A. We make the following additional assumptions.

1. The seller and all bidders are risk neutral and there is no collusive bidding by the bidders. Note that since our setting is one of private-value, the seller does not benefit from shill bidding and therefore

there is no shill bidding. Shill bidding may play a role in a common-value auction (Chakraborty & Kosmopoulou, 2004).

2. Coupons cannot be traded and the bidder who wins the auction always claims the product.
3. The seller's cost of organizing the auction, the cost of targeting coupons, and the bidder's cost of using a coupon are negligible.

2.1 Participation decision of bidders

In our model, the entry decision of the bidders' is endogenous and we now describe bidders' participation decisions. The low-valuation bidder's entry probability is denoted by p_L and the high-valuation bidder's entry probability is denoted by p_H. The low-valuation bidder enters the auction only if he expects non-negative surplus net of participation cost. He knows that if the high-valuation bidder enters the auction, he will not win. Therefore the low-valuation bidder's entry probability depends on his expectation of the high-valuation bidder's entry probability. On the other hand, the high-valuation bidder knows that if he enters the auction, he will win but his surplus depends on whether the low-valuation bidder enters the auction or not. Hence, the high-valuation bidder's probability of entry also depends on his expectation of the low-valuation bidder's entry probability. Note that our formulation of endogenous entry by bidders is consistent with Levin and Smith (1994) and Etzion et al. (2006).

The seller may or may not target the bidders with coupons. We begin with a description of a general model where the seller targets each bidder with a coupon. Later we will examine the cases where some or none of the bidders are targeted by setting the corresponding coupon values to zero. Let the draws of the participation cost of high-valuation and low-valuation bidders be denoted as $\tilde{c}_H$ and $\tilde{c}_L$ respectively.

The expected surplus of the low-valuation bidder is: $s_L = (1 - p_H)(v_L + c_L) - \tilde{c}_L$. To understand this expression, note that the low-valuation bidder wins the auction only when the high-valuation bidder does not enter the auction, and in that case his payoff is $(v_L + c_L)$.[4] The low-valuation bidder will enter the auction

[4] The case where the coupon to low-valuation bidder is so large that he wins even when the high-valuation bidder enters the auction is examined in Lemma 1a in the Appendix B. We show in Lemma 1a that it is never optimal for

when his expected surplus is non-negative or in other words, when $s_L \geq 0 \Leftrightarrow \tilde{t}_L \leq (1-p_H)(v_L + c_L)$. There exists a $\tilde{t}_L$ at which the low-valuation bidder is indifferent between entering and not entering the auction. We refer to this critical value of low-valuation bidder's participation cost as $\hat{t}_L$. The low-valuation bidder participates in the auction only when $\tilde{t}_L \leq \hat{t}_L$. Hence, the probability of entry of the low-valuation bidder is

$$p_L = \hat{t}_L / \kappa = \frac{(1-p_H)(v_L + c_L)}{\kappa} \tag{1}$$

The expected surplus of the high-valuation bidder is: $s_H = (1-p_L)(v_H + c_H) + p_L(v_H + c_H - (v_L + c_L)) - \tilde{t}_H$. To understand this expression, note that the high-valuation bidder wins the auction whenever he enters the auction. When the low-valuation bidder does not enter the auction, high-valuation bidder's payoff is $(v_H + c_H)$ and the probability of this event is $(1-p_L)$. When the low-valuation bidder enters the auction then the high-valuation bidder's payoff is $(v_H + c_H - (v_L + c_L))$ with probability p_L. The high-valuation bidder will enter the auction when his expected surplus is non-negative or in other words, when $s_H \geq 0$. There exists a $\tilde{t}_H$ at which the high-valuation bidder is indifferent between entering and not entering the auction. We refer to this critical value of high-valuation bidder's participation cost as $\hat{t}_H$. The high-valuation bidder participates in the auction only when $\tilde{t}_H \leq \hat{t}_H$. Hence, the probability of entry of the high-valuation bidder is

$$p_H = (1-p_L)(v_H + c_H) + p_L(v_H + c_H - (v_L + c_L)) \tag{2}$$

Under rational fulfilled expectations equilibrium, the equilibrium entry probabilities can be obtained by simultaneously solving the system of two equations (1) and (2):

$$p_L = \frac{(1-c_H - v_H)(c_L + v_L)}{\kappa - (c_L + v_L)^2} \tag{3}$$

$$p_H = 1 - \frac{\kappa(1-c_H - v_H)}{\kappa - (c_L + v_L)^2} \tag{4}$$

---

the seller to issue such a large coupon to the low-valuation bidder that he wins even when high-valuation bidder enters the auction.

Since our setting is of second-price auction, the optimal bidding strategy for a bidder of type i ($i \in \{L, H\}$) is to bid his true willingness to pay with the coupon, that is $v_i + c_i$ (Vickrey, 1961; McAfee & McMillan, 1987). The on-line auction ends with the highest bidder winning the auction and paying the second highest bid amount. We do not model the dynamics of the auction such as the sequence of bids and assume that the valuations are exogenous.

In order to find the expected profit of the seller, we need to consider four scenarios: (i) no bidder enters the auction, (ii) only low-valuation bidder enters the auction, (iii) only high-valuation bidder enters the auction, and (iv) both types of bidders enter the auction. The resulting expected profit can be written as:

$$\pi(c_H, c_L) = ((1-p_L)(1-p_H)*0 + p_L(1-p_H)*(-c_L) + (1-p_L)(p_H)*(-c_H) + p_L * p_H(v_L + c_L - c_H) \qquad (5)$$

We now provide an overview of the tradeoffs associated with the couponing strategy. As discussed before, in our conceptualization, targeted couponing implies that the seller has the ability to provide a coupon of a particular value to any one type or more types of bidders. Couponing can be beneficial to the seller because targeted couponing may make a coupon-bearing bidder raise the bid amount, which may generate higher profit from the winning bidder in a second-price auction. If the auction-winner is a bidder without a coupon, then the seller’s profit increases due to couponing. On the other hand, couponing may hurt the seller if only one bidder enters the auction because the winning bidder redeems the coupon without any increase in the amount paid by the winner. Further, coupons may impact the entry probabilities of bidders because each bidder’s decision to enter the auction is endogenous in our setting and coupons impact the expected gain of the bidder(s) who has (have) the coupon(s) from entering the auction. Therefore, it is clear that the optimality of a couponing strategy is critically dependent on the balance of tradeoffs between the potential gain and potential loss to the seller from targeted couponing. The seller has several strategies for targeted couponing: (i) she may target both bidder-types simultaneously; or (ii) she may target only one bidder-type. First, in Section 3, we consider the benchmark case in which the seller does not issue any coupons.

3. Benchmark Case: Seller does not issue any coupons

We start with a benchmark case to compute the seller's profit when no coupons are issued. We will compare the profit from various couponing strategies to this benchmark to determine whether the couponing strategies are profit increasing. When no coupons are issued, the profit of the seller can be obtained from Equation 3 by setting $c_H = 0$ and $c_L = 0$. The interior equilibrium entry probabilities of both types of bidders and expected profit of the seller from the auction are reported in Lemma 1 below:

LEMMA 1: **(Interior Region)** *The equilibrium entry probabilities of the bidders in the interior region are:* $p_L = \frac{v_L(1-v_H)}{\kappa - {v_L}^2}$, $p_H = 1 - \frac{\kappa(1-v_H)}{\kappa - {v_L}^2}$ *when* $\kappa > v_L(1+v_L-v_H)$ *and* $v_H < 1$. *The expected profit of the seller is*

$$\pi_N = \frac{v_L^2(1-v_H)(\kappa v_H - {v_L}^2)}{(\kappa - {v_L}^2)^2}.$$

All proofs of Lemmas and Propositions are provided in Appendix B.

Lemma 1 explores the expected profit of the seller in the interior region where there is some uncertainty about the entry of the bidders, and therefore, the entry probabilities of both types of bidders are strictly in (0, 1). The conditions for the interior region require that relative participation cost of low-valuation bidder (κ) is not too small relative to v_L because otherwise the low-valuation bidder always enters the auction when he draws his participation cost. This condition can also be rewritten as $v_H > 1 + v_L - \left(\kappa / v_L \right)$, this implies that the valuation of the high-valuation bidder (v_H) should be high relative to valuation of low-valuation bidder. The equilibrium entry probabilities of both types of bidders and expected profit of the seller in the boundary region are reported in Lemma 2 below:

LEMMA 2: **(Boundary Region)** *The equilibrium entry probabilities of the bidders on the boundaries are:*

(i) *If* $v_H = 1$, *then* $p_L = 0$ *and* $p_H = 1$. *The expected profit of the seller is* $\pi_N = 0$.

(ii) *If* $\kappa \le v_L(1+v_L-v_H)$ *and* $v_H < 1$, *then* $p_L = 1$ *and* $p_H = v_H - v_L$. *The expected profit of the seller is* $\pi_N = v_L(v_H - v_L)$.

Part (i) of Lemma 2, states the boundary condition when the high-valuation-bidder enters with certainty. Note that the high-valuation bidder always wins the auctions when he enters. Thus the low-valuation bidder does not enter when the high-valuation bidder's entry is certain. This situation is realized when $v_H = 1$, since the maximum participation cost that the high-valuation bidder can draw is 1. In this case the high-valuation bidder always has a non-negative surplus from entering the auction when the low-valuation bidder does not enter. Therefore the low-valuation bidder has no incentive to enter and this boundary condition is realized.

