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SPONTANEOUS BREAKING OF SUPERSYMMETRY

Bruno Zumino

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Bruno Zumino
Lawrence Berkeley Laboratory
and
Department of Physics
University of California
Berkeley, California 94720 U.S.A.
On leave from CERN, Geneva, Switzerland

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Bruno Zumino
Lawrence Berkeley Laboratory
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Department of Physics
University of California
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1. INTRODUCTION

There has been recently a revival of interest in supersymmetric gauge theories, stimulated by the hope that supersymmetry might help in clarifying some of the questions which remain unanswered in the so called Grand Unified Theories and in particular the gauge hierarchy problem. In a Grand Unified Theory[1] one has two widely different mass scales: the unification mass $M \simeq 10^{15}$GeV at which the unification group (e.g. SU(5)) breaks down to SU(3) × SU(2) × U(1) and the mass $\mu \simeq 100$ GeV at which SU(2) × U(1) is broken down to the U(1) of electromagnetism. There is at present no theoretical understanding of the extreme smallness of the ratio μ/M of these two numbers. This is the gauge hierarchy problem.

There is a more technical aspect to the hieracrchy problem.[2] In a Grand Unified Theory the two mass scales come from the vacuum expectation values of two Higgs fields, which in turn are related to the parameters entering the Higgs potential. For the gauge hierarchy to emerge, some Higgs fields must have a small mass close to μ while others must have a large mass close to M. This requires a "fine tuning" of the parameters of the Higgs potential which, however, is in general unstable under radiative corrections. As recently emphasized by Witten,[3] there are special properties of supersymmetric theories which could help in this connection, namely the absence of renormalization of some of the parameters entering the Lagrangian, for instance masses and scalar couplings.[4-7] More simply, one could hope that, in a supersymmetric theory, the smallness of a scalar mass is guaranteed by the smallness of the mass of its spinor superpartner, which in turn is guaranteed by an approximate chiral invariance. Of course, a solution of the numerical hierarchy puzzle itself will require more than these special naturalness properties of supersymmetric theories (called sometimes in jest "supernaturalness") and can be found perhaps in non-perturbative breaking of supersymmetry.[3]

I shall not review here the numerous recent papers attempting to construct realistic models of supersymmetric gauge theories. As in previous work mostly by Fayet, these papers use N = 1 supersymmetry and do not attempt unification with gravity. Supersymmetry must of course be broken, the scale of supersymmetry breaking being at least 15 to 20 GeV for consistency with experiment. In this lecture I shall attempt to review the various mechanisms for spontaneous supersymmetry breaking[9] in gauge theories. Most of the discussion will be concerned with the tree approximation but what is presently known about radiative correction will also be reviewed.

2. SCALAR-SPINOR SUPERMULTIPLETS

The supersymmetric Lagrangian[10] for n interacting chiral (spin 0 - spin $\frac{1}{2}$) supermultiplets ϕ_i $(i = 1,2,\ldots n)$ is the sum of the kinetic term plus an interaction which can be derived from a single function of the ϕ_i, which we shall call the superpotential. For a renormalizable theory the superpotential is a cubic polynomial

$$f(\phi) = a + b_i\phi_i + \frac{1}{2} m_{ij}\phi_i\phi_j + \frac{1}{3!} g_{ijk}\phi_i\phi_j\phi_k \tag{2.1}$$

(sum over repeated indices). The chiral superfields ϕ_i are complex and so are their scalar components A_i and the corresponding auxiliary fields F_i. The part of the Lagrangian which describes the scalar interactions is

$$\mathcal{L}_{S.I.} = F_i\bar{F}_i + F_i \frac{\partial f}{\partial A_i} + \bar{F}_i \overline{\frac{\partial f}{\partial A_i}} . \tag{2.2}$$

The equations of motion obtained by varying (2.2) with respect to F_i and $\bar{F}_i$ are

$$\bar{F}_i = - \frac{\partial f}{\partial A_i} \tag{2.3}$$

and their complex conjugates. Substituting (2.3) into (2.2) and changing the sign, one obtains the tree approximation scalar potential

$$V = \frac{\partial f}{\partial A_i} \overline{\frac{\partial f}{\partial A_i}} \geqslant 0. \tag{2.4}$$

