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Publication Date
1972-08-30

Submitted to Phys. Rev. -Comments and Addenda

LBL-1075
Preprint C.1

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August 30, 1972

AEC Contract No. W-7405-eng-48

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COMMENT ON POSITIVE REGGE CUT DISCONTINUITIES*

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August 30, 1972

ABSTRACT

Abarbanel has recently derived a formula for the discontinuity across the two reggeon branch point using direct channel unitarity. His derivation assumes that a "two-reggeon irreducible" kernel does not contain J-plane singularities in the vicinity of the branch point. We investigate this assumption for the two pomeron cut using an extension of the multi-Regge arguments of Finkelstein and Kajantie. We verify the assumption for $\alpha_P(0) = 1$, but the result is inconclusive otherwise.

While the original work[1] on Regge cuts in hadronic physics predicted that the discontinuity across a two-reggeon cut would be positive (i.e., that the cut contribution to cross sections would have the same sign as the pole), later studies[2,3] have led to the opposite conclusion. Recent papers by Abarbanel[4] and White[5] have sustained the controversy by obtaining a similar formula with positive and negative signs, respectively,[6] using different methods. In this note we examine a crucial assumption in Abarbanel's work, concerning the asymptotic behavior of "two-reggeon irreducible" kernels.

Abarbanel supposes that Regge cuts in the absorptive part of the two-reggeon scattering amplitude[8] are generated by direct channel unitarity when Regge poles are present in the production amplitudes appearing in the unitarity sum (see Fig. 1A; the dots refer to other contributions). When all terms are included, the result is the exact equation represented in Fig. 1B, where C is the full two-reggeon absorptive part and K is an "irreducible kernel." The latter is defined as that piece of the full absorptive part for which the Regge pole of interest does not appear in the intermediate integration. This can be made precise, for example, by requiring all relative rapidities in the intermediate cluster to be less than some fixed interval. It is assumed that the cut is obtained by integrating over the two-reggeons and that the cut does not appear in K.

Of course K will contain some singularities, so what is required for this procedure to be sensible is that they be less important asymptotically. In other words, the J-plane singularities of K should lie below the two-reggeon branch point. To study this question, we shall iterate the exact equation to obtain Fig. 2. By a straightforward extension of the arguments of Finkelstein and

Kajantie[7] it is possible to obtain a lower bound on the energy dependence of each term in the series, and to then perform the sum as $s \to \infty$. The result will have the asymptotic behavior $s^{\alpha_K + p}$ where α_K is the leading singularity in K and $p > 0$.

If we now restrict ourselves to pomeron singularities, there are two cases to be distinguished. If $\alpha_P(0) = 1$ then, since the cut is located as usual at $\alpha_C(t) = 2\alpha_P(t/4) - 1$, we have $\alpha_C(0) = 1$ and both pole and cut are above α_K. However, if $\alpha_P(0) = 1 - \Delta$, then $\alpha_C(0) = 1 - 2\Delta$ and (not having a reliable model of production amplitudes) we cannot rule out the possibility $\alpha_P(0) > \alpha_K \geq \alpha_C(0)$. The method gives no information on nonleading cuts, as that would require precise knowledge of the singularities of K.

We now turn to the calculation. The $\underline{n}$th term in the iteration of Abarbanel's equation has the form of Fig. 3. Momenta are parametrized à la Bali, Chew, and Pignotti[9] in terms of total cluster momenta p_i and invariant masses $u_i = p_i^2$, unspecified internal cluster variables $\{V_i\}$, momentum transfers $t_i = Q_i^2$, Toller angles ω_i, and relative boosts ζ_i. We are interested in a lower bound on the contribution of this term to the forward absorptive part, and by positivity we are free to work in the region of phase space where s, u_i, and ζ_i are large, and the t_i finite. In this limit the amplitude is assumed to have the form

$$T_{2\to n} = f(p_a^2, t_1, u_1, \{V_1\})(\mathrm{ch}\,\zeta_1)^{\alpha(t_1)}\, g(t_1, t_2, \omega_2, u_2, \{V_2\})$$

$$\otimes\ (\mathrm{ch}\,\zeta_2)^{\alpha(t_2)} \cdots f(t_{n-1}, p_b^2, u_n, \{V_n\}),$$

where $\alpha(t)$ is the pomeron trajectory.