On the other hand, when the relative participation cost of the low-valuation bidder is small relative to v_L, then the low-valuation bidder is more likely to enter the auction. When the high-valuation bidders valuation is small enough (the condition in part (ii) of Lemma 2 can be rewritten as $v_H \le 1 + v_L - \left(\kappa / v_L\right)$), the low-valuation bidder expects the high-valuation bidder to enter less often and that increases his entry probability. Hence when the stated conditions in part (ii) of Lemma 2 are satisfied, the low-valuation bidder always enters the auction, that is, $p_L = 1$.

PROPOSITION 1: ***Comparative statics of entry probabilities in the interior region:*** *When the seller does not issue any coupons, (i) the entry probability of the low-valuation bidder (p_L) increases with v_L, decreases with κ and decreases with v_H. (ii) the entry probability of the high-valuation bidder (p_H) decreases with v_L, increases with κ and increases with v_H.*

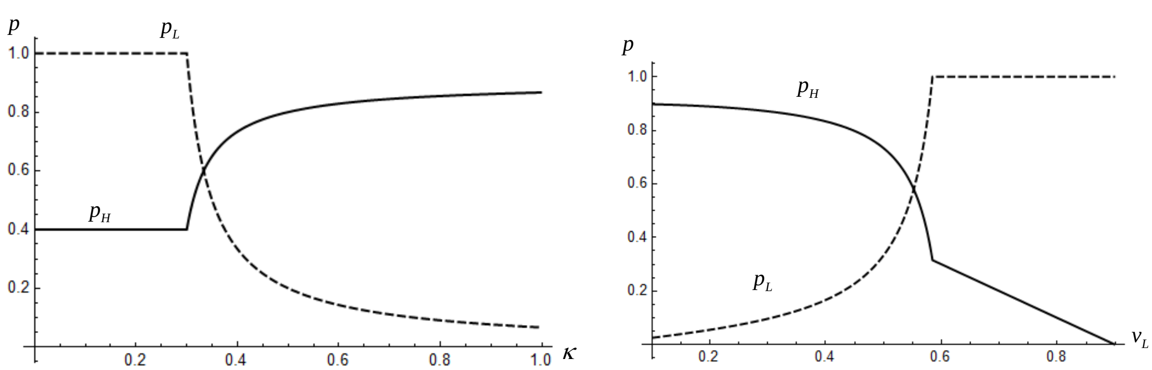


Figure 1: (a) Impact of low-valuation bidder's relative participation cost on the entry probabilities of both bidders $(v_H = 0.9,\ v_L = 0.5)$ and (b) Impact of valuation of low-valuation bidder on the entry probabilities of both bidders $(v_H = 0.9,\ \kappa = 0.4)$.

Figure 1(a) shows the entry probabilities of both types of bidders as a function of the relative participation cost of the low-valuation bidder (κ). The curved portion lies in the interior region as specified in Lemma 1 and the flat portion indicates the probabilities on the boundary as stated in Lemma 2. Similarly Figure 1 (b) shows the entry probabilities as a function of the valuation of the low-valuation bidder (v_L).

Note that the equilibrium entry probabilities in the interior region are determined by interplay of all the three parameters in our setting, namely v_H, v_L and κ and in turn the entry probabilities determine the seller's expected profit. The following Proposition describes the impact of different parameters on expected profit.

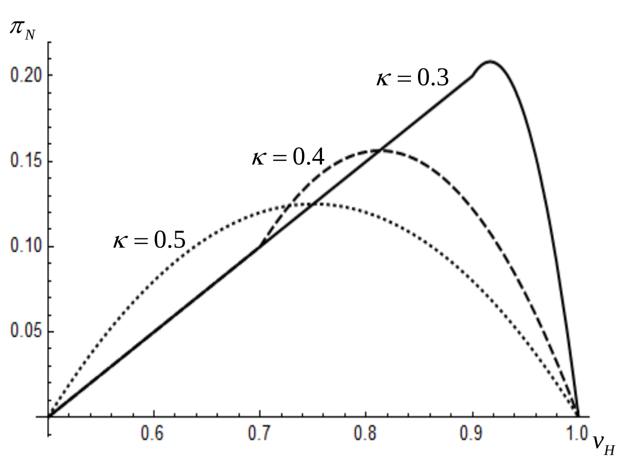
PROPOSITION 2: ***Comparative statics of profit in the interior region:*** *(i) When* $v_H < \frac{\kappa + v_L^2}{2\kappa}$*, the seller's profit increases with* v_H *and decreases otherwise. (ii) When* $\kappa < \frac{v_L^2(2-v_H)}{v_H}$*, the seller's profit increases with* κ *and decreases otherwise. (iii) When* $v_L > {v_H}/{2}$ *and* $v_L > \sqrt{\frac{\kappa v_H}{2-v_H}}$ *, the seller's profit decreases with* v_L *and increases otherwise.*

One would expect that an increase in the valuation of bidders will lead to increased profit to the seller. Surprisingly, the seller's expected profit may not be monotonically increasing in the valuation of high-valuation bidder (Proposition 2(i) and Figure 2(a)), valuation of low-valuation bidder and the relative participation cost of the low-valuation bidder. To understand the intuition for this result, one has to understand the underlying dynamics in the model.

The seller's expected profit is mediated by the entry probabilities as stated in Lemma 1. As v_H increases, the entry probability of high-valuation bidder increases, but the entry probability of the low-valuation bidder decreases because his expected surplus decreases when the high-valuation bidder enters more often. Replacing $c_L = c_H = 0$ in Equation 1, 2 and 3, we obtain the entry probabilities and the expected profit of the seller when no coupon is issued. Let the joint entry probability be $J(p_L, p_H) = p_L * p_H$. We have $\frac{\delta J}{\delta p_H} = p_L + p_H \frac{\delta p_L}{\delta p_H}$ which can be written as $\frac{\delta J}{\delta p_H} = \frac{v_L(1-p_H)}{\kappa} + p_H\left(\frac{-v_L}{\kappa}\right)$. Evaluating $\frac{\delta J}{\delta p_H}$ at the two limits: $p_H = 0$ and $p_H = 1$, we have: $\frac{\delta J}{\delta p_H}\Big|_{p_H=0} > 0$, $\frac{\delta J}{\delta p_H}\Big|_{p_H=1} < 0$. Hence $J(p_L, p_H)$ can be a non-monotone function of p_H when p_H is in the interior. Using Lemma 1, we have $\frac{\delta p_H}{\delta v_H} = \frac{\kappa}{\kappa - v_L^2}$ which can be positive or negative but does not change signs as v_H changes. Therefore p_H is a monotone function of v_H.

From Chain Rule, we can write: $\frac{\delta J(p_L, p_H)}{\delta v_H} = \frac{\delta J(p_L, p_H)}{\delta p_H}\frac{\delta p_H}{\delta v_H}$. Since $\frac{\delta J}{\delta p_H}\Big|_{p_H=0} > 0$ and $\frac{\delta J}{\delta p_H}\Big|_{p_H=1} < 0$, and since $\frac{\delta p_H}{\delta v_H}$ does not change sign, therefore the joint entry probability $J(p_L, p_H)$ can be a non-monotone function of v_H. Note that the expected profit of the seller is $p_L * p_H * v_L$ and it can be written as $J(p_L, p_H) * v_L$. Since $J(p_L, p_H)$ can be a non-monotone function of v_H, therefore, the seller's profit can be a non-monotone function of v_H for some parameter values.

In summary, the seller earns a profit only when both bidders enter the auction, and hence, seller's profit is determined by the joint entry probability of both bidders. Increased entry by one bidder causes the other bidder to enter less often. Thus it is possible that higher valuations that incentivize one bidder to enter more often may lead to a reduction in the joint entry probability and thus hurt the seller's profit. Hence the impact of v_H on seller's profit can be non-monotone.



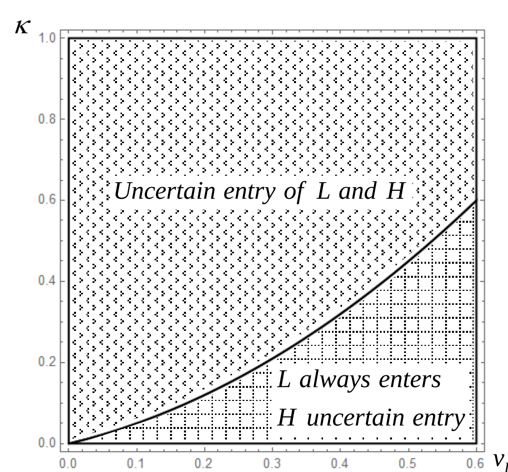


Figure 2: No Coupon Case: (a) Impact of valuation of high-valuation bidder on profit $\left(v_L = 0.5\right)$. (b) Interior and boundary regions with valuation of low-valuation bidder and low-valuation bidder's relative participation cost $\left(v_H = 0.6\right)$.

One would expect that an increase in the participation cost of any bidder would lead to a decrease in profit. Counter-intuitively, we find that an increase in the relative participation cost of the low-valuation bidder (κ) can increase seller's profit. Using Lemma 1, we have $\frac{\delta p_H}{\delta \kappa} = \frac{(1-v_H)v_L^2}{(k-v_L^2)^2} > 0$. Thus it can be shown that $J(p_L, p_H)$ can be a non-monotone function of the relative participation cost of the low-valuation bidder κ. Therefore, the seller's profit can be a non-monotone function of κ. To understand this result note that seller's expected profit is maximized only when both types of bidders enter the auction. As κ increases, the entry probability of the low-valuation bidder decreases and that of the high-valuation bidder increases. When κ is very small, the low-valuation bidder's entry probability is high causing the high-valuation bidder to enter less often leading to lower joint entry probability than the maximum level. Therefore, as κ increases, up to a critical value of $\kappa = \frac{(2-v_H)v_L^2}{v_H}$, the joint entry probability approaches the maximum level and the seller's expected profit increases. After this critical value, further increase in κ leads to reduction in the seller's profit.