The scalar potential is non-negative. If it is equal to zero at its minimum, supersymmetry is exact, if it is positive at its minimum supersymmetry is spontaneously broken. If supersymmetry is exact, the equations

$$\frac{\partial f}{\partial A_i} = 0 \tag{2.5}$$

must have a common solution $A_i = \overset{o}{A}_i$. Since (2.5) are n quadratic equations in n complex unknowns, in general they will have 2^n solutions but in special cases they may have no solutions[10,11] or they may have a continuous infinity of solutions, in which case there are massless scalars in the theory. It is not difficult to construct examples for all three situations. When there are more than one solution, any of them is equally acceptable as a vacuum. It can be shown[4,5] that, in perturbation theory, higher order corrections do not renormalize the second and third term in the right hand side of (2.2): the superpotential is unmodified by higher order corrections. Furthermore[12-18] higher order corrections cannot induce spontaneous breaking of supersymmetry nor can they remove the degeneracy when there are several acceptable zero-energy vacua at the tree approximation. If there are massless scalars they remain massless.

The reason for all this is that, for x-independent fields, all higher order corrections to the scalar interaction (2.2) have the form[12, 7]

$$\mathcal{L}'_{S.I.} = F_i \bar{F}_j h_{ij}(F, \bar{F}, A, \bar{A}), \tag{2.6}$$

where h_{ij} is a hermitean matrix, function of the indicated variables, which can be calculated in perturbation theory. One sees immediately that only the first term in the right hand side of (2.2) is renormalized and the wave function renormalization matrix is

$$\delta_{ij} + h_{ij}(0, 0, 0, 0). \tag{2.7}$$

If we add (2.6) to (2.2), the equations of motion for F_i and $\bar{F}_i$ become

$$\bar{F}_i + \frac{\partial f}{\partial A_i} + \bar{F}_j h_{ij} + F_j \bar{F}_k \frac{\partial h_{jk}}{\partial F_i} = 0, \tag{2.8}$$

plus the complex conjugates. On the other hand, the equations for A_i become

$$F_j \frac{\partial^2 f}{\partial A_i \partial A_j} + F_j \bar{F}_k \frac{\partial h_{jk}}{\partial A_i} = 0. \tag{2.9}$$

Clearly, a solution $A_i = \overset{o}{A}_i$ of (2.5), together with $F_i = 0$, satisfies both (2.8) and (2.9). The sum of (2.2) and (2.6) vanishes for those values of A_i and F_i. Therefore, a possible vacuum at the tree approximation is a possible vacuum to all orders.[18,19] Observe that, since the energy cannot become negative (this is a consequence of the supersymmetry algebra) all the solutions of (2.5) give true minima to all orders.

Let us now consider spontaneous breaking of supersymmetry.[20] At the tree approximation this means that (2.5) have no solutions and the F_i cannot all vanish. In this case one can show that the potential (2.4) cannot be "field-confining". We define a potential to be field-confining when it tends to infinity if A_i tends to infinity so that the fields A_i cannot become arbitrarily large. More precisely

$$V \to \infty \tag{2.10}$$

if

$$|A|^2 \equiv A_i \bar{A}_i \to \infty . \tag{2.11}$$

For a non confining potential, let us assume that there exists a positive number p such that

$$V \geqslant p > 0. \tag{2.12}$$

This excludes unphysical potentials which tends to zero when one of the scalar fields tends to infinity. Then one can show that the determinant of the second derivatives of the superpotential vanishes indentically[21] in A_i

$$\det \frac{\partial^2 f}{\partial A_i \partial A_j} \equiv 0, \tag{2.13}$$

and the matrix has therefore at least one vanishing eigenvalue. Calculated at the minimum of the potential (2.4), this matrix is the spinor mass matrix, which must have a zero eigenvalue corresponding to the Goldstone spinor of spontaneously broken supersymmetry (see (2.17) below). The fact that it has a zero eigenvalue for all values of the scalar fields A_i implies special properties of the Yukawa couplings.