This form is chosen so as to correspond to power behavior $(s_i/u_i u_{i+1})^{\alpha(t_i)}$, as expected for a Regge pole coupled to large masses. The total phase space is proportional to $\prod_{i=1}^{n} du_i\, d\{V_i\}$ and we assume that for large u_i,

$$\int d\omega_i\, d\{V_i\} |g(t_{i-1}, t_i, \omega_i, u_i, \{V_i\})|^2 \sim u^{\alpha_K} v(t_{i-1}, t_i)$$

and

$$\int d\{V\} |f(t, p^2, u, \{V\})|^2 \sim u^{\alpha_K} v'(p^2, t) .$$

Here α_K is the leading singularity of K and the vertices v and v' are unspecified for now. The remaining phase space element is

$$d\phi_n = \mathrm{ch}\,q_1\, \mathrm{ch}\,q_n \prod_2^{n-1} \mathrm{sh}\,q_i \prod_1^{n-1} dt_i \prod_1^{n-1} d(\mathrm{ch}\,\zeta_i)$$

$$\otimes \prod_1^{n} du_i\, \frac{1}{s}\, \delta\!\left(s - (p_a + p_b)^2\right) ,$$

up to overall constant factors which will be consistently dropped. The q_i are BCP[9] vertex boosts whose large u_i limits are

$$\mathrm{ch}\,q_1 \sim \frac{u_1}{(-t_1)^{\frac{1}{2}}} , \quad \mathrm{ch}\,q_n \sim \frac{u_n}{(-t_n)^{\frac{1}{2}}} , \quad \mathrm{sh}\,q_i \sim \frac{u_i}{(t_{i-1}\, t_i)^{\frac{1}{2}}} .$$

The constraint on s is

$$s \;\tilde{=}\; 2m_a\, m_b \;\mathrm{ch}\, q_1 \;\mathrm{ch}\, q_n \prod_2^{n-1} (\cos\,\omega_i + \mathrm{ch}\, q_i) \prod_1^{n-1} \mathrm{ch}\,\zeta_i$$

$$\sim 2m_a\, m_b \prod_1^{n} u_i \prod_1^{n-1} \mathrm{ch}\,\zeta_i \Big/ \prod_1^{n-1} (-t_i) \quad .$$

Defining $x = \log \frac{s}{2m_a\, m_b}$ and supposing the ζ_i to be large, phase space becomes

$$d\phi_n = \frac{1}{s} \prod_1^{n} du_i \prod_1^{n-1} d\zeta_i \prod_1^{n-1} dt_i \;\; \delta\left(x - \sum_1^{n-1} \zeta_i - \sum_1^{n} \log \frac{u_i}{u_0} + \sum_1^{n-1} \log(-t_i) + \kappa \right) \quad ,$$

where κ and u_0 are constants. We choose u_0 sufficiently large that the clusters are Regge behaved as above for $u > u_0$. For a lower bound on the n-cluster contribution, $K_n(s)$, integrate over the restricted region of phase space where $u > u_0$, $\zeta > \zeta_0$, where ζ_0 corresponds to the onset of Regge behavior, and $T_2 \leq t_i \leq T_1$ where the vertex functions v are nonvanishing in this interval. Letting $y_i \equiv \log \frac{u_i}{u_0}$ we have

$$K_n(s) \;>\; \frac{1}{s} \prod_1^{n-1} \int_{\zeta_0}^{\infty} d\zeta_i \prod_1^{n} \int_0^{\infty} dy_i \prod_1^{n-1} \int_{T_2}^{T_1} dt_i$$

$$\otimes \;\; \delta\left(x - \sum_1^{n-1} \zeta_i - \sum_1^{n} y_i + \sum_1^{n} \log(-t_i) + \kappa \right)$$

$$\otimes \;\; \exp\left[(\alpha_\kappa + 1) \sum_1^{n} y_i + \sum_1^{n-1} 2\alpha(t_i)\zeta_i \right] \otimes \prod_1^{n} v(t_{i-1}, t_i) \quad .$$

We now let $\bar{v}$ be a lower bound on the v's in the interval $T_2 \leq t_i \leq T_1$ and write

$$K_n(s) \;>\; \frac{1}{s}\, \bar{v}^n \prod_1^{n-1} \int_{\zeta_0}^{\infty} d\zeta_i \prod_1^{n} \int_0^{\infty} dy_i \prod_1^{n-1} \int_{T_2}^{T_1} dt_i$$

$$\otimes \;\; \delta\left(x - \sum_1^{n-1} \zeta_i - \sum_1^{n} y_i + (n-1)\lambda \right)$$

$$\otimes \;\; \exp\left[(\alpha_\kappa + 1) \sum_1^{n} y_i + \sum_1^{n-1} 2\alpha(t_i)\zeta_i \right] \quad .$$