The behavior of $J(p_L, p_H)$ with respect to the valuation of the low-valuation bidder, is similar and can be a non-monotone function of v_L because $\frac{\delta p_H}{\delta v_L} = -\frac{2\kappa(1-v_H)v_L}{(\kappa - v_L^2)^2} < 0$. However, the seller's profit is $J(p_L, p_H) * v_L$ which can be monotone increasing in v_L even when $J(p_L, p_H)$ is a non-monotone function of v_L. This can occur because the derivative of the seller's profit with v_L is $\frac{\delta\left(v_L * J(p_L, p_H)\right)}{\delta v_L} = J(p_L, p_H) + v_L \frac{\delta J(p_L, p_H)}{\delta v_L}$ which is monotone in the interior region when $v_L \leq {v_H}/{2}$ and non-monotone otherwise. Thus we find that the seller's profit can decrease even when the valuation of the low-valuation bidder increases.

Figure 2b shows the interior and the boundary region with the valuation and the relative participation cost of the low-valuation bidder. Please note that we have labelled the interior region as "Uncertain Entry of L and H" in Figure 2b and this region represents the space where the entry probability of each bidder is strictly between 0 and 1. The boundary region is labelled as "L Always Enters, H Uncertain Entry" and this region represents the space where the entry probability of the low-valuation bidder is 1 while the entry probability of the high-valuation bidder is strictly between 0 and 1. We can see that when κ is large, the interior region is realized, whereas small κ leads to the boundary where $p_L = 1$. The boundary region is larger when v_L increases, for a given κ.

Next, in Section 4 we consider the cases in which the seller issues a coupon to both bidders, or only to the high-valuation bidder or only to the low-valuation bidder. In Section 4.4 we evaluate the overall optimal couponing strategy.

4. Targeted couponing

In this section, we examine three cases: (i) the seller targets high-, and low-valuation bidders; (ii) the seller targets only the high-valuation bidder; and (iii) the seller targets only the low-valuation bidder. When the seller issues coupons, then the sequence of moves is as follows: first the seller determines her couponing strategy, which consists of determining the value and target of each coupon. Next, bidders enter the auction (with or without a coupon) with their respective entry probabilities. Finally, bidders place their bids and the winning bid is finalized.

4.1 Targeted couponing at both bidders

First, we analyze the case in which the seller targets a coupon of value c_L to the low-valuation bidder and c_H to the high-valuation bidder. The profit of the seller depends on the entry of different types of bidders and these entry probabilities are the same as those reported in Equations 1 to 4.

One might expect that since targeted couponing increases the entry probability of the targeted bidder, it may be optimal to issue coupons to both bidders under certain conditions. Interestingly this couponing strategy is never optimal as stated in Proposition 3 below.

PROPOSITION 3: *Simultaneously giving coupons to both low-valuation and high-valuation bidders is never optimal for the seller.*

To understand why it is not optimal to issue coupons to both bidders simultaneously in the interior region, note that a coupon issued to any bidder has two opposing effects on the benefits and costs of the coupon: (i) Effect on entry probability – it increases the entry probability of the recipient of the coupon but it reduces the entry probability of the other bidder (ii) Effect on value extraction – if the winner has a coupon then it has a weakly negative impact on profit otherwise it has a weakly positive impact. Issuing coupons to both bidders provides lower marginal benefit from changes in entry probability because the coupons work at cross-purposes. In addition, there is no improvement in value extraction from issuing coupons to both bidders. However the cost of couponing increases sharply as the winning bidder always cashes his coupon. Thus the net profit impact is negative and it is not optimal to issue coupons to both bidders.

Now we describe why it is not optimal to issue coupons to both bidders in the boundary region. If the boundary involves $p_L = 0,\ p_H = 1$ then the seller should not give a coupon to the high-valuation bidder as her entry probability is already maximized. Similarly, the seller should not give a coupon to the low-valuation bidder as well as to the high-valuation bidder when $p_L = 1$.

4.2 Targeted couponing to high-valuation bidder

Now we examine the seller's profit from targeting only the high-valuation bidder. We will then compare this profit to the profit in the absence of couponing to determine the parameter space in which targeted couponing to high-valuation bidder may be profit enhancing. Recall from Section 2.1 that couponing impacts seller's profit through two avenues: (i) it serves to influence the entry probabilities of the two bidders such that joint entry probability can be increased (ii) it allows the seller to extract more revenue from the high-valuation bidder by incentivizing the low-valuation bidder to bid a higher amount. When the seller targets only the high-valuation bidder, then she can use couponing to increase the entry probability of the high-valuation bidder but cannot increase value extraction. This is because, the high-valuation bidder wins the auction whenever he enters. The coupon amount is deducted from the winning bid so the revenue to the seller is $p_L * p_H(v_L - c_H) + (1 - p_L)(p_H)(-c_H)$. Therefore, the coupon does not improve value extraction but carries a cost. Proposition 4 states the solution in the interior region when the seller considers targeting only the high-valuation bidder.

PROPOSITION 4: (Interior Region) *When the seller targets an optimal coupon only to the high-valuation bidder, the equilibrium entry probabilities of the bidders in the interior region are:* $p_L = \frac{v_L(2 - v_H)}{2\kappa}$ *and* $p_H = v_H / 2$ *when* $2 - \frac{2\kappa}{v_L} < v_H < \frac{2v_L^2}{\kappa + v_L^2}$. *The optimal coupon value is* $c_H^* = \frac{2v_L^2 - v_H(\kappa + v_L^2)}{2\kappa}$ *and the expected profit of the seller is* $\pi_H^* = v_H^2 / 4$.

Proposition 4 shows that if the seller wants to follow a strategy of targeting only the high-valuation bidder, then it issues a coupon of strictly positive value when the valuation of the high-valuation bidder is moderate. When the seller targets a coupon to the high-valuation bidder, the entry probabilities of both bidders are affected. Note that the entry probability for the high-valuation bidder is no longer dependent on any characteristic of the low-valuation bidder such as, the valuation (v_L), and the relative participation cost (κ). From observation it is easy to see that p_H increases with v_H and is independent of v_L. p_L increases with v_L and decreases with v_H and κ. It is interesting to note that in Proposition 4 the seller's profit depends only on the valuation of the high-valuation bidder.

PROPOSITION 5: **(Boundary Region)** *(a) When the seller targets an optimal coupon only to the high-valuation bidder, the equilibrium entry probabilities of the bidders on the boundary are:* $p_L = 1$ *and* $p_H = v_H / 2$ *when* $v_H \le 2 - \frac{2\kappa}{v_L}$ *and* $v_H < 2v_L$. *The optimal coupon value is* $c_H^* = v_L - \left({v_H}/{2} \right)$ *and the expected profit of the seller is* $\pi_H^* = v_H^2 / 4$. *(b) The seller does not target the high-valuation bidder when (i)* $v_H \le 2 - \frac{2\kappa}{v_L}$ *and* $v_H \ge 2v_L$ *or (ii)* $v_H > 2 - \frac{2\kappa}{v_L}$ *and* $v_H \ge \frac{2v_L^2}{\kappa + v_L^2}$.

When the valuation of the high-valuation bidder is relatively small then the low-valuation bidder always enters the auction. The seller issues a coupon to the high-valuation bidder to increase his entry probability, however in this region, the low-valuation bidder always enters the auction ($p_L = 1$). It is interesting that when the seller issues an optimal coupon, the high-valuation bidder's entry probability is the same in the interior region as well as in the boundary region.

One interesting observation from Propositions 4 and 5 is that when high-valuation bidder is targeted, the optimal profit in the interior region and boundary region depends only on the valuation of the high-type bidder. The economic intuition for this result is as follows. The objective of targeting the high-valuation bidder is to optimize joint entry probability of both bidders, specifically by increasing the entry probability of the high-valuation bidder. When the valuation of the low-valuation bidder increases, it causes two separate impacts on the firm's profit, and these two impacts perfectly offset each other. First, increasing valuation of the low-type bidder increases profit because the winning bid is higher. Secondly, the increase in valuation of the low-type bidder reduces the entry probability of the high-valuation bidder, thus requiring a larger coupon which increases the cost of couponing. Thus these two effects offset each other and there is no net change in the seller's profit due to changes in the valuation of low-valuation bidder. Similarly, any increase in the relative participation cost of the low-valuation bidder leads to reduction in the joint entry probability because the entry probability of the low-valuation bidder is smaller. This increases the high-valuation bidder's entry probability. Thus the coupon amount to the high-valuation bidder decreases. The reduction in joint entry probability leads to lower revenue whereas the reduction in the cost of couponing leads to higher profit. These two effects cancel each other and there is no net impact of κ on the seller's profit.

The regions for interior and boundary solutions stated in Propositions 4 and 5 are shown in Figure 3 below. Please note that we refer to the interior region as "Uncertain Entry of L and H" and the boundary region as "L Always Enters, H Uncertain Entry" in Figure 3. It is optimal to target a coupon at the high-valuation bidder when the valuation of the low-valuation bidder is relatively large or the valuation of the high-valuation bidder is relatively small or when the relative participation cost of the low-valuation bidder is relatively small.

