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Title

Unit 43: Using Derivative Surface Operators

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UNIT 43: USING DERIVATIVE SURFACE OPERATORS

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Context

Surface models characterize attributes which are spatially continuous. Typically, exactly one attribute value is available for any location in space. One common surface variable is elevation. Other surface variables include mineral concentration, or human population density. In each of these examples, the height of the surface at any one location is the value of the variable at that location. This is a perspective view *[outdated link removed]* of a digital elevation model (DEM). This is a population density map *[outdated link removed]* of India.

In many cases, the variable alone is not of primary interest. Characteristics of the surface over local neighborhoods, known as surface derivatives, can be of more value. Finding answers to questions like, "how steep is the hillside?" or, "in which direction does the meadow face?" depend on deriving surface characteristics from the elevations, not just upon the elevations themselves. Scientists and engineers use surface derivatives to: describe and model the surface of the land

- study the flow of water and wind
- identify the habitats of plants and animals
- characterize the spread of pollutants
- determine optimal routes across the landscape
- identify potential sites for facility development

This unit identifies the fundamentals of surface derivatives that those familiar with GIS should know. It describes some methods by which they are calculated and measured, introduces ways in which they are used in GIS analysis, and identifies some challenges to using them.

Example Application

The Wyoming Mountain Stoat is a highly endangered species living in rugged, remote portions of the Rocky Mountains. Congress has allocated funding to identify and purchase tracts of land in which the remaining stoats live. A critical task for the Stoat Survey Group is to use a GIS to identify likely habitat locations prior to performing expensive field reconnaissance. Ecologists studying the stoat have determined several terrain-related factors to explain habitat location:

- **1. Stoats prefer altitudes between 2,500 and 3,000 meters.**
- **2. Stoat dens are always on south and west-facing slopes.**
- **3. Stoats can't live on slopes steeper than 35%, but they also suffer on slopes below 25%.**

The objective for the GIS-driven preliminary survey is to use these factors to identify potential stoat habitat in a region of northern New Mexico.

Learning Outcomes

Awareness:

Develop an understanding of what surface derivatives are, and what sorts of spatial data are appropriate for their use, and what applications require surface derivatives. Define in general terms what slope and aspect are.

Competency:

Calculate by hand the slope of a line segment, given the coordinates of its endpoints. Generate slope and aspect maps for a surface in a GIS.

Mastery:

Define the method by which slope is commonly calculated in a GIS. Execute a spatial analysis employing surface derivatives in a GIS. Identify on paper the steps that must be taken to accomplish the analysis and carry it out. Identify the meaning of "slope" for non-elevation surfaces, such as a population density map.

Preparatory Units

Recommended:

Unit 6 - Terrain Data

Complementary:

Unit 42 - Using Map Algebra

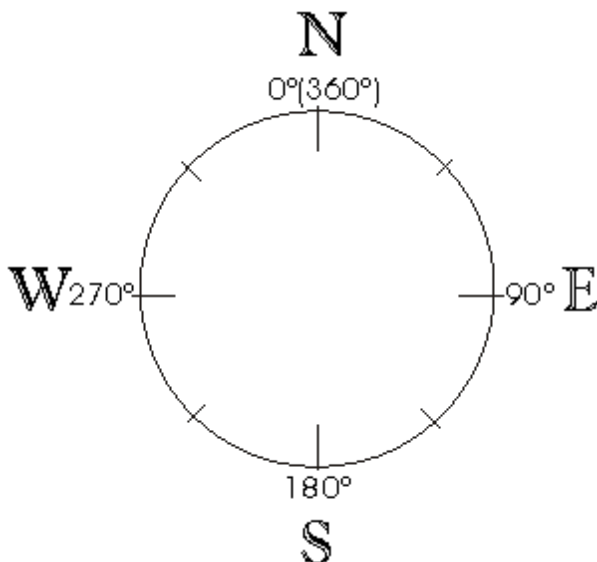
Awareness

Surfaces are usually modeled in a GIS using raster cells or triangular irregular networks (TINs). The derivatives discussed in this unit may be calculated on either model, but we will focus on rasters. The most commonly calculated surface derivatives are slope and aspect. All derivative measures, including slope and aspect, are only valid on a map using ground distance units like meters, feet, or miles. Slope estimates for data in coordinate systems using latitude and longitude are not interpretable.