We have also replaced the t_i dependence in the delta function by an average value λ and absorbed κ. This is harmless as the t_i integrations may be performed over an arbitrarily small interval away from the origin (see also Ref. 7). Integrating over y_n,

$$K_n(s) > \frac{1}{s} e^{(\alpha_K+1)x} \bar{v}^n \prod_1^{n-1} \int_{\zeta_0}^{\infty} d\zeta_i \prod_1^{n-1} \int_0^{\infty} dy_i$$

$$\otimes \prod_1^{n-1} \int_{T_2}^{T_1} dt_i \, \theta\left(x - \sum_1^{n-1} \zeta_i - \sum_1^{n-1} y_i + (n-1)\lambda \right)$$

$$\otimes \exp \sum_1^{n-1} 2\alpha(t_i)\zeta_i \, .$$

For a lower bound on $K_n(s)$, restrict the integration to the region $0 \leq y_i \leq L$ and $\zeta_0 \leq \zeta_i \leq L$ where $2(n-1)L \equiv x + (n-1)\lambda$. Then

$$K_n(s) > s^{\alpha_K} \left[L \bar{v} \int_{\zeta_0}^{\infty} d\zeta \int_{T_2}^{T_1} dt \, e^{2\alpha(t)\zeta} \right]^{n-1} .$$

For a lower bound on $C(s) = \sum_{n=1}^{\infty} K_n(x)$, let $n,x \to \infty$ at fixed L. This assumes that we can allow a logarithmically increasing number of clusters and still stay within the region of validity of our approximations. If K is defined as above in terms of a maximum rapidity interval, such is certainly the case for large enough L. Then as $s \to \infty$,

$$C(s) > K_n(s) \Big|_{n=1+\frac{x}{2L-\lambda}}$$

$$= s^{\alpha_K} \left[L \bar{v} \int\int d\zeta \, dt \cdots \right]^{x/2L-\lambda} = s^{\alpha_K+p}$$

where

$$p = \frac{1}{2L-\lambda} \log \left[L \bar{v} \int_{\zeta_0}^{L} d\zeta \int_{T_2}^{T_1} dt \, e^{2\alpha(t)\zeta} \right] .$$

Since L can be taken as large as desired, $p > 0$. Thus the leading singularity of $C(s)$ is above that of K.

ACKNOWLEDGMENT

I thank Geoffrey F. Chew for suggesting this investigation and for valuable advice and comments.

FOOTNOTES AND REFERENCES

* This work was supported in part by the U. S. Atomic Energy Commission.

† NSF Graduate Fellow.

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FIGURE CAPTIONS

Fig. 1. (A) Generation of the two-reggeon cut.

(B) Exact equation for the two-reggeon absorptive part.

Fig. 2. Iteration of Fig. 1B.

Fig. 3. Kinematics of the n-cluster contribution to the absorptive part.