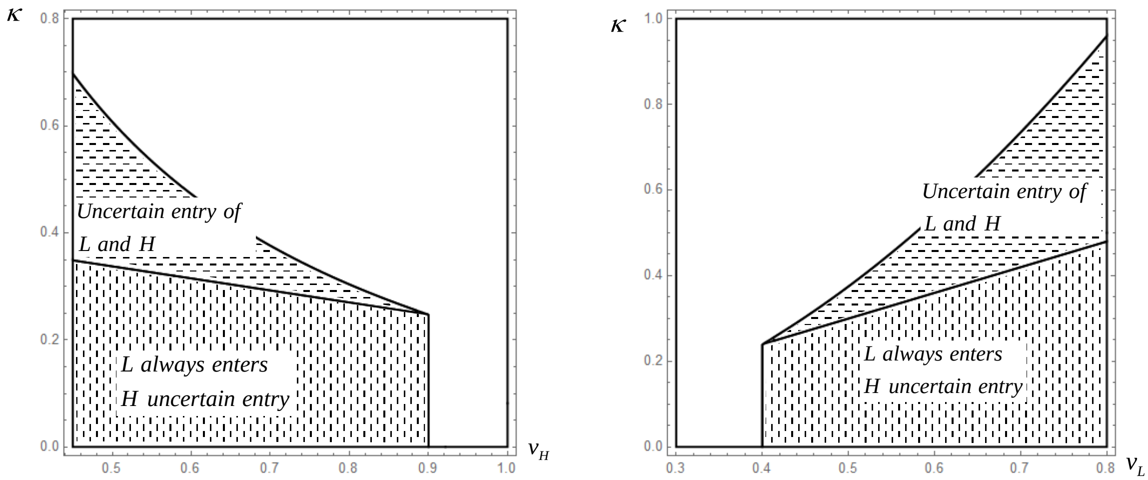


Figure 3: Target High-Valuation Bidder Case: (a) valuation of high-valuation bidder and low-valuation bidder's relative participation cost $(v_L = 0.45)$ (b) valuation of low-valuation bidder and low-valuation bidder's relative participation cost $(v_H = 0.8)$.

Now, we examine the impact of changes in bidder characteristics on the optimal coupon amount and the seller's profit in Proposition 6 below.

PROPOSITION 6: ***Comparative Statics of profit and coupon amount when high-valuation bidder is targeted***. *When the seller considers targeting the high-valuation* bidder *such that conditions in Proposition 3 are satisfied, (i) The optimal coupon amount increases with v_L and decreases with v_H and κ. (ii) Seller's profit is increasing with valuation of high-valuation bidder v_H and is independent of v_L and κ.*

To understand part (i) of Proposition 6, note that a coupon targeted at the high-valuation bidder serves to increase his entry probability and has no role in value extraction. When v_H is low or v_L is high or κ is low, the low-valuation bidder enters more often and that discourages the high-valuation bidder from entering the auction such that joint entry probability is lower than the optimal level. By issuing a coupon to the high-valuation bidder, the seller provides an incentive to the high-valuation bidder to enter more often, thus increasing his entry probability. As v_H decreases or v_L increases or κ decreases, the seller needs to provide stronger incentives and hence the optimal coupon amount increases (Figure 4b). Similarly, increasing v_H increases the entry probability of the high-valuation bidder reducing the need for a coupon, thus leading to a lower optimal coupon amount (Figure 4b). The expected profit of the seller increases as the valuation of the high-valuation bidder increases (Figure 4a).

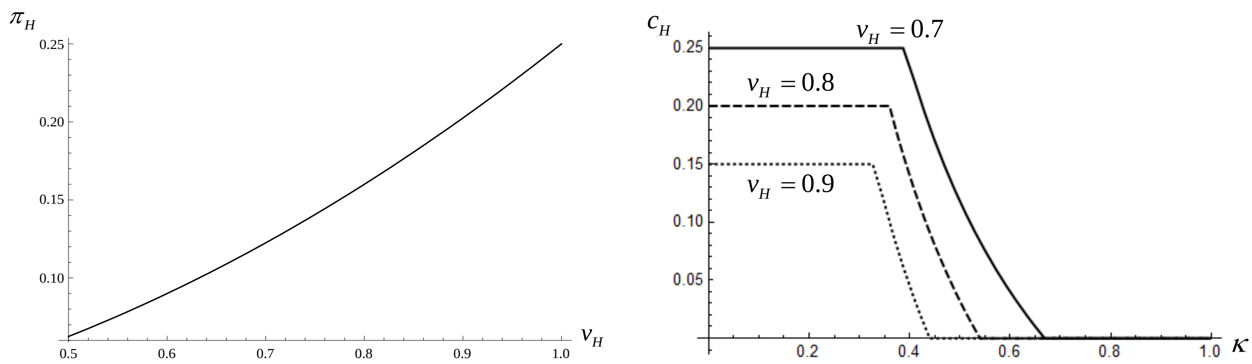


Figure 4: Target High-Valuation Bidder Case: (a) Impact of high-valuation bidder's valuation on profit $(\kappa = 0.2,\ v_L = 0.5)$ **and (b) Impact of low-valuation bidder's relative participation cost on the optimal coupon** $(v_L = 0.6)$**.**

Recall from Proposition 1 (ii) that the seller's profit is a non-monotone function of v_H, increasing with v_H when v_H is small and then decreasing with v_H. It is interesting to contrast this result with Proposition 4 (ii) where profit is monotonically increasing function of v_H. However, this result can be understood by examining the regions under which these results hold. When the seller's profit is decreasing with v_H, it is not optimal for the seller to target the high-valuation bidder. The region in which the seller targets the high-valuation bidder is a subset of the region where the seller's profit is increasing with v_H. The reason why the seller stops targeting the high-valuation bidder even when profit is increasing with v_H is that couponing carries a cost in terms of potential loss when only the high-valuation bidder enters the auction.

4.3 Targeted couponing to low-valuation bidder

In this subsection we examine the seller's strategy where she targets a coupon at the low-valuation bidder only. Recall that when the seller targets only the high-valuation bidder, then she can use couponing to increase the entry probability of the high-valuation bidder but cannot increase value extraction. In contrast when the seller targets the low-valuation bidder, she can use couponing to increase the entry probability of the low-valuation bidder and also enhance value extraction from the high-valuation bidder. In this case, the revenue to the seller is $(p_L * p_H)(v_L + c_L) + (p_L)(1 - p_H)(-c_L)$. The first term highlights the value extraction role of c_L when both bidders enter the auction. The first term also indicates the influence of joint entry probability $(p_L * p_H)$ which also serves to increase revenue. Targeting the low-valuation bidder can help the seller adjust the entry probabilities so as to increase the joint entry probability. Finally, the second term $(p_L)(1 - p_H)(-c_L)$ shows the cost of couponing when only the low-valuation bidder enters the auction. Proposition 7 states the solution in the interior region when the seller considers targeting only the low-valuation bidder.

PROPOSITION 7: **(Interior Region)** *When the seller targets an optimal coupon only to the low-valuation bidder, the equilibrium entry probabilities of the bidders in the interior region are:* $p_L = \frac{(1-v_H)(c_L^* + v_L)}{\kappa - (c_L^* + v_L)^2}$, $p_H = 1 - \frac{\kappa(1-v_H)}{\kappa - (c_L^* + v_L)^2}$ *when the high-valuation bidder's valuation is relatively large such that* $v_H > Max\left\{\frac{\kappa + 3v_L^2}{3\kappa + v_L^2},\ 1 + v_L + c_L^* - \frac{\kappa}{v_L + c_L^*}\right\}$. *The optimal coupon value is* $c_L^* = \frac{2^{1/3}(b^2 - 3ac)}{3ae} + \frac{e}{32^{1/3}a} - \frac{b}{3a}$ *where* $a = -2(3 - 2v_H)$, $b = -3(5 - 3v_H)v_L$, $c = 2(2\kappa v_H - \kappa) - 6(2 - v_H)v_L^2$, $d = v_L(v_H(3\kappa + v_L^2) - \kappa - 3v_L^2)$, *and* $e = \left(9abc - 2b^3 - 27a^2 d + \sqrt{4(3ac - b^2)^3 + (9abc - 2b^3 - 27a^2 d)^2}\right)^{1/3}$. *The expected profit of the seller is* $\pi_L^* = \frac{(1-v_H)(c_L^* + v_L)(kv_H v_L + c_L^*(2kv_H - k - 3v_L^2) - c_L^{*3} - 3c_L^{*2} v_L - v_L^3)}{((c_L^* + v_L)^2 - \kappa)^2}$.

The seller considers targeting the low-valuation bidder in the interior only when he does not enter often enough. The low-valuation bidder enters less often when his relative participation cost (κ) is high enough – the condition stated in Proposition 7 can be rewritten as $\left(\kappa > \frac{v_L^2(3 - v_H)}{3v_H - 1}\right)$ and, the high-valuation bidder enters more often when his valuation is high enough ($v_H > \frac{1 + 3v_L^2}{3 + v_L^2}$) as reported in Proposition 7.

Proposition 7 shows that if the seller follows a strategy of targeting only the low-valuation bidder, then it issues a coupon of strictly positive value when the valuation of the high-valuation bidder is relatively high. When the seller targets a coupon to the low-valuation bidder, the entry probabilities of both bidders are affected. From observation it is easy to see that p_H decreases with c_L^* whereas p_L increases with c_L^*.