Slope measures the steepness of the surface at any particular location. Slope is often measured in degrees or in percent rise. A flat region has zero slope. The steeper the surface, the higher the slope. Slope is an important variable for many analyses; here are just a few:

- drainage patterns on digital elevation models (DEMs)
- environmental modeling
- site selection
- cross-country movement models

Aspect measures the direction of steepest slope for a location on the surface. It is usually measured in degrees, where 0 degrees is due north, 90 degrees is due east, 180 degrees is due south, and 270 degrees is due west. The graphic below shows the directions. Note that zero and 360 degrees are equivalent.



Both slope and aspect are calculated using surrounding points. Usually these values are calculated for surface data in a raster grid, in which case the 3x3 set of neighbors are normally employed. The following diagram illustrates this.

A	B	C
D	E	F
G	H	I

Here, we are interested in calculating the slope and aspect for cell 'E'. The elevations of cell E's eight neighbors will be used to calculate these attributes. Usually cells B, D, F, and H will have more influence on the slope and aspect of cell E than the four diagonal cells.

Exercises to demonstrate awareness.

Roads which cross mountain ranges and hilly country often use passes. These passes are almost always lower than the ridgetops to either side. What was the road builders' reason for doing this, and why does it relate to this unit?

An airport is one example of a human-built feature that requires very low slopes. List three other such features.

Some human-built features require very steep slopes. Can you think of two?

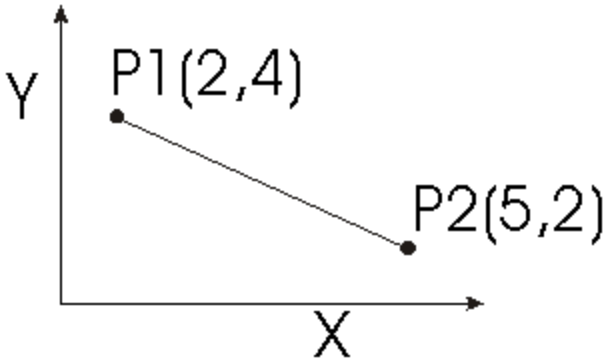
A southwest-facing hillside has an aspect of about how many degrees? How about the aspect of a southeast-facing hillside?

Why might aspect be an important factor if you were selecting a site for an array of solar energy collectors?

Why would the diagonal cells (A,C,G, and I in the diagram) be less influential than cell E's other neighbors in calculating slope?

Competency

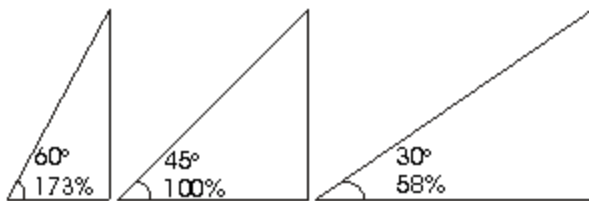
Recall the slope formula for a line segment: slope = rise/run. The diagram below will demonstrate this relationship more formally. Points P1 and P2 define a line segment. The coordinates for each point are given in the diagram.



If x_1 and y_1 refer to the x and y coordinates of point P1, and x_2 and y_2 refer to the coordinates of point P2, then slope can be calculated with the following formula:

$$\begin{aligned} \text{Slope} &= (y_2 - y_1) / (x_2 - x_1) \\ &= (2 - 4) / (5 - 2) \\ &= -2/3 \end{aligned}$$

Slope = -0.666, or negative 66 percent. Slope on surfaces is usually measured in degrees or percent rise, and the minus sign is dropped. The relationship between these two measures is a little complex.



The diagram above shows some common slopes and the relationship between the percent rise and the degrees of slope. They are related by the tangent, a trigonometric function, in the following manner: %-rise = $\tan(\text{°-rise})$. Degrees of slope can be calculated from percent slope, too: $\text{°-rise} = \arctan(\text{%-rise})$. As degrees slope approaches 90° , percent slope approaches infinity.

Generating and interpreting maps of aspect can be tricky, since the scale "wraps" around from 359 to 0 degrees. This makes establishing appropriate colors or shades more difficult than for

many surface maps.