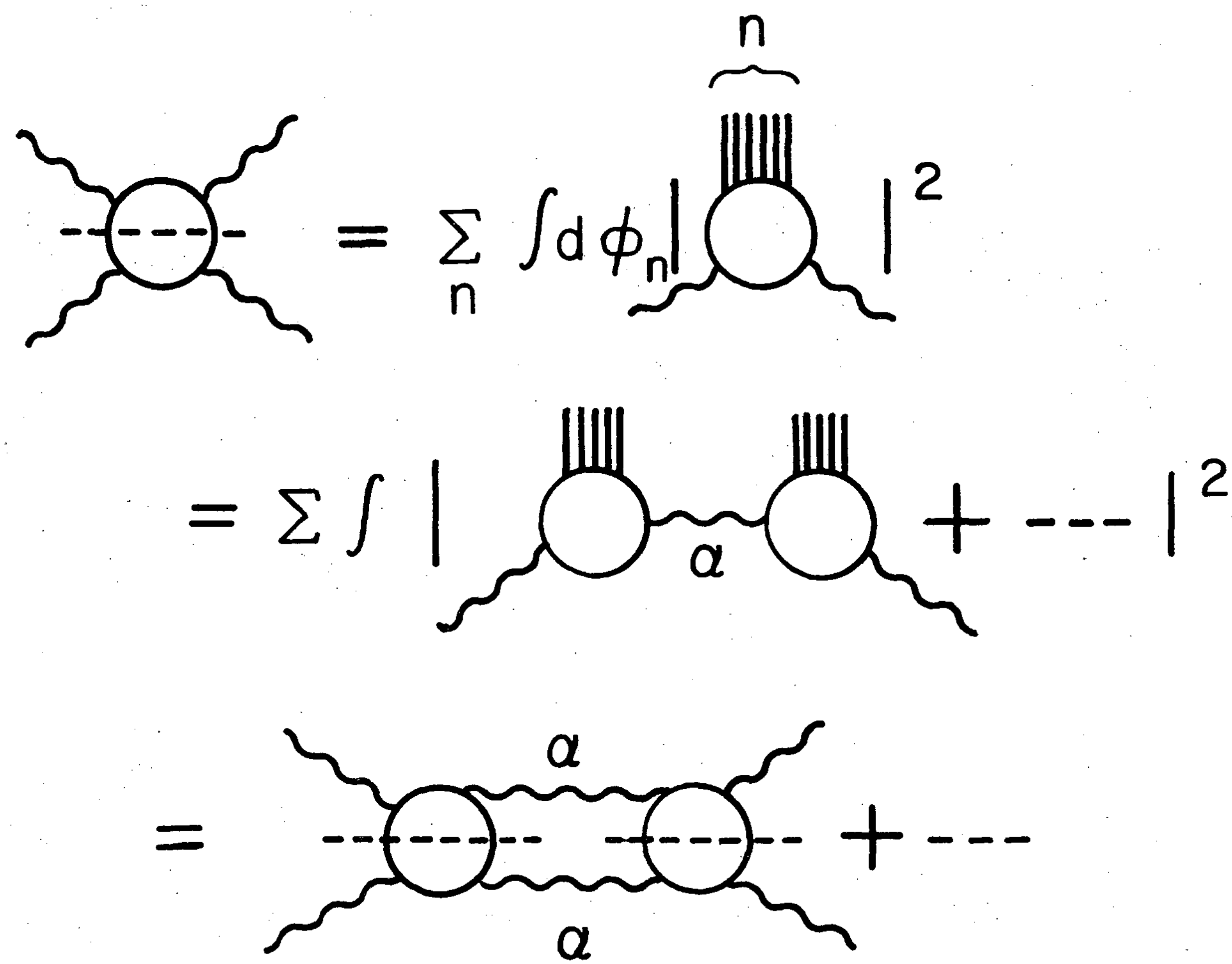

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Fig. 1A

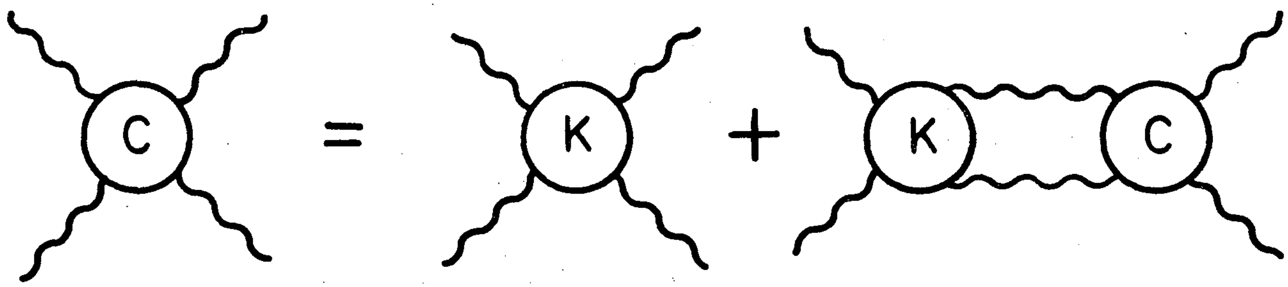

Fig. 1B

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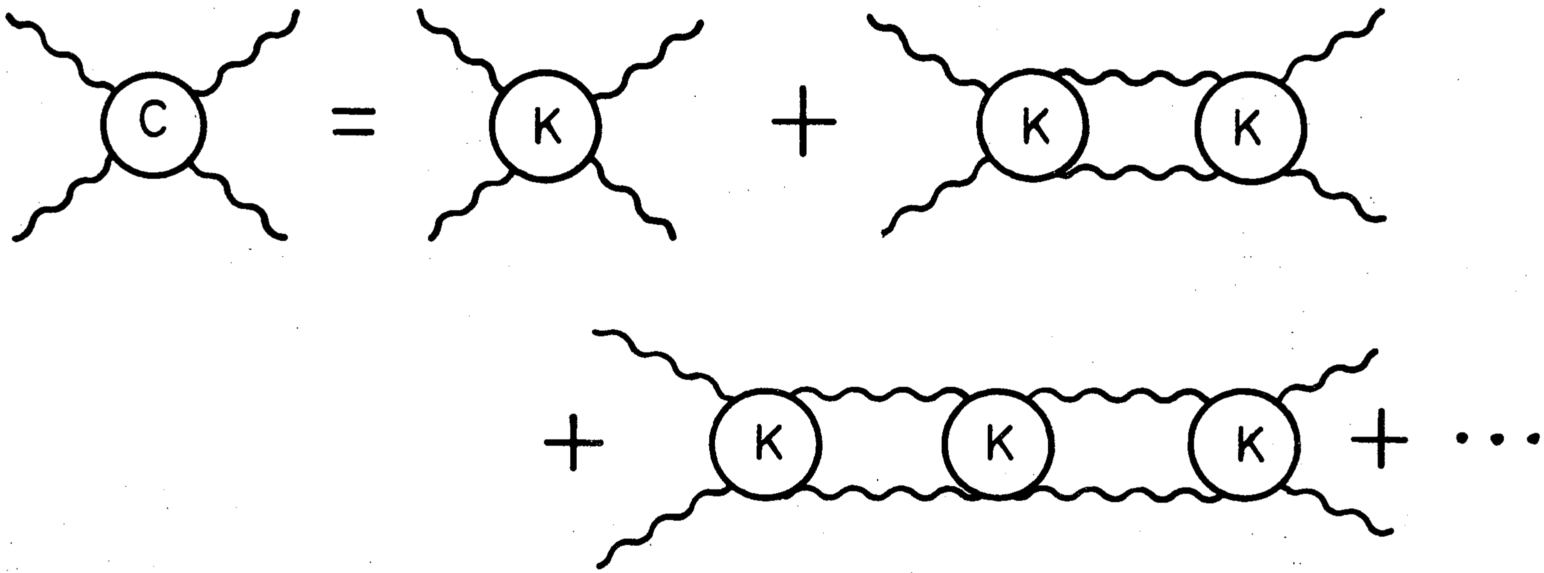