PROPOSITION 8: **(Boundary Region)** *The strategy of targeting the low-valuation bidder in the boundary is optimal when the high-valuation bidder's valuation is relatively large such that (i) When* $\frac{1}{2}(1+3v_L) < v_H \le 1+v_L+c_{LB1}^* - \frac{\kappa}{v_L+c_{LB1}^*}$, *the equilibrium entry probabilities of the bidders on the boundary are:* $p_L = 1$ *and* $p_H = 1-\frac{\kappa(1-v_H)}{\kappa-(c_{LB1}^*+v_L)^2}$. *The optimal coupon value at the boundary is* $c_{LB1}^* = \frac{1}{4}(-1+2v_H-3v_L)$ *and the expected profit of the seller is* $\pi_{LB1}^* = (v_H - v_L)v_L + \frac{1}{8}(1-2v_H+3v_L)^2$. *(ii) When,* $v_H \le$ *and* $1+c_{LB1}^*+v_L-\frac{\kappa}{c_{LB1}^*+v_L} < v_H \le 1+c_L^*+v_L-\frac{\kappa}{c_L^*+v_L}$ *, the equilibrium entry probabilities of the bidders on the boundary are:* $p_L = 1$ *and* $p_H = 1-\frac{\kappa(1-v_H)}{\kappa-(c_{LB2}^*+v_L)^2}$. *The optimal coupon value at the boundary is* $c_{LB2}^* = \frac{1}{2}(v_H - 1 + \sqrt{1+4\kappa+(v_H-2)v_H} - 2v_L)$ *and the expected profit of the seller is* $\pi_{LB2}^* = \frac{1}{2}(\sqrt{1+4\kappa+(-2+v_H)v_H} - 4\kappa - (1-v_H)(1-v_L) + \sqrt{1+4\kappa-(2-v_H)v_H}\,v_L)$.

The difference between the two boundary solutions (boundary 1 and boundary 2), stems from the formulation used to derive the optimal value of the coupon in these regions, and is therefore, technical. These technical differences are discussed in proof of Proposition 8, Appendix B. It is interesting to note in Proposition 8 that the seller sometimes finds it optimal to issue a coupon to the low-valuation bidder even when he always enters the auction. This occurs because the seller is motivated to target the low-valuation bidder for two reasons: (i) to optimize the joint entry probability and (ii) to improve revenue by increasing value extraction from the high-valuation bidder. When both bidders enter the auction, the high-valuation bidder has to pay a larger amount when the low-valuation bidder has a coupon. We refer to this as the value extraction role of couponing. It can also be seen that the optimal coupon amount in the boundary region is decreasing with the valuation of the low-valuation bidder and it is increasing with the valuation of the high-valuation bidder.

The regions for interior and boundary solutions stated in Propositions 7 and 8 are shown in Figure 5. Please note that we refer to the interior region as "Uncertain Entry of L and H" and the two boundary regions as "L Always Enters, H Uncertain Entry 1" and "L Always Enters, H Uncertain Entry 2" in Figures 5 and 6. We can see that the region for targeting the low-valuation bidder is larger when v_H is large and κ is relatively large or v_L is small and κ is relatively large. The boundary region where the low-valuation bidder always enters the auction $(p_L = 1)$ occurs when κ is small relative to v_H, or v_L is small. Note that the coupon amount in the boundary region decreases with v_L as stated in Proposition 8.

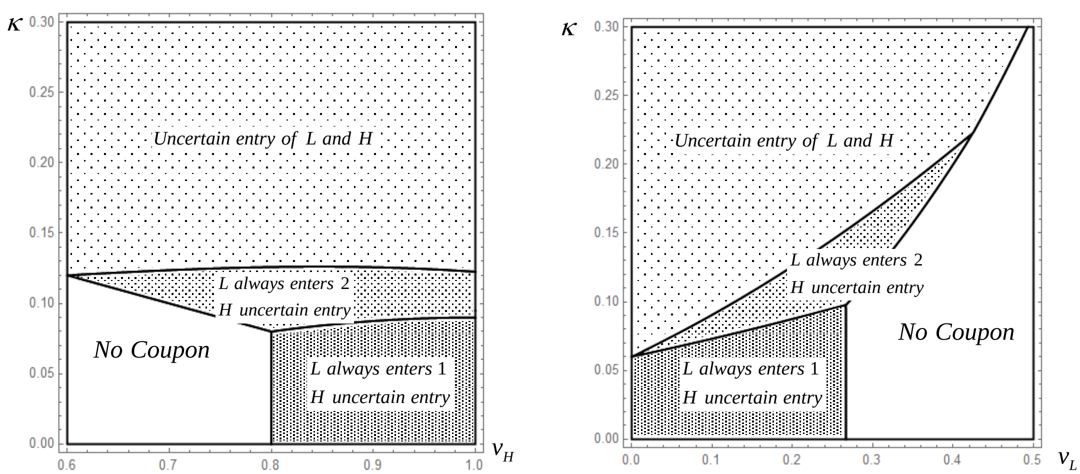


Figure 5: Target Low-Valuation Bidder Case: (a) valuation of high-valuation bidder and low-valuation bidder's relative participation cost $(v_L = 0.2)$ (b) low-valuation bidder's valuation and relative participation cost $(v_H = 0.9)$.

Figure 6(a) shows the seller's profit as a function of v_L. We can see the medium dashed curve in the interior region and the short dashed curve and long dashed curve in the boundary regions are both increasing with v_L. In these regions, it is optimal to target the low-valuation bidder. The very long dashed curve, shows where it is not optimal to issue any coupons. In this region, the optimal profit can be a non-monotone function of the valuation of the low-valuation bidder as discussed in Section 3. Increasing v_H, shifts the profit curves upwards as is shown in Figure 6(a).

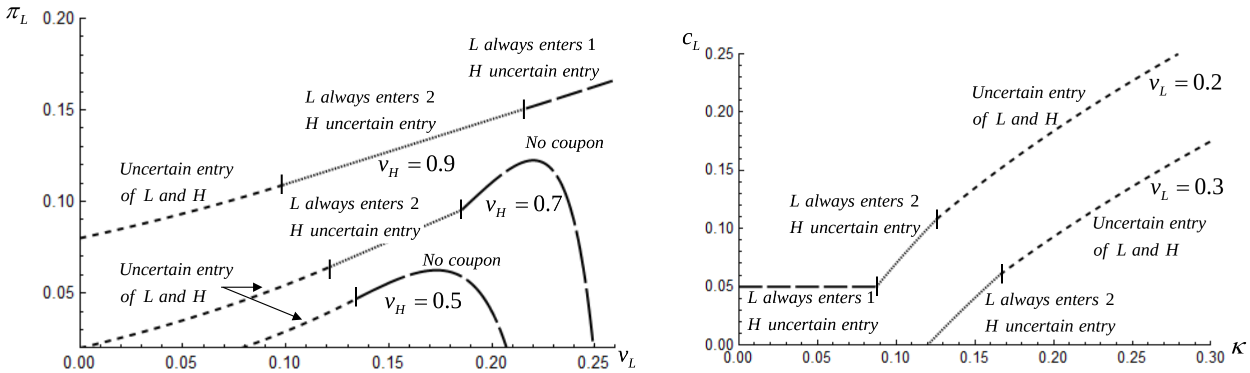


Figure 6: Target Low-Valuation Bidder Case: (a) Impact of low-valuation bidder's valuation on profit $(\kappa = 0.09)$ and (b) Impact of low-valuation bidder's relative participation cost on the optimal coupon amount $(v_H = 0.9)$.

From Figure 6(b), it is easy to see that the optimal coupon amount to the low-valuation bidder is increasing with κ. Note that in the case of issuing a coupon to the high-valuation bidder, the coupon amount decreases with κ (Figure 4(b)). This is so because as κ increases, the low-valuation bidder enters less often and therefore requires a larger coupon amount to induce more entry from the low-valuation bidder which increases the joint entry probability.

In the next subsection, we examine the overall optimal couponing strategy of the seller.

4.4 Optimal strategy for targeted couponing

Now we determine the seller's overall coupon targeting strategy by comparing the profit and feasibility conditions derived in Sections 3, 4.2 and 4.3. The feasible region for targeting only the high-valuation bidder is given in Propositions 4 and 5 and the feasible region for targeting only the low-valuation bidder is given in Propositions 7 and 8.

PROPOSITION 9: ***Optimal Strategy for Targeted Couponing:***

(i) the seller targets the high-valuation bidder when the high-valuation bidder's valuation is relatively small such that (a) $2-\dfrac{2\kappa}{v_L}<v_H<\dfrac{2v_L^2}{\kappa+v_L^2}$ or *(b)* $v_H \le 2-\dfrac{2\kappa}{v_L}$ *and* $v_H<2v_L$ *. The optimal value of the coupon for the interior region is stated in Proposition 4 and for the boundary region in Proposition 5;*

(ii) the seller targets the low-valuation bidder when the high-valuation bidder's valuation is relatively large such that (a) $v_H > Max\left\{\dfrac{\kappa+3v_L^2}{3\kappa+v_L^2},\ 1+v_L+c_L^*-\dfrac{\kappa}{v_L+c_L^*}\right\}$ *or (b)* $\dfrac{1}{2}(1+3v_L)<v_H \le 1+v_L+c_{LB1}^*-\dfrac{\kappa}{v_L+c_{LB1}^*}$ *or (c)* $v_H>1-\dfrac{\kappa}{v_L}+v_L$, $v_H \le 1+c_L^*+v_L-\dfrac{\kappa}{c_L^*+v_L}$ *and* $v_H>1+c_{LB1}^*+v_L-\dfrac{\kappa}{c_{LB1}^*+v_L}$*. The optimal value of the coupon for the interior region is stated in Proposition 7 and for the boundary region in Proposition 8;*

(iii) *The region in which the firm targets the low-valuation bidder does not overlap with the region in which the firm targets the high-valuation bidder.*

Why is it optimal to target the high-valuation bidder when his valuation is low to moderate relative to the valuation of the low-valuation bidder and κ whereas it is optimal to target the low-valuation bidder when the valuation of the high-valuation bidder is large relative to the valuation of the low-valuation bidder and κ. The intuition behind this result can be understood through the interplay of the two effects of couponing in our setup, namely, (i) Targeted couponing impacts the entry probability of the targeted as well as non-targeted bidder to increase the joint entry probability (ii) Targeted couponing can be used to extract value from the high-valuation bidder.