Exercises to demonstrate competency.

Draw five line segments, identify (x,y) coordinates for the end points, and calculate the slope for each by hand or with the aid of a calculator. Calculate both the percent slope and degrees of slope.

Develop a slope and aspect map for a digital elevation model (DEM). This may be done in any raster-based GIS like Arc/Info's Grid module, Idrisi, or Grass.

Slope derivatives can be used to generate attractive hillshaded maps. Arc/Info's SAI or HILLSHADE command, for one, will produce a hillshaded terrain map. Use a GIS to make a hillshaded map of a DEM. Experiment with different sun-angles to identify attractive options.

Mastery

There is no single correct slope algorithm for a raster cell surface. Many different algorithms exist; they will each produce different slope maps. The method used in Arc/Info's Grid module is only one version; it follows Burrough (1986). Their method uses a 3x3 grid centered on the cell for which you intend to calculate slope. In the diagram below, consider Z_a to be the elevation for cell A, Z_b the elevation for cell B, and so on. Then, the algorithm for slope in cell E is as follows:

1) calculate the east-west gradient (dz/dx) and north-south gradient (dz/dy) for cell E:

$$dz/dx = ((Z_a + 2*Z_d + Z_g) - (Z_c + 2*Z_f + Z_i)) / 8*\text{cell resolution}$$

$$dz/dy = ((Z_a + 2*Z_b + Z_c) - (Z_g + 2*Z_h + Z_i)) / 8*\text{cell resolution}$$

2) calculate the slope from these using the following:

$$\%slope(E) = \sqrt{(dz/dx)^2 + (dz/dy)^2}$$

A	B	C
D	E	F
G	H	I

Note that dz/dx and dz/dy are equivalent to rise/east-west run and rise/north-south run, respectively. They can be compared to the simple slope formula for a line segment.

Exercises to demonstrate mastery.

Conduct the preliminary survey described in the example application at the beginning of this unit. It is designed to work on the Aztec-east one degree DEM, available at no cost at the EROS 1:250 DEM Server. (In fact, you may download a one-degree DEM for any part of the United States at this location). Since this file is pretty large, it may be necessary to use a smaller one, perhaps a subset of Aztec-east.

Explore the effect grid resolution has on average slope:

- Use a GIS to calculate slope for an elevation surface. Write down the maximum slope, the mean slope, and the minimum slope.
- Resample the elevation surface to coarsen the resolution.
- Calculate slope again. Compare the maximum slope, average slope, and minimum slope to the values from the original map. How do they compare?

A slope map for a digital elevation model (DEM) is easy enough to interpret. How would you interpret a slope map for the population density map of India? How could such a map be used?

Follow-up Units

UNIT 44 - Using Hydrologic Models

UNIT 47 - On screen visualization - because of the 0° - 360° equivalency problem, aspect maps are particularly difficult to portray

Resources

Burrough, P.A., 1986. Principles of Geographical Information Systems for Land Resources Assessment. Chapter 3. Clarendon Press: Oxford.

Chapter three concentrates on digital elevation models and derived products.

Guth, P.L., 1995. Slope and aspect calculations on gridded digital elevation models: examples from a geomorphometric toolbox for personal computers. *Zeitschrift für Geomorphologie*, 101, 31-52.

This chapter different methods of calculating slope and aspect and identifies resolution effects on these calculations.

Moore, I.D., Grayson, R.B., Ladson, A.R., 1991. Digital terrain modelling: a review of hydrological, geomorphological, and biological applications. *Hydrological Processes*, 5, 3-30.

This paper identifies a large number of topographic attributes in addition to slope and aspect that are useful for a range of scientific applications.

USGS, 1987. Digital Elevation Models: Data Users Guide 5. US Dept. Interior, USGS, Reston, VA.

Most easily obtainable elevation data is from the U.S. Geological Survey; this is the best place to learn the technical details of their digital elevation models.

Weibel, R., Heller, M., 1991. Digital terrain modelling. In Maguire, D.J., Goodchild, M.F., Rhind, D.W. (Eds), Geographical Information Systems, Principles & Applications. Longman: London, 269-297.

This comprehensive review of the DEM literature includes discussions of surface derivatives.

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