It can be shown that the components of the eigenvector corresponding to zero eigenvalue

$$\frac{\partial^2 f}{\partial A_i \partial A_j} v_j(A) = 0 \tag{2.14}$$

are polynomials in A_i (independent of $\bar{A}_i$). The corresponding differential operator applied to the potential (2.4) gives zero identically

$$v_j(A) \frac{\partial V}{\partial A_j} = 0 \tag{2.15}$$

and the same is true of the complex conjugate differential operator. Along the complex curves defined by the differential equations

$$\frac{dA_i}{dt} = v_i(A) \tag{2.16}$$

the potential is constant. Assume that the potential reaches its minimum for a finite value of the scalar fields. Given any minumum of the potential, there is one of these curves (2.14) passing through it, which implies the presence of a complex massless scalar (actually these valleys of minima extend to infinity). Observe that, from (2.4),

$$\frac{\partial V}{\partial A_i} = \frac{\partial^2 f}{\partial A_i \partial A_j} \overline{\frac{\partial f}{\partial A_j}} . \tag{2.17}$$

At a minimum this must vanish (together with its complex conjugate). Therefore, if there is only one vanishing eigenvalue, one must have there the proportionality

$$v_i(A) \propto \overline{\frac{\partial f}{\partial A_i}} \tag{2.18}$$

between a polynomial vector whose components are functions of A_i only and one whose components are functions of $\bar{A}_i$ only. All these general results can be easily checked in the special examples of spontaneous breaking of supersymmetry discussed in Refs. 10, 11.

The necessity of massless scalars in addition to the Goldstone spinor may seem strange, but it is a property of the tree approximation only. When supersymmetry is spontaneously broken, the radiative corrections, which still have the form (2.6), change the situation in an essential way, because the F_i do not vanish. Already at the one-loop level the degeneracy of the valley of minima is lifted[22,23] and in general one has only one absolute minimum and no massless scalars. The potential increases with the scalar fields so that the minimum is for relatively small values of the fields. The value of the potential at the minimum also changes in the one loop approximation. All this has been verified in several special examples.[22]

3. SUPERSYMMETRIC GAUGE THEORIES

We consider now the case when there are gauge fields present. If the gauge group is simple, the tree approximation scalar Lagrangian (2.2) must be complemented by

$$\frac{1}{2}(D^a)^2 + gD^a\bar{A}T^aA \tag{3.1}$$

where the scalar fields A now belong to some representations of the guage group, T^a are the matrices which represent the generators of the group and g is the gauge coupling constant. If the gauge group is semi-simple, one has the sum of a number of terms like (3.1). If the gauge group contains U(1) factors, each U(1) factor contributes to the sum a term of the form

$$\frac{1}{2}D^2 + g_1 D\,\bar{A}YA + \xi D, \tag{3.2}$$

where g_1 is the U(1) gauge coupling constant and Y the U(1) charge of the scalar fields. The ξD term is the Fayet-Iliopoulos term,[24] which can induce spontaneous supersymmetry breaking. Eliminating the field D^a through its equations of motion, (3.1) gives a term of the form

$$-\frac{1}{2}g^2(\bar{A}T^aA)^2, \tag{3.3}$$

while (3.2) gives rise to

$$-\frac{1}{2}(g_1\bar{A}YA + \xi)^2. \tag{3.4}$$

In the scalar potential the negatives of (3.3) and (3.4) enter.