The impact of optimal couponing strategy is shown in Figure 7 which illustrates the joint entry probability with κ. We refer to the interior region where the seller targets the coupon to the high-valuation bidder and the low-valuation bidder as "Target H: Uncertain Entry of L and H" and "Target L: Uncertain Entry of L and H" respectively. The boundary region where the low-valuation bidder always enters and the seller targets the high-valuation bidder is labelled as "Target H: L Always Enters, H Uncertain Entry" in Figure 7. The long dotted portion of the curve (on the left) shows the joint entry probability when the seller targets the high-valuation bidder with an optimal coupon, long dotted portion of the curve (top right) shows the joint entry probability when the seller targets the low-valuation bidder. Finally the solid curve shows the joint entry probability when the seller does not target either bidder.

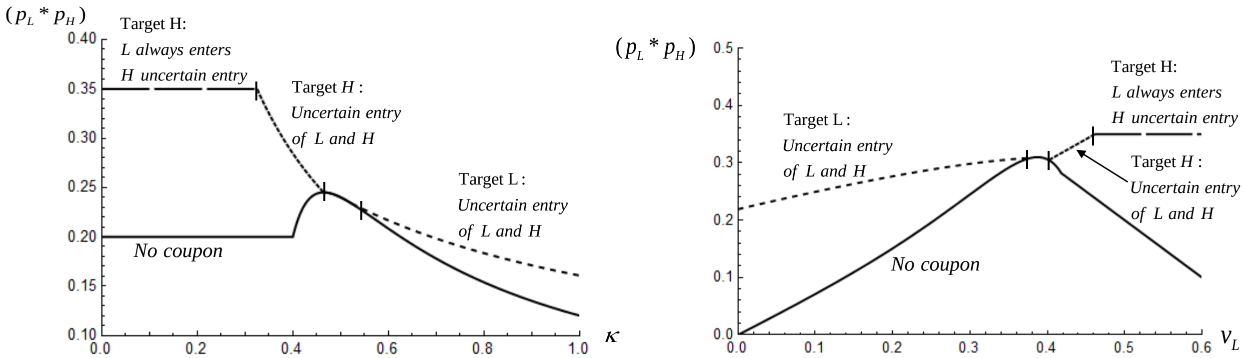


Figure 7: Joint entry probability as a function of (a) the low-valuation bidder's relative participation cost $(v_H = 0.7,\ v_L = 0.5)$ and (b) the low-valuation bidder's valuation $(\kappa = 0.3,\ v_H = 0.7)$.

We now describe the impact of couponing on entry probabilities. The entry probability of each bidder depends on their own valuation and participation cost and also on the valuation and participation cost of the other bidder. The entry probability of the bidder who receives the coupon (targeted bidder) increases because his expected surplus from entering the auction increases. At the same time the entry probability of the bidder without coupon (non-targeted bidder) decreases because his expected surplus is a decreasing function of the entry probability of the targeted bidder. This is because entry of the targeted bidder can reduce both the probability of winning and the surplus when non-targeted bidder wins. Since the entry probabilities of the bidders in our setup are interdependent, targeted couponing impacts the targeted as well as the non-targeted bidder. The seller obtains positive profit only when both bidders enter the auction. Therefore the joint entry probability of the two bidders is key to maximizing the seller's profit. Thus profit can be improved by adjusting the entry probability of the two bidders through the use of targeted couponing. When the entry probabilities are such that one bidder enters much more often than the other, then targeted couponing can be used to increase the entry probability of the bidder who enters less often.

The second effect of couponing is value extraction. When both bidders enter the auction, the high-valuation bidder is forced to bid higher if the low-valuation bidder has a coupon. Thus the seller may obtain an additional benefit from targeting the low-valuation bidder. The value extraction benefit is moderated by the impact of couponing on entry probabilities because targeting the low-valuation bidder will lead to reduced entry by the high-valuation bidder which can lead to reduction in profit. Note that there is no value extraction benefit when the coupon is targeted to the high-valuation bidder.

The optimal couponing strategy is determined by balancing these two effects of couponing with the potential revenue loss from couponing. This revenue loss from couponing is incurred when a bidder who has a coupon is the only one who enters the auction or he wins the auction (if high-valuation bidder is targeted). When this happens the bidder is able to cash his coupon without any gain to the seller. Figure 8 shows the regions in which it is optimal to target the coupon either to the low- or the high-valuation bidder within the parameter space formed by v_L, v_H, κ. Please note that we refer to the interior region where the seller targets the coupon to the high-valuation bidder as "Target H: Uncertain Entry of L and H" and where she targets the low-valuation bidder as "Target L: Uncertain Entry of L and H". The boundary regions where the low-valuation bidder always enters and the seller targets the high-valuation bidder is labelled as "Target H: L Always Enters, H Uncertain Entry" in Figures 8 and 9. Similarly, the two boundary regions where the seller targets the low-valuation bidder and the low-valuation bidder always enters are labelled as "Target L: L Always Enters, H Uncertain Entry 1" and "Target L: L Always Enters, H Uncertain Entry 2".

These regions can be explained by noting that when κ is relatively small or v_L is relatively large, the entry probability of the low-valuation bidder is high and this decreases the entry probability of the high-valuation bidder. In order to maximize profit, the seller seeks to optimize the joint entry probability by targeting the high-valuation bidder. When κ is relatively large or v_L is relatively small then the entry probability of the low-valuation bidder is low. The seller seeks to optimize the joint entry probability by targeting the low-valuation bidder. When the relative participation cost for the low-valuation bidder (κ) is relatively moderate then the bidders entry probabilities are such that the gain to the seller from couponing is not sufficient to overcome the potential revenue loss from the coupon. Hence when κ is relatively moderate, the seller prefers not to issue any coupons. As noted in Proposition 9, part (iii), the regions where couponing is optimal (Figure 8) do not overlap. However in a model with more than two bidders, it may be optimal to issue coupons to more than one bidder at a time.

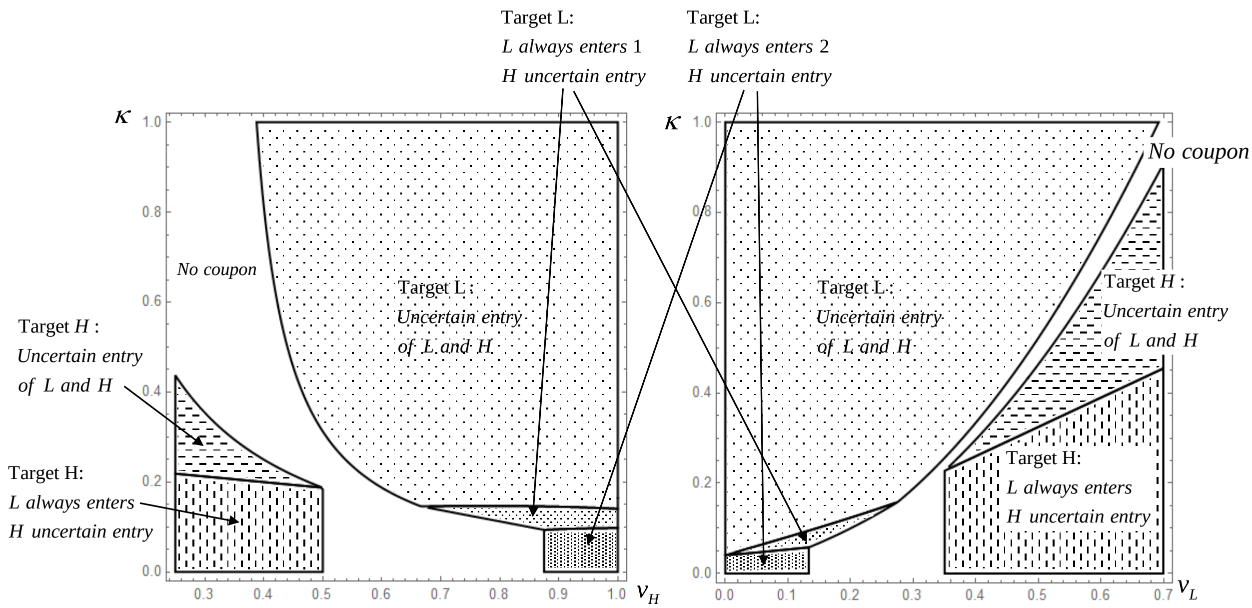


Figure 8: Interior and boundary regions for targeting low- and high-valuation bidder with (a) valuation of high-valuation bidder and low-valuation bidder's relative participation cost $(v_L = 0.25)$ (b) valuation of low-valuation bidder and low-valuation bidder's relative participation cost $(v_H = 0.7)$.