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Fig. 2

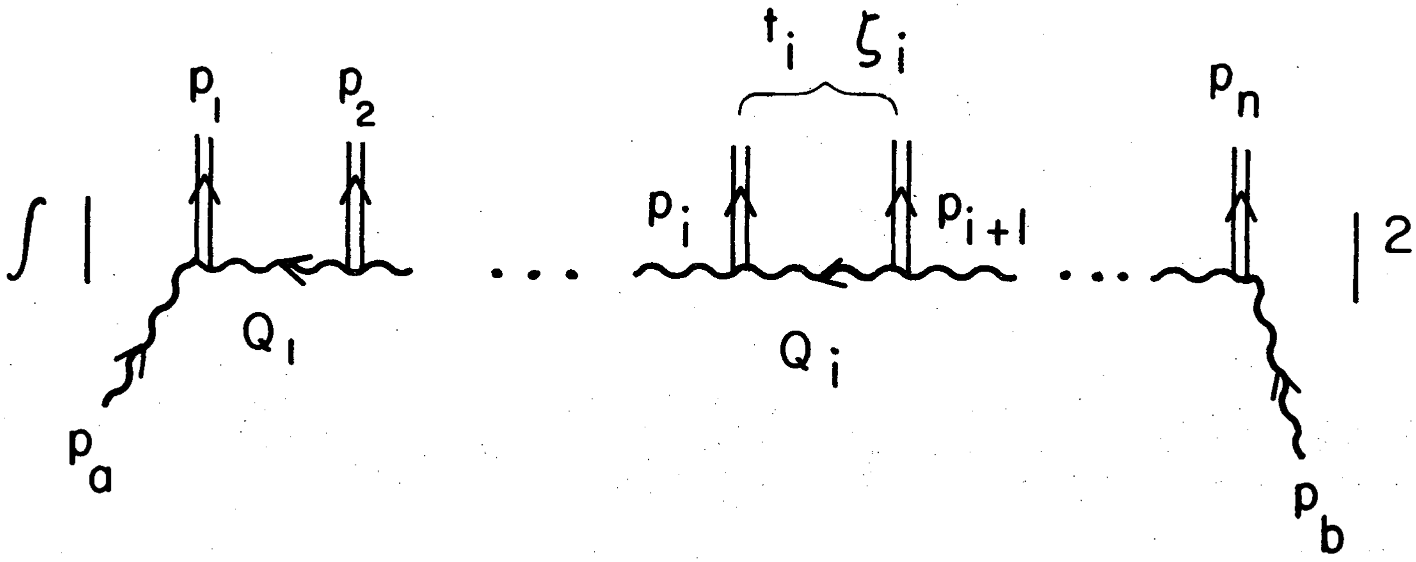

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Fig. 3

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