So, when gauge fields are present, the scalar potential consists of (2.4) plus a sum of terms like the negatives of (3.3) and (3.4)

$$V = \frac{\partial f}{\partial A_i}\overline{\frac{\partial f}{\partial A_i}} + \Sigma\frac{1}{2}g^2(\bar{A}T^aA)^2 + \Sigma\frac{1}{2}(g_1\bar{A}YA + \xi)^2 \geqslant 0. \tag{3.5}$$

If for a value $A_i = \overset{o}{A}_i$ of the scalar fields the potential in (3.5) vanishes, supersymmetry is exact in the tree approximation. Again one can show[13-16,18] that higher order corrections will not break supersymmetry and will not remove any degeneracy which may exist in the tree approximation. Since the effective potential, to any order, is determined by a knowledge of the renormalization group functions[25] this fact can be related to the special properties of supersymmetric gauge theories with respect to renormalization. The only renormalization constants needed are:[6] a wave function renormalization for each chiral superfield, a wave function renormalization for each gauge superfield and a gauge coupling renormalization for each gauge coupling constant. No separate mass and scalar coupling renormalizations are necessary, which gives relations among the renormalization group functions. Of course, those superfields which belong to the same irreducible representation of the gauge group have the same wave function renormalization.

If the chiral superpotential gives rise to spontaneous breaking of supersymmetry in the tree approximation, which means that (2.5) have no solution, the presence of gauge fields does not change the fact that supersymmetry is spontaneously broken, since the additional terms in (3.5) are positive. On the other hand, let us assume that the first term in the right hand side of (3.5) vanishes for some value $A_i = \overset{o}{A}_i$ of the scalar fields. We distinguish several cases.

Let us first consider the case when there are no U(1) factors, so that the last term in the right hand side of (3.5) is missing. If $\overset{o}{A}_i = 0$, the second term vanishes: supersymmetry is exact. If not all the $\overset{o}{A}_i$ vanish, the second term in (3.5) does not vanish in general, however this does not necessarily mean that supersymmetry is broken. The superpotential f(A) is invariant under the semi-simple gauge group; as it was first pointed out by Ovrut and Wess,[26] this means that f(A) is also invariant under the complex extension of the group (same generators, but the parameters are allowed to be complex instead of being restricted to be real). This complex invariance can be used to find other values of A_i where the first term in (3.5) still vanishes. The second term is not invariant under the complex extension of the group and one can show that it can be transformed to zero by using a transformation of the complex extension of the group. In conclusion, for a semisimple gauge group, if the chiral part of the scarlar potential (the f dependent part) reaches the value zero for some value of the scalar fields, even if the gauge term does not vanish at that point, one can find another value of the scalar fields where both terms vanish. This is then a true minimum and supersymmetry is exact.[27]

This result is also valid if there is one U(1) factor even with non vanishing ξ, provided the chiral part of the potential vanishes for non zero scalar field. In this case supersymmetry cannot be spontaneously broken if it is not already broken by the chiral superpotential. However, if there is more than one U(1) factor, one cannot prove an analogous result in general, although, if there are enough chiral supermultiplets in the theory the statement tends to be correct anyway in concrete examples.

For gauge theories also (with no Fayet-Iliopoulos term), if supersymmetry is spontaneously broken at the tree level by the chiral superpotential and the potential has the same minimum value along a valley, higher order corrections will remove the degeneracy. However now the effective potential does not necessarily increase with the scalar fields[23] and can in some cases reach its minimum for large values of the fields. This fact has led Witten to suggest a possible "inverse" solution of the hierarchy problem, in which the small mass scale μ is put into the theory at the start and the large mass scale M is generated by radiative corrections.

In a gauge theory with a U(1) factor and no Fayet-Iliopoulos term, can one be generated in perturbation theory and cause spontaneous breaking of supersymmetry? In the one-loop approximation a D tadpole is quadratically divergent and proportional to the trace of the U(1) charge Y. It has been shown[28] by the supergraph method that all higher loop contributions cancel. Therefore, if Tr Y = 0 no Fayet-Iliopoulos term is generated. It should be possible to understand this non-renormalization result as a consequence of combined supersymmetry and gauge invariance.[29] In the so called Wess-Zumino gauge, where only the physical fields and the auxiliary field D of the vector supermultiplet remain, the D tadpole, including all radiative corrections, can be related to the D-scalar-scalar vertex, by cutting a line. This vertex, in turn, is related by supersymmetry to the vector-scalar-scalar vertex for which gauge invariance provides a non-renormalization statement.

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