Figure 9(a) shows how the optimal coupon value changes with the relative participation cost of the low-valuation bidder. When the relative participation cost of the low-valuation bidder is low then the firm targets the high-valuation bidder, when it is moderate, the firm does not issue coupons and when it is high, the firm targets the low-valuation bidder. Figure 9(b) shows the expected profit of the firm with the valuation of the low-valuation bidder under optimal couponing strategy. When the valuation of the low-valuation bidder is low then the firm targets the low-valuation bidder, when it is moderate, the firm does not issue coupons and when it is high, the firm targets the high-valuation bidder.

Recall from Proposition 2 that when the seller does not use couponing, the joint entry probability of the two bidders as well as the seller's profit is a non-monotone function of the bidders' valuations and the participation cost parameter of the low-valuation bidder. It is interesting to note in Figure 9b that when the seller issues coupons, the seller's profit becomes a monotone function of the valuation of the low-valuation bidder. We have verified that this observation also holds true with respect to the valuation of the high-valuation bidder and the relative participation cost of the low-valuation bidder (κ). Thus we find that when couponing is introduced, the non-monotonicity result is no longer present. This occurs because couponing allows the seller to internalize the externality caused by the entry of one bidder on other bidders.

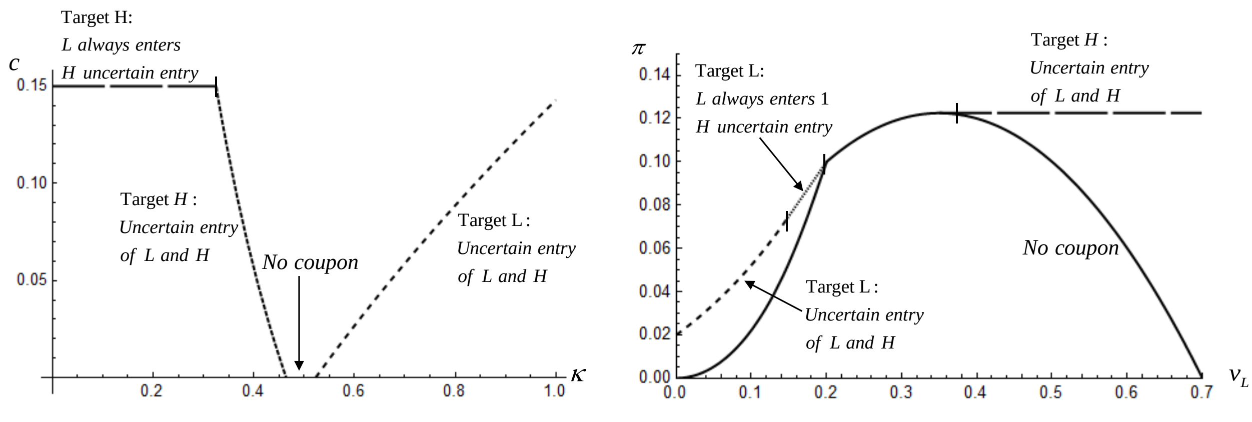


Figure 9: (a) Impact of low-valuation bidder's relative participation cost on the optimal coupon amount to low- and high-valuation bidder $(v_H = 0.7,\ v_L = 0.5)$ and (b) Impact of low-valuation bidder's valuation on the profit from optimal targeting $(\kappa = 0.1,\ v_H = 0.7)$

5. Discussion

In this paper, we analyze the impact of targeted coupons issued by a seller to bidders in an auction setting. First we provide an overview of our results and then discuss our contributions and highlight some managerial implications that follow from our analysis.

We find that under endogenous entry of bidders, the seller's revenue can be a non-monotone function of the valuations and participation cost of the bidders. The optimal couponing strategy of the seller depends on the relative values of the valuations of the bidders and the relative participation cost of the low-valuation bidder. We find that it is optimal for the firm to target the high-valuation bidder when his valuation is low, the relative participation cost of the low-valuation bidder is low and the valuation of the low-valuation bidder is relatively high. On the other hand, the firm should target the low-valuation bidder when the high-valuation bidder's valuation is moderate to high, and the relative participation cost of the low-valuation bidder is moderate to low. The seller earns a profit only when both bidders enter the auction, yet couponing may be optimal for the seller even when it leads to a reduction in the joint entry probability of the two bidders. We find that it is never optimal for the firm to simultaneously target both bidders with coupons. Finally, we also determine the optimal coupon values under each targeting strategy and find that the value of the low-valuation bidder's coupon increases when the relative participation cost of the low-valuation bidder increases.

We find novel results that are different from prior literature on couponing. In the context of online auctions it can be optimal for the seller to issue targeted coupons to the high-valuation bidder. In contrast, prior literature recommends issuing coupons to low-valuation consumers for price discrimination or to marginal consumers to poach them from competitors (Narasimhan, 1984; Levedahl, 1984; Sweeney, 1984; Varian, 1989; Shaffer & Zhang, 1995; Fudenberg and Tirole, 2000). Another counterintuitive result is that there exist conditions under which it can be optimal to target the low-valuation bidder even when he always enters the auction. This is also different from the literature where coupons are typically used to entice consumers who may not otherwise purchase.

These novel findings can be understood in the context of the two key forces driving couponing in auctions which differ from the key forces in couponing in the retail context. The two forces are (i) the value extraction role of coupons and (ii) the influence of coupons on the entry probability of the bidders. The value extraction role of coupons allows the seller to extract value from the winning bidder by making him pay a higher amount when the coupon increases the second highest bid amount. This dynamic does not exist in couponing literature in the retail context where there is no supply-side constraint and hence, consumers do not compete for a unit of the product. The second force of couponing in auctions influences bidders entry probability in two ways – there is a positive impact on the entry probability of the bidder who receives the coupon and a negative impact on the entry probability of other bidders. In contrast, in couponing in the retail context, there is only a positive impact on purchase probability of the recipient of the coupon.

Our analysis generates insights that have implications for online auctioneers and platforms when there is endogenous entry based on valuations and asymmetric participation costs. Participation costs include the cost of inspecting auction goods, time cost of monitoring and participating in auction, search cost and delay cost from waiting for the auction to close. The extent of impact of participation costs on different bidders is likely to be different. The high-valuation bidders are likely to incur greater participation costs due to their higher opportunity cost of time. These higher participation costs may result in lower entry probability for high-valuation bidders. Therefore managers who sell items that appeal to high-valuation buyers such as expensive artwork and real-estate may need to provide incentives to high-valuation buyers. For example, Sotheby's is a leading auction house for high-value collectable items and they offer value added services only to "Sotheby's Preferred" members where membership is by invitation. This lowers the participation cost for high-valuation bidders and its impact is similar to issuing coupons to high-valuation bidders in our model. Similarly Auction.com which auctions residential and commercial real-estate has a VIP program for high-valuation buyers. Their VIP program provides several benefits including dedicated accounts associate, dedicated closing team, waiver of deposit and earnest money, and streamlined process for post-auction activities.

Our findings related to the non-monotonicity of the joint entry probability of bidders have important implications for managers. The presence of a very high valuation buyer can make it unattractive for others to participate in the auction. Such concerns may be addressed by limiting the number of items that a single bidder can win in an auction. For example, the auctioneer may limit any single bidder to winning at most twenty percent of the auction inventory. This would reassure bidders with moderate or low valuation that they have a reasonable probability of winning an auction.

Managers must be careful in using coupons in auctions because they lead to externalities on bidders who do not have coupons. Sometimes, this property of auction coupons can be leveraged to optimize the joint entry probability of multiple bidders. We also find that it can be optimal to issue coupons to low-valuation bidders even when they always enter the auction. Online automobile auction firms such as Copart.com and CarMax.com should consider offering coupons to low-valuation bidders who regularly attend their auctions but rarely win the auction. Such a strategy is beneficial because of the value extraction benefit to the seller – the low valuation bidder who receives a coupon can cause an increase in the amount realized from the bidder who wins the auction.

Targeted couponing strategy can inform managers at eBay, which is one of the largest online auction platforms. It runs a wide variety of auctions every day ranging from high-value industrial items to low-value consumer items. The auction platform used by eBay is the same across all these items. However bidder valuations and participation costs are likely to be very different in different categories of auctions. eBay can actively reduce the participation costs faced by high-value bidders by providing value-added services related to search and authentication to high-valuation bidders. They can also monitor the items listed in the high-value categories such as “Business and Industrial” to prevent low-value items being listed there. eBay could also partner with targeted couponing firms such as Aucser.com, Criteo, and Highco to make it easier for sellers to issue targeted coupons. High-valuation bidders may face large participation costs when they are either new to the auction site and thus need to invest time and effort to learn the interface and the rules for auctions or when the auction duration is relatively long. We recommend that when high-valuation bidders face relatively large participation cost, managers should provide coupons targeted at them.

6. Conclusion and Limitations

This paper analyzes the role of targeted couponing in online auctions. We find that the role of couponing in auctions is substantially different from other retail settings. Our stylized model assumes a private-value auction. How would the results change in the context of common-value auctions? If the bidder valuations were to be common value, then the impact of couponing will be different on the bidding behavior and the entry probability of bidders. In common-value auctions, the bidder without coupon may bid higher because the bidder with coupon bids higher and this may impact the entry probabilities of the bidders. One of the limitations of our model is that it features two bidders. Future research can study the conditions under which these forces generalize to multiple bidder auctions. There is some empirical evidence that the key forces discussed in our model may remain relevant even when the number of bidders is large (Wang et al. 2008). Moreover, we have also assumed that bidder valuations are common knowledge and this research can be extended by allowing bidder valuations to be private information.

We studied a model of a single seller, however coupons could be used to attract bidders and obtain competitive advantage over other sellers in a setting with competing sellers. Another extension could be the analysis of couponing by an auction platform owner. For example, eBay has an “eBay Bucks” program that awards a 2 percent cash-back to auction winners, which could be studied. Future research can also study the impact of relaxing our assumption of zero reserve price and the role of coupons in multi-unit auctions.

References

Ariely, D., & Simonson, I. (2003). Buying, bidding, plying, or competing? Value assessment and decision dynamics in online auctions. *Journal of Consumer Psychology*, 13:1–2, 38–53.

Bapna, R., Goes, P., & Gupta, A. (2003). Analysis and design of business-to-consumer online auctions. *Management Science*, 49:1 (January), 85–101.

Bapna, R., Goes, P., Gupta, A., & Karuga, G. (2008). Predicting bidders' willingness to pay in online multiunit ascending auctions: Analytical and empirical insights. *INFORMS Journal on Computing*, 20(3), 345-355.

Bapna, R., Goes, P., Gupta, A., & Yiwei, J. (2004). User heterogeneity and its impact on electronic auction market design: An empirical exploration. *Management Information Systems Quarterly,* 28:1, 21–43.

Bapna, R., Seokjoo, A. C., Goes, P., & Gupta, A. (2009). Overlapping online auctions: Empirical characterization of bidder strategies and auction prices. *Management Information Systems Quarterly,* 10:4, 763–783.

Bester, H., & Petrakis, E. (1996). Coupons and oligopolistic price discrimination. *International Journal of Industrial Organization*, 14, 227–242.

Budish, E. B., & Takeyama, L. N. (2001). Buy prices in online auctions: irrationality on the internet? *Economics letters*, 72(3), 325-333.

Cao, X., & Tian, G. (2010). Equilibria in first price auctions with participation costs. *Games and Economic Behavior*, 69(2), 258-273.

Chakraborty, I., & Kosmopoulou, G. (2004). Auctions with shill bidding. *Economic Theory,* 24:2, 271–87.

Chen, Y., & Iyer, G. (2002). Consumer addressability and customized pricing. *Marketing Science,* 21:2, 197–208.

Choudhary, V., Ghose, A., Mukhopadhyay, T., & Rajan, U. (2005). Personalized pricing and quality differentiation. *Management Science,* 51:7, 1120–1130.

Etzion, H., Pinker, E., & Seidmann, A. (2006). Analyzing the simultaneous use of auctions and posted prices for online selling. *Manufacturing & Service Operations Management,* 8:1, 68–91.

Fong, Y., & Liu, Q. (2012). Using coupons to intertemporally price discriminate and tacitly collude. Forthcoming in *Journal of Economics and Management Strategy*.

Fundenberg, D., & Tirole, J. (2000). Customer poaching and brand switching. *RAND Journal of Economics,* 31, 634-657.

Gale, I. L., & Hausch, D. B. (1994). Bottom-fishing and declining prices in sequential auctions. *Games and Economic Behavior*, 7(3), 318-331.

Gallien, J. & Gupta, S. (2007). Temporary and permanent buyout prices in online auctions. *Management Science*, 53:5, 814–833.

Geldman, A. (2007). Statistics You Won't Find on eBay. http://www.webretailer.com/articles/ebay-statistics.asp#howmanybids. Accessed April 24, 2014.

Gerstner, E., Hess, J. D., & Holthausen, D.M. (1994). Price discrimination through a distribution channel: Theory and evidence. *The American Economic Review,* 84:5, 1437–45.

Goes, P. B., Karuga, G. G., & Tripathi, A. (2010). Understanding willingness-to-pay formation of repeat bidders in sequential online auctions. *Information Systems Research,* 21:4, 907–924.

Hou, J., & Rego, C. (2007). A classification of online bidders in a private value auction: Evidence from eBay. *International Journal of Electronic Marketing and Retailing,* 1:4, 322–338.

Kannan, K. N. (2010). Declining prices in sequential auctions with complete revelation of bids. *Economics Letters*. 108, 49–51.

Kannan, K. N. (2012). Effects of information revelation policies under cost uncertainty. *Information Systems Research,* 23:1, 75–92.

Klemperer, P. (1999). Auction theory: A guide to the literature. *Journal of Economic Surveys,* 13:3, 227–286.

Kuruzovich, J., Viswanathan, S., & Agarwal, R. (2010). Seller search and market outcomes in online auctions. *Information Systems Research,* 56:10, 1702–17.

Levin, D., & Smith, J. L. (1994). Equilibrium in auctions with entry. *The American Economic Review,* 84:3, 585–599.

Levedahl, W. (1984). Marketing, price discrimination, and welfare: Comment. *Southern Economic Journal,* 50, 886–91.

Liu, D., & Chen, J. (2006). Designing online auctions with performance information. *Decision Support Systems,* 42:3, 1307–1320.

Liu, D., Chen, J., & Whinston, A. B. (2010). Ex ante information and the design of keyword auctions. *Information Systems Research,* 21:1, 133–153.

Maskin, E. S., & Riley, J. G. (1985). Auction theory with private values. *The American Economic Review*, 75(2), 150-155.

McAfee, R. P., & McMillan, J. (1987). Auctions and bidding. *Journal of Economic Literature,* 25:2 (June), 699–738.

Milgrom, P., & Weber, R. J. (1982b). The value of information in a sealed-bid auction. *Journal of Mathematical Economics*, 10(1), 105-114.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of Operations Research,* 6, 58–73.

Narasimhan, C. (1982). Coupons as Price Discrimination Devices - A Theoretical Perspective and Empirical Analysis. Unpublished Ph.D. dissertation, Graduate School of Management, University of Rochester.

Narasimhan, C. (1984). A price discrimination theory of coupons. *Marketing Science,* 3, 128–147.

Ockenfels, A., & Roth, A. E. (2006). Late and multiple bidding in second price Internet auctions: Theory and evidence concerning different rules for ending an auction. *Games in Economic Behavior,* 55:2, 397–320.

Overby, E., & Jap, S. (2009). Electronic and physical market channels: A multiyear investigation in a market for products of uncertain quality. *Management Science,* 55:6, 940–957.

Rao, L. (2012). Confirmed: Google Acquires Incentive Targeting To Power Super Targeted, Personalized Coupon Programs. *TechCrunch.* http://techcrunch.com/2012/11/28/google-acquires-incentive-targeting-to-power-targeted-coupon-programs/ Accessed on August 30, 2013.

Riley, J. G., & Samuelson, W. F. (1981). Optimal auctions. *American Economic Review,* 71, 381–92.

Roth, A. E., & Ockenfels, A. (2002). Last minute bidding and the rules for ending second-price auctions: Evidence from eBay and Amazon auctions on the Internet. *American Economic Review,* 92:4, 1093–1103.

Shaffer, G., & Zhang, Z. J. (1995). Competitive coupon targeting. *Marketing Science*, 14, 395–415.

Shaffer, G., & Zhang, Z. J. (2002). Competitive one-to-one promotions. *Management Science,* 48, 1143–1160.

Shapiro, C., & Varian, H. (1999). *Information Rules.* Boston: HBS Press, 40–43.

Sweeney, G. (1984). Marketing, price discrimination, and welfare: Comment. *Southern Economic Journal,* 50, 892–99.

Van Heck, E., & Vervest, P. (1998). How should CIOs deal with web-based auctions? *Communication of ACM,* 41:7, 99–100.

Varian, H. (1989). Price discrimination. In *Handbook of Industrial Organization,* edited by R. Schmalensee and R. D. Willig. Amsterdam, The Netherlands: Elsevier Science Publishers B.V., 597–654.

Vickrey, W. (1961). Counter speculation, auctions, and competitive sealed tenders. *Journal of Finance,* 16, 8–37.

Wang, X., Montgomery, A., & Srinivasan, K. (2008). When auction meets fixed price: a theoretical and empirical examination of buy-it-now auctions. *Quantitative Marketing and Economics*, 6(4), 339-370.

Yao, S., & Mela, C. F. (2008). Online auction demand. *Marketing Science*, 27:5 (September–October), 861–885.

Zeithammer, R. (2006). Forward-looking bidding in online auctions. *Journal of Marketing Research,* 43:3, 462–76.

Appendix A: Summary of Notations

Notation	Definition	Notation	Definition
v_H	Valuation of high-valuation bidder	c_H	Coupon amount targeted to high-valuation bidder
v_L	Valuation of low-valuation bidder	c_H^*	Optimal coupon amount targeted to high-valuation bidder
p_H	Entry probability of high-valuation bidder	c_L	Coupon amount targeted to low-valuation bidder
p_L	Entry probability of low-valuation bidder	c_L^*	Optimal coupon amount targeted to low-valuation bidder
κ	Relative participation cost of low-valuation bidder	π_N^*	Expected profit of seller with no couponing
$\tilde{t}_H$	Draw of participation cost of high-valuation bidder	π_H^*	Expected profit of seller with optimal targeted couponing to high-valuation bidder
$\tilde{t}_L$	Draw of participation cost of low-valuation bidder	π_L^*	Expected profit of seller with optimal targeted couponing to low-valuation bidder
s_H	Expected surplus of high-valuation bidder	$J(p_L, p_H)$	Joint entry probability of both types of bidders
s_L	Expected surplus of low-valuation bidder	c_{LB2}^*	Optimal coupon targetd to low-valuation bidder in Boundary 2
c_{LB1}^*	Optimal coupon targetd to low-valuation bidder in Boundary 1		